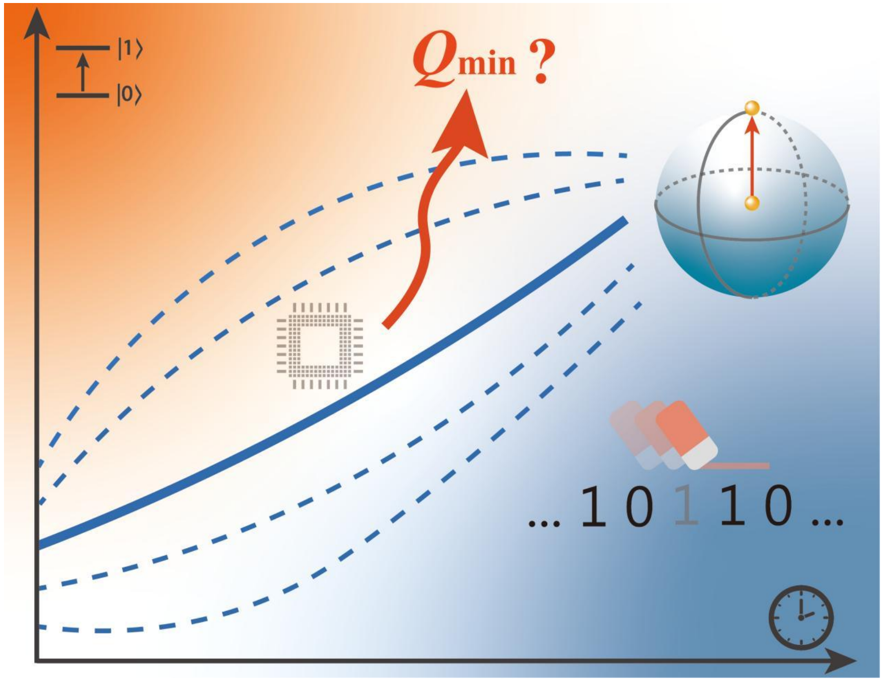
# Optimally Fast Qubit Reset

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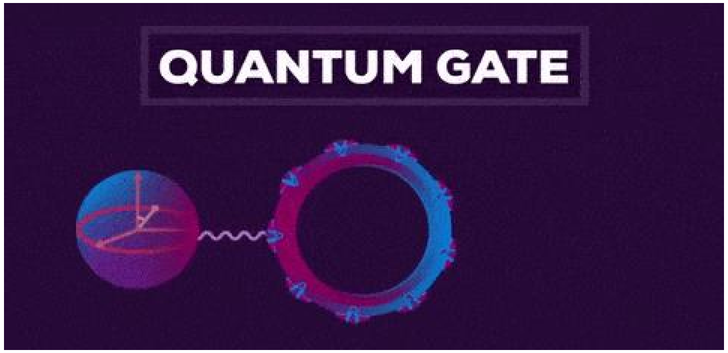
2025.12.12
QTST

# Introduction

# Quantum computing


this year




rapid development

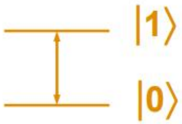
**energy consumption?**

will use one-fifth of global energy consumption

need urgent attention

# DiVincenzo's criteria for quantum computation

- Scalable system with well-defined qubits




- Initializing qubits to desired states

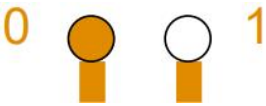
$$|00...0\rangle$$

- Long coherence times

$$|00...0\rangle + |11...1\rangle$$

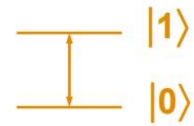
- Universal set of quantum gates


- Qubit-specific measurement



# DiVincenzo's criteria for quantum computation

- Scalable system with well-defined qubits




- Initializing qubits to desired states (qubit reset)

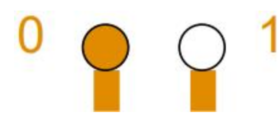
$$|00...0\rangle$$

- Long coherence times

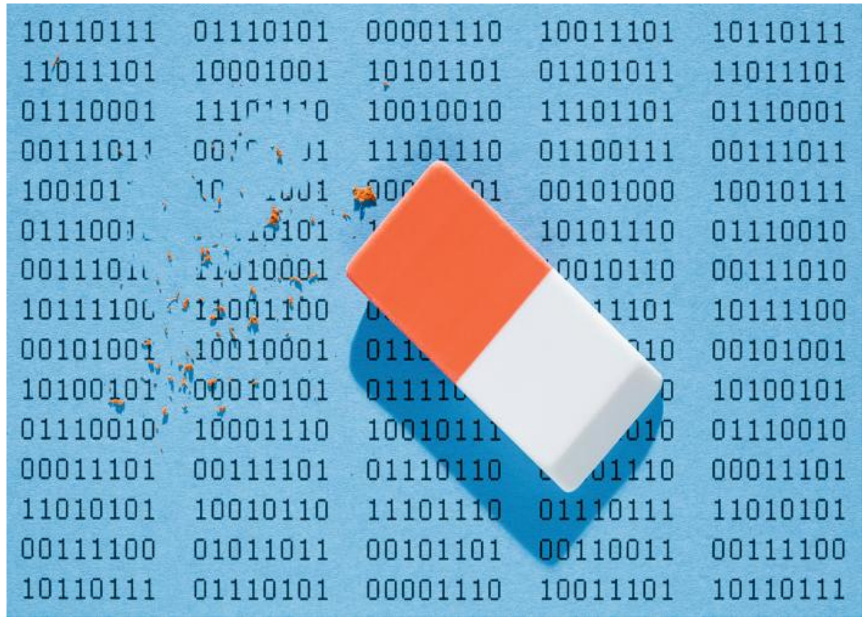
$$|00...0\rangle + |11...1\rangle$$

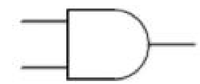
- Universal set of quantum gates


- Qubit-specific measurement



# Landauer principle





**AND**

| A | B | Output |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

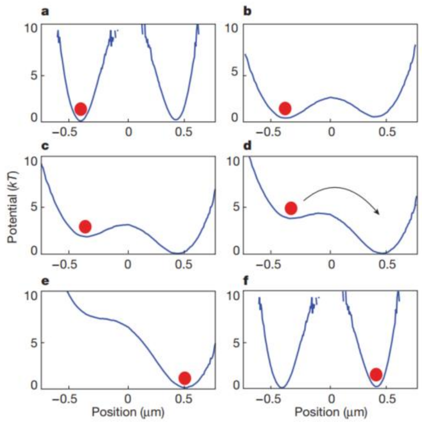
The minimal thermodynamic cost of per qubit reset

$$Q = k_{\mathrm{B}} T \ln 2$$

R. Landauer, IBM J. Res. Dev. 5, 183 (1961).
J. M. R. Parrondo, J. M. Horowitz, and T. Sagawa, Nat. Phys. 11, 131 (2015).
I. Georgescu, Nat. Rev. Phys. 3, 770 (2021)

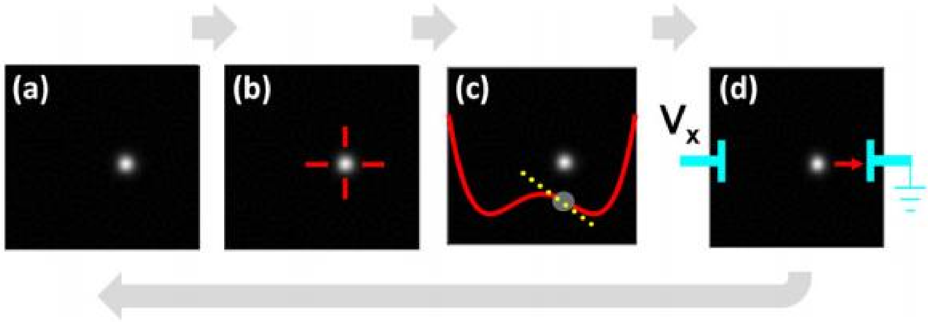
# Experimental verification

## Optical tweezers



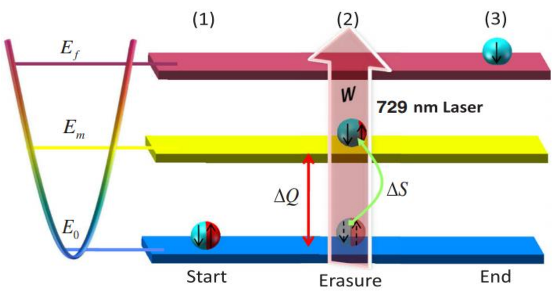

A. Berut *et al.*, Nature 483, 187 (2012).

## Virtual potential



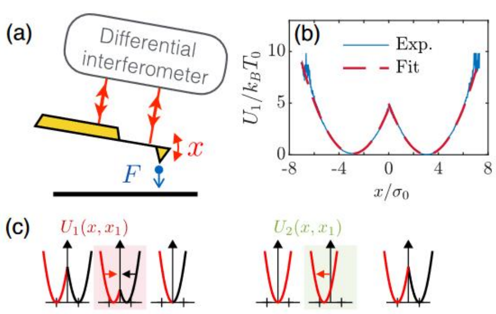

Y. Jun *et al.*, PRL113, 190601 (2014).

## Trapped ultracold ion



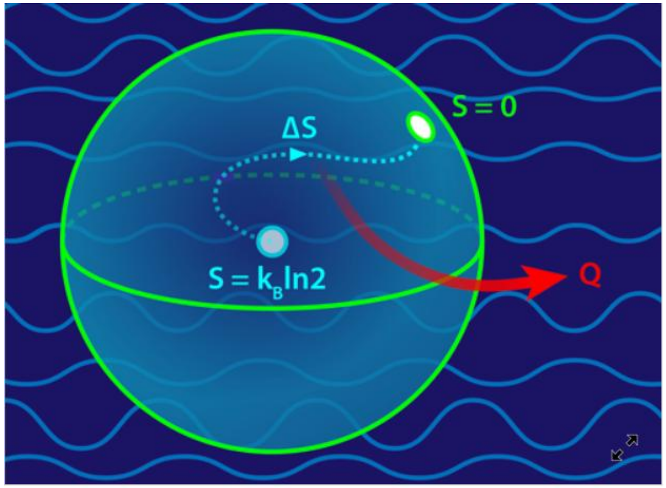

L. L. Yan *et al.*, PRL120, 210601 (2018).

## Conductive cantilever




S. Dago *et al.*, PRL126, 170601 (2021)

# Limitation




$$Q = k_B T \ln 2$$

applies only to quasi-static processes of infinite time

- Fluctuations on the micro and nano scale cannot be ignored;
- Information erasing is always done in finite time, which will lead to an increase in power consumption or a decrease in accuracy.

# Finite-time Landauer principle

According to the thermodynamic trade-off relations (e.g., speed limit)

$$\frac{L(p(0),p(\tau))^2}{2\Sigma\langle A\rangle_\tau} \le \tau$$

Phys. Rev. Lett 121, 070601 (2018).

$$Q \ge -T\Delta S_{sys} + \frac{T(p(0),p(\tau))^2}{\tau\beta\langle a\rangle_\tau/2}$$

Phys. Rev. Lett 128, 010602 (2022).
Phys. Rev. X 13, 011013 (2023).

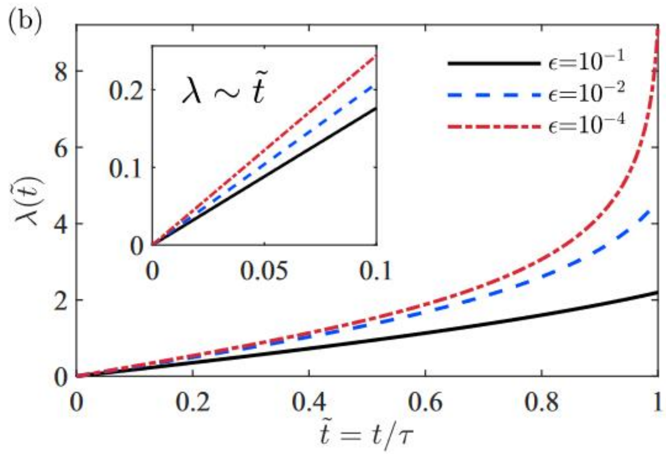
$$\frac{l}{2A_{\mathrm{tot}}} \le f\left(\frac{\Sigma}{A_{\mathrm{tot}}}\right) \Longrightarrow \frac{Q}{T} \ge \ln 2 + \tanh^{-1} v$$

Phys. Rev. Lett 129, 120603 (2022).

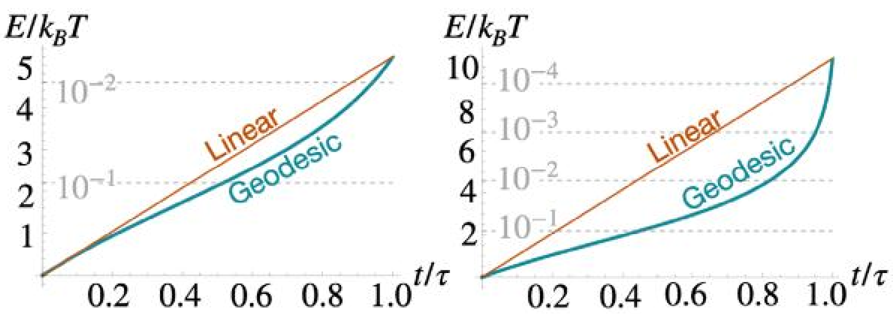
not tightest, may not achievable

# Optimal protocol

the minimal thermodynamical cost




Y. Ma *et al.*, Phys. Rev. E 106, 034112 (2022)

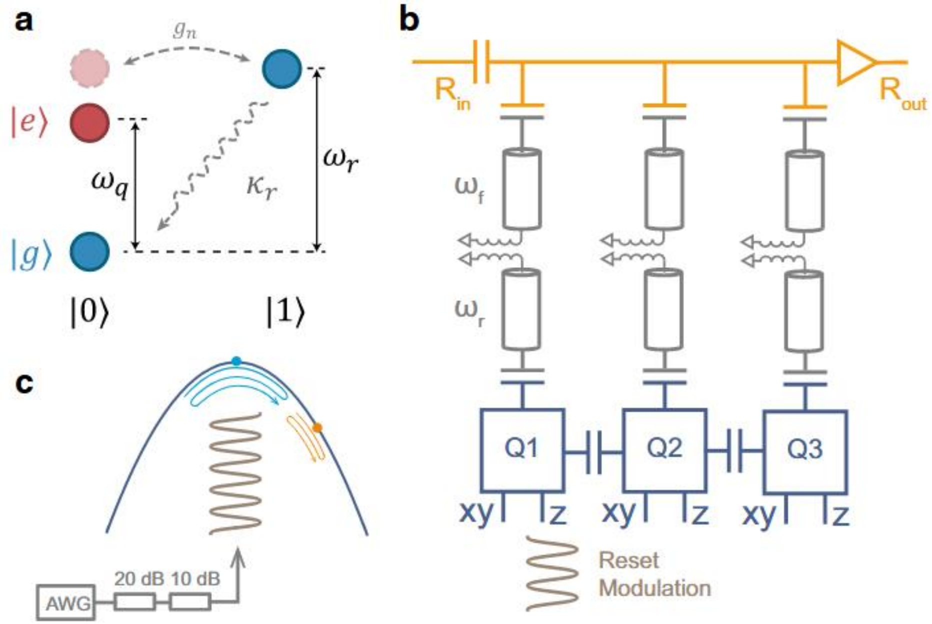



M. Scandi *et al.*, Phys. Rev. Lett 129, 270601 (2022)

by thermodynamic geometry

restricted in the slow-driving regime

# Experiment




Phys. Rev. Lett. 121, 060502 (2018)
Nat. Commun. 12, 5924 (2021)
Phys. Rev. X 12, 041008 (2022)
Nat. Nanotechnol. 19, 605 (2024)
Phys. Rev. Lett. 132, 230601 (2024)

achieving fidelity exceeding 99% within 300 ns

a deeper theoretical understanding of qubit reset in the fast-driving regime is crucial

# Questions

1. A unifying form of the minimal thermodynamic cost across time scales for general thermal environments.

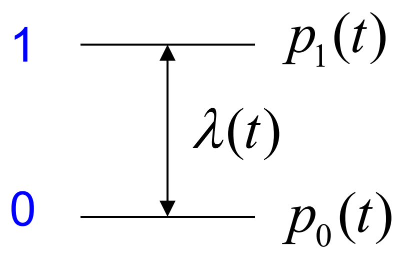
2. Optimal qubit reset in the fast-driving regime, which is more practically relevant.

Concretely, at present, we have a very limited understanding of why thermodynamic costs in both natural and artificial computers are many orders of magnitude above the minimum possible.

PNAS 121, e2321112121 (2024).
Is stochastic thermodynamics the key to understanding the energy costs of computation?

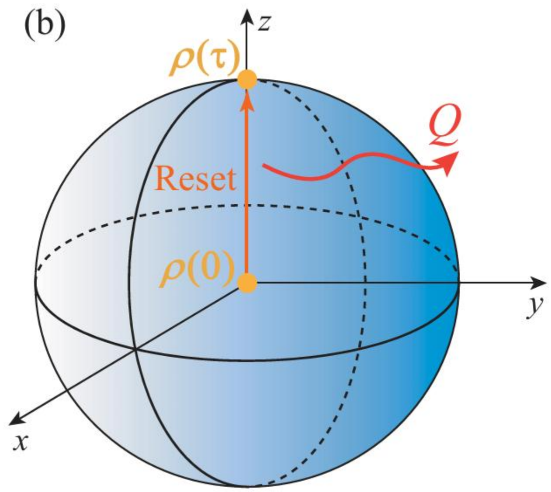
# Setup

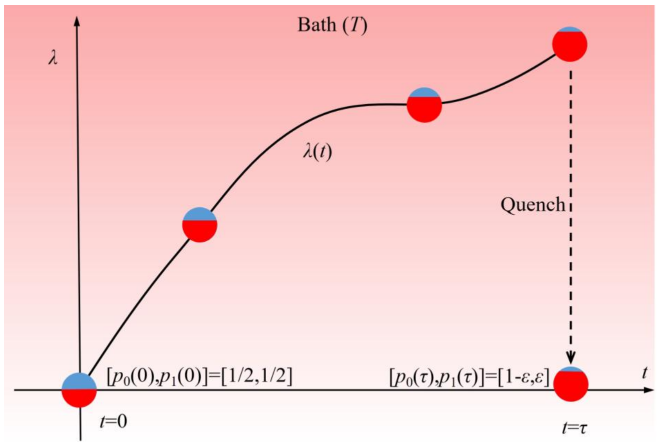
# Memory erasure




$$H_t = \frac{\lambda(t)}{2}\sigma_z$$

$$\rho = p_0|0\rangle\langle 0| + p_1|1\rangle\langle 1|$$




## Step I

$\lambda(0) = 0$

$\rho(0) = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$

## Step II

$H_t$ is controlled by a protocol

the Lindblad equation

$$\dot{\rho} = -i[H_t, \rho] + \sum_{k=0}^{1} L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

Heat production $Q = \int_0^\tau \mathrm{Tr}(\dot{\rho} H_t)\mathrm{d}t$

## Step III

$H_\tau$ is quenched to $H_0$

$\rho(\tau) \approx |0\rangle\langle 0|$

$\rho(\tau) = (1-\varepsilon)|0\rangle\langle 0| + \varepsilon|1\rangle\langle 1|$

with an error probability

$R := \langle 1|L_0^\dagger|0\rangle$ due to detailed balance

# Optimal protocol and minimal heat production

# General Form

**heat functional respect to $p_1$**

$$Q = \int_{\varepsilon}^{\frac{1}{2}} \lambda(p_1)\, \mathrm{d}p_1$$

boundary conditions $p_1(0) = 1/2$ and $p_1(\tau) = \varepsilon$

⇓ **solving the Euler-Lagrange equation**

parametric equations

$$\begin{cases} \lambda(p_1) \\ t(p_1) \end{cases} \quad p_1 \in [1/2, \varepsilon] \quad \Rightarrow \quad \lambda(t)$$

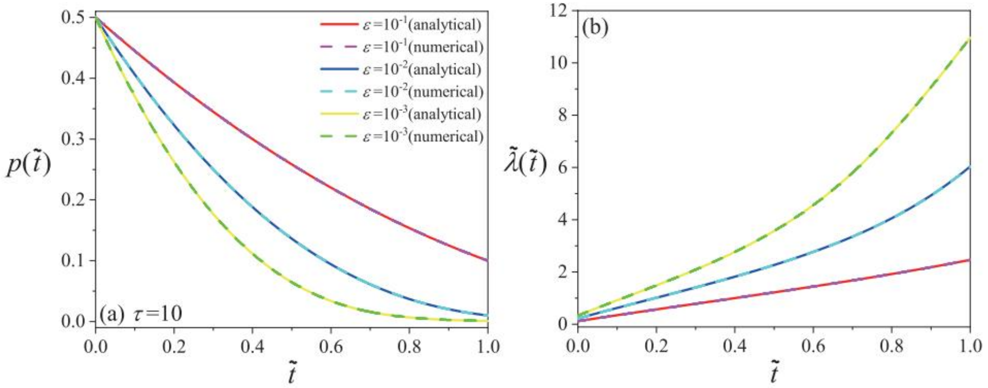
determine the optimal protocol

Minimal heat production

$$Q_{\min} = \beta^{-1} \int_{\varepsilon}^{\frac{1}{2}} \tilde{\lambda}(p)\mathrm{d}p = -T\Delta S + T\Delta\Sigma_{\min}$$

Once the jump operator $R$ is given, one can obtain the optimal protocol and minimal thermodynamic cost.

# Remarks




The optimizing is in the probability space rather than parametric space

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Our result is algebraic equations, which is convenient for asymptotic analysis

In the slow-driving regime $\mu\tau \gg 1,$ we have

$$\beta Q_{\min} = -\Delta S + \frac{H(\varepsilon; R)}{\mu\tau}$$

**1/τ behavior**

# Two classes of thermal environments

# Two classes

We classify thermal environments into two classes based on the convergence or divergence of the rescaled transition rate $R(\tilde{\lambda})$.

**Class I:**

$$\lim_{\tilde{\lambda}\to+\infty} R(\tilde{\lambda}) < +\infty$$

**Examples:**

Fermionic heat bath:

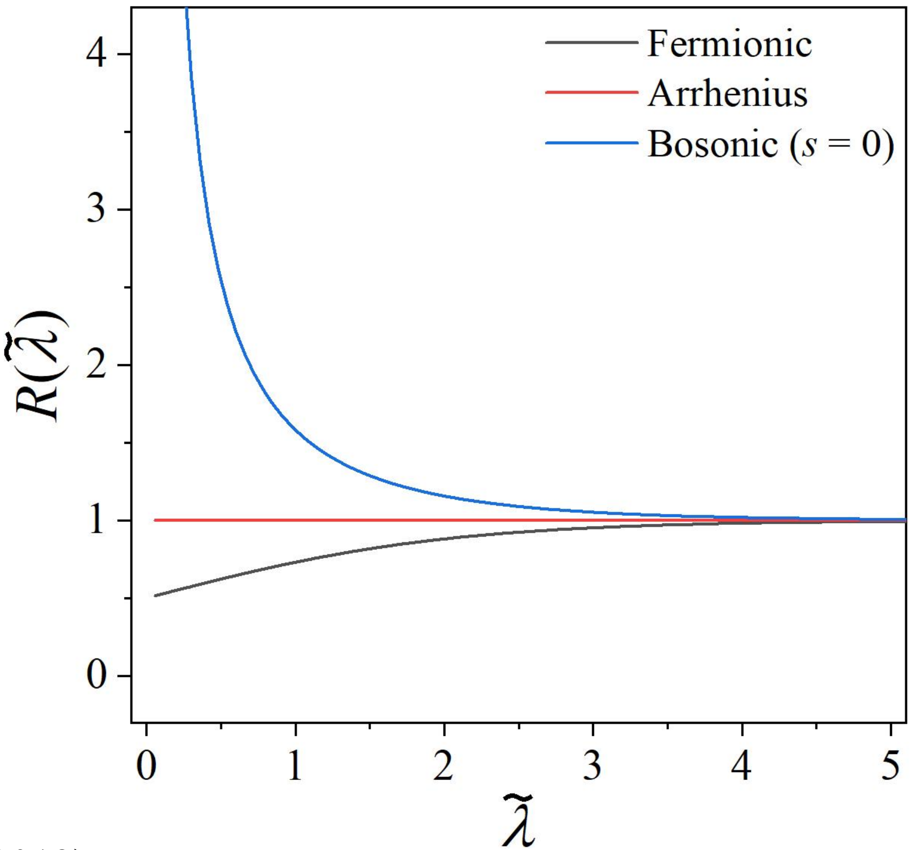
$$R(\tilde{\lambda}) \equiv \frac{1}{1+e^{-\tilde{\lambda}}}$$

Arrhenius case:

$$R(\tilde{\lambda}) \equiv 1$$

Bosonic heat bath ($s$=0):

$$R(\tilde{\lambda}) \equiv \frac{1}{1-e^{-\tilde{\lambda}}}$$




G. Diana, *et al.*, Phys. Rev. E 87, 012111 (2013).
M. Scandi, *et al.*, Phys. Rev. Lett. 129, 270601 (2022).

limits the speed of the transition

# Two classes

We classify thermal environments into two classes based on the convergence or divergence of the rescaled transition rate $R(\tilde{\lambda})$.

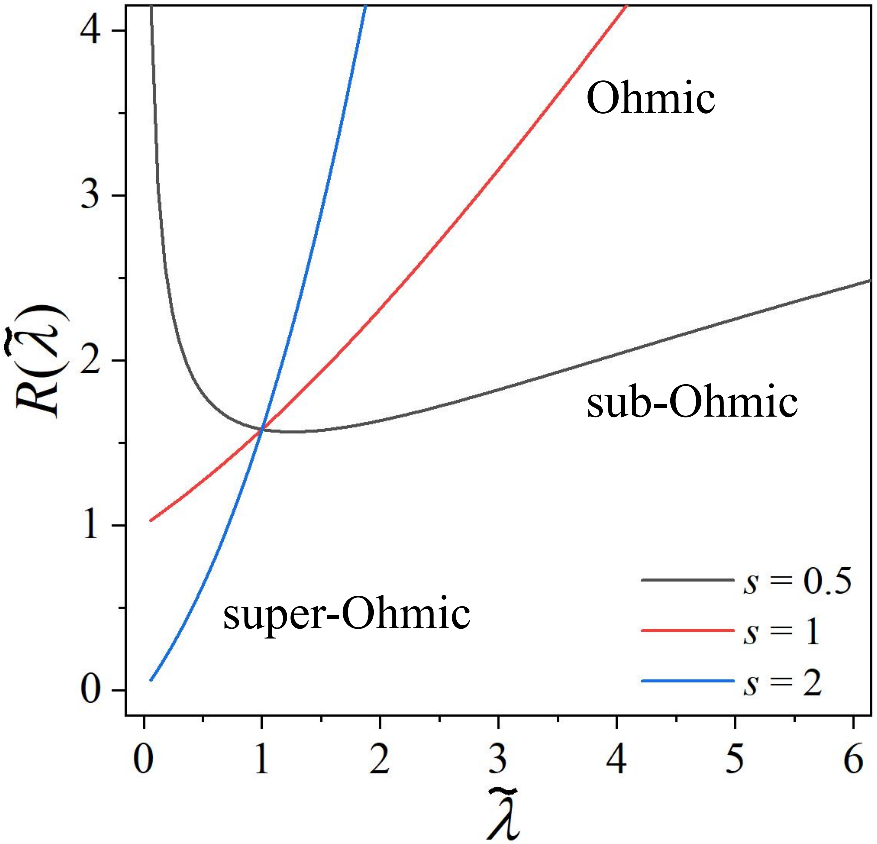
**Class II:**

$$\lim_{\tilde{\lambda}\to+\infty} R(\tilde{\lambda}) = +\infty$$

**Examples:**

bosonic baths with different spectral densities:

$$R(\tilde{\lambda}) \equiv \frac{\tilde{\lambda}^s}{1-e^{-\tilde{\lambda}}}, \quad s>0$$




arbitrarily fast transition

Y. Ma, *et al.*, Phys. Rev. E 106, 034112 (2022).

# The minimal error probability and erasure time

**Class I:**

The solvability condition of the above boundary-value problem for the first class is $\varepsilon > \varepsilon_{\min}$ or $\tau > \tau_{\min}$

**minimum error probability**

$$\varepsilon_{\min} = \frac{e^{-\mu\tau}}{2}$$

**minimum erasure time**

$$\tau_{\min} = -\frac{\ln 2\varepsilon}{\mu}$$

This suggests that the arbitrarily small error probability cannot be achieved in a finite time for the first class.

This corresponds to the quench protocol $\lambda \to \infty$, resulting in the fastest decrease of the population of the "1" logical state and a divergent thermodynamic cost.

# The fast-driving regime

**Class I:**

For the case of convergence of $R(+\infty)$ $\quad \tau \to \tau_{\min}$

$$\Delta\Sigma_{\min} = O\left(\ln \frac{1}{\mu(\tau - \tau_{\min})}\right)$$

exhibiting **logarithmic behavior**

---

**Class II:**

For the case of divergence of $R(+\infty)$, $\tau_{\min} \to 0$, one can use the asymptotic expansion

$$R(\tilde{\lambda}) = \sum_n c_n \phi_n(\tilde{\lambda}) \text{ for } \tilde{\lambda} \to +\infty$$

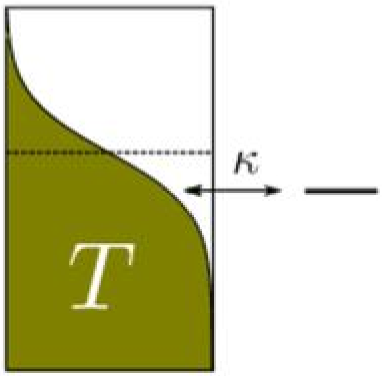
$$\Delta\Sigma_{\min} = O\left(\phi_0^{-1}\left[\frac{1}{c_0 \mu \tau}\left(\frac{1}{2\varepsilon} - 1\right)\right]\right)$$

one can determine the minimal heat dissipation
on the results for a longer erasure time

# Examples

# Example

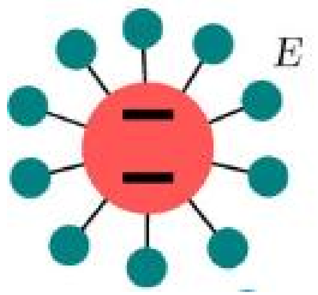
**quantum dot**




$$L_0 = \sqrt{\mu(1-N_{\rm FD})}|0\rangle\langle 1|$$
$$L_1 = \sqrt{\mu N_{\rm FD}}|1\rangle\langle 0|$$

$$N_{\rm FD} = 1/(e^{\beta\lambda}+1)$$

**spin-boson model**




$$L_0 = \sqrt{\mu\lambda^s(1+N_{\rm BE})}|0\rangle\langle 1|$$
$$L_1 = \sqrt{\mu\lambda^s N_{\rm BE}}|1\rangle\langle 0|$$

$$N_{\rm BE} = 1/(e^{\beta\lambda}-1)$$

$s < 1$ sub-Ohmic

$s = 1$ Ohmic

$s > 1$ super-Ohmic

# Example

**fermionic bath**

$$R(\tilde{\lambda}) = \frac{1}{1+e^{-\tilde{\lambda}}}$$

**quantum dot**

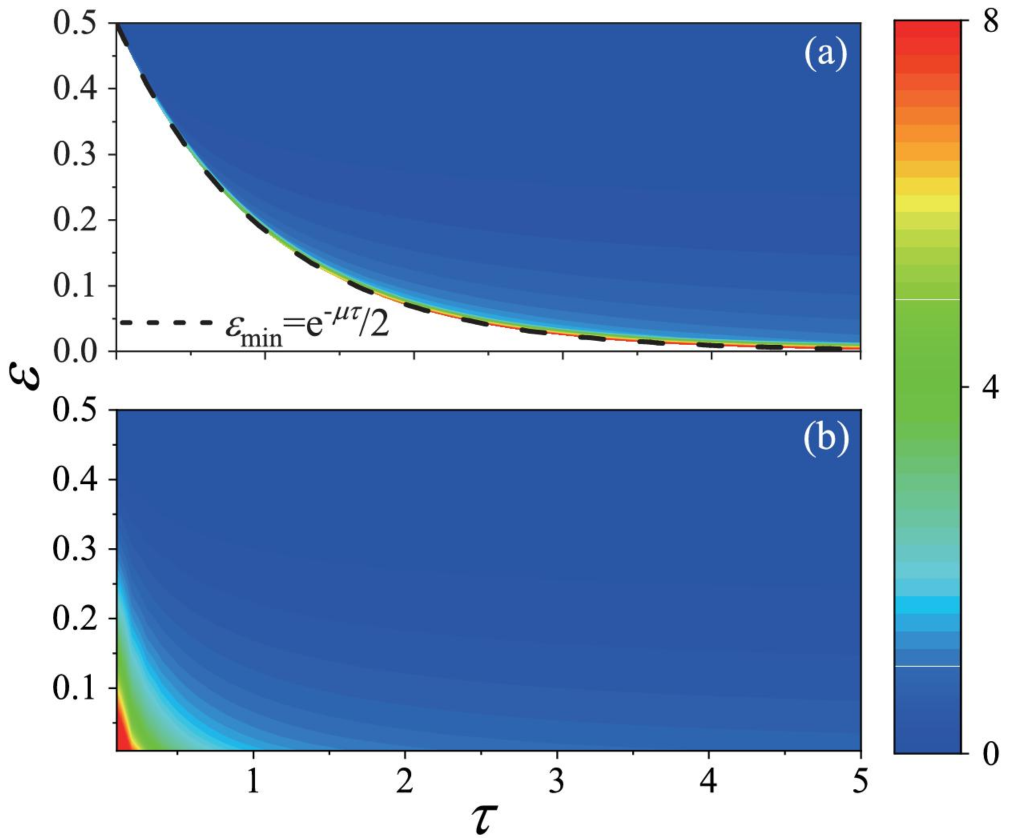
**There exists minimal error probability**

**bosonic bath**

$$R(\tilde{\lambda}) = \frac{\tilde{\lambda}^s}{1-e^{-\tilde{\lambda}}}$$

**spin-boson model**

**There is no minimal error probability**

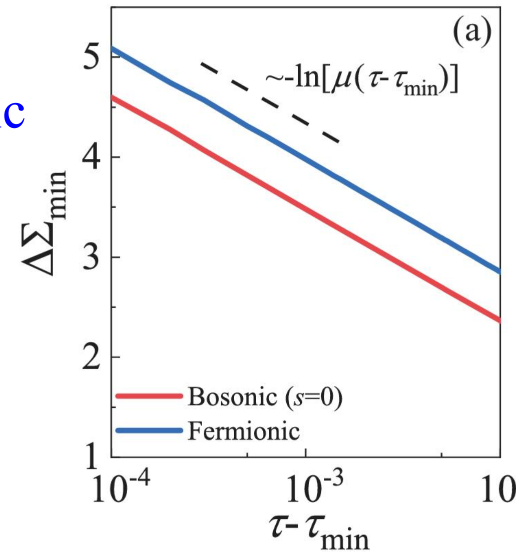


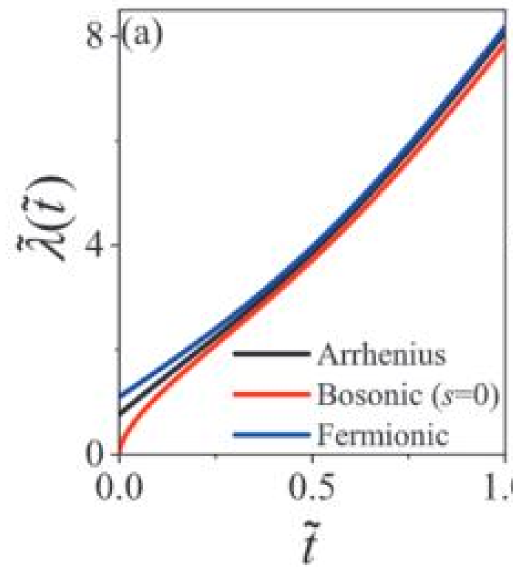

Minimal heat production for (a) the quantum dot and (b) the bosonic heat bath with $s = 1$. The black dashed line in (a) represents the minimal error probability for a given erasure time.

# Fast-driving regime

The first class
For the fermionic bath and bosonic bath with s=0

$$\Delta\Sigma_{\min} = O\left(-\ln\mu(\tau-\tau_{\min})\right)$$




One can find/prove that

1. Initial discontinuity of the optimal protocol
2. Bosonic bath have advantage over fermionic bath

# Fast-driving regime

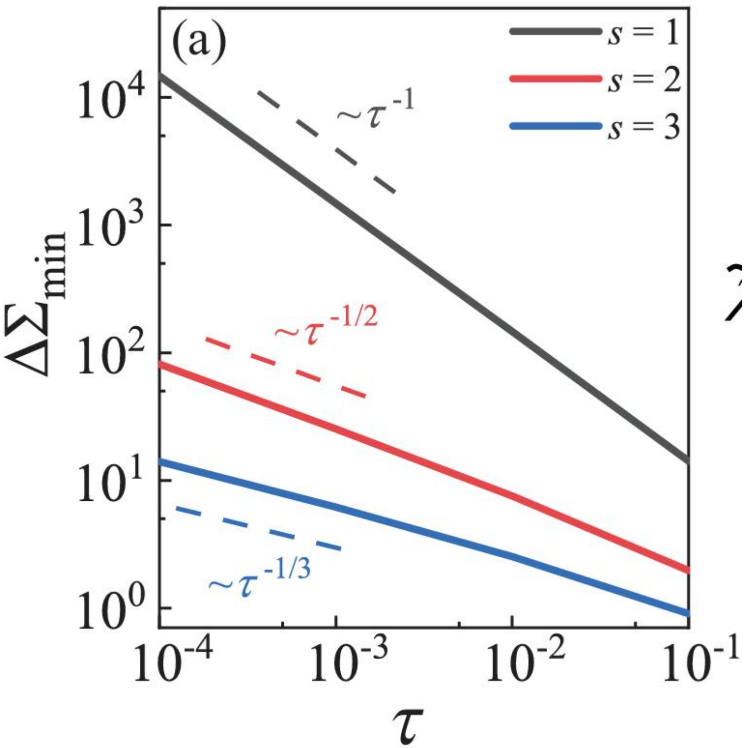
The second class

For the bosonic bath with s≠0

$$R(\tilde{\lambda}) = \frac{\tilde{\lambda}^s}{1-e^{-\tilde{\lambda}}} \text{ with } s \neq 0$$

$$c_0 = 1, \ \phi_0(\tilde{\lambda}) = \tilde{\lambda}^s$$

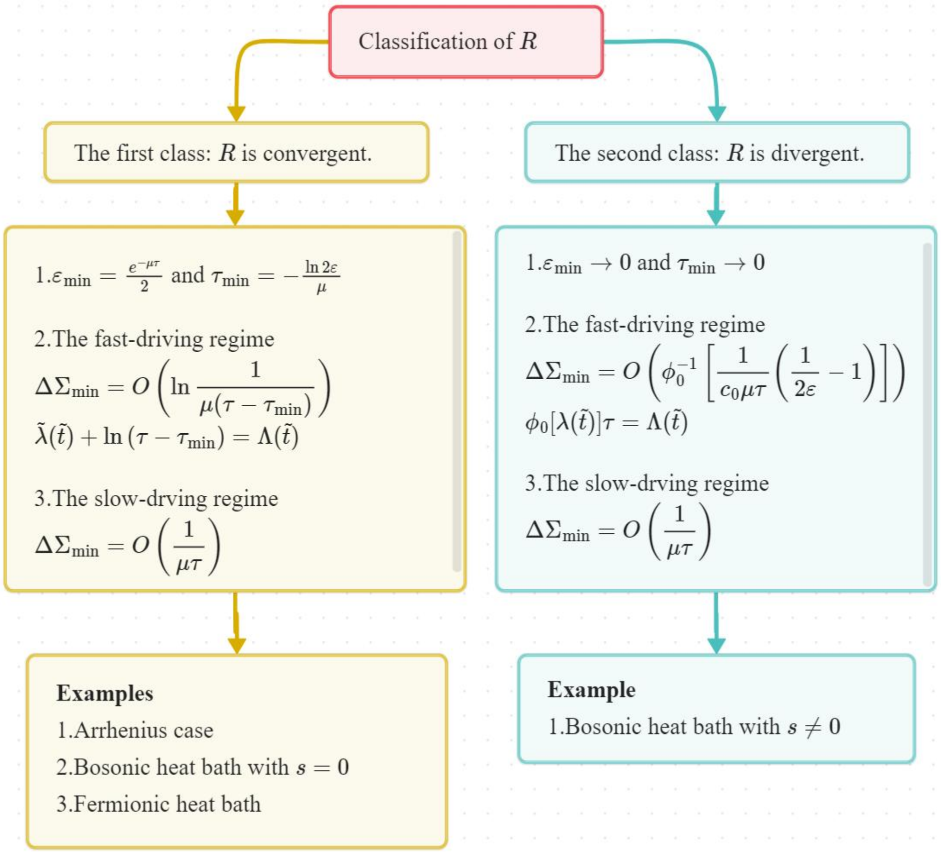
$$\Delta\Sigma_{\min} = O\left[(\mu\tau)^{-\frac{1}{s}}\right]$$




One can find/prove that

Advantage of the super-Ohmic case, arising from the spectral properties.

# Summary




1. We develop a general framework for optimal qubit reset with arbitrary erasure time.
2. In the fast-driving regime, the minimal entropy production exhibits the scaling behavior, and the optimal protocol shows the similar transformation.

# Questions

Concretely, at present, we have a very limited understanding of why thermodynamic costs in both natural and artificial computers are many orders of magnitude above the minimum possible.

PNAS 121, e2321112121 (2024)

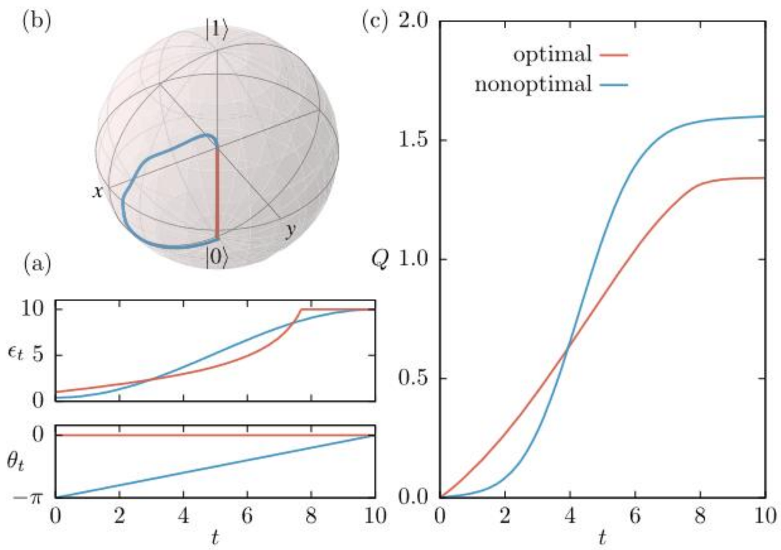
Is stochastic thermodynamics the key to understanding the energy costs of computation?

$$\Delta\Sigma_{\min} = O\left[(\mu\tau)^{-\frac{1}{s}}\right]$$

power-law divergent as the time shorten

# Quantum effect for qubit reset

**quantum coherence**



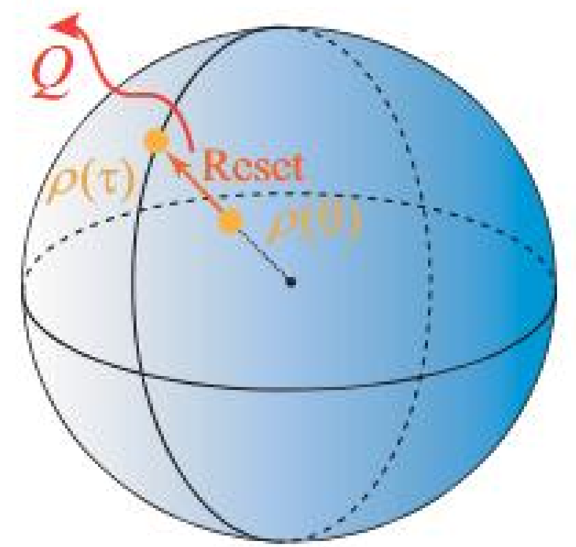

$$\beta Q \geq \underbrace{\Delta S_{\rm cl} + \frac{\|\Lambda(\varrho_0) - \Lambda(\varrho_\tau)\|_1^2}{2\tau\bar{\gamma}_\tau}}_{\text{classical}} + \underbrace{C_{\rm rel} + \frac{C_{\rm res}}{2\tau\bar{\gamma}_\tau}}_{\text{quantum}}.$$

**"quantum friction"**

H. J. D. Miller *et al.*, Phys. Rev. Lett. 125, 160602 (2020)
T. V. Vu and K. Saito, Phys. Rev. Lett. 128, 010602 (2022)

# Memory erasure

(a)




$$\rho = \frac{1}{2}(\boldsymbol{I} + \boldsymbol{r}\cdot\boldsymbol{\sigma})$$

$$S(\rho) = -\frac{1-r}{2}\ln\frac{1-r}{2} - \frac{1+r}{2}\ln\frac{1+r}{2}$$

Hamiltonian

$$H_t = \frac{\lambda(t)}{2}\boldsymbol{\sigma}\cdot\boldsymbol{n}(t)$$

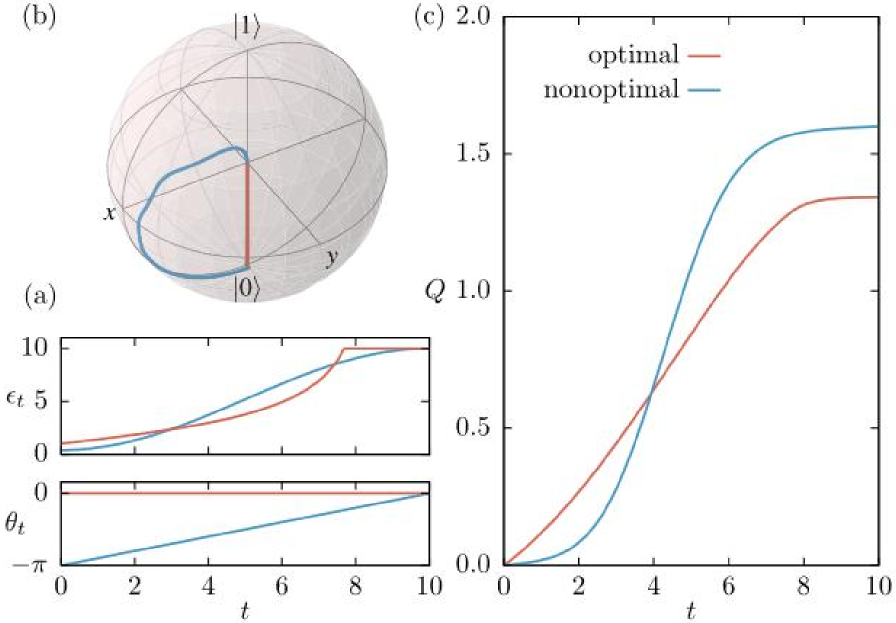
Lindblad equation

$$\dot{\rho} = -i[H_t,\rho] + \sum_{k=1}^{2} L_k\rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k,\rho\}$$

Heat production

$$Q = \int_0^\tau \mathrm{Tr}(\dot{\rho}H_t)\mathrm{d}t$$

# Coherence




$$\beta Q \geq \underbrace{\Delta S_{\rm cl} + \frac{\|\Lambda(\varrho_0) - \Lambda(\varrho_\tau)\|_1^2}{2\tau\bar{\gamma}_\tau}}_{\text{classical}} + \underbrace{C_{\rm rel} + \frac{C_{\rm res}}{2\tau\bar{\gamma}_\tau}}_{\text{quantum}}.$$

"quantum friction"

H. J. D. Miller, G. Guarnieri, M. T. Mitchison, and J. Goold, Phys. Rev. Lett. 125, 160602 (2020)
Tan Van Vu and Keiji Saito, Phys. Rev. Lett. 128, 010602 (2022)

# Slow-driving regime

For the fermionic bath $R(\tilde{\lambda}) = \frac{1}{1+e^{-\tilde{\lambda}}}$

$$\Downarrow$$

$$Q_{\min} = -T\Delta S + \beta^{-1} \frac{\arcsin^2(1-2\varepsilon)}{\mu\tau}$$

Previous study gives

$$Q_{\min} = -T\Delta S + \beta^{-1} \frac{\pi^2}{4\mu\tau}$$

M. Scandi et al., Phys. Rev. Lett 129, 270601 (2022)

Our result reduces to their result by $\varepsilon=0$.

# Example

**fermionic bath**

$$R(\tilde{\lambda}) = \frac{1}{1+e^{-\tilde{\lambda}}}$$

**quantum dot**

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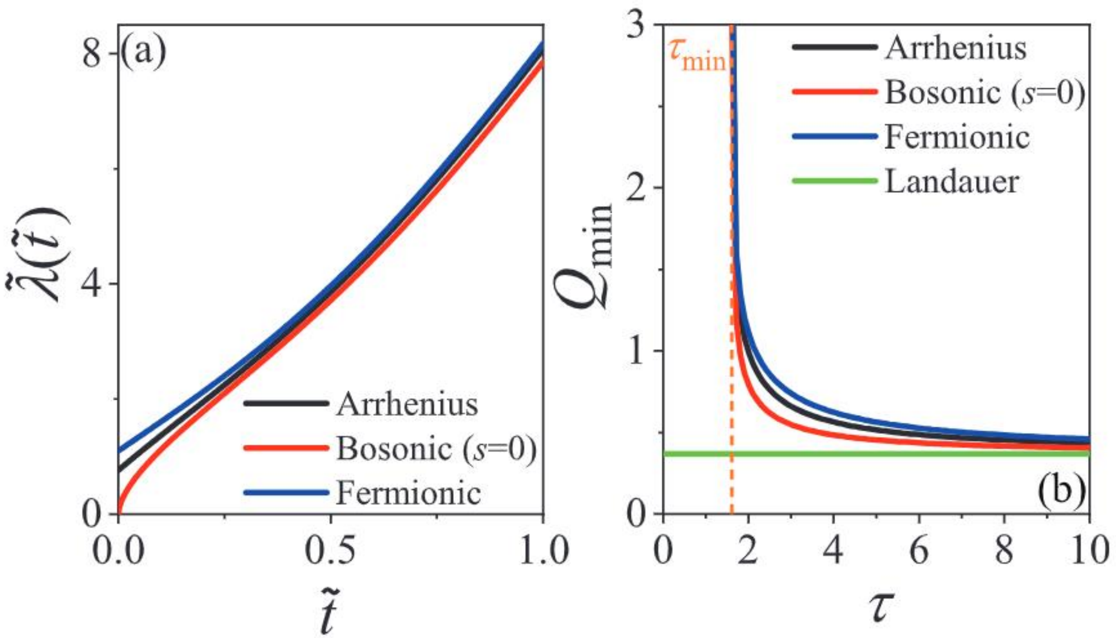
**bosonic bath**

$$R(\tilde{\lambda}) = \frac{\tilde{\lambda}^s}{1-e^{-\tilde{\lambda}}}$$

**spin-boson model**




One can find/prove that

1. Initial discontinuity of the optimal protocol
2. Bosonic bath have advantage over fermionic bath

# Quasi-static

$$W_{\rm qs}^{(1)} = \int_0^{\lambda_m} p_1^{\rm eq}(\lambda){\rm d}\lambda = \beta^{-1}\ln\left(\frac{2}{1+e^{-\beta\lambda_m}}\right)$$

$$W_{\rm qs}^{(2)} = \int_{\lambda_m}^{0} p_1^{\rm eq}(\lambda_m){\rm d}\lambda = -\frac{e^{-\beta\lambda_m}}{1+e^{-\beta\lambda_m}}\lambda_m$$

$$p_1^{\rm eq}(E) = \frac{e^{-\beta E}}{1+e^{-\beta E}}$$

$$W = W_{\rm qs}^{(1)} + W_{\rm qs}^{(2)} = \beta^{-1}[\ln 2 - S(\varepsilon)] = \Delta F_{\rm neq}$$

Non-equilibrium free energy

Phys. Rev. E.106, 034112 (2022)

$$S(\varepsilon) = -\varepsilon\ln\varepsilon - (1-\varepsilon)\ln(1-\varepsilon)$$

When $\lambda_m \to \infty,\ \varepsilon \to 0$, reduced to the traditional Landauer principle：

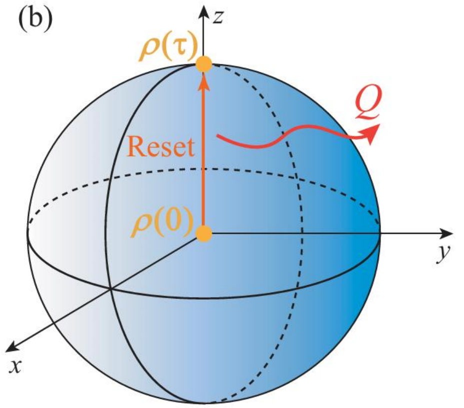
$$W = k_B T\ln 2$$

# Acknowledgements

刘越　　黄晨龙　　张星宇

***Thank You for your attention!***

# Qubit reset




Bloch sphere $$\rho = \frac{1}{2}(\boldsymbol{I} + \boldsymbol{r} \cdot \boldsymbol{\sigma}) = \frac{1}{2}\begin{bmatrix} 1+z & x-iy \\ x+iy & 1-z \end{bmatrix}$$

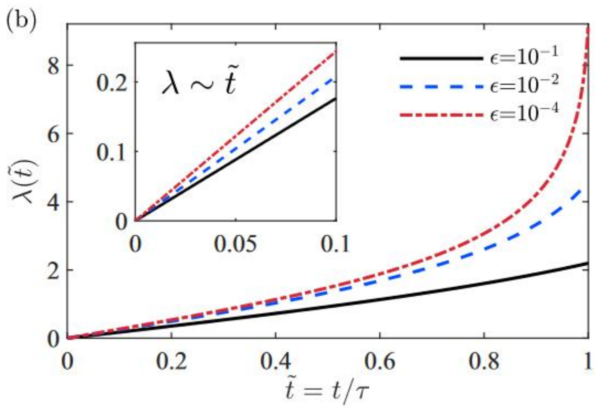
Von Neumann entropy production $$S(\rho) = -\frac{1-r}{2}\ln\frac{1-r}{2} - \frac{1+r}{2}\ln\frac{1+r}{2}$$

Maximally mixed state $$r = 0,\ \rho = \frac{1}{2}\boldsymbol{I},\ S = \ln 2$$

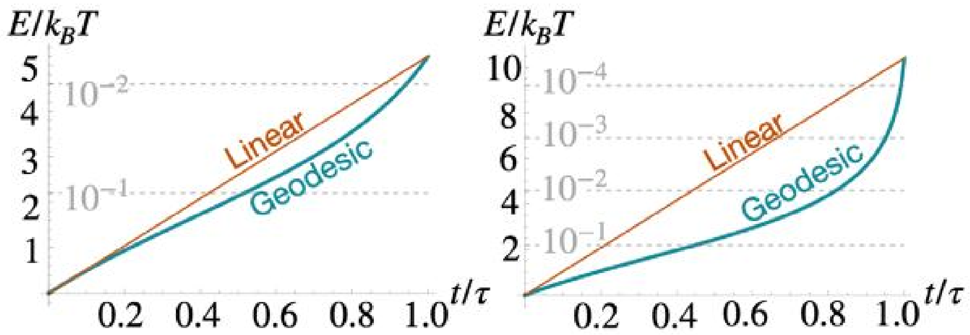
Pure state $$r = 1,\ \rho = \frac{1}{2}(\boldsymbol{I} + \boldsymbol{n} \cdot \boldsymbol{\sigma}),\ S = 0$$

# Optimal protocol

parameter optimization → minimum irreversible work




Y. Ma *et al.*, Phys. Rev. E 106, 034112 (2022)




M. Scandi *et al.*, Phys. Rev. Lett 129, 270601 (2022)

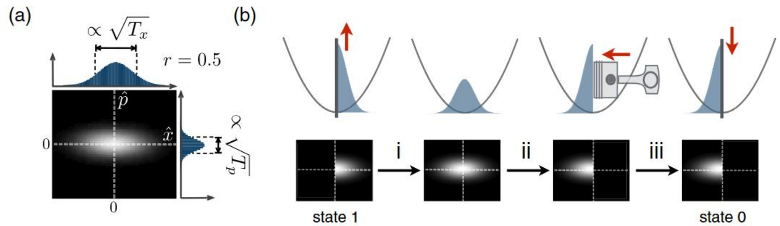
Question:

1) slow-driving
2) non-optimal

→ Tightest bound for any finite time interval?

# Quantum effect for qubit reset

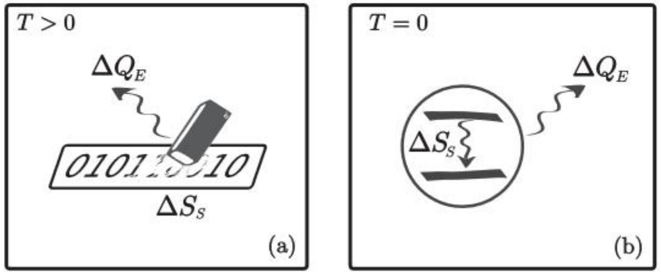
## Squeezed reservoir




$$W = k_B T e^{-2r} \ln 2$$

J. Klaers, PRL 122, 040602 (2019)

## Zero temperature effect




$$\Delta Q_E \geq -T\Delta S_S + \frac{3\hbar c}{\pi L}\Delta S_S^2,$$

A. M. Timpanaro, *et al*., PRL 124, 240601 (2020)

# Qubit reset

**Density matrix**

$$\rho = p_0|0\rangle\langle 0| + p_1|1\rangle\langle 1|$$

**Master equation**

$$\dot{p_0}(t) = k_{01}(t)p_1(t) - k_{10}(t)p_0(t),$$
$$\dot{p_1}(t) = k_{10}(t)p_0(t) - k_{01}(t)p_1(t),$$

$$[p_0(0), p_1(0)] = [\tfrac{1}{2}, \tfrac{1}{2}] \Rightarrow [p_0(\tau), p_1(\tau)] = [1-\varepsilon, \varepsilon]$$

**2nd Law** $$Q \geq -T\Delta S(\varepsilon) = \beta^{-1}[\ln 2 + \varepsilon \ln \varepsilon + (1-\varepsilon)\ln(1-\varepsilon)]$$

**Detailed balance condition**

$$k_{01} = \mu R(\beta\lambda),\ \ k_{10} = \mu R(\beta\lambda)e^{-\beta\lambda}$$

↓

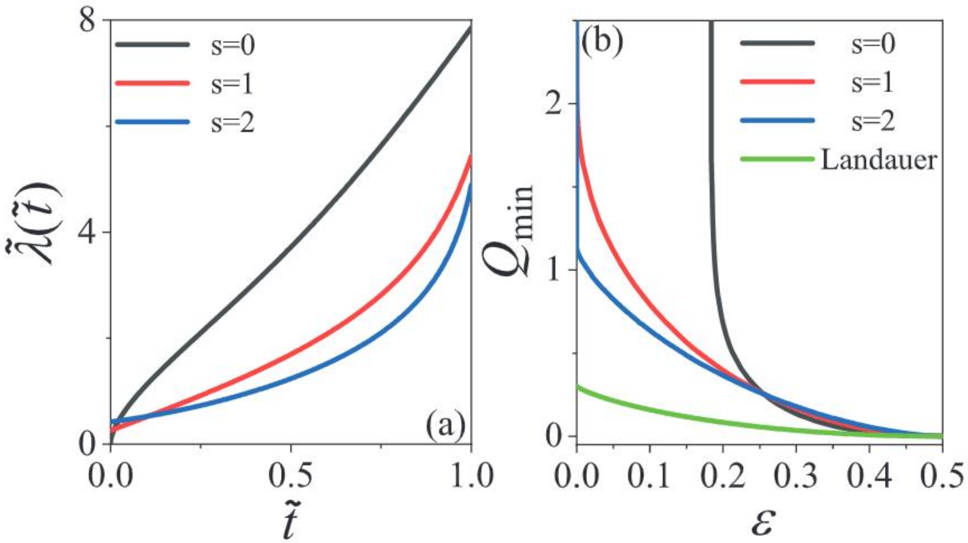
rescaled transition rate

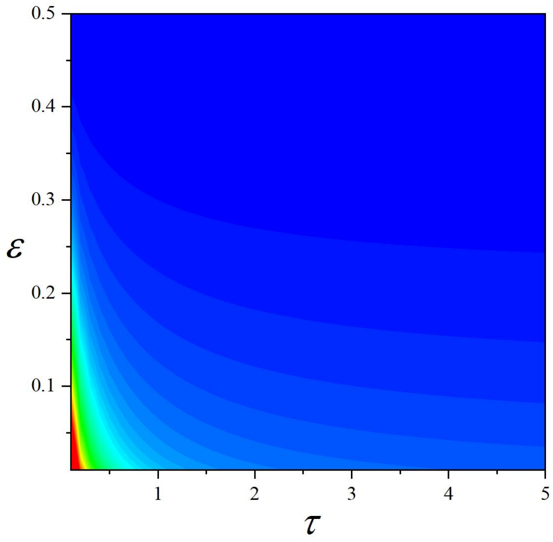
# Bosonic bath with s≠0 (2nd class)

$$R(\tilde{\lambda}) = \frac{\tilde{\lambda}^s}{1-e^{-\tilde{\lambda}}} \qquad R(+\infty) \text{ is divergent}$$

Slow-driving regime: Y. Ma *et al.*, Phys. Rev. E 106, 034112 (2022).

Our general framework can extend their study beyond the slow-driving regime.




The case of s≠0 have advantages over s=0

# One-direction driving

**without coherence**

**density matrix** $\rho = p_0|0\rangle\langle 0| + p_1|1\rangle\langle 1|$

**master equation**

$$\dot{p_0}(t) = k_{01}(t)p_1(t) - k_{10}(t)p_0(t),$$
$$\dot{p_1}(t) = k_{10}(t)p_0(t) - k_{01}(t)p_1(t),$$

$$[p_0(0), p_1(0)] = [\tfrac{1}{2}, \tfrac{1}{2}] \Rightarrow [p_0(\tau), p_1(\tau)] = [1-\varepsilon, \varepsilon]$$

**The Second Law** $Q \geq -T\Delta S(\varepsilon) = \beta^{-1}[\ln 2 + \varepsilon \ln \varepsilon + (1-\varepsilon)\ln(1-\varepsilon)]$

**detailed balance condition** $k_{01} = \mu R(\beta\lambda),\ k_{10} = \mu R(\beta\lambda)e^{-\beta\lambda}$

↓

**rescaled transition rate**