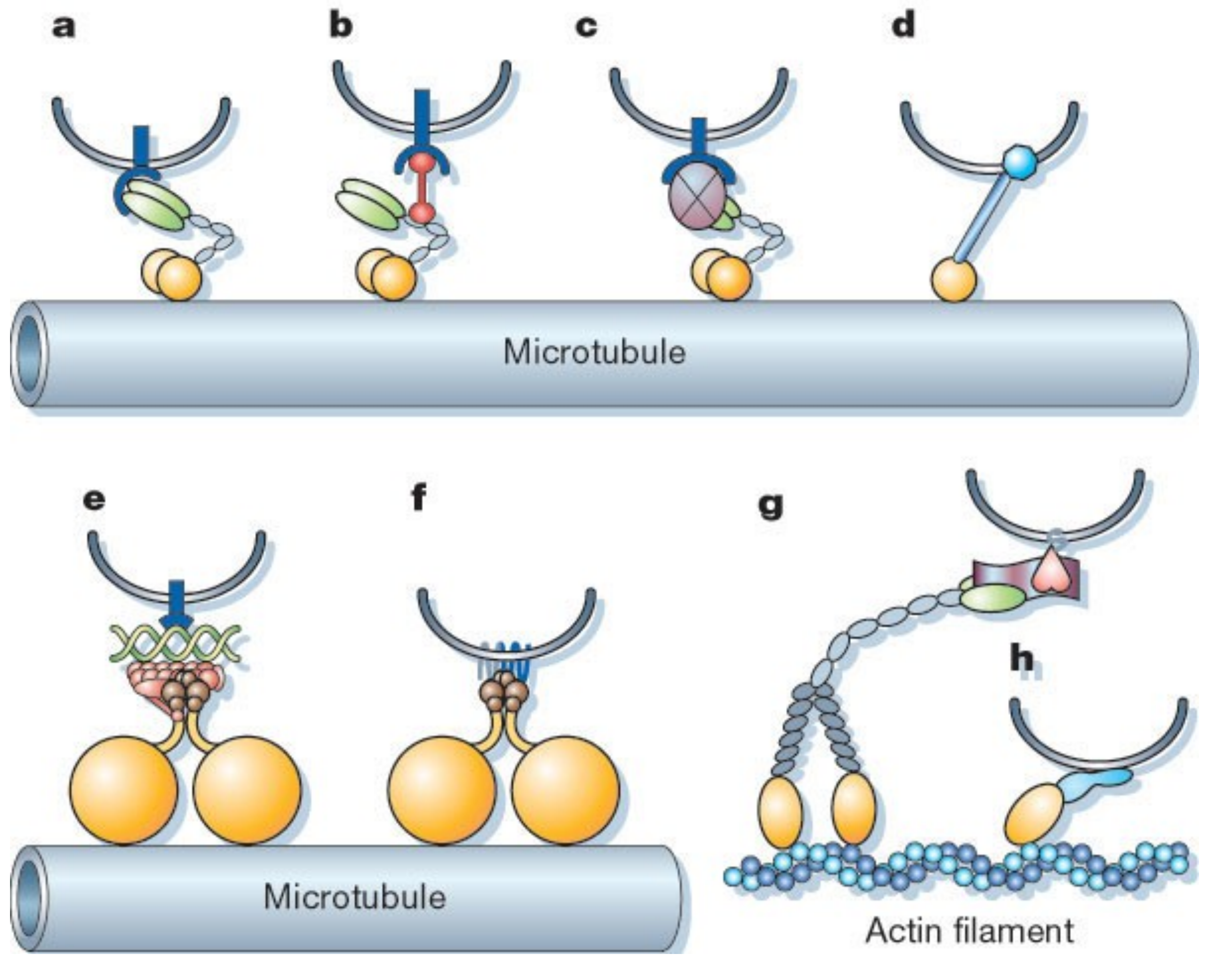
Thermodynamics of Precision in Open Quantum Systems

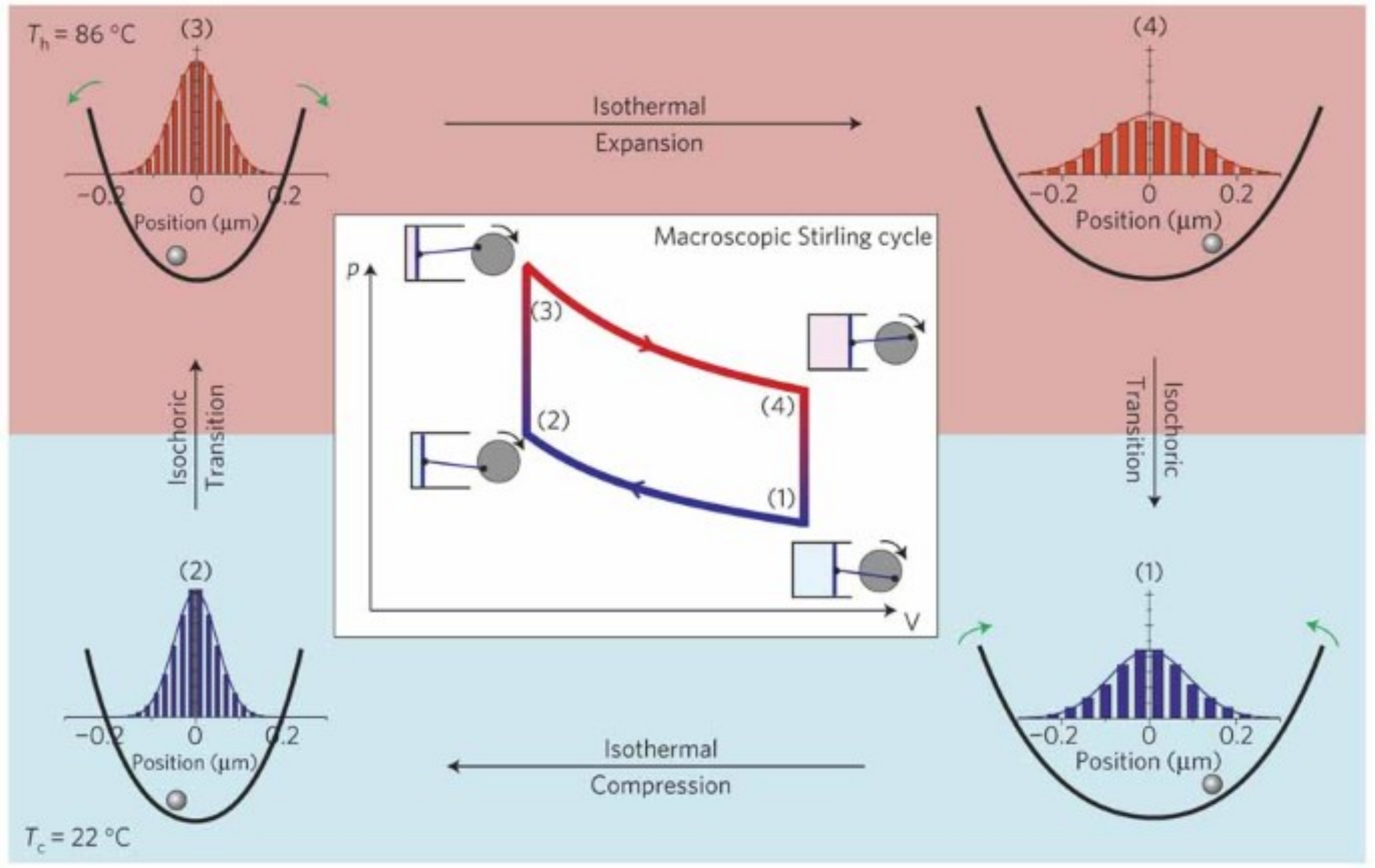
Tan Van Vu
Yukawa Institute for Theoretical Physics, Kyoto University

Kyoto Workshop on Quantum Thermodynamics and Stochastic Thermodynamics
Dec 8-12 2025, Kyoto University

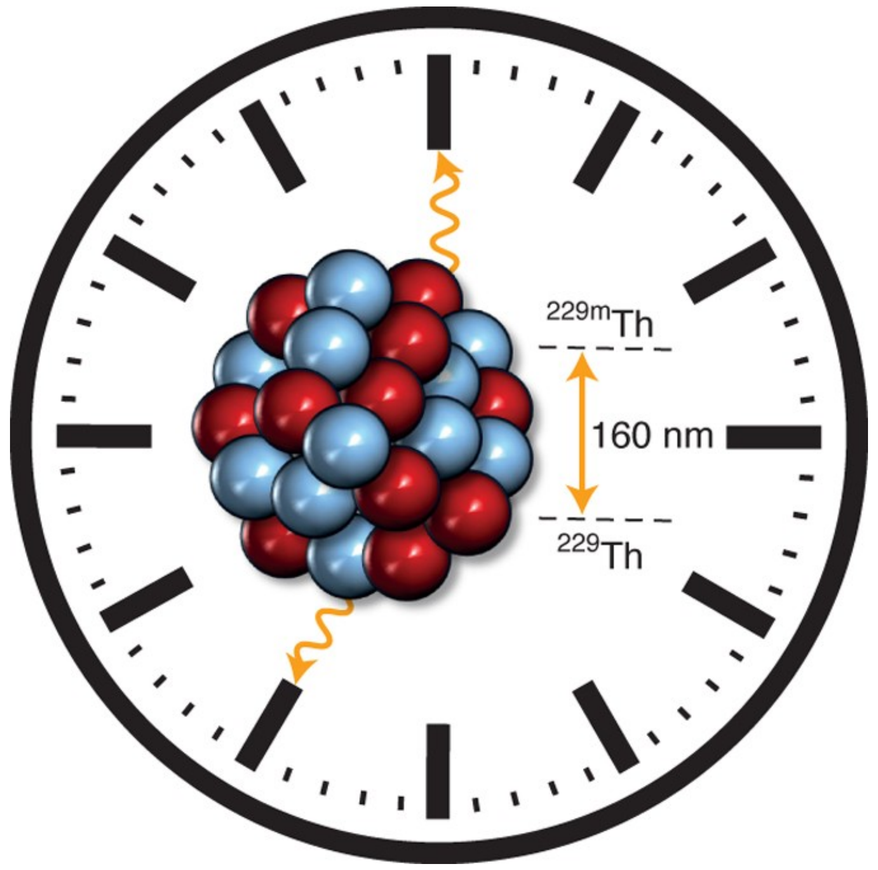
Thermodynamics of precision



Molecular motor, Nature (2003)

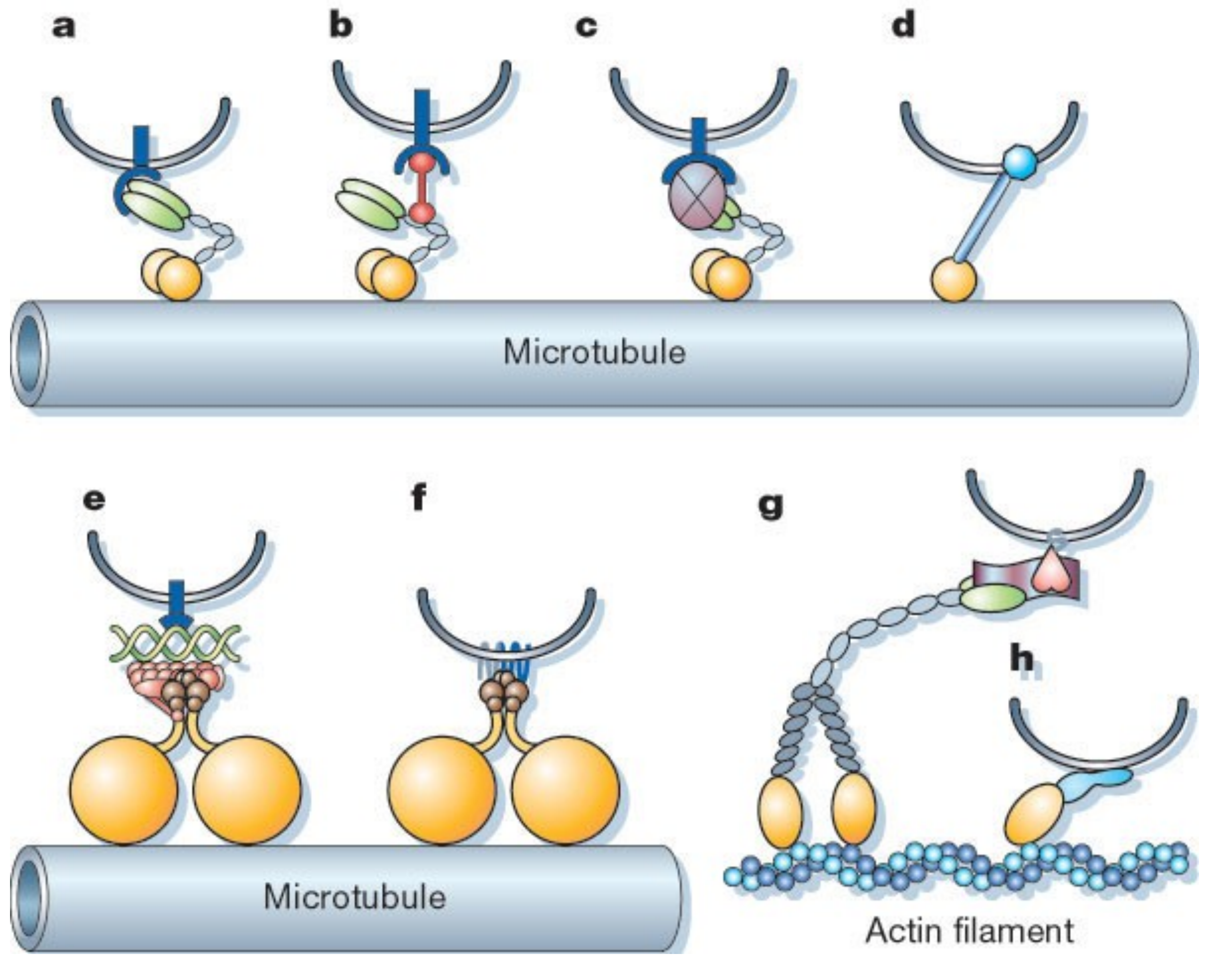


Heat engine, Nat. Phys. (2012)

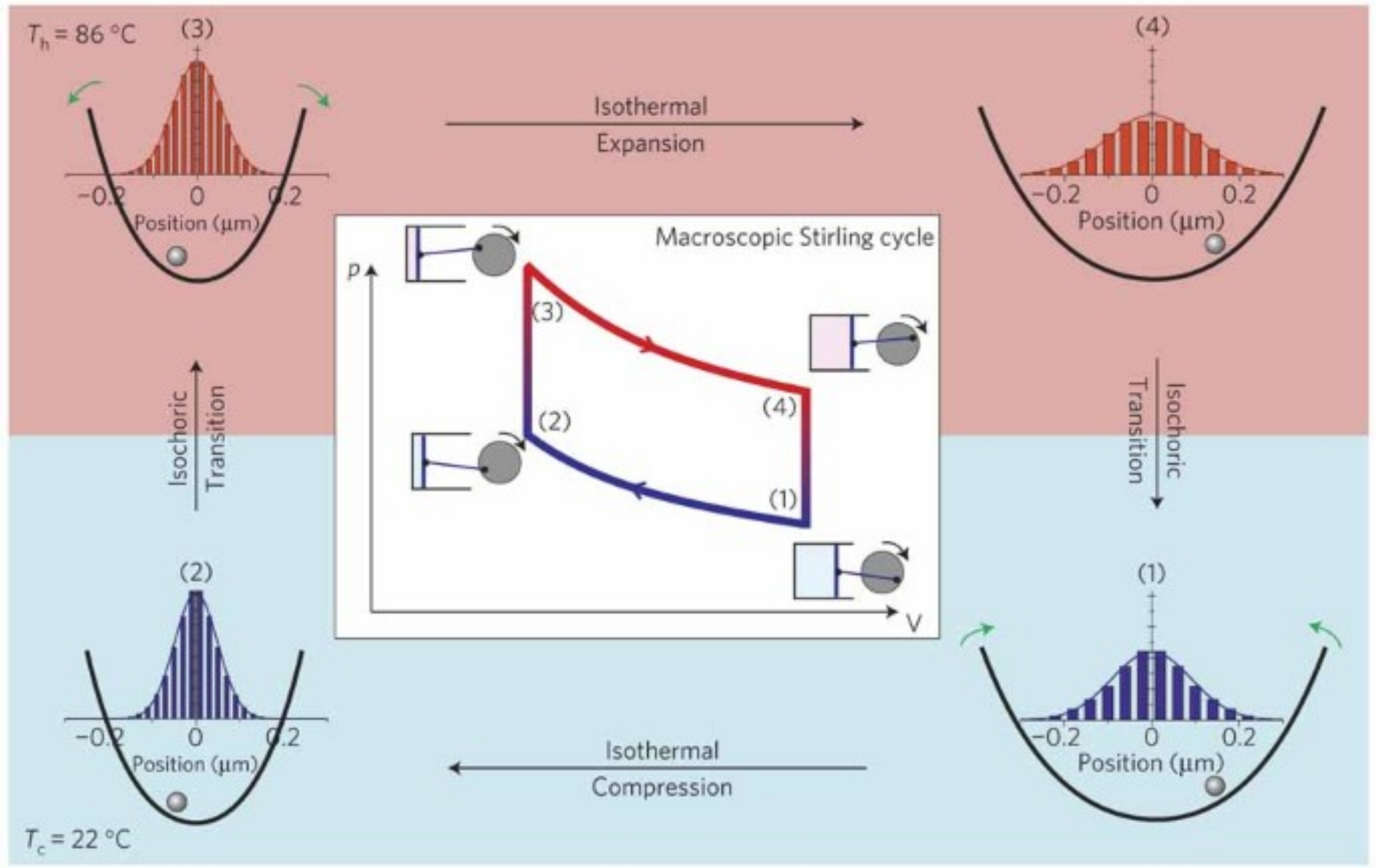


Clock, Nat. Phys. (2018)

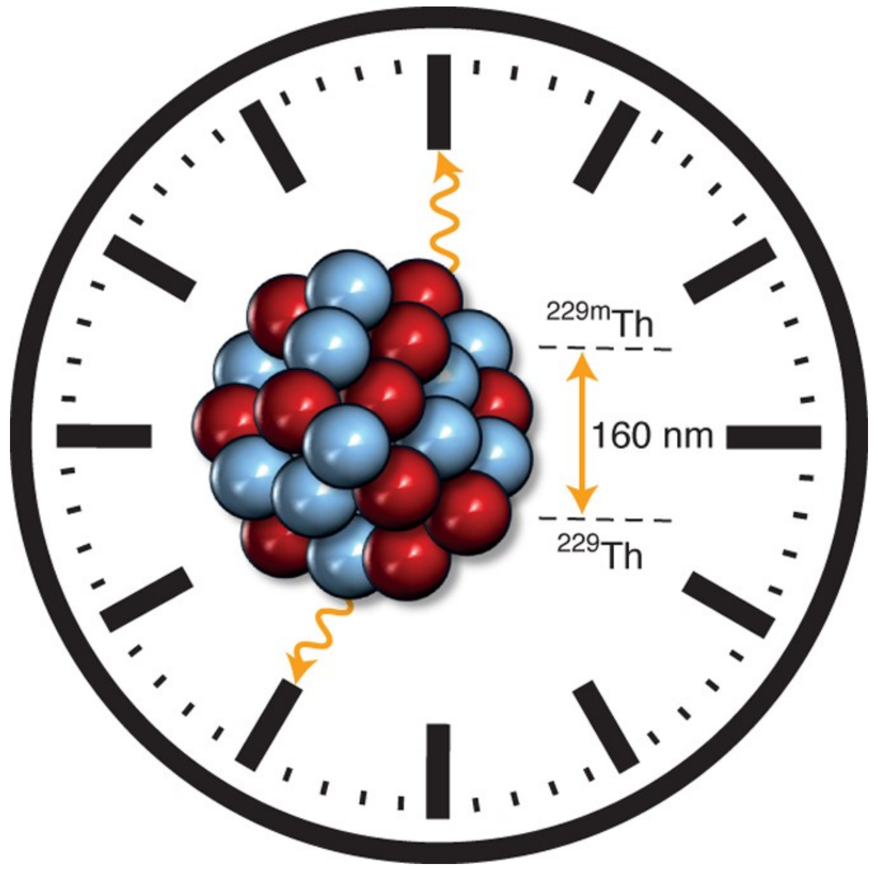
Thermodynamics of precision



Molecular motor, Nature (2003)



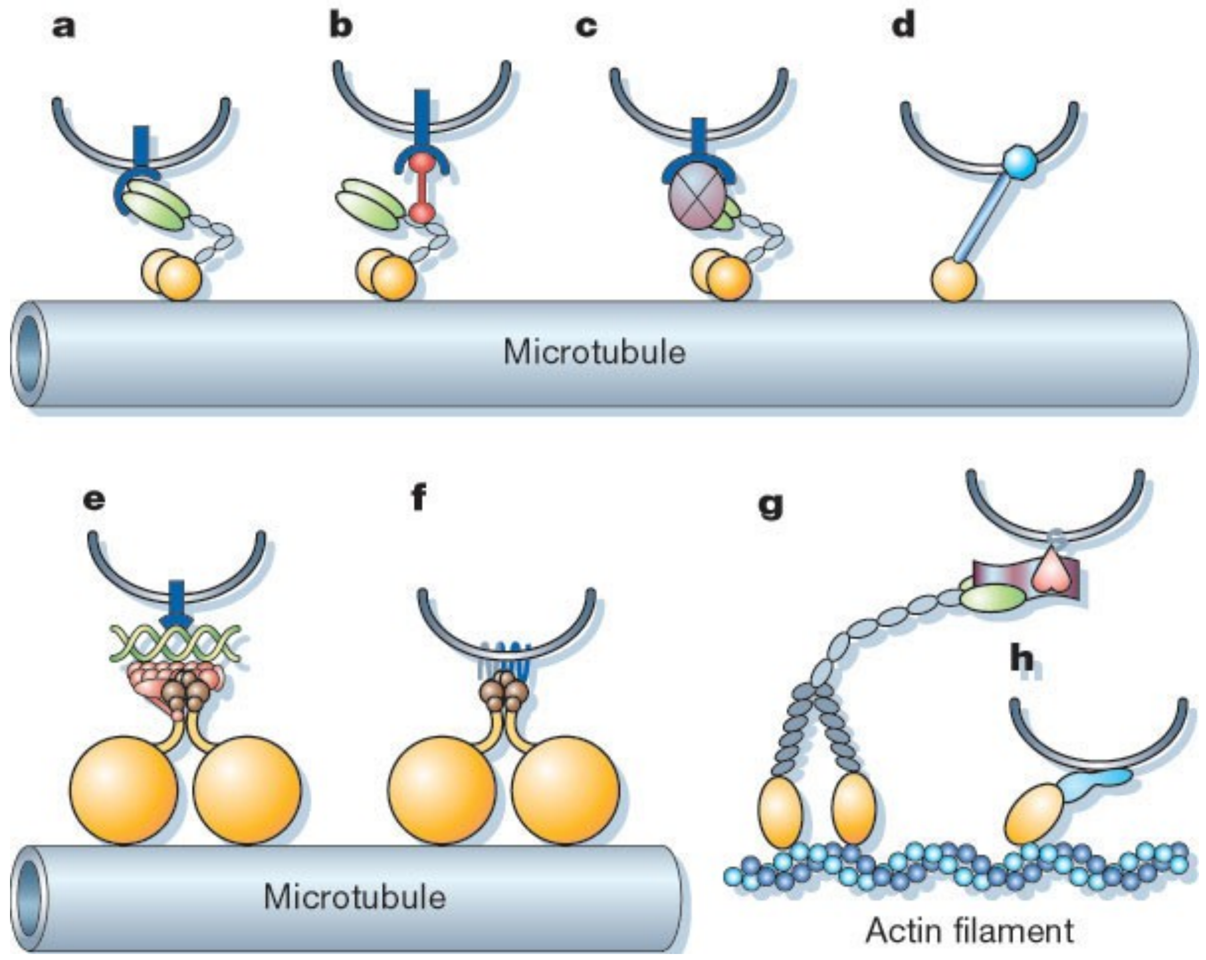
Heat engine, Nat. Phys. (2012)



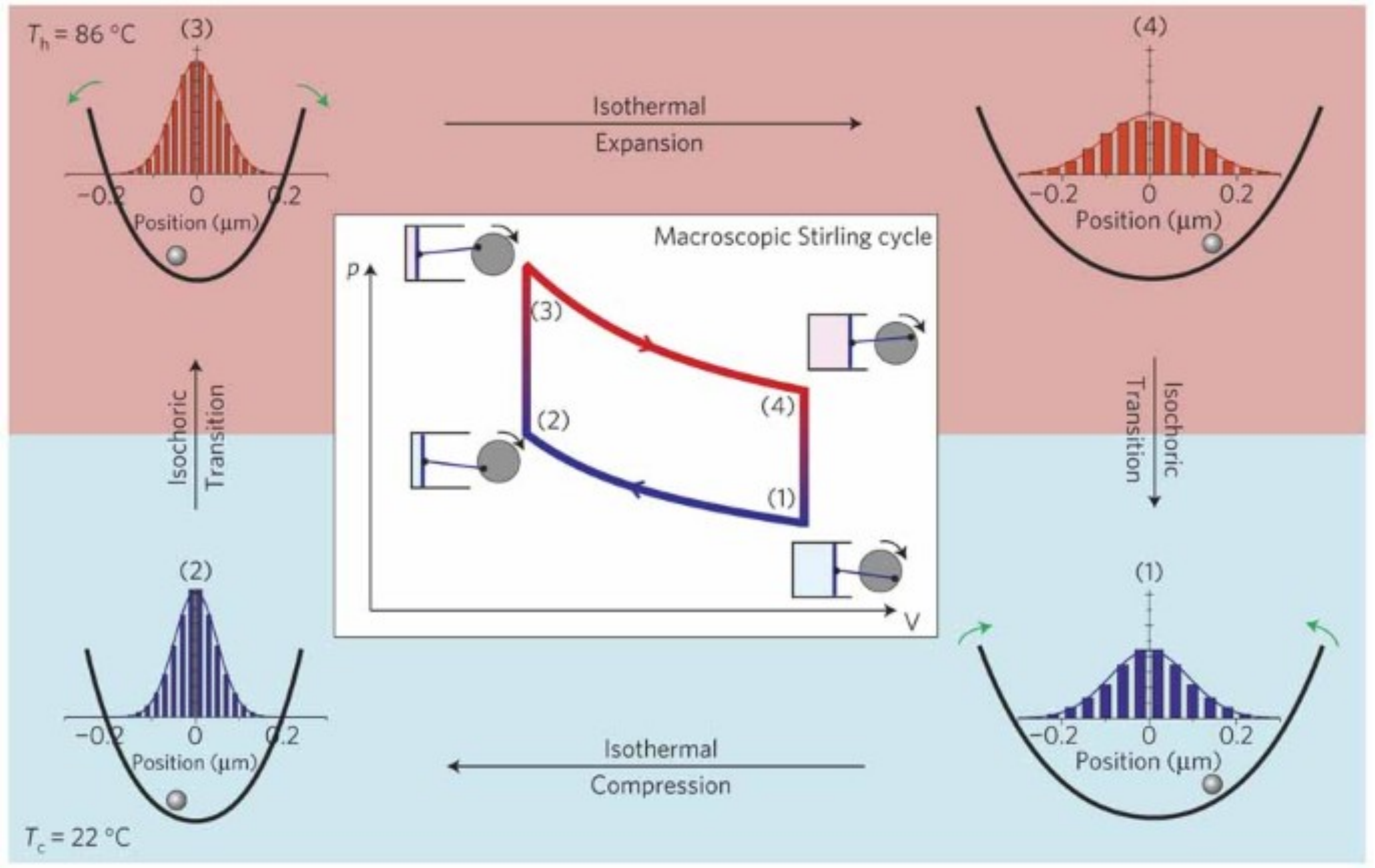
Clock, Nat. Phys. (2018)

traveled distance

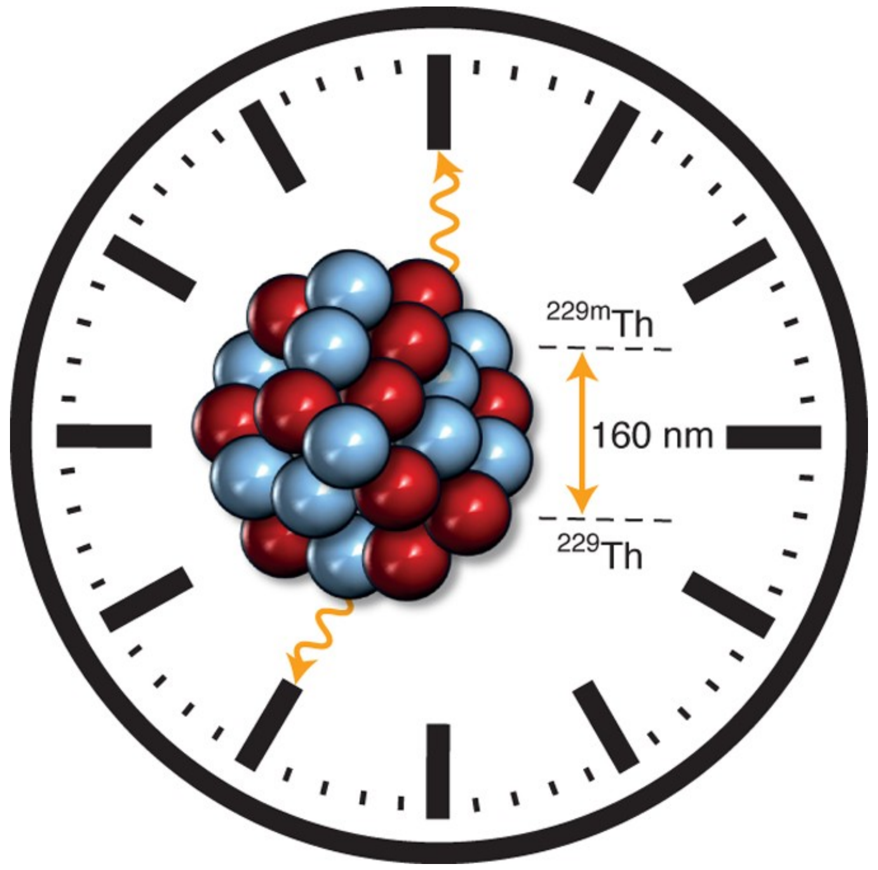
Thermodynamics of precision



Molecular motor, Nature (2003)



Heat engine, Nat. Phys. (2012)

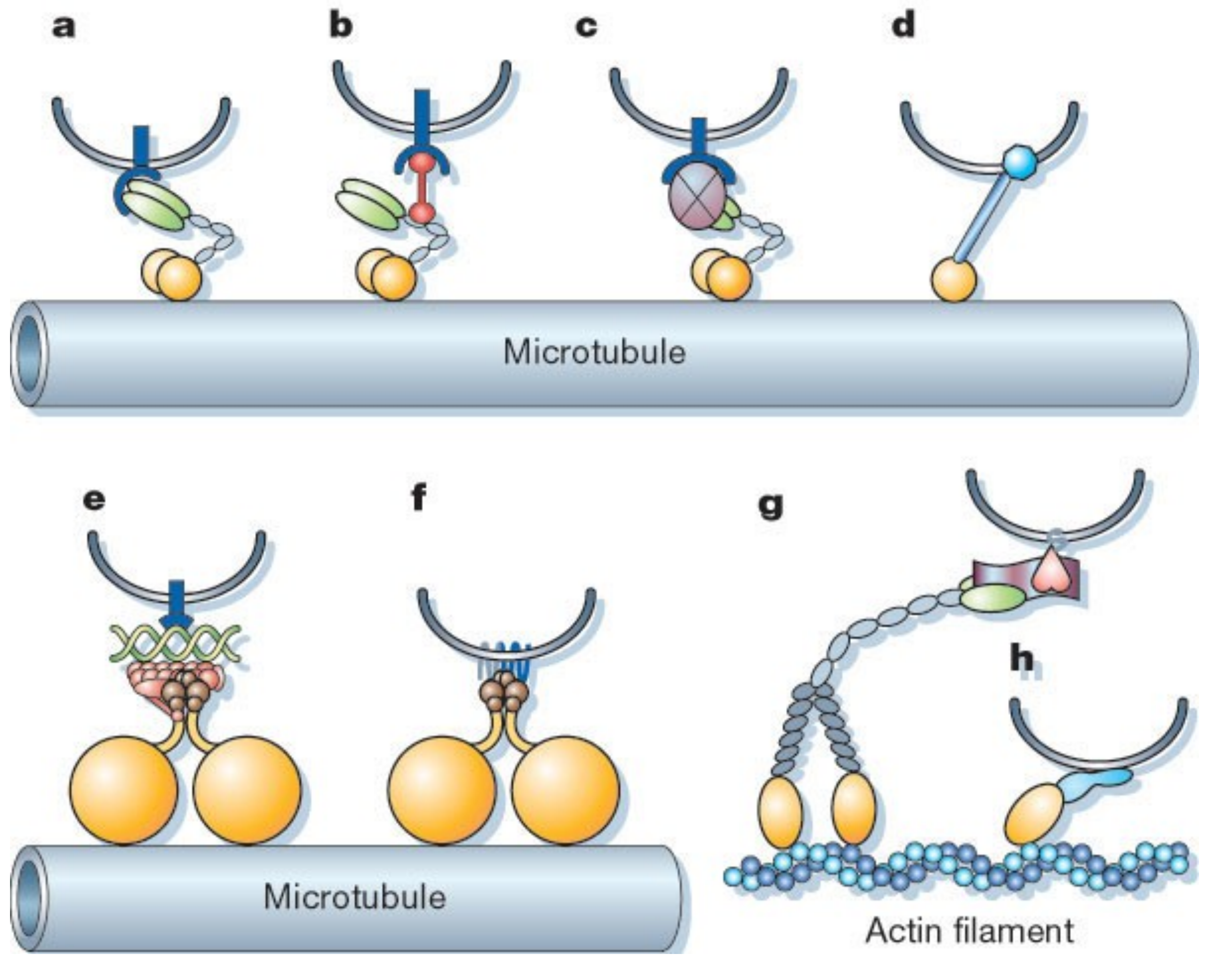


Clock, Nat. Phys. (2018)

traveled distance

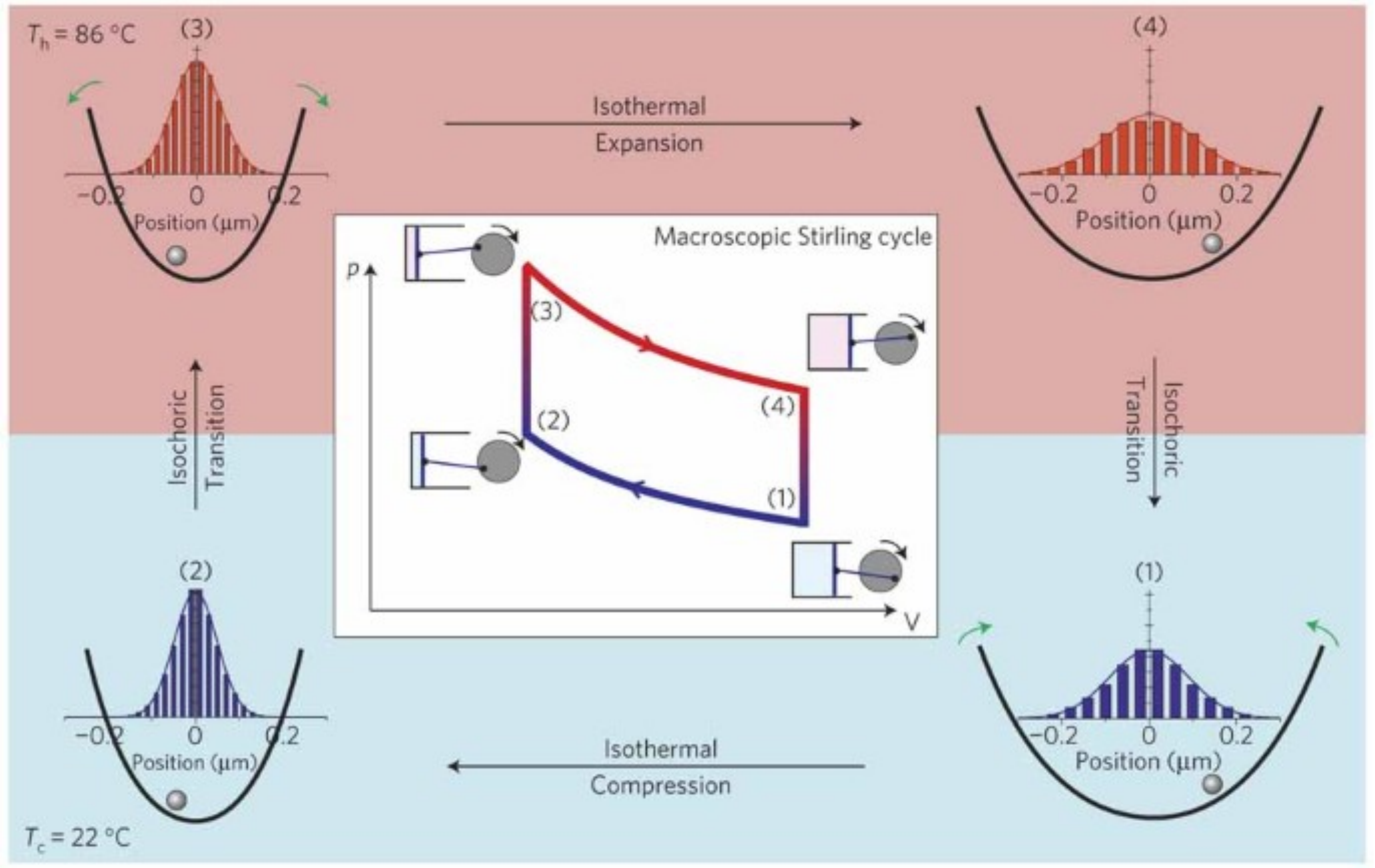
power current

Thermodynamics of precision



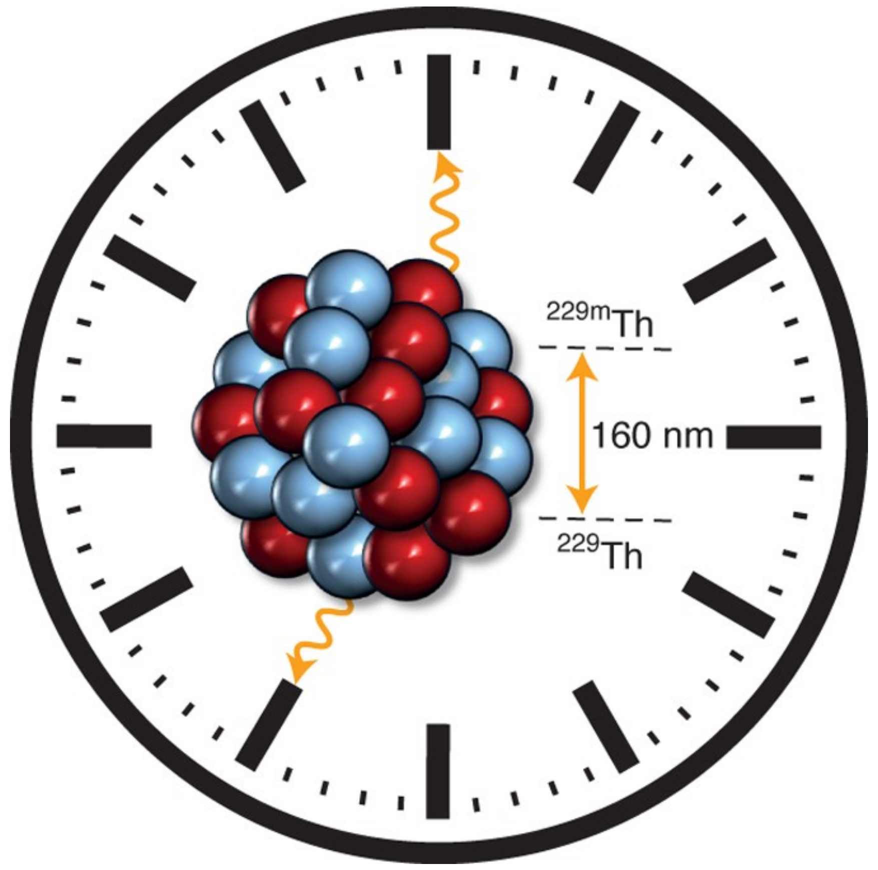
Molecular motor, Nature (2003)

traveled distance



Heat engine, Nat. Phys. (2012)

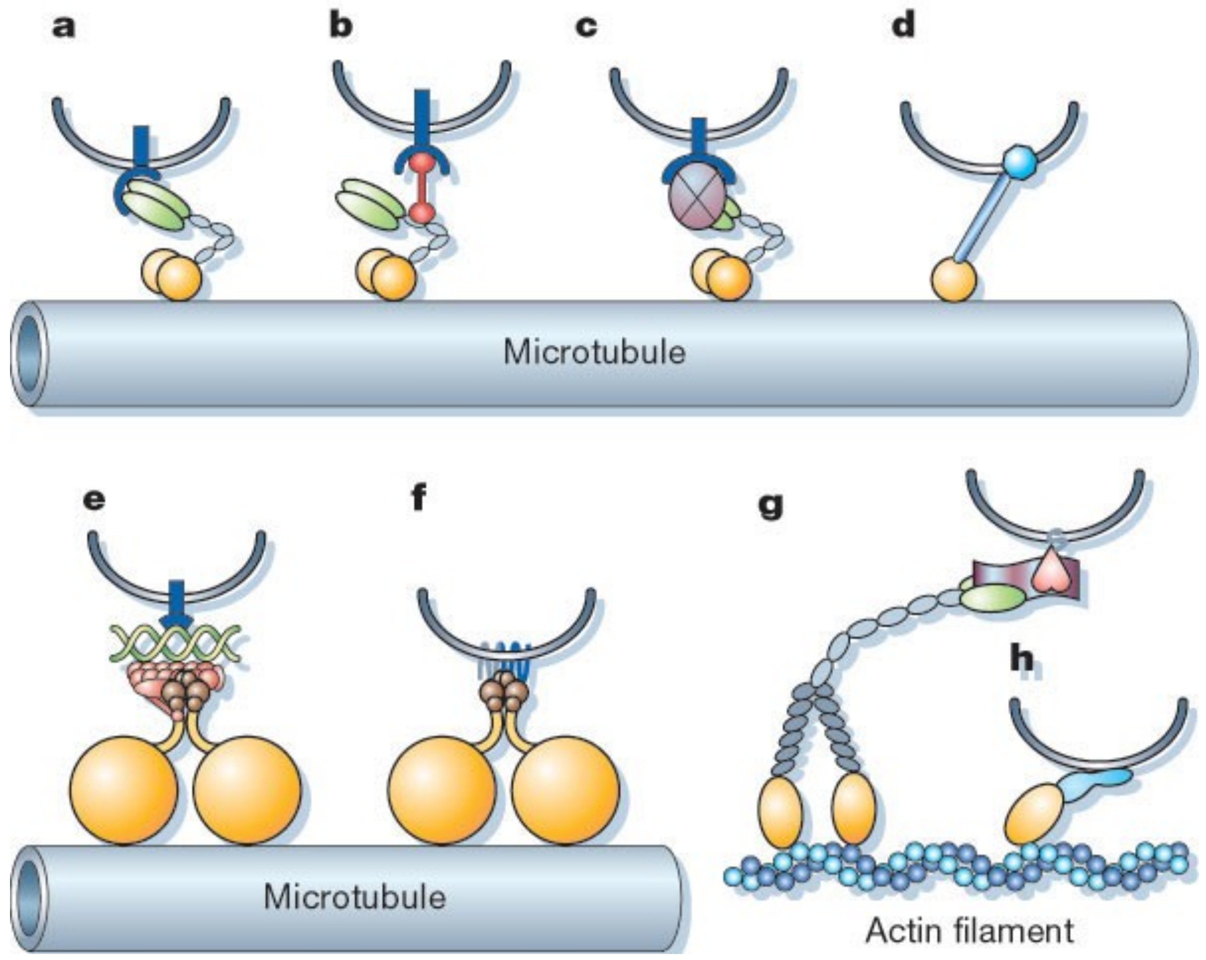
power current



Clock, Nat. Phys. (2018)

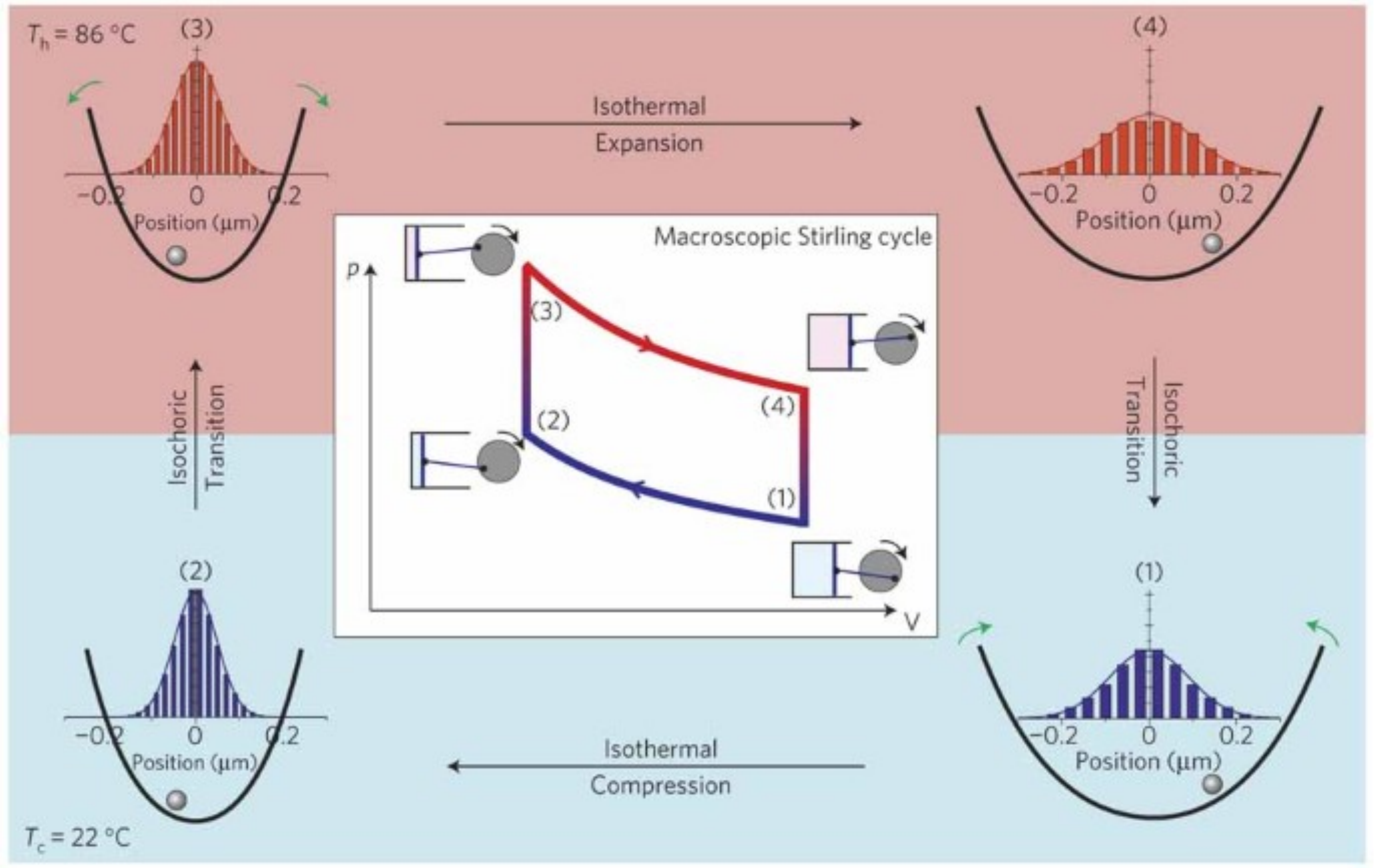
tick current

Thermodynamics of precision



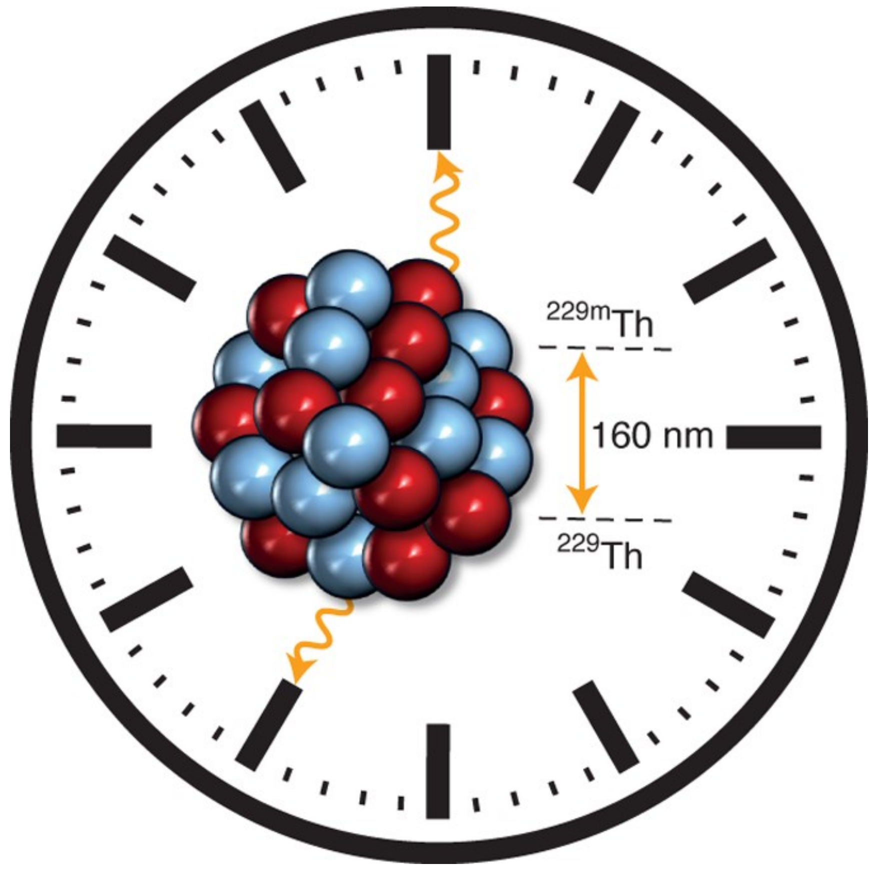
Molecular motor, Nature (2003)

traveled distance



Heat engine, Nat. Phys. (2012)

power current



Clock, Nat. Phys. (2018)

tick current

- How precise can these currents be?
- What is the cost of being precise?

Outline

- Background & motivation
- Markovian dynamics [PRX Quantum 6, 010343 (2025)]
- General open quantum dynamics [arXiv:2508.21567]
- Summary

Classical precision bounds

- Thermodynamic and kinetic uncertainty relations

$$F_\phi := \tau \frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2}$$

ϕ: time-integrated current

e.g., particle & heat current,
motor's displacement

Classical precision bounds

- Thermodynamic and kinetic uncertainty relations

$$F_\phi := \tau \frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2}$$

TUR: $F_\phi \geq \dfrac{2}{\sigma}$

Barato+, PRL (2015)

ϕ: time-integrated current

σ: entropy production rate

e.g., particle & heat current,
motor's displacement

Classical precision bounds

- Thermodynamic and kinetic uncertainty relations

$$F_\phi := \tau \frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2}$$

TUR: $F_\phi \geq \frac{2}{\sigma}$

Barato+, PRL (2015)

KUR: $F_\phi \geq \frac{1}{a}$

Garrahan, PRE (2017)

ϕ: time-integrated current

e.g., particle & heat current, motor's displacement

σ: entropy production rate

a: dynamical activity rate

Classical precision bounds

- Thermodynamic and kinetic uncertainty relations

$$F_\phi := \tau \frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2}$$

ϕ: time-integrated current

e.g., particle & heat current, motor's displacement

TUR: $F_\phi \geq \dfrac{2}{\sigma}$

Barato+, PRL (2015)

σ: entropy production rate

KUR: $F_\phi \geq \dfrac{1}{a}$

Garrahan, PRE (2017)

a: dynamical activity rate

TKUR: $F_\phi \geq \dfrac{4a}{\sigma^2}\Phi\left(\dfrac{\sigma}{2a}\right)^2 \geq \max\left(\dfrac{2}{\sigma}, \dfrac{1}{a}\right)$

Vo, TVV, Hasegawa, J. Phys. A (2022)

Φ: inverse function of $x \tanh x$

Classical precision bounds

- Thermodynamic and kinetic uncertainty relations

$$F_\phi := \tau \frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2}$$

TUR: $F_\phi \geq \dfrac{2}{\sigma}$

Barato+, PRL (2015)

KUR: $F_\phi \geq \dfrac{1}{a}$

Garrahan, PRE (2017)

ϕ: time-integrated current

e.g., particle & heat current, motor's displacement

σ: entropy production rate

a: dynamical activity rate

TKUR: $F_\phi \geq \dfrac{4a}{\sigma^2}\Phi\left(\dfrac{\sigma}{2a}\right)^2 \geq \max\left(\dfrac{2}{\sigma}, \dfrac{1}{a}\right)$

Vo, TVV, Hasegawa, J. Phys. A (2022)

- Valid only for overdamped and Markov jump processes

Φ: inverse function of $x \tanh x$

Motivation

- TKUR can be violated for quantum systems due to quantum coherence

Motivation

- TKUR can be violated for quantum systems due to quantum coherence

  x classical: $1 \to 0,\ 1 \to 0,\ \ldots$

Motivation

- TKUR can be violated for quantum systems due to quantum coherence

x classical: $1 \to 0,\ 1 \to 0,\ \dots$

✓ quantum: $|1\rangle \to |0\rangle, |1\rangle \to |0\rangle,\ \dots$

Motivation

- TKUR can be violated for quantum systems due to quantum coherence

× classical: $1 \to 0,\ 1 \to 0,\ \ldots$

✓ quantum: $|1\rangle \to |0\rangle, |1\rangle \to |0\rangle,\ \ldots$

consecutive occurrence of identical jumps ← precision can be enhanced

Motivation

- TKUR can be violated for quantum systems due to quantum coherence

✗ classical: $1 \to 0,\ 1 \to 0,\ \ldots$

✓ quantum: $|1\rangle \to |0\rangle, |1\rangle \to |0\rangle,\ \ldots$

consecutive occurrence of identical jumps ← precision can be enhanced

- How precision limits are modified in the presence of quantum effects?

Motivation

- TKUR can be violated for quantum systems due to quantum coherence

  ✗ classical: $1 \to 0,\ 1 \to 0,\ \ldots$

  ✓ quantum: $|1\rangle \to |0\rangle, |1\rangle \to |0\rangle,\ \ldots$

  consecutive occurrence of identical jumps ← precision can be enhanced

- How precision limits are modified in the presence of quantum effects?

  - Understand effects of quantum features (coherence & entanglement)

Motivation

- TKUR can be violated for quantum systems due to quantum coherence

  ✗ classical: $1 \to 0,\ 1 \to 0,\ \ldots$

  ✓ quantum: $|1\rangle \to |0\rangle, |1\rangle \to |0\rangle,\ \ldots$

  consecutive occurrence of identical jumps ← precision can be enhanced

- How precision limits are modified in the presence of quantum effects?
  - Understand effects of quantum features (coherence & entanglement)
  - Utilize quantum features to enhance precision

PRX QUANTUM **6,** 010343 (2025)

IYQ Collection

Fundamental Bounds on Precision and Response for Quantum Trajectory Observables

Tan Van Vu*

Center for Gravitational Physics and Quantum Information, Yukawa Institute for Theoretical Physics, Kyoto University, Kitashirakawa Oiwakecho, Sakyo-ku, Kyoto 606-8502, Japan

(Received 10 December 2024; revised 4 February 2025; accepted 13 February 2025; published 6 March 2025)

Markovian dynamics - setup

- GKSL equation

$$\dot{\varrho}_t = \mathscr{L}(\varrho_t),$$

$$\mathscr{L}(\circ) := -i[H, \circ] + \sum_{k\geq 1} (L_k \circ L_k^\dagger - \{L_k^\dagger L_k, \circ\}/2)$$

Markovian dynamics - setup

- GKSL equation

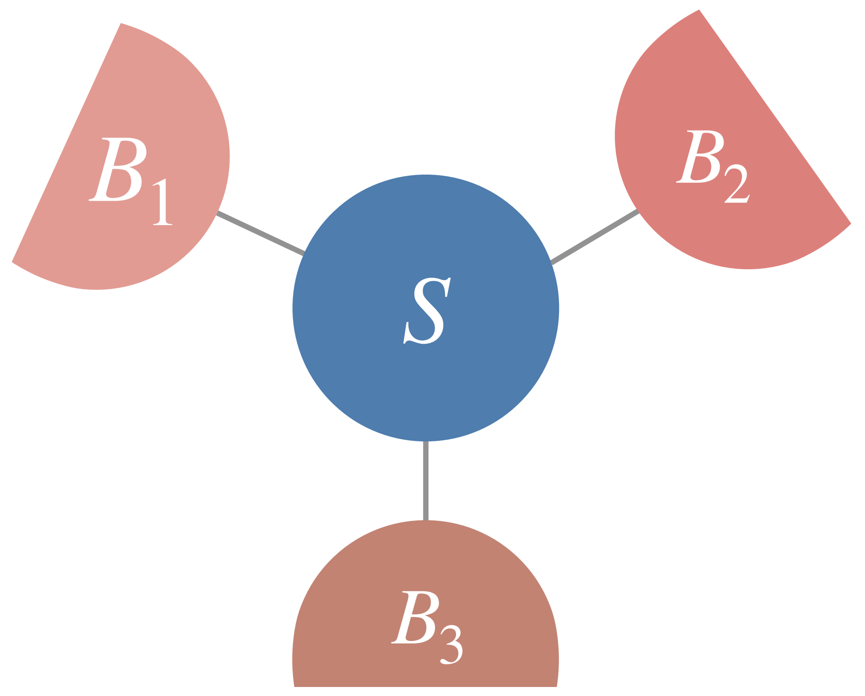
$$\dot{\varrho}_t = \mathscr{L}(\varrho_t),$$

$$\mathscr{L}(\circ) := -i[H, \circ] + \sum_{k \geq 1} (L_k \circ L_k^\dagger - \{L_k^\dagger L_k, \circ\}/2)$$

- Thermodynamic processes

local detailed balance condition $L_k = e^{\Delta s_k/2} L_{k^*}^\dagger$

Δs_k: environmental entropy change due to jump L_k



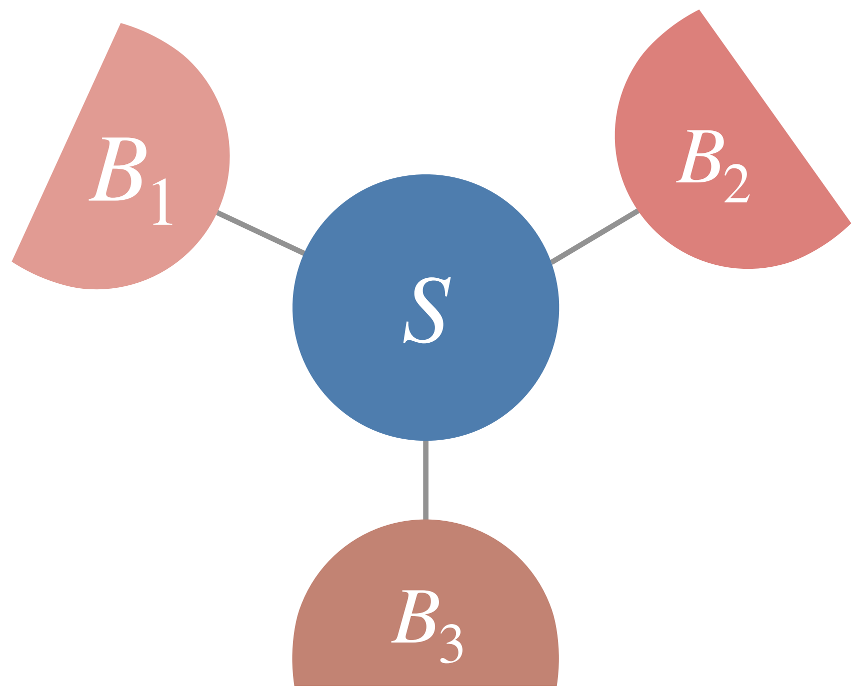
Markovian dynamics - setup

- GKSL equation

$$\dot{\varrho}_t = \mathscr{L}(\varrho_t),$$

$$\mathscr{L}(\circ) := -i[H, \circ] + \sum_{k\geq 1} (L_k \circ L_k^\dagger - \{L_k^\dagger L_k, \circ\}/2)$$

- Thermodynamic processes



local detailed balance condition $L_k = e^{\Delta s_k/2} L_{k*}^\dagger$

Δs_k: environmental entropy change due to jump L_k

- Entropy production & dynamical activity

$$\sigma = \sum_{k\geq 1} \mathrm{tr}(L_k \pi L_k^\dagger) \Delta s_k \qquad a = \sum_{k\geq 1} \mathrm{tr}(L_k \pi L_k^\dagger)$$

π: steady state

Unraveling & observables

- Quantum jump unraveling

$$\varrho_{t+dt} = \sum_{k\geq 0} M_k \varrho_t M_k^\dagger \qquad M_0 = \mathbf{1} - (iH + \sum_{k\geq 1} L_k^\dagger L_k/2)dt, \quad M_k = L_k\sqrt{dt}\ (k \geq 1)$$

no jump

kth jump

Unraveling & observables

- Quantum jump unraveling

$$\varrho_{t+dt} = \sum_{k\geq 0} M_k \varrho_t M_k^\dagger \qquad M_0 = \mathbf{1} - (iH + \sum_{k\geq 1} L_k^\dagger L_k/2)dt, \quad M_k = L_k\sqrt{dt} \ (k \geq 1)$$

no jump (M_0) — kth jump (M_k)

- Observables defined for stochastic trajectory $\Gamma = \{(t_1, k_1), \ldots, (t_J, k_J)\}$

$$\phi(\Gamma) := \sum_{i=1}^{J} c_{k_i}$$

c_k: coefficient associated with kth jump

e.g., entropy flux ($c_k = \Delta s_k$), number of jumps ($c_k = 1$)

Unraveling & observables

- Quantum jump unraveling

$$\varrho_{t+dt} = \sum_{k\geq 0} M_k \varrho_t M_k^\dagger \qquad \underbrace{M_0 = \mathbf{1} - (iH + \sum_{k\geq 1} L_k^\dagger L_k/2)dt}_{\text{no jump}}, \quad \underbrace{M_k = L_k\sqrt{dt}}_{k\text{th jump}} \ (k \geq 1)$$

- Observables defined for stochastic trajectory $\Gamma = \{(t_1, k_1), \ldots, (t_J, k_J)\}$

$$\phi(\Gamma) := \sum_{i=1}^{J} c_{k_i}$$

c_k: coefficient associated with kth jump

e.g., entropy flux $(c_k = \Delta s_k)$, number of jumps $(c_k = 1)$

- Currents: $c_k = -c_{k^*}$ (time-antisymmetry)

Quantum thermokinetic uncertainty relation

Quantum thermokinetic uncertainty relation

- Quantum TKUR for currents

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

Quantum thermokinetic uncertainty relation

- Quantum TKUR for currents

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

$$\delta_\phi = -\frac{(\mathbf{1}|\widehat{\mathscr{C}}\widehat{\mathscr{L}}^+\widehat{\mathscr{D}}_\ell|\pi)}{(\mathbf{1}|\widehat{\mathscr{C}}|\pi)}$$: quantum correction term

$$\widehat{\mathscr{D}}_\ell = \sum_{k\geq 1} \ell_k \left[L_k \otimes L_k^* - \frac{1}{2}(L_k^\dagger L_k) \otimes \mathbf{1} - \frac{1}{2}\mathbf{1} \otimes (L_k^\dagger L_k)^\top\right]$$

$$\widehat{\mathscr{C}} = \sum_{k\geq 1} c_k L_k \otimes L_k^*$$

$\widehat{\mathscr{L}}^+$: pseudoinverse of $\widehat{\mathscr{L}}$

Quantum thermokinetic uncertainty relation

- Quantum TKUR for currents

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

$$\delta_\phi = -\frac{(\mathbf{1}|\widehat{\mathscr{C}}\,\widehat{\mathscr{L}}^{+}\widehat{\mathscr{D}}_\ell|\pi)}{(\mathbf{1}|\widehat{\mathscr{C}}|\pi)}$$: quantum correction term

$$\widehat{\mathscr{D}}_\ell = \sum_{k\geq 1} \ell_k \left[L_k \otimes L_k^* - \frac{1}{2}(L_k^\dagger L_k) \otimes \mathbf{1} - \frac{1}{2}\mathbf{1} \otimes (L_k^\dagger L_k)^\top\right]$$

$$\widehat{\mathscr{C}} = \sum_{k\geq 1} c_k L_k \otimes L_k^*$$

$\widehat{\mathscr{L}}^{+}$: pseudoinverse of $\widehat{\mathscr{L}}$

- Remarks

Quantum thermokinetic uncertainty relation

- Quantum TKUR for currents

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

$$\delta_\phi = -\frac{(\mathbf{1} | \widehat{\mathscr{C}} \widehat{\mathscr{L}}^+ \widehat{\mathscr{D}}_\ell | \pi)}{(\mathbf{1} | \widehat{\mathscr{C}} | \pi)}$$: quantum correction term

$$\widehat{\mathscr{D}}_\ell = \sum_{k\geq 1} \ell_k \left[L_k \otimes L_k^* - \frac{1}{2}(L_k^\dagger L_k) \otimes \mathbf{1} - \frac{1}{2}\mathbf{1} \otimes (L_k^\dagger L_k)^\top \right]$$

$$\widehat{\mathscr{C}} = \sum_{k\geq 1} c_k L_k \otimes L_k^*$$

$\widehat{\mathscr{L}}^+$: pseudoinverse of $\widehat{\mathscr{L}}$

- Remarks
  - δ_ϕ vanishes in the classical limit

Quantum thermokinetic uncertainty relation

- Quantum TKUR for currents

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

$$\delta_\phi = -\frac{(\mathbf{1}|\widehat{\mathscr{C}}\,\widehat{\mathscr{L}}^+\widehat{\mathscr{D}}_\ell|\pi)}{(\mathbf{1}|\widehat{\mathscr{C}}|\pi)}$$: quantum correction term

$$\widehat{\mathscr{D}}_\ell = \sum_{k\geq 1} \ell_k \left[L_k \otimes L_k^* - \frac{1}{2}(L_k^\dagger L_k)\otimes \mathbf{1} - \frac{1}{2}\mathbf{1}\otimes (L_k^\dagger L_k)^\top\right]$$

$$\widehat{\mathscr{C}} = \sum_{k\geq 1} c_k L_k \otimes L_k^*$$

$\widehat{\mathscr{L}}^+$: pseudoinverse of $\widehat{\mathscr{L}}$

- Remarks
  - δ_ϕ vanishes in the classical limit
  - classical TKUR can be violated if $\delta_\phi \in (-2,0)$

Quantum thermokinetic uncertainty relation

- Quantum TKUR for currents

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2}\Phi\left(\frac{\sigma}{2a}\right)^2$$

$$\delta_\phi = -\frac{(\mathbf{1}|\widehat{\mathscr{C}}\widehat{\mathscr{L}}^+\widehat{\mathscr{D}}_\ell|\pi)}{(\mathbf{1}|\widehat{\mathscr{C}}|\pi)}$$: quantum correction term

$$\widehat{\mathscr{D}}_\ell = \sum_{k\geq 1}\ell_k\left[L_k\otimes L_k^* - \frac{1}{2}(L_k^\dagger L_k)\otimes\mathbf{1} - \frac{1}{2}\mathbf{1}\otimes(L_k^\dagger L_k)^\top\right]$$

$$\widehat{\mathscr{C}} = \sum_{k\geq 1} c_k L_k\otimes L_k^*$$

$\widehat{\mathscr{L}}^+$: pseudoinverse of $\widehat{\mathscr{L}}$

- Remarks
  - δ_ϕ vanishes in the classical limit
  - classical TKUR can be violated if $\delta_\phi \in (-2,0)$
  - generalizable to time-dependent driving

Quantum thermokinetic uncertainty relation

- Quantum TKUR for currents

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

$$\delta_\phi = -\frac{(\mathbf{1}|\widehat{\mathscr{C}}\widehat{\mathscr{L}}^{+}\widehat{\mathscr{D}}_\ell|\pi)}{(\mathbf{1}|\widehat{\mathscr{C}}|\pi)}$$: quantum correction term

$$\widehat{\mathscr{D}}_\ell = \sum_{k\geq 1} \ell_k \left[L_k \otimes L_k^* - \frac{1}{2}(L_k^\dagger L_k) \otimes \mathbf{1} - \frac{1}{2}\mathbf{1} \otimes (L_k^\dagger L_k)^\top\right]$$

$$\widehat{\mathscr{C}} = \sum_{k\geq 1} c_k L_k \otimes L_k^*$$

$\widehat{\mathscr{L}}^{+}$: pseudoinverse of $\widehat{\mathscr{L}}$

- Remarks
  - δ_ϕ vanishes in the classical limit
  - classical TKUR can be violated if $\delta_\phi \in (-2,0)$
  - generalizable to time-dependent driving
  - similar result can be obtained for quantum diffusion unraveling

Application to heat engines

Application to heat engines

- Power-efficiency trade-off for steady-state heat engines

$$P \frac{\eta}{\eta_C - \eta} \frac{T_c (1 + \delta_P)^2}{D_P} \leq \frac{1}{2}$$

P: power, D_P: power fluctuation

η: efficiency, η_C: Carnot efficiency

Application to heat engines

- Power-efficiency trade-off for steady-state heat engines

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c(1+\delta_P)^2}{D_P}\leq\frac{1}{2}$$

cf. classical trade-off

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c}{D_P}\leq\frac{1}{2}$$

Pietzonka+, PRL (2018)

P: power, D_P: power fluctuation

η: efficiency, η_C: Carnot efficiency

Application to heat engines

- Power-efficiency trade-off for steady-state heat engines

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c(1+\delta_P)^2}{D_P}\le\frac{1}{2}$$

P: power, D_P: power fluctuation

η: efficiency, η_C: Carnot efficiency

cf. classical trade-off

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c}{D_P}\le\frac{1}{2}$$

Pietzonka+, PRL (2018)

$\mapsto D_P\to\infty$ as $\eta\to\eta_C$ at $P=O(1)$

Application to heat engines

- Power-efficiency trade-off for steady-state heat engines

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c(1+\delta_P)^2}{D_P} \leq \frac{1}{2}$$

P: power, D_P: power fluctuation

η: efficiency, η_C: Carnot efficiency

$$(1+\delta_P)^2 = O(\eta_C-\eta)$$

cf. classical trade-off

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c}{D_P} \leq \frac{1}{2}$$

Pietzonka+, PRL (2018)

$\mapsto D_P \to \infty$ as $\eta \to \eta_C$ at $P = O(1)$

Application to heat engines

- Power-efficiency trade-off for steady-state heat engines

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c(1+\delta_P)^2}{D_P}\leq\frac{1}{2}$$

P: power, D_P: power fluctuation

η: efficiency, η_C: Carnot efficiency

cf. classical trade-off

$$P\frac{\eta}{\eta_C-\eta}\frac{T_c}{D_P}\leq\frac{1}{2}$$

Pietzonka+, PRL (2018)

$\mapsto D_P\to\infty$ as $\eta\to\eta_C$ at $P=O(1)$

$(1+\delta_P)^2=O(\eta_C-\eta)$ $\mapsto$ possible to achieve Carnot efficiency at finite power without divergent power fluctuation?

Application to quantum clocks

- Precision of quantum clocks is limited by both dissipation and coherence

ϕ: number of clock cycles

$$\mathcal{N} := \frac{\langle \phi \rangle}{\mathrm{Var}[\phi]}$$: clock precision

$$\Sigma_{\mathrm{tick}} := \frac{\tau\sigma}{\langle \phi \rangle}$$: entropy production per tick

Application to quantum clocks

- Precision of quantum clocks is limited by both dissipation and coherence

ϕ: number of clock cycles

$$\mathcal{N} := \frac{\langle\phi\rangle}{\mathrm{Var}[\phi]}$$: clock precision

$$\Sigma_{\mathrm{tick}} := \frac{\tau\sigma}{\langle\phi\rangle}$$: entropy production per tick

$$\frac{F_\phi}{(1+\delta_\phi)^2} \geq \frac{2}{\sigma} \quad \Rightarrow \quad \mathcal{N} \leq \frac{\Sigma_{\mathrm{tick}}}{2(1+\delta_\phi)^2}$$

Application to quantum clocks

- Precision of quantum clocks is limited by both dissipation and coherence

ϕ: number of clock cycles

$\mathcal{N} := \frac{\langle \phi \rangle}{\mathrm{Var}[\phi]}$: clock precision

$\Sigma_{\mathrm{tick}} := \frac{\tau\sigma}{\langle \phi \rangle}$: entropy production per tick

$$\frac{F_\phi}{(1+\delta_\phi)^2} \geq \frac{2}{\sigma} \quad \Rightarrow \quad \mathcal{N} \leq \frac{\Sigma_{\mathrm{tick}}}{2(1+\delta_\phi)^2}$$

$\mapsto$ precision can be arbitrarily large as $1+\delta_\phi \to 0$

Application to quantum clocks

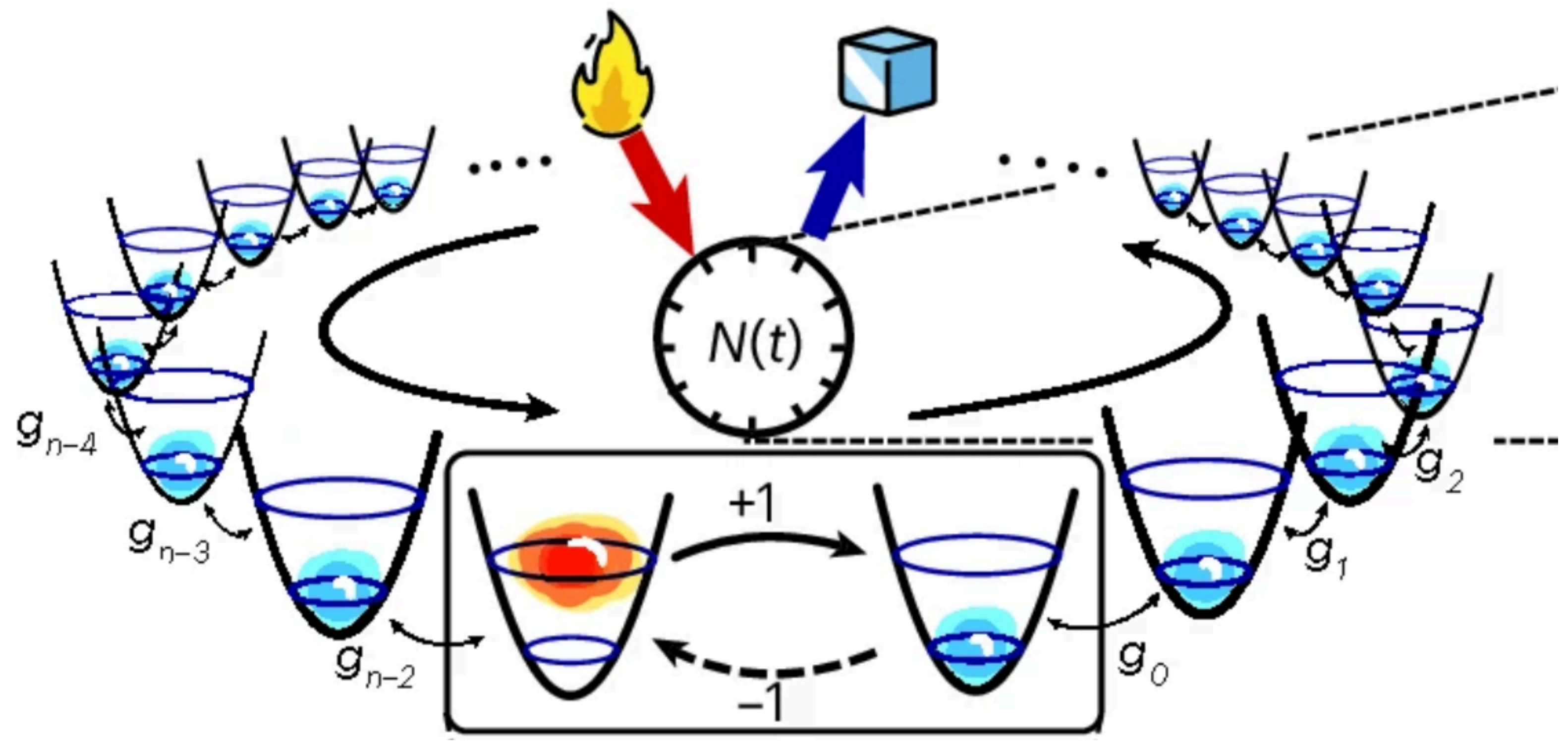
- Precision of quantum clocks is limited by both dissipation and coherence

ϕ: number of clock cycles

$\mathcal{N} := \dfrac{\langle \phi \rangle}{\mathrm{Var}[\phi]}$: clock precision

$\Sigma_{\mathrm{tick}} := \dfrac{\tau\sigma}{\langle \phi \rangle}$: entropy production per tick

$$\frac{F_\phi}{(1+\delta_\phi)^2} \geq \frac{2}{\sigma} \quad \Rightarrow \quad \mathcal{N} \leq \frac{\Sigma_{\mathrm{tick}}}{2(1+\delta_\phi)^2}$$



$\mapsto$ precision can be arbitrarily large as $1+\delta_\phi \to 0$

cf. $\mathcal{N} = O(e^{\Sigma_{\mathrm{tick}}})$ Meier+, Nat. Phys. (2025)

arXiv > quant-ph > arXiv:2508.21567

Universal Precision Limits in General Open Quantum Systems

Tan Van Vu,[1,*] Ryotaro Honma,[1] and Keiji Saito[2]

[1] *Center for Gravitational Physics and Quantum Information, Yukawa Institute for Theoretical Physics, Kyoto University, Kitashirakawa Oiwakecho, Sakyo-ku, Kyoto 606-8502, Japan*

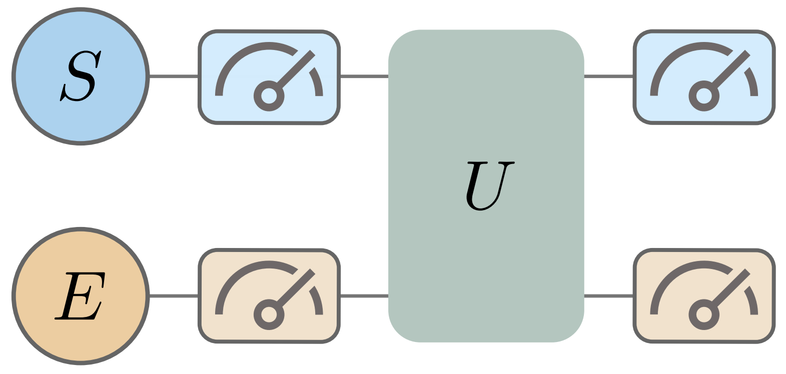
[2] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

General open quantum dynamics - setup

- General unitary dynamics

$$\dot{\varrho}_t = -[H, \varrho_t]$$
$$H = H_S + H_E + H_I$$

(a)

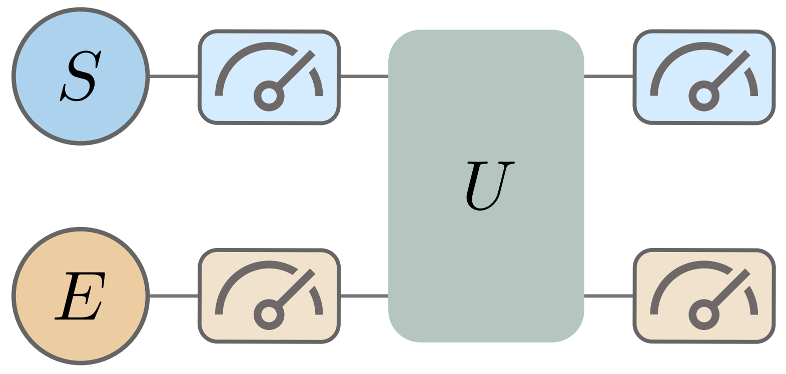


General open quantum dynamics - setup

- General unitary dynamics

$$\dot{\varrho}_t = -[H, \varrho_t]$$
$$H = H_S + H_E + H_I$$

(a)



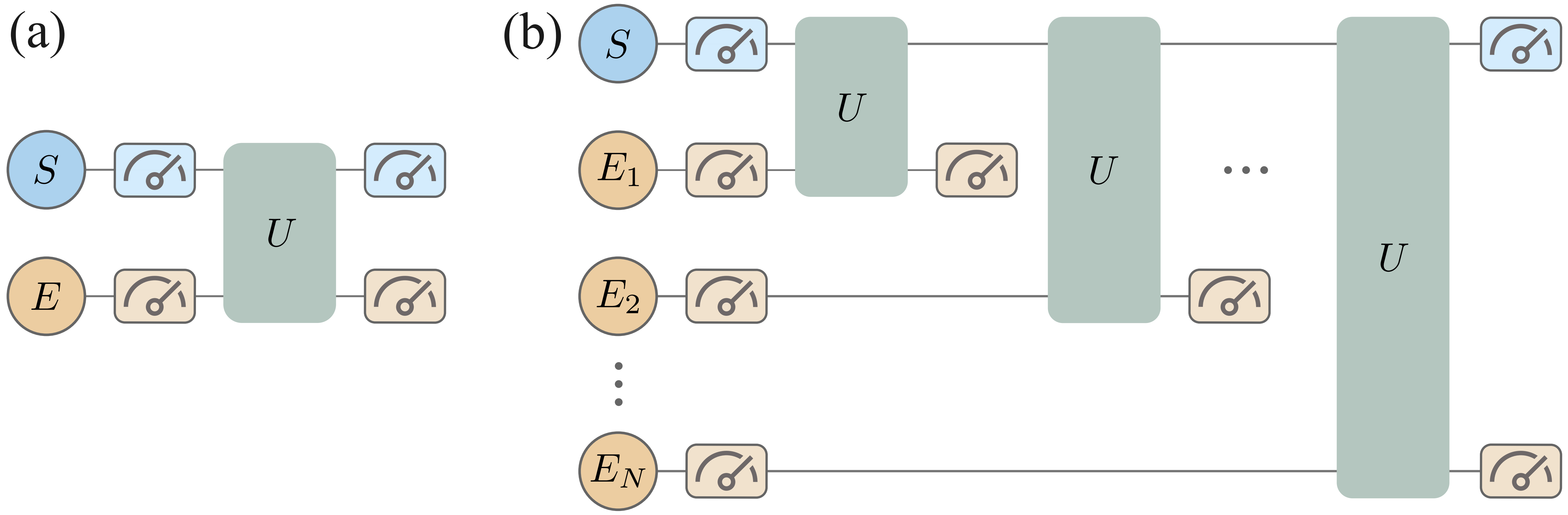
- Projective measurements on both system and environments

$$\varrho_S = \sum_n p_n |n\rangle\langle n|, \quad \varrho_E = \sum_\mu p_\mu |\mu\rangle\langle \mu| \ (= e^{-\beta H_E}/Z)$$

General open quantum dynamics - setup

- General unitary dynamics

$$\dot{\varrho}_t = -[H, \varrho_t]$$
$$H = H_S + H_E + H_I$$



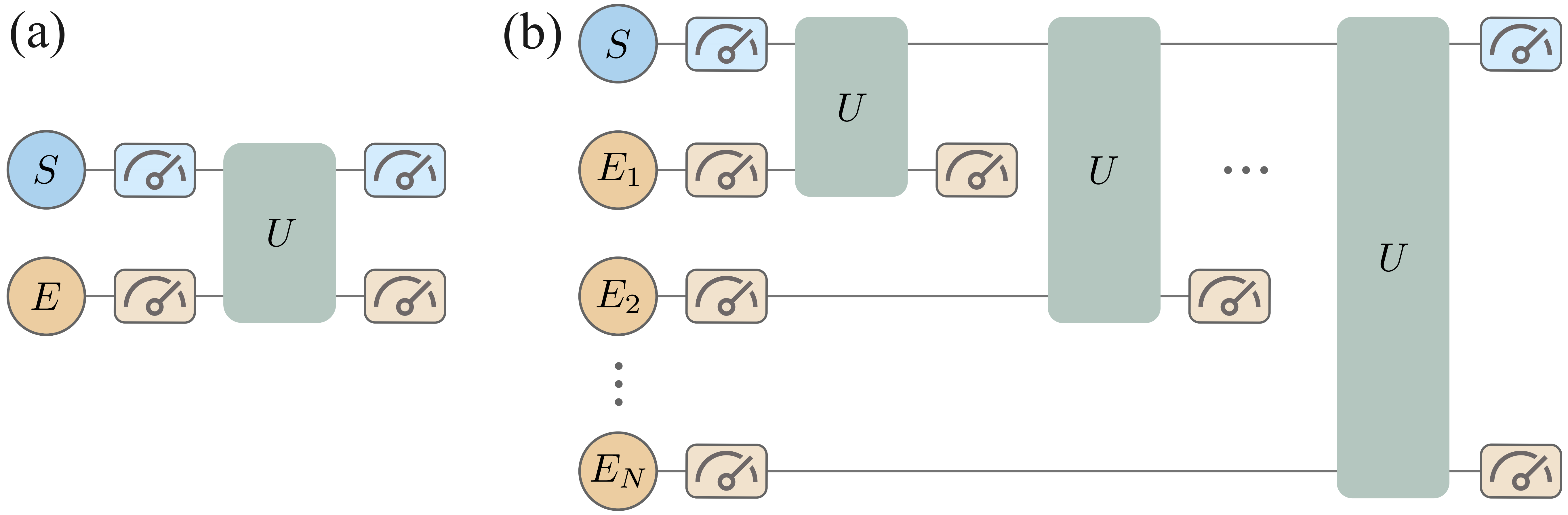
- Projective measurements on both system and environments

$$\varrho_S = \sum_n p_n |n\rangle\langle n|, \quad \varrho_E = \sum_\mu p_\mu |\mu\rangle\langle \mu| \;(= e^{-\beta H_E}/Z)$$

General open quantum dynamics - setup

- General unitary dynamics

$$\dot{\varrho}_t = -[H, \varrho_t]$$
$$H = H_S + H_E + H_I$$



- Projective measurements on both system and environments

$$\varrho_S = \sum_n p_n |n\rangle\langle n|, \quad \varrho_E = \sum_\mu p_\mu |\mu\rangle\langle \mu| \; (= e^{-\beta H_E}/Z)$$

- Stochastic trajectory $\gamma = \{n, (\nu_1, \mu_1), \ldots, (\nu_N, \mu_N), m\}$

Symmetry-breaking quantities

Symmetry-breaking quantities

- Entropy production

$$\Sigma := \underline{\Delta S} + \underline{\beta Q}$$

ΔS: von Neumann entropy change

Q: heat dissipation

Symmetry-breaking quantities

- Entropy production

$$\Sigma := \underline{\Delta S} + \underline{\beta Q} = \left\langle \ln \frac{\mathrm{P}(\gamma)}{\widetilde{\mathrm{P}}(\widetilde{\gamma})} \right\rangle$$

ΔS: von Neumann entropy change

Q: heat dissipation

$\widetilde{\gamma} = \{m, (\mu_N, \nu_N), \ldots, (\mu_1, \nu_1), n\}$: time-reversed trajectory

$\mathrm{P}, \widetilde{\mathrm{P}}$: path probabilities in forward and backward processes

Symmetry-breaking quantities

- Entropy production

$$\Sigma := \underline{\Delta S} + \underline{\beta Q} = \left\langle \ln \frac{\mathrm{P}(\gamma)}{\widetilde{\mathrm{P}}(\widetilde{\gamma})} \right\rangle$$

ΔS: von Neumann entropy change

Q: heat dissipation

$\widetilde{\gamma} = \{m, (\mu_N, \nu_N), \ldots, (\mu_1, \nu_1), n\}$: time-reversed trajectory

$\mathrm{P}, \widetilde{\mathrm{P}}$: path probabilities in forward and backward processes

- Forward-backward asymmetry

$$\Sigma_* := \left\langle \ln \frac{\mathrm{P}(\gamma)}{\widetilde{\mathrm{P}}(\gamma)} \right\rangle$$

Symmetry-breaking quantities

- Entropy production

$$\Sigma := \underline{\Delta S} + \underline{\beta Q} = \left\langle \ln \frac{\mathsf{P}(\gamma)}{\widetilde{\mathsf{P}}(\widetilde{\gamma})} \right\rangle$$

ΔS: von Neumann entropy change

Q: heat dissipation

$\widetilde{\gamma} = \{m, (\mu_N, \nu_N), \ldots, (\mu_1, \nu_1), n\}$: time-reversed trajectory

$\mathsf{P}, \widetilde{\mathsf{P}}$: path probabilities in forward and backward processes

- Forward-backward asymmetry

$$\Sigma_* := \left\langle \ln \frac{\mathsf{P}(\gamma)}{\widetilde{\mathsf{P}}(\gamma)} \right\rangle$$

- difference between forward and backward processes

Symmetry-breaking quantities

- Entropy production

$$\Sigma := \underline{\Delta S} + \underline{\beta Q} = \left\langle \ln \frac{\mathrm{P}(\gamma)}{\widetilde{\mathrm{P}}(\widetilde{\gamma})} \right\rangle$$

ΔS: von Neumann entropy change

Q: heat dissipation

$\widetilde{\gamma} = \{m, (\mu_N, \nu_N), \ldots, (\mu_1, \nu_1), n\}$: time-reversed trajectory

$\mathrm{P}, \widetilde{\mathrm{P}}$: path probabilities in forward and backward processes

- Forward-backward asymmetry

$$\Sigma_* := \left\langle \ln \frac{\mathrm{P}(\gamma)}{\widetilde{\mathrm{P}}(\gamma)} \right\rangle$$

- difference between forward and backward processes
- vanishes when $\mathrm{P} \equiv \widetilde{\mathrm{P}}$

Symmetry-breaking quantities

- Entropy production

$$\Sigma := \underline{\Delta S} + \underline{\beta Q} \; = \left\langle \ln \frac{\mathrm{P}(\gamma)}{\widetilde{\mathrm{P}}(\widetilde{\gamma})} \right\rangle$$

ΔS: von Neumann entropy change

Q: heat dissipation

$\widetilde{\gamma} = \{m, (\mu_N, \nu_N), \ldots, (\mu_1, \nu_1), n\}$: time-reversed trajectory

$\mathrm{P}, \widetilde{\mathrm{P}}$: path probabilities in forward and backward processes

- Forward-backward asymmetry

$$\Sigma_* := \left\langle \ln \frac{\mathrm{P}(\gamma)}{\widetilde{\mathrm{P}}(\gamma)} \right\rangle$$

- difference between forward and backward processes
- vanishes when $\mathrm{P} \equiv \widetilde{\mathrm{P}}$
- arises due to dynamical factors

Origin of forward-backward asymmetry

Origin of forward-backward asymmetry

- External magnetic fields $\mathsf{P} \neq \widetilde{\mathsf{P}} \to \Sigma_* > 0$

Origin of forward-backward asymmetry

- External magnetic fields $\mathsf{P} \neq \widetilde{\mathsf{P}} \rightarrow \Sigma_* > 0$

- Quantum coherence in Markovian dynamics

Origin of forward-backward asymmetry

- External magnetic fields $\mathsf{P} \neq \widetilde{\mathsf{P}} \to \Sigma_* > 0$

- Quantum coherence in Markovian dynamics

$$\Sigma_* \geq \frac{8}{9}\frac{|\langle[H,\mathsf{L}]\rangle_S|^2}{\langle 2H^2 + \mathsf{L}^2/2\rangle_S}T^2 \text{ for } T \ll 1 \qquad \mathsf{L} := \sum_k L_k^\dagger L_k, \quad \langle \circ \rangle_S := \mathrm{tr}(\circ \varrho_S)$$

Origin of forward-backward asymmetry

- External magnetic fields $\mathsf{P} \neq \widetilde{\mathsf{P}} \to \Sigma_* > 0$

- Quantum coherence in Markovian dynamics

$$\Sigma_* \geq \frac{8}{9} \frac{|\langle [H, \mathsf{L}] \rangle_S|^2}{\langle 2H^2 + \mathsf{L}^2/2 \rangle_S} T^2 \text{ for } T \ll 1 \qquad \mathsf{L} := \sum_k L_k^\dagger L_k, \quad \langle \circ \rangle_S := \mathrm{tr}(\circ \varrho_S)$$

$$\Sigma_* \geq D(\boldsymbol{p} \| \boldsymbol{q}) \qquad \varrho_S = \sum_n p_n |n\rangle\langle n|, \; q_n = \langle n | e^{\mathscr{L}' T}(\varrho_S) | n \rangle \;, \; \mathscr{L}'(\circ) - \mathscr{L}(\circ) = 2i[H, \circ]$$

Origin of forward-backward asymmetry

- External magnetic fields $\mathsf{P} \neq \widetilde{\mathsf{P}} \to \Sigma_* > 0$

- Quantum coherence in Markovian dynamics

$$\Sigma_* \geq \frac{8}{9}\frac{|\langle [H, \mathsf{L}]\rangle_S|^2}{\langle 2H^2 + \mathsf{L}^2/2\rangle_S} T^2 \text{ for } T \ll 1 \qquad \mathsf{L} := \sum_k L_k^\dagger L_k, \quad \langle \circ \rangle_S := \mathrm{tr}(\circ \varrho_S)$$

$$\Sigma_* \geq D(\boldsymbol{p} \| \boldsymbol{q}) \qquad \varrho_S = \sum_n p_n |n\rangle\langle n|,\ q_n = \langle n | e^{\mathscr{L}'T}(\varrho_S) | n\rangle,\ \mathscr{L}'(\circ) - \mathscr{L}(\circ) = 2i[H, \circ]$$

- Quantum entanglement between system and environment

Origin of forward-backward asymmetry

- External magnetic fields $\quad \mathsf{P} \neq \widetilde{\mathsf{P}} \to \Sigma_* > 0$

- Quantum coherence in Markovian dynamics

$$\Sigma_* \geq \frac{8}{9}\frac{|\langle[H,\mathsf{L}]\rangle_S|^2}{\langle 2H^2+\mathsf{L}^2/2\rangle_S}T^2 \text{ for } T \ll 1 \qquad \mathsf{L} := \sum_k L_k^\dagger L_k, \quad \langle \circ \rangle_S := \mathrm{tr}(\circ \varrho_S)$$

$$\Sigma_* \geq D(\boldsymbol{p}||\boldsymbol{q}) \qquad \varrho_S = \sum_n p_n |n\rangle\langle n|,\ q_n = \langle n|e^{\mathscr{L}'T}(\varrho_S)|n\rangle\ ,\ \mathscr{L}'(\circ) - \mathscr{L}(\circ) = 2i[H,\circ]$$

- Quantum entanglement between system and environment

No ability of generating entanglement $\to \Sigma_* = 0$

Origin of forward-backward asymmetry

- External magnetic fields $\quad \mathsf{P} \neq \widetilde{\mathsf{P}} \rightarrow \Sigma_* > 0$

- Quantum coherence in Markovian dynamics

$$\Sigma_* \geq \frac{8}{9} \frac{|\langle [H, \mathsf{L}] \rangle_S|^2}{\langle 2H^2 + \mathsf{L}^2/2 \rangle_S} T^2 \text{ for } T \ll 1 \qquad \mathsf{L} := \sum_k L_k^\dagger L_k, \quad \langle \circ \rangle_S := \mathrm{tr}(\circ \varrho_S)$$

$$\Sigma_* \geq D(\boldsymbol{p} \| \boldsymbol{q}) \qquad \varrho_S = \sum_n p_n |n\rangle\langle n|, \; q_n = \langle n | e^{\mathscr{L}' T}(\varrho_S) | n \rangle \;, \; \mathscr{L}'(\circ) - \mathscr{L}(\circ) = 2i[H, \circ]$$

- Quantum entanglement between system and environment

No ability of generating entanglement $\rightarrow \Sigma_* = 0$

Σ_*: asymmetry induced by dynamical factors (e.g., coherence, entanglement, magnetic fields)

Generalized TUR

Generalized TUR

- Time-antisymmetric currents $\phi(\gamma) = -\phi(\widetilde{\gamma})$ (time-extensive & time-intensive)

Generalized TUR

- Time-antisymmetric currents $\phi(\gamma) = -\phi(\widetilde{\gamma})$ (time-extensive & time-intensive)

- Generalized TUR

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq f(\Sigma + \Sigma_*)$$

$$f(x) = 4[\Phi(x/2)/x]^2 - 1 \in (0, +\infty)$$

Φ: inverse function of $x \tanh(x)$

Generalized TUR

- Time-antisymmetric currents $\phi(\gamma) = -\phi(\tilde{\gamma})$ (time-extensive & time-intensive)

- Generalized TUR

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq f(\Sigma + \Sigma_*)$$

$$f(x) = 4[\Phi(x/2)/x]^2 - 1 \in (0, +\infty)$$

Φ: inverse function of $x \tanh(x)$

↦ high precision of currents requires large dissipation or strong asymmetry

Generalized TUR

- Time-antisymmetric currents $\phi(\gamma) = -\phi(\tilde{\gamma})$ (time-extensive & time-intensive)

- Generalized TUR

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq f(\Sigma + \Sigma_*)$$

$$f(x) = 4[\Phi(x/2)/x]^2 - 1 \in (0, +\infty)$$

Φ: inverse function of $x \tanh(x)$

↦ high precision of currents requires large dissipation or strong asymmetry

✓ tight and saturable

✓ fundamental thermodynamic limit on current precision

Remarks on generalized TUR

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq f(\Sigma + \Sigma_*)$$

Remarks on generalized TUR

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq f(\Sigma + \Sigma_*)$$

- Behavior of lower-bound function

$f(x) \simeq 2/x \ (x \ll 1)$ and $f(x) \propto e^{-x} \ (x \gg 1)$

Remarks on generalized TUR

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq f(\Sigma + \Sigma_*)$$

- Behavior of lower-bound function

$$f(x) \simeq 2/x \ (x \ll 1) \text{ and } f(x) \propto e^{-x} \ (x \gg 1)$$

- Recover conventional fluctuation-theorem TUR when $\Sigma_* = 0$

$$\frac{P(\sigma, \phi)}{P(-\sigma, -\phi)} = e^{\sigma} \to \frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq f(\Sigma)$$

PRL 123, 110602 (2019)
PRL 123, 090604 (2019)

Remarks on generalized TUR

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq f(\Sigma + \Sigma_*)$$

- Behavior of lower-bound function

$$f(x) \simeq 2/x \ (x \ll 1) \text{ and } f(x) \propto e^{-x} \ (x \gg 1)$$

- Recover conventional fluctuation-theorem TUR when $\Sigma_* = 0$

$$\frac{P(\sigma, \phi)}{P(-\sigma, -\phi)} = e^{\sigma} \to \frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq f(\Sigma)$$

PRL 123, 110602 (2019)
PRL 123, 090604 (2019)

- Generalization to arbitrary initial states

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq f(\Sigma + \Sigma_* + \mathfrak{b})$$

$\mathfrak{b}$: boundary term

Generalized KUR

Generalized KUR

- Generic observables $\phi(\gamma) = 0 \ \forall \gamma \in I, \quad I = \{\gamma \,|\, \mu_i = \nu_i \forall i\}$

  trajectories that have no jump

Generalized KUR

- Generic observables $\phi(\gamma) = 0 \ \forall \gamma \in I, \quad I = \{\gamma \,|\, \mu_i = \nu_i \forall i\}$

  trajectories that have no jump

- Generalized KUR

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq \frac{1}{\mathscr{P}^{-1} - 1}$$

$\mathscr{P} := \sum_{\gamma \in I} \mathrm{P}(\gamma)$: probability of inactivity

→ $\mathscr{P}^{-1} - 1$ can be interpreted as an activity term

Generalized KUR

- Generic observables $\phi(\gamma) = 0 \ \forall \gamma \in I, \quad I = \{\gamma \,|\, \mu_i = \nu_i \forall i\}$

  trajectories that have no jump

- Generalized KUR

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq \frac{1}{\mathscr{P}^{-1} - 1}$$

$\mathscr{P} := \sum_{\gamma \in I} \mathrm{P}(\gamma)$: probability of inactivity

$\rightarrow \mathscr{P}^{-1} - 1$ can be interpreted as an activity term

$\mapsto$ high precision of observables requires high activity

Generalized KUR

- Generic observables $\phi(\gamma) = 0 \ \forall \gamma \in I$, $I = \{\gamma \mid \mu_i = \nu_i \forall i\}$

  trajectories that have no jump

- Generalized KUR

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq \frac{1}{\mathscr{P}^{-1} - 1}$$

$\mathscr{P} := \sum_{\gamma \in I} \mathrm{P}(\gamma)$: probability of inactivity

$\rightarrow$ $\mathscr{P}^{-1} - 1$ can be interpreted as an activity term

$\mapsto$ high precision of observables requires high activity

✓ saturable for $\phi(\gamma) = 1 \ \forall \gamma \notin I$

Generalized KUR

- Generic observables $\phi(\gamma) = 0 \ \forall \gamma \in I,$ $\quad I = \{\gamma \,|\, \mu_i = \nu_i \forall i\}$

  trajectories that have no jump

- Generalized KUR

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq \frac{1}{\mathscr{P}^{-1} - 1}$$

$\mathscr{P} := \sum_{\gamma \in I} \mathrm{P}(\gamma)$: probability of inactivity

$\rightarrow \mathscr{P}^{-1} - 1$ can be interpreted as an activity term

$\mapsto$ high precision of observables requires high activity

✓ saturable for $\phi(\gamma) = 1 \ \forall \gamma \notin I$

✓ holds for arbitrary measurements and initial states

Generalized KUR

- Generic observables $\phi(\gamma) = 0 \ \forall \gamma \in I, \quad I = \{\gamma \,|\, \mu_i = \nu_i \forall i\}$

  trajectories that have no jump

- Generalized KUR

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq \frac{1}{\mathscr{P}^{-1} - 1}$$

$\mathscr{P} := \sum_{\gamma \in I} \mathrm{P}(\gamma)$: probability of inactivity

→ $\mathscr{P}^{-1} - 1$ can be interpreted as an activity term

↦ high precision of observables requires high activity

✓ saturable for $\phi(\gamma) = 1 \ \forall \gamma \notin I$

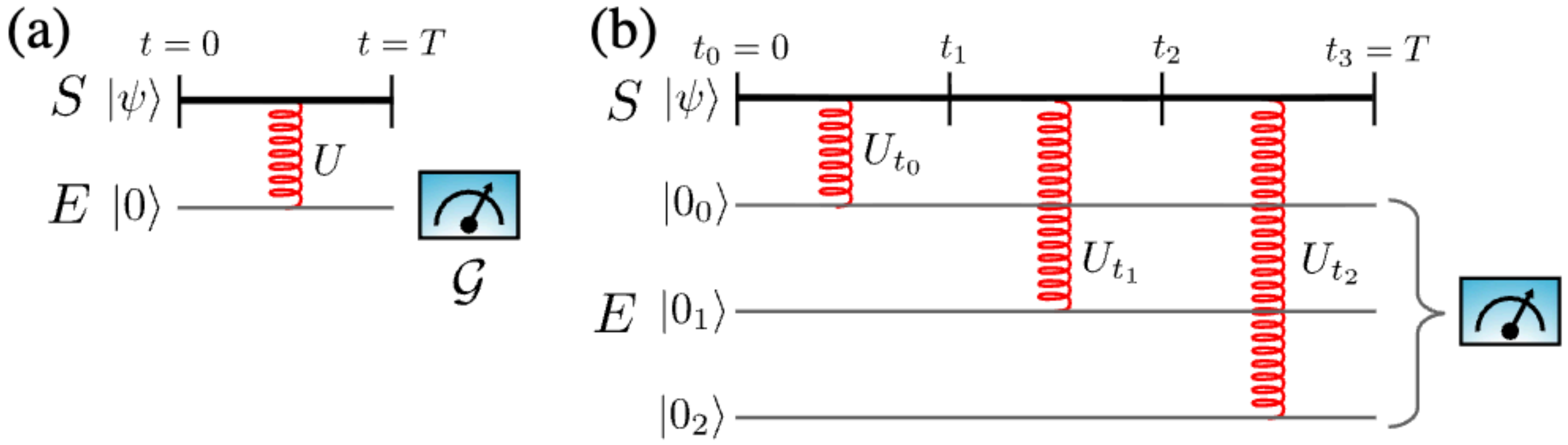
✓ holds for arbitrary measurements and initial states

✓ tightens and generalizes previous results

PRL 126, 010602 (2021)
PRL 127, 240602 (2021)

Comparison with previous results

- Previous result for general open quantum systems PRL 126, 010602 (2021)



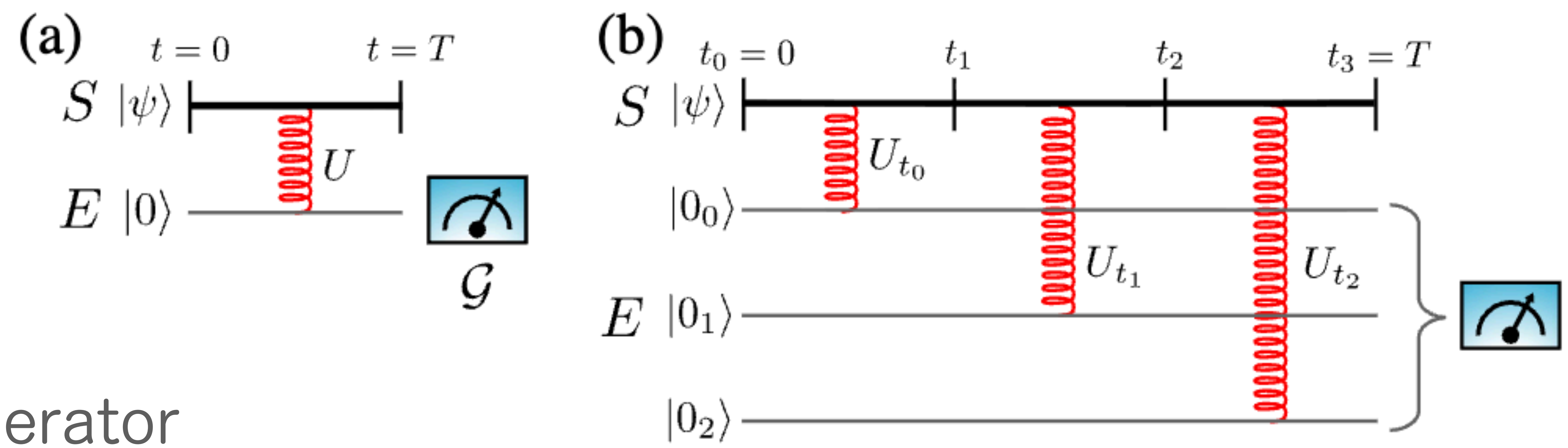
Comparison with previous results

- Previous result for general open quantum systems PRL 126, 010602 (2021)

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\mathscr{A}^{-1} - 1}$$

$$\mathscr{A} := \mathrm{tr}\{(V_0^\dagger V_0)^{-1}\varrho_S\}$$

$V_0 = \langle 0 | U | 0 \rangle^N$: no-jump Kraus operator



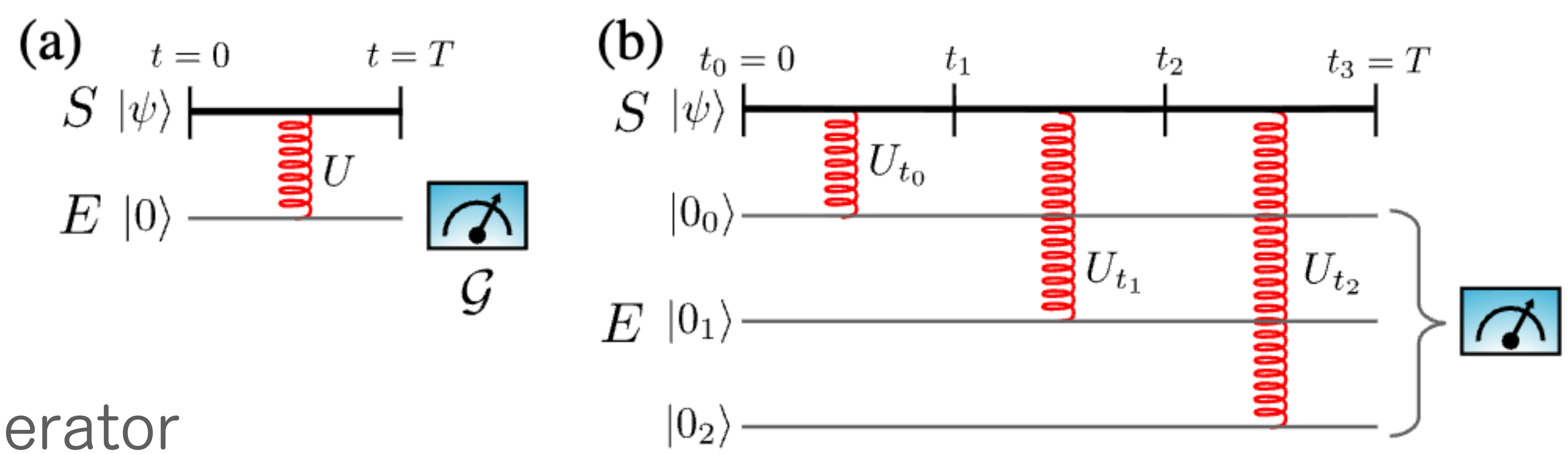
Comparison with previous results

- Previous result for general open quantum systems PRL 126, 010602 (2021)

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\mathscr{A}^{-1}-1}$$

$$\mathscr{A} := \mathrm{tr}\{(V_0^\dagger V_0)^{-1}\varrho_S\}$$

$V_0 = \langle 0|U|0\rangle^N$: no-jump Kraus operator



$$\begin{aligned} 1 &= \mathrm{tr}(\varrho_S)^2 \\ &\leq \mathrm{tr}\{(V_0^\dagger V_0)^{-1}\varrho_S\}\mathrm{tr}\{(V_0^\dagger V_0)\varrho_S\} \\ &= \mathscr{A}\mathscr{P} \end{aligned}$$

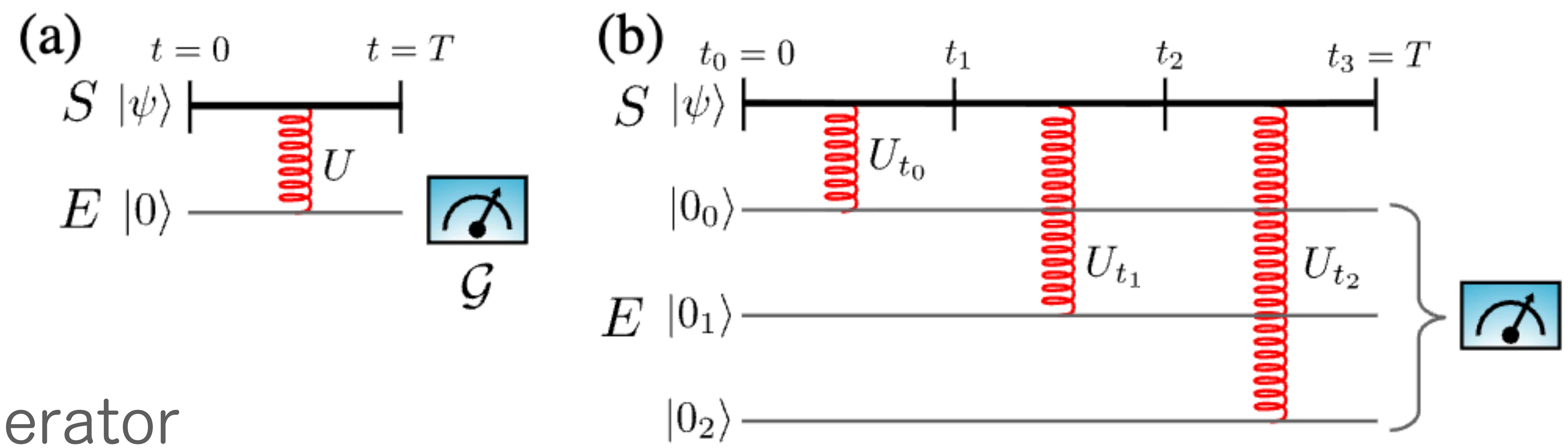
Comparison with previous results

- Previous result for general open quantum systems   PRL 126, 010602 (2021)

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\mathscr{A}^{-1}-1}$$

$$\mathscr{A} := \mathrm{tr}\{(V_0^\dagger V_0)^{-1}\varrho_S\}$$

$V_0 = \langle 0|U|0\rangle^N$: no-jump Kraus operator



$$\begin{aligned} 1 &= \mathrm{tr}(\varrho_S)^2 \\ &\leq \mathrm{tr}\{(V_0^\dagger V_0)^{-1}\varrho_S\}\mathrm{tr}\{(V_0^\dagger V_0)\varrho_S\} \\ &= \mathscr{A}\mathscr{P} \end{aligned}$$

$\Rightarrow$

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\mathscr{P}^{-1}-1} \geq \frac{1}{\mathscr{A}-1}$$

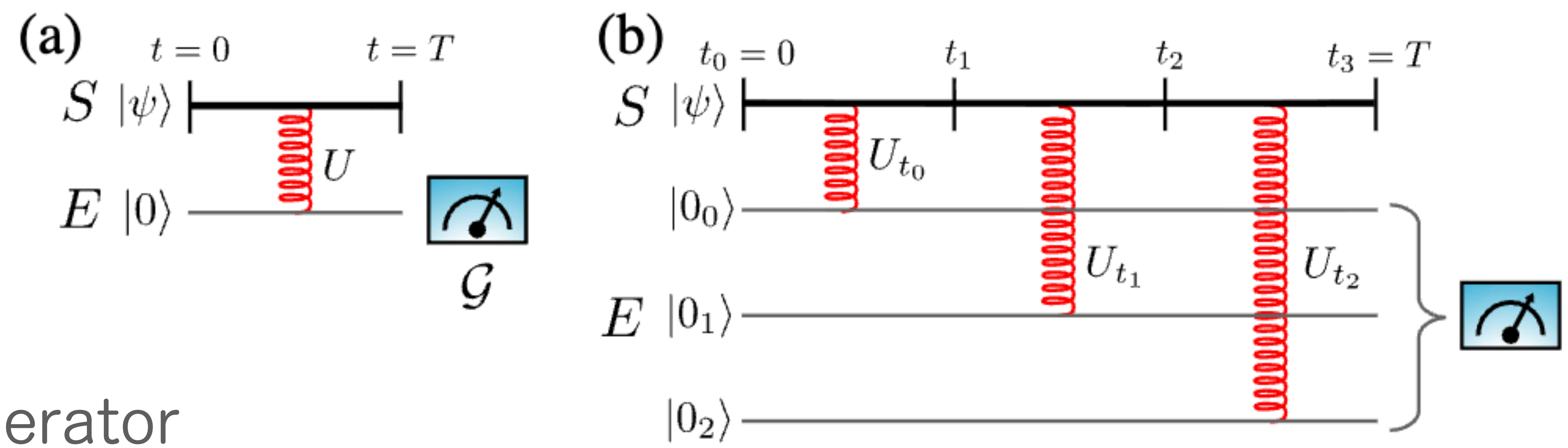
Comparison with previous results

- Previous result for general open quantum systems   PRL 126, 010602 (2021)

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\mathscr{A}^{-1}-1}$$

$$\mathscr{A} := \mathrm{tr}\{(V_0^\dagger V_0)^{-1}\varrho_S\}$$

$V_0 = \langle 0|U|0\rangle^N$: no-jump Kraus operator



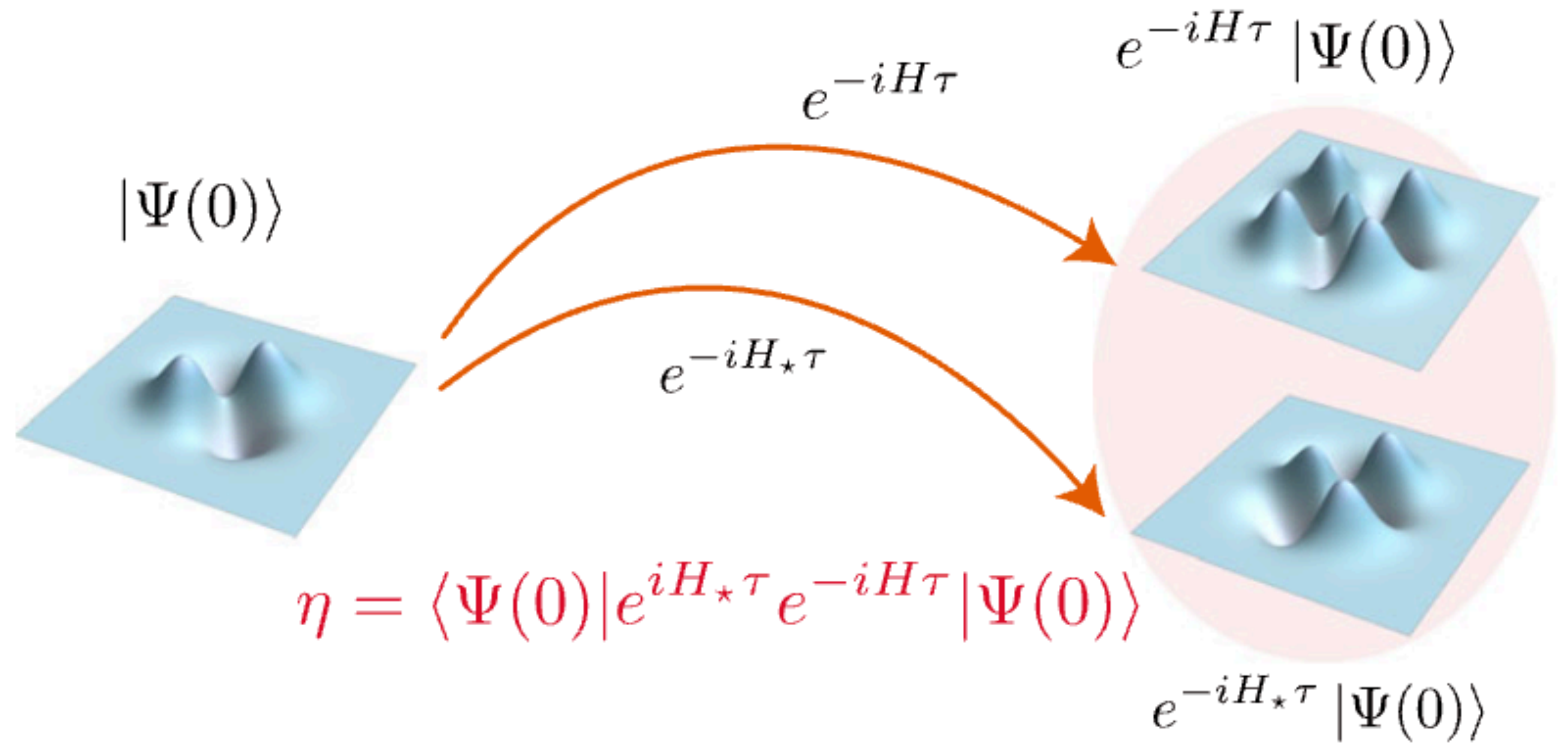
$$\begin{aligned} 1 &= \mathrm{tr}(\varrho_S)^2 \\ &\leq \mathrm{tr}\{(V_0^\dagger V_0)^{-1}\varrho_S\}\mathrm{tr}\{(V_0^\dagger V_0)\varrho_S\} \\ &= \mathscr{A}\mathscr{P} \end{aligned}$$

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\mathscr{P}^{-1}-1} \geq \frac{1}{\mathscr{A}-1}$$

$\mapsto$ generalize and sharpen the previous result

Comparison with previous results

- Previous result for Markovian dynamics PRL 127, 240602 (2021)



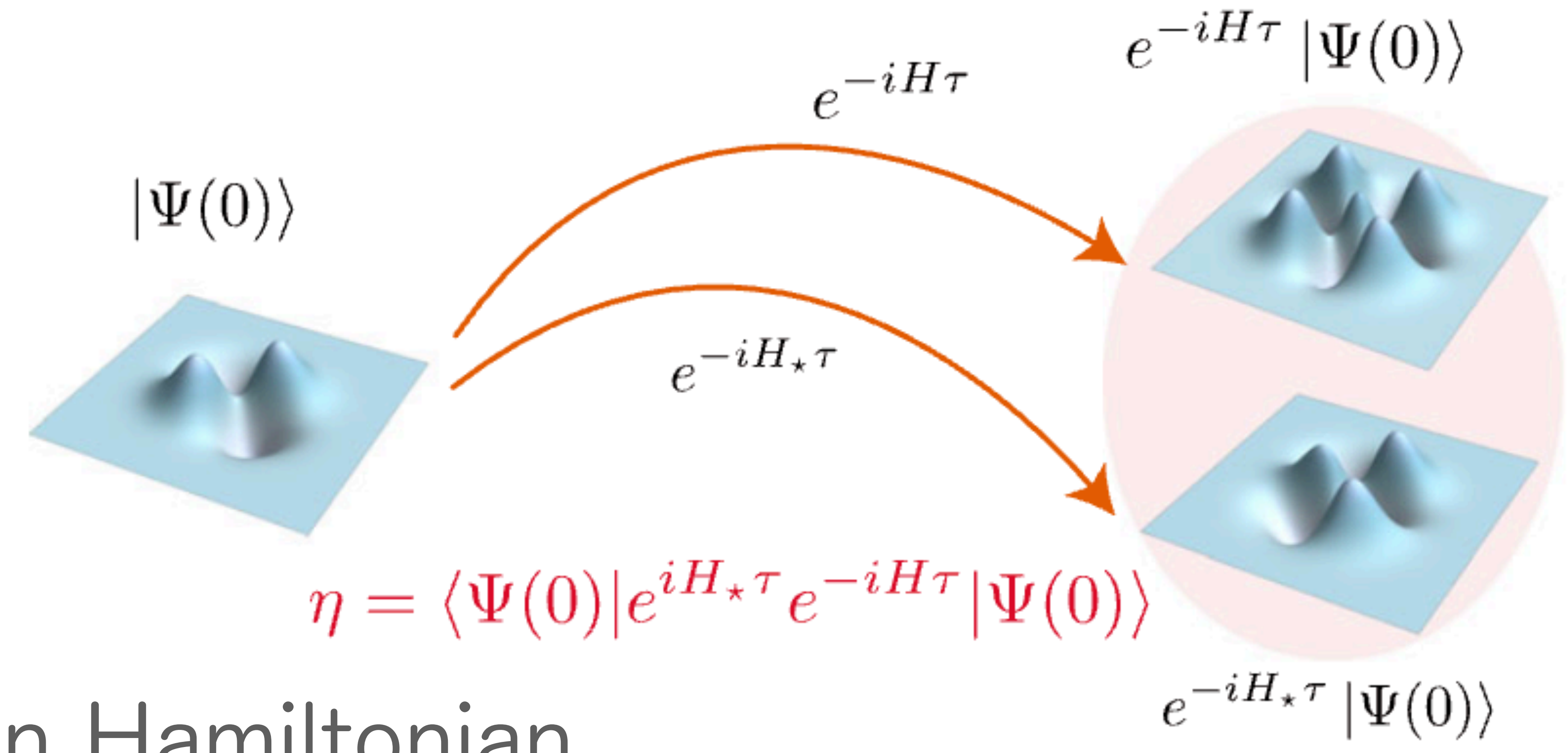
Comparison with previous results

- Previous result for Markovian dynamics PRL 127, 240602 (2021)

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\eta^{-1}-1}$$

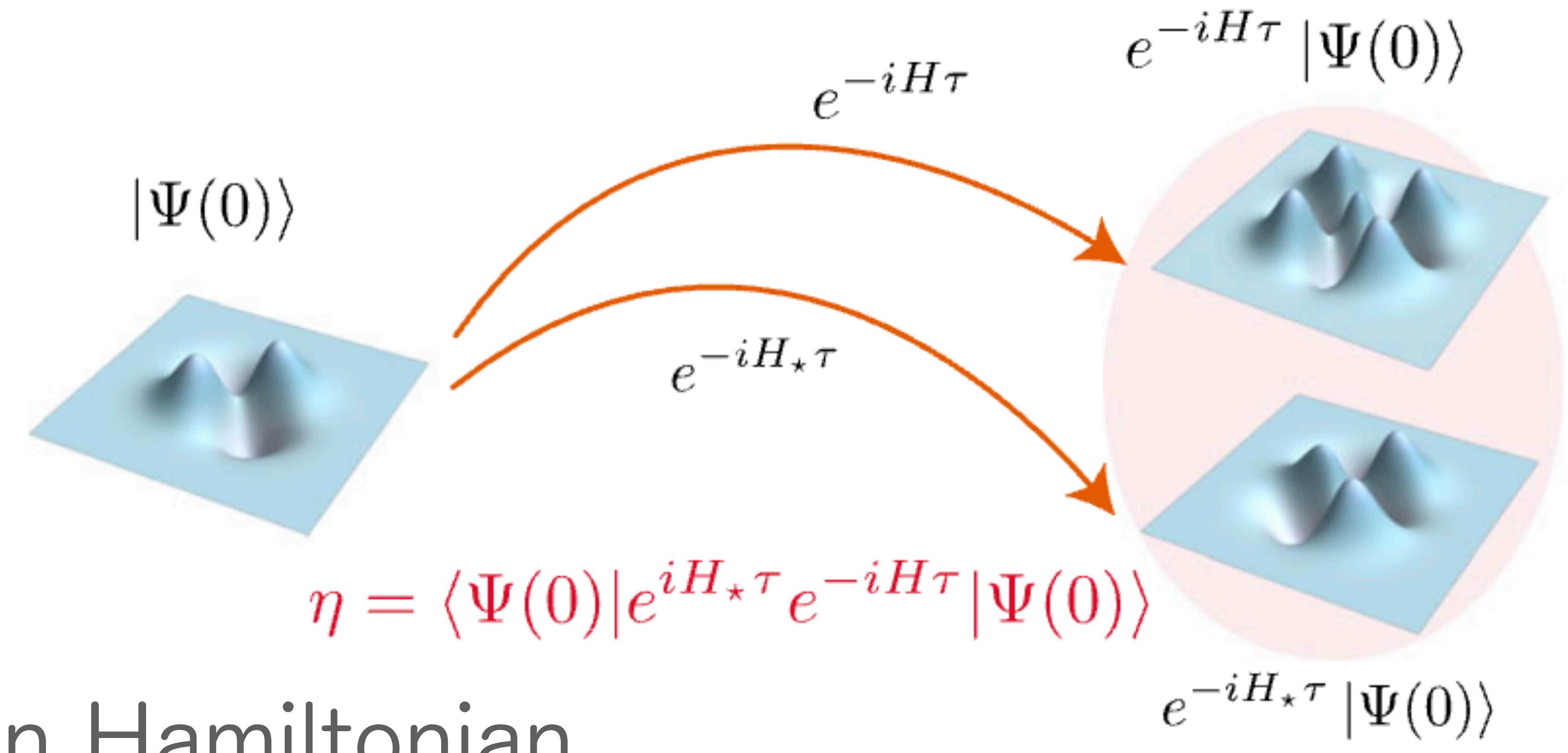
$\eta := |\mathrm{tr}(e^{-iH_{\mathrm{eff}}\tau}\varrho_S)|^2$: Loschmidt echo

$H_{\mathrm{eff}} := H - (i/2)\sum_k L_k^\dagger L_k$: effective non-Hermitian Hamiltonian



Comparison with previous results

- Previous result for Markovian dynamics   PRL 127, 240602 (2021)



$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\eta^{-1}-1}$$

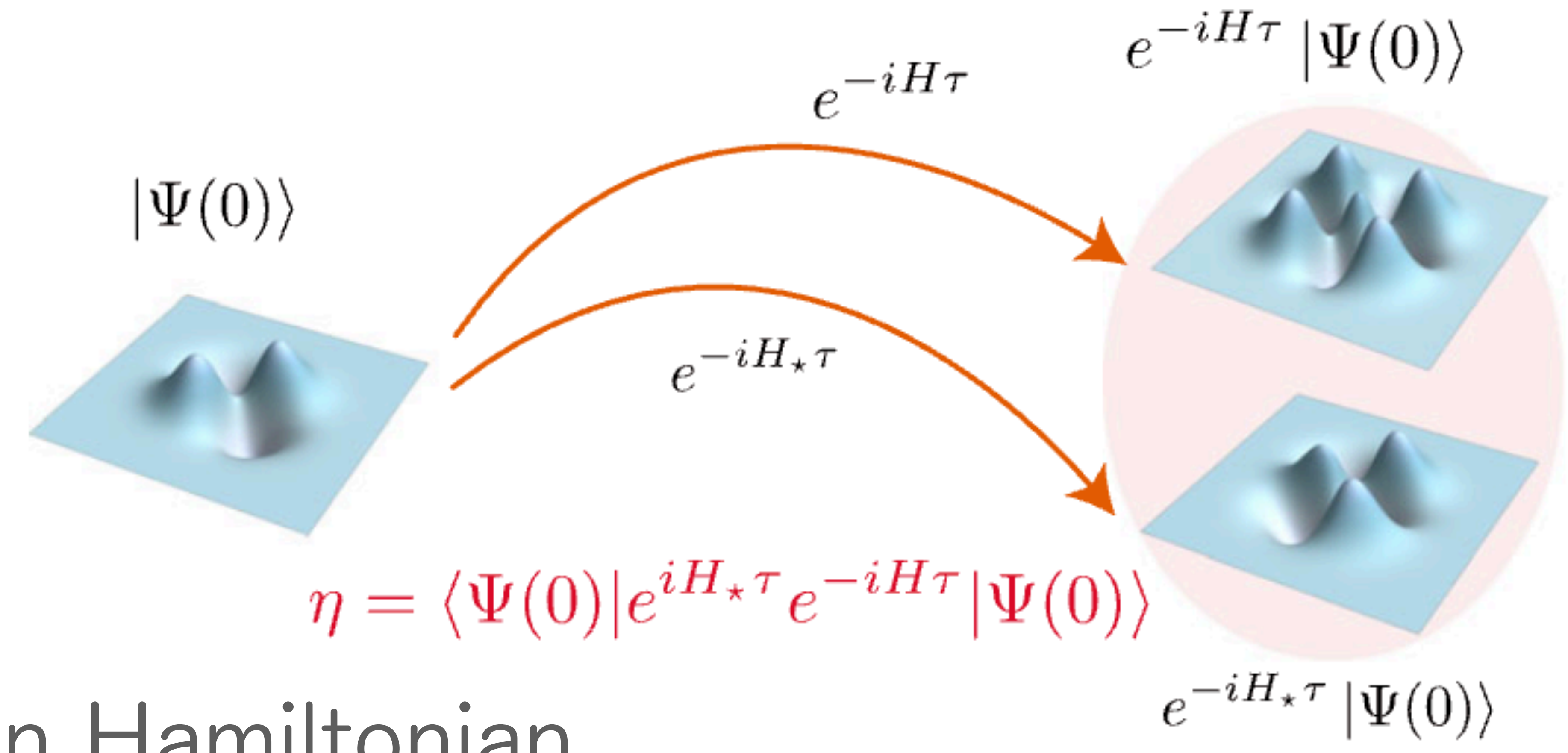
$\eta := |\mathrm{tr}(e^{-iH_{\mathrm{eff}}\tau}\varrho_S)|^2$: Loschmidt echo

$H_{\mathrm{eff}} := H - (i/2)\sum_k L_k^\dagger L_k$: effective non-Hermitian Hamiltonian

$$\begin{aligned}\eta &= |\mathrm{tr}(e^{-iH_{\mathrm{eff}}\tau}\varrho_S)|^2 \\ &\leq \mathrm{tr}\{e^{-iH_{\mathrm{eff}}\tau}\varrho_S e^{iH_{\mathrm{eff}}^\dagger\tau}\}\mathrm{tr}\{\varrho_S\} \\ &= \mathscr{P}\end{aligned}$$

Comparison with previous results

- Previous result for Markovian dynamics PRL 127, 240602 (2021)



$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\eta^{-1}-1}$$

$\eta := |\mathrm{tr}(e^{-iH_{\mathrm{eff}}\tau}\varrho_S)|^2$: Loschmidt echo

$H_{\mathrm{eff}} := H - (i/2)\sum_k L_k^\dagger L_k$: effective non-Hermitian Hamiltonian

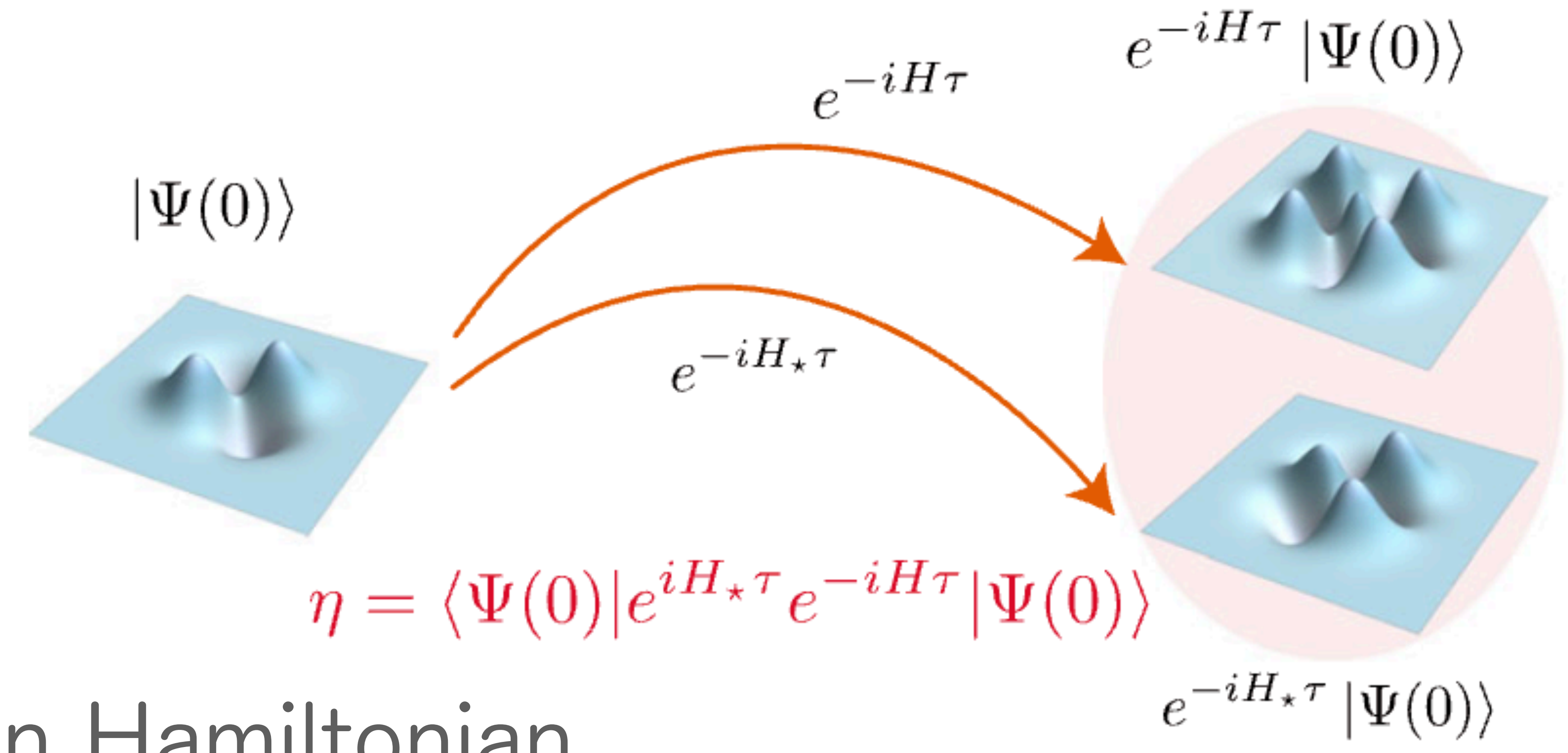
$$\begin{aligned}\eta &= |\mathrm{tr}(e^{-iH_{\mathrm{eff}}\tau}\varrho_S)|^2 \\ &\leq \mathrm{tr}\{e^{-iH_{\mathrm{eff}}\tau}\varrho_S e^{iH_{\mathrm{eff}}^\dagger\tau}\}\mathrm{tr}\{\varrho_S\} \\ &= \mathscr{P}\end{aligned}$$

$$\Rightarrow \quad \frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\mathscr{P}^{-1}-1} \geq \frac{1}{\eta^{-1}-1}$$

Comparison with previous results

- Previous result for Markovian dynamics   PRL 127, 240602 (2021)

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\eta^{-1}-1}$$



$\eta := |\mathrm{tr}(e^{-iH_{\mathrm{eff}}\tau}\varrho_S)|^2$: Loschmidt echo

$H_{\mathrm{eff}} := H - (i/2)\sum_k L_k^\dagger L_k$: effective non-Hermitian Hamiltonian

$$\begin{aligned}\eta &= |\mathrm{tr}(e^{-iH_{\mathrm{eff}}\tau}\varrho_S)|^2 \\ &\leq \mathrm{tr}\{e^{-iH_{\mathrm{eff}}\tau}\varrho_S e^{iH_{\mathrm{eff}}^\dagger\tau}\}\mathrm{tr}\{\varrho_S\} \\ &= \mathscr{P}\end{aligned}$$

$$\frac{\mathrm{Var}[\phi]}{\langle\phi\rangle^2} \geq \frac{1}{\mathscr{P}^{-1}-1} \geq \frac{1}{\eta^{-1}-1}$$

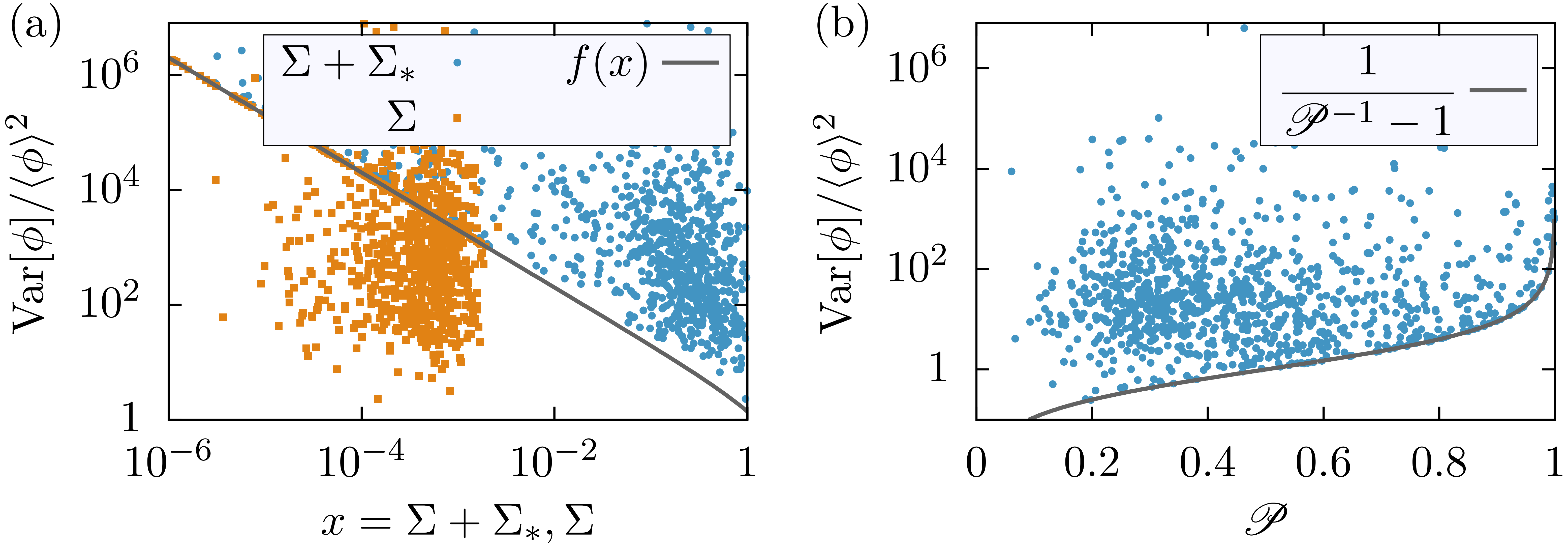
$\mapsto$ tighter than the previous result

Demonstration - Qubits

$$H = \frac{1}{2}(\omega_z \sigma_z + \omega_x \sigma_x) \otimes \mathbf{1}_E + \mathbf{1}_S \otimes H_E + \lambda V_S \otimes V_E$$

Demonstration - Qubits

$$H = \frac{1}{2}(\omega_z\sigma_z + \omega_x\sigma_x) \otimes \mathbf{1}_E + \mathbf{1}_S \otimes H_E + \lambda V_S \otimes V_E$$

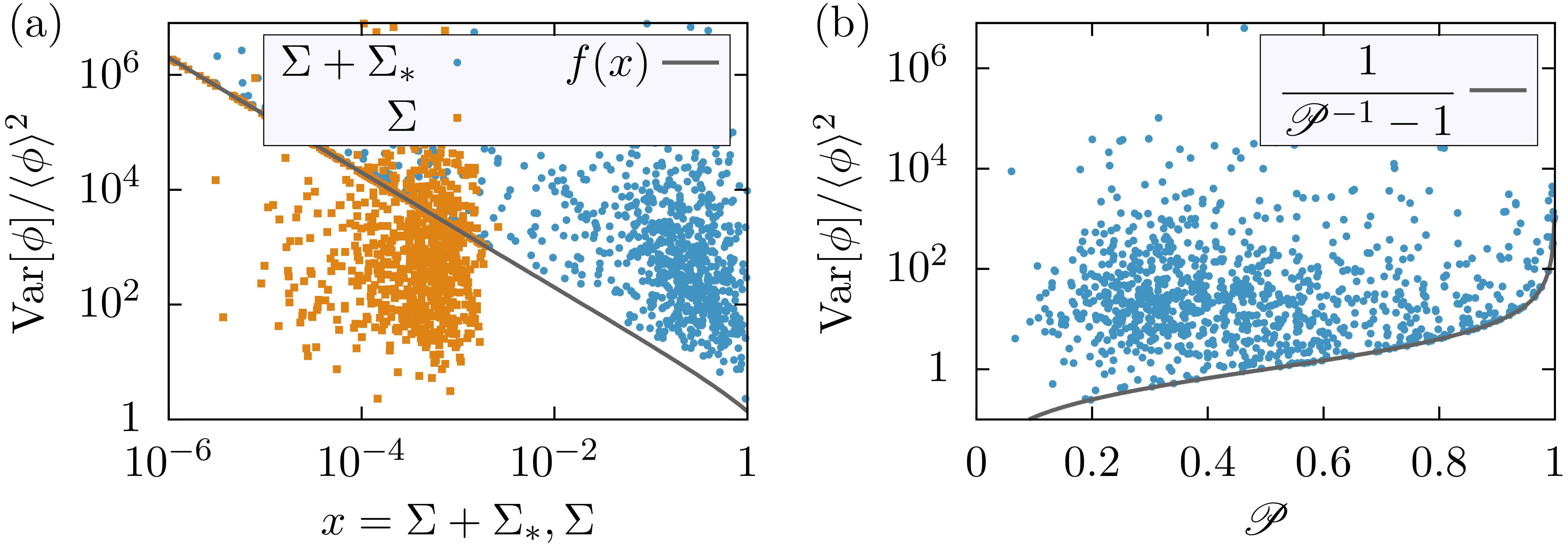


Demonstration - Qubits

$$H = \frac{1}{2}(\omega_z \sigma_z + \omega_x \sigma_x) \otimes \mathbf{1}_E + \mathbf{1}_S \otimes H_E + \lambda V_S \otimes V_E$$

$\bar{S}_{\mathrm{EE}} := \sum_{n,\nu} p_n p_\nu S_{\mathrm{EE}}(\varrho'_{n,\nu})$: average entanglement entropy

$\mathcal{Q} := (\mathrm{Var}[\phi]/\langle\phi\rangle^2)/f(\Sigma)$: quality factor

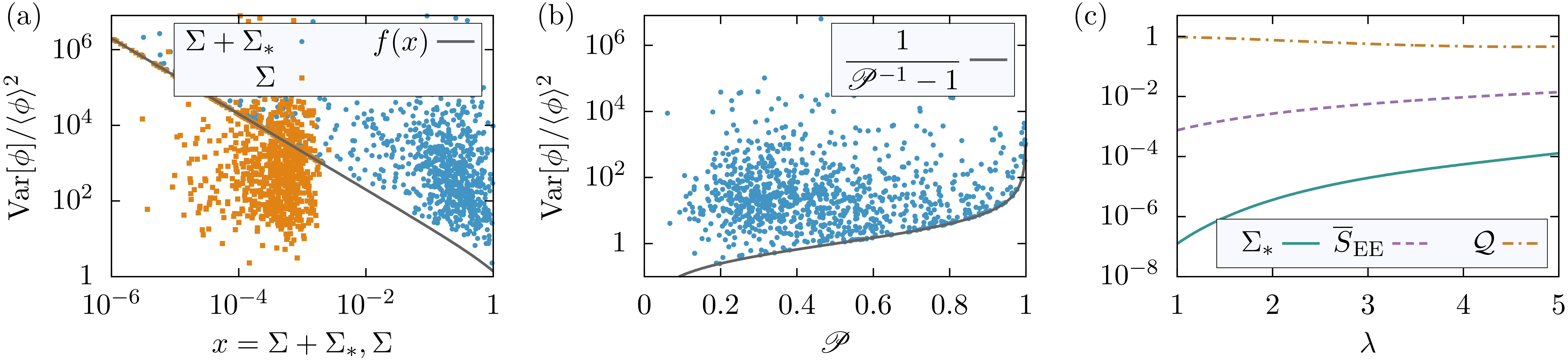


Demonstration - Qubits

$$H = \frac{1}{2}(\omega_z \sigma_z + \omega_x \sigma_x) \otimes \mathbf{1}_E + \mathbf{1}_S \otimes H_E + \lambda V_S \otimes V_E$$

$\overline{S}_{\mathrm{EE}} := \sum_{n,\nu} p_n p_\nu S_{\mathrm{EE}}(\varrho'_{n,\nu})$: average entanglement entropy

$\mathcal{Q} := (\mathrm{Var}[\phi]/\langle\phi\rangle^2)/f(\Sigma)$: quality factor



Open problems

Open problems

- Extension to infinite-dimensional environments

Open problems

- Extension to infinite-dimensional environments

- Improved TUR for only time-extensive currents

Open problems

- Extension to infinite-dimensional environments

- Improved TUR for only time-extensive currents

- Generalization to incorporate quantum feedback control

Summary

Summary

- Quantum TKUR: quantum coherent dynamics constrains current precision along with dissipation and activity

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

Summary

- Quantum TKUR: quantum coherent dynamics constrains current precision along with dissipation and activity

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

- Generalized TUR: current precision is limited by both dissipation and asymmetry

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq f(\Sigma + \Sigma_*)$$

Summary

- Quantum TKUR: quantum coherent dynamics constrains current precision along with dissipation and activity

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

- Generalized TUR: current precision is limited by both dissipation and asymmetry

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq f(\Sigma + \Sigma_*)$$

- Generalized KUR: observable precision is limited by a generalized activity

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq \frac{1}{\mathscr{P}^{-1} - 1}$$

Summary

Thank you for your attention!

- Quantum TKUR: quantum coherent dynamics constrains current precision along with dissipation and activity

$$F_\phi \geq (1+\delta_\phi)^2 \frac{4a}{\sigma^2} \Phi\left(\frac{\sigma}{2a}\right)^2$$

- Generalized TUR: current precision is limited by both dissipation and asymmetry

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq f(\Sigma + \Sigma_*)$$

- Generalized KUR: observable precision is limited by a generalized activity

$$\frac{\mathrm{Var}[\phi]}{\langle \phi \rangle^2} \geq \frac{1}{\mathscr{P}^{-1} - 1}$$