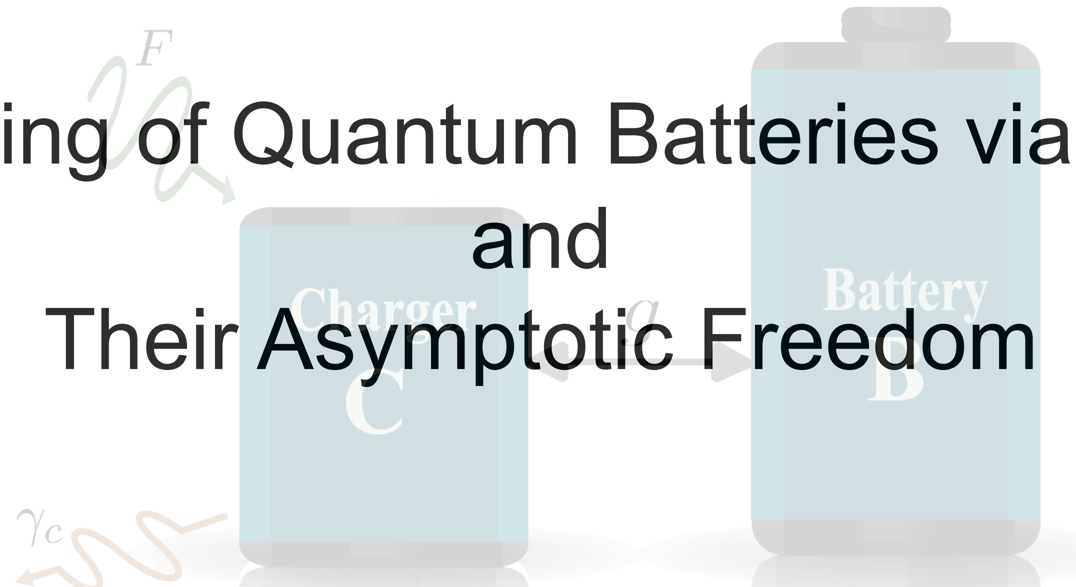
Fast Charging of Quantum Batteries via Dephasing and Their Asymptotic Freedom



Shastri, Jiang, Xu, Venkatesh & GW, npj Quantum Inf. **11**, 9 (2025)

Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

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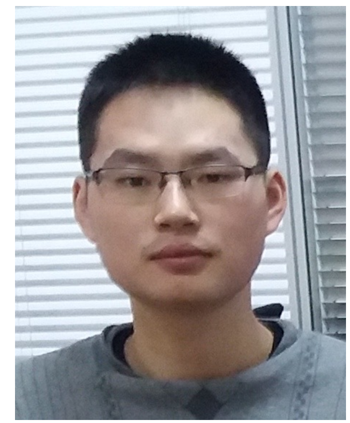

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Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

Batteries (Energy storage)

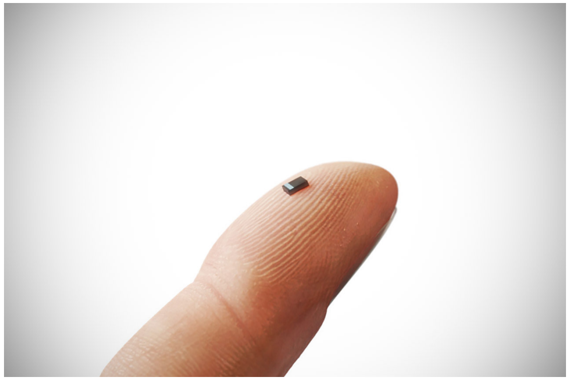
Batteries: energy storage & power supply to other machines

For remote (spatial and/or temporal) usage.

Ubiquitous device with a large variety in size and purpose.

For power plants

For daily-life gadgets



For microdevices

Quantum batteries (QBs)

Quantum batteries: Quantum systems used to store energy.

Ex.: Two-level sys. (TLS), harmonic osc. etc.

Reviews: Bhattacharjee & Dutta, EPJ B **94**, 239 (2021)
Quach, Cerullo & Virgili, Joule **7**, 2195 (2023)
Campaioli *et al*., ROMP **96**, 031001 (2024)

Unlike classical battery, quantum batteries are usually non-electric.

"Work reservoir"

Incentive:

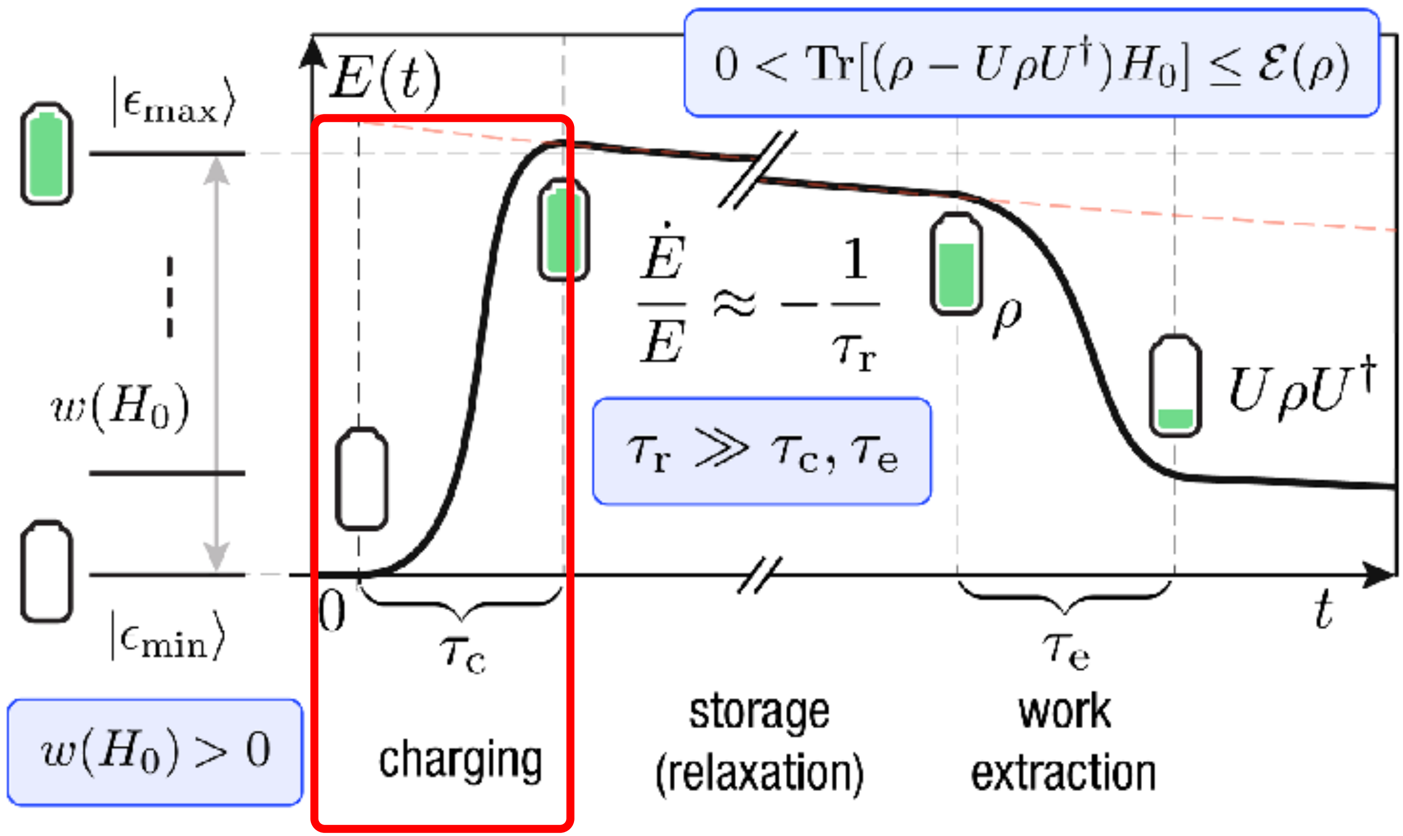
Improving the battery performance using quantum resources (quantum coherence, entanglement, etc.).

Alicki & Fannes, PRE **87**, 042123 (2013)
Hovhannisyan *et al*., PRL **111**, 240401 (2013)
Binder *et al*., NJP **17**, 075015 (2015)
Campaioli *et al*., PRL **118**, 150601 (2017)
Julia-Farre *et al*., PRR **2**, 023113 (2020)

Andolina, Polini *et al*.; Gyhm, Safranek & Rosa; Barra, …

An; Son, Talker, Thingna; …

Quantum batteries in operation



Campaioli *et al*., ROMP **96**, 031001 (2024)

1. Charging: Higher power, larger energy, smaller fluctuation
2. Storage: Smaller loss (leakage) for longer time
3. Discharging: Larger energy, higher/const. power, smaller fluct.

Decoherence

Quantum coherence: Nonzero off-diag. elements of $\hat{\rho}$ in $\widehat{H}$-basis.

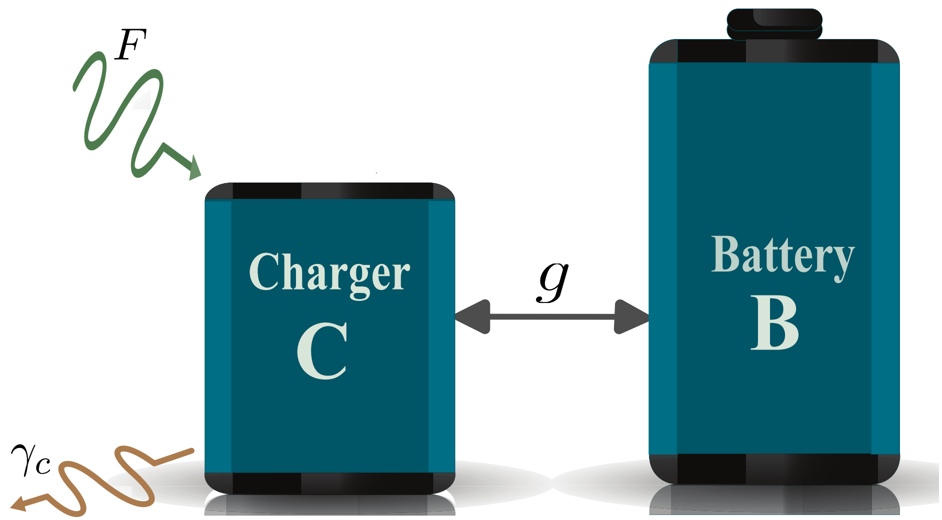
Originated from superposition of energy eigenst.

Such superposition is easy to destroyed by the interaction with environment.

Decoherence
- Dissipation: Involving energy exchange with environment.
- Dephasing: Without energy exchange with environment.

Open models of quantum batteries.

Farina *et al.*, PRB **99**, 035421 (2019)
Ghosh et al. PRA **104**, 032207 (2021)
Saha *et al.*, arXiv:2309.15634 (2023)
Shaghaghi *et al.*, Entropy **25**, 430 (2023)



Usually, decoherence is "thing to avoid".

Decoherence (dephasing) as a resource?

Shastri, Jiang, Xu, Venkatesh & GW, npj Quantum Inf. **11**, 9 (2025)

Dephasing: Decay of off-diagonal elements in $\hat{H}$-basis.

(coherence: nonzero off-diag. elements in $\hat{H}$-basis.)

Example: For a TLS

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|e\rangle + e^{i\theta}|g\rangle\right)$$

$\{|e\rangle, |g\rangle\}$: basis of $\hat{H}$

$$\hat{\rho} = |\psi\rangle\langle\psi| = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\theta} \\ e^{i\theta} & 1 \end{pmatrix}$$

: pure st.

Dephasing →

$$\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

: statistical mix. $\begin{cases} |e\rangle & 50\% \\ |g\rangle & 50\% \end{cases}$

$\langle \hat{H}^n \rangle \equiv \mathrm{Tr}\left[\hat{\rho}\, \hat{H}^n\right]$ unchanged.

Dephasing: Decay of off-diagonal elements in $\hat{H}$-basis.

(coherence: nonzero off-diag. elements in $\hat{H}$-basis.)

Corresponding jump op. $\hat{L} \propto \hat{H} \quad \left(\hat{L} = \hat{H}/\varepsilon\right)$.

$$\frac{d\hat{\rho}}{dt} = -i\left[\hat{H}, \hat{\rho}\right] + \frac{\tilde{\Gamma}}{2}\left(2\hat{H}\hat{\rho}\hat{H}^{\dagger} - \hat{H}^{\dagger}\hat{H}\hat{\rho} - \hat{\rho}\hat{H}^{\dagger}\hat{H}\right) \qquad \left(\tilde{\Gamma} \equiv \Gamma/\varepsilon^2\right)$$

$$\hat{\rho}(t) = \sum_{i,j} \rho_{ij}(t)\left|\varepsilon_i\right\rangle\left\langle\varepsilon_j\right|$$

$$\frac{d\rho_{ij}}{dt} = -i\left(\varepsilon_i - \varepsilon_j\right)\rho_{ij} - \frac{\tilde{\Gamma}}{2}\left(\varepsilon_i - \varepsilon_j\right)^2\rho_{ij}$$

$\Rightarrow$ off-diag. $(i \neq j)$: $\rho_{ij}(t) \propto \rho_{ij}(0)\, e^{-\frac{\tilde{\Gamma}}{2}\left(\varepsilon_i - \varepsilon_j\right)^2 t}$

diag. $(i = j)$: $\rho_{ii}(t) = \rho_{ii}(0)$

Master eq. describing dephasing:

$$\frac{d\hat{\rho}(t)}{dt} = \frac{\gamma}{2}\left(2\hat{L}\hat{\rho}(t)\hat{L} - \{\hat{L}^2, \hat{\rho}(t)\}\right) \quad \text{with} \quad [\hat{L}, \hat{H}] = 0$$

For a TLS with $\hat{H} = \omega\hat{\sigma}^+\hat{\sigma}^-$

$$\hat{L} = \hat{\sigma}^+\hat{\sigma}^-$$

Example:

1. Continuous weak energy measurement

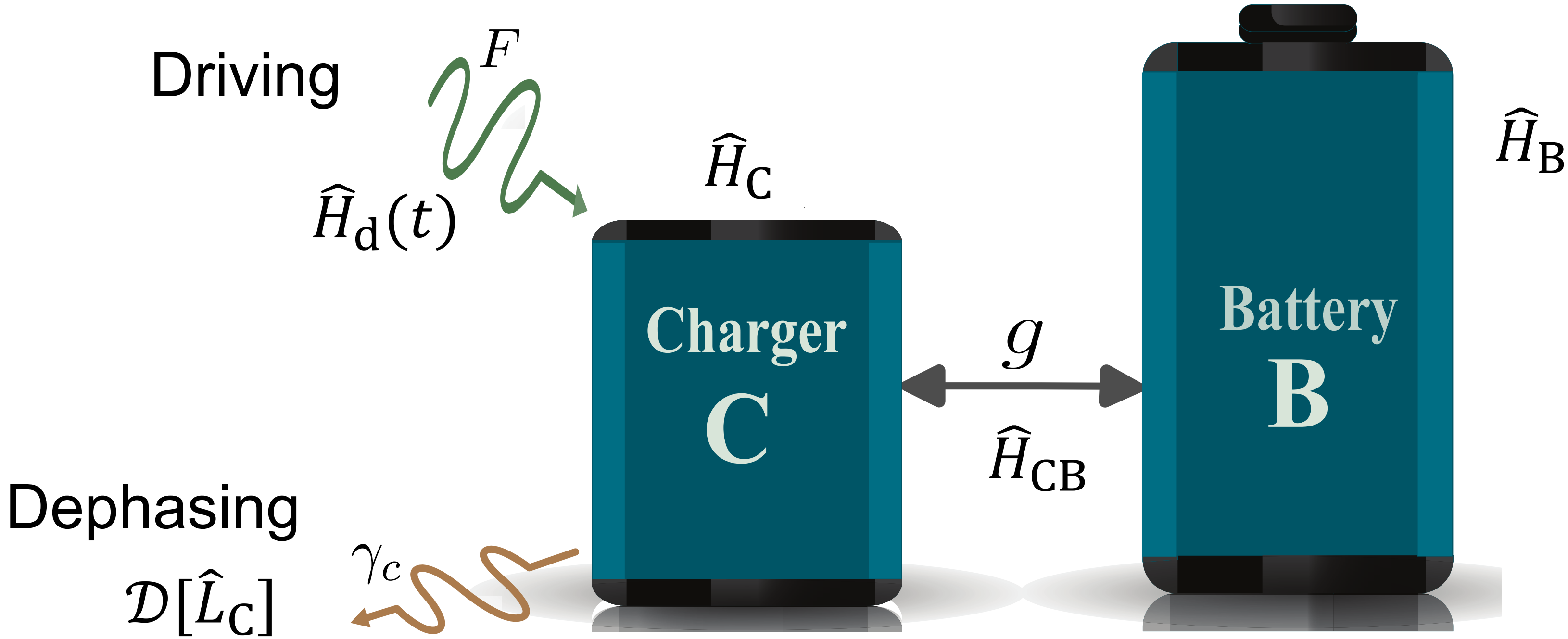
   Initial st. with coherence reduces to one of the energy eigenst.

2. Spins under a noisy magnetic field

   $$\hat{H}'(t) = B(t)\,\hat{\sigma}_z \text{ with } B(t) = \sqrt{\gamma/2}\,\xi(t)$$

   Relative phase in the initial superposition is randomized stochastically.

   Ensemble avr. of $\hat{\rho} = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\theta} \\ e^{i\theta} & 1 \end{pmatrix}$ with random θ.



$$[\hat{L}_{\mathrm{C}}, \hat{H}_{\mathrm{C}}] = 0$$

See also:

Farina *et al*. PRB **99**, 035421 (2019)

Saha *et al*. arXiv:2309.15634 (2023)

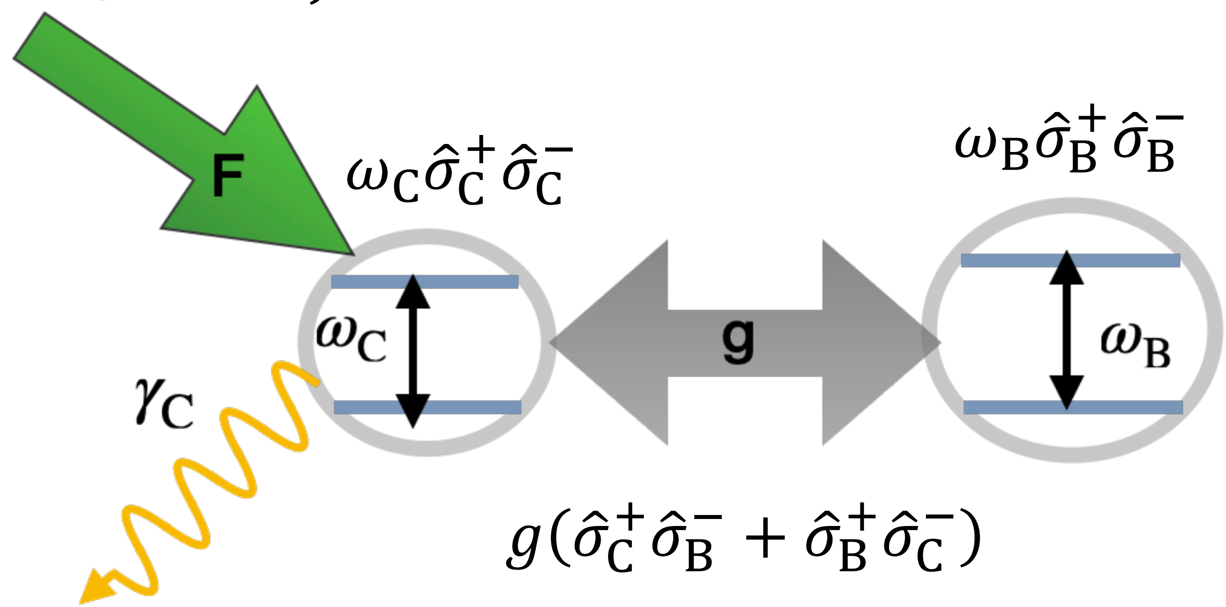
$$\hat{H}(t) = \hat{H}_{\mathrm{C}} + \hat{H}_{\mathrm{d}}(t) + \hat{H}_{\mathrm{B}} + \hat{H}_{\mathrm{CB}}$$

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}, \hat{\rho}(t)] + \frac{\gamma_C}{2}\left(2\hat{L}_{\mathrm{C}}\hat{\rho}(t)\hat{L}_{\mathrm{C}} - \{\hat{L}_{\mathrm{C}}^2, \hat{\rho}(t)\}\right)$$

$$\equiv -i[\hat{H}, \hat{\rho}(t)] + \mathcal{D}[\hat{L}_{\mathrm{C}}]\,\hat{\rho}(t)$$

Two-level systems (TLSs)

$$F\left(\hat{\sigma}_{\mathrm{C}}^{-}e^{i\omega_{\mathrm{d}}t}+\hat{\sigma}_{\mathrm{C}}^{+}e^{-i\omega_{\mathrm{d}}t}\right)$$



$$\mathcal{D}[\hat{L}_{\mathrm{C}}]$$

$$g\left(\hat{\sigma}_{\mathrm{C}}^{+}\hat{\sigma}_{\mathrm{B}}^{-}+\hat{\sigma}_{\mathrm{B}}^{+}\hat{\sigma}_{\mathrm{C}}^{-}\right)$$

$$\hat{L}_{\mathrm{C}}=\hat{\sigma}_{\mathrm{C}}^{+}\hat{\sigma}_{\mathrm{C}}^{-}=(\mathbb{I}+\hat{\sigma}_{\mathrm{C}}^{z})/2$$

initial st.: $\hat{\rho}(0)=|g\rangle\langle g|_{\mathrm{B}}\otimes|g\rangle\langle g|_{\mathrm{C}}$

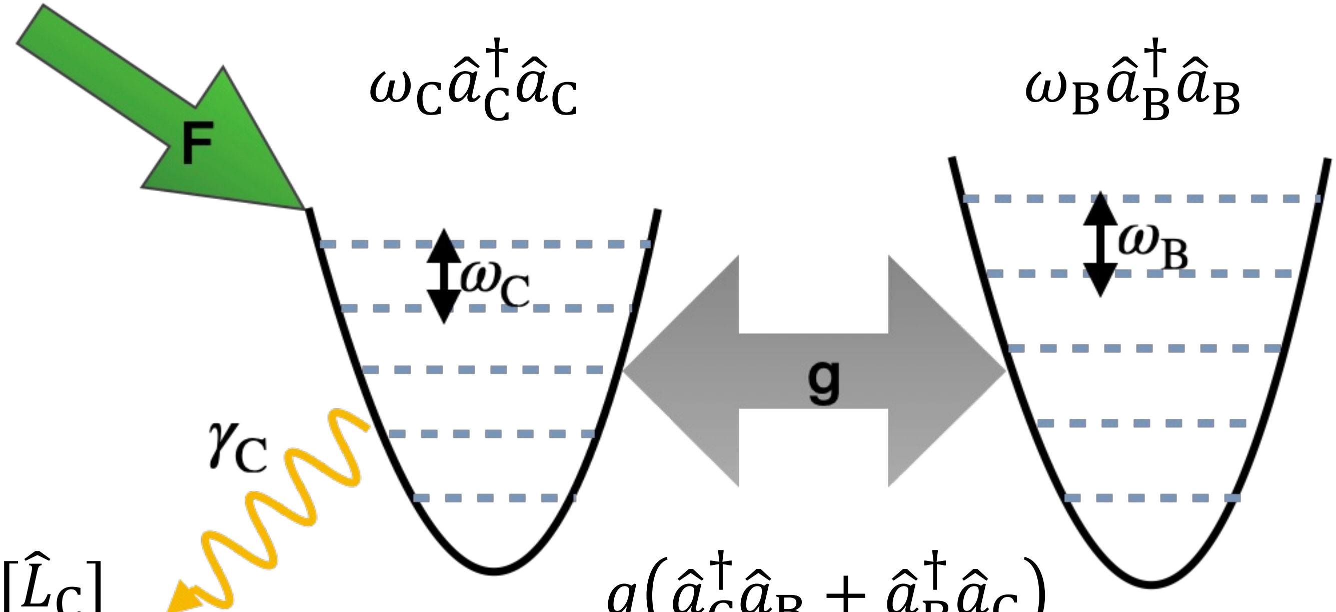
$$\hat{H}=\omega_{\mathrm{C}}\hat{\sigma}_{\mathrm{C}}^{+}\hat{\sigma}_{\mathrm{C}}^{-}+\omega_{\mathrm{B}}\hat{\sigma}_{\mathrm{B}}^{+}\hat{\sigma}_{\mathrm{B}}^{-}+g\left(\hat{\sigma}_{\mathrm{C}}^{+}\hat{\sigma}_{\mathrm{B}}^{-}+\hat{\sigma}_{\mathrm{B}}^{+}\hat{\sigma}_{\mathrm{C}}^{-}\right)$$
$$+F\left(\hat{\sigma}_{\mathrm{C}}^{-}e^{i\omega_{\mathrm{d}}t}+\hat{\sigma}_{\mathrm{C}}^{+}e^{-i\omega_{\mathrm{d}}t}\right)$$

$$\omega_{\mathrm{C}}=\omega_{\mathrm{B}}\implies\left[\hat{H}_{\mathrm{C}}+\hat{H}_{\mathrm{B}},\hat{H}_{\mathrm{CB}}\right]=0$$

Harmonic oscillators (HOs)

$F\left(\hat{a}_{\mathrm{C}} e^{i\omega_{\mathrm{d}} t} + \hat{a}_{\mathrm{C}}^{\dagger} e^{-i\omega_{\mathrm{d}} t}\right)$

$\omega_{\mathrm{C}} \hat{a}_{\mathrm{C}}^{\dagger} \hat{a}_{\mathrm{C}}$ $\omega_{\mathrm{B}} \hat{a}_{\mathrm{B}}^{\dagger} \hat{a}_{\mathrm{B}}$



$\mathcal{D}[\hat{L}_{\mathrm{C}}]$

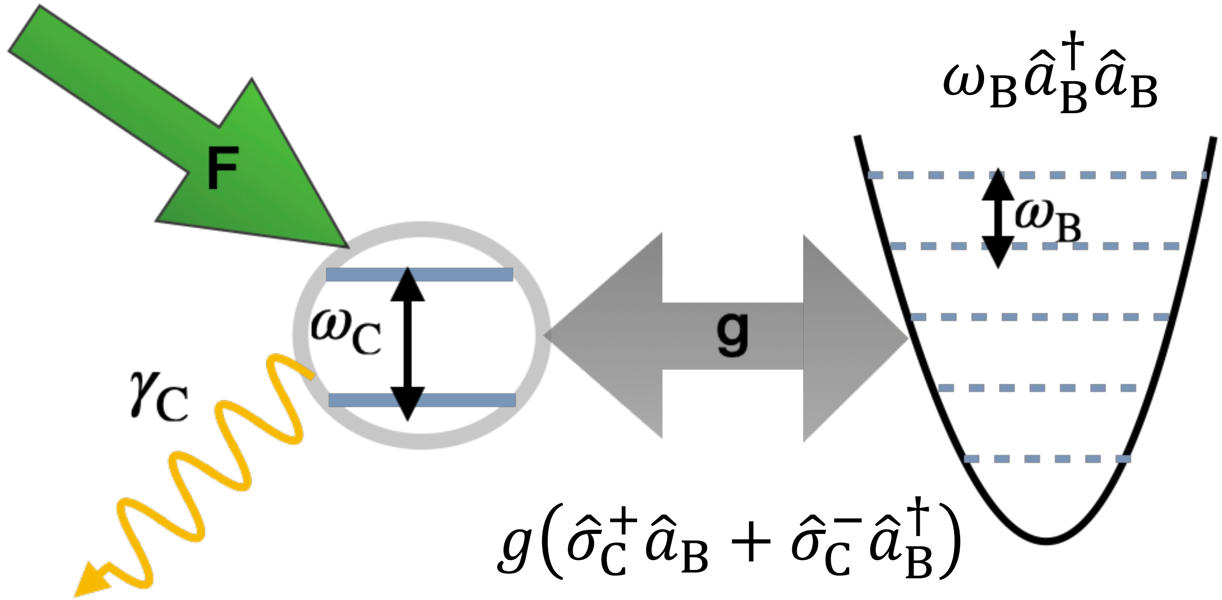
$g\left(\hat{a}_{\mathrm{C}}^{\dagger} \hat{a}_{\mathrm{B}} + \hat{a}_{\mathrm{B}}^{\dagger} \hat{a}_{\mathrm{C}}\right)$

$\hat{L}_{\mathrm{C}} = \hat{a}_{\mathrm{C}}^{\dagger} \hat{a}_{\mathrm{C}}$

$$\hat{H} = \omega_{\mathrm{C}} \hat{a}_{\mathrm{C}}^{\dagger} \hat{a}_{\mathrm{C}} + \omega_{\mathrm{B}} \hat{a}_{\mathrm{B}}^{\dagger} \hat{a}_{\mathrm{B}} + g\left(\hat{a}_{\mathrm{C}}^{\dagger} \hat{a}_{\mathrm{B}} + \hat{a}_{\mathrm{B}}^{\dagger} \hat{a}_{\mathrm{C}}\right) + F\left(\hat{a}_{\mathrm{C}} e^{i\omega_{\mathrm{d}} t} + \hat{a}_{\mathrm{C}}^{\dagger} e^{-i\omega_{\mathrm{d}} t}\right)$$

$$\omega_{\mathrm{C}} = \omega_{\mathrm{B}} \implies \left[\hat{H}_{\mathrm{C}} + \hat{H}_{\mathrm{B}}, \hat{H}_{\mathrm{CB}}\right] = 0$$

$$F\left(\hat{\sigma}_{\mathrm{C}}^{-} e^{i\omega_{\mathrm{d}}t} + \hat{\sigma}_{\mathrm{C}}^{+} e^{-i\omega_{\mathrm{d}}t}\right)$$

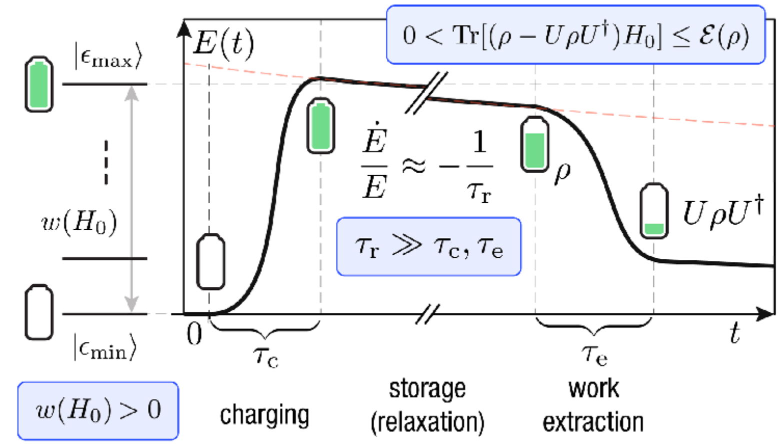


$$\mathcal{D}[\hat{L}_{\mathrm{C}}]$$

$$\hat{L}_{\mathrm{C}} = \hat{\sigma}_{\mathrm{C}}^{+} \hat{\sigma}_{\mathrm{C}}^{-} = (\mathbb{I} + \hat{\sigma}_{\mathrm{C}}^{z})/2$$

$$\begin{aligned}\hat{H} = {} & \omega_{\mathrm{C}} \hat{\sigma}_{\mathrm{C}}^{+} \hat{\sigma}_{\mathrm{C}}^{-} + \omega_{\mathrm{B}} \hat{a}_{\mathrm{B}}^{\dagger} \hat{a}_{\mathrm{B}} + g\left(\hat{\sigma}_{\mathrm{C}}^{+} \hat{a}_{\mathrm{B}} + \hat{\sigma}_{\mathrm{C}}^{-} \hat{a}_{\mathrm{B}}^{\dagger}\right) \\ & + F\left(\hat{\sigma}_{\mathrm{C}}^{-} e^{i\omega_{\mathrm{d}}t} + \hat{\sigma}_{\mathrm{C}}^{+} e^{-i\omega_{\mathrm{d}}t}\right)\end{aligned}$$

$$\omega_{\mathrm{C}} = \omega_{\mathrm{B}} \implies \left[\hat{H}_{\mathrm{C}} + \hat{H}_{\mathrm{B}}, \hat{H}_{\mathrm{CB}}\right] = 0$$



[Campaioli *et al*., ROMP **96**, 031001 (2024)]

Energy of the battery E_B

$$E_{\mathrm{B}} = \mathrm{Tr}_{\mathrm{B}}[\hat{\rho}_{\mathrm{B}}\hat{H}_{\mathrm{B}}]$$

Ergotropy of the battery $\mathcal{E}_B$

$$\mathcal{E}_{\mathrm{B}} = E_{\mathrm{B}} - \min_{\hat{U}_{\mathrm{B}} \in \mathrm{SU}(d)} \mathrm{Tr}_B[\hat{U}_{\mathrm{B}}\hat{\rho}_{\mathrm{B}}\hat{U}_{\mathrm{B}}^\dagger \hat{H}_{\mathrm{B}}]$$

[Allahverdyan *et al*. EPL **67**, 565 (2004)]

Maximum extractable energy by all possible unitary.

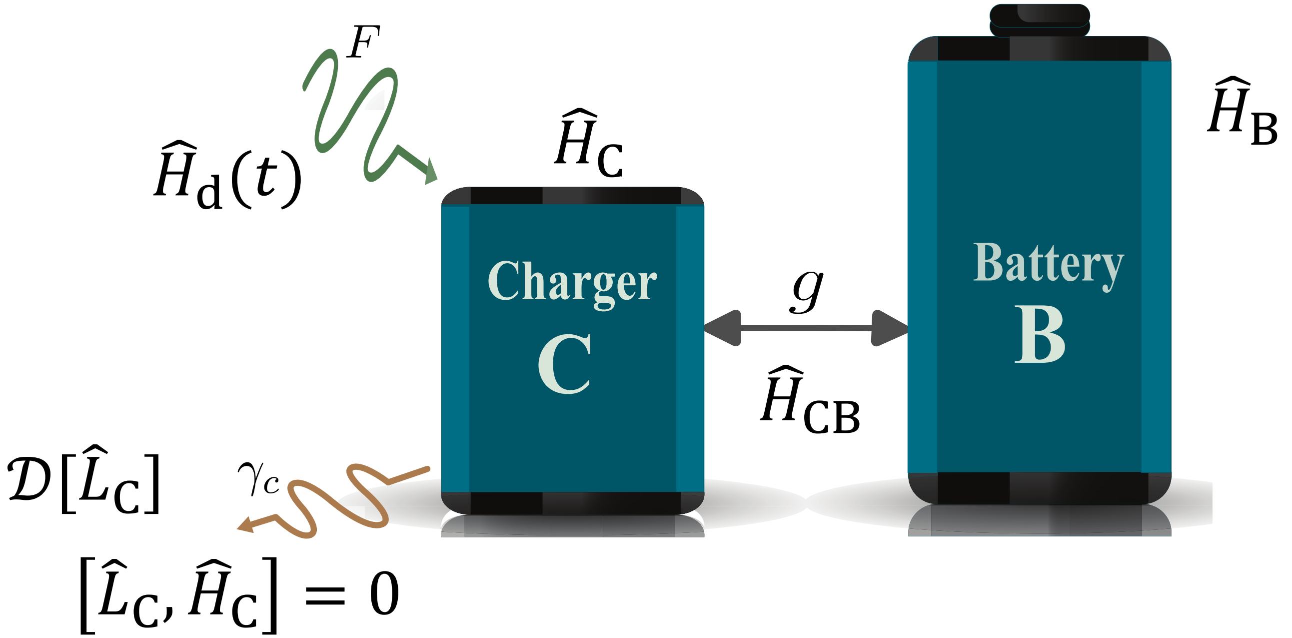
“quality” of the energy

Charging time τ

$$\left|\frac{E_{\mathrm{B}}(\tau) - E_{\mathrm{B}}(\infty)}{E_{\mathrm{B}}(0) - E_{\mathrm{B}}(\infty)}\right| = e^{-n}$$

(standard exp decay: $n = 1$)

Setup of the numerical simulations



Initial state: $\hat{\rho}(0) = |g\rangle\langle g|_{\rm B} \otimes |g\rangle\langle g|_{\rm C}$

Both battery & charger are “empty”.

Evolution: $$\frac{d\hat{\rho}(t)}{dt} = -i\left[\hat{H}(t), \hat{\rho}(t)\right] + \mathcal{D}\left[\hat{L}_{\rm C}\right]\hat{\rho}(t)$$

Battery will be charged up by the charger under driving $\hat{H}_{\rm d}(t)$.

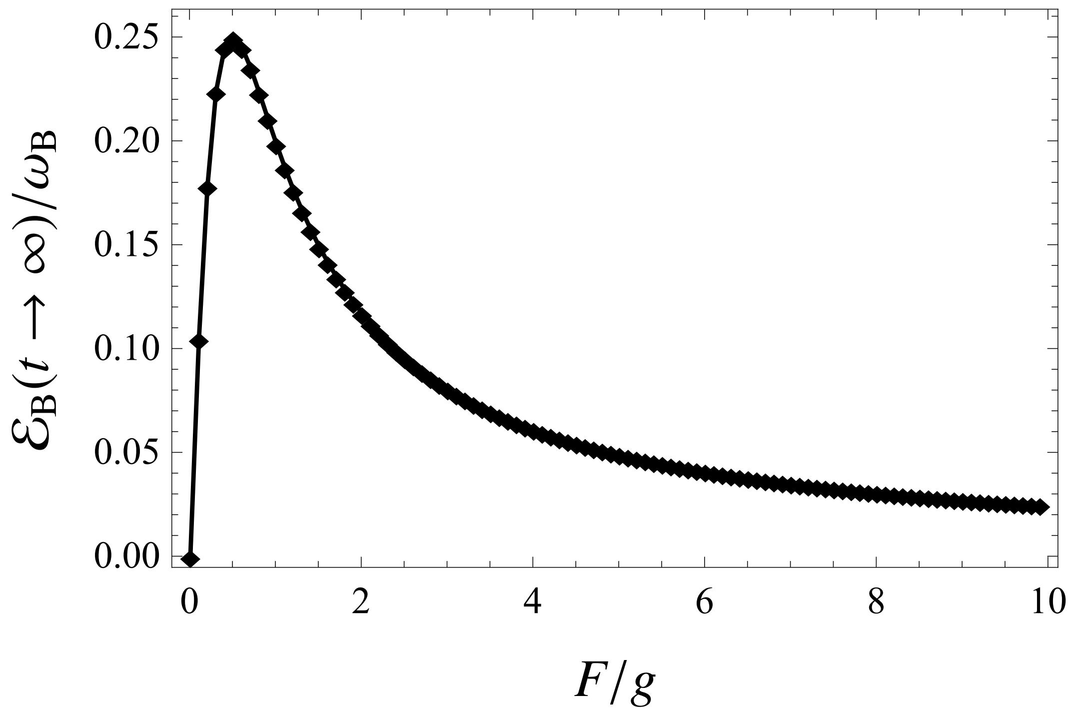
Results

Shastri, Jiang, Xu, Venkatesh & GW, npj Quantum Inf. **11**, 9 (2025)

Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

TLSs at resonance: Steady state

At resonance: $\omega_{\mathrm{B}} = \omega_{\mathrm{C}} = \omega_{\mathrm{d}}$



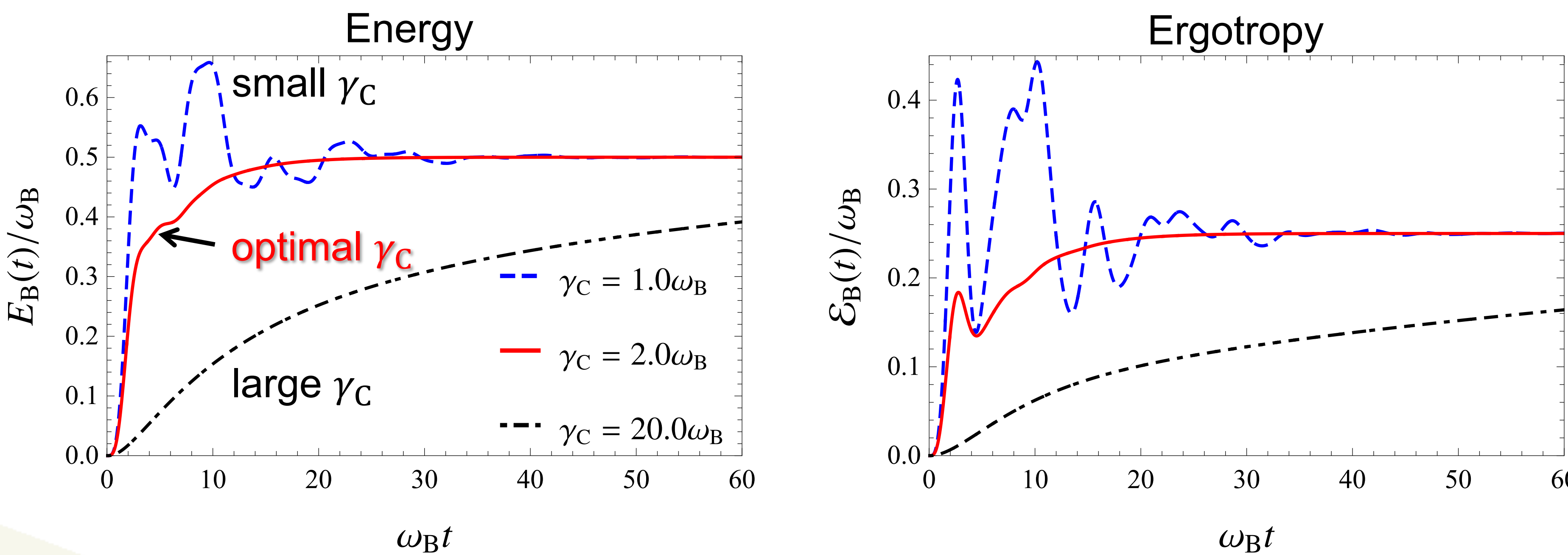
Steady st. E_{B} & $\mathcal{E}_{\mathrm{B}}$ are indep. of γ_{C}.

$$E_{\mathrm{B}}(\infty)/\omega_{\mathrm{B}} = \frac{1}{2} \qquad \mathcal{E}_{\mathrm{B}}(\infty)/\omega_{\mathrm{B}} = \frac{F/g}{1 + (F/g)^2}$$

$\mathcal{E}_{\mathrm{B}}(\infty)$ takes max $(0.25\omega_{\mathrm{B}})$ when $F/g = 0.5$.

TLSs at resonance: Charging dynamics

For intermediate (optimal) driving: $F/g = 0.5$ $(g = 1.0\omega_{\mathrm{B}})$



Moderate dephasing leads to fast stabilization to steady state!

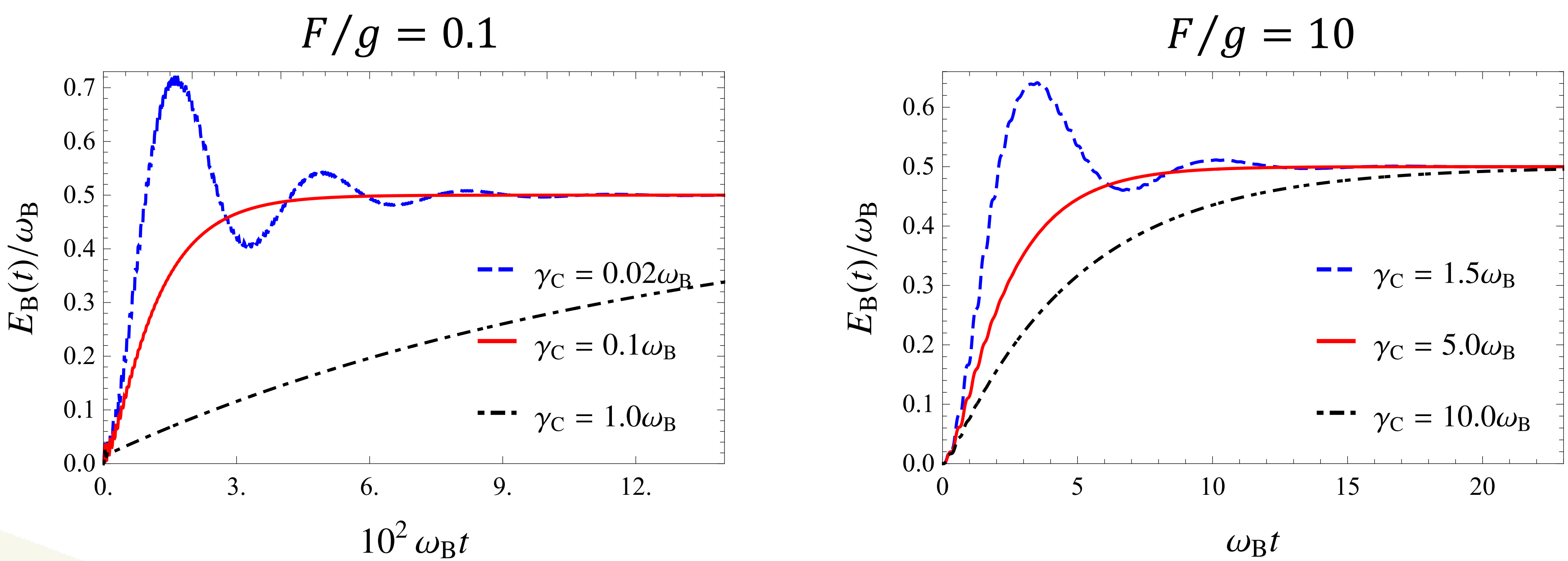
Transient maxima are **impractical**:

1. Fine temporal control is required.
2. Disturbance by sudden parameter change for decoupling is inevitable.

TLSs at resonance: Charging dynamics

Weak & strong driving

$(g = 1.0\omega_B)$



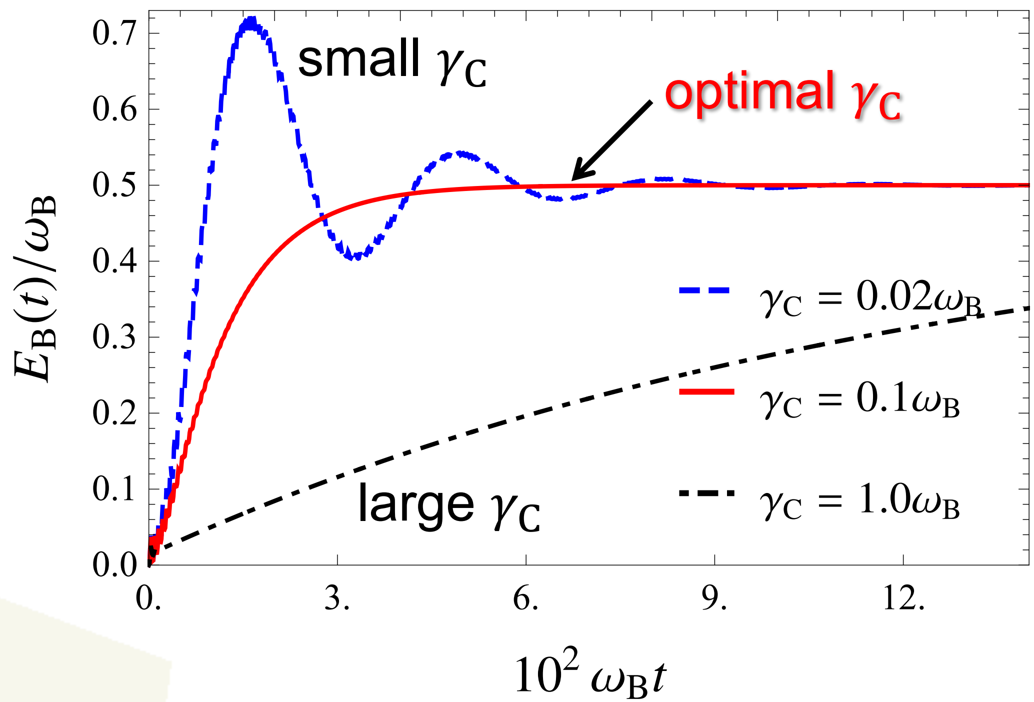
Moderate dephasing leads to fast stabilization to steady state!

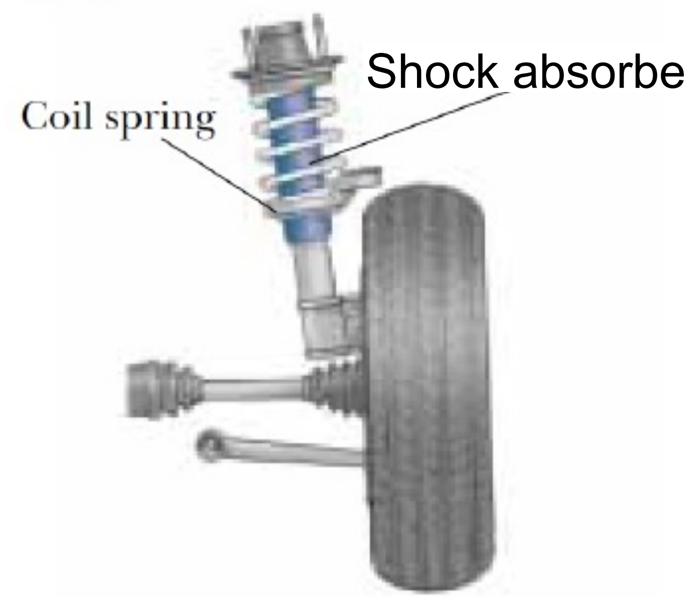
Transient maxima are **impractical**:

1. Fine temporal control is required.
2. Disturbance by sudden parameter change for decoupling is inevitable.

Coherent osc. vs quantum-Zeno

Moderate dephasing leads to fast charging!





- weak dephasing $(\gamma_C \lesssim F, g)$: underdamped coherent oscillation
- strong dephasing $(\gamma_C \gg F, g)$: quantum Zeno effect ➡ slow energy flow

➡ Optimal γ_C with fast relaxation.

Dephasing works as something similar to a shock absorber of the car.

Universal competition under dephasing

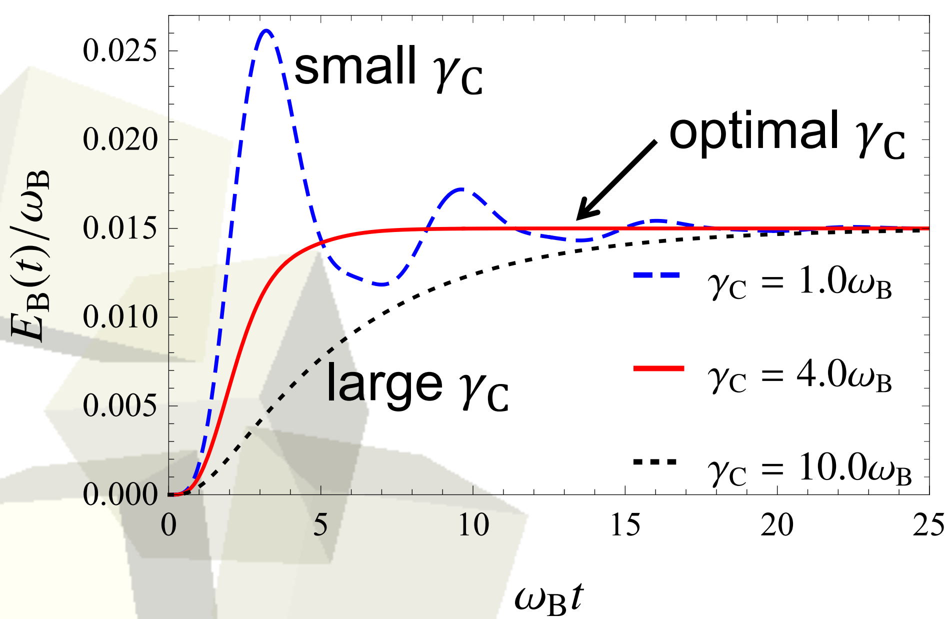
Competition:

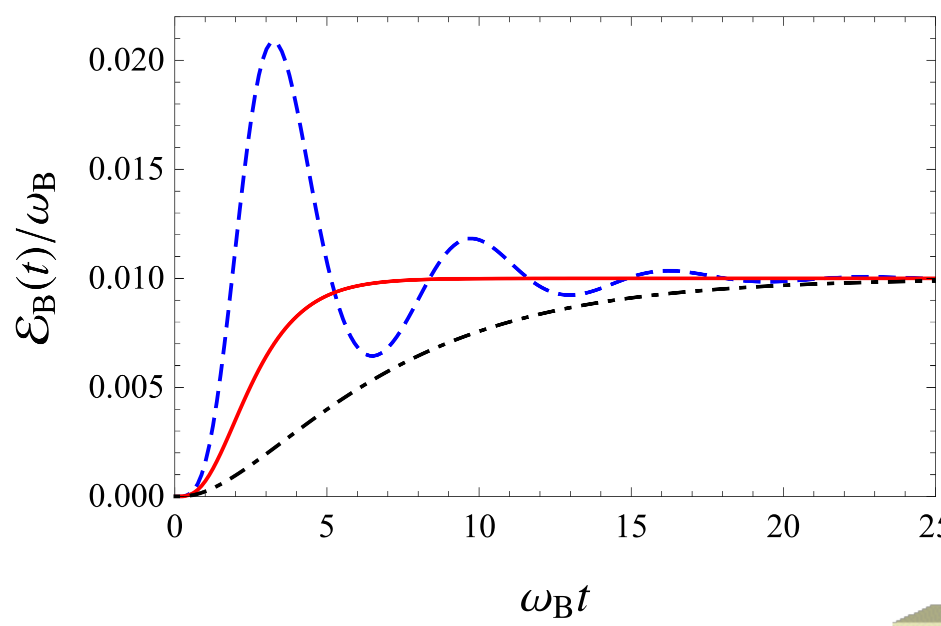
Coherent oscillation vs Quantum Zeno freezing

Universal for systems under dephasing.

HOs at resonance

$(F/g = 0.1, g = 1.0\omega_B)$





Universal competition under dephasing

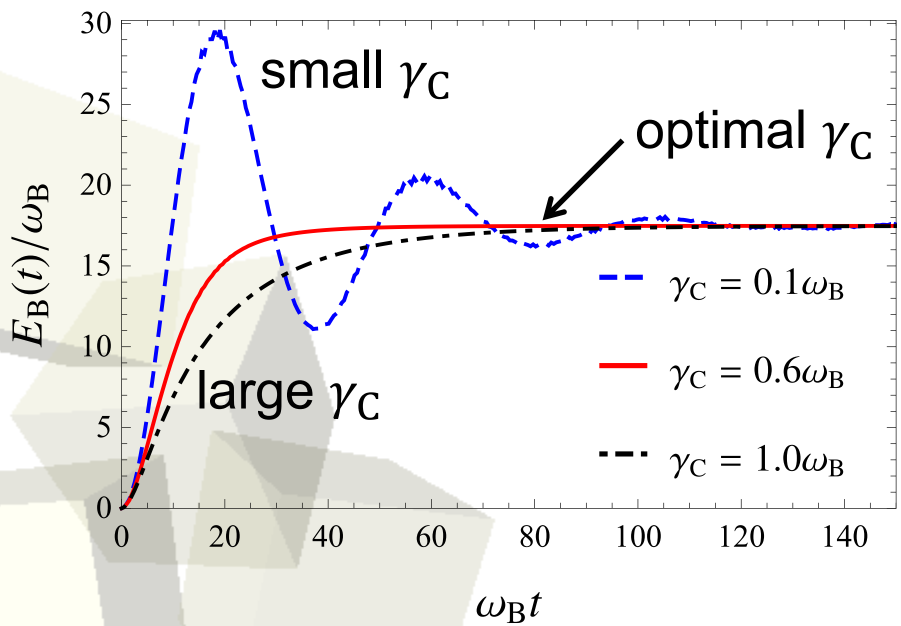
Competition:

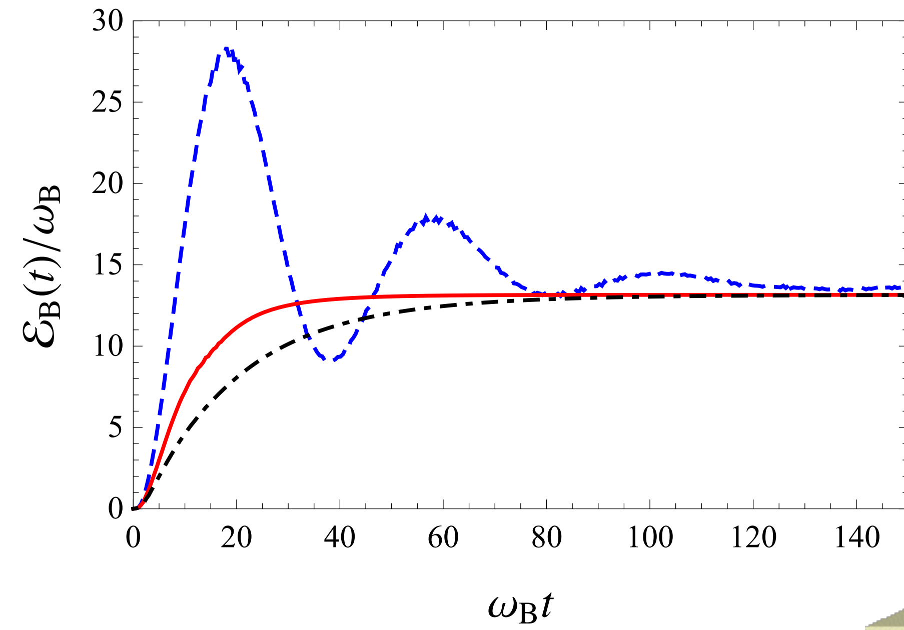
Coherent oscillation vs Quantum Zeno freezing

Universal for systems under dephasing.

TLS-HO at resonance

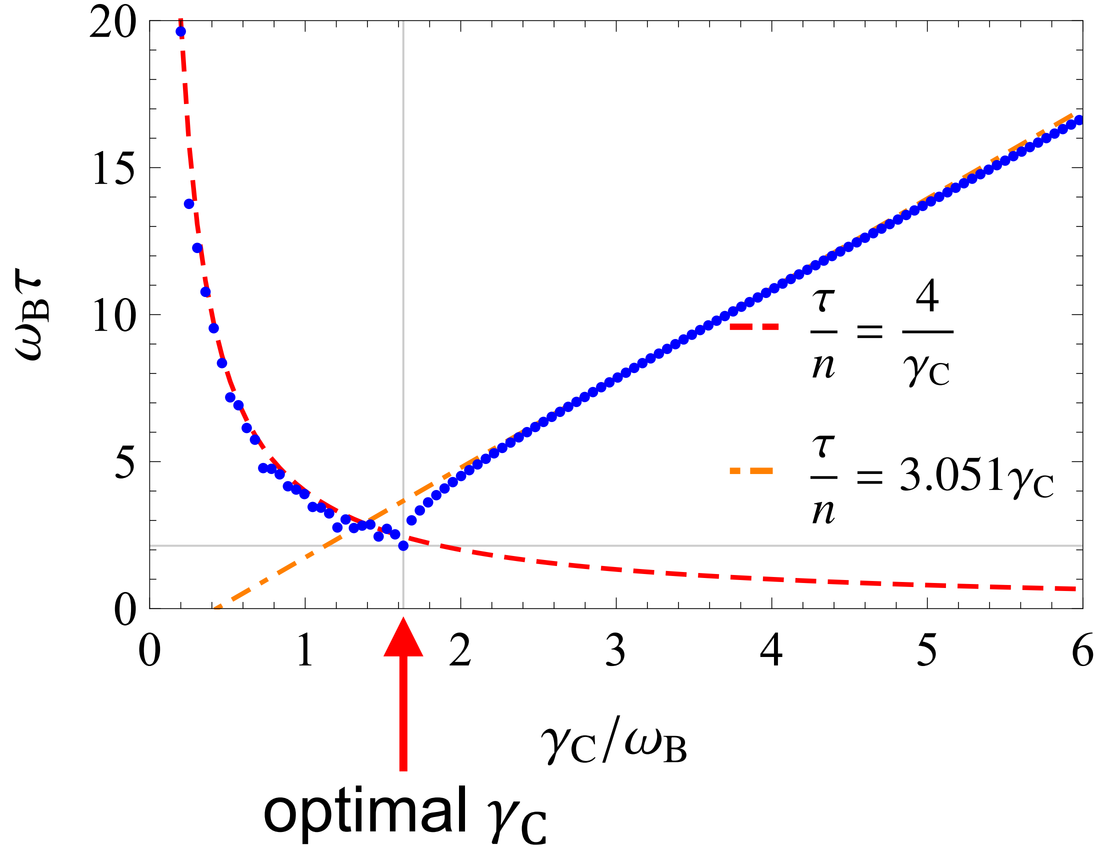
$(F/g = 3.0, g = 1.0\omega_B)$





Charging time (TLSs at resonance)

For intermediate driving: $F/g = 0.5$



weak dephasing ($\gamma_C \ll g, F$) : $\tau \sim 1/\gamma_C$

strong dephasing ($\gamma_C \gg g, F$) : $\tau \sim \gamma_C/[E^2]$

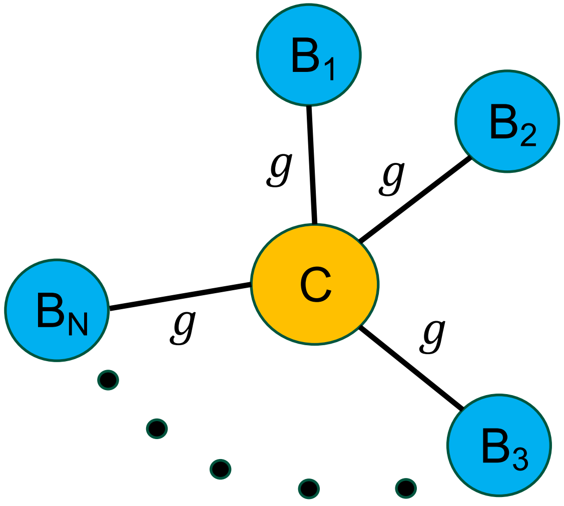
Minimum τ when

$$\frac{1}{\gamma_C} \sim \frac{\gamma_C}{[E^2]}$$

Batteries in star configuration

To enhance steady-state ergotropy:

→ Multiple batteries in star configuration



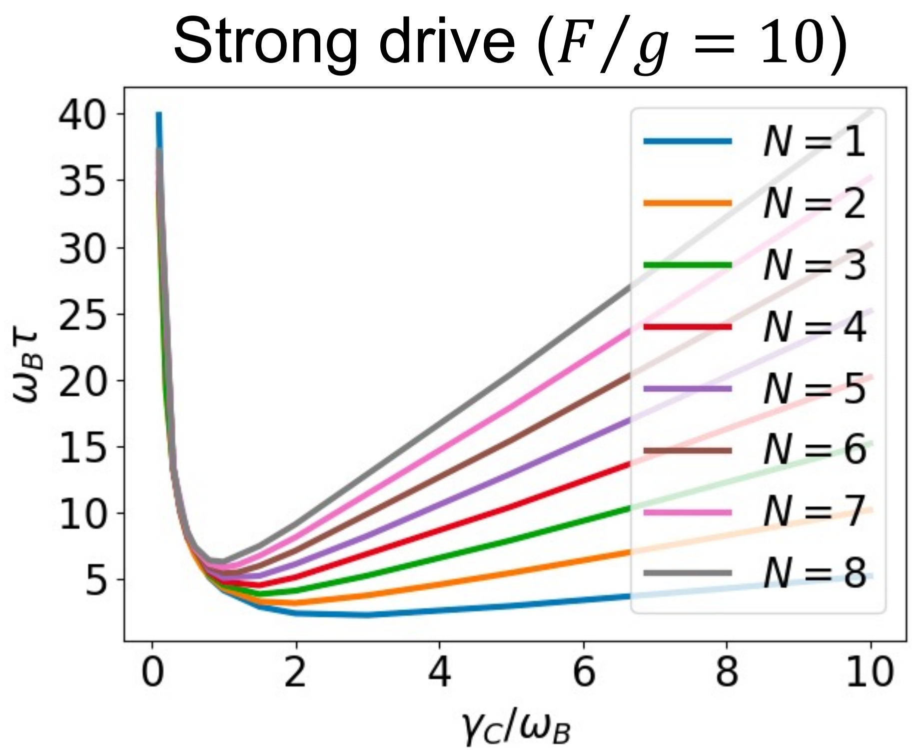
$$\hat{H}_{\mathrm{C}} = \omega_{\mathrm{C}} \hat{\sigma}_{\mathrm{C}}^{+} \hat{\sigma}_{\mathrm{C}}^{-}$$

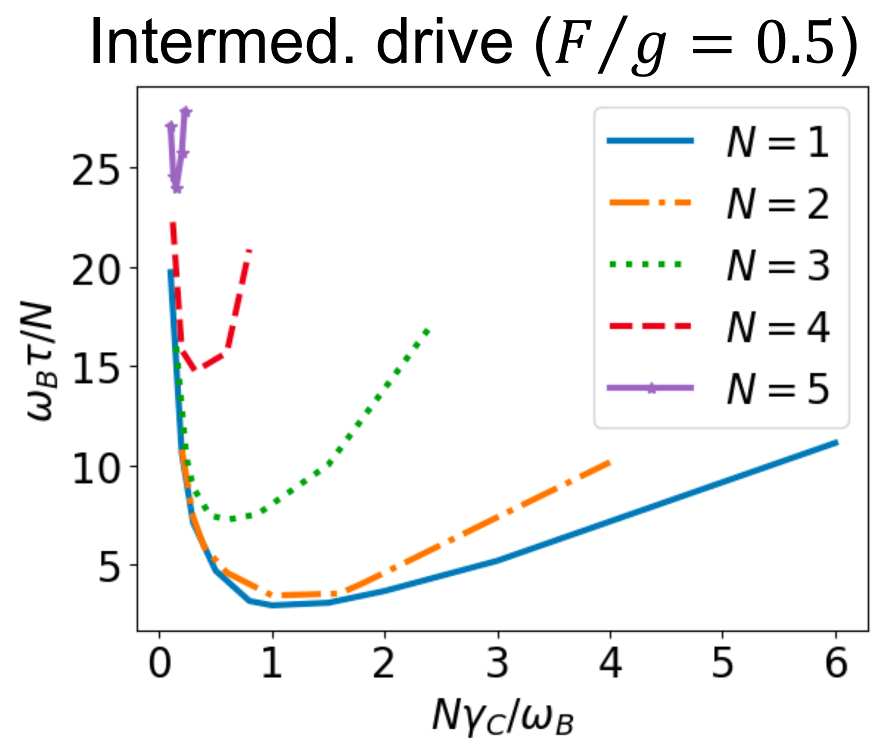
$$\hat{H}_{\mathrm{B}} = \sum_{i=1}^{N} \hat{H}_{\mathrm{B}_i} \quad \text{with} \quad \hat{H}_{\mathrm{B}_i} = \omega_{\mathrm{B}} \hat{\sigma}_{\mathrm{B}_i}^{+} \hat{\sigma}_{\mathrm{B}_i}^{-}$$

$$\hat{H}_{\mathrm{CB}} = g\left(\hat{\sigma}_{\mathrm{C}}^{-} \sum_{i=1}^{N} \hat{\sigma}_{\mathrm{B}_i}^{+} + \hat{\sigma}_{\mathrm{C}}^{+} \sum_{i=1}^{N} \hat{\sigma}_{\mathrm{B}_i}^{-}\right)$$: "Star" config.

$$\hat{H}_{\mathrm{d}} = F\left(\hat{\sigma}_{\mathrm{C}}^{-} e^{i\omega_{\mathrm{d}} t} + \hat{\sigma}_{\mathrm{C}}^{+} e^{-i\omega_{\mathrm{d}} t}\right)$$

Charging time τ





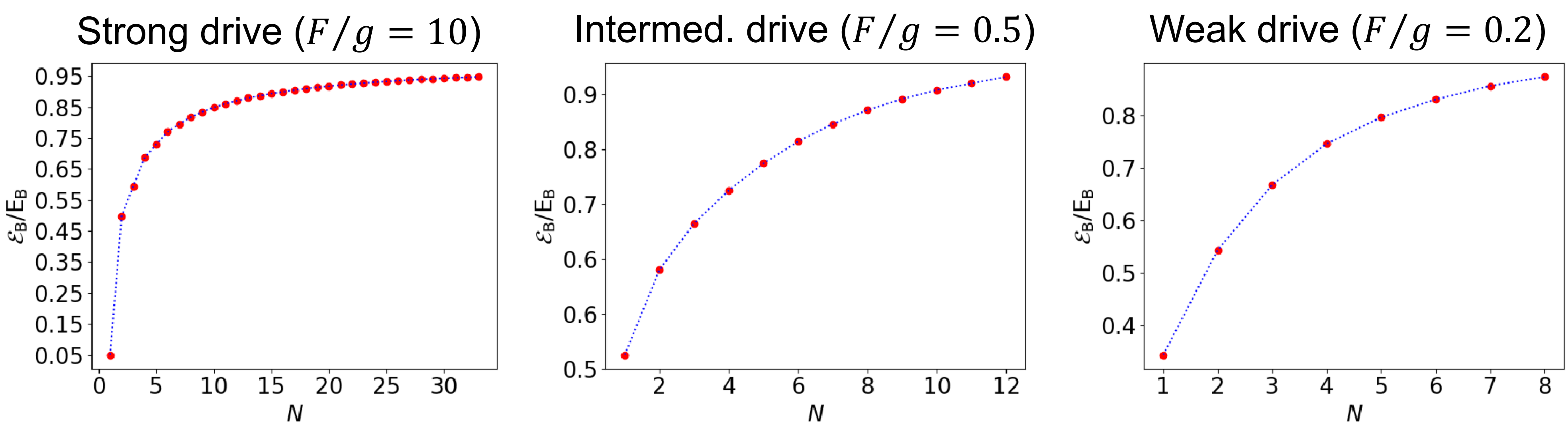
In all the cases, there is an optimum γ_C for fast charging.

Strong driving regime: minimum $\tau \sim N$

Intermediate driving regime: minimum $\tau \sim N^{3.2}$

Asympt. freedom: "Locked" energy $E_B - \mathcal{E}_B$ vanishes as $N \to \infty$.
(passive energy)

Andolina *et al.*, PRL **122**, 047702 (2019)



Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

$$\frac{\mathcal{E}_B}{E_B} \sim 1 - \frac{a}{N} \quad \text{as } N \to \infty \quad \text{with a const. } a = O(1)$$

Asymptotic freedom

$$\frac{\mathcal{E}_B}{E_B} \sim 1 - \frac{a}{N} \quad \text{as } N \to \infty \quad \text{with a const. } a = O(1)$$

Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

Rough proof: $E_B - \mathcal{E}_B$ is upper & lower bounded by a const.

$$\Rightarrow \quad 1 - \frac{\mathcal{E}_B}{E_B} \sim 1/N$$

Upper bound ⇐ Dimensional reduction because of the permutation sym.

Lower bound ⇐ Battery state is non-pure even in the $N \to \infty$ limit.

More generally, one can prove:

For any d-level systems, $\mathcal{E}_B / E_B$ approaches unity as

$$\frac{\mathcal{E}_B}{E_B} \sim 1 - \frac{a}{N}$$

or faster.

GW, Venkatesh, Chen (in preparation)

Physical mechanism of asymptotic freedom

Asymptotic steady state is mixed st.

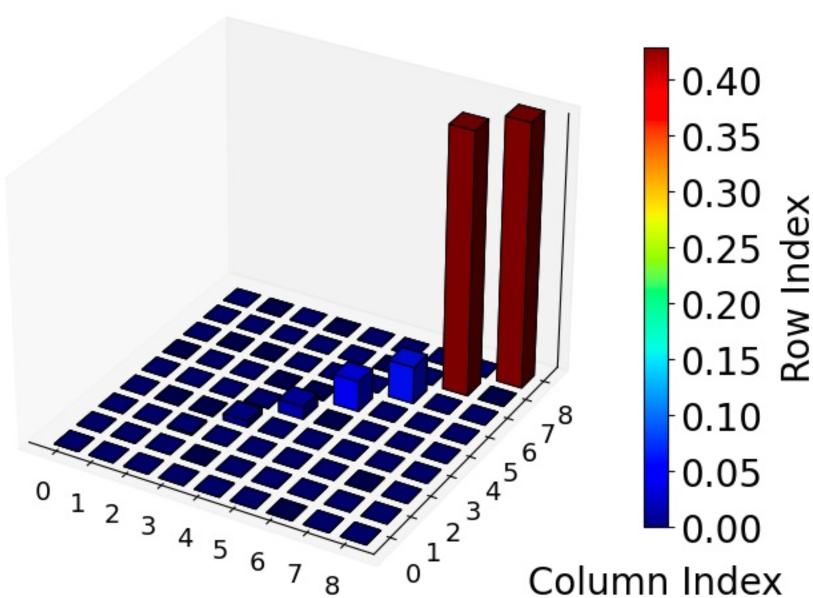
Ground state is unique: $|0\rangle^{\otimes N}$

Q: Why asymptotic free even though the state is mixed?

A: Emergent approximate ground st. degeneracy in $N \to \infty$.

Relevant quantity for discussion of $\mathcal{E}_B/E_B$: Δ_g/E_B

Δ_g: energy gap btwn. ground & 1st excited st.



$\Delta_g \sim \omega_B$, $E_B \sim N\omega_B$ ➡ $\Delta_g/E_B \sim 1/N$

Gap becomes negligible as $N \to \infty$.

Summary & conclusion

Decoherence can be a resource.

- Moderate dephasing helps fast charging!

- General behavior: Tradeoff between
  Coherent oscillations vs Zeno freezing

- Multiple batteries in star configuration:
  Asymptotic free due to approx. ground st. degeneracy.

Shastri, Jiang, Xu, Venkatesh & GW, npj Quantum Inf. **11**, 9 (2025)

Purkait, Venkatesh & GW, arXiv:2508.13497 (2025)

Charging time & optimal dephasing

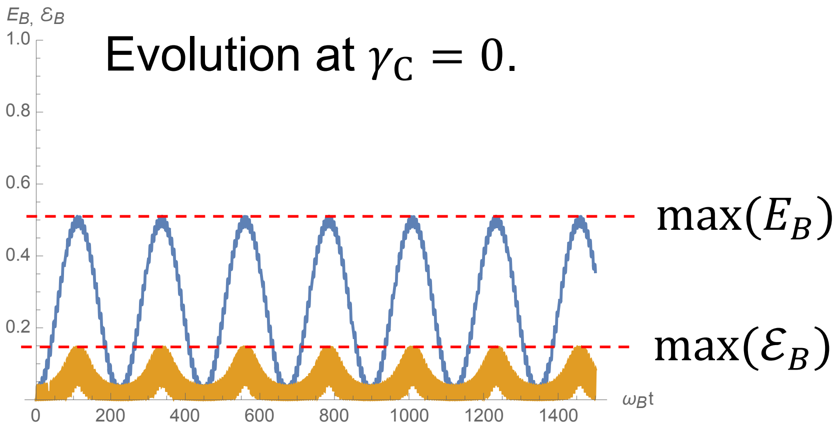
	$F/g \gg 1$	$F/g \ll 1$
$\gamma_C \ll g$	$\tau \sim \frac{4}{\gamma_C}$	$\tau \sim \frac{4}{\gamma_C}$
$\gamma_C \gg g, F$	$\tau \sim \frac{1}{2g^2}\gamma_C$	$\tau \sim \frac{g^2}{F^4}\gamma_C$
Optimal dephasing	$\gamma_C^* \approx 2\sqrt{2}\, g$	$\gamma_C^* \approx \frac{8F^2}{\sqrt{2}g}$

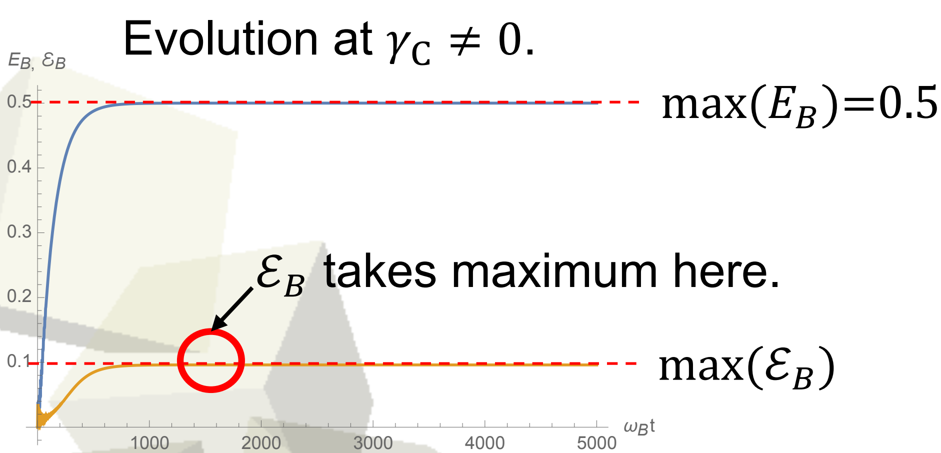
Charging strategy

1. Set $F/g = 0.5$, maximizing the steady state $\mathcal{E}_B$.

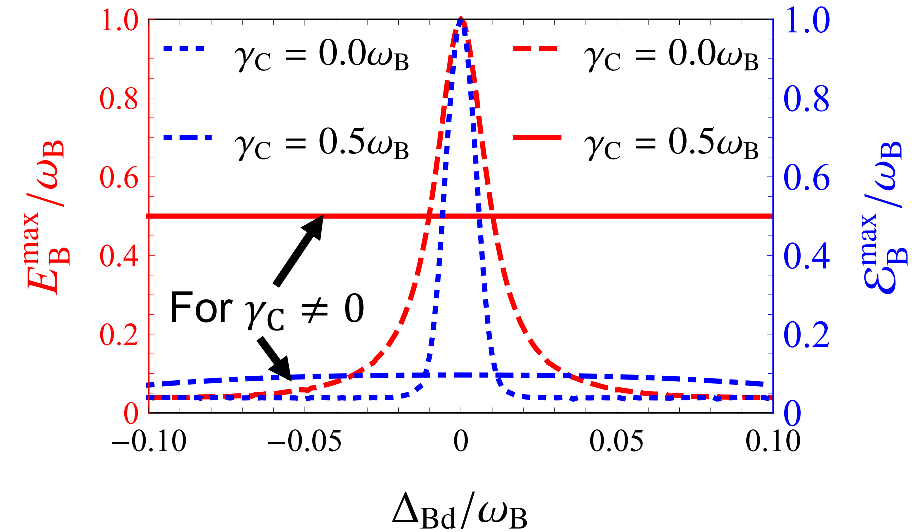
2. Since $\gamma_C^* \approx \frac{8}{\sqrt{2}}\frac{F}{g}F = 2\sqrt{2}F$ and $\tau \sim \frac{4}{\gamma_C} \approx \sqrt{2}/F$, make F, g as large as possible.

Robustness against detuning

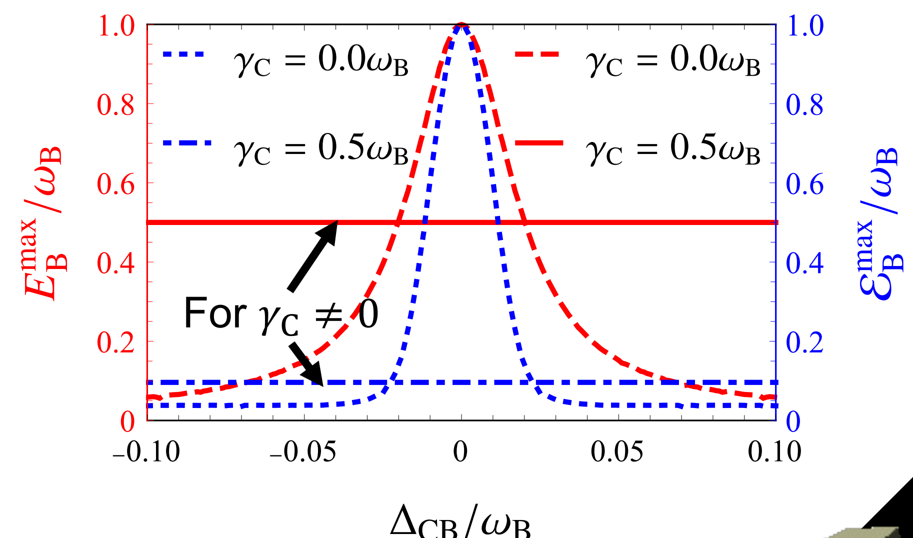




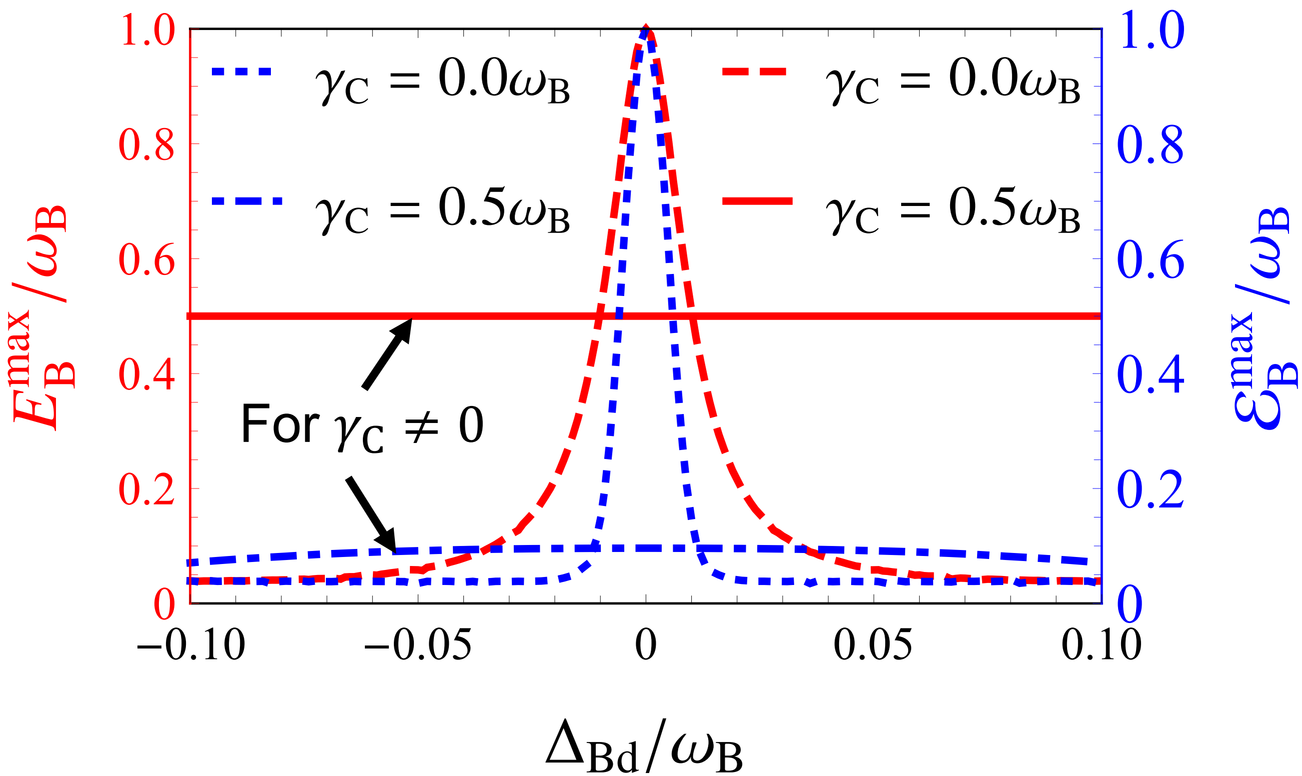
Detuning of ω_d: $\Delta_{Bd} \equiv \omega_B - \omega_d$



Detuning of ω_C: $\Delta_{CB} \equiv \omega_C - \omega_B$



Robustness against detuning



- Nonzero γ_C leads to robust charging performance against the detuning!
- Set $F/g = 0.5$ and $\gamma_C \approx 2\sqrt{2}F$, and take F & g as large as possible.

→ Fast & robust charging.

Asymptotic freedom: proof

Bounds of the passive energy: $\omega_B \delta \le (E_B - \mathcal{E}_B) < \omega_B$

1. Dimensional reduction because of the permutation symmetry of the Liouvillian & initial state. $[\, 2^N \longrightarrow N+1 \,]$

$$\mathrm{Tr}\big[\hat{H}_B \hat{\rho}^{\downarrow}\big] = \sum_{i=1}^{N+1} r_i^{\downarrow} \varepsilon_i^{\uparrow} \le \frac{1}{N+1} \sum_{i=1}^{N+1} \varepsilon_i^{\uparrow}$$
$$= \frac{1}{N+1}\left[0\binom{N}{0} + \omega_B \binom{N}{1}\right] = \frac{N}{N+1}\omega_B < \omega_B$$

➡ Upper bound: $(E_B - \mathcal{E}_B) < \omega_B$

2. Nonzero asymptotic passive energy because of the mixture.

Mixed st.: $r_1^{\downarrow} < 1$ and some $r_{i\neq 1}^{\downarrow} > 0$ ➡ Define $\delta \equiv 1 - r_1^{\downarrow}\big|_{N\to\infty} > 0$

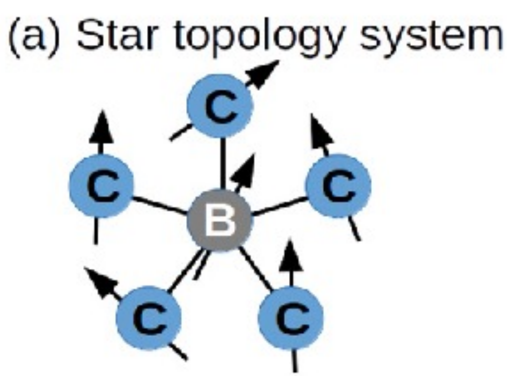
$$\mathrm{Tr}\big[\hat{H}_B \hat{\rho}^{\downarrow}\big] = \sum_{i=1} r_i^{\downarrow} \varepsilon_i^{\uparrow} = \sum_{i=2} r_i^{\downarrow} \varepsilon_i^{\uparrow} \ge \varepsilon_2^{\uparrow} \delta$$

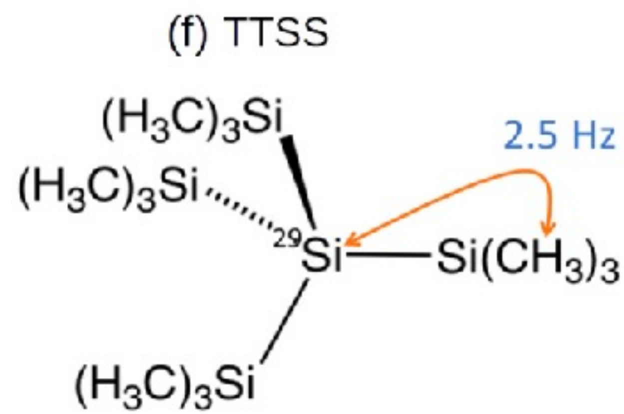
➡ Lower bound: $\omega_B \delta \le (E_B - \mathcal{E}_B)$

Together with $E_B = N\omega_B/2$, we get: $\frac{\mathcal{E}_B}{E_B} \sim 1 - \frac{a}{N}$ with $2\delta \le a \le 2$

Joshi & Maheshi, PRA **106**, 042601 (2022)

Implementation of charger-battery sys. in NMR spin sys.





1 battery coupled to multiple chargers

Battery: central nuclear spin $\left(I = \frac{1}{2}\right)$

Charger: ^{1}H nuclear spin $\left(I = \frac{1}{2}\right)$

$$\hat{H}_{\mathrm{BC}} = J(\hat{S}_x \hat{I}_x + \hat{S}_y \hat{I}_y)$$

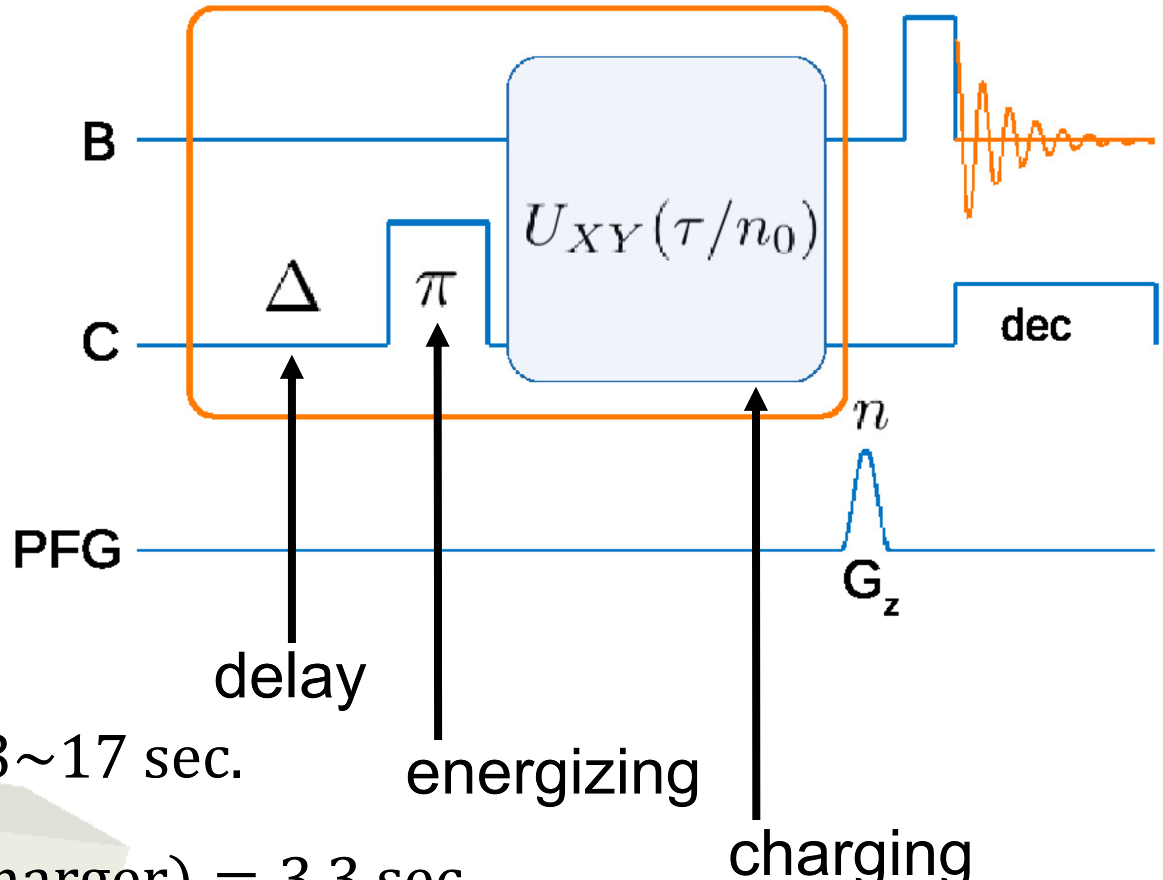
$\hat{S}_{x,y,z}$: battery spin

$\hat{I}_{x,y,z} = \sum_i \hat{I}^i_{x,y,z}$: battery spins

Experimental feasibility

Joshi & Maheshi, PRA **106**, 042601 (2022)

Mimic dephasing by iteratively reenergizing chargers after delay.



$\Delta = 3\sim17$ sec.

$T_1(\text{charger}) = 3.3$ sec.
$T_1(\text{battery}) = 115.4$ sec.