(Super)Conformal defects and boundaries in holography

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Contents

Geometry of conformal interfaces, boundaries, and defects

Conformal symmetry and anti-de Sitter space

- Consider holographic dual of a d-dimensional CFT
- Lorentz symmetry enhanced to conformal symmetry

$$SO(1, d-1) \rightarrow SO(2, d)$$

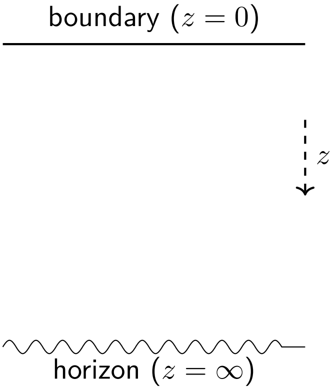
Realize holographic dual by embedding geometry in $\mathbb{R}^{2,d}$. The dual spacetime (anti-de Sitter space) is then determined by the $SO(2, d)$ invariant constraint equation:

$$-X_{-1}^2 - X_0^2 + X_1^2 + ... + X_d^2 = -L^2$$

Poincaré patch

- Introduce parametrization: $z, t, x_1, ..., x_{d-1}$
- Solve constraint preserving $SO(1, d-1)$ Lorentz symmetry and translations.

$$
\begin{aligned}
X_{-1} =& \frac{z}{2}\left(1 + \frac{1}{z^2}(L^2 - t^2 + x_1^2 + ... + x_{d-1}^2)\right)\\
X_0 =& \frac{L}{z} t,\\
X_i =& \frac{L}{z} x_i,\\
X_d =& \frac{z}{2}\left(1 + \frac{1}{z^2}(-L^2 - t^2 + x_1^2 + ... + x_{d-1}^2)\right)
\end{aligned}
$$



AdS$_d$ slicing of AdS$_{d+1}$

- A conformal interface/boundary preserves $SO(2, d-1) \subset SO(2, d)$
- Return to constraint equation:

$$-X_{-1}^2 - X_0^2 + X_1^2 + ... + X_d^2 = -L^2$$

- Partially solve constraint:

$$X_i = L\cosh(x)\,\hat{X}_i, \qquad X_d = L\sinh(x)$$

where the $\hat{X}_i$ satisfy the constraint equation

$$-\hat{X}_{-1}^2 - \hat{X}_0^2 + \hat{X}_1^2 + ... + \hat{X}_{d-1}^2 = -1$$

- The $\hat{X}_i$ parametrize an AdS$_d$ slice of AdS$_{d+1}$.

AdS_d slicing of AdS_{d+1}

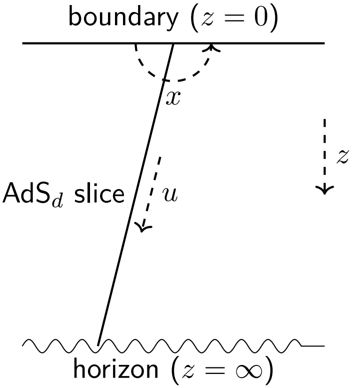
- Metric:

$$ds^2_{\mathsf{AdS}_{d+1}} = L^2 \left(\cosh^2(x) ds^2_{\mathsf{AdS}^2_d} + dx^2\right)$$

- Map to AdS_{d+1} FG-coordinates:

$$x_{d-1} = u \tanh(x)$$
$$z = \frac{u}{\cosh(x)}$$

- Introduce u as FG coordinate along AdS_d slice



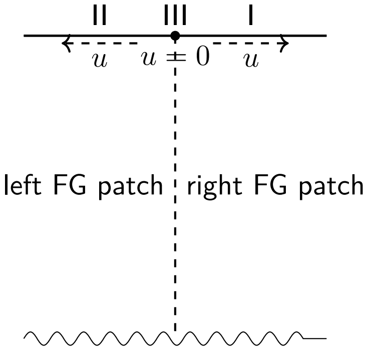
Identity interface

- Introduce left and right FG patches

$$x_{\perp,R} = +\,u\tanh(|x|) \qquad z = \frac{u}{\cosh(x)}$$

$$x_{\perp,L} = -\,u\tanh(|x|)$$

- The boundary defined by $z = 0$ is split into three-components:
  - Component I: $x \to \infty$ with $x_\perp \geq 0$
  - Component II: $x \to -\infty$ with $x_\perp \leq 0$
  - Component III: $u \to 0$ with $x_\perp = 0$
- Components I and II each have a boundary located at $x_\perp = 0$. These two boundaries are glued together along component III.

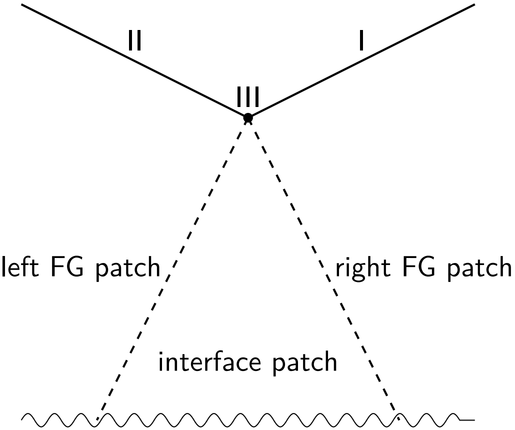


General conformal interface

- General interface:

$$ds^2 = L^2 \left(f^2(x) ds^2_{\mathsf{AdS}_d^2} + dx^2 \right)$$

- $f(x)$ asymptotes to $\cosh(x)$ as $x \to \pm\infty$
- Geometry can no longer be covered using a single patch of FG-coordinates
- Boundary is "locally asymptotically AdS_{d-1}", but not globally
- Interface "develops thickness" as we move into the bulk



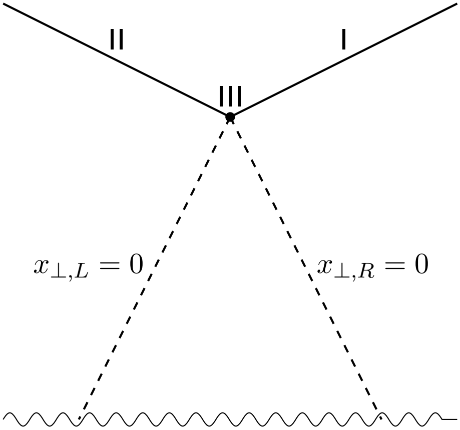
Example: interface

- Karch-Randall [1]
- Metric:

$$ds^2 = L^2 \left(\cosh^2(|x| - c) ds^2_{\mathsf{AdS}^2_d} + dx^2 \right)$$

- Space is locally AdS$_{d+1}$
- Source located at $x = 0$
- Map to AdS_{d+1} FG-coordinates:

$$x_{\perp,R} = +\, u \tanh(|x| - c)$$
$$x_{\perp,L} = -\, u \tanh(|x| - c)$$
$$z_{L,R} = \frac{u}{\cosh(|x| - c)}$$

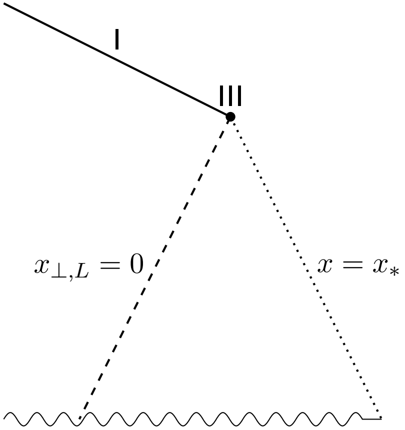


Example: boundary

- Takayangi [2]
- Metric:

$$ds^2 = L^2 \left(\cosh^2(x) ds^2_{\mathsf{AdS}^2_d} + dx^2\right)$$

- x is restricted to the range $-\infty \leq x \leq x_*$
- Brane source located at $x = x_*$
- Brane tension chosen to balance with extrinsic curvature and enforce Neumann boundary conditions



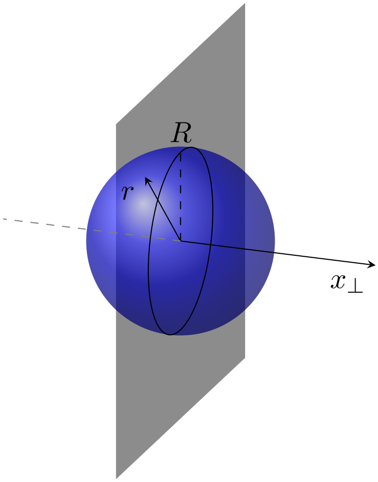
Entanglement entropy: field theory

- Decompose system into two subsystems A and B
- Consider spherical entangling surface centered around interface
  - A is the space inside the sphere
  - B is the space outside the sphere
- Introduce reduced density matrix ρ_A, entanglement entropy is then

$$S_A = -\mathrm{Tr}_A\, \rho_A \ln \rho_A$$

- Introduce cylindrical coordinates: $x_\perp, r, S^{d-3}$
- Entangling surface defined by

$$r^2 + x_\perp^2 = R^2$$



Entanglement entropy: geometry

- Determine minimal surface γ with boundary given by the entangling surface on the boundary of AdS (Ryu-Takayanagi[3])
- Metric:

$$ds^2 = L^2 \left(f^2 \frac{du^2 - dt^2 + dr^2 + r^2 ds^2_{S^{d-3}}}{u^2} + dx^2 \right)$$

- Minimal surface wraps S^{d-3} and is given by a two-dimensional surface in the space spanned by u, r and x. Choose u and x as a coordinates. Find $r(u, x)$.
- Area functional for surface

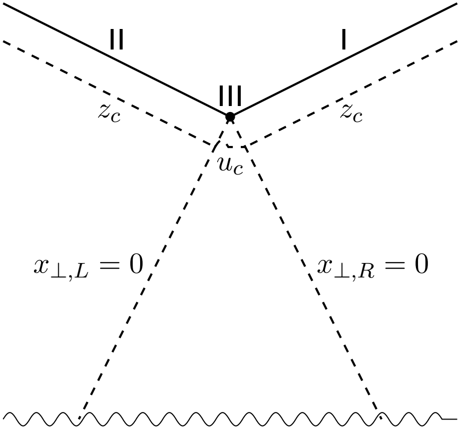
$$S_A = \frac{\text{vol}(S^{d-3}) L^{d-1}}{4G_N^{d+2}} \int du\, dr\, r^{d-3} \frac{f^{d-2}(x)}{u^{d-2}} \sqrt{1 + (\partial_u r)^2 + \frac{f^2(x)}{u^2} (\partial_x r)^2}$$

- Universal saddle point

$$r^2 + u^2 = R^2$$

Entanglement entropy: cutoff/regularization

- Reduction of Lorentz symmetry leads to many choices for cutoff surface
- Impose FG-cutoff in left and right FG patches with $z_c = \varepsilon$
- Interface region impose FG-cutoff with with $u_c = \varepsilon h(x)$
- Function $h(x)$ chosen so that cutoff surface is continuous

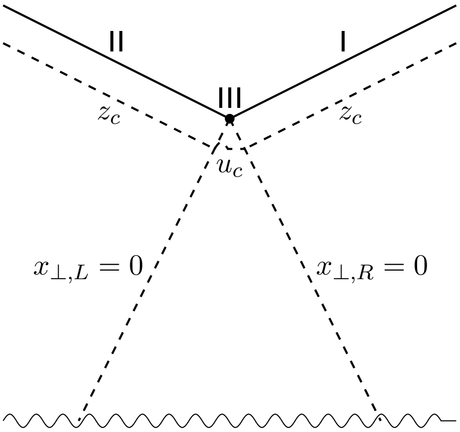


Entanglement entropy: cutoff/regularization

- Use background subtraction to isolate interface/boundary contributions
- Background subtracted entanglement entropy takes the form

$$\Delta S_A = \begin{cases} I_{(log)} \ln\left(\frac{2R}{\varepsilon}\right) + I_0, & d = 3 \\ I_1 \frac{R}{\varepsilon} + I_0, & d = 4 \end{cases}$$

- $I_{(log)}$ and I_0 are independent of the choice of interpolating function
- Double cutoff prescription (simple alternative): separate cutoffs u_c and x_c



Planar defect

- Let m denote the dimension of the defect and n the co-dimension
- Symmetry: $SO(2, m+1) \times SO(n-1) \subset SO(2, d)$
- Return to constraint equation:

$$-X_{-1}^2 - X_0^2 + X_1^2 + \ldots + X_d^2 = -L^2$$

- Partially solve constraint:

$$X_i = L \cosh(x) \, \hat{X}_i, \qquad X_a = L \sinh(x) \, \hat{Y}_a$$

- The $\hat{X}_i$ parametrize AdS_{m+1}, while the $\hat{Y}_a$ parameterize S^{n-1}
- Metric:

$$ds^2 = L^2 \left(\cosh^2(x) ds^2_{\mathsf{AdS}_{m+1}} + \sinh^2(x) ds^2_{S^{n-1}} + dx^2 \right)$$

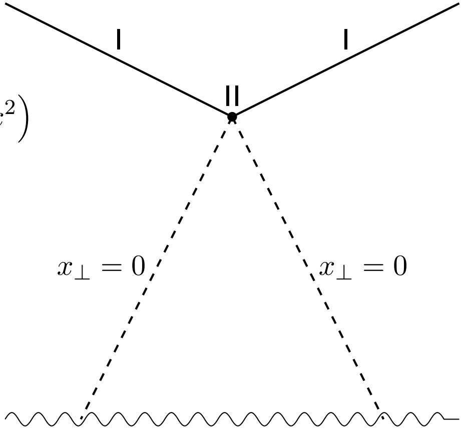
Planar defect

- Metric:

$$ds^2 = L^2 \left(f_1^2(x) ds^2_{\mathsf{AdS}_{m+1}} + f_2^2(x) ds^2_{S^{n-1}} + dx^2 \right)$$

- $f_1(x)$ and $f_2(x)$ asymptote to $\cosh(x)$ as $x \to +\infty$
- $f_2(x)$ has a zero so the geometry caps off smoothly

$$f(x) \simeq (x - x_*)$$



Conformal anomalies

Weyl anomaly for CFTs

- Under a Weyl rescaling of the metric $\delta_\omega g_{\mu\nu} = 2 g_{\mu\nu} \delta\omega$, the effective action W transforms anomalously

$$\delta_\omega W = -\int \sqrt{g}\, \delta\omega\, \langle T^\mu{}_\mu \rangle$$

- The trace of the stress tensor takes the form

$$T^\mu{}_\mu = \frac{1}{(4\pi)^{\frac{d}{2}}} \left((-1)^{\frac{d}{2}+1}\, a\, E + \sum_n c_n\, I_n \right)$$

  where E is the Euler density and the I_n are conformal invariants built from the Riemann tensor

- The Euler density is referred to as A-type, while the I_n are referred to as B-type
- For $d = 2$ there are no I_n, for $d = 4$ there are two, and for $d = 6$ there are three
- Under RG-flow, typically $a_{UV} \geq a_{IR}$ (often referred to as an a-theorem or c-theorem)

Weyl anomaly with a defect or boundary

- Introduce dependence on embedding function X^μ and corresponding operator $\mathcal{D}$, referred to as the displacement operator

$$\delta W = -\frac{1}{2}\int_{\mathcal{M}} \sqrt{g}\,\delta g_{\mu\nu}\,\langle T^\mu{}_\mu\big|_{\mathcal{M}}\rangle - \frac{1}{2}\int_{\Sigma}\sqrt{g_\Sigma}\left(\delta g_{\mu\nu}\,\langle T^\mu{}_\mu\big|_{\Sigma}\rangle + 2\delta X^i\langle \mathcal{D}_i\rangle\right)$$

- Broken Ward identities for transational normal to the defect

$$D_\mu T^{\mu i} = \delta(x_\perp)\mathcal{D}^i$$

- The trace of the stress tensor picks up contributions localized to the defect or boundary

$$T^\mu{}_\mu = T^\mu{}_\mu\big|_{\mathcal{M}} + \delta(x_\perp)T^\mu{}_\mu\big|_\Sigma$$

4d defect

- 29 possible anomaly coefficients (Chalabi, Herzog, O'Bannon, Robinson, Sisti [4])
  - 1 A-type term,
  - 22 parity even B-type terms
  - 6 parity odd B-type terms

$$T^{\mu}{}_{\mu}\big|_{\Sigma} \supset \frac{1}{(4\pi)^2}\left(-a_{\Sigma}E_4 + d_2\mathcal{J}_2 + \ldots\right)$$

- E_4 is the 4-dimensional Euler density intrinsic to the defect
- $\mathcal{J}_2$ is a complicated function built out of objects which depend on both the background geometry and embedding

4d defect: $\mathcal{J}_2$

- W is the Weyl tensor
- N is a projection onto the normal bundle of the defect
- Π is the second fundamental form characterizing the extrinsic curvature, and $\mathring{\Pi}$ the corresponding traceless version
- D is the Levi-Civita connection and $\bar{D}$ the pullback to the defect

$$\begin{aligned}
\mathcal{J}_2 = & \frac{d-4}{d-2} W_{ab}{}^{ab} N^{\mu\nu} R_{\mu\nu} - \frac{d-4}{d-1} R W_{ab}{}^{ab} \\
& + \frac{4(d-5)}{3(d-1)} R_{ab} W_c{}^{acb} - \frac{5(d-4)}{48} W_{ab}{}^{ab} \Pi^i \Pi_i + \frac{2(d-5)}{3} W^c{}_{ica} \bar{D}^b \mathring{\Pi}^i_{ab} \\
& + \frac{4(d+1)}{9} \mathring{\Pi}^{iab} D_i W^c{}_{acb} - \frac{1}{3} W_{ic}{}^{ac} \bar{D}_a \Pi^i - \frac{2(d-5)}{3} \Pi^i \mathrm{Tr} \mathring{\Pi}_i \mathring{\Pi}^j \mathring{\Pi}_j \\
& + \frac{(d-10)}{12} \Pi^i D_i W_{ab}{}^{ab} + \frac{1}{3} D^i D_i W_{ab}{}^{ab}
\end{aligned}$$

Observables: stress-energy tensor

- Stress-energy tensor for defect of dimension m

$$\langle T^{ab}\rangle = -h_T\frac{(d-m-1)\delta^{ab}}{|x_\perp|^d}, \qquad \langle T^{ij}\rangle = h_T\frac{(m+1)\delta^{ij} - d\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^d}$$

- Generally, h_T is determined by the defect Weyl anomaly. For a 4d defect it is given by

$$h_T = -\frac{\Gamma(\frac{d}{2}-1)}{\pi^{\frac{d}{2}}(d-1)}d_2$$

- Average null energy condition

$$\int \langle T_{\mu\nu}\rangle v^\mu v^\nu \geq 0$$

- Orient null ray v^μ to be parallel to the defect and separated in the normal direction

$$h_T \geq 0 \qquad \Rightarrow \qquad d_2 \leq 0$$

Observables: spherical entanglement entropy

- Spherical entanglement entropy for 4d defect

$$S[\Sigma]\Big|_{\ln} \supset -\left(4a_\Sigma + \frac{(d-4)(d-5)}{d-1}d_2\right)\ln\left(\frac{R}{\epsilon}\right)$$

- Employ background subtraction to isolate defect terms and deal with divergences from bulk physics

$$4a_\Sigma + \frac{(d-4)(d-5)}{d-1}d_2 = -R\partial_R\left(S[\Sigma] - S[\varnothing]\right)\big|_{R\to 0}$$

- To determine a_Σ from entanglement entropy, we also need d_2
- No constraint on sign of a_Σ
- a_Σ obeys a weak defect a-theorem (Wang[5] and Casini-Salazar-Torroba[6])

1/2 BPS 4d defects in M-theory

4d $\mathcal{N} = 2$ SCFTs from M5-branes

- Superconformal symmetry: $SU(2,2|2)$
- Bosonic subgroup: $SO(2,4) \otimes SO(3) \otimes U(1)$
- General solutions of M-theory with $SU(2,2|2)$ symmetry: Lin-Lunin-Maldacena [7]
- 4d theories obtained from M5-branes wrapping a Riemann surface
  - Explicit constructions: Maldacena-Nunez [8], Gauntlett-Martelli-Sparks-Waldram [9], Maldacena-Gaiotto [10], Petropoulos-Sfetsos-Siampos [11, 12], Bah-Bonetti-Federico-Waddleton [13, 14, 15], Couzens-Kim-Kim-Lee [16], ...
- General solutions determined by Toda type equation

$$\frac{1}{\rho}\partial_\rho\left(\rho\partial_\rho D\right) + \frac{1}{\rho^2}\partial_\beta^2 D + \partial_\xi^2 e^D = 0$$

  - 3d base space has boundary at $\xi = 0$, boundary conditions are given on the 2d plane
  - non-linear, few solutions constructed without the help of an additional isometry

Electrostatic formulation

- Assume additional $U(1)_\beta$ isometry.
- Introduce variables η and r, and electrostatic potential V by

$$\rho^2 e^D = r^2, \qquad \xi = r\partial_r V, \qquad \ln\rho = \partial_\eta V$$

- Potential V satisfies 3d Laplace equation with axial symmetry

$$r\partial_r\left(r\partial_r V\right) + r^2\partial_\eta^2 V = 0$$

- Spacetime structure: $\mathrm{AdS}_5 \times S^2 \times T^2 \times \Sigma_2$
  - Σ_2 is two dimensional base space, typically the upper-right-quadrant
  - T^2 is a torus and can have a non-trivial fibration over Σ_2
  - Along the r-axis V satisfies conducting boundary conditions ($\partial_r V|_{\eta=0} = 0$). In the space-time, S^2 shrinks to zero size on the r-axis.
  - Along the η-axis, an S^1 shrinks to zero size. Typically a mixture of $U(1)$ and $U(1)_\beta$.
  - Introduce line-charge density along η-axis

$$\bar\omega(\eta) = \lim_{r\to 0^+} r\partial_r V(r,\eta)$$

Electrostatic formulation: 4d theories

- $\bar{\omega}(\eta)$ is a linear continuous function with discontinuities in the slope

$$\bar{\omega}(\eta) \begin{cases} p_1\eta + \delta_1, & \eta \in [0, \eta_1] \\ p_2\eta + \delta_2, & \eta \in [\eta_1, \eta_2], \\ & \dots \end{cases}$$

- Regular punctures: $\bar{\omega}(\eta)$ is monotonic and asymptotes to a constant at large η
- Irregular puncture: $\bar{\omega}(\eta)$ has a segment along which it decreases

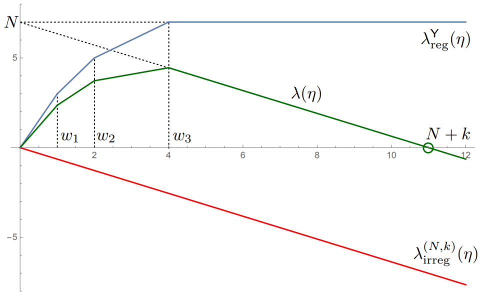


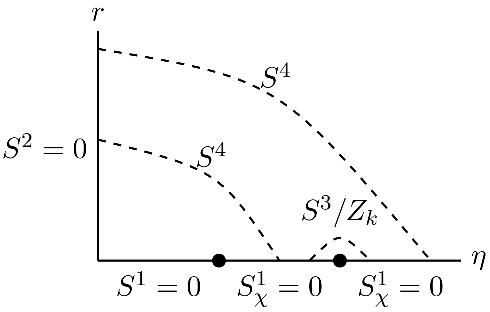
Figure: Bah-Bonetti-Federico-Waddleton [15]

Electrostatic formulation: topology of 4d theories

- Metric: $V' = \partial_\eta V$, $\dot{V} = r\partial_r V$, and $ds^2_{\hat{\beta}} = (d\beta + \frac{2\dot{V}\dot{V}'}{2V} d\chi)^2$

$$ds^2 = 4f_1^2 ds^2_{\mathsf{AdS}_5} + f_2^2 ds^2_{S^2} + f_3^2 ds^2_{\hat{\beta}} + \frac{2V''}{\dot{V}}\left(dr^2 + \frac{2\dot{V}}{2\dot{V} - \ddot{V}} r^2 d\chi^2 + d\eta^2\right)$$

- Geometry has non-trivial cycles:
- $S^4 = S^2 \times S^1_{\hat{\beta}} \times \mathcal{C}$
- $S^3/Z_k = S_{\hat{\beta}^1} \times_k (S^1_\chi \times \mathcal{C})$

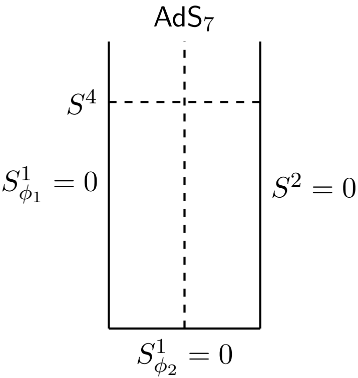


4d $\mathcal{N} = 2$ slicing of $\mathrm{AdS}_7 \times S^4$

- Superconformal symmetry: $SU(2,2|2) \subset OSp(8^*|2)$
- Metric written as an $\mathrm{AdS}_5 \times S^1$ slicing of AdS_7 and $S^2 \times S^1$ slicing of S^4

$$ds^2 = 4L^2\left(\cosh^2(x) ds^2_{\mathrm{AdS}_5} + \sinh^2(x) d\varphi_1^2 + dx^2\right) + L^2\left(\cos^2(\theta) ds^2_{S^2} + \sin^2(\theta) d\varphi_1^2 + d\theta^2\right)$$

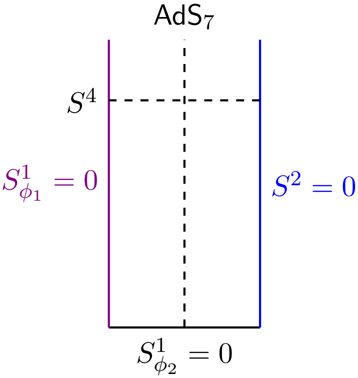
  where $0 \le \varphi_{1(2)} \le 2\pi$ and $0 \le \theta \le \pi/2$.
- Trivial fibration of T^2 over Σ_2 in this case.

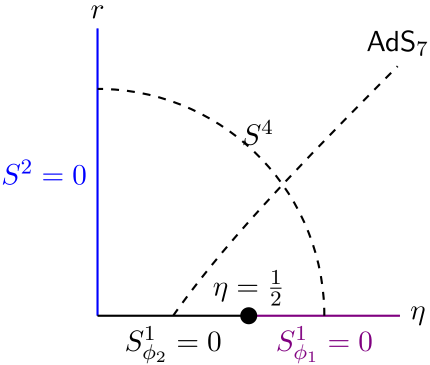


$\mathrm{AdS}_7 \times S^4$ in electrostatic coordinates

- Map: $(x, \theta) \to (r, \eta)$

$$r = \frac{1}{2}\sinh(2x)\sin(\theta), \qquad \eta = \frac{1}{2}\cosh(2x)\cos(\theta)$$





$\mathbf{AdS_7} \times S^4$ as solution to electrostatic problem

- Electrostatic potential

$$V = \frac{N}{2}\left(\cos(\theta) + 2\eta \ln r + \ln \tan \frac{\theta}{2}\right)$$

- Line-charge density

$$\bar{\omega}(\eta) = N \begin{cases} 2\eta, & \eta \in [0, \frac{1}{2}] \\ \eta + \frac{1}{2}, & \eta \in [\frac{1}{2}, \infty) \end{cases}$$

- Convenient to make a scaling transformation under which $\bar{\omega} \to N\bar{\omega}$ and $\eta \to \eta/N$

$$\bar{\omega}(\eta) = \begin{cases} 2\eta, & \eta \in [0, \frac{N}{2}] \\ \eta + \frac{N}{2}, & \eta \in [\frac{N}{2}, \infty) \end{cases}$$

1/2 BPS 4d defect in 6d $\mathcal{N} = (2, 0)$ SCFT

- Generalized to include co-dimension 2 defects by Gutperle-Klein-Rathore using $d = 7$ gauged supergravity and uplifting to M-theory [17, 18]
- Can increase size of kink at $\eta = N/2$. Denote the change in slope as k. This introduces an $\mathbb{R}^4/\mathbb{Z}_k$ orbifold with k the corresponding monopole charge

$$\bar{\omega}(\eta) = \begin{cases} (1+k)\eta, & \eta \in [0, \frac{N}{2k}] \\ \eta + \frac{N}{2}, & \eta \in [\frac{N}{2k}, \infty) \end{cases}$$

- $AdS_7 \times S^4$ is then the case $k = 1$
- For $k > 1$ the solution has an A_{k-1} singularity, corresponding to a $SU(k)$ global symmetry in the field theory

1/2 BPS 4d defect in 6d $\mathcal{N} = (2,0)$ SCFT

- Can introduce additional kinks with unit slope change. Denote the number of kinks as n and the location of the kinks as η_a, with $a \in 0, 1, ..., n$.

$$\bar{\omega}(\eta) = \begin{cases} (n+1)\eta, & \eta \in [0, \eta_1] \\ n\,\eta + \eta_1, & \eta \in [\eta_1, \eta_2] \\ (n-1)\,\eta + \eta_1 + \eta_2, & \eta \in [\eta_2, \eta_3] \\ ... & \\ \eta + \frac{N}{2}, & \eta \in [\eta_n, \infty) \end{cases}$$

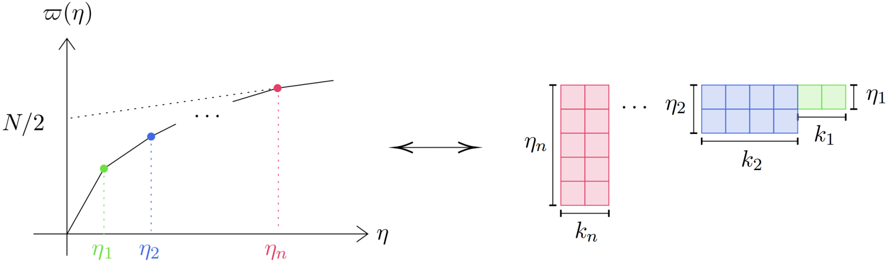
  where $N = \sum_a 2\eta_a$.
- Corresponds to the partition $N = N_1 + N_2 + N_3 + ... + N_n$ where $N_a = 2\eta_a$
- Geometries are smooth.
- The additional kinks correspond to an $U(1)^{n-1}$ global symmetry in the dual theory.

1/2 BPS 4d defect in 6d $\mathcal{N} = (2,0)$ SCFT

- In general, we can have additional kinks with $\mathbb{R}^4/\mathbb{Z}_k$ orbifolds
- On the interval $[\eta_a, \eta_{a+1}]$ the line-charge density is

$$\bar{\omega}_a(\eta) = \left(1 + \sum_{b=a+1}^{n} k_b\right)\eta + \sum_{b=1}^{a} \eta_b k_b$$

- Generic line-charge distribution and partition $N = \sum_a N_a$ with $N_a = 2k_a\eta_a$



Stress energy tensor one-point function and d_2

- Stress-energy tensor for 4d defect in six dimensions

$$\langle T^{ij} \rangle = -\frac{d_2}{5\pi^3} \frac{5\delta^{ij} - 6\frac{x_\perp^i x_\perp^j}{|x_\perp|^2}}{|x_\perp|^6}$$

- Reduce on the S^4 and employ standard holographic renormalization methods
- Use background subtraction to isolate defect contributions
- Final result

$$d_2 = -\frac{1}{24} \left(N^3 - \sum_a \frac{N_a^3}{k_a^2} \right)$$

- $d_2 \leq 0$ in agreement with average null energy condition
- Inequality saturates only when $n = 1$ and $k_1 = 1$, corresponding to no defect

Defect supersymmetric Casimir energy and d_2

- Supersymmetric Casimir energy for 6d $\mathcal{N} = (2,0)$ SCFT with the presence of a co-dimension 2 defect computed using localization in Bullimore-Kim [19]
- Defect introduces a change in the exponential prefactor. Conjectured this change can be interpreted as the defect supersymmetric Casimir energy and is related to defect conformal anomalies [4]
- Defect supersymmetric Casimir energy computed using background subtraction

$$E_C[\Sigma]_{\vartheta,\vec{\mathfrak{w}}} - E_C[\varnothing] = -\frac{1}{6}\left(N^3 - \sum_{a=1}^{n} N_a^3 - 3(\vec{\mathfrak{w}},\vec{\mathfrak{w}})\right)$$

- Setting the monodromy parameters $\vec{\mathfrak{w}} = 0$, we see agreement for the $k_a = 1$ case

$$E_C[\Sigma]_{\vartheta,\vec{0}} - E_C[\varnothing] = 4d_2$$

- Similar mach found for co-dimension 4 defects

Entanglement entropy and A-type anomaly

- Spherical entanglement entropy for 4d defect in six dimensions

$$S[\Sigma]\Big|_{\ln} = -\left(4a_\Sigma + \frac{2}{5}d_2\right)\ln\left(\frac{R}{\epsilon}\right)$$

- Final result

$$a_\Sigma = \frac{N^3}{32} - \frac{1}{96}\sum_{a=1}^{n}\left(\frac{1+2k_a}{k_a^2}N_a^3 + \sum_{b=a+1}^{n} N_a k_b\left(\frac{N_a^2}{k_a^2} + 3\frac{N_b^2}{k_b^2}\right)\right)$$

- Result is similar to the compact electrostatic solutions describing 4d theories

$$a_\Sigma \simeq \frac{N^3}{32} + c_{4d}$$

- $a_\Sigma \geq 0$
- Inequality saturates only when $n = 1$ and $k_1 = 1$, corresponding to no defect

1/4 BPS 4d defect in 6d $\mathcal{N} = (2,0)$ SCFT

- Uplift of two parameter domain wall solution from 7d gauged gravity
- 7d metric takes the form

$$ds^2 = f_1^2(x) ds^2_{\mathsf{AdS}_5} + f_2(x)^2 d\theta^2 + f_3(x)^2 dx^2$$

- Two charges q_1 and q_2
- These determine the range of x as $x_+ \leq x < \infty$ where x_+ is the largest root of

$$Q(x) = \frac{1}{2}(x^2 + q_1)(x^2 + q_2) - x^3$$

- The geometry is either smooth or has a conical deficit of $2\pi(n-1)/n$ with n determined by

$$nQ'(y_+) = y_+^2$$

1/4 BPS 4d defect in 6d $\mathcal{N} = (2,0)$ SCFT

- The two defect Weyl anomaly coefficients are

$$a_\Sigma = \frac{N^3}{24}(1 - x_+^2), \qquad d_2 = -\frac{N^3}{6}(q_1 + q_2)$$

- On-shell action (background subtracted)

$$S|_{\mathsf{log}} = N^3 \frac{4q_1 q_2 - 2(q_1 + q_2)x_+(1 - x_+) + 5y_+(1 - x_+^2)}{1920 x_+}$$

- Agreed with 7d computation up to a discrepency involving large gauge transformations

Thank you!

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