Universal tradeoff relations between resource cost and irreversibility of channels: General-resource Wiger-Araki-Yanase theorems and beyond

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Transformative research area (B): Quantum Energy Innovation

MOONSHOT RESEARCH & DEVELOPMENT PROGRAM

The main question of this talk

How much "resource" do we need to realize a given quantum dynamics?

Why “resource” matter in physics

Any physical task requires resource.

Example:

Thermodynamics :

We generate *work* from thermodynamic resource such as free energy, chemical potential, etc.

Quantum devices:

We obtain *quantum advantage* from *quantum resource*:
e.g. quantum computers: magic, entanglement, coherence⋯
quantum engines: entanglement, coherence, asymmetry,⋯

Key question :

How much resource is required to achieve a given task?

Results and messsages

A general lower bound of resource cost for an arbitrary quantum dynamics written as a CPTP map:

$$M_{\mathrm{c}}(\Lambda) \geq \frac{\Delta M^2}{16 K_A \delta(\Lambda, \Omega_p)} - c'$$

resource cost of Λ

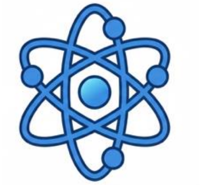
irreversibility of Λ for Ω_p

It works for various resources:

Energy, free energy, coherence, asymmetry, magic, etc···

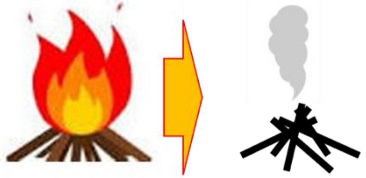
Message :

Resource



Inversely proportional

Irreversibility



→**Resource-irreversibility tradeoff**

Applications

$$M_{\mathrm{c}}(\Lambda) \geq \frac{\Delta M^2}{16K_A\delta\left(\Lambda, \Omega_p\right)} - c'$$

・General lower bound of **energy cost** and **free energy cost** of arbitrary quantum operations

→ e.g., zero-error measurements of an observable non-commuting with energy requires infinite energy

・a generalization of the Wigner-Araki-Yanase theorem, a symmetry-induced limitation on measurements, to **various class of resources** and **arbitrary quantum dynamics**

・In many resource theories, there exist resource-non-increasing channels requiring infinite amount of resource cost.

Related papers and collaborators

Main paper:

H. Tajima, K. Yamaguchi, R. Takagi and Y. Kuramochi, arXiv:2507.23760 (2025) ➡ **QIP 2026 regular talk**



Implementation cost of arbitrary channels under general resource theory

Other related papers:

H. Tajima* and R. Takagi*, PRL, 121, 110403 (2025)
➡**QIP 2025 regular talk**

H. Tajima, R. Takagi and Y. Kuramochi, arXiv:2206.11086 (2022)
➡**QIP 2023 regular talk**

Y. Kuramochi and H. Tajima, PRL 131 210201 (2023)
➡**Featured in physics**

H. Tajima, N. Shiraishi and K. Saito, PRR, 2, 043374 (2020)

H. Tajima, N. Shiraishi and K. Saito, PRL, 121, 110403 (2018)
➡**QIP 2020 regular talk**


Implementation cost of various channels under global symmetry

R. Takagi and H. Tajima, PRA, 101, 022315 (2020)

Implementation cost of unitary channels under general resource theory

Outline of this talk

- Detailed Background & brief summary of main results

- Framework and details of the results

- Applications

- Summary

Background & brief summary of main results

Background: quantifying "difficulty" of realization of quantum operations

The limitations on some "costs" of quantum information processing have been studied for a long time, well before the advent of modern resource theories.

One of the oldest results: Wigner-Araki-Yanase theorem

・An archetype of various relations of limitations of quantum information processing.

・The main result can be seen as a generalization of WAY theorem

Therefore, we first review the development of the WAY-type theorems.

WAY theorem (1/3)

E. P. Wigner, Z. Phys. **133**, 101 (1952).

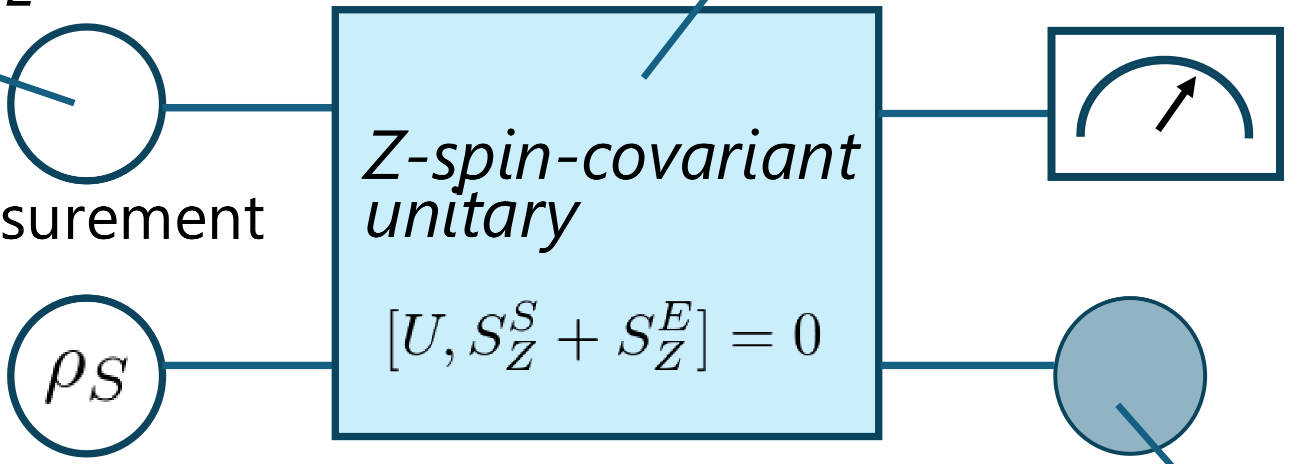
E. P. Wigner (1952)

Situation: indirect measurement on a qubit *S*

Under the conservation law of *z-spin*,

we need *infinite size* of *E* (so it is impossible)

External measurement device *E*



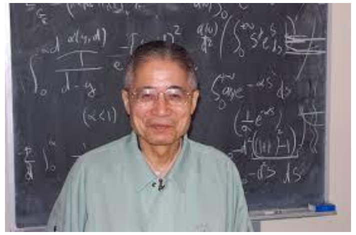
a qubit system *S*

To implement the *x-direction spin measurement* without error

Under conservation law of Z-spin, we can't measure X-spin without error!

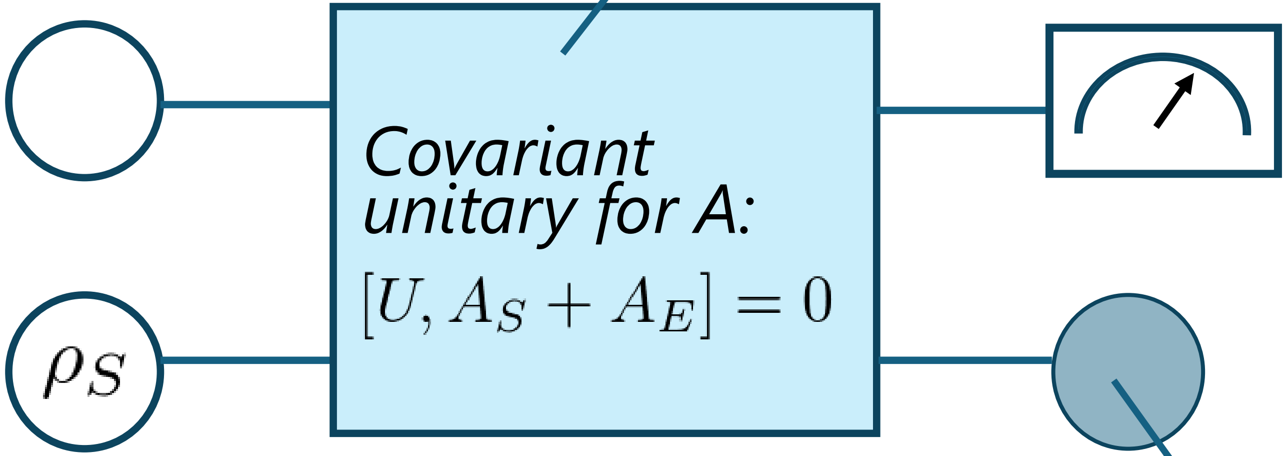
WAY theorem (2/3)

H. Araki and M. M. Yanase, Phys. Rev. **120**, 622 (1960).



H. Araki and M. M. Yanase (1960): generalization of Wigner's result

Under the conservation law of *A*,



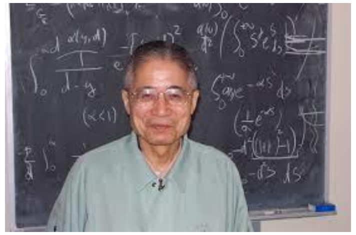
a d-level system *S*

we can't measure *B* without error, if [A,B]≠0.

Wigner-Araki-Yanase (WAY) theorem !

WAY theorem (2/3)

H. Araki and M. M. Yanase, Phys. Rev. **120**, 622 (1960).



H. Araki and M. M. Yanase (1960): generalization of Wigner's result

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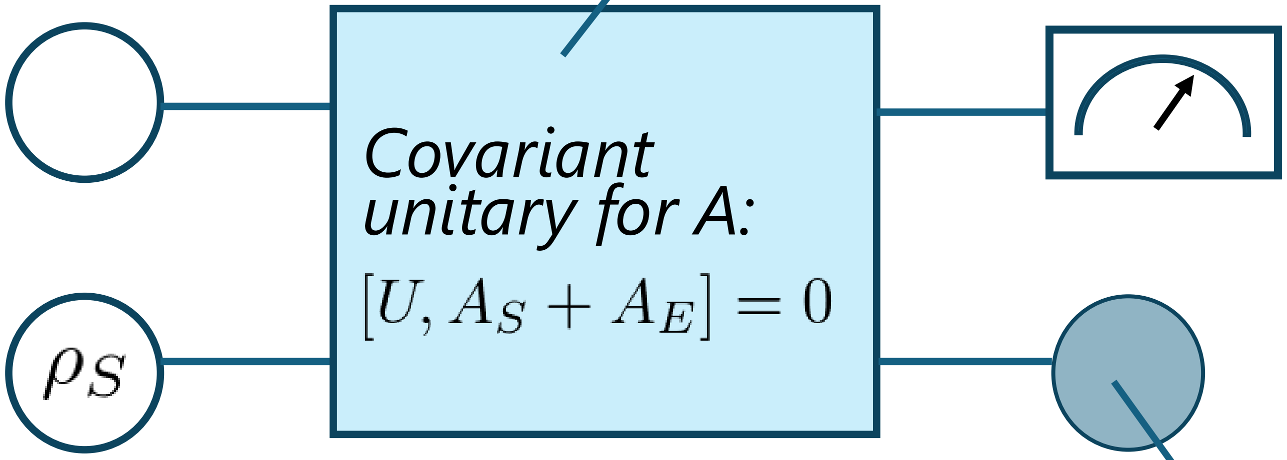


a d-level system *S*

we can't measure *B* without error, if [A,B]≠0.

Extension to unbounded operators including momentum:

Y. Kuramochi and **HT**, PRL 131 210201 (2023)

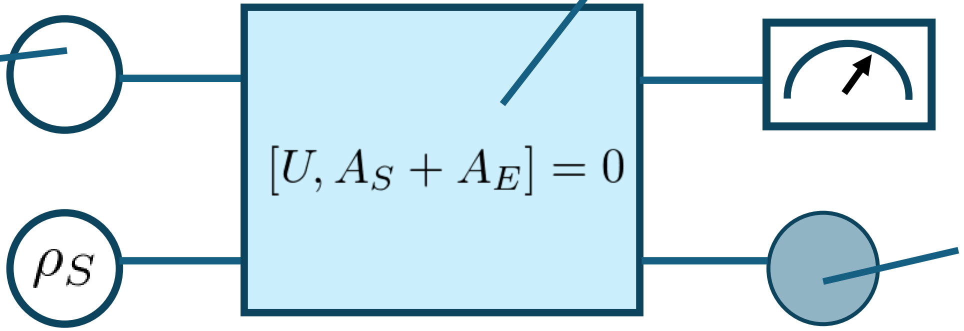
➡**Featured in physics**

WAY theorem (3/3)

M. Ozawa (2002): **quantitative WAY theorem** M. Ozawa, Phys. Rev. Lett. 88 050402 (2002).

Under the conservation law of *A*

we need variance of A_E
inversely proportional to δ



in order to measure *B_S*
within error δ

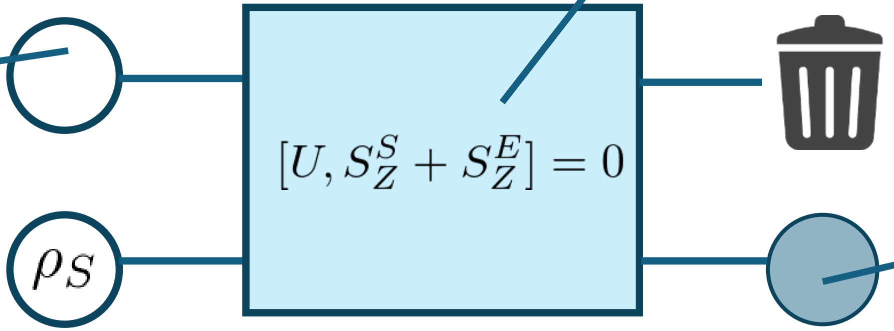
d-level system

M. Ozawa (2002): **quantitative WAY theorem for C-NOT gate**

M. Ozawa, Phys. Rev. Lett. 89 507902 (2002).

Under the conservation law of *z-spin*

we need variance of z-spin
inversely proportional to δ



in order to realize C-NOT
gate within error δ

two qubit system

WAY-type theorems

WAY theorem: measurements

M. Ozawa, Phys. Rev. Lett. 88 050402 (2002).

$$\text{Variance of conserved quantity} \propto \frac{1}{\text{measurement error}}$$

WAY theorem for C-NOT gate

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Natural questions :

- Wouldn't a similar relationship hold for general unitary gates?
- Is variance really good? （Thermal noise improves accuracy? ）

WAY-type theorems

WAY theorem: measurements

M. Ozawa, Phys. Rev. Lett. 88 050402 (2002).

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WAY theorem for C-NOT gate general unitary gates

M. Ozawa, Phys. Rev. Lett. 89 057902 (2002).

HT, N. Shiraishi and K. Saito PRL 121 110403 (2018)

$$\text{Variance of conserved quantity} \propto \frac{1}{\text{implementation error}}$$

Natural questions :

- Wouldn't a similar relationship hold for general unitary gates?
- Is variance really good? （Thermal noise improves accuracy? ）

QFI as quantum fluctuation

We can use the SLD-Quantum Fisher information for state family $\{e^{-iXt}\rho e^{iXt}\}$ as the measure of quantum fluctuation of X:

$$\mathcal{F}_\rho(X) \coloneqq -4\partial_t^2 F(\rho, e^{-iXt}\rho e^{iXt})$$

Property A: SLD-Quantum Fisher information $\mathcal{F}_\rho(X)$ is a standard measure of quantum fluctuation of X:

S. Yu, arXiv:1302.5311 (2013).

$$\mathcal{F}_\rho(X) = 4 \min_{\{q_j, |\phi_j\rangle\}:\rho=\sum_j q_j \phi_j} \sum_j q_j V_{\phi_j}(X)$$

So, we can consider $\mathcal{F}_\rho(X)/4$ as the "quantum part" of the variance.

Property B: SLD-Quantum Fisher information is a standard measure of resource in the resource theory of asymmetry.

C. Zhang, et al., Physical Review A 96, 042327 (2017). I. Marvian, Nature Communications 11, 25 (2020).
R. Takagi, Scientific Reports 9, 14562 (2019).

And it works as a good quantifier of symmetry breaking!

S Yamashika, S Endo, **HT,** arXiv:2509.07468 (2025)

WAY-type theorems

WAY theorem: measurements

M. Ozawa, Phys. Rev. Lett. 88 050402 (2002).
HT and H. Nagaoka, arXiv:1909.02904 (2019).

Quantum fluctuation of conserved charge
=quantum Fisher information

$$\text{Variance of conserved quantity} \propto \frac{1}{\text{measurement error}}$$

WAY theorem for general unitary gates

HT, N. Shiraishi and K. Saito, PRL, 121, 110403 (2018)

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WAY-type theorems

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WAY theorem for general unitary gates

HT, N. Shiraishi and K. Saito, PRL, 121, 110403 (2018)

Quantum fluctuation of conserved charge =quantum Fisher information $\propto \dfrac{1}{\text{implementation error}}$

A common error-coherence tradeoff actually exists!

Isn't there a more general restriction beyond "measurement and unitary"…?

WAY-type theorems

WAY theorem: measurements

HT and H. Nagaoka, arXiv:1909.02904 (2019).

Quantum fluctuation of conserved charge
=quantum Fisher information $\propto \dfrac{1}{\text{measurement error}}$

WAY theorem for general unitary gates

HT, N. Shiraishi and K. Saito, PRL, 121, 110403 (2018)

Quantum fluctuation of conserved charge
=quantum Fisher information $\propto \dfrac{1}{\text{implementation error}}$

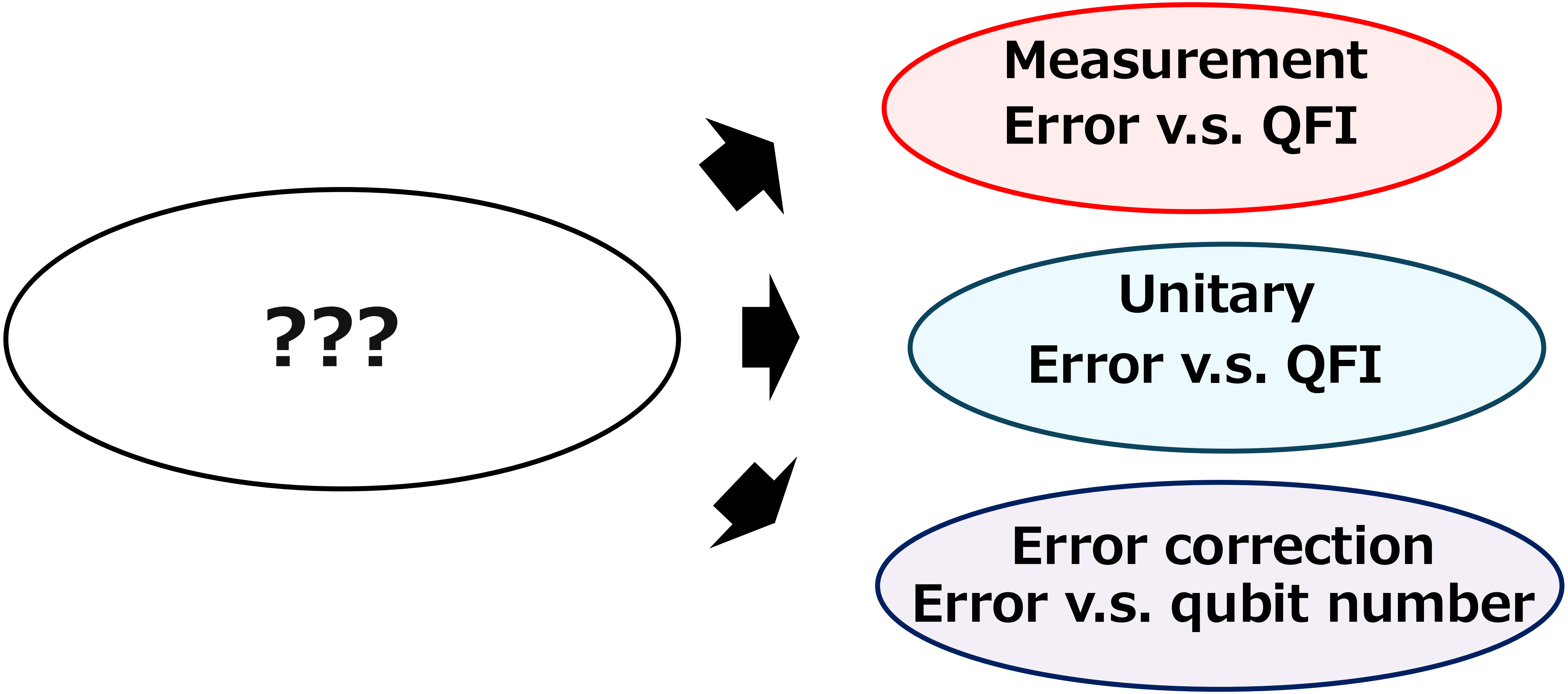
Eastin-Knill theorem: error correcting codes

B. Eastin and E. Knill, PRL 102, 110502 (2009)
P. Faist, et al., PRX 10, 041018 (2020).

$$\text{number of code qubits} \propto \frac{1}{\text{error}}$$

A unified theorem?

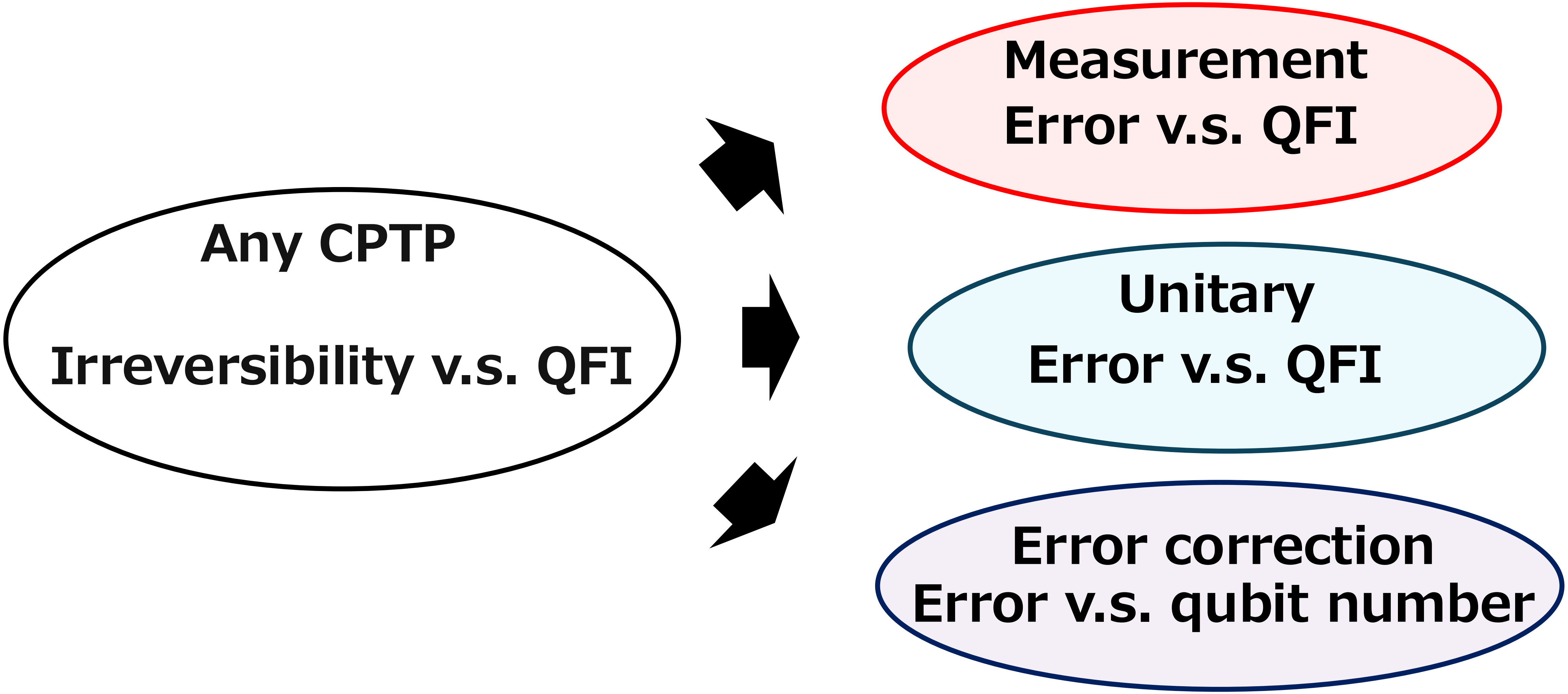
Three very similar restrictions are known in different fields:



Question:
Are they *special aspects* of a single unified theorem?

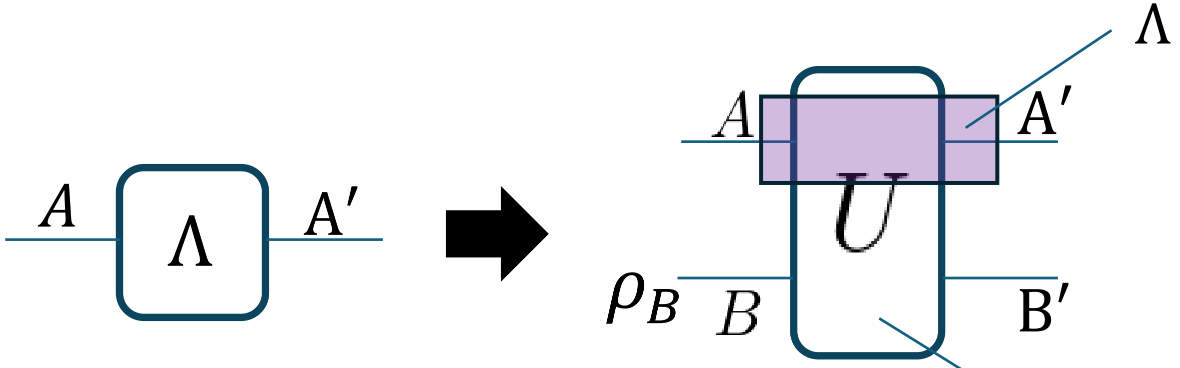
A unified theorem

There is a unified theorem of the WAY-type theorems:



Symmetry–Irreversibility–Quantumness (SIQ) tradeoff

Setup:



$$H_A + H_B = U^\dagger (H_{A'} + H_{B'}) U$$

Inequality:

$$\sqrt{\mathcal{F}_c(\Lambda)} \geq \frac{\mathcal{C}}{\delta(\Lambda, \Omega_p)} - \Delta$$

minimal QFI required in ρ_B

irreversibility of Λ for Ω_p

HT, R. Takagi and Y. Kuramochi, arXiv:2206.11086 (2022)

Message

Quantum fluctuation of conserved charge =quantum Fisher information $\propto \frac{1}{\text{irreversibility}}$

Symmetry–Irreversibility–Quantumness (SIQ) tradeoff

$$\sqrt{\mathcal{F}_{\mathrm{c}}(\Lambda)} \geq \frac{\mathcal{C}}{\delta\left(\Lambda, \Omega_p\right)} - \Delta$$

・It applies to arbitrary CPTP Λ.

・δ gives lower bounds for various irreversibility measures.

$$\delta \leq \sqrt{\Sigma}, \qquad \delta \leq \delta_Q, \qquad \text{and } \delta \leq \frac{\delta_P}{\sqrt{2}},$$

e.g., it bounds the entropy production, the entanglement-fidelity recovery error, and the error of Petz map recovery, etc.

・δ also gives bounds (almost) arbitrary existing error and disturbance of measurement, and OTOC.

So, there are various applications.

Applications of SIQ tradeoff

1. Unification of WAY, Unitary-WAY, and Eastin–Knill Theorems
➡all become corollaries of SIQ tradeoff

2. Proving existence of Gibbs-preserving operations requiring infinite QFI
HT and R. Takagi, PRL, 121, 110403 (2025), QIP 2025 regular talk

3. Applications to measurement errors, disturbances, and OTOC
H Emori, **HT**, arXiv:2309.14172 (2023)

➡ δ gives bounds (almost) arbitrary existing error and disturbance of measurement, and OTOC.

e.g. Ozawa error, Arthur-Kelly-Goodman error, Watanabe-Sagawa-Ueda error etc…

M. Ozawa, PRA 67, 042105 (2003).
E. Arthurs and J. L. Kelly jr, Bell Syst. Tech. J. 44, 725 (1965).
Y. Watanabe, T. Sagawa, and M. Ueda, PRA 84, 042121 (2011).

➡SIQ predicts limitations on them
e.g. WAY theorem for arbitrary error and disturbance

4. Speed-accuracy tradeoff for measurements and computations
➡**zero error requires infinite measurement time**
S Nakajima, **HT**, arXiv:2405.15291 (2024)

More generalization?

$$\delta(\Lambda,\rho) \geq \frac{\mathcal{C}}{\sqrt{\mathcal{F}_{\rho_B}(X_B)}} - \Delta$$

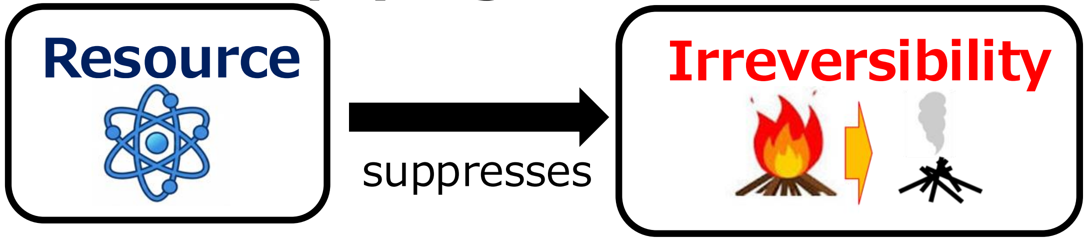
$$\sqrt{\mathcal{F}_{c}(\Omega(\Lambda))} \geq \frac{\mathcal{C}}{\delta(\Lambda,\Omega(\rho)) + \Delta}$$

Under global symmetry,

QFI(=symmetry breaking) → suppresses → irreversibility

But the irreversibility suppression by quantum effect can be seen even when there is no symmetry…

Is there a general relation implying that



in general resource theory?

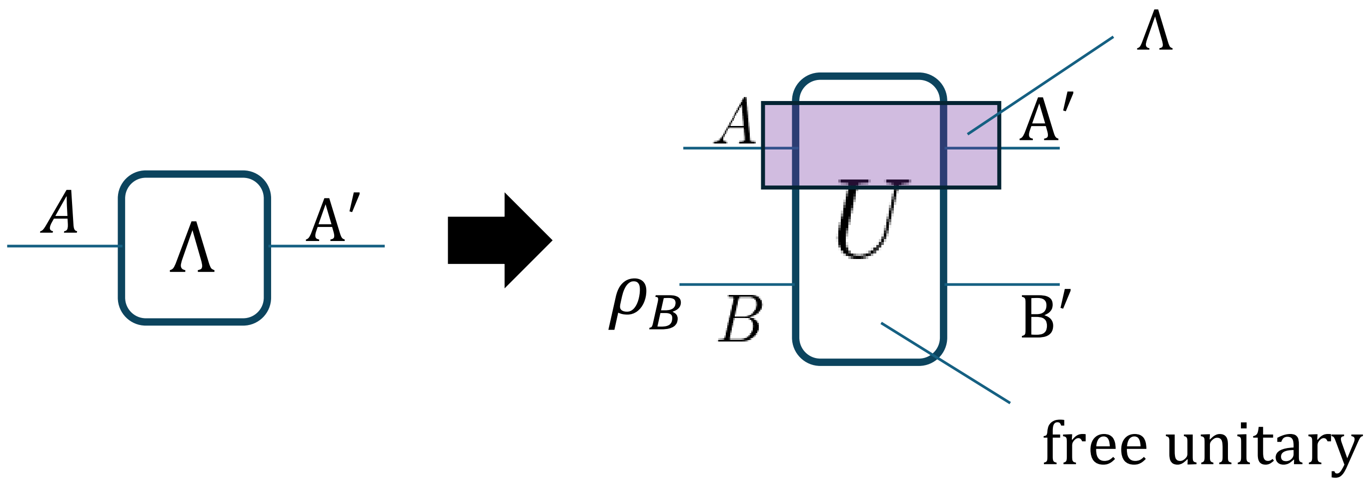
In this talk, I report…

- **this conjecture is true.**
- **there are several applications to quantum thermodynamics.**

Framework and Details of the results

Setup

Setup



Question:

When U is a free unitary in some resource theory, how much resource do we need in ρ_B to realize a given CPTP map Λ?

Answer: Resource-Irreversibility tradeoff

When the resource theory satisfies three natural conditions, the following inequality holds:

HT, K. Yamaguchi, R. Takagi and Y. Kuramochi, arXiv:2507.23760 (2025)

$$M_{\mathrm{c}}(\Lambda) \geq \frac{\Delta M^2}{16 K_A \delta(\Lambda, \Omega_p)} - c'$$

minimal resource required in ρ_B

irreversibility of Λ for Ω_p

Resource Theory (RT)

Theory of quantification (and manipulation) of “resource”

Resource : something allowing you to do some desired operations that you cannot do without it.

➡**A good image is gasoline: you can run your car when you have it.**

Examples of quantum resource theories:

Entanglement Theory: quantify quantum correlation

Quantum Thermodynamics: quantify extractable work

RT of non-Gaussianity: quantify non-Gaussianity

RT of Asymmetry: quantify symmetry breaking

Merit: RT can give a common format to quantify abstract properties of quantum states like quantum correlation, non-Gaussianity, and symmetry breaking.

Resource Theory (RT)

Theory of quantification (and manipulation) of “resource”

Resource : something allowing you to do some desired operations that you cannot do without it.

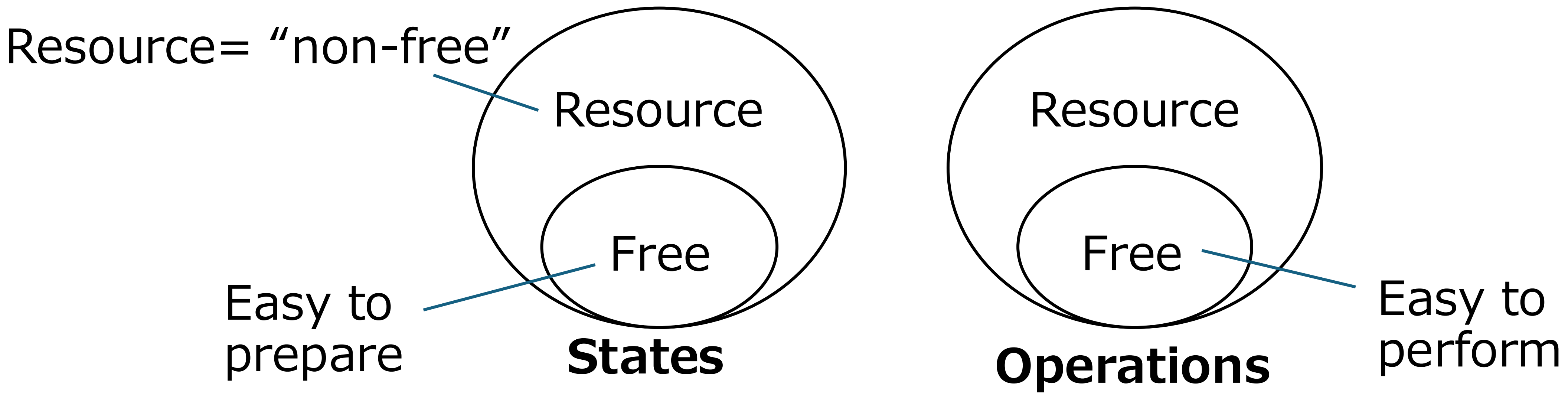
➡**A good image is gasoline: you can run your car when you have it.**

Minimal steps to quantify resource:

① Define the class of “free/resource states”, “free/resource operations”

② Define resource measures as functions of states

Free vs Resource



Minimal requirements:

・free operations include the identity operation

Doing nothing is free.

・Compositions of free operations are free

・free operations map free states only to free states

Free operations cannot make resource from free states

Every resource theory satisfies them.

Resource measure

When a function of state $M(\rho)$ *is* satisfies the following conditions, we call it a resource measure:

- $M(\rho) \geqq 0$
- $M(\rho) > 0 \Rightarrow \rho$ is a resource state.

  If the converse is also valid, *M* is a faithful measure.
- Λ is free $\Rightarrow M(\rho) \geqq M(\Lambda(\rho))$

  Free operations cannot increase resource.

Scope of application of main results

$$M_{\mathrm{c}}(\Lambda) \geq \frac{\Delta M^2}{16 K_A \delta(\Lambda, \Omega_p)} - c'$$

This form is for the case of $a_S + b_S = 1$

Whenever a RT has a resource measure M satisfies the following conditions, the above inequality holds:

(i) monotonicity

M is nonincreasing under
(a) adding a free state: $\rho \to \rho \otimes \eta$, where η is free.
(b) $\mathrm{id} \otimes \mathcal{E}$, where is $\mathcal{E}$ is a free unitary or its dual
(c) a partial trace

(ii) Additivity for product states: $M(\rho \otimes \sigma) = M(\rho) + M(\sigma)$

(iii) Hölder continuity

For any system S, there exist positive constants K_S, a_S and b_S, s.t. $a_S + b_S > 0$ and

$$\left|M(\rho^{\otimes m}) - M(\sigma_m)\right| \leq m^{a_S} K_S \epsilon_m^{b_S} + c(\epsilon_m)$$

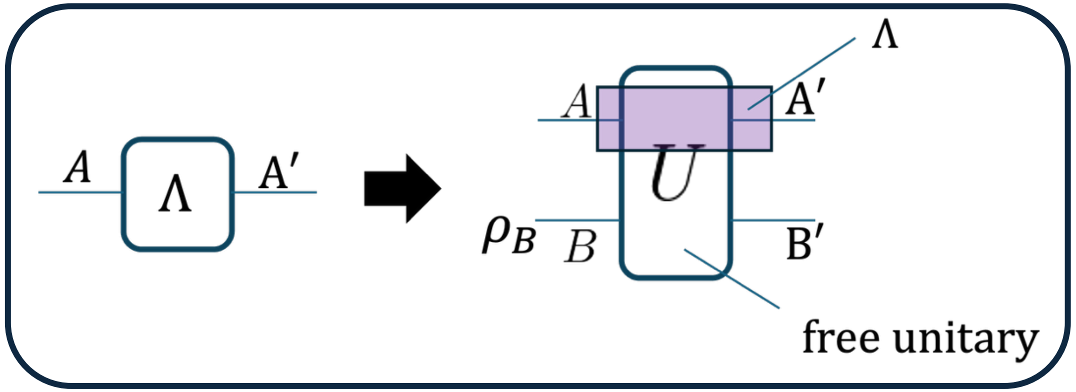
where $\rho^{\otimes m}$ and σ_m are on m-copies of S, and, $\epsilon_m \coloneqq \left|\left|\rho^{\otimes m} - \sigma_m\right|\right|_1$,

$$\lim_{x \to 0} c(x) = 0 \text{ and } c(x) \leq c_{\max}.$$

Various resource theories satisfy (i)-(iii):
e.g. Energy, athermality, coherence, asymmetry, magic, etc.

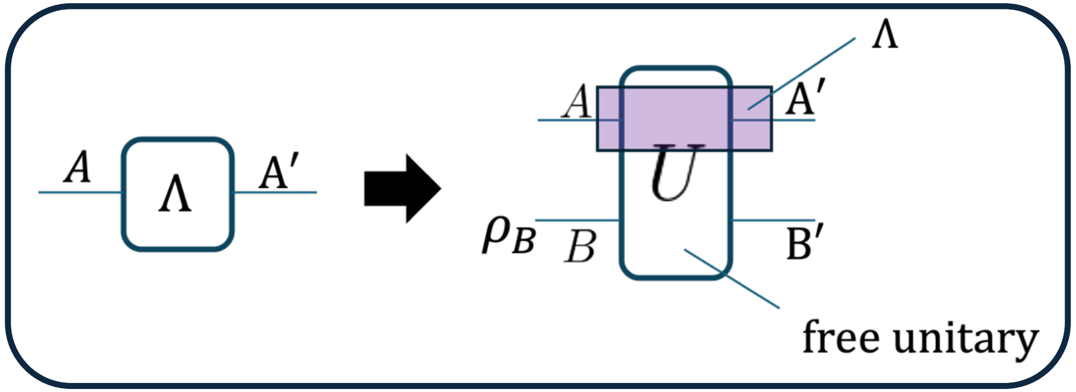
Properties of key quantities

$$M_{\mathrm{c}}(\Lambda) \geq \frac{\Delta M^2}{16K_A\delta\left(\Lambda,\Omega_p\right)} - c'$$



Properties of key quantities

$$M_c(\Lambda) \geq \frac{\Delta M^2}{16 K_A \delta(\Lambda, \Omega_p)} - c'$$



Irreversibility δ : function of Λ and a "test ensemble" Ω_p

$$\Omega_p := (\rho_1, \rho_2)_p$$

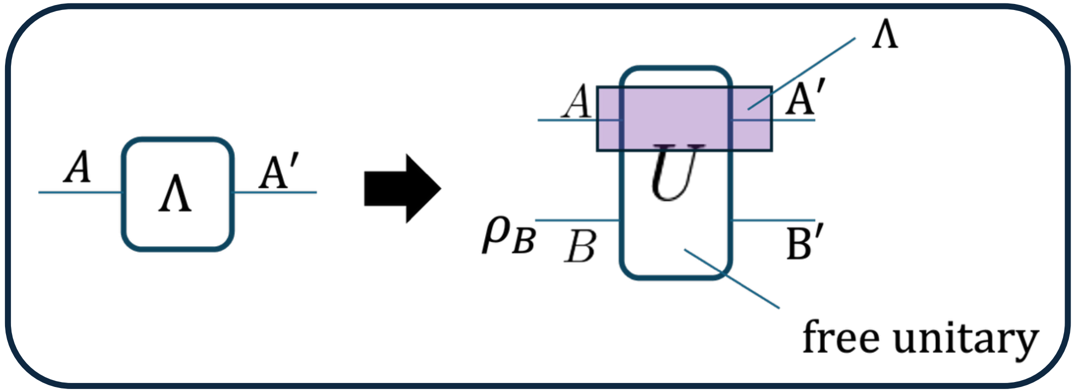
Preparing ρ_1 with the probability p and ρ_2 with $1-p$

The states ρ_1 and ρ_2 are orthogonal to each other

i.e. $F(\rho_1, \rho_2) = 0$

Properties of key quantities

$$M_c(\Lambda) \geq \frac{\Delta M^2}{16 K_A \delta(\Lambda, \Omega_p)} - c'$$



Irreversibility δ : function of Λ and a "test ensemble" Ω_p

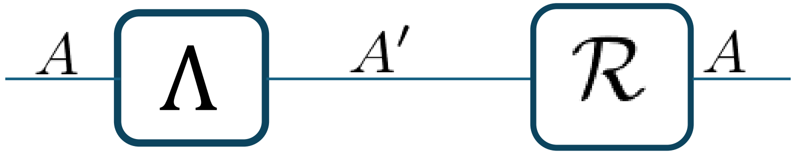
$$\delta := \min_{\mathcal{R}:A' \to A} \sqrt{\sum_{k=1,2} p_k \delta_k^2}$$

$$\delta_k := D_F(\rho_k, \mathcal{R} \circ \mathcal{E}(\rho_k))$$

recovery error for ρ_k

$$D_F(\rho, \sigma) := \sqrt{1 - F^2(\rho, \sigma)}$$

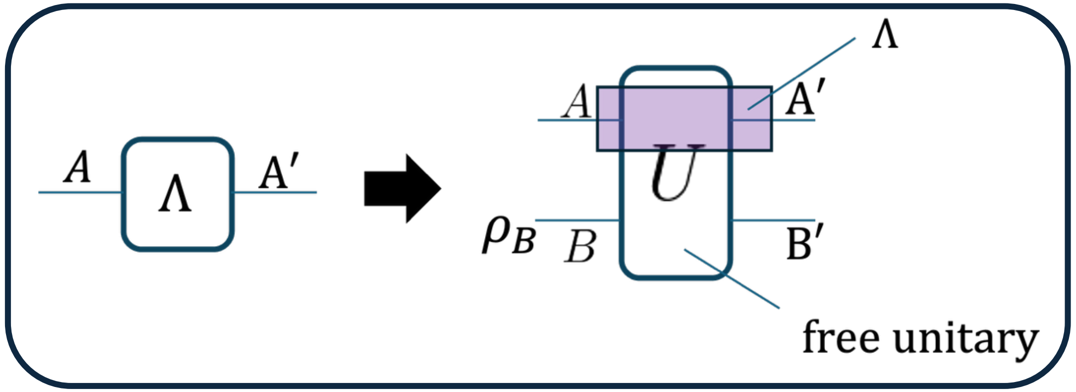
$(\rho_1, \rho_2)_p$



$(\mathcal{R} \circ \Lambda(\rho_1), \mathcal{R} \circ \Lambda(\rho_2))_p$

Properties of key quantities

$$M_{\mathrm{c}}(\Lambda) \geq \frac{\Delta M^2}{16K_A\,\delta(\Lambda,\Omega_p)} - c'$$



Irreversibility δ : function of $\mathcal{E}$ and a "test ensemble"Ω_p

$$\delta := \min_{\mathcal{R}:A'\to A} \sqrt{\sum_k p_k \delta_k^2}$$

Property: δ gives lower bounds for various irreversibility measures.

$$\delta \leq \sqrt{\Sigma}, \quad \delta \leq \delta_Q, \quad \text{and } \delta \leq \frac{\delta_P}{\sqrt{2}},$$

e.g., it bounds the entropy production, the entanglement-fidelity recovery error, and the error of Petz map recovery, etc.

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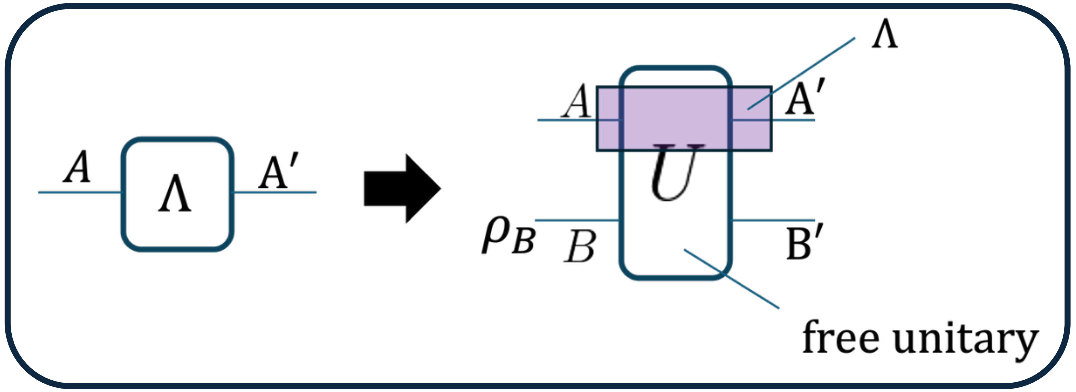


Resource cost M_c :

$$M_c(\Lambda) \coloneqq \min\{M(\rho_B) | \Lambda(\ldots) = \mathrm{Tr}_{B'}[U(\ldots \otimes \rho_B)U^\dagger], \rho_B \in \mathcal{U}_F\}$$

Resource Cost for approximate implementation:

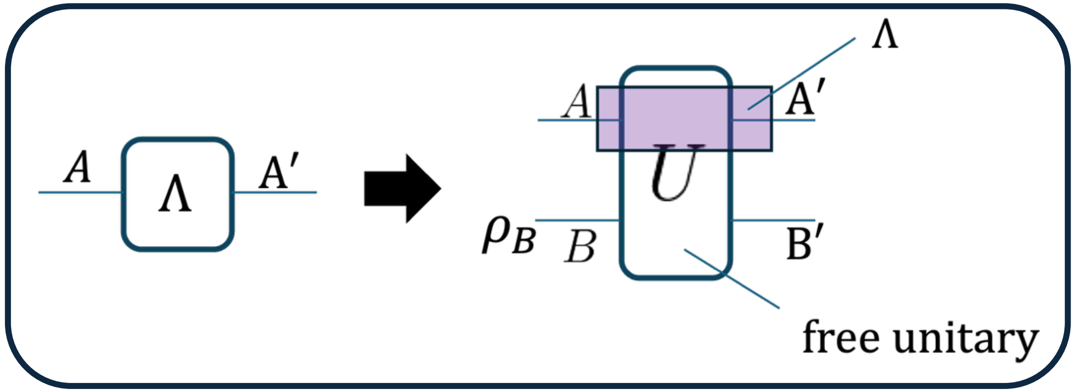
$$M_c^\epsilon(\Lambda) \coloneqq \min\{M(\rho_B) | \Lambda(\ldots) \approx_\epsilon \mathrm{Tr}_{B'}[U(\ldots \otimes \rho_B)U^\dagger], \rho_B \in \mathcal{U}_F\}$$

Error ϵ is measured by the purified distance

$D_F(\rho, \sigma) := \sqrt{1 - F^2(\rho, \sigma)}$

Properties of key quantities

$$M_c(\Lambda) \geq \frac{\Delta M^2}{16 K_A \delta(\Lambda, \Omega_p)} - c'$$



$$\Delta M := \left|M_g - M_r\right|_+$$

$|x|_+ = x$ when $x > 0$
and $= 0$ otherwise

Resource gain by discriminating ρ_1 and ρ_2 in Ω_p

Minimal resource-increasing power of channel to discriminate $\Lambda(\rho_1)$ and $\Lambda(\rho_2)$ optimally

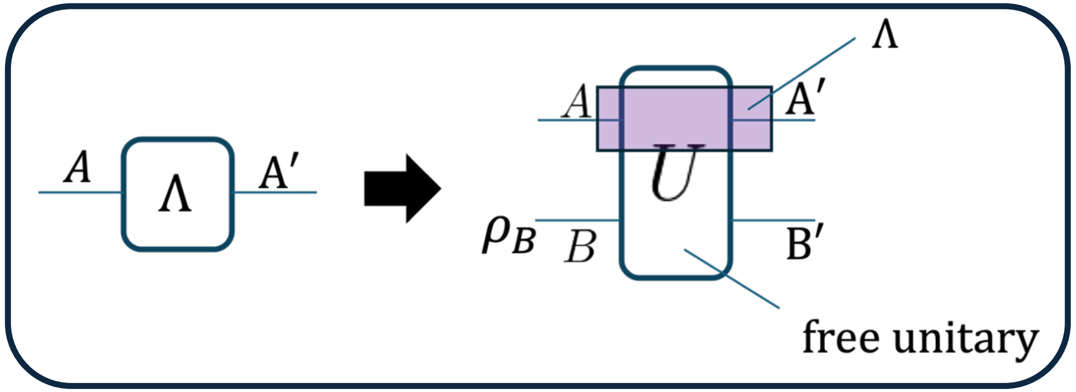
※The success probability of the optimal discrimination between $\Lambda(\rho_1)$ and $\Lambda(\rho_2)$ satisfies

$$P_{\mathrm{succ}} = 1 - \delta(\Lambda, \Omega_p)^2$$

Properties of key quantities

$$M_c(\Lambda) \geq \frac{\Delta M^2}{16 K_A \delta(\Lambda, \Omega_p)} - c'$$

ΔM^2: $|M_g - M_r|_+^2$



Formal definition of M_g and M_r :

Resource gain by discriminating ρ_1 and ρ_2 in Ω_p:

$$M_g(\Omega_p|K) \coloneqq \max_\rho\{M\left(\sum_{k=1,2} p_k\rho_k \otimes |k\rangle\langle k|_K\right) - M(\rho):$$
$$\exists \text{ a PVM } \{P_k\},\ \sum_k P_k \ldots P_k \otimes |k\rangle\langle k| = \sum_{k=1,2} p_k\rho_k \otimes |k\rangle\langle k|_K\}$$

Minimal resource-increasing power of channel to discriminate $\Lambda(\rho_1)$ and $\Lambda(\rho_2)$ optimally

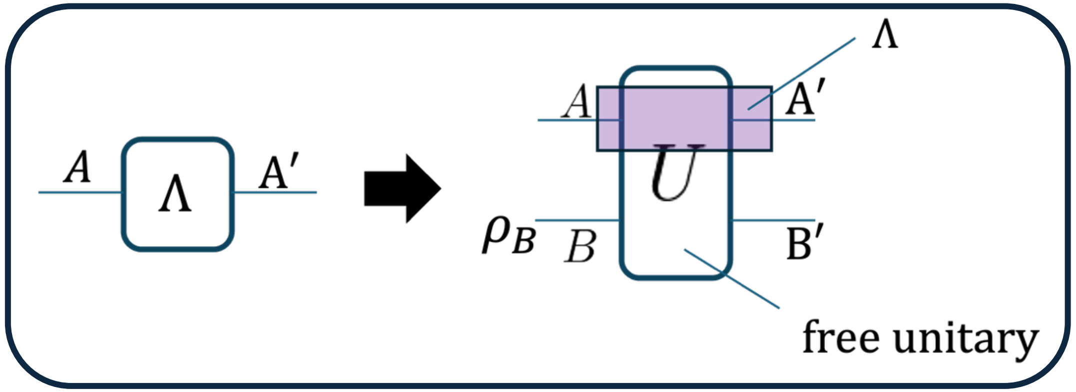
$$M_r(\Omega_p|K) \coloneqq \min_{\text{POVM } \mathbb{Q}=\{Q_k\}}\{M(\Lambda_\mathbb{Q}): P_{\text{fail}}\left(\left(\Lambda(\rho_1), \Lambda(\rho_2)\right)_p, \mathbb{Q}\right) = \delta(\Lambda, \Omega_p)^2\}$$

$$\Lambda_\mathbb{Q}(\ldots) \coloneqq \sum_k \sqrt{Q_k} \ldots \sqrt{Q_k} \otimes |k\rangle\langle k|_K$$

$M(\Lambda_\mathbb{Q}) \coloneqq \max_\sigma(M(\text{id} \otimes \Lambda(\sigma)) - M(\sigma))$: resource-increasing power of $\Lambda_\mathbb{Q}$

Messages of the main result

$$M_{\mathrm{c}}(\Lambda) \geq \frac{\Delta M^2}{16 K_A \delta(\Lambda, \Omega_p)} - c'$$



About resource cost:

1. $\Delta M > 0$ ➡ $\mathrm{Cost} \propto \frac{1}{\mathrm{irreversibility}}$

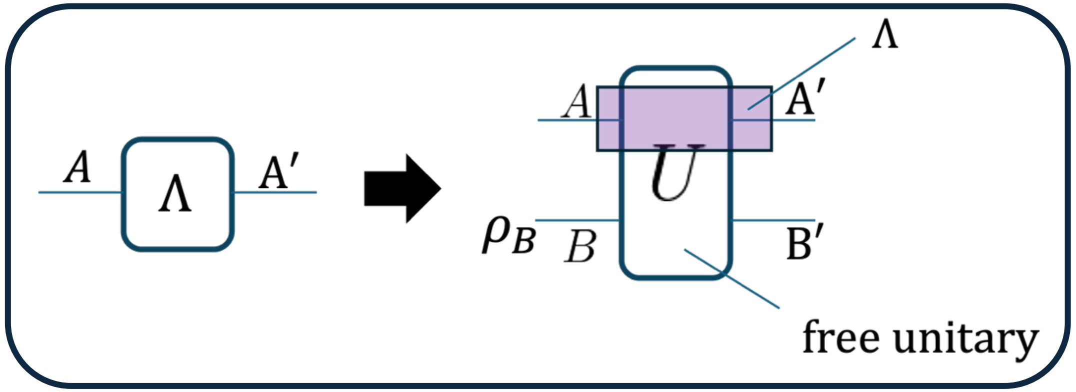
2. $\Delta M > 0 \wedge \delta = 0$ ➡ $M_{\mathrm{c}}(\Lambda) = \infty$

In this case, we can also obtain

$$M_{\mathrm{c}}^{\epsilon}(\Lambda) \geq \frac{\Delta M^2}{16 K_A \epsilon} - c'. \quad \text{i.e.} \quad \mathrm{Cost} \propto \frac{1}{\mathrm{error}}$$

Messages of the main result

$$\delta(\Lambda, \rho) \geq \frac{\Delta M^2}{16 K_A M(\rho_B) + c'}$$

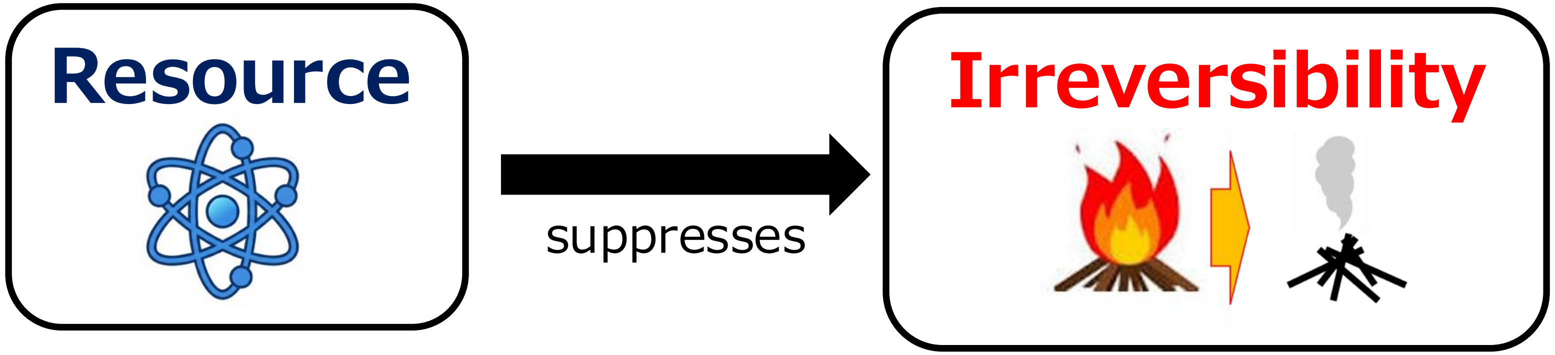


About relation between irreversibility and resource:

1. $\Delta M > 0$ ➡ $\delta > 0$

2. The resource in B $(= M(\rho_B))$ can mitigate the irreversibility.

Take home message :

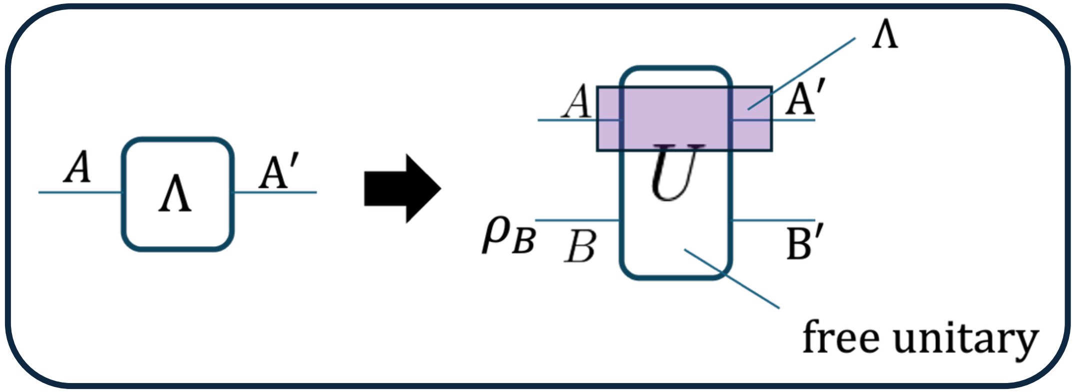


→Resource-irreversibility tradeoff

Relation between RI tradeoff and SIQ tradeoff

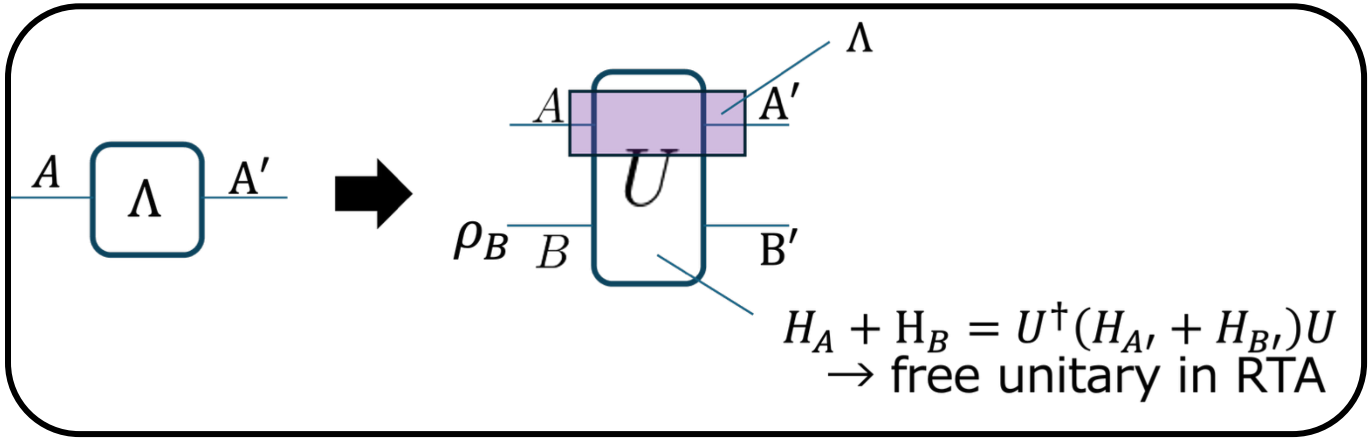
Resource-irreversibility tradeoff: general resource theory

$$\delta(\Lambda, \Omega_p) \geq \frac{\Delta M^2}{16 K_A (M(\rho_B) + c')}$$



SIQ tradeoff: resource theory of asymmetry (RTA)

$$\delta(\Lambda, \Omega_p) \geq \frac{\mathcal{C}}{\sqrt{\mathcal{F}_{\rho_B}(H_B) + \Delta}}$$



Qualitatively, the SIQ trade-off is a specialization of the RI trade-off within the resource theory of asymmetry, that is, "Assymetry-Irrversibility tradeoff".

Applications

How to use

$$M_{\mathrm{c}}(\Lambda) \geq \frac{\Delta M^2}{16 K_A \delta(\Lambda, \Omega_p)} - c'$$

1. To evaluate the cost of Λ, we pick a “good” state ensemble Ω_p
   --ideally with large $\Delta \mathrm{M}$ and small δ.

2. If there is a ensemble satisfying $\Delta M > 0 \wedge \delta = 0$, the channel Λ cannot be implemented; it would require infinite resource.

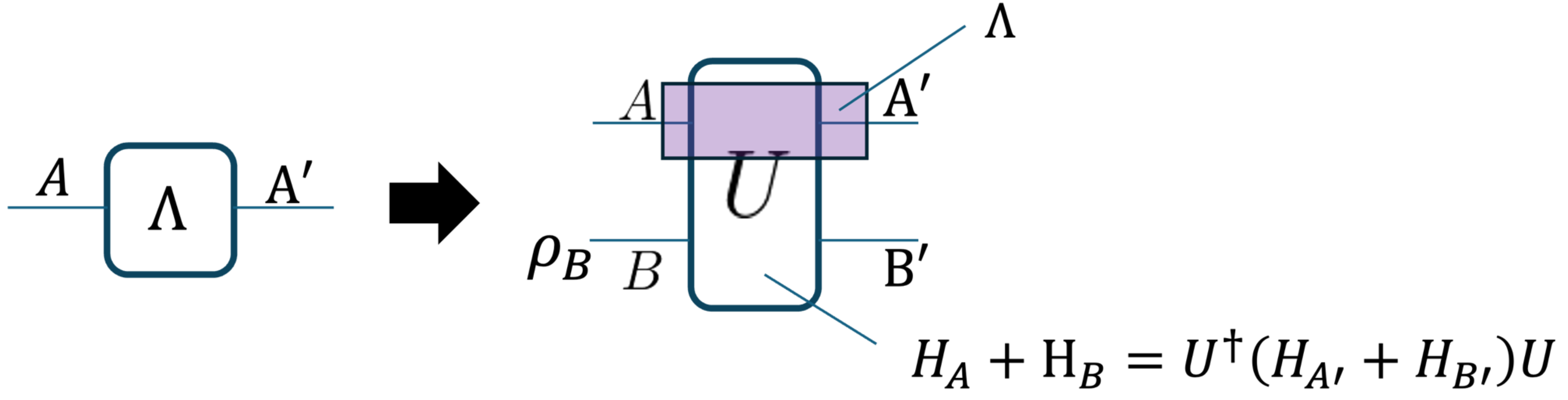
3. Furthermore, in that case, the cost for approximate implementation is inversely proportional to the error.

applicable to various resources: energy, athermality (=non-equilibrium free energy), coherence, asymmetry, magic, etc.

Application 1: lower bound for energy cost of arbitrary channels

Our bound applies to the resource theory of energy, and thus it provides a universal bound for energy cost of arbitrary channels:

Setup



Result

For any Λ and $\Omega_{1/2} := (|\psi_1\rangle, |\psi_2\rangle)_{1/2}$,

$$\mathcal{C}(\Lambda, \Omega_{1/2}) := \left|\langle\psi_2|H - \Lambda^\dagger(H')|\psi_1\rangle\right|$$

$$E_{\mathrm{c}}(\Lambda) \geq \frac{\mathcal{C}\left(\Lambda, \Omega_{1/2}\right)^2}{8(\Delta_H + \Delta_{H'})\delta\left(\Lambda, \Omega_{1/2}\right)} - 2\mathcal{C}\left(\Lambda, \Omega_{1/2}\right)$$

required energy expectation value in ρ_B

difference between maximum and minimum eigenvalues of H

Application 1: lower bound for energy cost of arbitrary channels

$$E_{\mathrm{c}}(\Lambda) \geq \frac{\mathcal{C}\left(\Lambda, \Omega_{1/2}\right)^2}{8(\Delta_H + \Delta_{H'})\delta\left(\Lambda, \Omega_{1/2}\right)} - 2\mathcal{C}(\Lambda, \Omega_{1/2})$$

Implications:

・projective measurements $\{P_k\}$ requires infinite energy cost if $[P_k, H] \neq 0$.

We can furthermore show that to make $\epsilon(A{:}\mathcal{M}) = 0$, implementation of $\mathcal{M}$ requires infinite energy cost if $[\mathrm{A}, H] \neq 0$

・unitary operations U requires infinite energy cost if $[U, H] \neq 0$.

Remark:

We can also obtain a similar bound for **free energy cost** for arbitrary channels.

→bound of free energy cost for projective measurements and unitary operations

Application 2: general-resource WAY-type theorems

Main result provides extensions of WAY-type theorems to the general resource theory.

For example, for measurements, the following theorem holds:

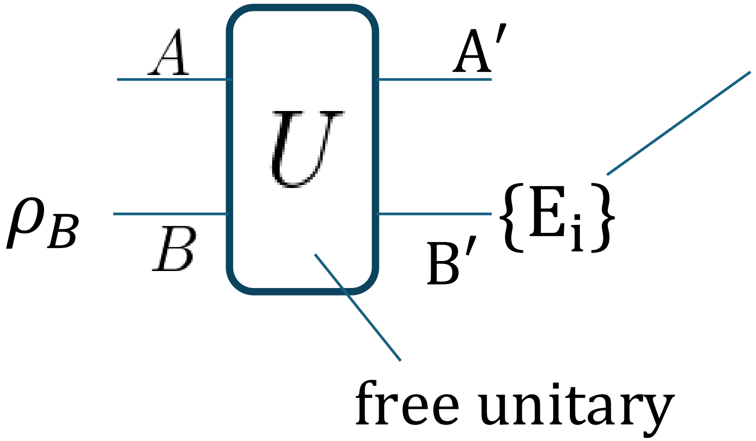
Theorem: Assume a resource theory to satisfy the conditions (i)-(iii).

Let two states ρ_1 and ρ_2 be orthogonal to each other satisfying

$M(p\rho_1 \otimes |1\rangle\langle 1| + (1-p)\rho_2 \otimes |2\rangle\langle 2|) > M(p\rho_1 + (1-p)\rho_2)$ for a $0 \leq p \leq 1$.

Then, if the failure probability of discrimination between ρ_1 and ρ_2 by the following indirect measurement is smaller than ϵ^2,

$$M(\rho_B) \geq \max_p \frac{M_g(\Omega_p | K)}{16 K_A \epsilon} - const.$$



id $\otimes \Lambda_{\{E_i\}}$ is free

$$\Lambda_{\{E_i\}}(\ldots) := \sum \sqrt{E_i} \ldots \sqrt{E_i} \otimes |i\rangle\langle i|\Big|_K$$

Application 3: cost-diverging resource-non-increasing operations

Our tradeoff also shows that in many resource theories, there are cost-diverging resource-non-increasing operations.

energy, athermality (=non-equilibrium free energy), coherence, asymmetry, magic, etc.

The cost-diverging operations are measurement-and-prepare channels, which are used in resource distillation and dilution tasks.

→It is necessary to classify operations with explicit attention to there physical expense.

Application to thermodynamics:

There is a Gibbs-preserving operation requiring infinite costs in terms of asymmetry, energy, and nonequilibrium free energy at the same time!

Summary

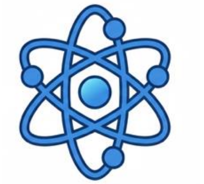
Summary

HT, K. Yamaguchi, R. Takagi and Y. Kuramochi, arXiv:2507.23760 (2025)

We find a "resource-irreversibility" tradeoff for arbitrary quantum channels:

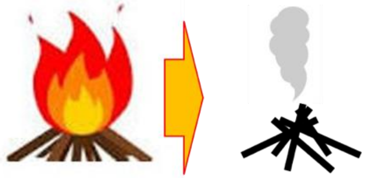
$$M_{\mathrm{c}}(\mathcal{E}) \geq \frac{\Delta M^2}{16 K_A \delta(\Lambda, \Omega_p)} - c'$$

Resource



⟷ Inversely proportional

Irreversibility



It applies to various resource theories such as energy, coherence, asymmetry, athermality and magic.

It has various applications, e.g., bounds of energy cost and free energy cost of arbitrary channels, WAY-type theorems in general resource theories, etc.