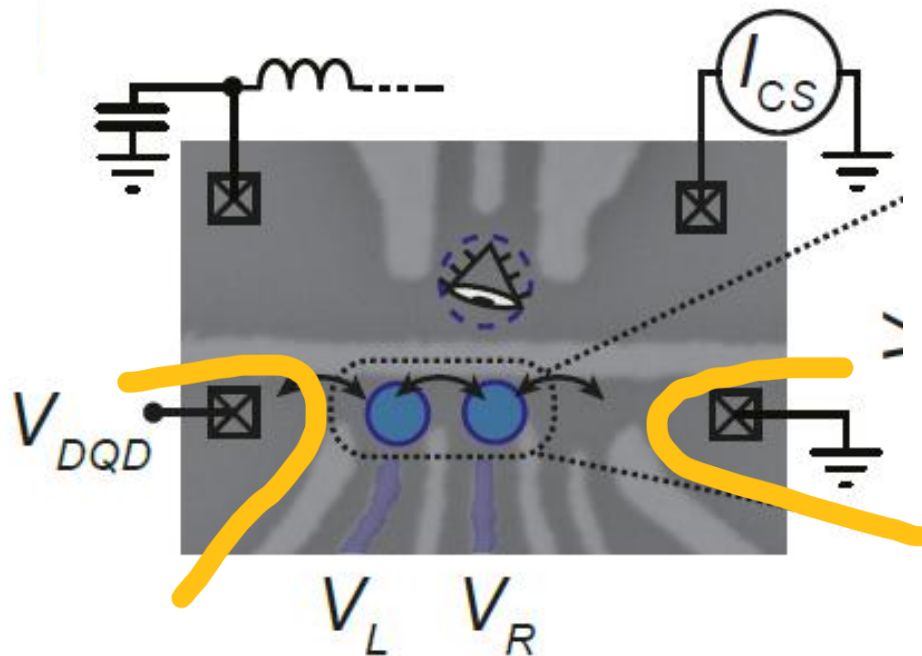


Quantum precision limits in mesoscopic conductors

Géraldine Haack

Department of Applied Physics, University of Geneva



$$\frac{I_{\alpha}^2}{S_{\alpha\alpha}} \leq \xi$$

How to bound the
signal-to-noise ratio?

Department of Applied Physics, University of Geneva, Switzerland



Theory

Nicolas Brunner
Géraldine Haack

Q. networks, Q.memories,
Q. comm., Q. tech

Wolfgang Tittel
Boris Korzh
Mikaël Afzelius
Rob Thew

Non-linear optics

Jean-Pierre Wolf
Jérôme Kasparian
Luigi Bonacina

Topics of interest : Quantum dynamics
Quantum thermodynamics
Quantum transport / Mesoscopic physics
Quantum information theory



Geneva Quantum Centre



**UNIVERSITÉ
DE GENÈVE**

This project



Gianmichele Blasi



**UNIVERSITÉ
DE GENÈVE**



IFISC*

**Universitat de les
Illes Balears**



Ricard Rodriguez

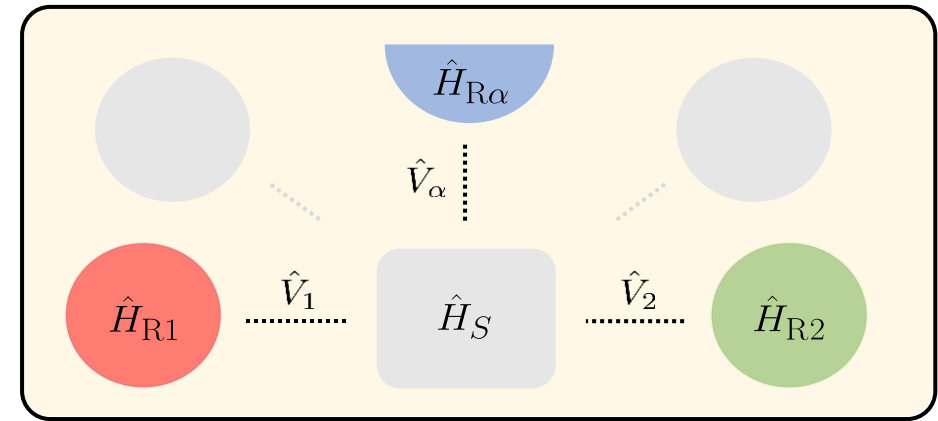
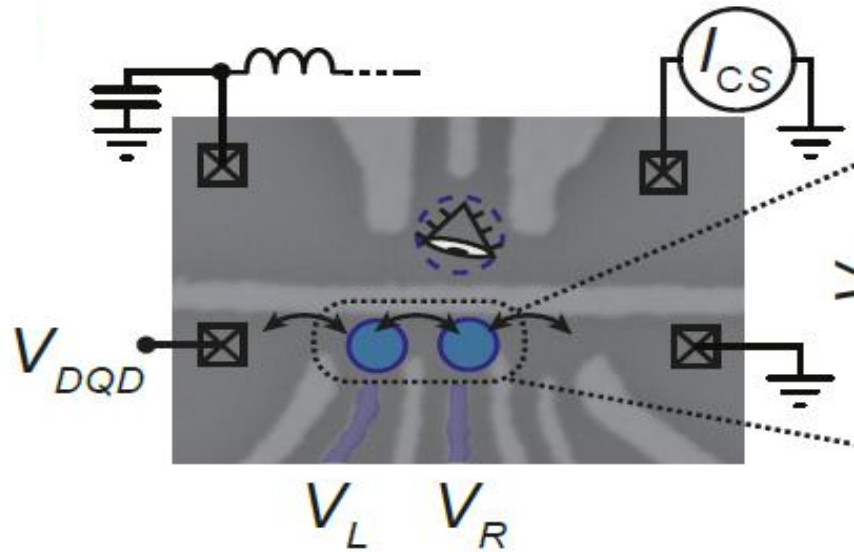


Michael Moskalets



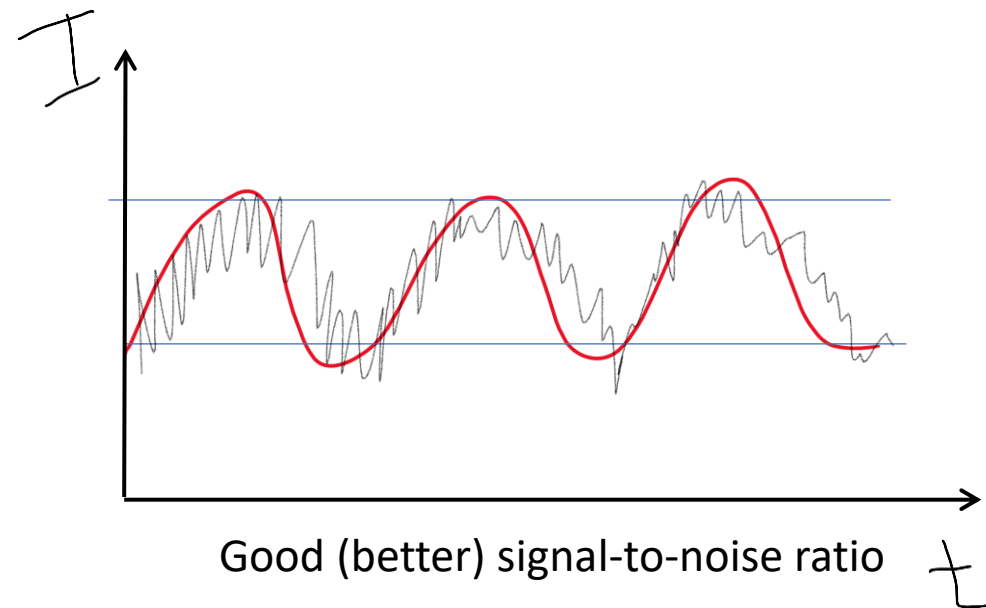
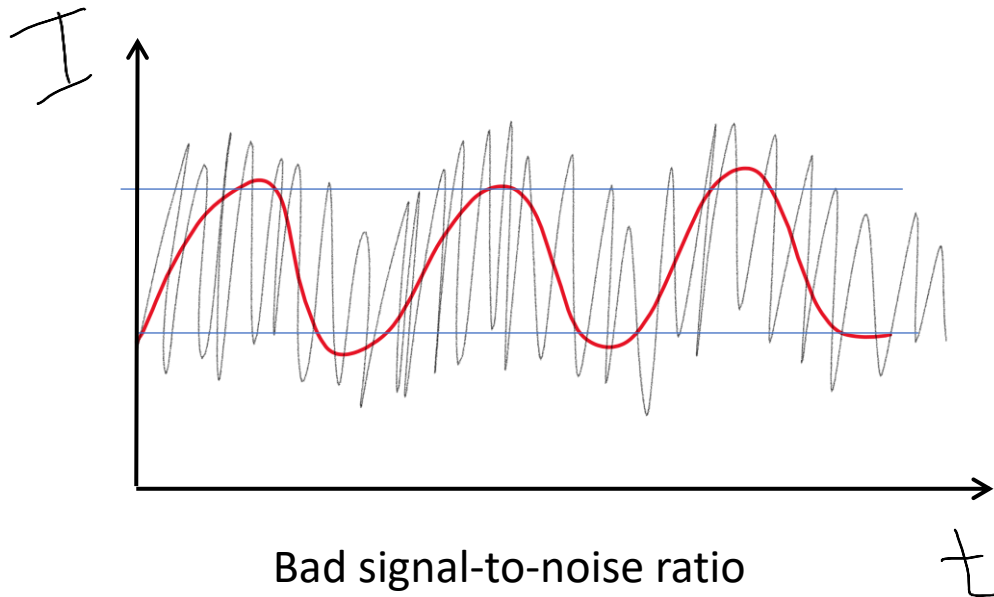
Rosa Lopez

Transport experiments and signal-to-noise ratio

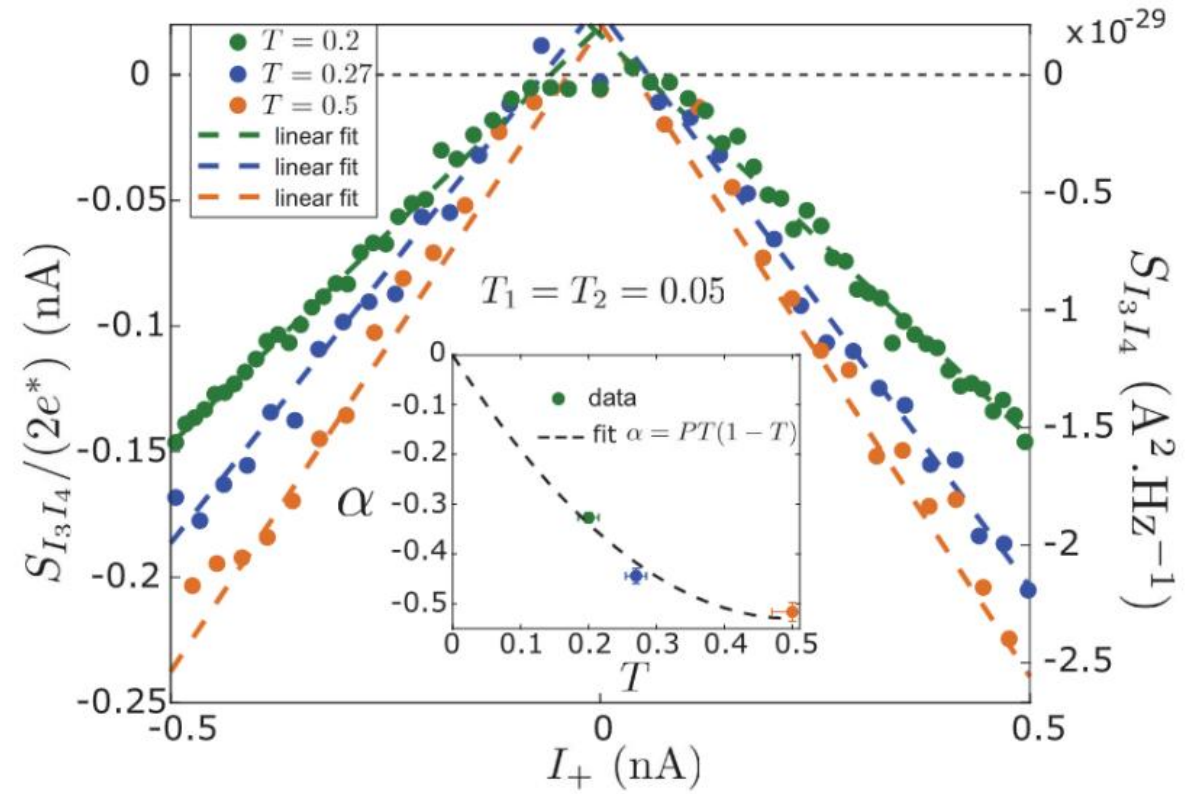
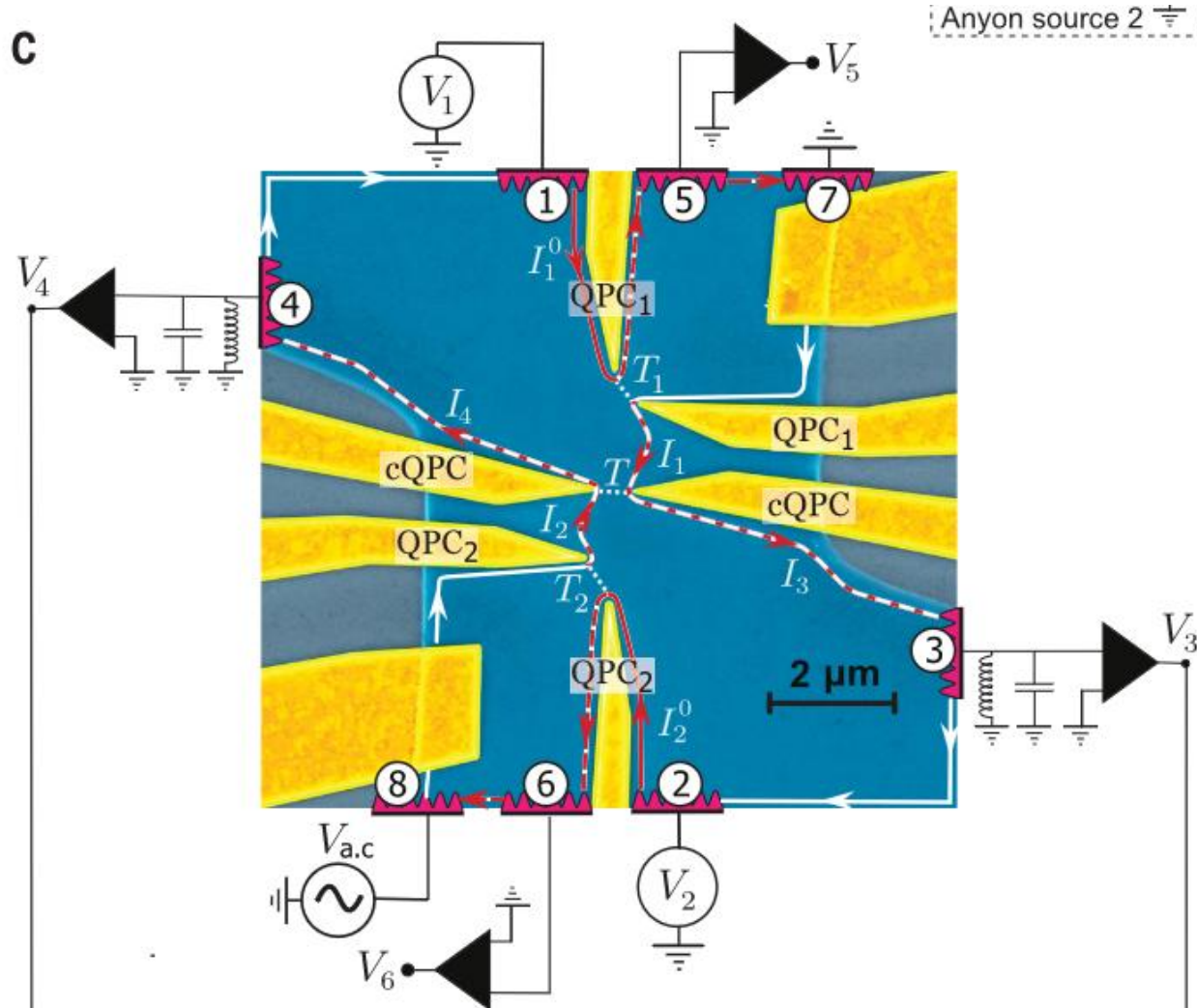


Model for an open quantum system

Wadhia et al., arXiv:2502.00096 Ares lab, Oxford

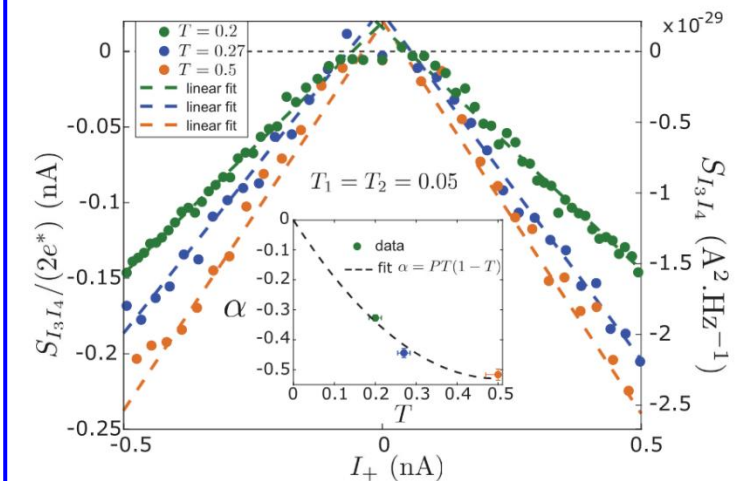
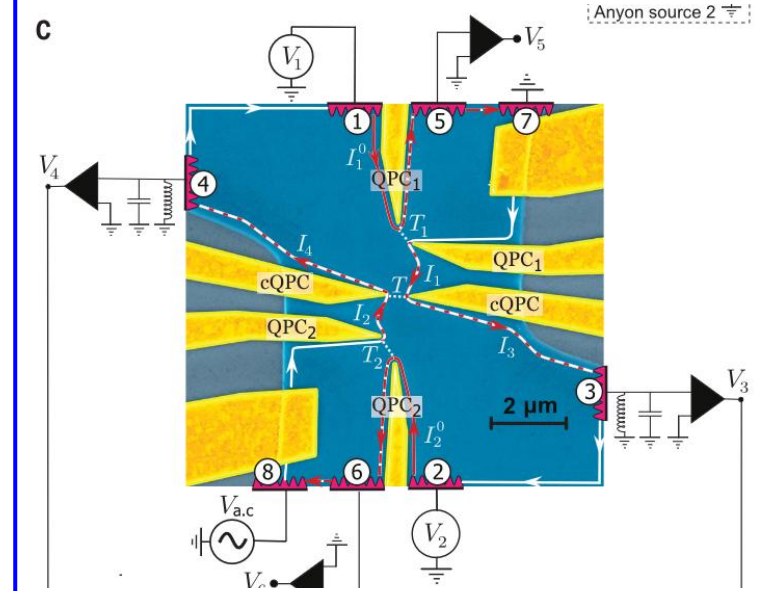


Signal-to-noise ratio (SNR) in quantum systems

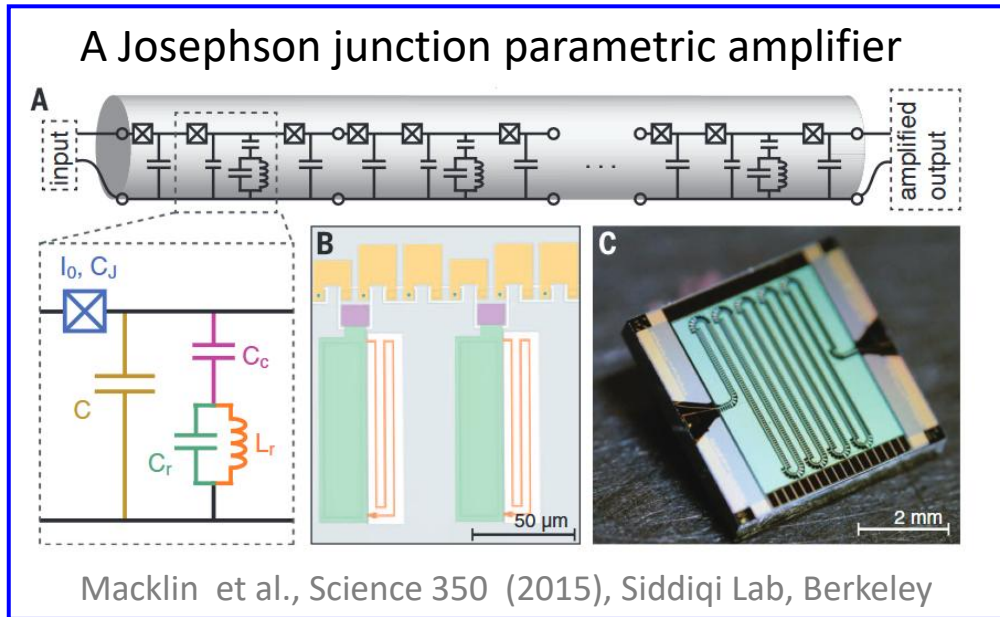
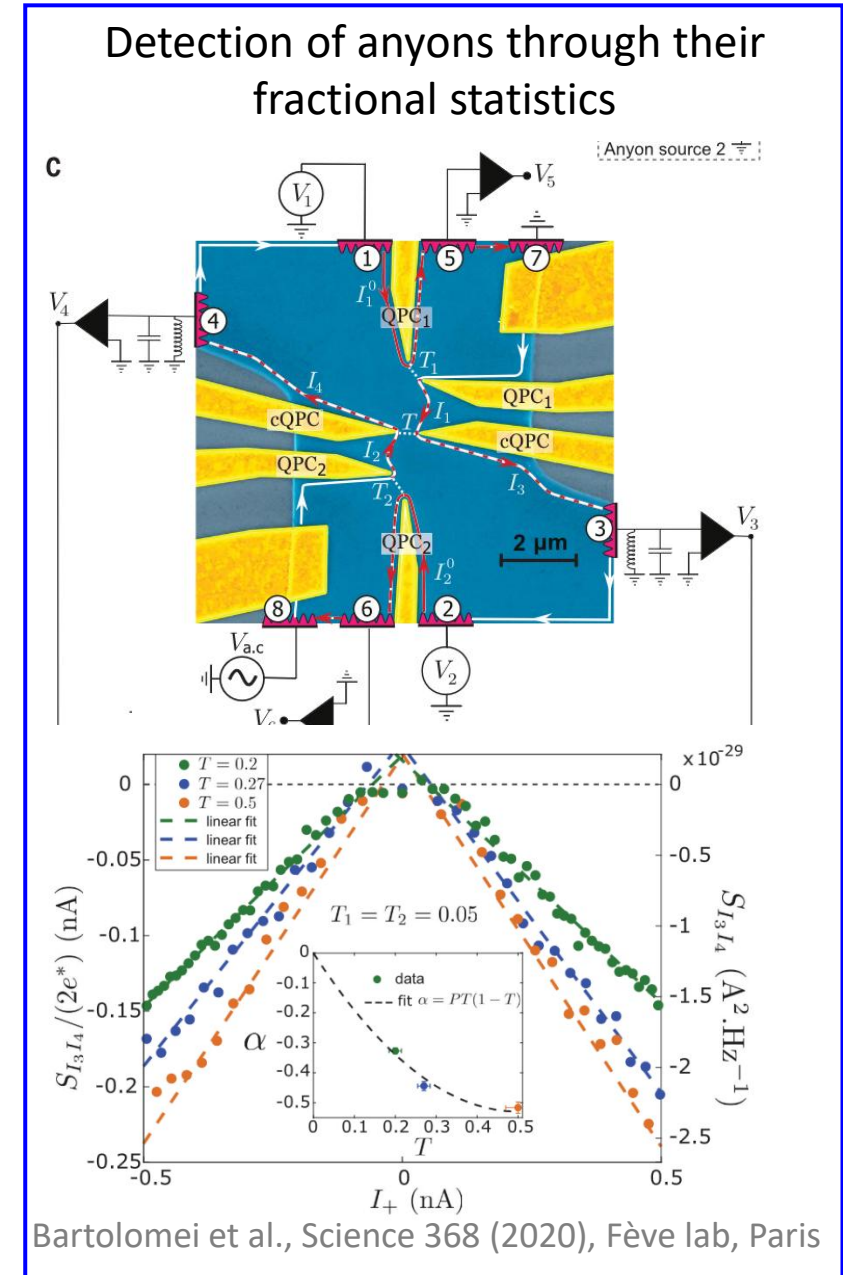
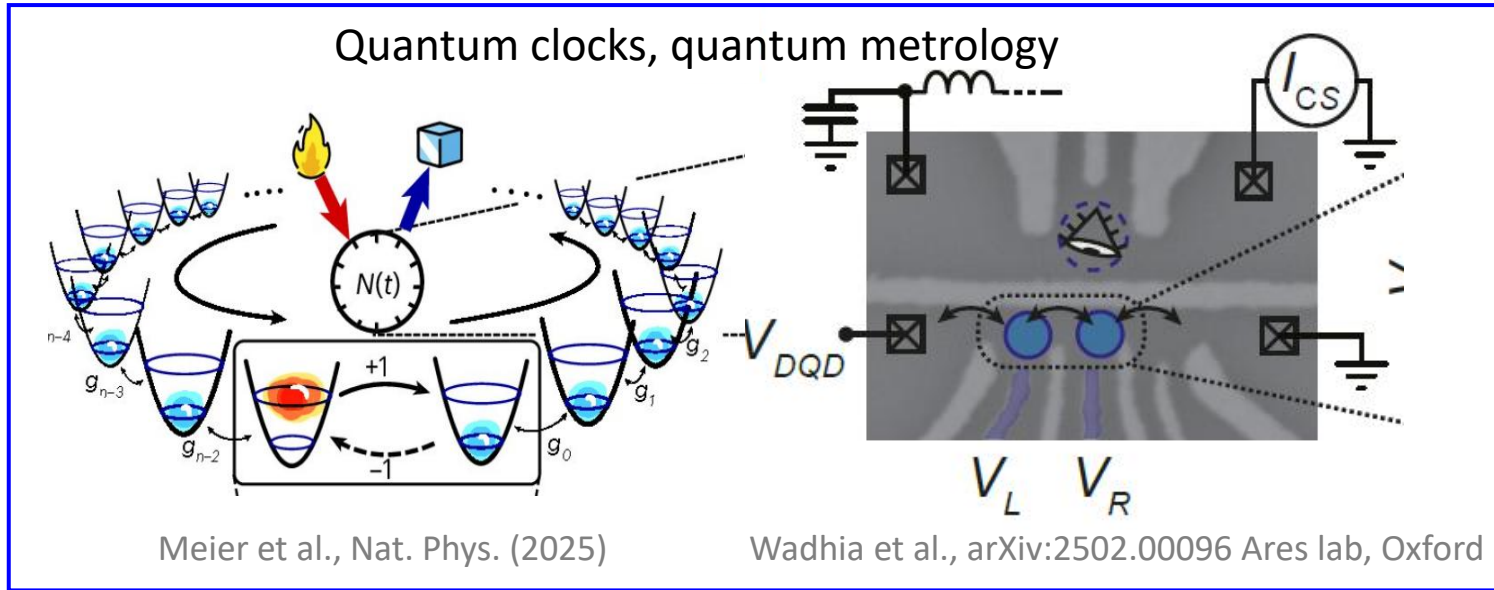


Signal-to-noise ratio (SNR) in quantum systems

Detection of anyons through their fractional statistics



Signal-to-noise ratio (SNR) in quantum systems



Quantum batteries

Quantum thermal machines

Quantum state tomography

Uncertainty relations for signal-to-noise ratio

- General fluctuation-dissipation inequality

Dechant, Sasa, J. Stat. Mech. 063209 (2018)

Dechant, Sasa, PNAS 117, 6430 (2020)

$$\frac{I_{\alpha}^2}{S_{\alpha\alpha}} \leq \xi$$

Uncertainty relations for signal-to-noise ratio

- General fluctuation-dissipation inequality

Dechant, Sasa, J. Stat. Mech. 063209 (2018)

Dechant, Sasa, PNAS 117, 6430 (2020)

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \xi$$

- Thermodynamic Uncertainty Relations (TUR) -> bound as a function of entropy production

$$\xi_{TUR} = \sigma/2k_B$$

Barato, Seifert, Phys. Rev. Lett. 114, 158101 (2015)

Gingrich et al., Phys. Rev. Lett. 116, 120601 (2016)

Review: Horowitz, Gingrich, Nat. Phys. 16, 15 (2019)

$$\xi_{QTUR} = I_\alpha \sinh(\sigma/2k_B I_\alpha)$$

Brandner, Saito, Phys. Rev. Lett. 135 (2025)

Uncertainty relations for signal-to-noise ratio

- General fluctuation-dissipation inequality

Dechant, Sasa, J. Stat. Mech. 063209 (2018)

Dechant, Sasa, PNAS 117, 6430 (2020)

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \xi$$

- Thermodynamic Uncertainty Relations (TUR) -> bound as a function of entropy production

$$\xi_{TUR} = \sigma / 2k_B$$

Barato, Seifert, Phys. Rev. Lett. 114, 158101 (2015)

Gingrich et al., Phys. Rev. Lett. 116, 120601 (2016)

Review: Horowitz, Gingrich, Nat. Phys. 16, 15 (2019)

$$\xi_{QTUR} = I_\alpha \sinh(\sigma / 2k_B I_\alpha)$$

Brandner, Saito, Phys. Rev. Lett. 135 (2025)

Role of external drives, superconducting leads, quantum coherence, TUR for general open quantum systems

Vo et al., Phys. Rev. E 102 (2020)

Koyuk, Seifert, Phys. Rev. Lett. 125 (2020)

Miller et al., Phys. Rev. Lett. 126 (2021)

Hasegawa, Nat. Comm. 14 (2023)

Yunoki and Hasegawa, arXiv:2502.09081 (2025)

Potts and Samuelsson, PRE 100 (2019)

Lopez et al., Phys. Rev. Res. 5 (2023)

Taddei and Fazio, Phys. Rev. B 108 (2023)

Manzano and Lopez, Phys. Rev. Res. 5 (2023)

Agarwalla, Segal, Phys. Rev. B 98 (2018)

Ptaszynski, Phys. Rev. B 98 (2018)

Liu and Segal, Phys. Rev. E 99 (2019)

Cangemi et al., Phys. Rev. B 102 (2020)

Liu et al., Phys. Rev. Lett. 125 (2020)

Kalaei et al., Phys. Rev. E 104 (2021)

Menczel, Journal of Physics A: Mathematical and Theoretical 54 (2021)

Ehrlich and Schaller, Phys. Rev. B 104 (2021)

Prech et al., Phys. Rev. Lett. 134 (2025)

Funo and Tajima, Phys. Rev. Lett. 134 (2025)

Uncertainty relations for signal-to-noise ratio

- General fluctuation-dissipation inequality

Dechant, Sasa, J. Stat. Mech. 063209 (2018)

Dechant, Sasa, PNAS 117, 6430 (2020)

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \xi$$

- Thermodynamic Uncertainty Relations (TUR) -> bound as a function of entropy production

$$\xi_{TUR} = \sigma / 2k_B$$

Barato, Seifert, Phys. Rev. Lett. 114, 158101 (2015)

Gingrich et al., Phys. Rev. Lett. 116, 120601 (2016)

Review: Horowitz, Gingrich, Nat. Phys. 16, 15 (2019)

$$\xi_{QTUR} = I_\alpha \sinh(\sigma / 2k_B I_\alpha)$$

Brandner, Saito, Phys. Rev. Lett. 135 (2025)

Role of external drives, superconducting leads, quantum coherence, TUR for general open quantum systems

- Kinetic Uncertainty Relations (KUR) -> bound as a function of «dynamical activity»

$$\xi_{KUR} = \mathcal{A}_\alpha$$

Terlizzi, Baiesi, J. Phys. A 52, 02LT03 (2018)

Uncertainty relations for signal-to-noise ratio

- General fluctuation-dissipation inequality

Dechant, Sasa, J. Stat. Mech. 063209 (2018)

Dechant, Sasa, PNAS 117, 6430 (2020)

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \xi$$

- Thermodynamic Uncertainty Relations (TUR) -> bound as a function of entropy production

$$\xi_{TUR} = \sigma / 2k_B$$

Barato, Seifert, Phys. Rev. Lett. 114, 158101 (2015)

Gingrich et al., Phys. Rev. Lett. 116, 120601 (2016)

Review: Horowitz, Gingrich, Nat. Phys. 16, 15 (2019)

$$\xi_{QTUR} = I_\alpha \sinh(\sigma / 2k_B I_\alpha)$$

Brandner, Saito, Phys. Rev. Lett. 135 (2025)

Role of external drives, superconducting leads, quantum coherence, TUR for general open quantum systems

- Kinetic Uncertainty Relations (KUR) -> bound as a function of «dynamical activity»

$$\xi_{KUR} = \mathcal{A}_\alpha$$

Terlizzi, Baiesi, J. Phys. A 52, 02LT03 (2018)

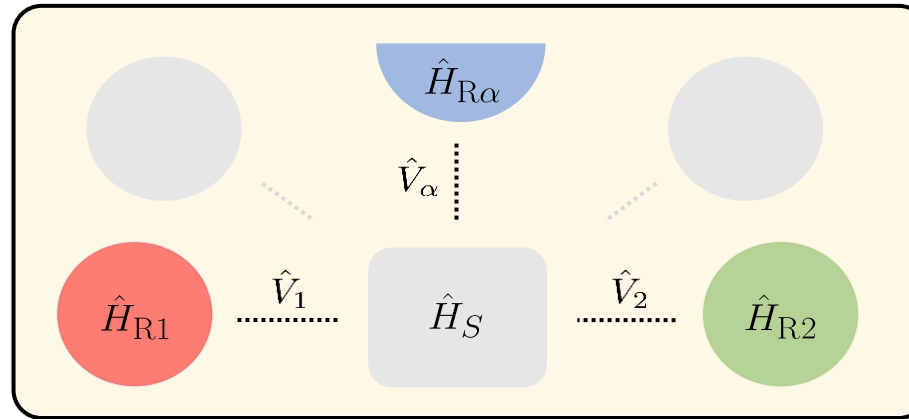
What is activity ? We need to assess the dynamics of open quantum systems.

Theoretical frameworks for assessing the dynamics of open quantum systems

Exact approach
Heisenberg equations

$$\frac{d}{dt}\hat{d}_j = +i[\hat{H}, \hat{d}_j]$$

$$\frac{d}{dt}\hat{c}_{k\alpha} = +i[\hat{H}, \hat{c}_{k\alpha}]$$



Generic open quantum system

$$\hat{H} = \hat{H}_S + \sum_{\alpha} \hat{H}_{R\alpha} + \sum_{\alpha} \hat{V}_{\alpha}$$

Particle current: $I_{\alpha}(t) = - \left\langle \frac{d\hat{N}_{\alpha}}{dt} \right\rangle$

Energy current: $J_{\alpha}(t) = - \left\langle \frac{d\hat{H}_{R\alpha}}{dt} \right\rangle$

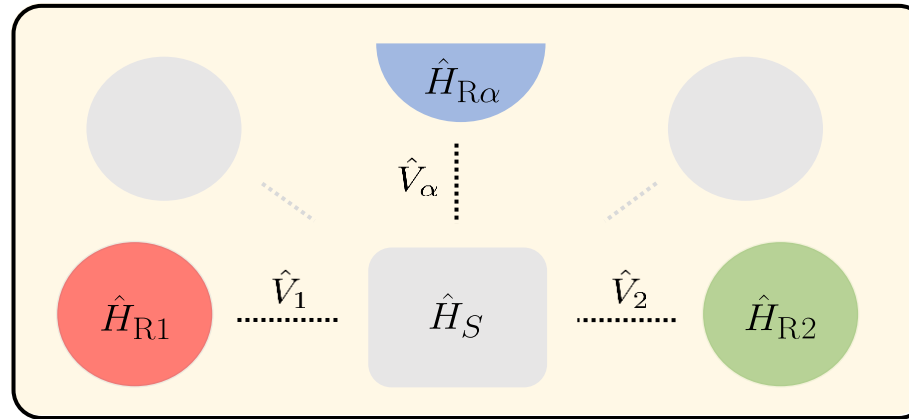
Noise: $S_{\alpha\alpha'}(t, t') = \left\langle \hat{I}_{\alpha}(t)\hat{I}_{\alpha'}(t') \right\rangle - \left\langle \hat{I}_{\alpha}(t) \right\rangle \left\langle \hat{I}_{\alpha'}(t') \right\rangle$

Theoretical frameworks for assessing the dynamics of open quantum systems

Exact approach
Heisenberg equations

$$\frac{d}{dt}\hat{d}_j = +i[\hat{H}, \hat{d}_j]$$

$$\frac{d}{dt}\hat{c}_{k\alpha} = +i[\hat{H}, \hat{c}_{k\alpha}]$$



Generic open quantum system

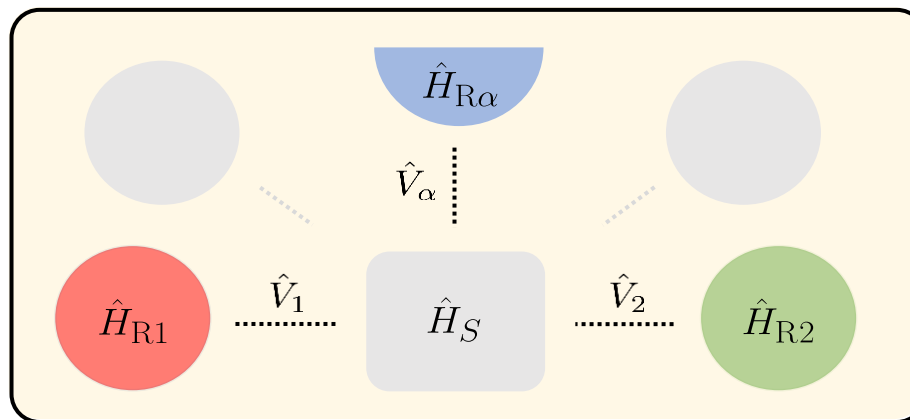
$$\hat{H} = \hat{H}_S + \sum_{\alpha} \hat{H}_{R\alpha} + \sum_{\alpha} \hat{V}_{\alpha}$$

Theoretical frameworks for assessing the dynamics of open quantum systems

Exact approach
Heisenberg equations

$$\frac{d}{dt} \hat{d}_j = +i[\hat{H}, \hat{d}_j]$$

$$\frac{d}{dt} \hat{c}_{k\alpha} = +i[\hat{H}, \hat{c}_{k\alpha}]$$

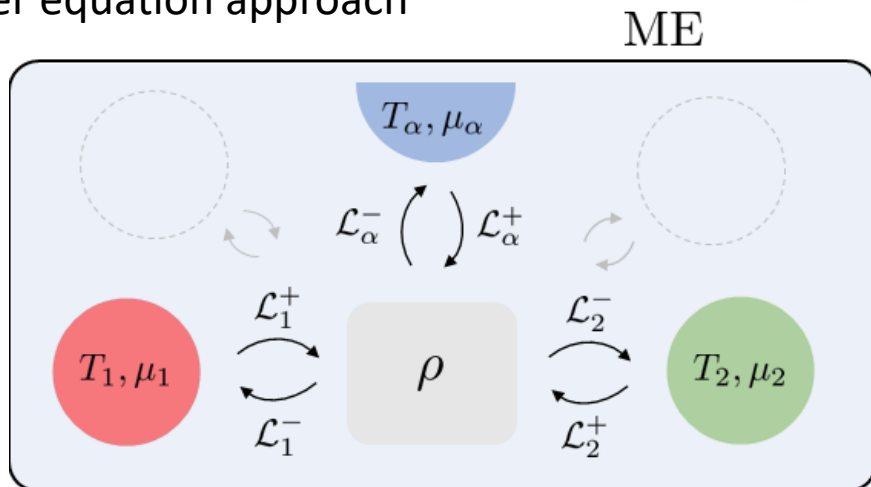


HE

Generic open quantum system

$$\hat{H} = \hat{H}_S + \sum_{\alpha} \hat{H}_{R\alpha} + \sum_{\alpha} \hat{V}_{\alpha}$$

Weak-coupling limit
Master equation approach



ME

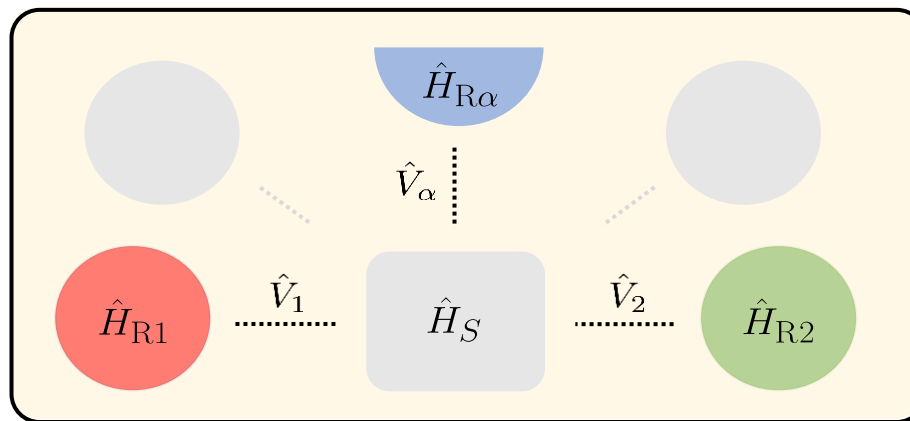
$$\dot{\rho}(t) = \mathcal{L} \rho(t) = \left(\mathcal{L}_0 + \sum_{j\alpha} (\mathcal{L}_{j\alpha}^+ + \mathcal{L}_{j\alpha}^-) \right) \rho(t)$$

Theoretical frameworks for assessing the dynamics of open quantum systems

Exact approach
Heisenberg equations

$$\frac{d}{dt} \hat{d}_j = +i[\hat{H}, \hat{d}_j]$$

$$\frac{d}{dt} \hat{c}_{k\alpha} = +i[\hat{H}, \hat{c}_{k\alpha}]$$



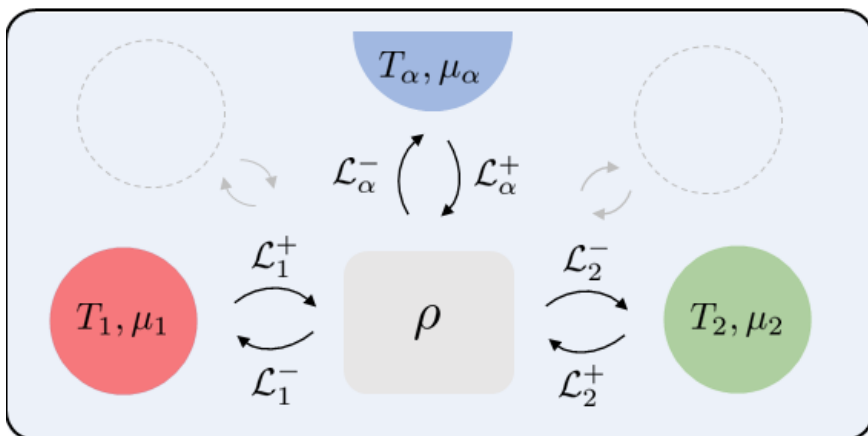
HE

Generic open quantum system

$$\hat{H} = \hat{H}_S + \sum_{\alpha} \hat{H}_{R\alpha} + \sum_{\alpha} \hat{V}_{\alpha}$$

Weak-coupling limit
Master equation approach

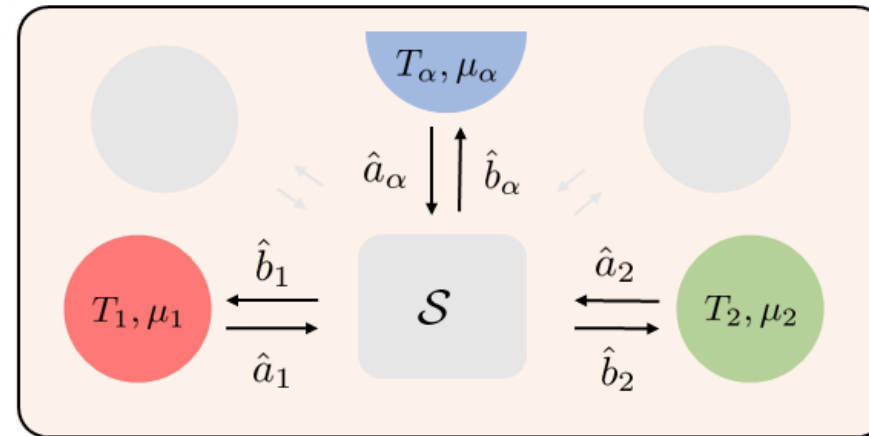
ME



$$\dot{\rho}(t) = \mathcal{L} \rho(t) = \left(\mathcal{L}_0 + \sum_{j\alpha} (\mathcal{L}_{j\alpha}^+ + \mathcal{L}_{j\alpha}^-) \right) \rho(t)$$

Steady state
Landauer-Büttiker approach

LB



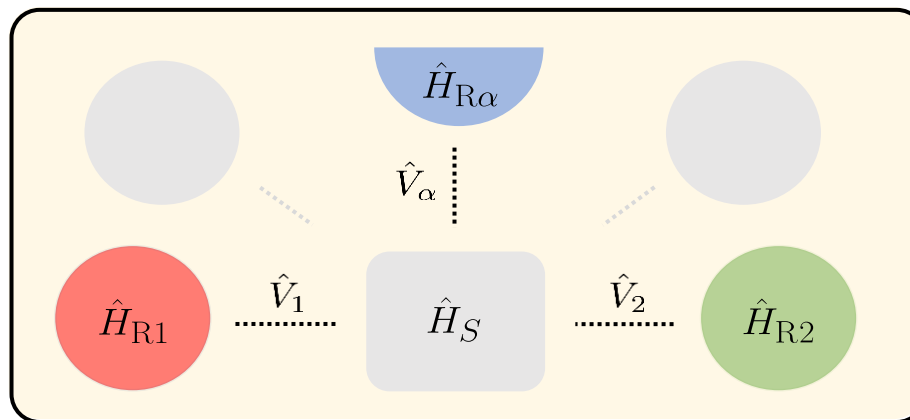
$$I_{\alpha}^{LB} = \sum_{\beta} \int \frac{d\epsilon}{2\pi} \mathcal{T}(\epsilon) [f_{\alpha}(\epsilon) - f_{\beta}(\epsilon)]$$

Theoretical frameworks for assessing the dynamics of open quantum systems

Exact approach
Heisenberg equations

$$\frac{d}{dt} \hat{d}_j = +i[\hat{H}, \hat{d}_j]$$

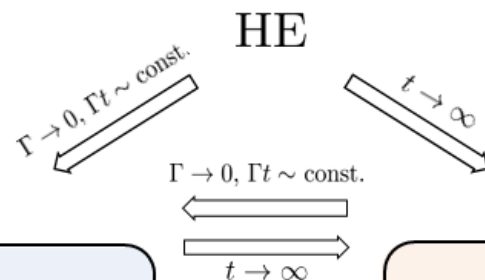
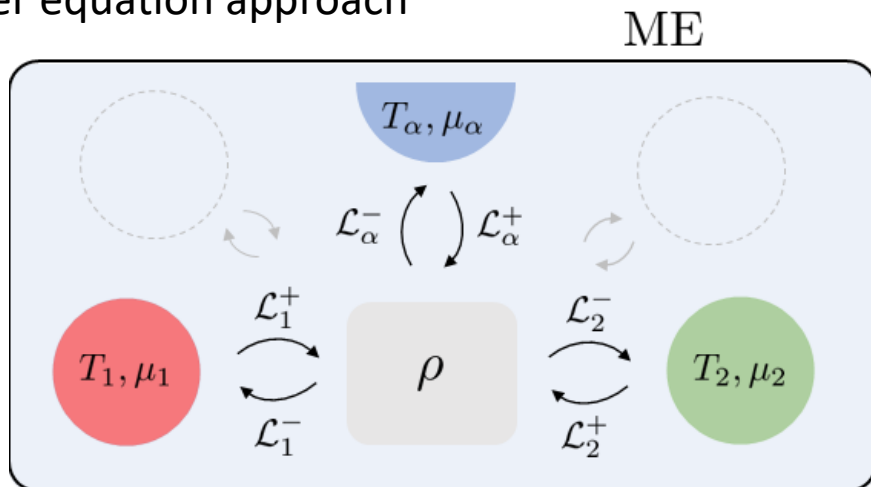
$$\frac{d}{dt} \hat{c}_{k\alpha} = +i[\hat{H}, \hat{c}_{k\alpha}]$$



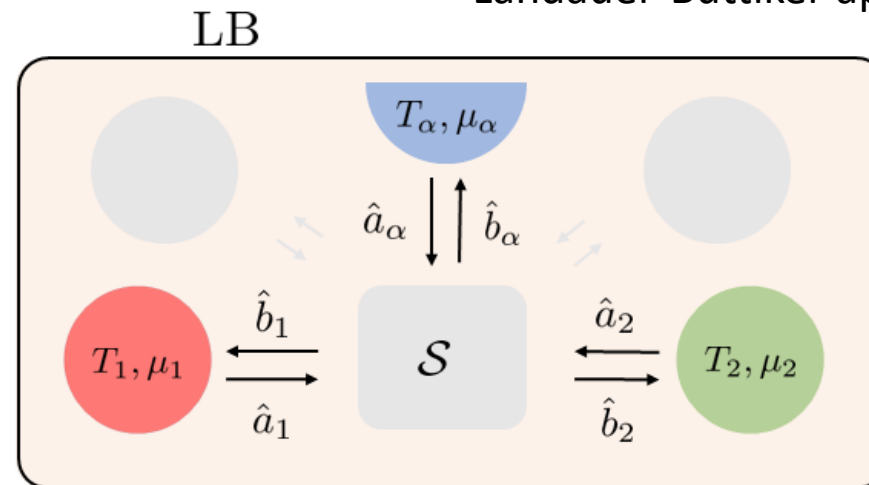
Generic open quantum system

$$\hat{H} = \hat{H}_S + \sum_{\alpha} \hat{H}_{R\alpha} + \sum_{\alpha} \hat{V}_{\alpha}$$

Weak-coupling limit
Master equation approach



Steady state
Landauer-Büttiker approach



Theoretical frameworks for assessing the dynamics of open quantum systems

	Master Equation	$\xleftarrow[\Gamma t \sim \text{const.}]{\Gamma \rightarrow 0}$	Heisenberg Equation	$\xrightarrow{t \rightarrow \infty}$	Landauer Büttiker
$I_\alpha(t)$	$\text{Tr} \left\{ \mathcal{I}_\alpha e^{\mathcal{L}(t-t_0)} \rho_0 \right\}$		$-i \left\langle [\hat{H}, \hat{N}_\alpha] \right\rangle$		$\int \frac{d\epsilon}{2\pi} \mathcal{T} (f_\alpha - f_{\bar{\alpha}})$
$J_\alpha(t)$	$\text{Tr} \left\{ \mathcal{J}_\alpha e^{\mathcal{L}(t-t_0)} \rho_0 \right\}$		$-i \left\langle [\hat{H}, \hat{H}_{R_\alpha}] \right\rangle$		$\int \frac{d\epsilon}{2\pi} \epsilon \mathcal{T} (f_\alpha - f_{\bar{\alpha}})$
$S_{\alpha\alpha'}(t, t')$	$\begin{aligned} & \delta_{\alpha\alpha'} \delta(t-t') \text{Tr} \left\{ \mathcal{A}_\alpha e^{\mathcal{L}(t-t_0)} \rho_0 \right\} \\ & + \Theta(t'-t) \text{Tr} \left\{ \mathcal{I}_{\alpha'} e^{\mathcal{L}(t'-t)} \mathcal{I}_\alpha e^{\mathcal{L}(t-t_0)} \rho_0 \right\} \\ & + \Theta(t-t') \text{Tr} \left\{ \mathcal{I}_\alpha e^{\mathcal{L}(t-t')} \mathcal{I}_{\alpha'} e^{\mathcal{L}(t'-t_0)} \rho_0 \right\} \\ & - \text{Tr} \left\{ \mathcal{I}_\alpha e^{\mathcal{L}(t-t_0)} \rho_0 \right\} \text{Tr} \left\{ \mathcal{I}_{\alpha'} e^{\mathcal{L}(t'-t_0)} \rho_0 \right\} \end{aligned}$		$\begin{aligned} & \left\langle \hat{I}_\alpha(t) \hat{I}_{\alpha'}(t') \right\rangle \\ & - \left\langle \hat{I}_\alpha(t) \right\rangle \left\langle \hat{I}_{\alpha'}(t') \right\rangle \end{aligned}$		$\begin{aligned} & \int \frac{d\epsilon}{2\pi} \mathcal{T} \sum_\beta f_\beta (1-f_\beta) \\ & + \mathcal{T}(1-\mathcal{T}) (f_L - f_R)^2 \end{aligned}$

Theoretical frameworks for assessing the dynamics of open quantum systems

	Master Equation	$\xleftarrow[\Gamma t \sim \text{const.}]{\Gamma \rightarrow 0}$ Heisenberg Equation $\xrightarrow{t \rightarrow \infty}$	Landauer Büttiker
$I_\alpha(t)$	$\text{Tr} \left\{ \mathcal{I}_\alpha e^{\mathcal{L}(t-t_0)} \rho_0 \right\}$	$-i \left\langle [\hat{H}, \hat{N}_\alpha] \right\rangle$	$\int \frac{d\epsilon}{2\pi} \mathcal{T} (f_\alpha - f_{\bar{\alpha}})$
$J_\alpha(t)$	$\text{Tr} \left\{ \mathcal{J}_\alpha e^{\mathcal{L}(t-t_0)} \rho_0 \right\}$	$-i \left\langle [\hat{H}, \hat{H}_{R\alpha}] \right\rangle$	$\int \frac{d\epsilon}{2\pi} \epsilon \mathcal{T} (f_\alpha - f_{\bar{\alpha}})$
$S_{\alpha\alpha'}(t, t')$	$\begin{aligned} & \delta_{\alpha\alpha'} \delta(t-t') \text{Tr} \left\{ \mathcal{A}_\alpha e^{\mathcal{L}(t-t_0)} \rho_0 \right\} \\ & + \Theta(t'-t) \text{Tr} \left\{ \mathcal{I}_{\alpha'} e^{\mathcal{L}(t'-t)} \mathcal{I}_\alpha e^{\mathcal{L}(t-t_0)} \rho_0 \right\} \\ & + \Theta(t-t') \text{Tr} \left\{ \mathcal{I}_\alpha e^{\mathcal{L}(t-t')} \mathcal{I}_{\alpha'} e^{\mathcal{L}(t'-t_0)} \rho_0 \right\} \\ & - \text{Tr} \left\{ \mathcal{I}_\alpha e^{\mathcal{L}(t-t_0)} \rho_0 \right\} \text{Tr} \left\{ \mathcal{I}_{\alpha'} e^{\mathcal{L}(t'-t_0)} \rho_0 \right\} \end{aligned}$	$\begin{aligned} & \left\langle \hat{I}_\alpha(t) \hat{I}_{\alpha'}(t') \right\rangle \\ & - \left\langle \hat{I}_\alpha(t) \right\rangle \left\langle \hat{I}_{\alpha'}(t') \right\rangle \end{aligned}$	$\begin{aligned} & \int \frac{d\epsilon}{2\pi} \mathcal{T} \sum_\beta f_\beta (1-f_\beta) \\ & + \mathcal{T}(1-\mathcal{T}) (f_L - f_R)^2 \end{aligned}$

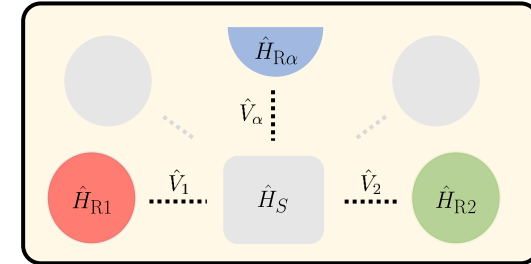
Exact expressions (Heisenberg equations) for the noise at all times for paradigmatic (and simple) open quantum systems



Uncertainty relations for signal-to-noise ratio

- Upper bound for the Signal-to-Noise Ratio (SNR)

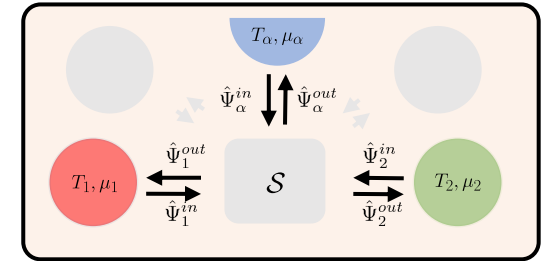
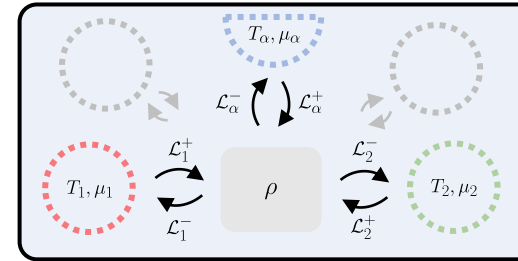
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \xi$$



- Thermodynamic Uncertainty Relations (TUR)

$$\xi_{TUR} = \sigma / 2k_B$$

$$\xi_{QTUR} = I_\alpha \sinh(\sigma / 2k_B I_\alpha)$$



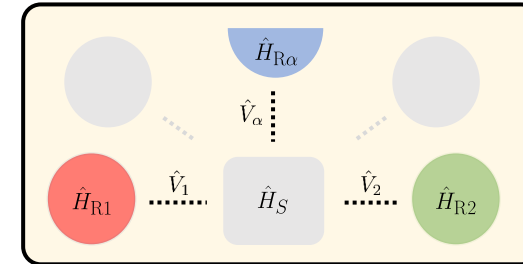
- Kinetic Uncertainty Relations (KUR)

$$\xi_{KUR} = \mathcal{A}_\alpha$$

Uncertainty relations for signal-to-noise ratio

- Upper bound for the Signal-to-Noise Ratio (SNR)

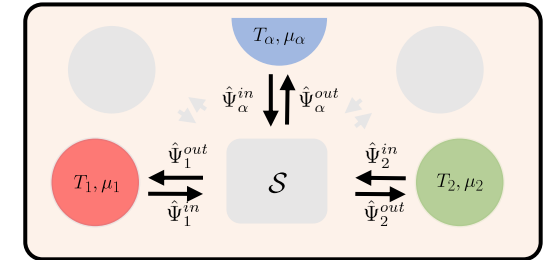
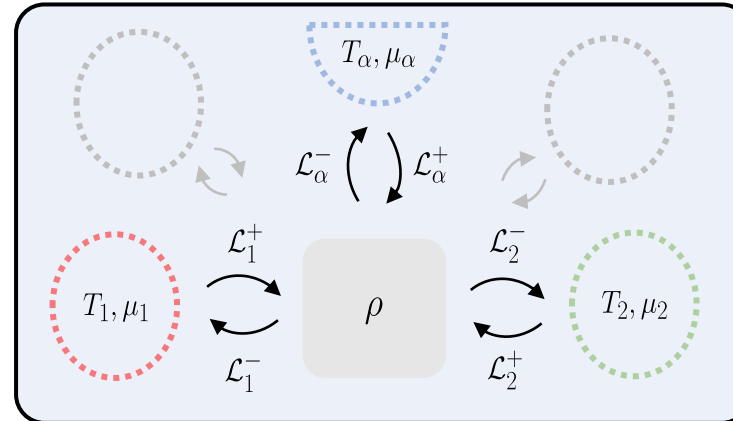
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \xi$$



- Thermodynamic Uncertainty Relations (TUR)

$$\xi_{TUR} = \sigma / 2k_B$$

$$\xi_{QTUR} = I_\alpha \sinh(\sigma / 2k_B I_\alpha)$$



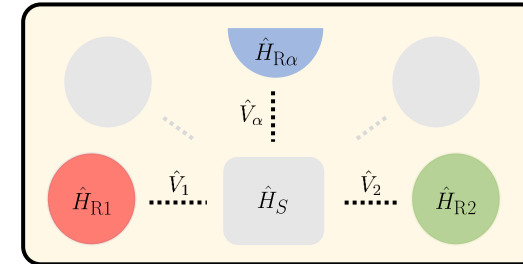
- Kinetic Uncertainty Relations (KUR)

$$\xi_{KUR} = \mathcal{A}_\alpha$$

Uncertainty relations for signal-to-noise ratio

- Upper bound for the Signal-to-Noise Ratio (SNR)

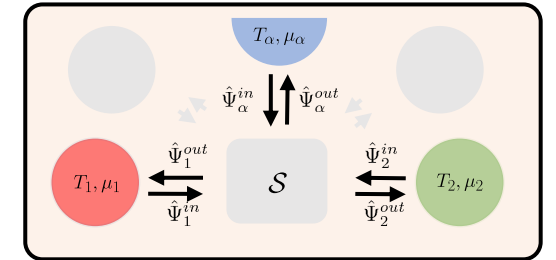
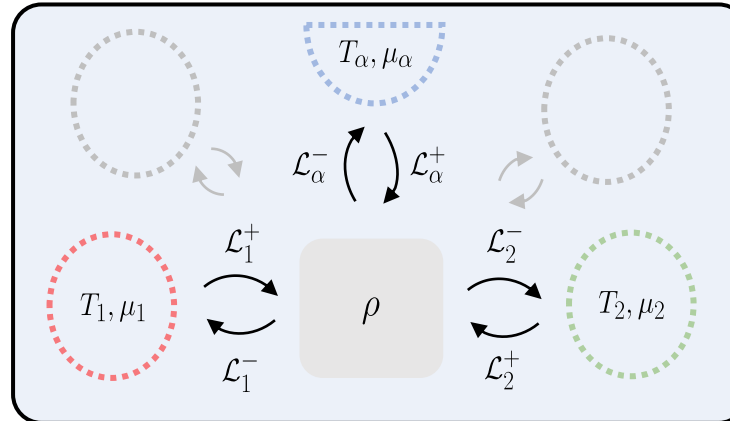
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \xi$$



- Thermodynamic Uncertainty Relations (TUR)

$$\xi_{TUR} = \sigma / 2k_B$$

$$\xi_{QTUR} = I_\alpha \sinh(\sigma / 2k_B I_\alpha)$$



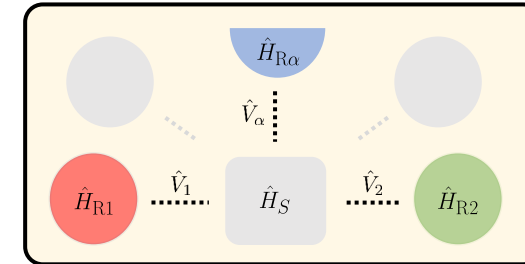
- Kinetic Uncertainty Relations (KUR)

$$\xi_{KUR} = \mathcal{A}_\alpha = \text{Tr}\{(\mathcal{L}_\alpha^+ + \mathcal{L}_\alpha^-) \rho\}$$

Uncertainty relations for signal-to-noise ratio

- Upper bound for the Signal-to-Noise Ratio (SNR)

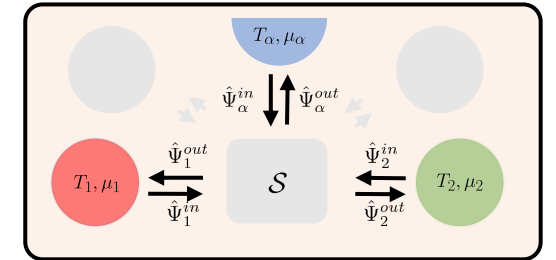
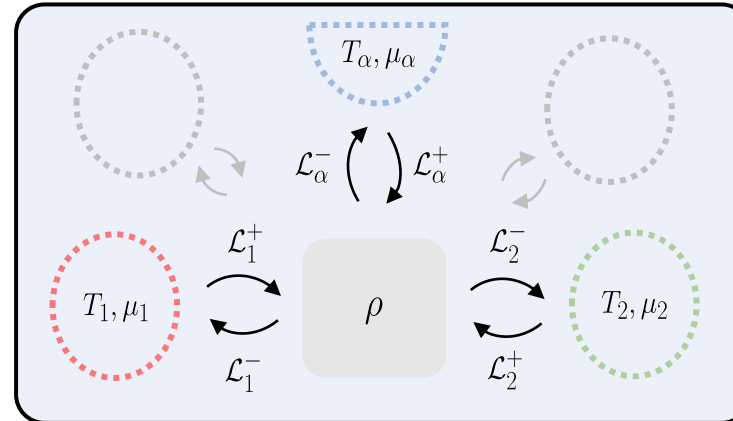
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \xi$$



- Thermodynamic Uncertainty Relations (TUR)

$$\xi_{TUR} = \sigma / 2k_B$$

$$\xi_{QTUR} = I_\alpha \sinh(\sigma / 2k_B I_\alpha)$$



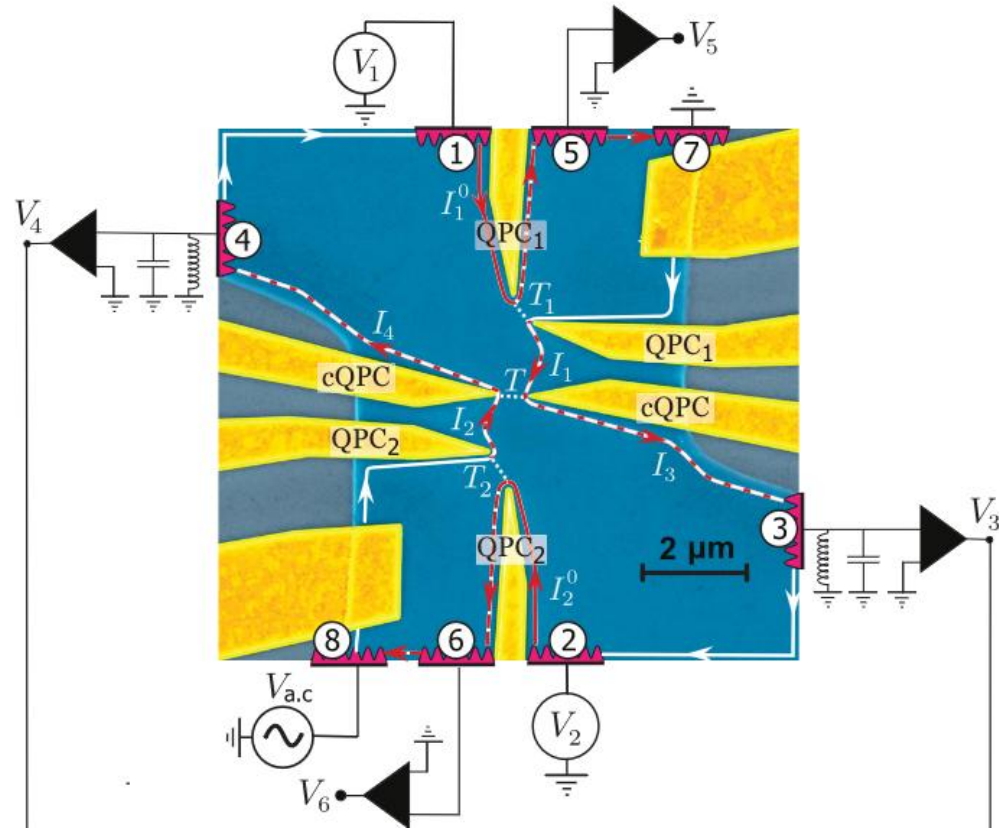
- Kinetic Uncertainty Relations (KUR)

$$\xi_{KUR} = \mathcal{A}_\alpha = \text{Tr}\{(\mathcal{L}_\alpha^+ + \mathcal{L}_\alpha^-) \rho\}$$

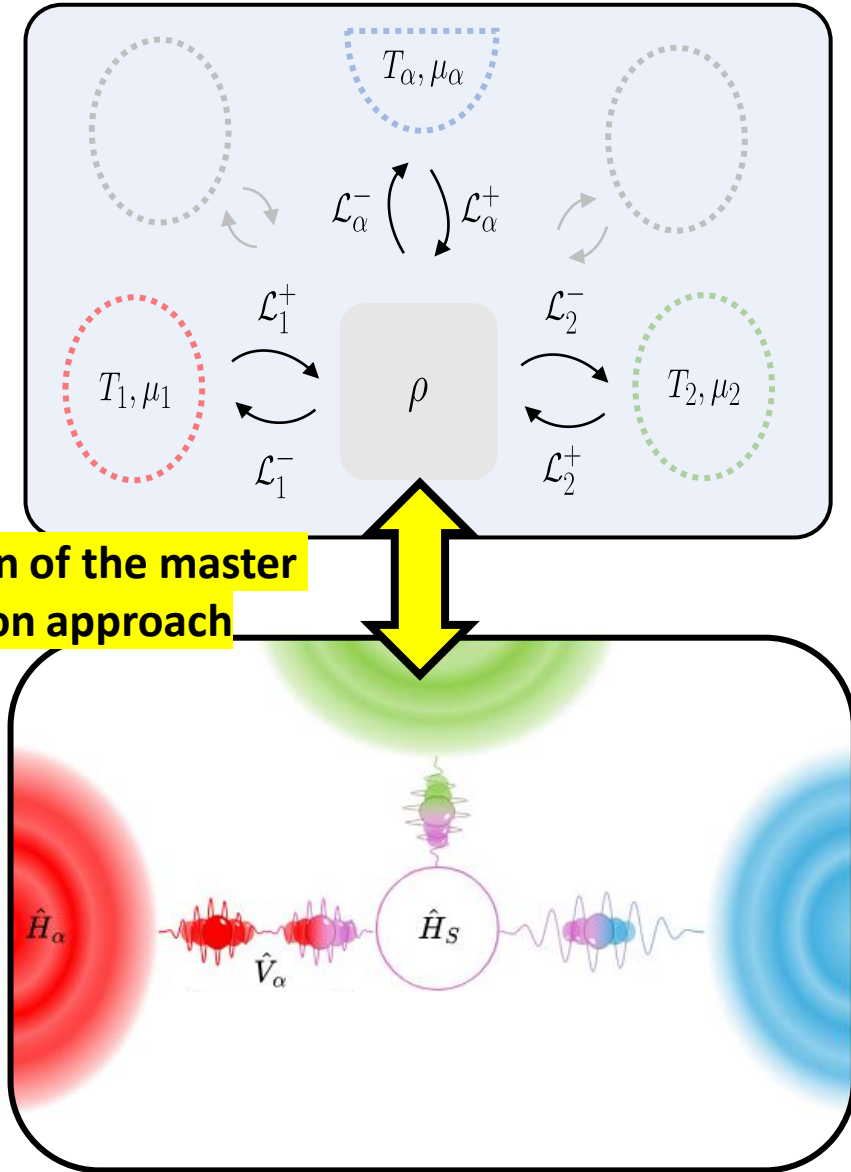
Activity = Total rate of jumps

Only defined within the master equation approach,
e.g. at weak coupling

When the notion of quantum jumps is challenged ...



Breakdown of the master equation approach



- What is the activity beyond the weak-coupling regime ?
- Can we define KUR-type relations in the strong coupling regime ?

Outline for the rest of this talk

1. Generalized definition of activity

- ❖ valid at arbitrary coupling strength
- ❖ beyond master equation approach
- ❖ Explicit expressions in the steady-state regime

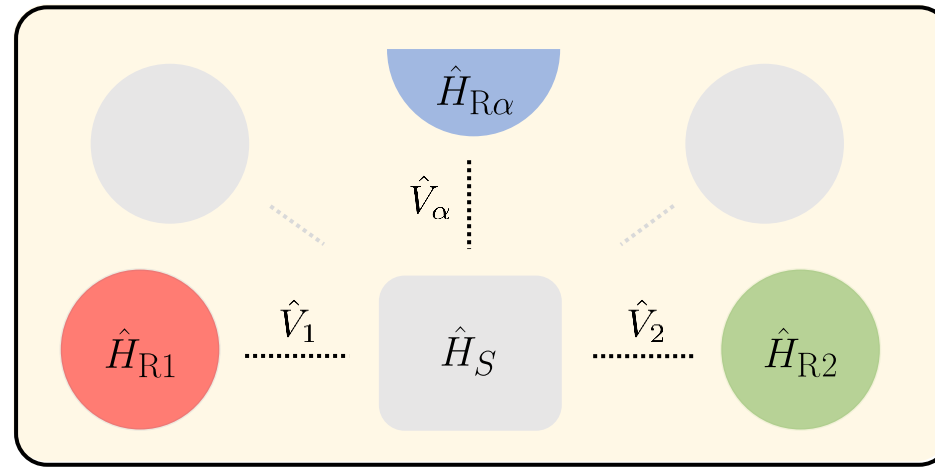
2. Quantum KUR valid at arbitrary coupling strengths

3. Paradigmatic examples

- ❖ Double quantum dot
- ❖ Quantum point contact in a two-terminal device

Generalized activity valid at arbitrary coupling strength

Generic open quantum system



$$\hat{H} = \hat{H}_S + \sum_{\alpha} \hat{H}_{R\alpha} + \sum_{\alpha} \hat{V}_{\alpha}$$

$$\hat{V}_{\alpha} = \sum_k \left(t_{k\alpha}^* \hat{c}_{k\alpha}^{\dagger} \hat{d} + t_{k\alpha} \hat{d}^{\dagger} \hat{c}_{k\alpha} \right)$$

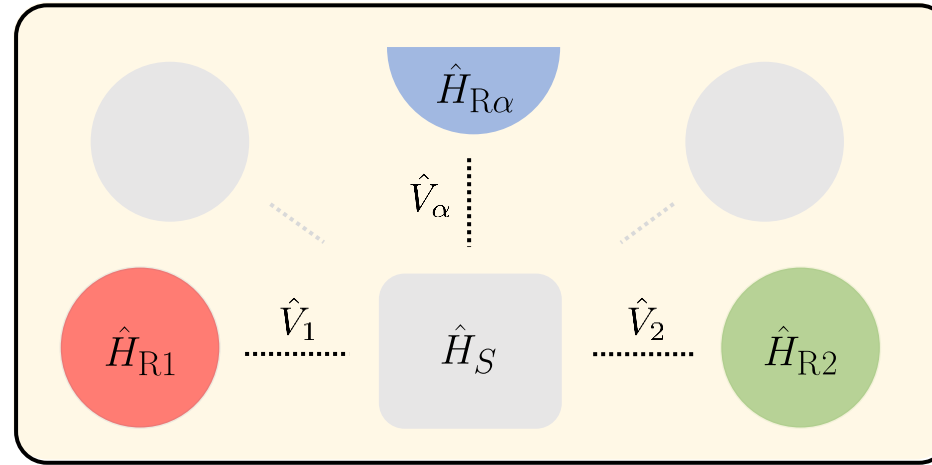
$$\mathcal{A}_{\alpha}(t) = \frac{1}{2\hbar^2} \int_{-t}^t d\tau \left\langle \left\langle \left\{ \hat{V}_{\alpha}(t), \hat{V}_{\alpha}(t + \tau) \right\} \right\rangle \right\rangle$$

Generalized activity corresponds to the zero-frequency component of spectrum of exchange rate fluctuations.

Blasi, Rodriguez, Moskalets, Lopez, Haack, arXiv:2505.13200 (2025)

Generalized activity valid at arbitrary coupling strength

Generic open quantum system



$$\hat{H} = \hat{H}_S + \sum_{\alpha} \hat{H}_{R\alpha} + \sum_{\alpha} \hat{V}_{\alpha}$$

$$\hat{V}_{\alpha} = \sum_k \left(t_{k\alpha}^* \hat{c}_{k\alpha}^{\dagger} \hat{d} + t_{k\alpha} \hat{d}^{\dagger} \hat{c}_{k\alpha} \right)$$

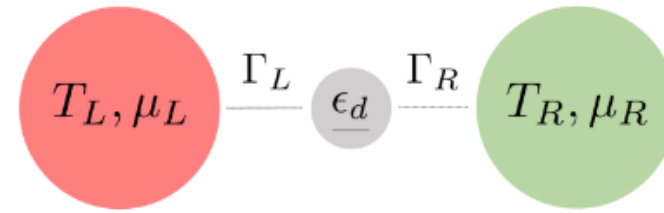
$$\mathcal{A}_{\alpha}(t) = \frac{1}{2\hbar^2} \int_{-t}^t d\tau \left\langle\left\langle \left\{ \hat{V}_{\alpha}(t), \hat{V}_{\alpha}(t + \tau) \right\} \right\rangle\right\rangle$$

Generalized activity corresponds to the zero-frequency component of spectrum of exchange rate fluctuations.

Blasi, Rodriguez, Moskalets, Lopez, Haack, arXiv:2505.13200 (2025)

How did we arrive to this definition ? Where does it come from ?

Interlude : derivation of the definition of the generalized activity (via identification)



- Start from the model of a single-level quantum dot
- Compute the exact expression of the noise from Heisenberg equations

❖ Hamiltonian $\hat{H} = \epsilon_d \hat{d}^\dagger \hat{d} + \sum_{k\alpha} \epsilon_{k\alpha} \hat{c}_{k\alpha}^\dagger \hat{c}_{k\alpha} + \sum_{k\alpha} [t_{k\alpha}^* \hat{c}_{k\alpha}^\dagger \hat{d} + t_{k\alpha} \hat{d}^\dagger \hat{c}_{k\alpha}].$

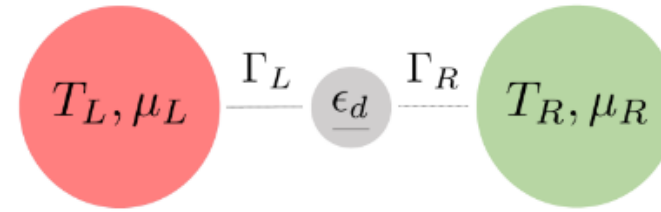
❖ Heisenberg equations $\frac{d}{dt} \hat{d} = i[\hat{H}, \hat{d}] = -i\epsilon_d \hat{d} - i \sum_{k\alpha} t_{k\alpha} \hat{c}_{k\alpha},$

$\frac{d}{dt} \hat{c}_{k\alpha} = i[\hat{H}, \hat{c}_{k\alpha}] = -i\epsilon_{k\alpha} \hat{c}_{k\alpha} - i t_{k\alpha}^* \hat{d}.$

Solvable

Blasi, Khandelwal, Haack, Phys. Rev. Res. 6 (2024)

Interlude : derivation of the definition of the generalized activity (via identification)



- Start from the model of a single-level quantum dot
- Compute the exact expression of the noise from Heisenberg equations

❖ Hamiltonian $\hat{H} = \epsilon_d \hat{d}^\dagger \hat{d} + \sum_{k\alpha} \epsilon_{k\alpha} \hat{c}_{k\alpha}^\dagger \hat{c}_{k\alpha} + \sum_{k\alpha} [t_{k\alpha}^* \hat{c}_{k\alpha}^\dagger \hat{d} + t_{k\alpha} \hat{d}^\dagger \hat{c}_{k\alpha}].$

❖ Heisenberg equations $\frac{d}{dt} \hat{d} = i[\hat{H}, \hat{d}] = -i\epsilon_d \hat{d} - i \sum_{k\alpha} t_{k\alpha} \hat{c}_{k\alpha},$

$\frac{d}{dt} \hat{c}_{k\alpha} = i[\hat{H}, \hat{c}_{k\alpha}] = -i\epsilon_{k\alpha} \hat{c}_{k\alpha} - i t_{k\alpha}^* \hat{d}.$

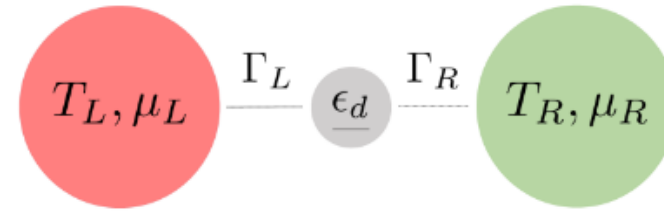
Solvable

Blasi, Khandelwal, Haack, Phys. Rev. Res. 6 (2024)

❖ Noise $S_{\alpha\alpha'}^{\text{HE}}(t, t') = - \sum_{kk'} \{ t_{k\alpha}^* t_{k'\alpha'}^* \langle \hat{c}_{k\alpha}^\dagger(t) \hat{d}(t') \rangle \langle \hat{d}(t) \hat{c}_{k'\alpha'}^\dagger(t') \rangle$
 $- t_{k\alpha}^* t_{k'\alpha'} \langle \hat{c}_{k\alpha}^\dagger(t) \hat{c}_{k'\alpha'}(t') \rangle \langle \hat{d}(t) \hat{d}^\dagger(t') \rangle$
 $- t_{k\alpha} t_{k'\alpha'}^* \langle \hat{d}^\dagger(t) \hat{d}(t') \rangle \langle \hat{c}_{k\alpha}(t) \hat{c}_{k'\alpha'}^\dagger(t') \rangle$
 $+ t_{k\alpha} t_{k'\alpha'} \langle \hat{d}^\dagger(t) \hat{c}_{k'\alpha'}(t') \rangle \langle \hat{c}_{k\alpha}(t) \hat{d}^\dagger(t') \rangle \}.$

Interlude : derivation of the definition of the generalized activity (via identification)

- Start from the model of a single-level quantum dot

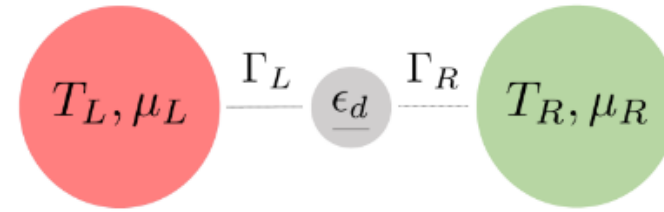


- Compute the exact expression of the noise from Heisenberg equations

$$\begin{aligned}
 S_{\alpha\alpha'}^{\text{HE}}(t, t') &= \Gamma_{\alpha}\Gamma_{\alpha'} \sum_{\beta, \beta'} \frac{\Gamma_{\beta}\Gamma_{\beta'}}{\Gamma^2} [\Lambda_0(t, t') - \Lambda_{\alpha}^{(1)}(t, t') + \Lambda_{\beta}^{(2)}(t, t')] [\bar{\Lambda}_0(t', t) - \bar{\Lambda}_{\alpha'}^{(1)}(t', t) + \bar{\Lambda}_{\beta'}^{(2)}(t', t)] \\
 &+ \Gamma_{\alpha}\Gamma_{\alpha'} \sum_{\beta, \beta'} \frac{\Gamma_{\beta}\Gamma_{\beta'}}{\Gamma^2} [\Lambda_0(t, t') - \Lambda_{\alpha}^{(1)}(t, t') - \Lambda_{\alpha'}^{(1)}(t', t)^* + \Lambda_{\beta}^{(2)}(t, t')] [\bar{\Lambda}_0(t', t) + \bar{\Lambda}_{\beta'}^{(2)}(t', t)] \\
 &+ \Gamma_{\alpha}\delta_{\alpha\alpha'} \sum_{\beta} \Gamma_{\beta}\Lambda_{\alpha}(t, t') [\bar{\Lambda}_0(t', t) + \bar{\Lambda}_{\beta}^{(2)}(t', t)] + \{\text{c.c. with } \Lambda \leftrightarrow \bar{\Lambda}\},
 \end{aligned}$$

Interlude : derivation of the definition of the generalized activity (via identification)

- Start from the model of a single-level quantum dot



- Compute the exact expression of the noise from Heisenberg equations

$$\begin{aligned}
 S_{\alpha\alpha'}^{\text{HE}}(t, t') &= \Gamma_{\alpha}\Gamma_{\alpha'} \sum_{\beta, \beta'} \frac{\Gamma_{\beta}\Gamma_{\beta'}}{\Gamma^2} [\Lambda_0(t, t') - \Lambda_{\alpha}^{(1)}(t, t') + \Lambda_{\beta}^{(2)}(t, t')] [\bar{\Lambda}_0(t', t) - \bar{\Lambda}_{\alpha'}^{(1)}(t', t) + \bar{\Lambda}_{\beta'}^{(2)}(t', t)] \\
 &+ \Gamma_{\alpha}\Gamma_{\alpha'} \sum_{\beta, \beta'} \frac{\Gamma_{\beta}\Gamma_{\beta'}}{\Gamma^2} [\Lambda_0(t, t') - \Lambda_{\alpha}^{(1)}(t, t') - \Lambda_{\alpha'}^{(1)}(t', t)^* + \Lambda_{\beta}^{(2)}(t, t')] [\bar{\Lambda}_0(t', t) + \bar{\Lambda}_{\beta'}^{(2)}(t', t)] \\
 &+ \Gamma_{\alpha}\delta_{\alpha\alpha'} \sum_{\beta} \Gamma_{\beta}\Lambda_{\alpha}(t, t') [\bar{\Lambda}_0(t', t) + \bar{\Lambda}_{\beta}^{(2)}(t', t)] + \{\text{c.c. with } \Lambda \leftrightarrow \bar{\Lambda}\},
 \end{aligned}$$

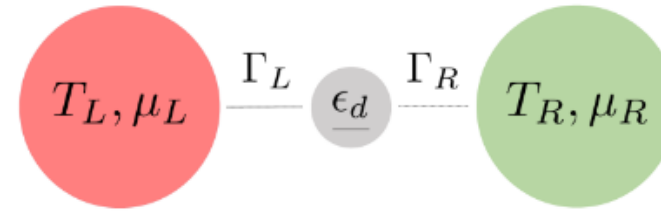
- Take the weak coupling protocol and compare with the expression obtained from a master equation approach

Blasi, Khandelwal, Haack, Phys. Rev. Res. 6 (2024)

$$S_{\alpha\alpha'}^{\text{ME}}(t, t') = \delta_{\alpha\alpha'}\delta(t - t') \sum_{\beta} \frac{\Gamma_{\alpha}\Gamma_{\beta}}{\Gamma} \{ e^{-\Gamma t} (f_{\beta} - n_d) [f_{\alpha} - (1 - f_{\alpha})] + [f_{\beta}(1 - f_{\alpha}) + f_{\alpha}(1 - f_{\beta})] \} + \dots \quad \text{A}$$

Interlude : derivation of the definition of the generalized activity (via identification)

- Start from the model of a single-level quantum dot



- Compute the exact expression of the noise from Heisenberg equations

$$\begin{aligned}
 S_{\alpha\alpha'}^{\text{HE}}(t, t') &= \Gamma_{\alpha}\Gamma_{\alpha'} \sum_{\beta, \beta'} \frac{\Gamma_{\beta}\Gamma_{\beta'}}{\Gamma^2} [\Lambda_0(t, t') - \Lambda_{\alpha}^{(1)}(t, t') + \Lambda_{\beta}^{(2)}(t, t')] [\bar{\Lambda}_0(t', t) - \bar{\Lambda}_{\alpha'}^{(1)}(t', t) + \bar{\Lambda}_{\beta'}^{(2)}(t', t)] \\
 &+ \Gamma_{\alpha}\Gamma_{\alpha'} \sum_{\beta, \beta'} \frac{\Gamma_{\beta}\Gamma_{\beta'}}{\Gamma^2} [\Lambda_0(t, t') - \Lambda_{\alpha}^{(1)}(t, t') - \Lambda_{\alpha'}^{(1)}(t', t)^* + \Lambda_{\beta}^{(2)}(t, t')] [\bar{\Lambda}_0(t', t) + \bar{\Lambda}_{\beta'}^{(2)}(t', t)] \\
 &+ \Gamma_{\alpha}\delta_{\alpha\alpha'} \sum_{\beta} \Gamma_{\beta}\Lambda_{\alpha}(t, t') [\bar{\Lambda}_0(t', t) + \bar{\Lambda}_{\beta}^{(2)}(t', t)] + \{\text{c.c. with } \Lambda \leftrightarrow \bar{\Lambda}\},
 \end{aligned}$$

- Take the weak coupling protocol and compare with the expression obtained from a master equation approach

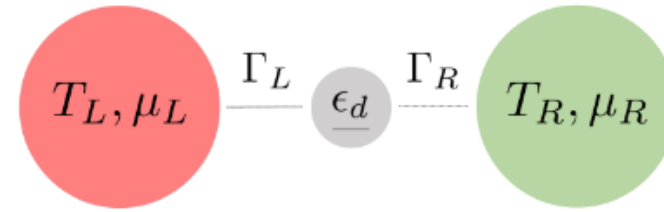
Blasi, Khandelwal, Haack, Phys. Rev. Res. 6 (2024)

$$S_{\alpha\alpha'}^{\text{ME}}(t, t') = \delta_{\alpha\alpha'}\delta(t - t') \sum_{\beta} \frac{\Gamma_{\alpha}\Gamma_{\beta}}{\Gamma} \{ e^{-\Gamma t} (f_{\beta} - n_d) [f_{\alpha} - (1 - f_{\alpha})] + [f_{\beta}(1 - f_{\alpha}) + f_{\alpha}(1 - f_{\beta})] \} \quad \text{A}$$

+ . . .

- Identify which terms lead to the activity in the weak coupling regime

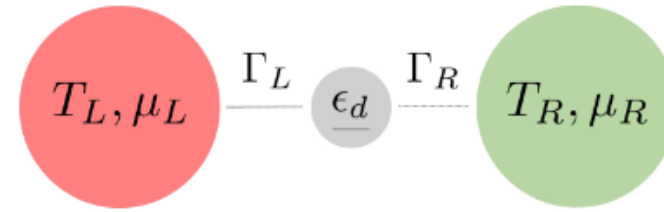
Interlude : derivation of the definition of the generalized activity (via identification)



- Start from the model of a single-level quantum dot
- Compute the exact expression of the noise from Heisenberg equations
- Take the weak coupling protocol and compare with the expression obtained from a master equation approach
- Identify which terms lead to the activity in the weak coupling regime

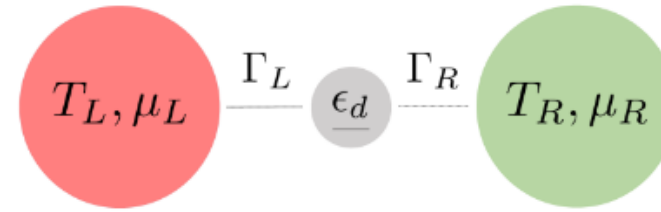
Blasi, Khandelwal, Haack, Phys. Rev. Res. 6 (2024)

Interlude : derivation of the definition of the generalized activity (via identification)



- Start from the model of a single-level quantum dot
- Compute the exact expression of the noise from Heisenberg equations
- Take the weak coupling protocol and compare with the expression obtained from a master equation approach
Blasi, Khandelwal, Haack, Phys. Rev. Res. 6 (2024)
- Identify which terms lead to the activity in the weak coupling regime
- Understand the origin of the different terms from the total Hamiltonian. The key quantity turns out to be the correlator of the system-bath interaction Hamiltonian.

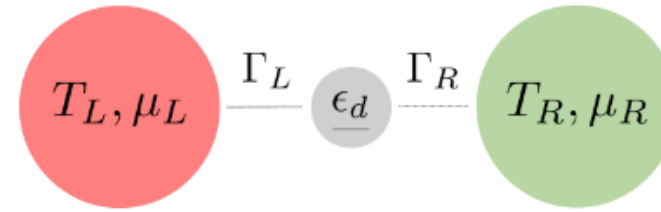
Interlude : derivation of the definition of the generalized activity (via identification)



- Start from the model of a single-level quantum dot
- Compute the exact expression of the noise from Heisenberg equations
- Take the weak coupling protocol and compare with the expression obtained from a master equation approach
Blasi, Khandelwal, Haack, Phys. Rev. Res. 6 (2024)
- Identify which terms lead to the activity in the weak coupling regime
- Understand the origin of the different terms from the total Hamiltonian. The key quantity turns out to be the correlator of the system-bath interaction Hamiltonian.

$$\begin{aligned} \langle\langle \hat{V}_\alpha(t) \hat{V}_\alpha(t + \tau) \rangle\rangle &= \sum_{kk'} t_{k\alpha}^* t_{k'\alpha}^* \langle c_{k\alpha}^\dagger(t) d(t + \tau) \rangle \langle d(t) c_{k'\alpha}^\dagger(t + \tau) \rangle + \sum_{kk'} t_{k\alpha}^* t_{k'\alpha} \langle c_{k\alpha}^\dagger(t) c_{k'\alpha}(t + \tau) \rangle \langle d(t) d^\dagger(t + \tau) \rangle \\ &+ \sum_{kk'} t_{k\alpha} t_{k'\alpha}^* \langle d^\dagger(t) d(t + \tau) \rangle \langle c_{k\alpha}(t) c_{k'\alpha}^\dagger(t + \tau) \rangle + \sum_{kk'} t_{k\alpha} t_{k'\alpha} \langle d^\dagger(t) c_{k\alpha}(t + \tau) \rangle \langle c_{k\alpha}(t) d^\dagger(t + \tau) \rangle \end{aligned}$$

Interlude : derivation of the definition of the generalized activity (via identification)



- Start from the model of a single-level quantum dot
- Compute the exact expression of the noise from Heisenberg equations
- Take the weak coupling protocol and compare with the expression obtained from a master equation approach
- Identify which terms lead to the activity in the weak coupling regime
- Understand the origin of the different terms from the total Hamiltonian. The key quantity turns out to be the correlator of the system-bath interaction Hamiltonian.

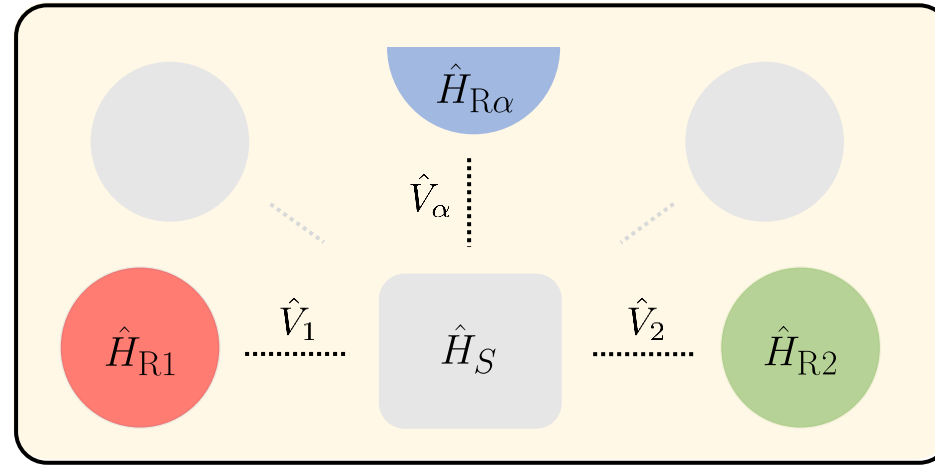
Blasi, Khandelwal, Haack, Phys. Rev. Res. 6 (2024)

$$\begin{aligned} \langle\langle \hat{V}_\alpha(t) \hat{V}_\alpha(t + \tau) \rangle\rangle &= \sum_{kk'} t_{k\alpha}^* t_{k'\alpha}^* \langle c_{k\alpha}^\dagger(t) d(t + \tau) \rangle \langle d(t) c_{k'\alpha}^\dagger(t + \tau) \rangle + \sum_{kk'} t_{k\alpha}^* t_{k'\alpha} \langle c_{k\alpha}^\dagger(t) c_{k'\alpha}(t + \tau) \rangle \langle d(t) d^\dagger(t + \tau) \rangle \\ &+ \sum_{kk'} t_{k\alpha} t_{k'\alpha}^* \langle d^\dagger(t) d(t + \tau) \rangle \langle c_{k\alpha}(t) c_{k'\alpha}^\dagger(t + \tau) \rangle + \sum_{kk'} t_{k\alpha} t_{k'\alpha} \langle d^\dagger(t) c_{k\alpha}(t + \tau) \rangle \langle c_{k\alpha}(t) d^\dagger(t + \tau) \rangle \end{aligned}$$

- Definition of the generalized activity $\mathcal{A}_\alpha(t) = \frac{1}{2\hbar^2} \int_{-t}^t d\tau \langle\langle \{ \hat{V}_\alpha(t), \hat{V}_\alpha(t + \tau) \} \rangle\rangle$

New definition: Generalized activity

Generic open quantum system



$$\hat{H} = \hat{H}_S + \sum_{\alpha} \hat{H}_{R\alpha} + \sum_{\alpha} \hat{V}_{\alpha}$$

$$\hat{V}_{\alpha} = \sum_k \left(t_{k\alpha}^* \hat{c}_{k\alpha}^{\dagger} \hat{d} + t_{k\alpha} \hat{d}^{\dagger} \hat{c}_{k\alpha} \right)$$

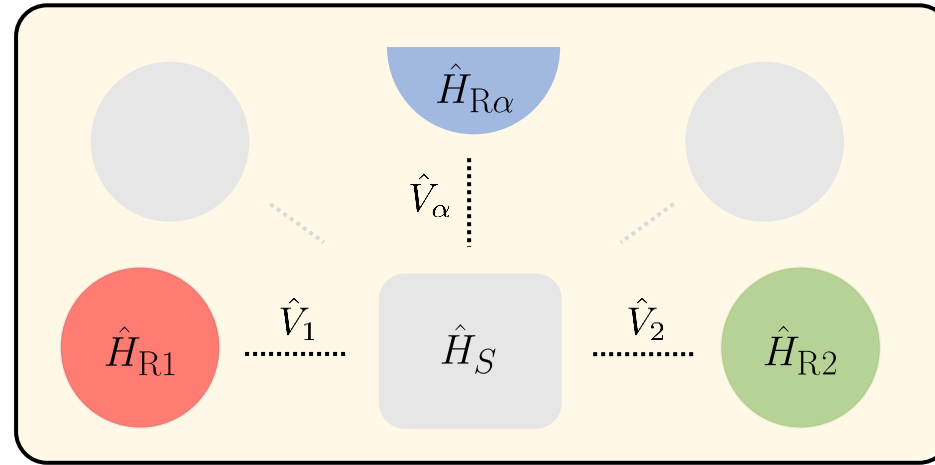
$$\mathcal{A}_{\alpha}(t) = \frac{1}{2\hbar^2} \int_{-t}^t d\tau \left\langle \left\langle \left\{ \hat{V}_{\alpha}(t), \hat{V}_{\alpha}(t + \tau) \right\} \right\rangle \right\rangle$$

Generalized activity corresponds to the zero-frequency component of spectrum of exchange rate fluctuations.

Blasi, Rodriguez, Moskalets, Lopez, Haack, arXiv:2505.13200 (2025)

New definition: Generalized activity

Generic open quantum system



$$\hat{H} = \hat{H}_S + \sum_{\alpha} \hat{H}_{R\alpha} + \sum_{\alpha} \hat{V}_{\alpha}$$

$$\hat{V}_{\alpha} = \sum_k \left(t_{k\alpha}^* \hat{c}_{k\alpha}^{\dagger} \hat{d} + t_{k\alpha} \hat{d}^{\dagger} \hat{c}_{k\alpha} \right)$$

$$\mathcal{A}_{\alpha}(t) = \frac{1}{2\hbar^2} \int_{-t}^t d\tau \left\langle \left\langle \left\{ \hat{V}_{\alpha}(t), \hat{V}_{\alpha}(t + \tau) \right\} \right\rangle \right\rangle$$

Generalized activity corresponds to the zero-frequency component of spectrum of exchange rate fluctuations.

Blasi, Rodriguez, Moskalets, Lopez, Haack, arXiv:2505.13200 (2025)

- This result could not have been derived without the analytical results we got for the noise.

Blasi, Khandelwal, Haack, Phys. Rev. Res. 6 (2024)

- Benchmark: Strong-coupling \longrightarrow Weak-coupling
Exchange-rate fluctuations \longrightarrow Rate of jumps

New definition: Generalized activity

- Explicit expressions in the steady-state regime
- Using Green function's approach and scattering-matrix approach: Valid for quadratic Hamiltonians

New definition: Generalized activity

- Explicit expressions in the steady-state regime
- Using Green function's approach and scattering-matrix approach: Valid for quadratic Hamiltonians

$$A_{\alpha}^{ss} = \frac{1}{\hbar} \int \frac{d\epsilon}{2\pi} \text{Tr} \left\{ 4\mathbf{T}_{\alpha\alpha}(\epsilon) - \left(\sum_{\beta} \mathbf{T}_{\alpha\beta}(\epsilon) \right)^2 \right\} F_{\alpha\alpha}(\epsilon) + \sum_{\beta \neq \alpha} \text{Tr} \{ \mathbf{T}_{\alpha\beta}(\epsilon) \} [F_{\alpha\beta}(\epsilon) + F_{\beta\alpha}(\epsilon)]$$

Definitions
(Green functions formalism)

$$\left\{ \begin{array}{l} \mathbf{T}_{\alpha\beta}(\epsilon) = \mathbf{\Gamma}_{\alpha}(\epsilon) \mathbf{G}^r(\epsilon) \mathbf{\Gamma}_{\beta}(\epsilon) \mathbf{G}^a(\epsilon) \\ [\mathbf{\Gamma}_{\alpha}(\epsilon)]_{ij} = 2\pi \sum_k t_{ik\alpha} t_{jk\alpha}^* \delta(\epsilon - \epsilon_{k\alpha}) \\ F_{\alpha\beta}(\epsilon) = f_{\alpha}(\epsilon) (1 - f_{\beta}(\epsilon)) \\ f_{\alpha}(\epsilon) = \frac{1}{e^{(\epsilon - \mu_{\alpha})/k_B T_{\alpha}} + 1} \end{array} \right.$$

Crepieux, Duong, Lavagna, J. of Physics: Condensed Matter 37, 075302 (2024).

New definition: Generalized activity

- Explicit expressions in the steady-state regime
- Using Green function's approach and scattering-matrix approach: Valid for quadratic Hamiltonians

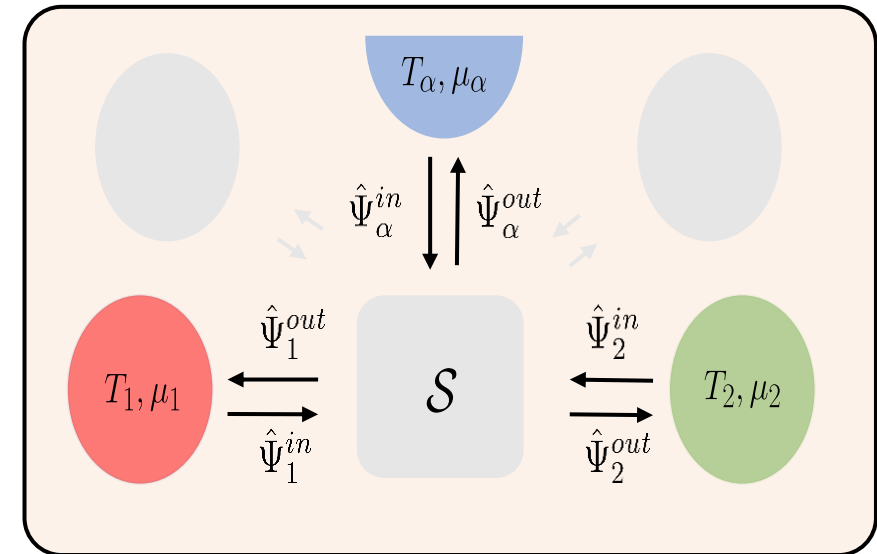
New definition: Generalized activity

- Explicit expressions in the steady-state regime
- Using Green function's approach and scattering-matrix approach: Valid for quadratic Hamiltonians

In terms of scattering amplitudes

Fisher, Lee, Phys. Rev. B 23 (1981)

$$s_{\alpha\beta}(\epsilon) = \delta_{\alpha\beta} - i\sqrt{\Gamma_\alpha\Gamma_\beta} \mathcal{G}_{\alpha\beta}^r(\epsilon)$$



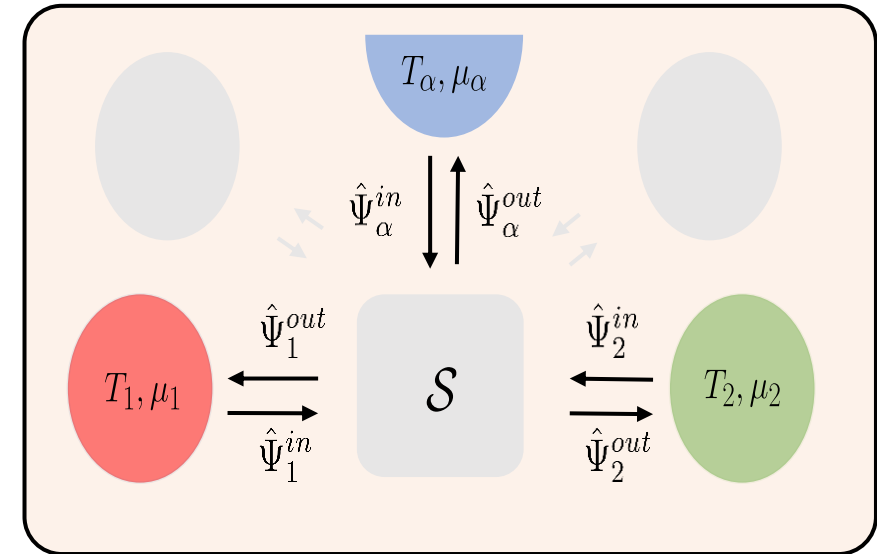
New definition: Generalized activity

- Explicit expressions in the steady-state regime
- Using Green function's approach and scattering-matrix approach: Valid for quadratic Hamiltonians

In terms of scattering amplitudes

Fisher, Lee, Phys. Rev. B 23 (1981)

$$s_{\alpha\beta}(\epsilon) = \delta_{\alpha\beta} - i\sqrt{\Gamma_\alpha\Gamma_\beta} \mathcal{G}_{\alpha\beta}^r(\epsilon)$$



$$\mathcal{A}_\alpha^{ss} = \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \quad \text{with} \quad \begin{cases} \mathcal{A}_\alpha^{th} = \frac{1}{\hbar} \int \frac{d\epsilon}{2\pi} [1 + \mathcal{R}_{\alpha\alpha}(\epsilon) (1 - 2 \cos(\phi_\alpha))] F_{\alpha\alpha}(\epsilon) + \sum_{\beta \neq \alpha} \mathcal{T}_{\alpha\beta}(\epsilon) F_{\beta\beta}(\epsilon), \\ \mathcal{A}_\alpha^{sh} = \frac{1}{\hbar} \sum_{\beta \neq \alpha} \int \frac{d\epsilon}{2\pi} \mathcal{T}_{\alpha\beta}(\epsilon) (f_\alpha(\epsilon) - f_\beta(\epsilon))^2. \end{cases}$$

New definition: Generalized activity

- Explicit expressions in the steady-state regime
- Using Green function's approach and scattering-matrix approach: Valid for quadratic Hamiltonians
- We can distinguish different types of contributions, similar to the contributions to the noise.

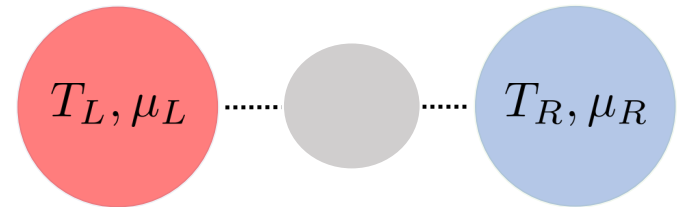
Thermal part

$$\mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$

Shot part

One reservoir

Two reservoirs



Outline for the rest of this talk

1. Generalized definition of activity

- ❖ valid at arbitrary coupling strength
- ❖ beyond master equation approach
- ❖ Explicit expressions in the steady-state regime

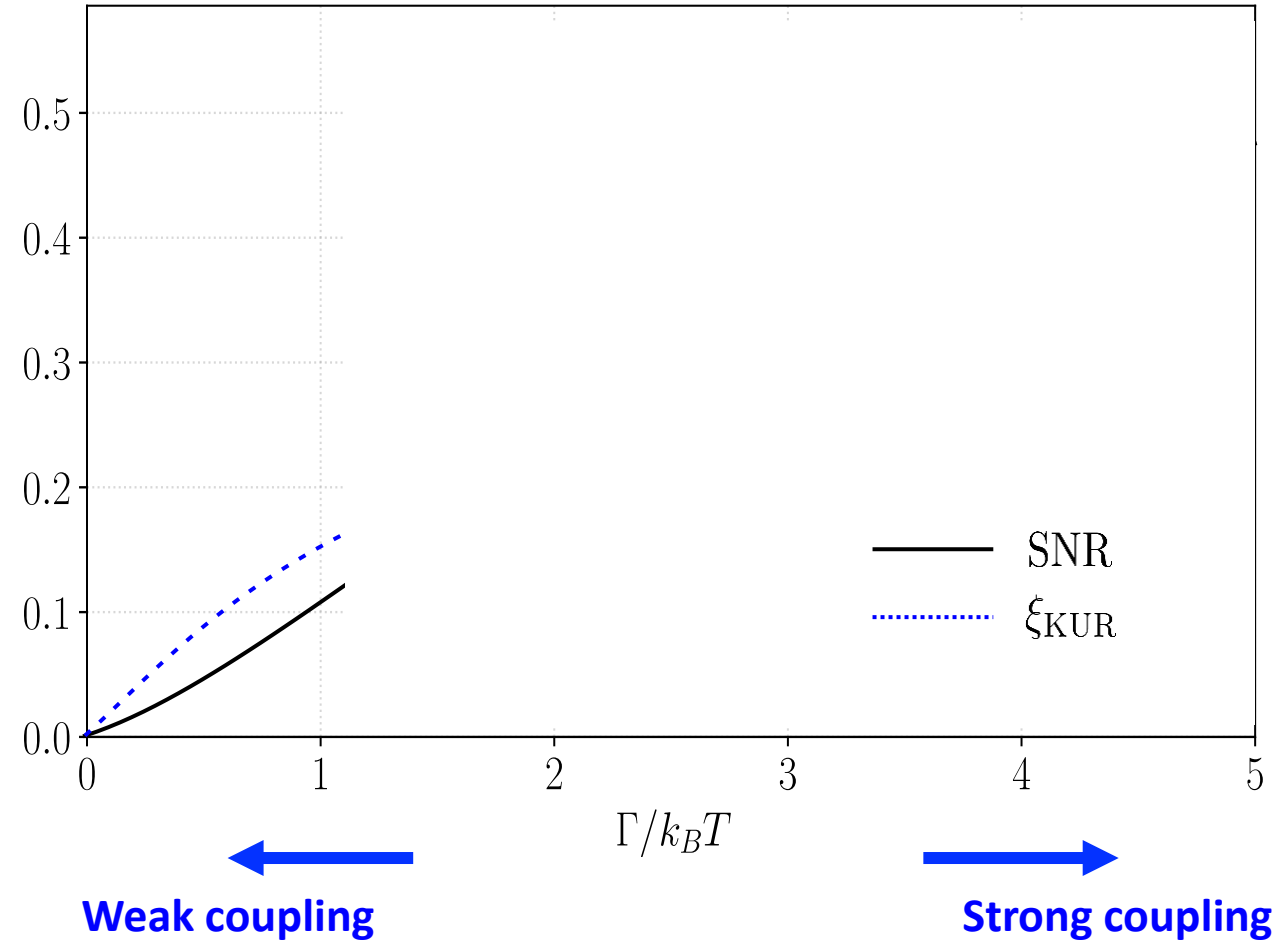
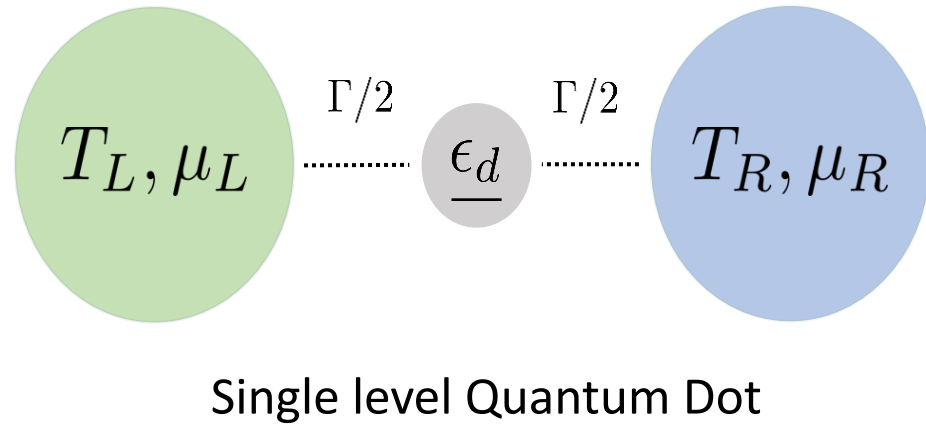
2. Quantum KUR valid at arbitrary coupling strengths

3. Paradigmatic examples

- ❖ Double quantum dot
- ❖ Quantum point contact in a two-terminal device

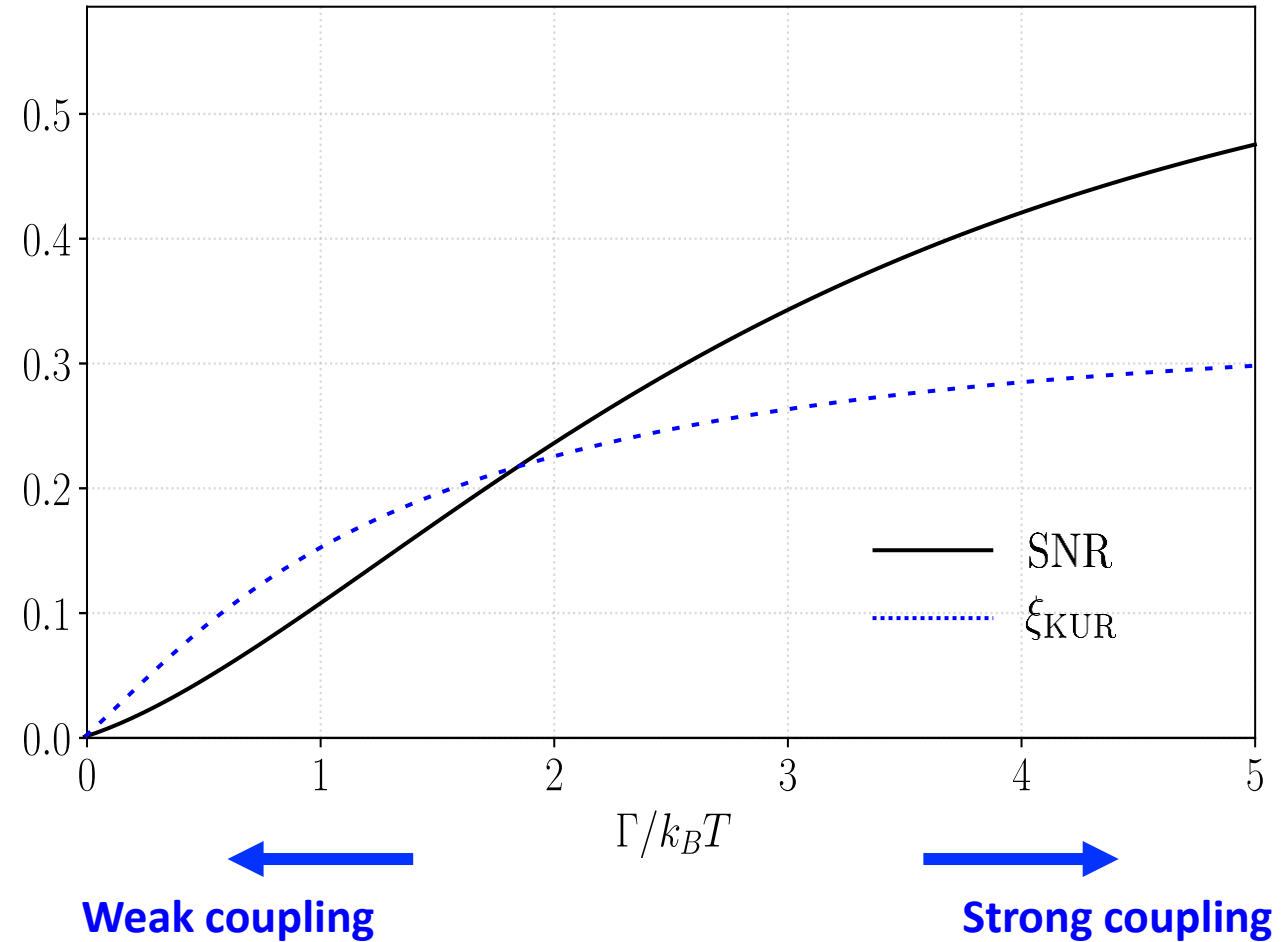
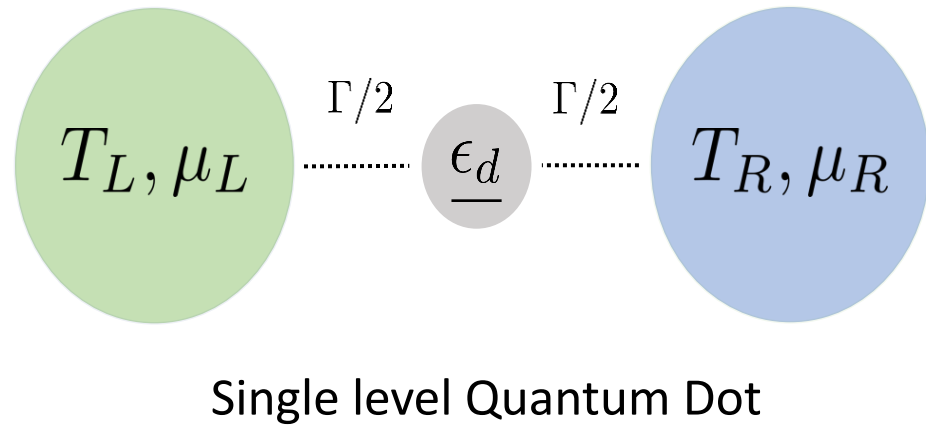
Quantum Kinetic Uncertainty Relations

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$



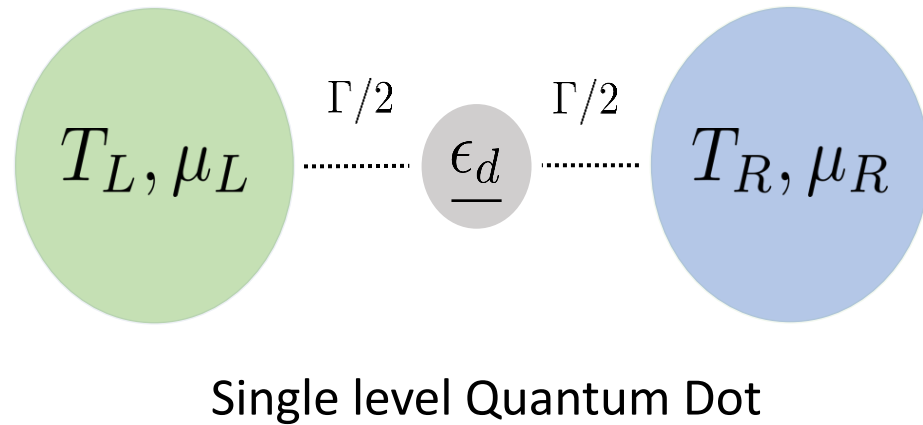
Quantum Kinetic Uncertainty Relations

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$



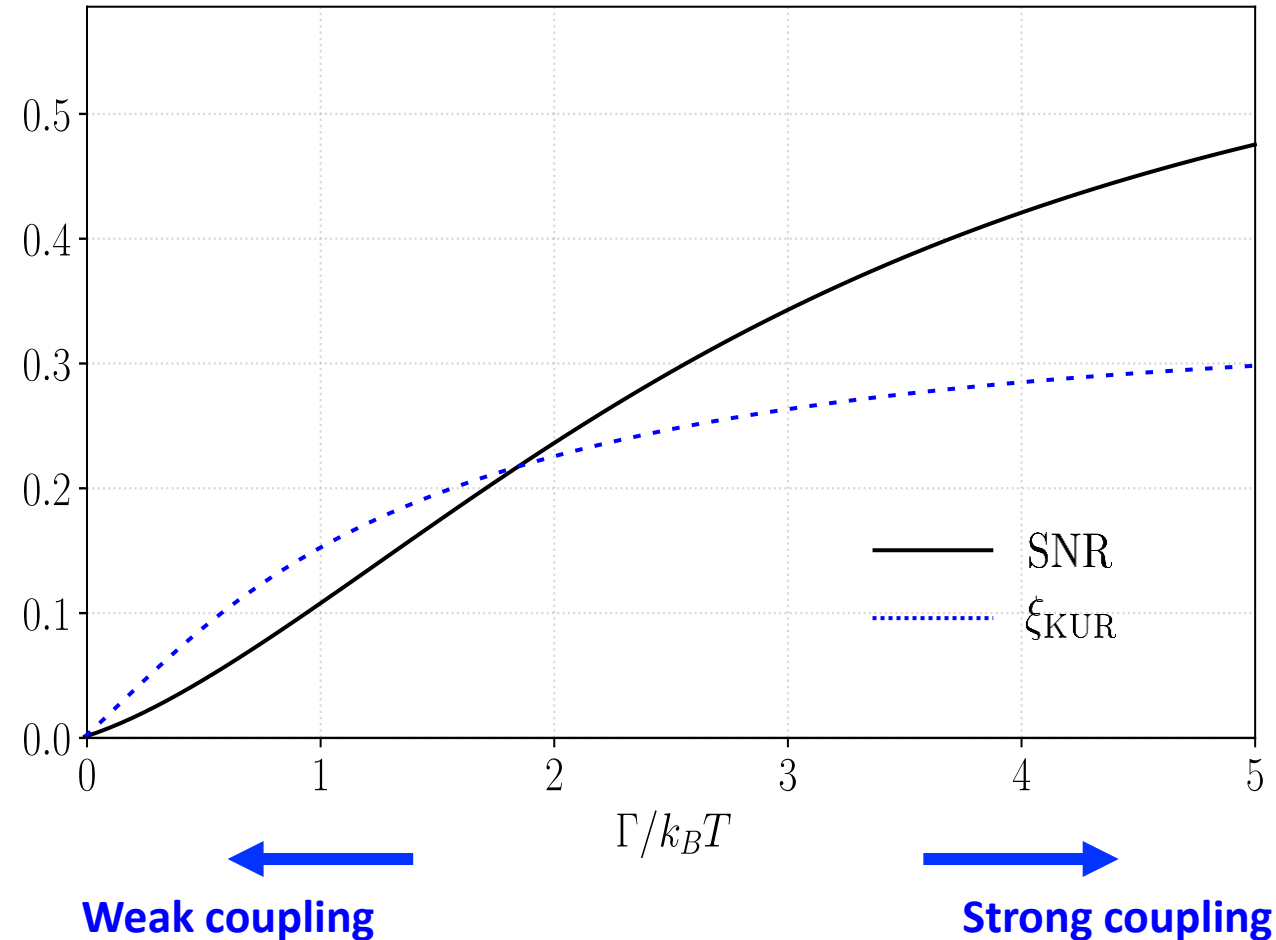
Quantum Kinetic Uncertainty Relations

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$



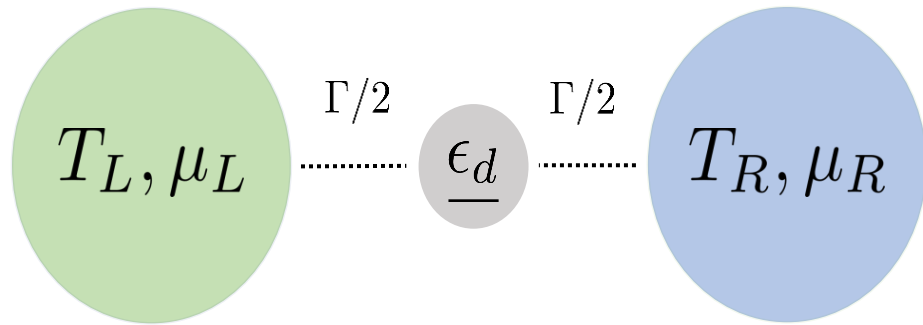
Conclusion: The generalized activity is not a good bound.

Can we find a valid bound at strong coupling ?



Quantum Kinetic Uncertainty Relations

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$

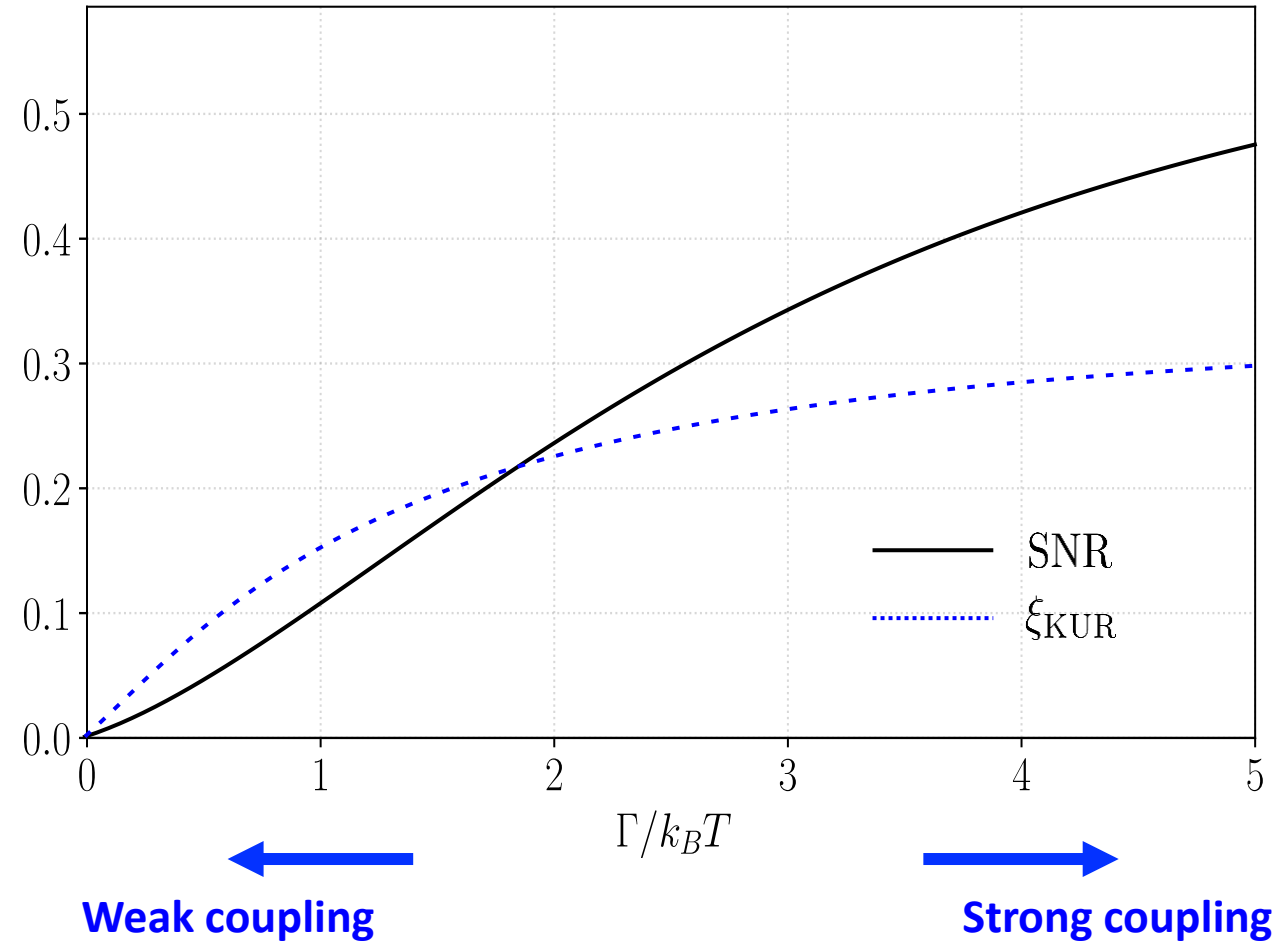


Single level Quantum Dot

NEW BOUND

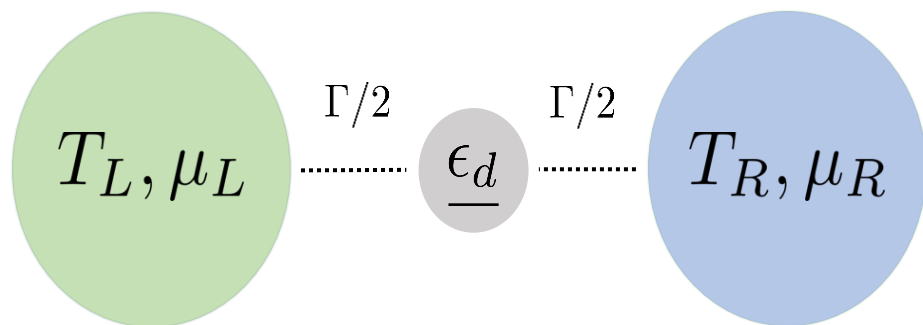
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \frac{(\mathcal{A}_\alpha^{cross})^2}{\mathcal{A}_\alpha^{cross} - \mathcal{A}_\alpha^{sh}} \equiv \xi_{\text{QKUR}}$$

Proof in the paper



Quantum Kinetic Uncertainty Relations

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$

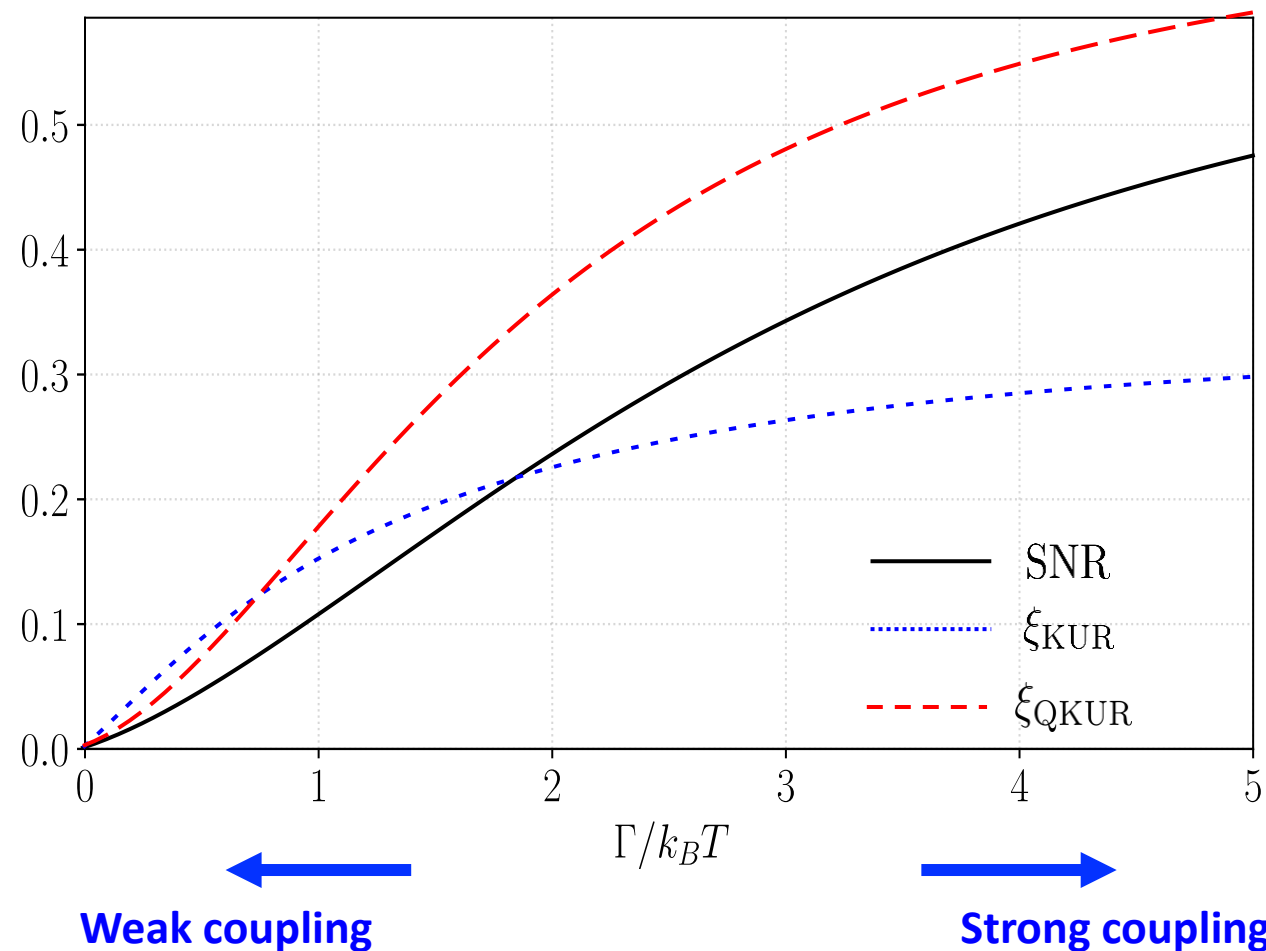


Single level Quantum Dot

NEW BOUND

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \frac{(\mathcal{A}_\alpha^{cross})^2}{\mathcal{A}_\alpha^{cross} - \mathcal{A}_\alpha^{sh}} \equiv \xi_{\text{QKUR}}$$

Proof in the paper



Quantum Kinetic Uncertainty Relations

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$

NEW BOUND

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \frac{(\mathcal{A}_\alpha^{cross})^2}{\mathcal{A}_\alpha^{cross} - \mathcal{A}_\alpha^{sh}} \equiv \xi_{\text{QKUR}}$$

Proof in the paper

Quantum Kinetic Uncertainty Relations

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$

NEW BOUND

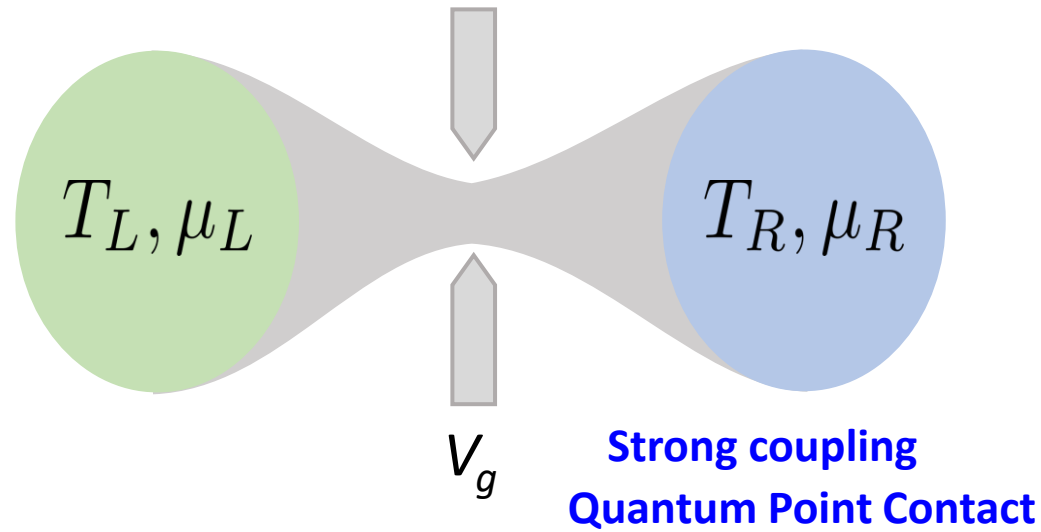
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \frac{(\mathcal{A}_\alpha^{cross})^2}{\mathcal{A}_\alpha^{cross} - \mathcal{A}_\alpha^{sh}} \equiv \xi_{\text{QKUR}}$$

Proof in the paper

Remark: the KUR bound recently derived by Splettstoesser's group corresponds to our bound under equilibrium conditions.
Palmqvist et al., Phys. Rev. Lett. 135 (2025)

Quantum Kinetic Uncertainty Relations

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$



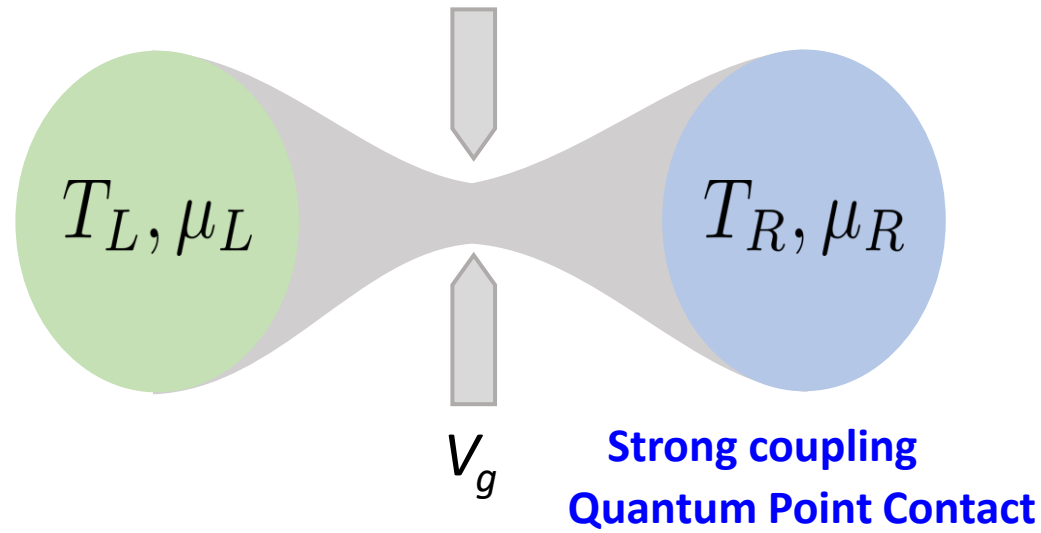
NEW BOUND

$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \frac{(\mathcal{A}_\alpha^{cross})^2}{\mathcal{A}_\alpha^{cross} - \mathcal{A}_\alpha^{sh}} \equiv \xi_{\text{QKUR}}$$

Proof in the paper

Quantum Kinetic Uncertainty Relations

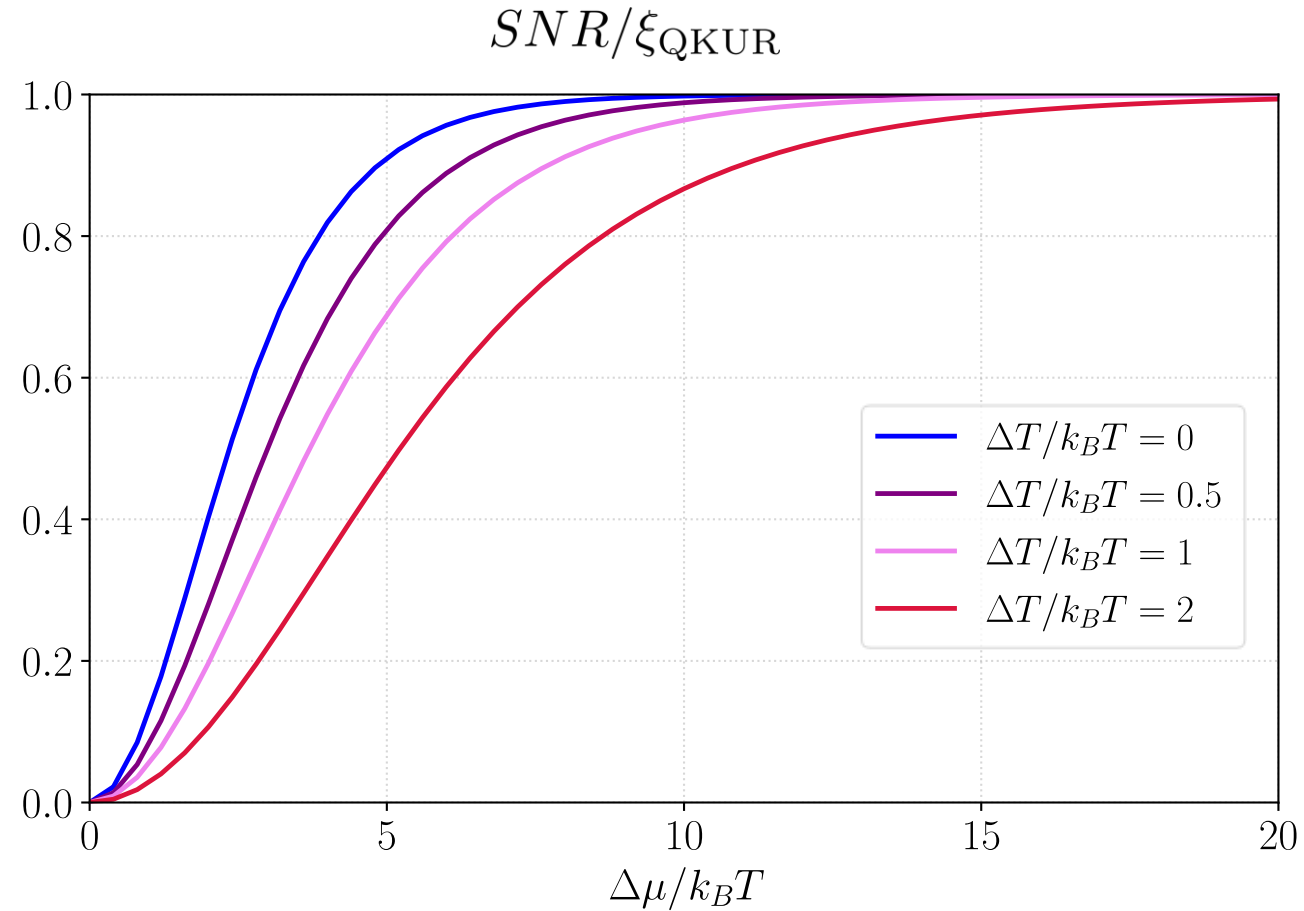
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$



NEW BOUND

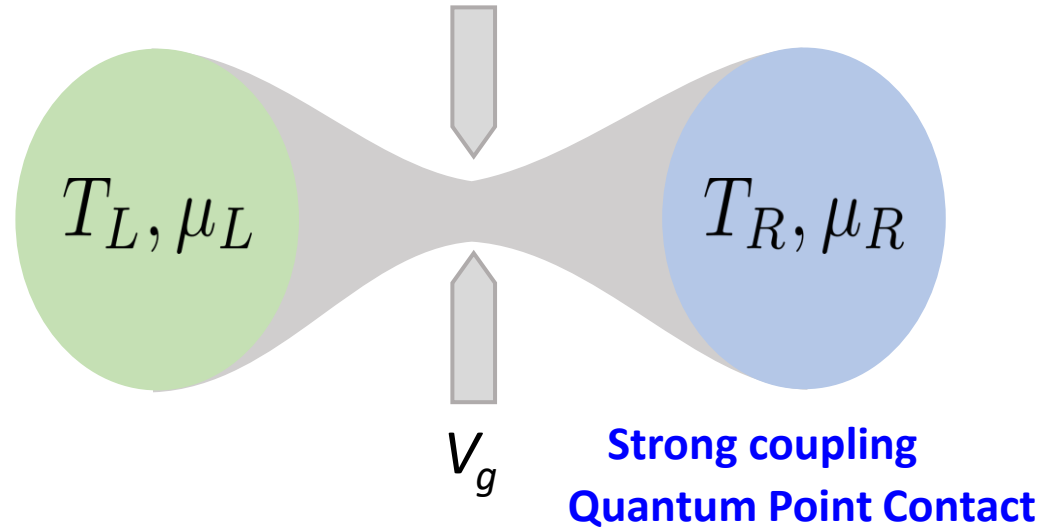
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \frac{(\mathcal{A}_\alpha^{cross})^2}{\mathcal{A}_\alpha^{cross} - \mathcal{A}_\alpha^{sh}} \equiv \xi_{\text{QKUR}}$$

Proof in the paper



Quantum Kinetic Uncertainty Relations

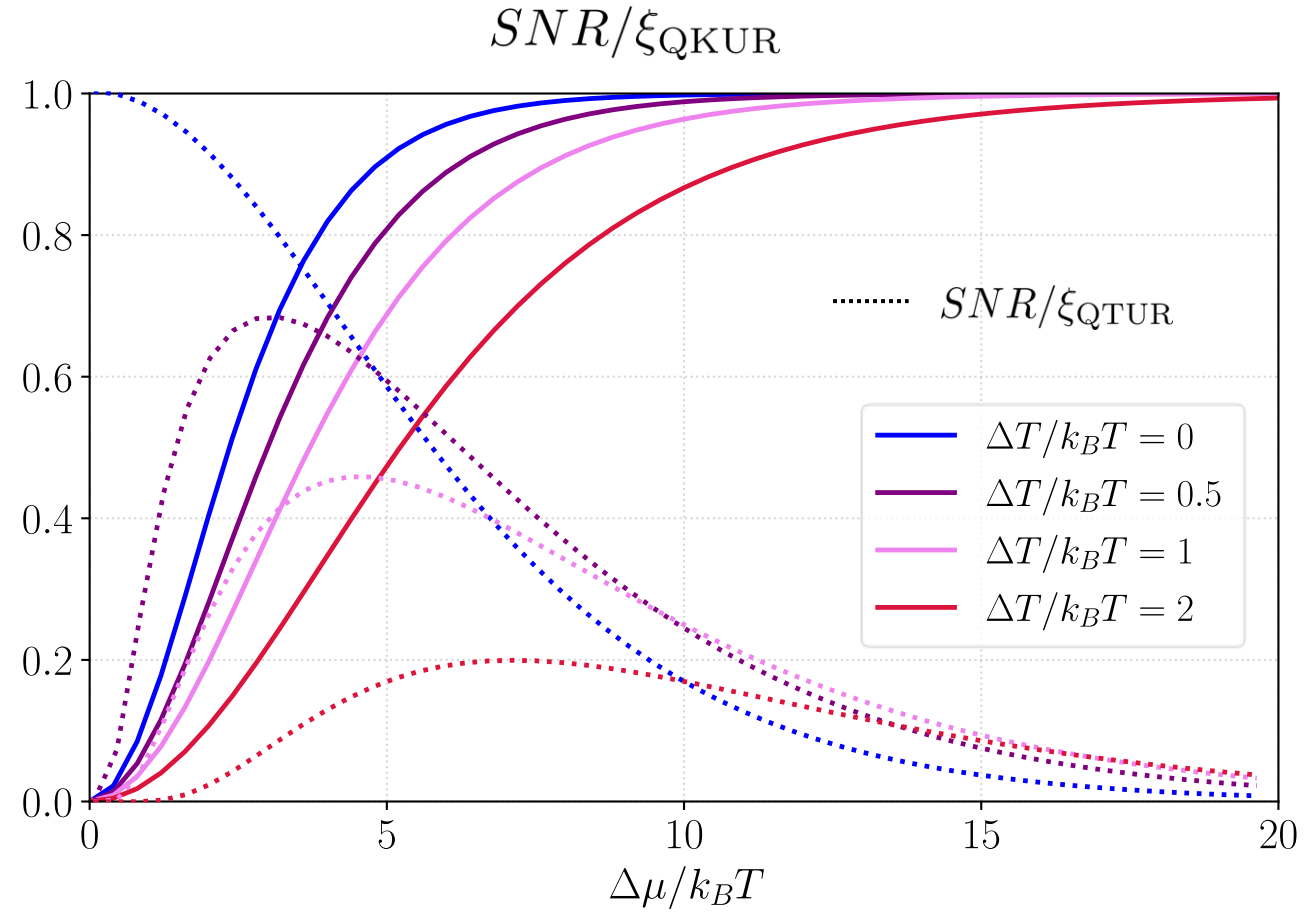
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$



NEW BOUND

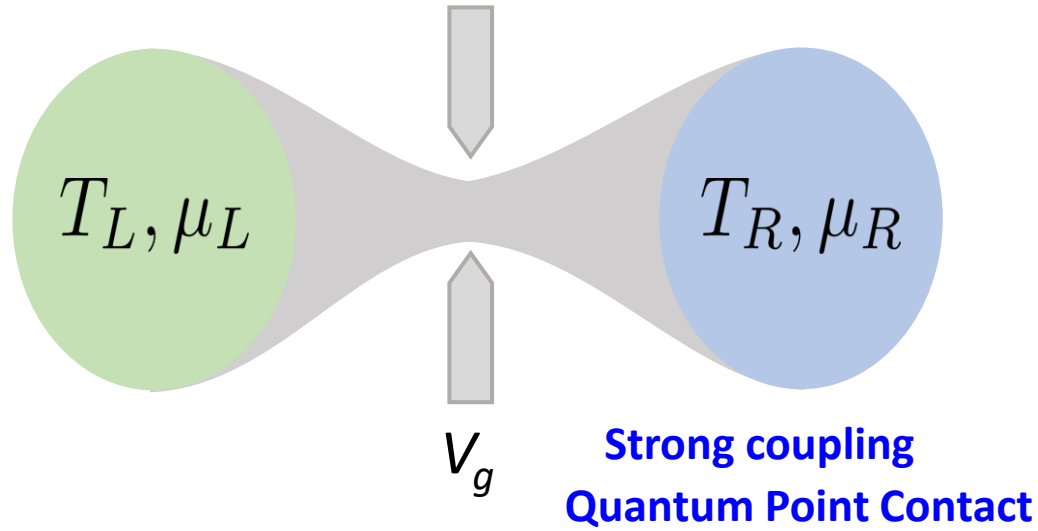
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \frac{(\mathcal{A}_\alpha^{cross})^2}{\mathcal{A}_\alpha^{cross} - \mathcal{A}_\alpha^{sh}} \equiv \xi_{\text{QKUR}}$$

Proof in the paper



Quantum Kinetic Uncertainty Relations

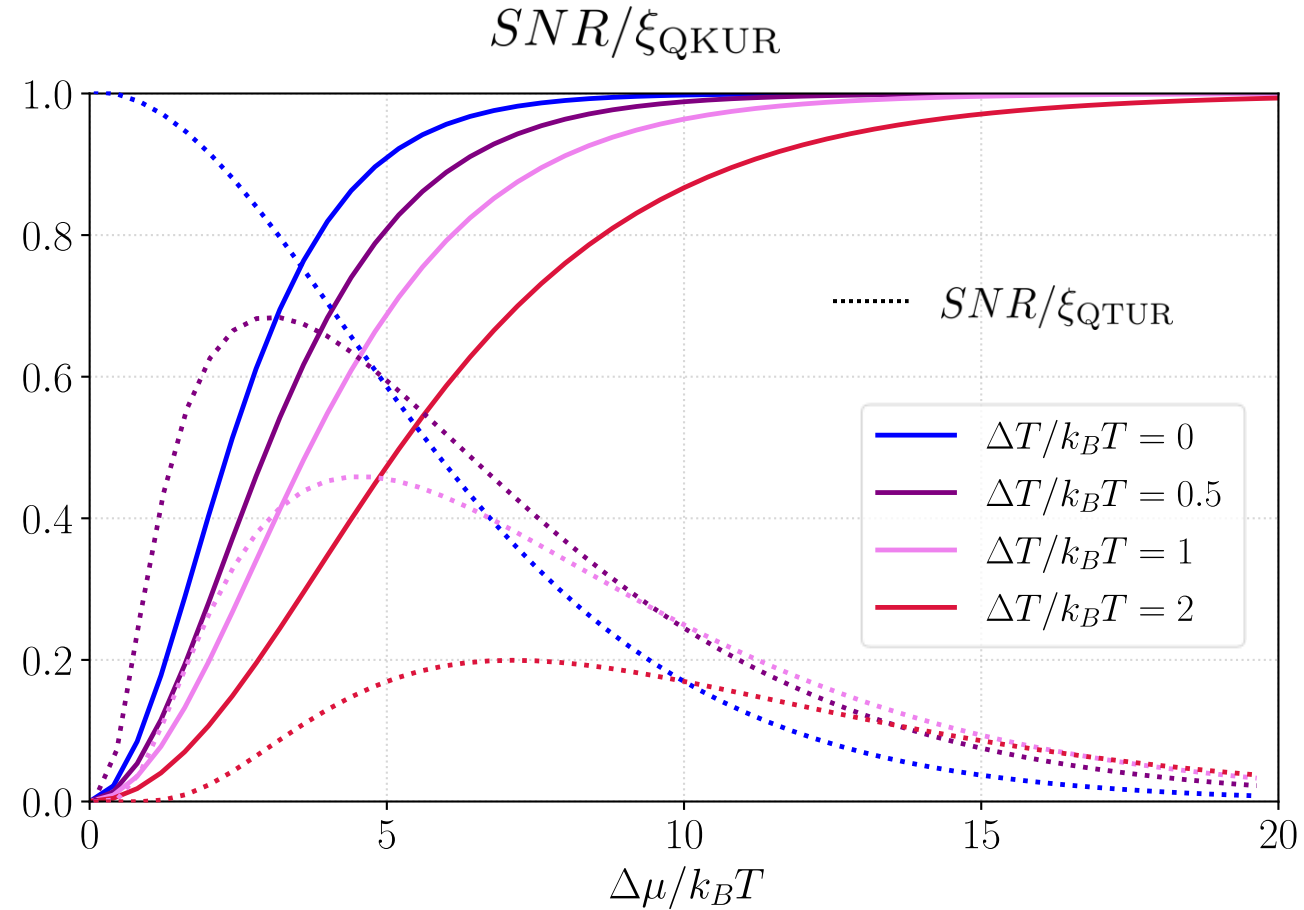
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \mathcal{A}_\alpha^{ss} \equiv \xi_{\text{KUR}} \quad \text{with} \quad \mathcal{A}_\alpha^{ss} = \begin{cases} \mathcal{A}_\alpha^{th} + \mathcal{A}_\alpha^{sh} \\ \mathcal{A}_\alpha^{auto} + \mathcal{A}_\alpha^{cross} \end{cases}$$



NEW BOUND

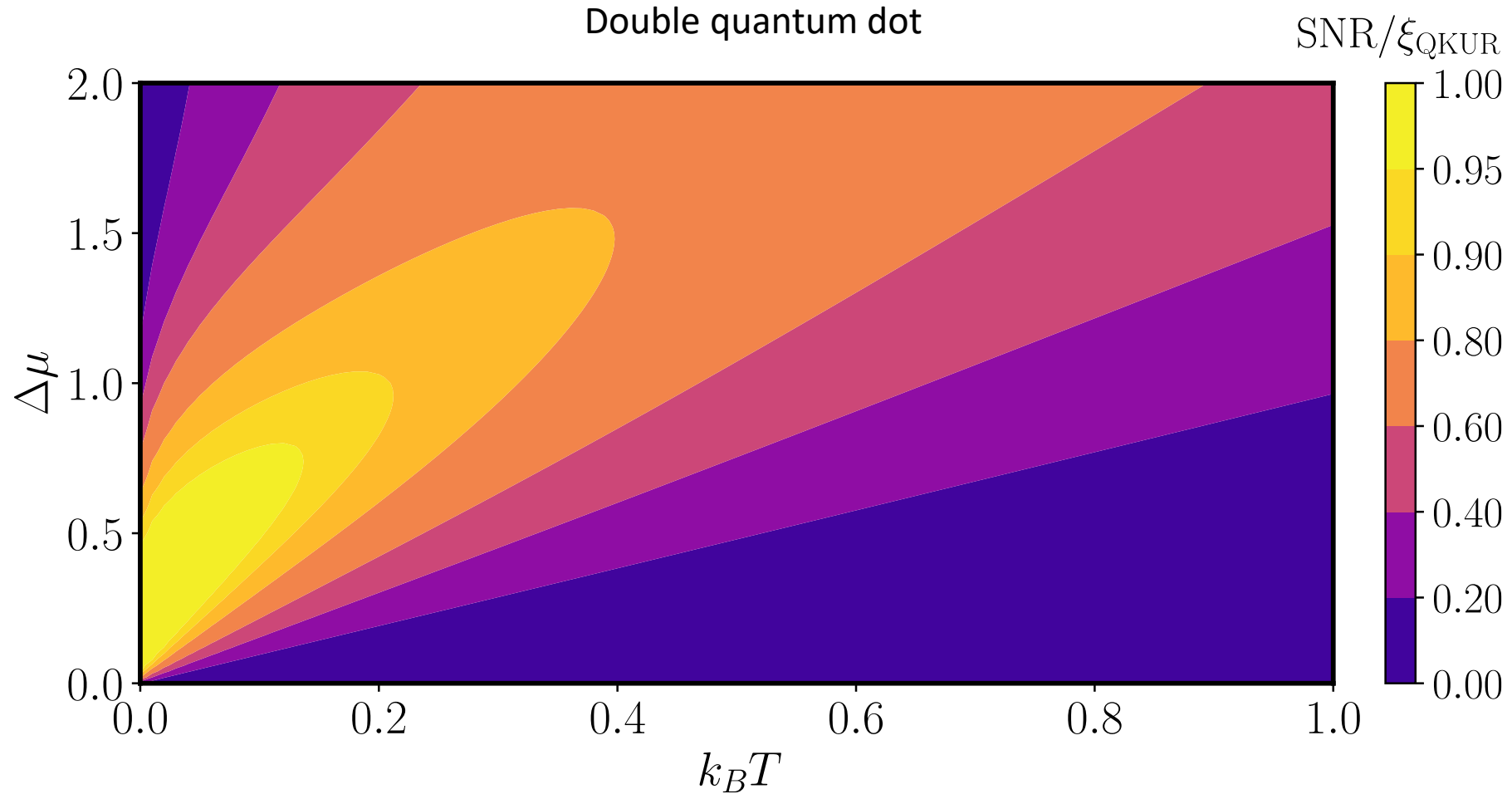
$$\frac{I_\alpha^2}{S_{\alpha\alpha}} \leq \frac{(\mathcal{A}_\alpha^{cross})^2}{\mathcal{A}_\alpha^{cross} - \mathcal{A}_\alpha^{sh}} \equiv \xi_{\text{QKUR}}$$

Proof in the paper



- QKUR tight far from equilibrium
- QTUR tight close to equilibrium

Quantum Kinetic Uncertainty Relations



Take-home message & Perspectives

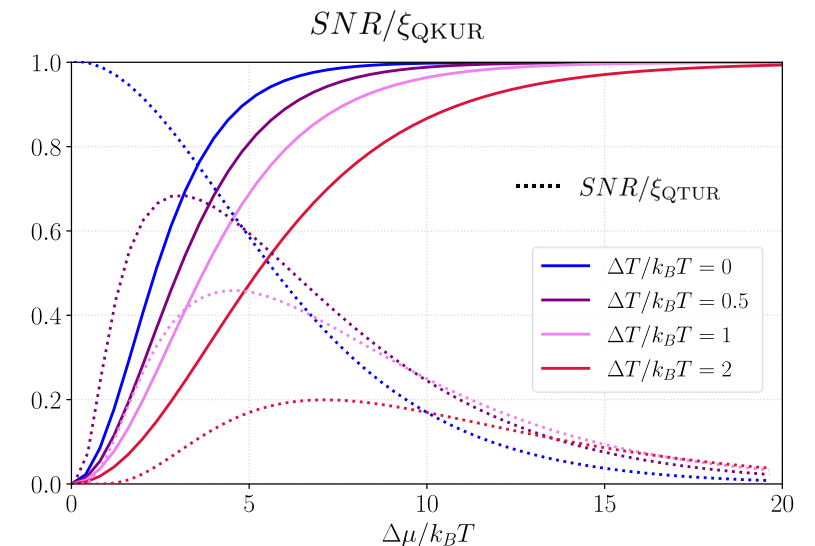
Novel theoretical framework for investigating uncertainty relations in quantum coherent transport setups

- Generalized activity captures the physics contains in the interaction between system and environment
- Derivation of a novel kinetic uncertainty relation, valid in the fully quantum regime (strong coupling, zero temperature)
- Illustrations with paradigmatic quantum transport setups, showing how the bound can be saturated.

Quantum kinetic uncertainty relation

$$\frac{I_{\alpha}^2}{S_{\alpha\alpha}} \leq \frac{(\mathcal{A}_{\alpha}^{cross})^2}{\mathcal{A}_{\alpha}^{cross} - \mathcal{A}_{\alpha}^{sh}} \equiv \xi_{\text{QKUR}}$$

Blasi, Rodriguez, Moskalets, Lopez, Haack, arXiv:2505.13200 (2025)



Follow-ups: Role of quantum statistics, role of quantum coherence, role of many-body interactions, non-Markovian effects, interplay with TUR (link with entropy production),

