Entanglement Suppression Due to Black Hole Scattering

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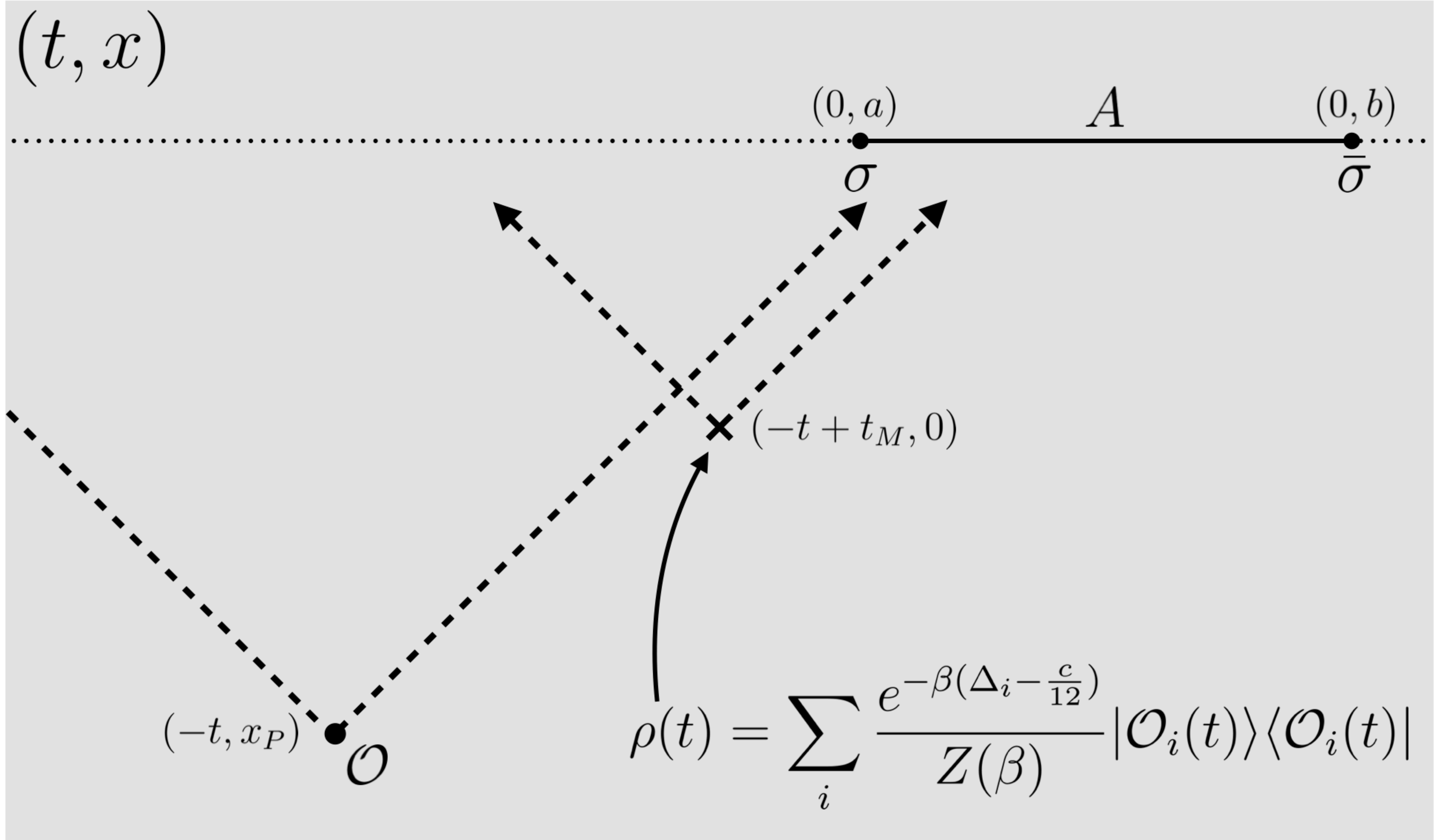
Setup of interest



[Bhattacharyya-Takayanagi-Umemoto, 2019]

2D holographic CFT

- a local operator quench $\mathcal{O}$ w/ regulator δ
  ➡ dual to a defect

- a mixed-state quench ρ w/ regulator s, inv. temp. β
  ➡ dual to a localized black hole

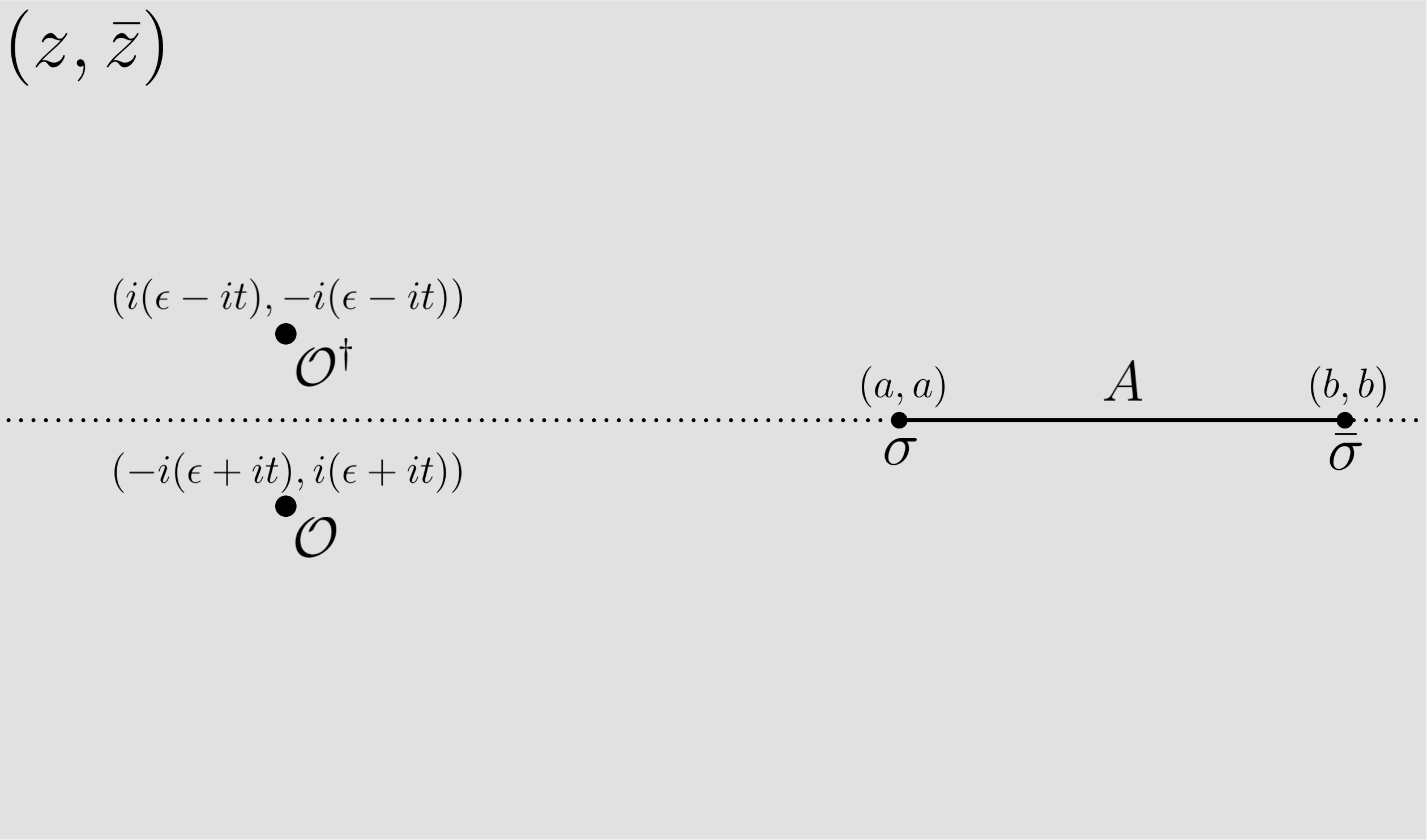
entanglement entropy S_A?

Why this setup?

- In general, we are interested in the behavior of quenches and how they interact with each other.
  ➡ Single local operator quenches give rise to excitation $\Delta S_A \sim \frac{c}{6}\log t$, so naively, n quenches $\rightarrow n\Delta S_A$?
- Previous work on single local operator quenches boils down to the computation of the four-point function $\langle \mathcal{O}^{\dagger\otimes n}\mathcal{O}^{\otimes n}\sigma_n\bar{\sigma}_n\rangle$, which is already difficult to do in holographic CFTs, but adding more of them only complicates things further as one has to consider higher-order correlation functions.
- This motivates us to consider mixed-state quenches instead, as we can treat these geometrically (not as operator insertions).
  ➡ This allows us to consider multiple quenches without having to compute higher-order correlation functions.

Single local operator insertion

[Asplund-Bernamonti-Galli-Hartman, 2015]



Replica method $\longrightarrow$ $\langle \mathcal{O}^{\dagger \otimes n} \mathcal{O}^{\otimes n} \sigma_n \bar{\sigma}_n \rangle$

Heavy-heavy-light-light (HHLL) approximation

n-th Rényi entropy of the excited state relative to the vacuum $\Delta S_A^{(n)}$

$$\Delta S_A^{(n)} = \frac{1}{1-n} \log \left(\frac{\langle \mathcal{O}^{\dagger \otimes n} \mathcal{O}^{\otimes n} \sigma_n \bar{\sigma}_n \rangle}{\langle \mathcal{O}^{\dagger \otimes n} \mathcal{O}^{\otimes n} \rangle \langle \sigma_n \bar{\sigma}_n \rangle} \right)$$

$$= \frac{1}{1-n} \log \left(|z|^{4h_{\sigma_n}} G(z, \bar{z}) \right)$$

where

$$z = \frac{\left(z_{\sigma_n} - z_{\bar{\sigma}_n}\right)\left(z_{\mathcal{O}^{\dagger \otimes n}} - z_{\mathcal{O}^{\otimes n}}\right)}{\left(z_{\sigma_n} - z_{\mathcal{O}^{\dagger \otimes n}}\right)\left(z_{\bar{\sigma}_n} - z_{\mathcal{O}^{\otimes n}}\right)} \quad \text{(cross ratio)}$$

$$G(z, \bar{z}) \equiv \frac{\langle \mathcal{O}^{\dagger \otimes n}(1) \mathcal{O}^{\otimes n}(\infty) \sigma_n(0) \bar{\sigma}_n(z, \bar{z}) \rangle}{\langle \mathcal{O}^{\dagger \otimes n}(1) \mathcal{O}^{\otimes n}(\infty) \rangle} \quad \text{(normalized 4-pt func.)}$$

Heavy-heavy-light-light (HHLL) approximation - cont.

For holographic CFTs, we expect the identity conformal block to dominate

$$G(z,\bar{z}) = \left| \mathcal{F}(0, h_{\mathcal{O}^{\otimes n}}, h_{\sigma_n} | z) \right|^2$$

two operators are heavy ($h_{\mathcal{O}} \sim c$) and two are light ($h_\sigma \to 0$ as $n \to 1$)

$$\mathcal{F}(0, h_{\mathcal{O}^{\otimes n}}, h_{\sigma_n} | z) = \left(\frac{\alpha_H}{1-(1-z)^{\alpha_H}} \right)^{2h_{\sigma_n}} (1-z)^{-h_{\sigma_n}(1-\alpha_H)} \quad \text{where} \quad \alpha_H = \sqrt{1-24h_{\mathcal{O}}/c}$$

[Fitzpatrick-Kaplan-Walters, 2014]

substitute and take limit $n \to 1$

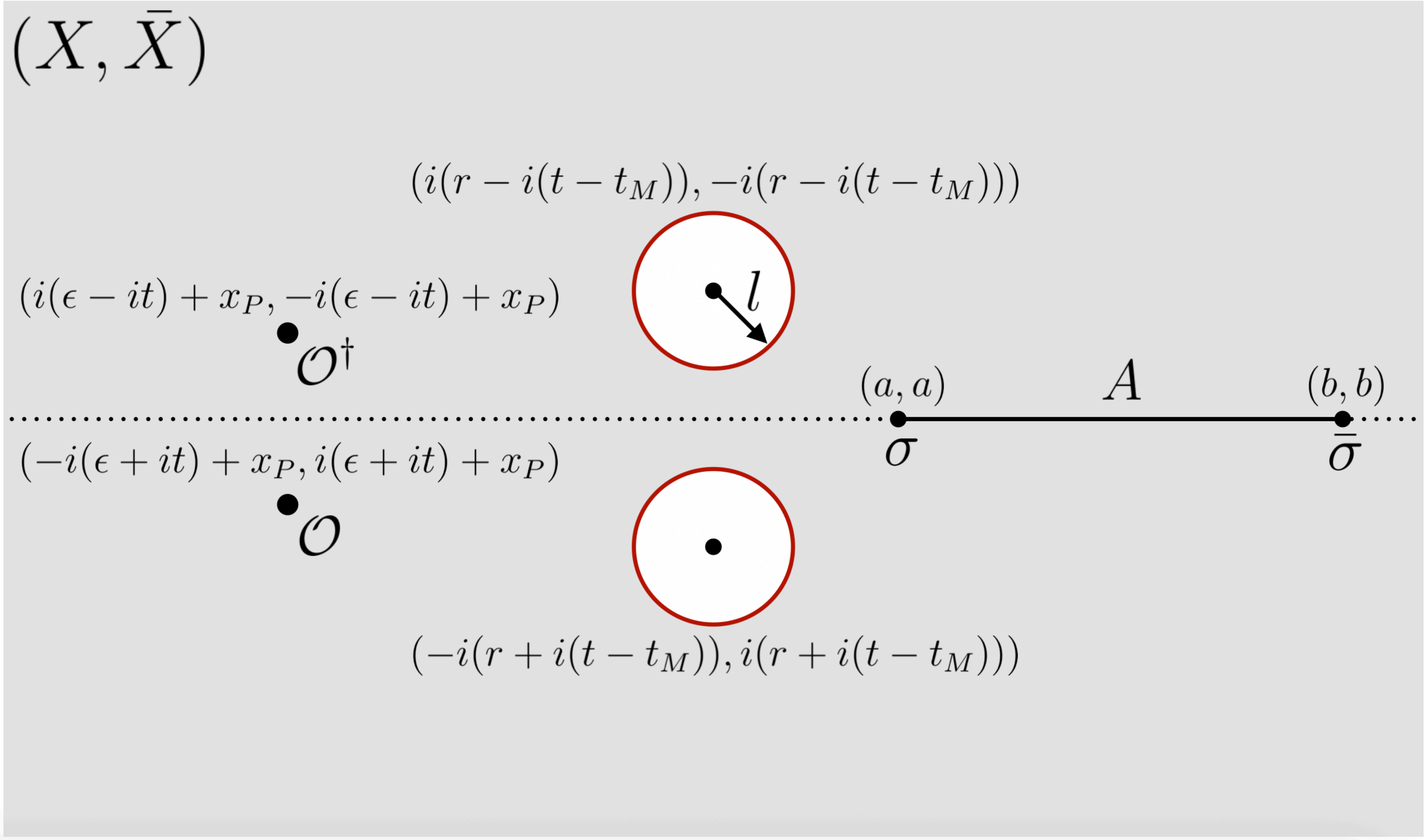
$$\Delta S_A = \frac{c}{6} \log \left(\frac{1}{|z|^2} \left| \frac{1-(1-z)^{\alpha_H}}{\alpha_H} \right|^2 |1-z|^{1-\alpha_H} \right)$$

For single local operator quenches, this comes out as

$$\Delta S_A = \frac{c}{6} \log \left(\frac{\sin(\alpha_H \pi) t}{\alpha_H \delta} \right)$$

if we take the subsystem to be semi-infinite

w/ mixed-state excitation



[Bhattacharyya-Takayanagi-Umemoto, 2019]

The mixed-state quench can be recast in the Euclidean path integral by considering this geometry and setting

$$r = \frac{s}{\tanh(\beta/2)}$$

$$l = \frac{s}{\sinh(\beta/2)}$$

The two holes are identified, giving rise to a torus geometry

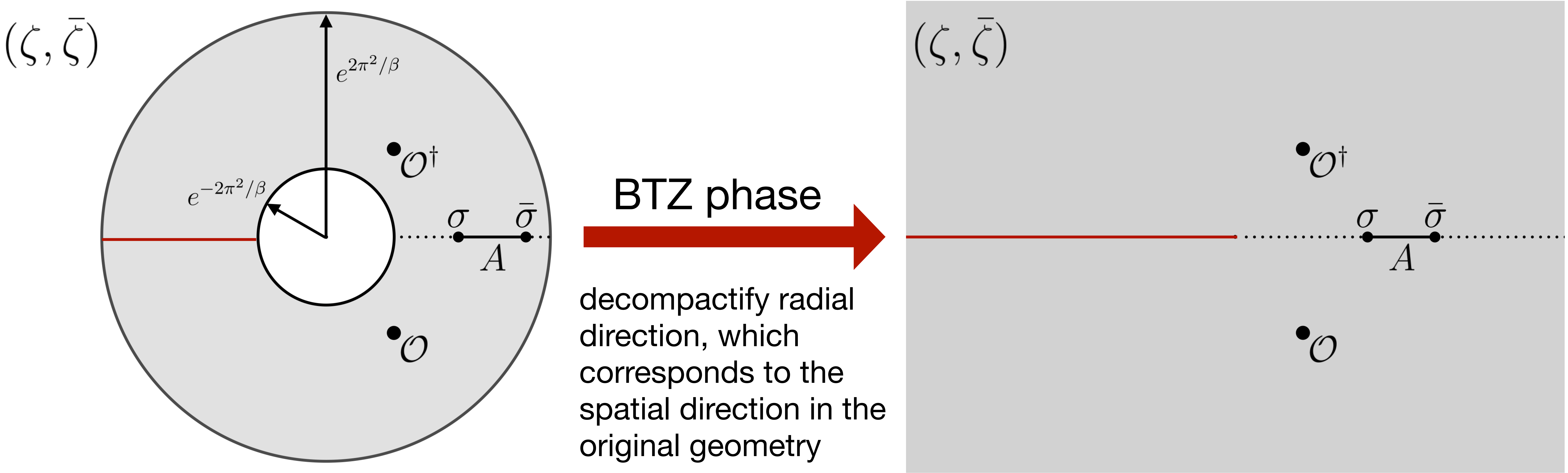
Apply the conformal map

$$\zeta = \left(-\frac{X - t + t_M + is}{X - t + t_M - is}\right)^{\frac{2\pi i}{\beta}}$$

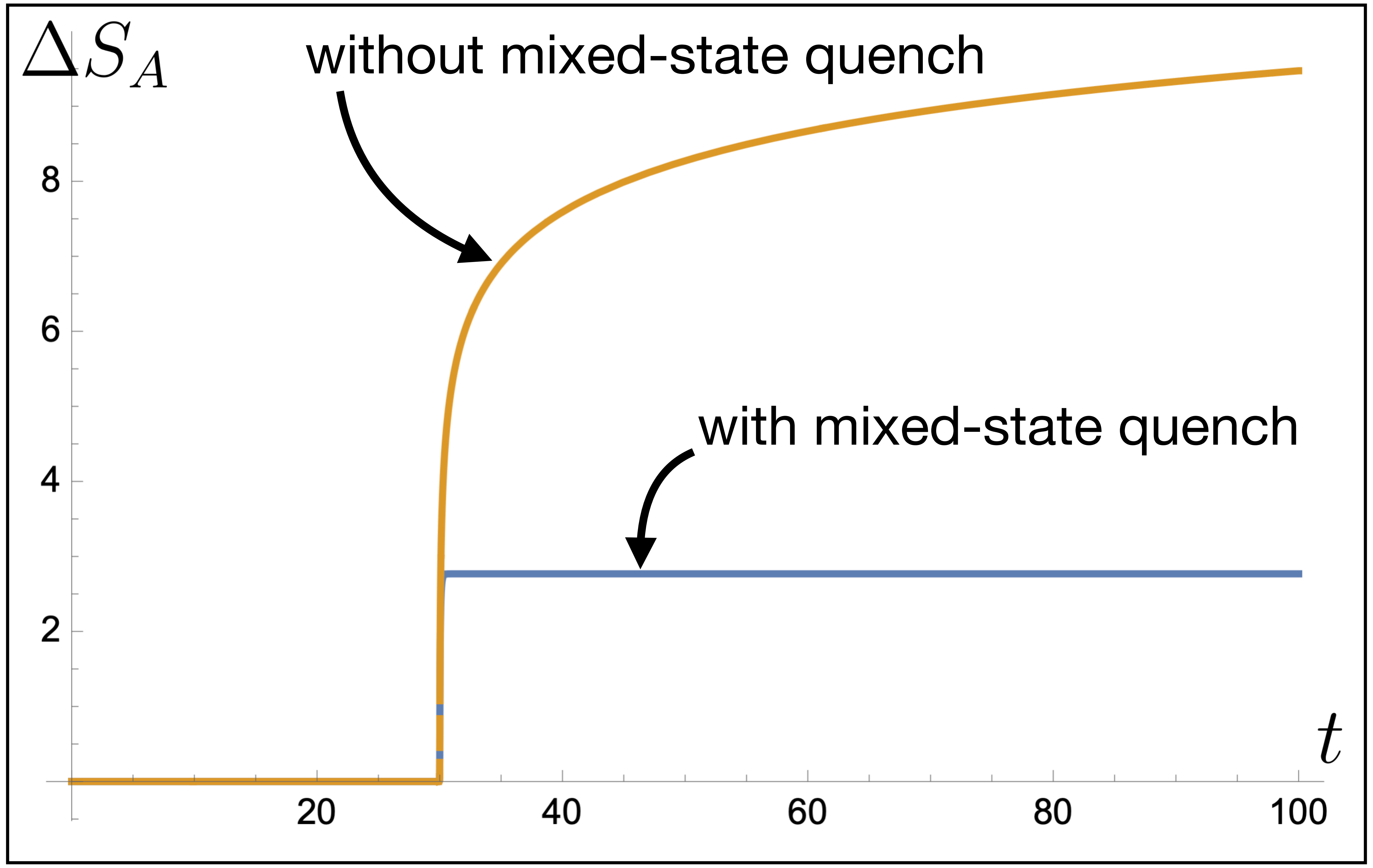
Conformal mapping of BTZ phase onto a Euclidean plane

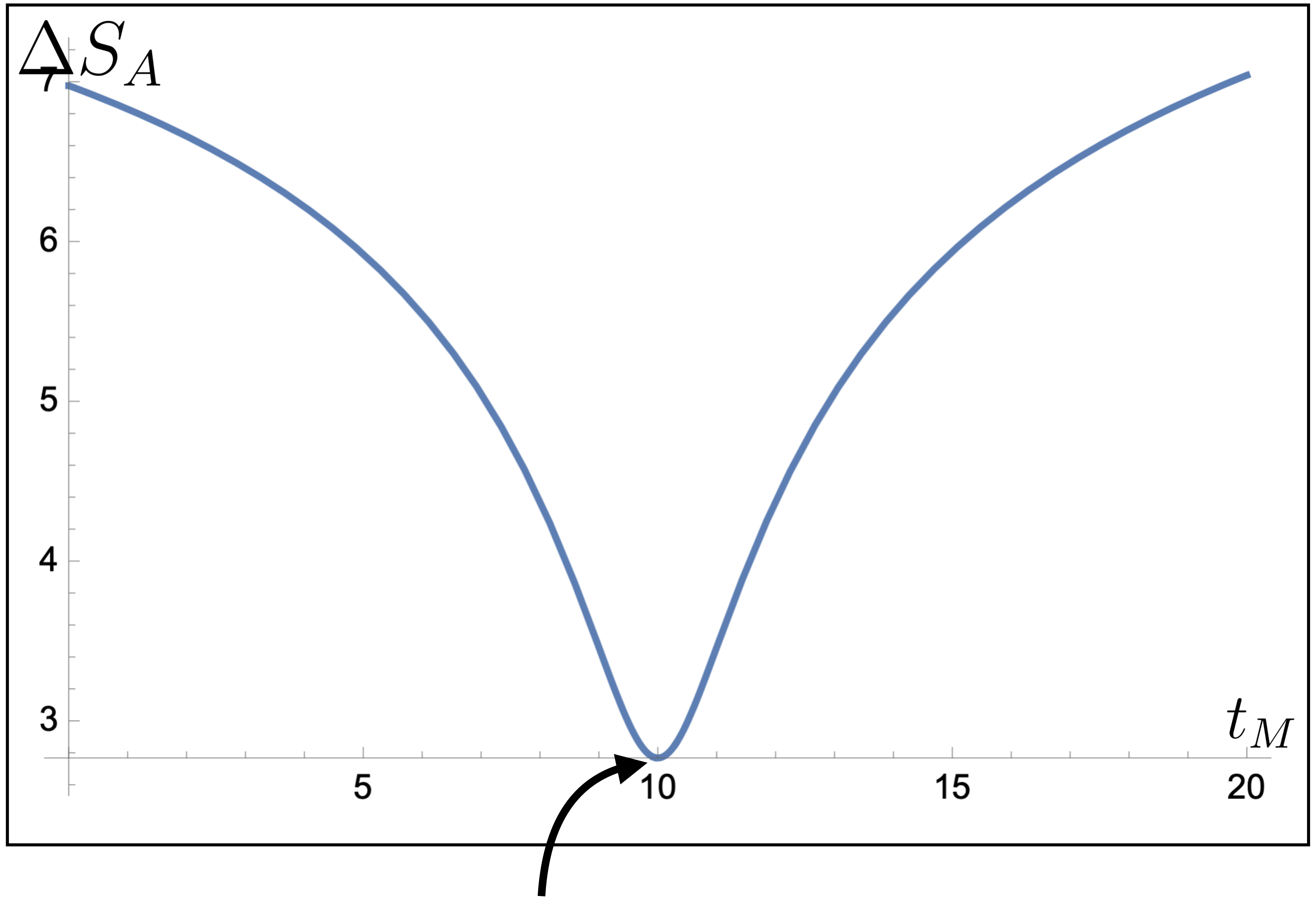
Holographic CFTs → Two phases: i) thermal AdS, **ii) BTZ black hole**

We can treat the torus as an infinitely long cylinder by decompactifying a direction (the phases correspond to the different choice of directions)



Results (when $\delta \ll s$)

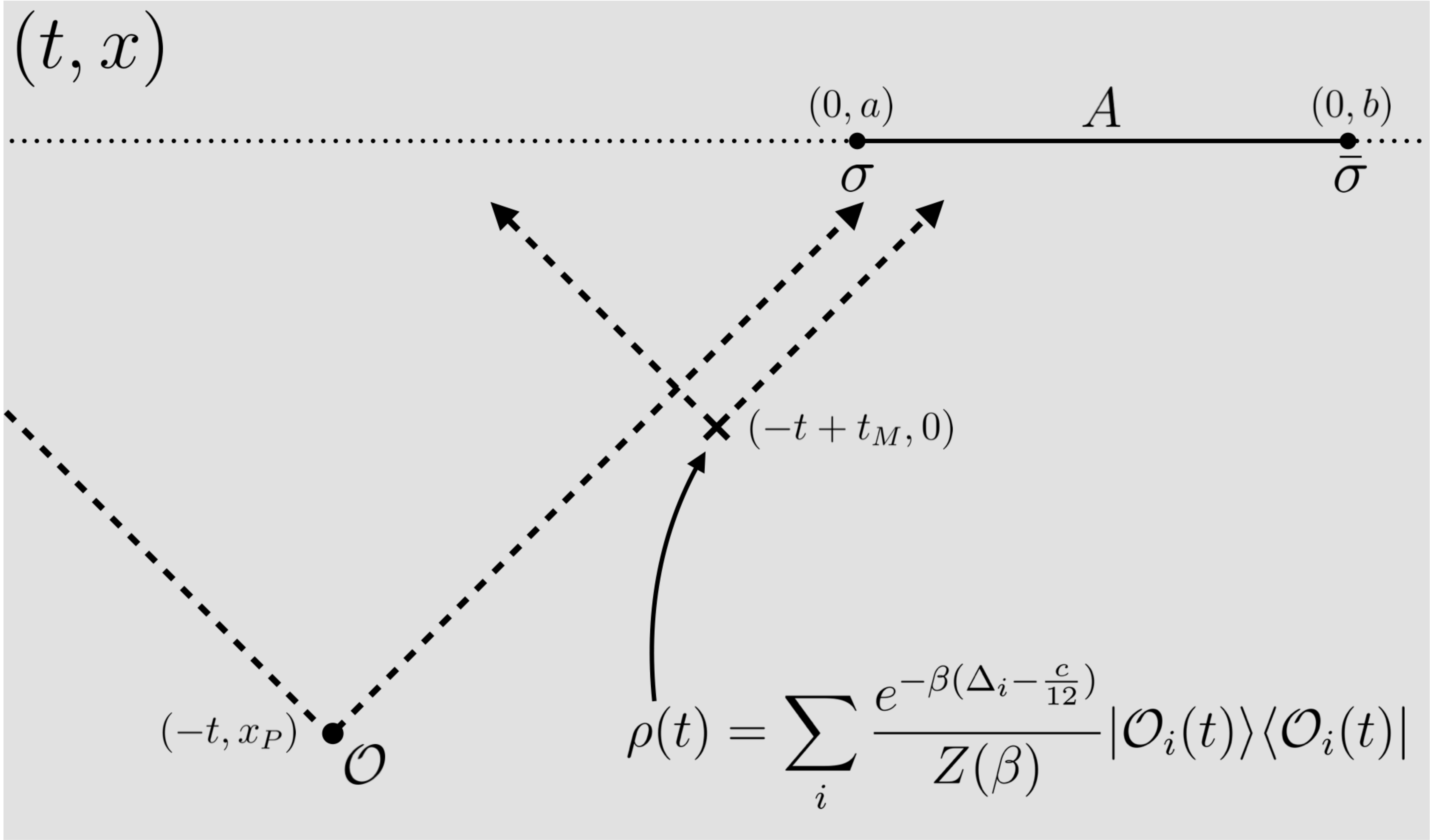




➡ entanglement suppression!

$$\Delta S_A = \frac{c}{6} \log \left(\frac{\beta \sqrt{r^2 - l^2}}{4\pi \alpha_H \epsilon} \sin(\alpha_H \pi) \tanh\left(\frac{\pi^2}{\beta} \right) \right)$$

Insertion of a local operator creates an entangled pair of modes that propagate in opposite directions at the speed of light. In the presence of a mixed-state quench, one of the modes gets absorbed by the quench (converted to an excitation in a separate Hilbert space obtained through purification), which leads to a suppression.



(t, x)
$(0, a)$
A
$(0, b)$
σ
$\bar{\sigma}$
$(-t + t_M, 0)$
$(-t, x_P)$
$\mathcal{O}$
$\rho(t) = \sum_i \frac{e^{-\beta(\Delta_i - \frac{c}{12})}}{Z(\beta)} |\mathcal{O}_i(t)\rangle\langle\mathcal{O}_i(t)|$

Summary & future directions

- An additional insertion of a mixed-state local quench suppresses the contribution to entanglement entropy by a pure-state local operator (& vice versa).
  ➡ An originally logarithmic time-dependence is reduced to a constant bump.

- Considering other limits such as $s \ll \delta$ yields similar qualitative results, but extra care must be taken in picking out correct branches.

- Once we depart nice limits, we see a breakdown of our method as we start to observe singularities.
  ➡ Proper treatment most likely requires better understanding of conformal blocks on a torus.

Thank you!

Preprint coming out soon!