Anomalies and Entanglement: Twist field approach to non-invertible symmetries in RCFTs

Based on - arXiv [hep-th]:

- 2402.06322, 2409.02162 (BCFT approach)
- 2507.xxxxx (Twist field approach)

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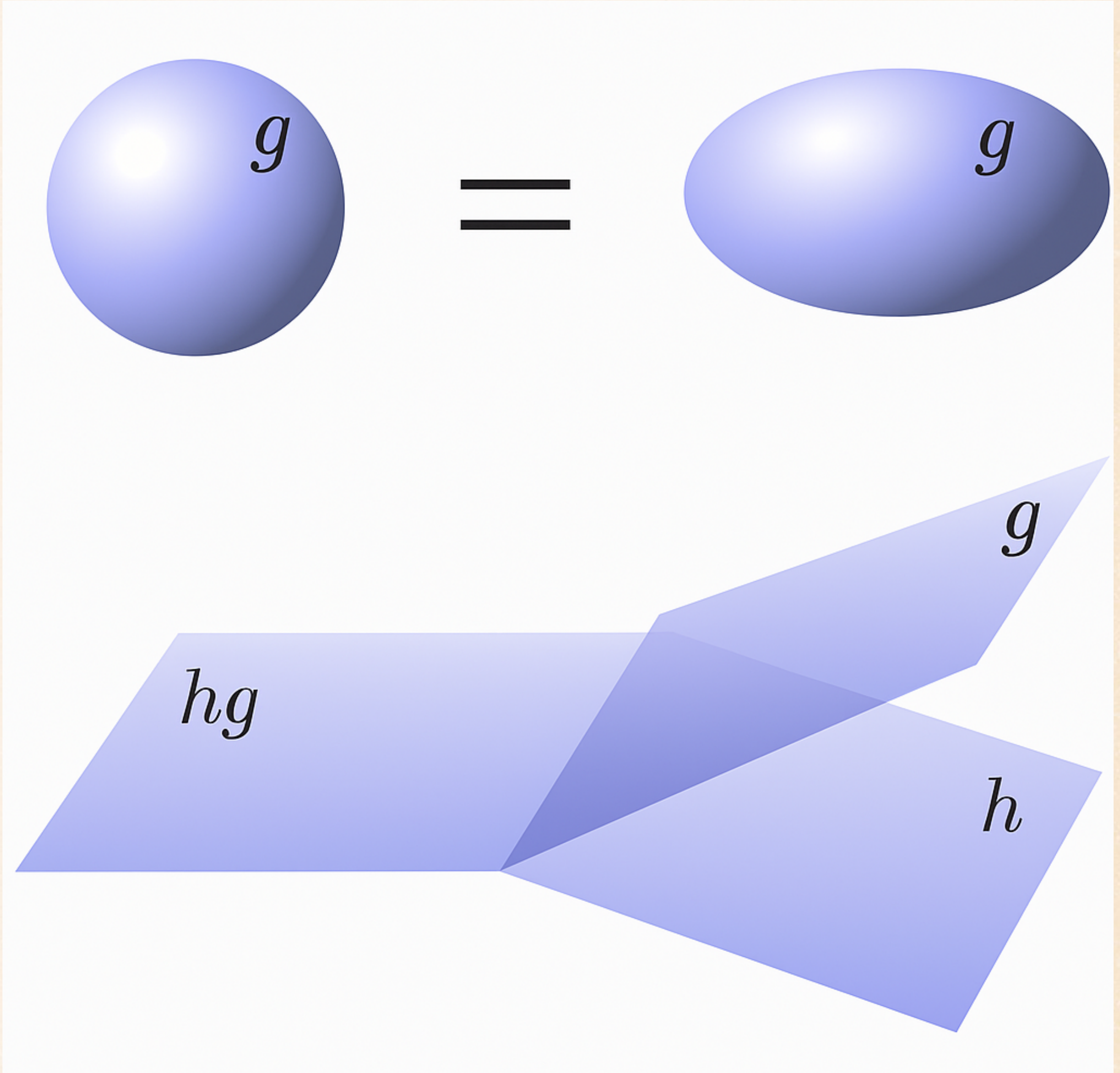
Global symmetries = Topological Ops.

[Gaiotto, Kapustin, Seiberg, Willet]

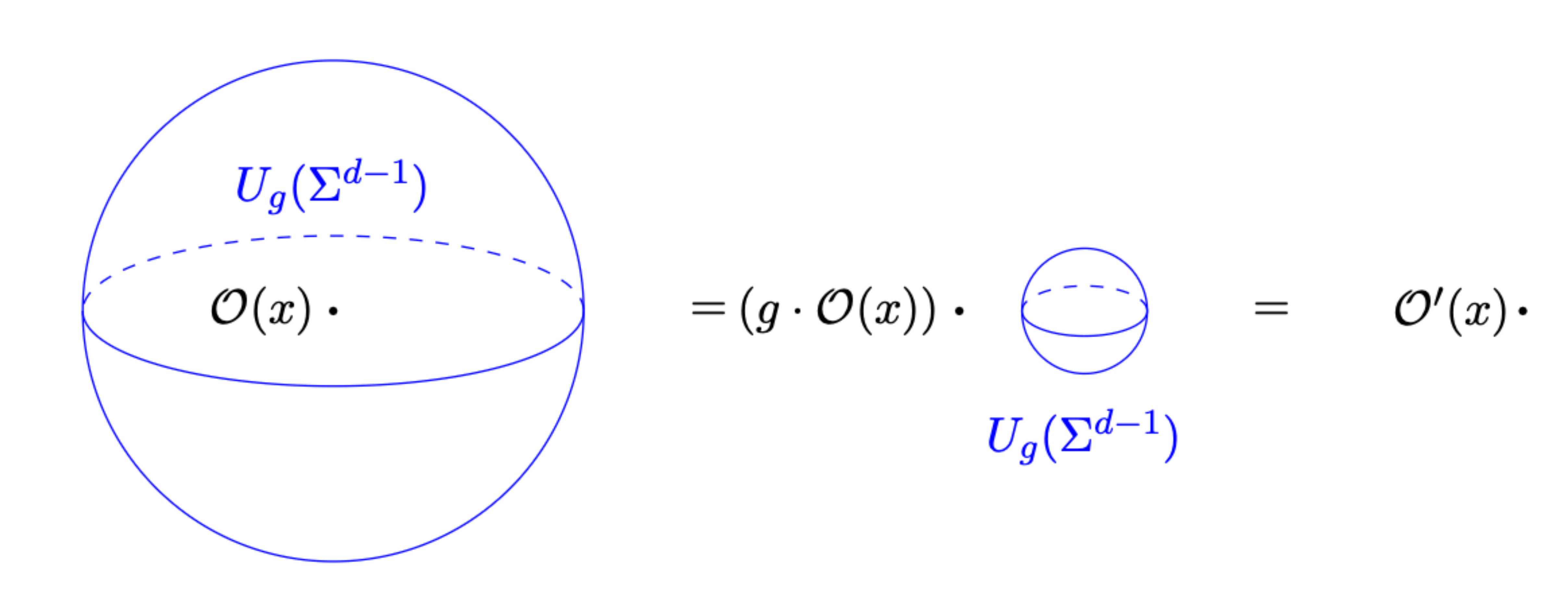
- Defn.: a p-form global symmetry is associated with a topological op. $U_g(\Sigma_{d-p-1})$ defined on a co-dim. $p+1$ sub-manifold associated to a group element g



- Topological: deformations of Σ do not change correlators involving $U_g(\Sigma)$

- Group mult.: Composition as – $U_g(\Sigma) \times U_h(\Sigma) = U_{gh}(\Sigma)$

- Linking: Action on local ops. via linking

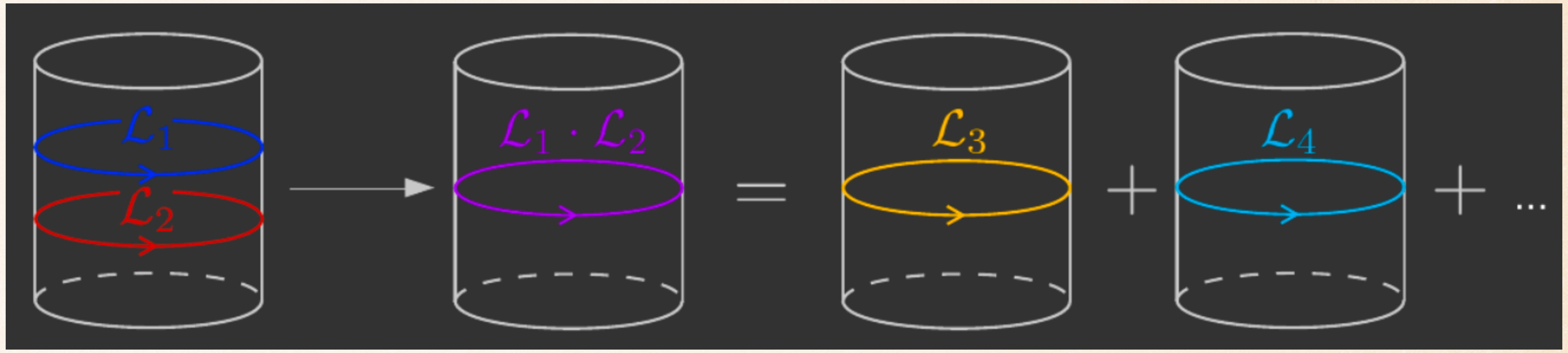


[Bhardwaj et. al.]

Non-invertible symmetries

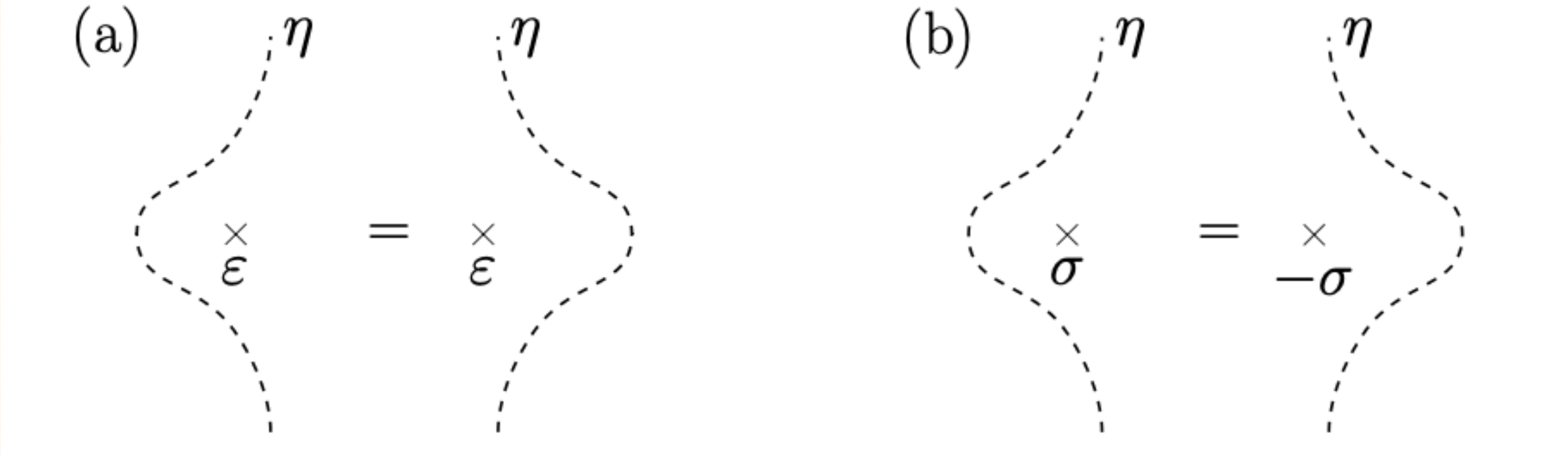
- Generalisations: Composition law is given by fusion rules of a fusion category [Verlinde et. al.; Shao et. al.; Cordova et. al.]

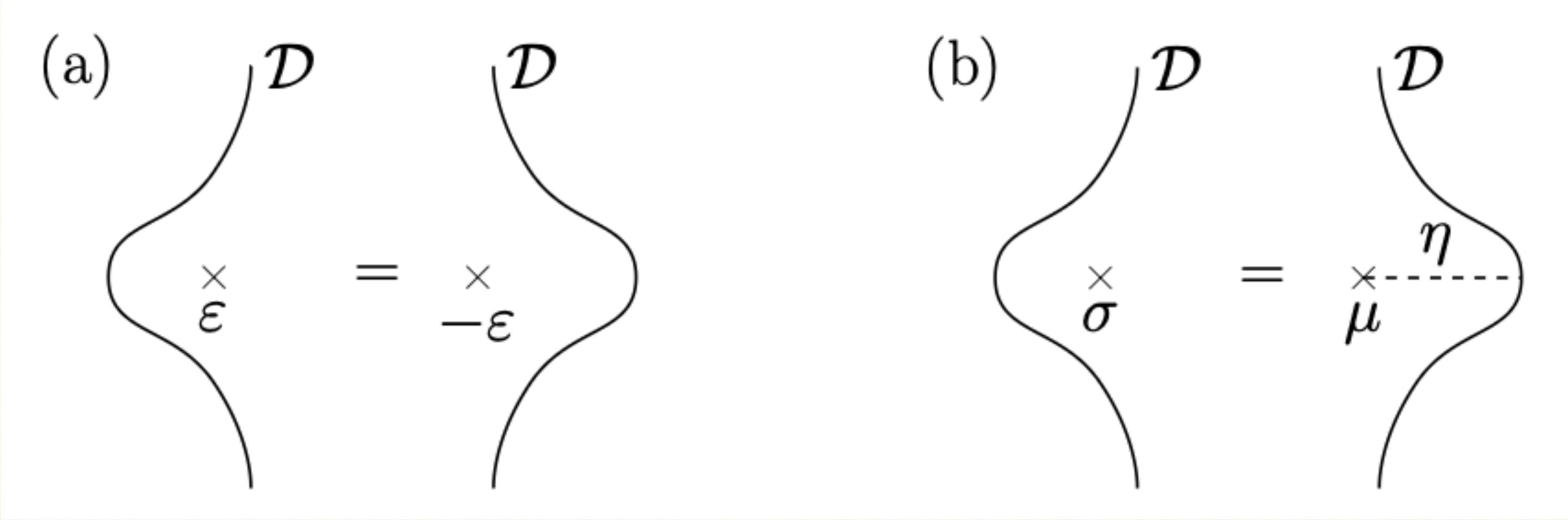
$$a \times b = \sum_{c \in \mathscr{C}} N^{c}_{ab}\, c \quad \leftrightarrow$$



- E.g.: 2D Ising – $\mathbf{1}_{(0,0)}; \epsilon_{(\frac{1}{2},\frac{1}{2})}; \sigma_{(\frac{1}{16},\frac{1}{16})} \leftrightarrow \mathbf{1}; \eta; \mathscr{D}$ with fusion rule as: $\eta^2 = \mathbf{1}; \eta\,\mathscr{D} = \mathscr{D}\,\eta = \mathscr{D}; \mathscr{D}^2 = \mathbf{1} + \eta$

- Diagnostic: non-integral quantum dimension – $\langle 0 | \mathscr{N} | 0 \rangle = d_{\mathscr{D}} = \sqrt{2} \notin \mathbb{Z}$, but $d_\eta = 1$ [Shao]





Entanglement resolution w/ symmetries (SREE)

- QFT w/ global symmetry: decompose its Hilbert sp. into irreps of the symmetry

- Entanglement in QFT w/ symmetries → Symmetry Resolved Entanglement Entropy (SREE) which quantifies amt. of entanglement for different irreps/charged sectors

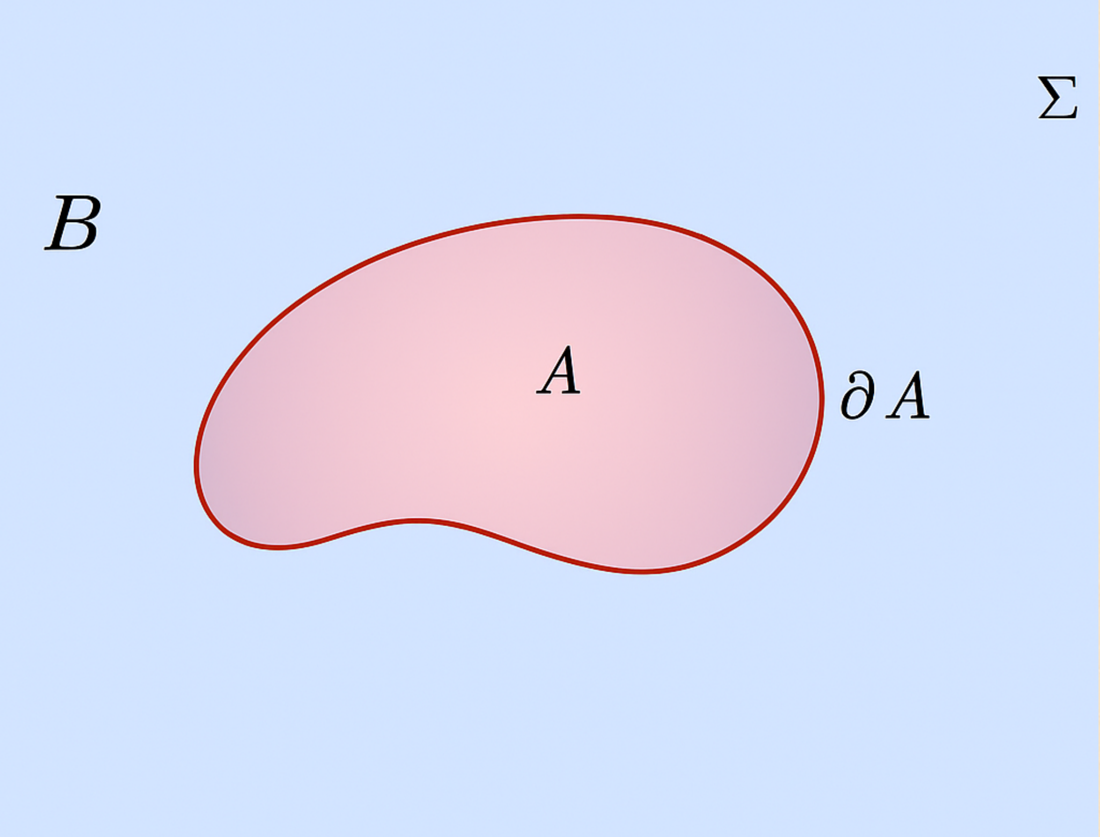
- Results for grp-like symmetries → extend for categorical symmetries – probe universal features pertaining to structure of the CFT

  [Goldstein et. al.; Calabrese et. al.] [Saura-Bastida et. al.; Das et. al.]

- Recall, ent. entropy of a region A of length ℓ is: [Cardy et. al.]

$$S_A^n = \frac{1}{1-n}\log \mathrm{Tr}\,\rho_A^n = \frac{1}{1-n}\log\frac{Z_n[\mathfrak{q}]}{Z_1^n[\mathfrak{q}]} \to S_A = \lim_{n\to 1} S_A^n = -\,\mathrm{Tr}\,\rho_A \log \rho_A$$

$(Z_n = \mathrm{Tr}\,\rho_A^n)$



- Work w/ (diagonal) rational CFTs: finite no. of primaries → $c, h_i \in \mathbb{Q}$

[BPZ; Moore-Seiberg; Verlinde et. al.; WZW]

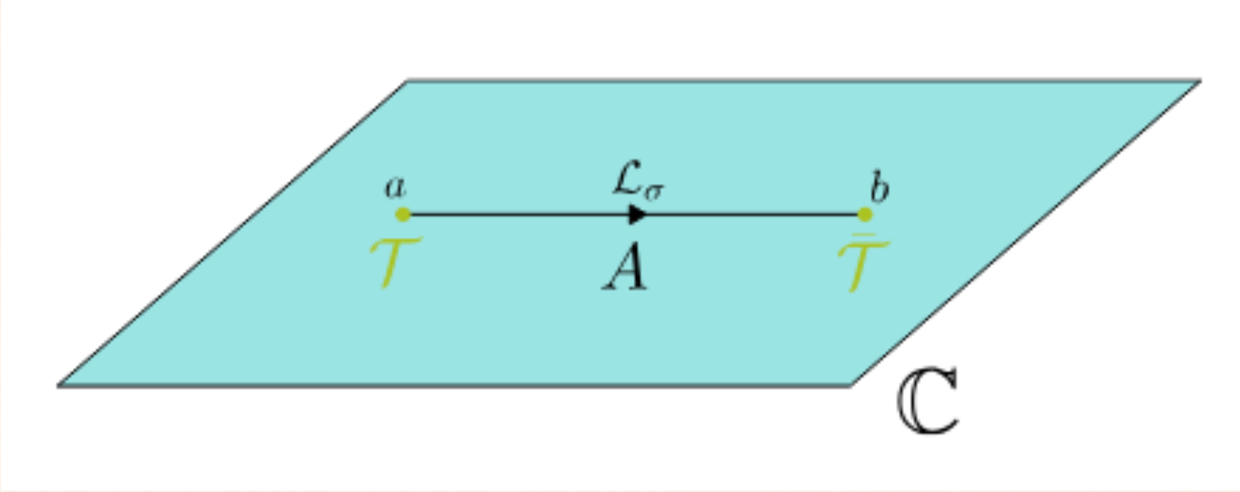
SREE for groups

$[\mathbb{C} \cong \mathscr{R}_{n,A}/\mathbb{Z}_n]$

- For ent. entropy: $\langle \mathscr{T}(0,a)\,\tilde{\mathscr{T}}(0,b)\rangle_{\mathscr{L}^{(n)},\,\mathbb{C}} = \ell^{-2\Delta_{\mathscr{T}}}$

$$\Delta_{\mathscr{T}} = \frac{c}{12}\left(n - \frac{1}{n}\right) = \Delta_{\tilde{\mathscr{T}}}$$

$$Z_n[\mathscr{L}^{(n)}, \mathbb{C}] \sim \langle \mathscr{T}(0,a)\tilde{\mathscr{T}}(0,b)\rangle_{\mathscr{L}^{(n)},\,\mathbb{C}}$$



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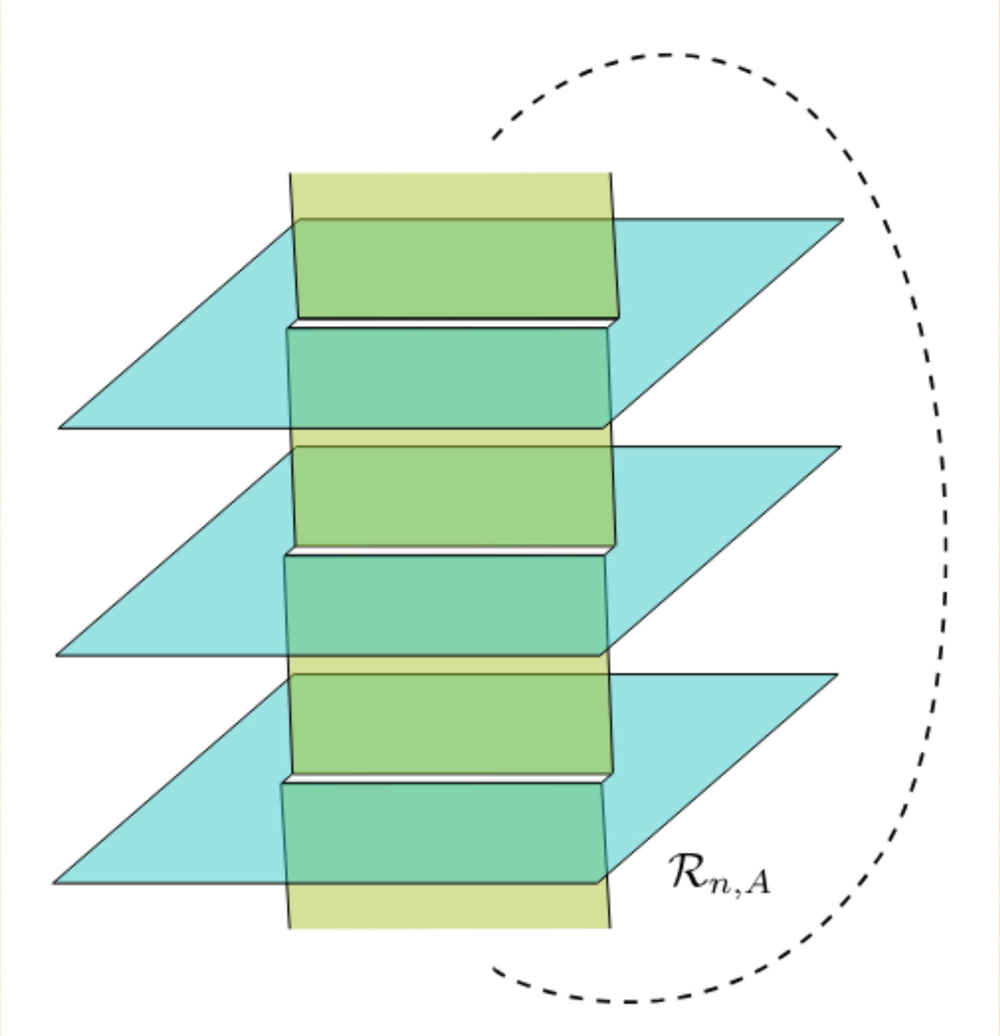
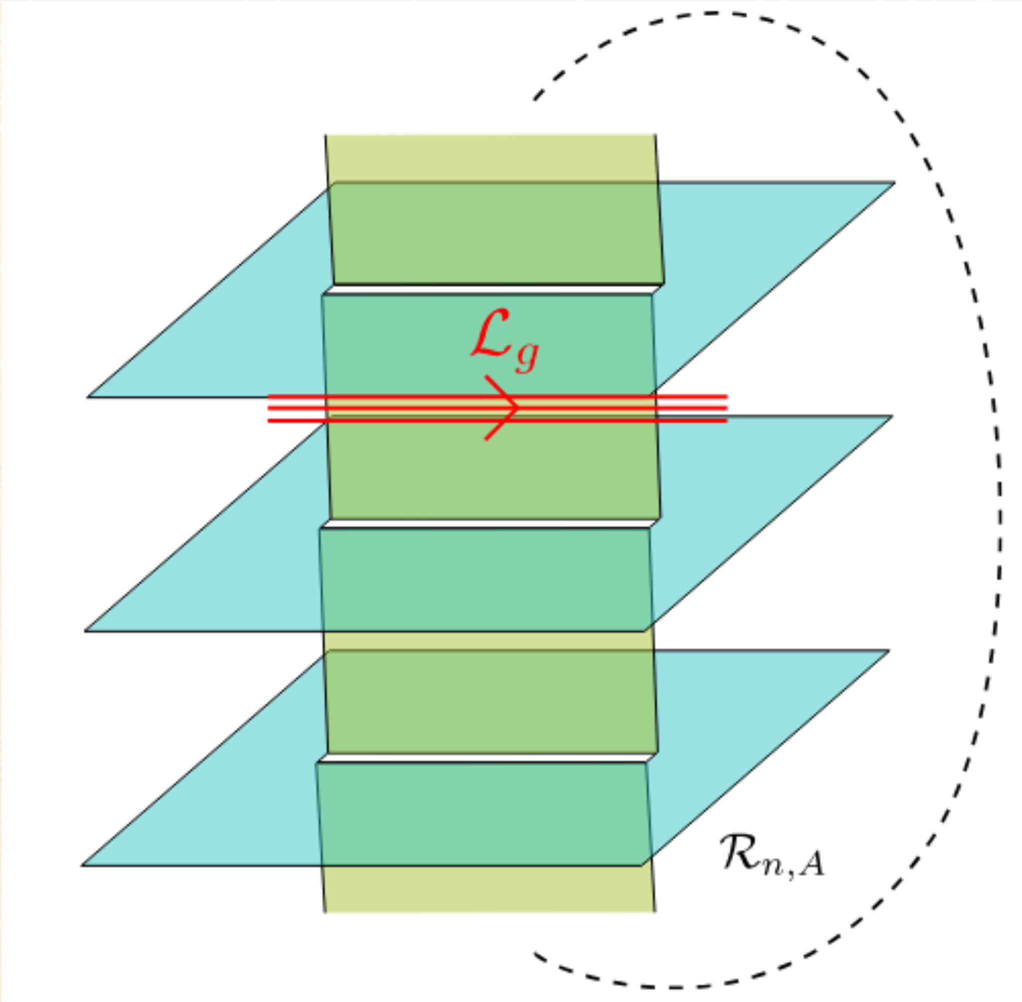


- For the grp-like case: $\mathscr{L}_g = e^{\mathrm{i}\alpha}$,

$$\langle \mathscr{T}_g(0,a)\,\tilde{\mathscr{T}}_g(0,b)\rangle_{\mathscr{L}^{(n)},\mathbb{C}} = \ell^{-2\Delta_{\mathscr{T}}-2\frac{\Delta_{\mathscr{O}_g}}{n}}$$

$$\Delta_{\mathscr{T}_g} = \frac{c}{12}\left(n - \frac{1}{n}\right) + \frac{\Delta_{\mathscr{O}_g}}{n}$$

$$Z_n[g] \sim \langle \mathscr{T}_g(0,a)\tilde{\mathscr{T}}_g(0,b)\rangle_{\mathscr{L}^{(n)},\mathbb{C}}$$



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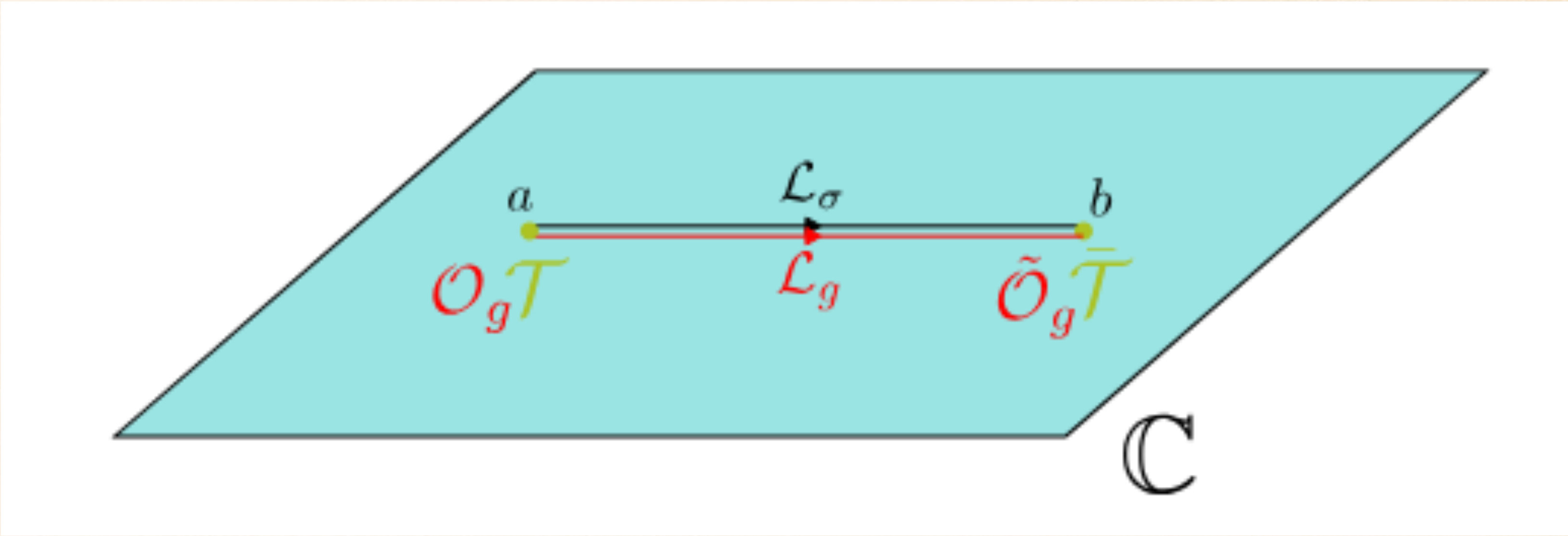


[Goldstein et. al.]

Properties of twist fields

- The composite twist fields must satisfy the following properties:

  - $\Delta_{\mathscr{T}_g} = \Delta_{\tilde{\mathscr{T}}_g}$ [Goldstein et. al.; Bianchini et. al.]
  - $\Delta_{\mathscr{T}_g}$ should be the (unique) lowest scaling dimension of all primaries
  - $\Delta_{\mathscr{T}_g} = \bar{\Delta}_{\mathscr{T}_g} \Rightarrow$ it belongs to a scalar primary

- Grp-like results:

$$S_A[r] = \frac{c}{3}\log\frac{\ell}{\epsilon} + \log\frac{d_r^2}{|G|} + \mathcal{O}\left(\left(\frac{\epsilon}{\ell}\right)^{2\Delta_{\mathcal{O}_g}}\right)$$

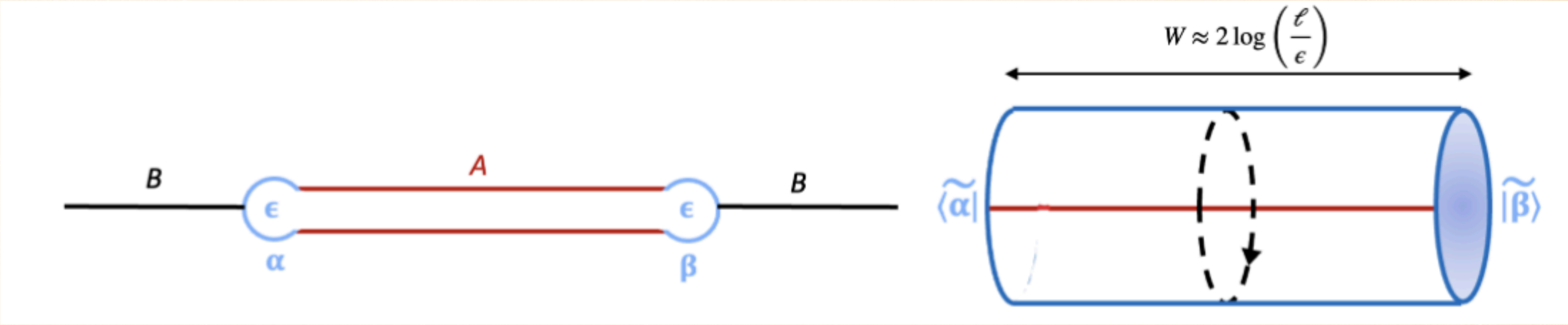
{Entanglement equipartition @ sub-leading $\mathcal{O}(1)$ order for Abelian groups} [Xavier et. al.]

SREE via BCFT

- From torus to annulus:

$$\mathcal{H}_1 = \mathcal{H}_{A,\alpha\beta} \otimes \mathcal{H}_{B,\beta\alpha}$$

$$\Pi_r : \mathcal{H}_1 \to \mathcal{H}_r$$



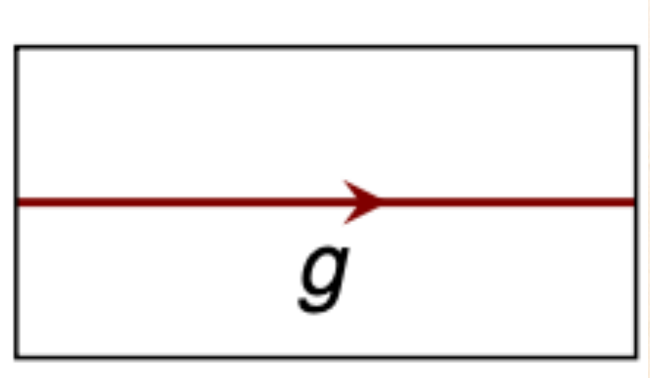
[Ohmori, Tachikawa]

- Using character-orthogonality theorem:

$$\Pi_r = \frac{d_r}{|G|}\sum_{g\in G}\chi_r^*(g)\,\widehat{\mathcal{L}}_g, \forall\, g \in G, \quad \rho = \frac{\mathfrak{q}^{L_0-\frac{c}{24}}}{Z(\mathfrak{q})}$$

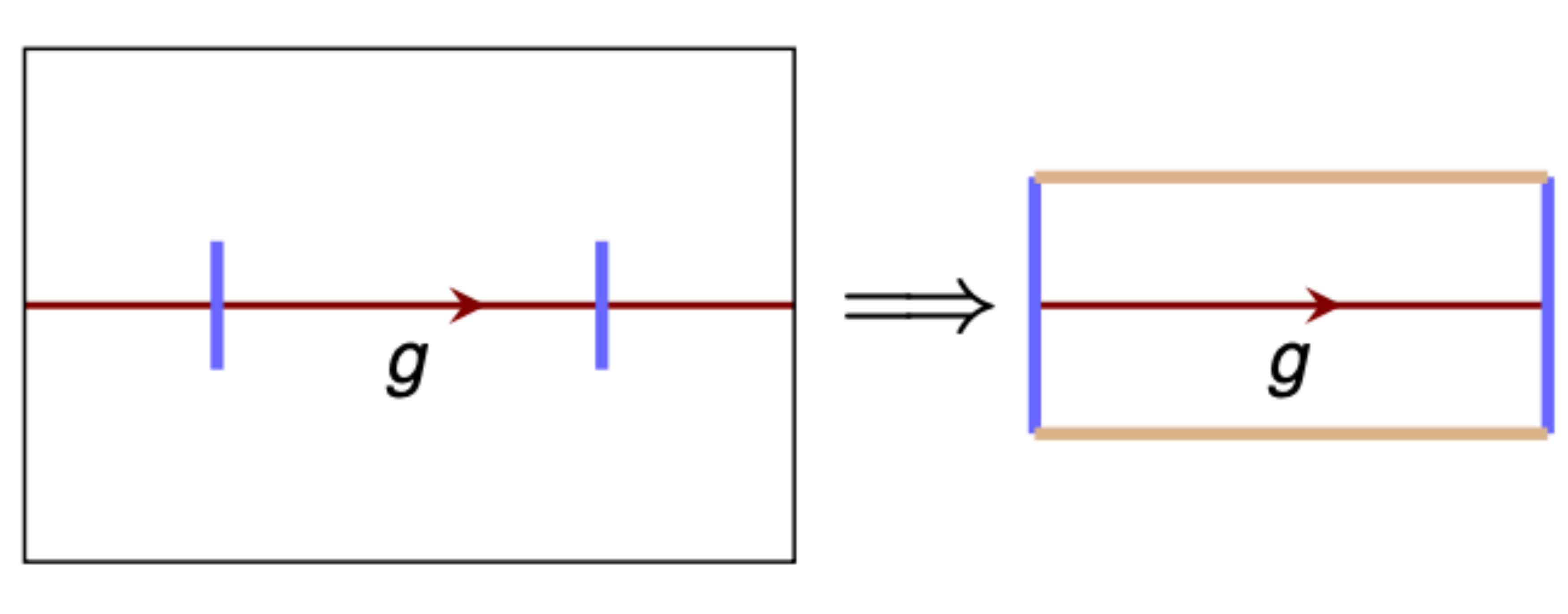
[Kusuki et. al.]

$$\mathrm{Tr}\,\Pi_r\,\rho = \frac{d_r}{|G|}\sum_{g\in G}\chi_r^*(g)$$

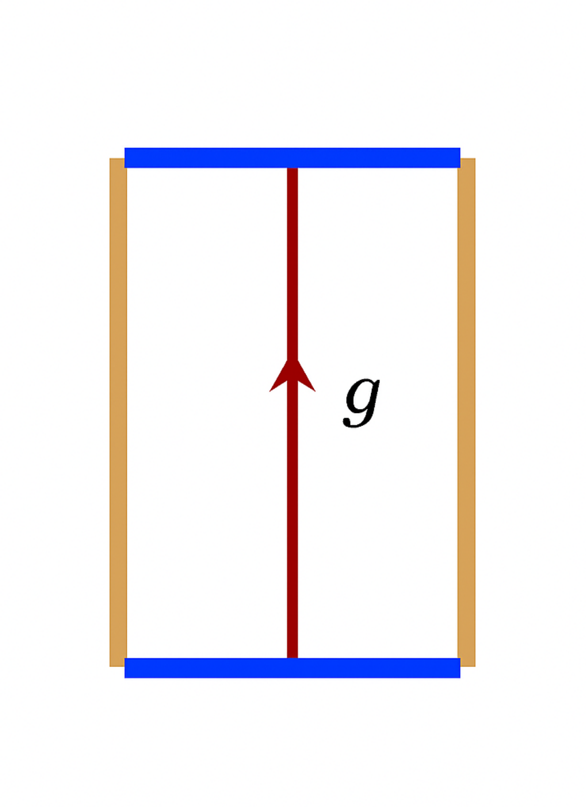


SREE via BCFT (contd.)

- From torus to annulus:



$\equiv Z_{\alpha\beta}[q, g] \xrightarrow{S}$



$= {}_g\langle \widetilde{\alpha} | \tilde{q}^{L_0 - \frac{c}{24}} | \widetilde{\beta} \rangle_g$

- Twisted Cardy states: in above $|\widetilde{\alpha}\rangle_g, |\widetilde{\beta}\rangle_g$ are G-twisted Cardy states, where their untwisted analogues are G-invariant,

$$g|\widetilde{\alpha}\rangle_e = |\widetilde{\alpha}\rangle_e, \; \forall\, g \in G$$

$$\boxed{S_A[q, r] = \frac{c}{3}\log\frac{\ell}{\epsilon} + \log\frac{d_r^2}{|G|} + (g_\alpha^* + g_\beta) + \mathcal{O}\left(\frac{\epsilon}{\ell}\right)}$$ [Kusuki et. al.]

[where $g_\alpha \equiv \log\langle 0|\widetilde{\alpha}\rangle_e$ is the Affleck-Ludwig boundary entropy] [Affleck et. al.]

Categorical symmetries

- For generic topological defect lines (TDLs) in a category $\mathscr{C}$, need notion of $\mathscr{C}$-symmetric Cardy states:

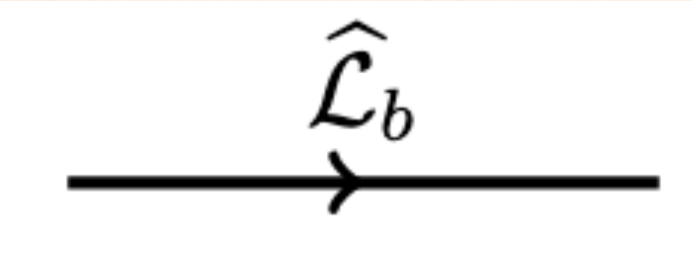
{gaugeable} ← $a|\tilde{\alpha}\rangle_e = |\tilde{\alpha}\rangle_e + \ldots$ {weakly-symmetric Cardy states}

{non-anomalous: compatible w/ trivial gapped phase} ← $a|\tilde{\alpha}\rangle_e = d_a|\tilde{\alpha}\rangle_e$ {strongly-symmetric Cardy states}

[Konechny; Choi et. al.]

- Build projectors compatible w/ bulk symmetries of CFT involving TDLs which can "end" topologically at entangling points: map projectors from torus to annulus demanding *topological endability*, [Lin et. al.]

$$\Pi^c = \frac{d_c}{|\mathscr{C}|}\sum_{b\in\mathscr{C}}\chi_c^*(b)$$



{w/ $|\mathscr{C}| = \sum_{c\in\mathscr{C}} d_c^2, \quad \chi_c^*(b) = \frac{\bar{S}_{bc}}{S_{00}}$}

{equipartiton breaks at sub-leading order for non-invertible TDL w/ $d_c > 1$}

- CaT-SREE:

[Das et. al.]

$$S_A[q,c] = \frac{c}{3}\log\frac{\ell}{\epsilon} + \log\frac{d_c^2}{|\mathscr{C}|} + (g_{\tilde{\alpha}}^* + g_\beta) + \mathcal{O}\left(\frac{\epsilon}{\ell}\right)$$

{where, $|\tilde{\alpha}\rangle_e$ is a weakly-symmetric Cardy state}

Anomalies & defect Hilbert sp.

- If the defect Hilbert space contains no scalars, then no twisted Ishibashi state and hence no twisted Cardy state can be constructed → no weakly-symmetric Cardy states → cannot symmetry resolve!

- No weakly-symmetric Cardy states → TDL is non-gaugeable! [how to relate to anomalous TDL?]

[Lin et. al.; Li et. al.; Choi et. al.]

- Symmetry resolution possible only if atleast one of the defect primaries is a scalar. [upcoming]

- If $\mathscr{C}$ contains only invertible lines then no symmetry resolution is possible: e.g. $SU(2)_1$ WZW CFT has no non-invertible TDL and hence its $\mathbb{Z}_2$-line is anomalous! [Das et. al.]

- Composite twist field approach produces same result as BCFT approach for CaT-SREE but obviously no Affleck-Ludwig boundary term is present. [upcoming]

Summary

- CaT-SREE computed for tri-critical Ising $(c = 7/10)$ which has a Fibonacci sub-category: $\mathscr{C}_{\text{Fib}} = \{\mathbf{1}, W\}$,

$$W^2 = \mathbf{1} + W \quad \rightarrow \quad \mathscr{H}_W \text{ has 3 defect scalar primaries}$$

- For non-invertible TDL entanglement equipartition is broken @ sub-leading order while for invertible TDL equipartition holds @ sub-leading order in ϵ_{UV}. Possibility to measure this equipartition breaking in lattice models owing to recent constructions of non-invertible symmetries on the lattice. [Seiberg et. al.; Seifnashri et. al.]

- Breakdown of entanglement equipartition is "categorical" in nature and doesn't generically happen due to non-Abelianity, e.g., $SU(2)_k$ WZW CFT has weakly-symmetric Cardy states w.r.t. a sub-category called $(A_1, k)_{\frac{1}{2}}$ which includes sub-set of TDLs w/ integer spins only. So, CaT-SREE of these lines will lead to entanglement breaking due to non-invertibility and not due to non-Abelianity since the full category is anomalous and no symmetry resolution occurs w.r.t. it.

Arigatou gozaimasu!!!

ありがとうございます。