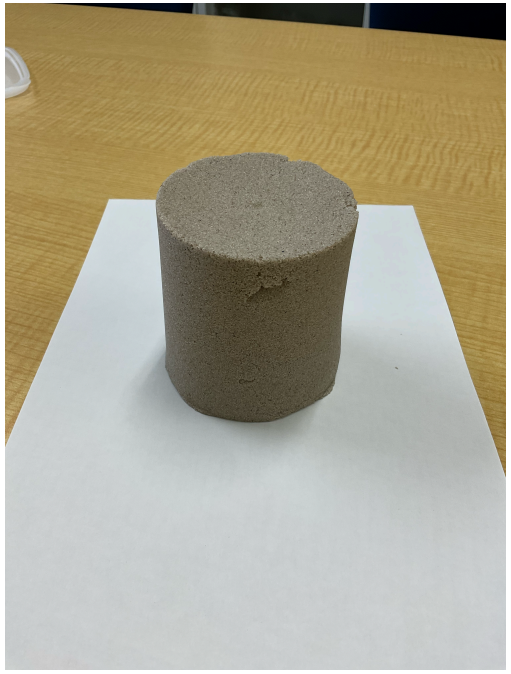
Hysteresis and Marginal Stability of Cohesive Grains

Michio Otsuki (Shimane Univ.)

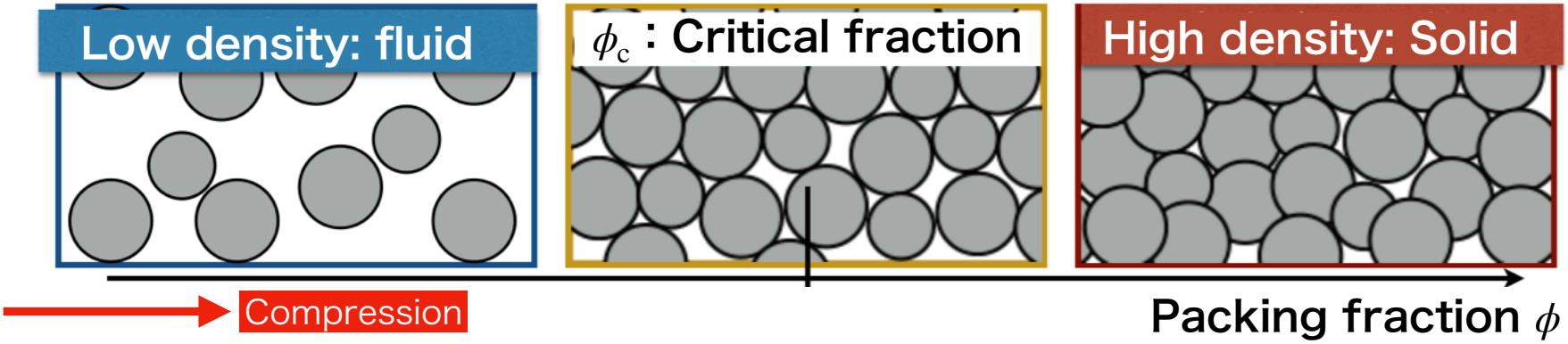


Collaborators:
K. Yoshii (Tokyo Univ. of Sci.),
H. Mizuno (Tokyo Univ.)

Mechanical properties of repulsive grains

▸ Jamming transition

M. van Hecke, J. Phys.: Condens. Matter (2010)



▸ Critical behavior ($\phi > \phi_c$)

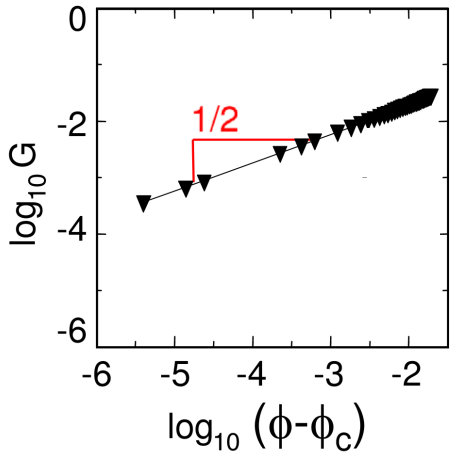
C. S. O'Hern et al., PRE (2003)

Shear modulus: $G \propto (\phi - \phi_c)^{\alpha}$

Pressure: $p \propto (\phi - \phi_c) \geq 0$

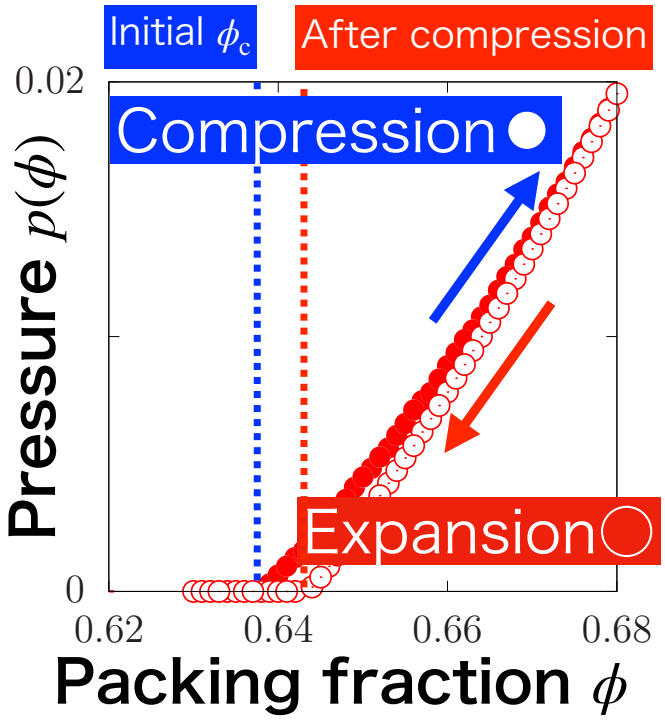
Coordination number: $Z - Z_{iso} \sim (\phi - \phi_c)^{1/2}$

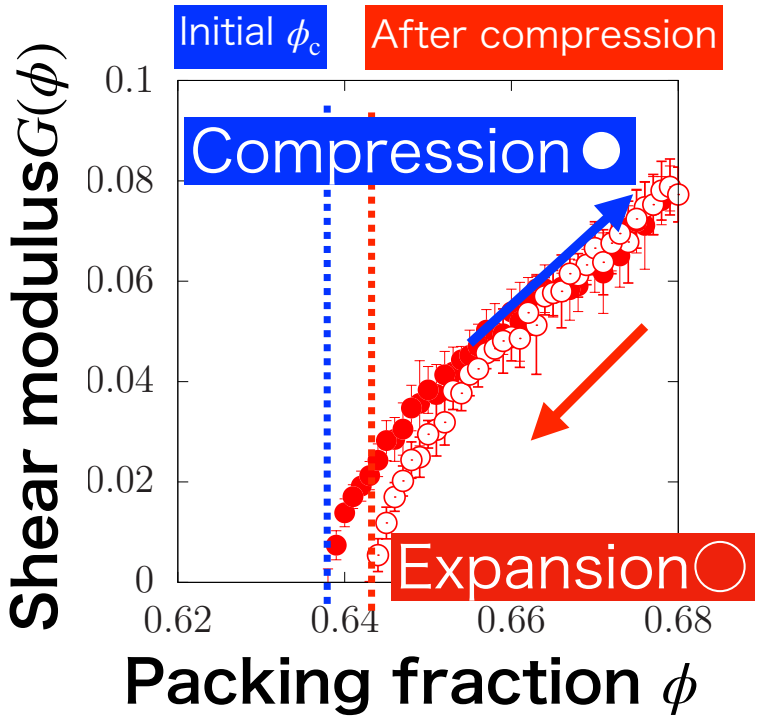
Isostatic value: $Z_{iso} = 2D$ (D dimensions)



Hysteresis of repulsive grains

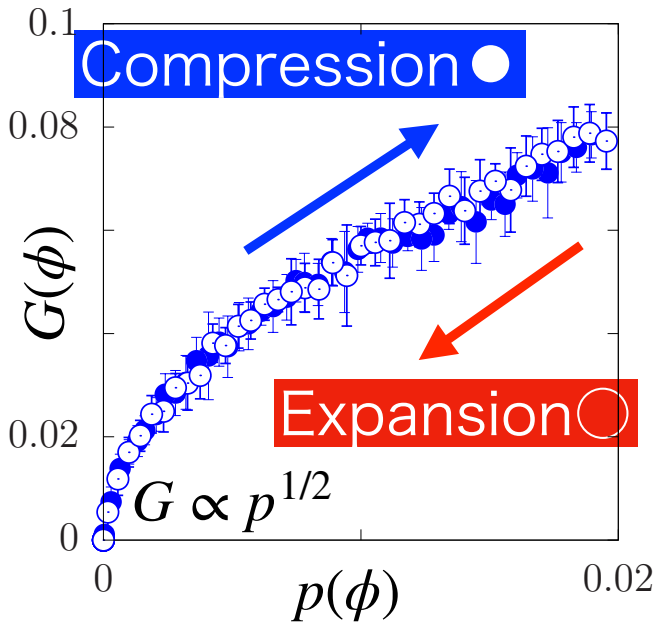
▸ Hysteresis by mechanical training





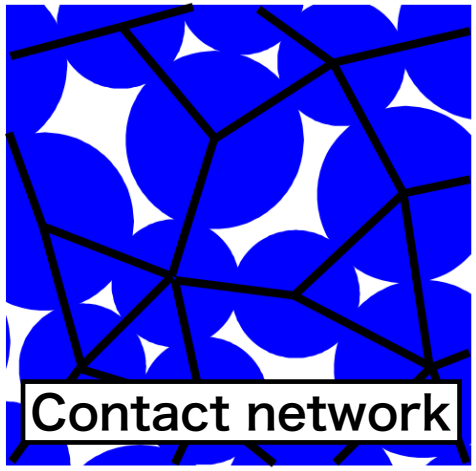
Kumar, Luding, Granular Matter(2016), Kawasaki, Miyazaki, PRL (2024)

▸ Scaling using p



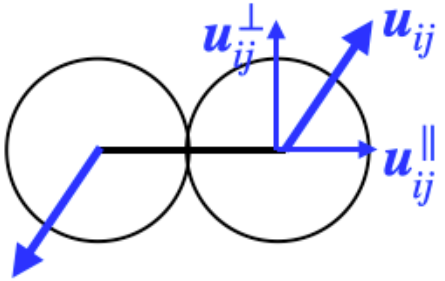
Hysteresis is removed by a scaling plot using p.

Theoretical explanation (repulsive grains)



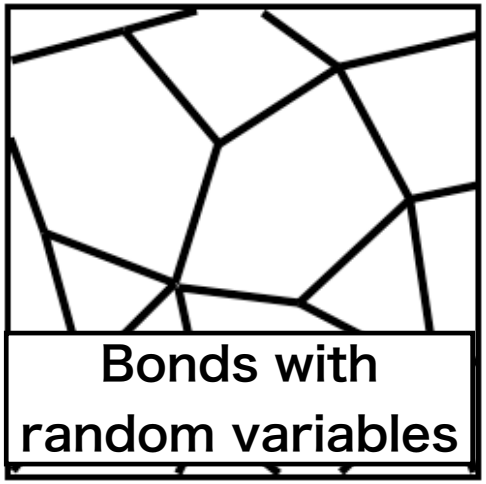
▸ Hessian H (linear repulsive force F_{ij})

Relative displacement u_{ij}



Energy variation:

$$\Delta E \simeq \sum_{(i,j)} \frac{1}{2} \left\{ k \left(u_{ij}^{\parallel} \right)^2 - \frac{F_{ij}}{r_{ij}} \left(u_{ij}^{\perp} \right)^2 \right\}$$
$$= \boldsymbol{u} \cdot \boldsymbol{H} \cdot \boldsymbol{u}$$



▸ Effective medium theory, random matrix

H → Random matrix $H^{\mathrm{R}}(Z,p)$ → Density of state $D(\omega;Z,p)$

Marginal stability condition ([Minimum eigen-value of H^{R}] $=0$)

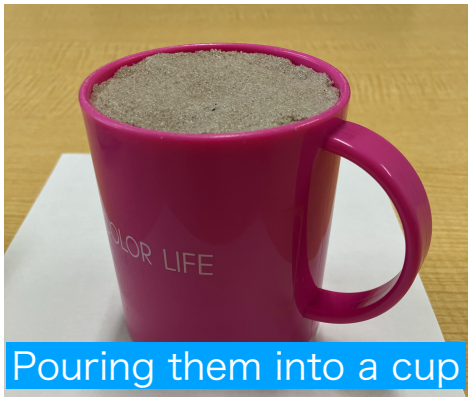
$Z - Z_{\mathrm{iso}} \propto p^{1/2}$

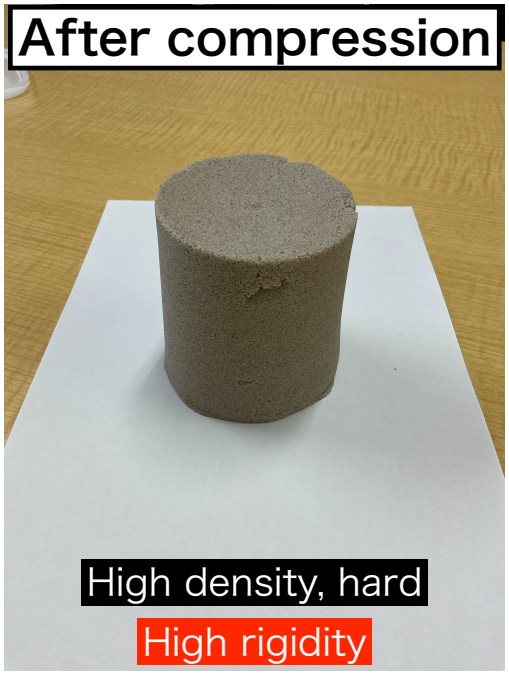
$G \propto Z - Z_{\mathrm{iso}}$

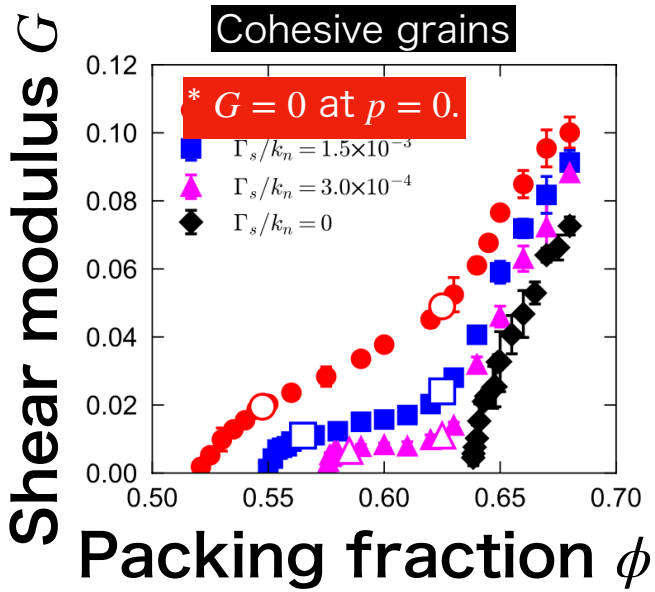
→ $G \propto p^{1/2}$

H. Mizuno and A. Ikeda, arXiv 2407.15323,
H. Ikeda and M. Shimada, PRE (2022)

Problem: Scaling and hysteresis for cohesive grains





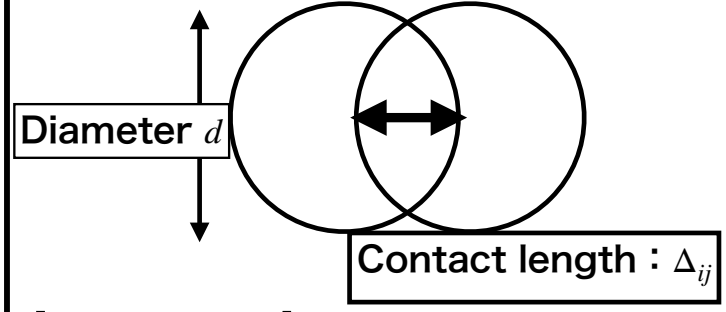


Koeze, et al., PRR (2020)
Yoshii, MO, JPSJ (2025)

- Due to initial compression, the shear modulus G changes.
- At the surface of the column, $p = 0. \rightarrow G = 0$ for repulsive grains.
- For cohesive grains, G exhibits hysteresis and $G > 0$ even for $p = 0$?

Setup : 3D-Discrete Element Method

3D mono-disperse system



Interaction:

Static force for ij grains $F_{ij}^{(\mathrm{s})}$
(Repulsion+Cohesion)

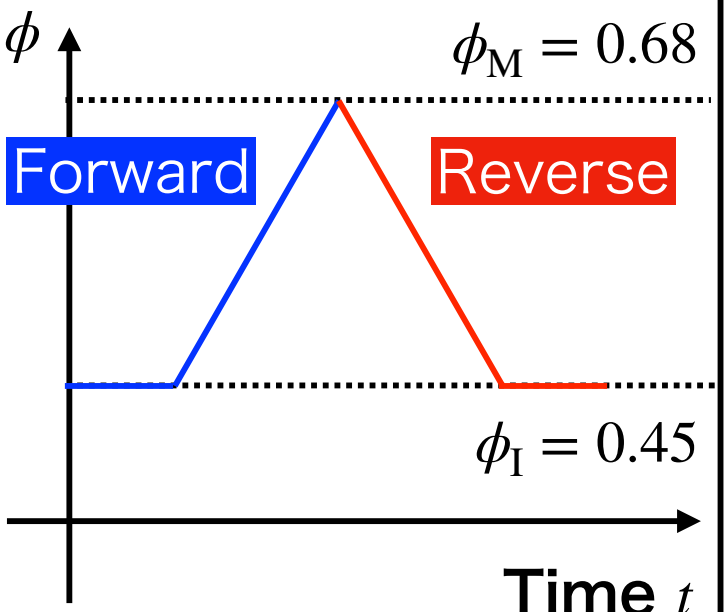
Dissipative force $F_{ij}^{(\mathrm{d})}$

Time evolution:

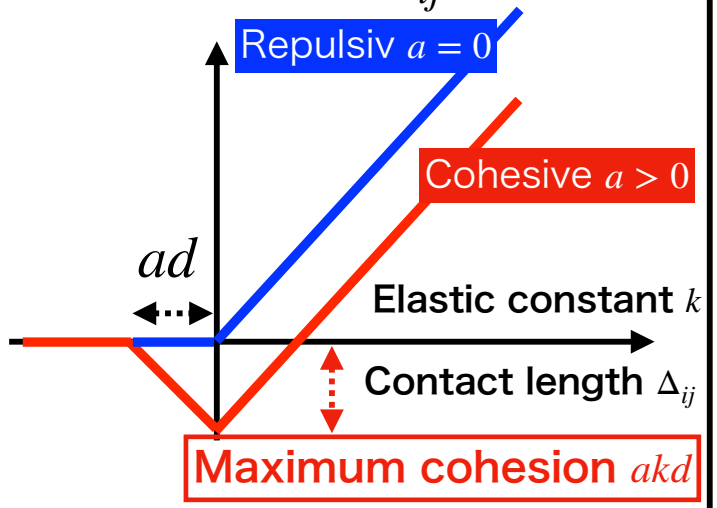
$$m\ddot{\boldsymbol{r}}_i = \sum_j \left(F_{ij}^{(\mathrm{s})} + F_{ij}^{(\mathrm{d})} \right) \boldsymbol{n}_{ij}$$

Position $\boldsymbol{r}_i$, normal vector $\boldsymbol{n}_{ij}$

Cyclic compression



Static force $F_{ij}^{(\mathrm{s})}$



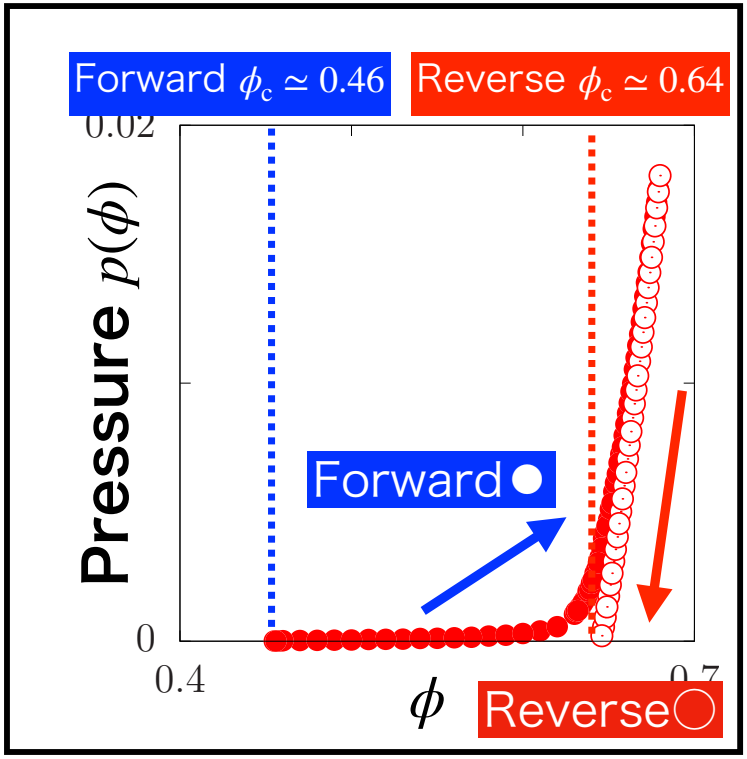
Cohesiveness a

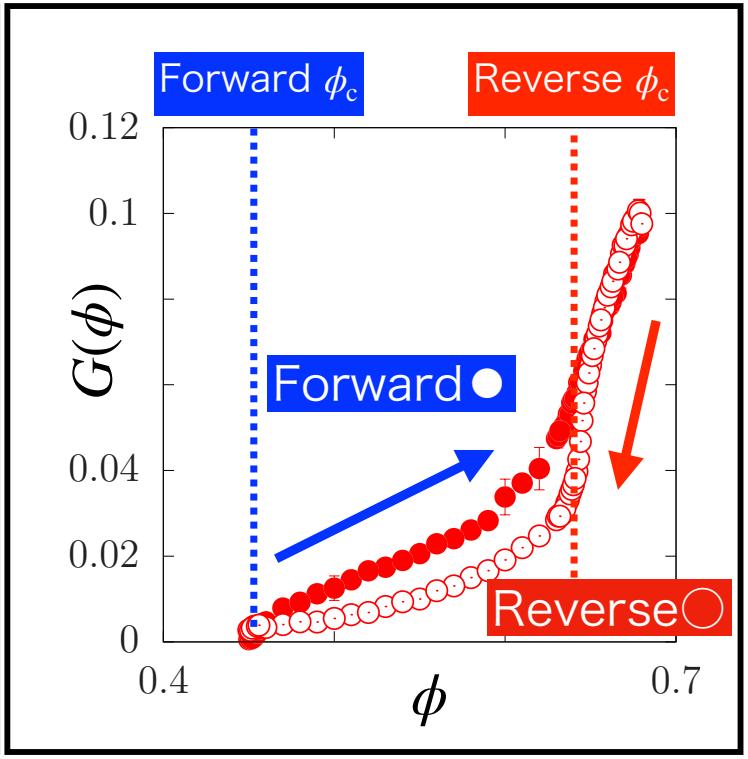
Koeze, et al., (2018)

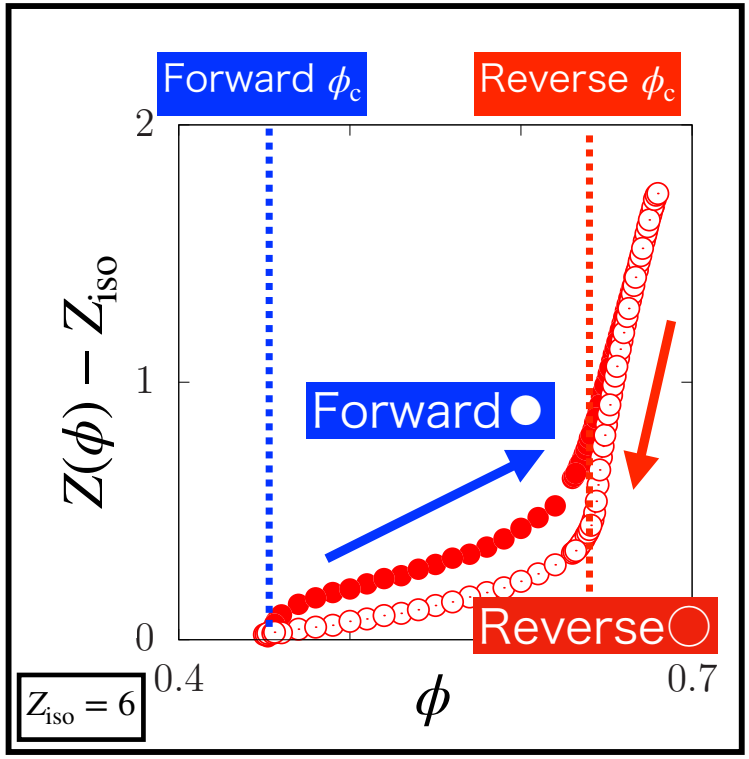
- $p(\phi), G(\phi),$ and $Z(\phi)$ are measured in both processes (Forward, Reverse) for cohesive grains ($a > 0$).

Mechanical Hysteresis for Cohesive Grains ($a = 10^{-3}$)

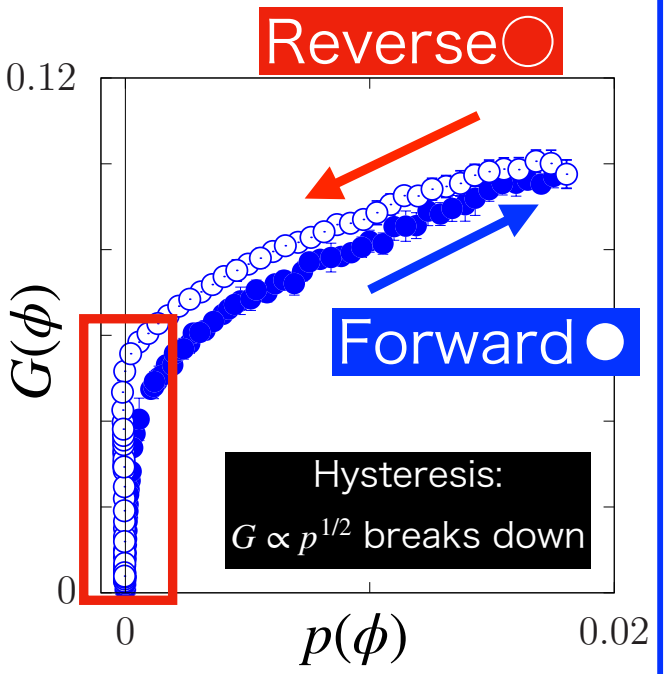
- ϕ_c(packing fraction for $p = 0$) changes.
- Hysteresis : $G(\phi)$ decreases in the reverse process ($G > 0$ for $\phi < \phi_c$.)

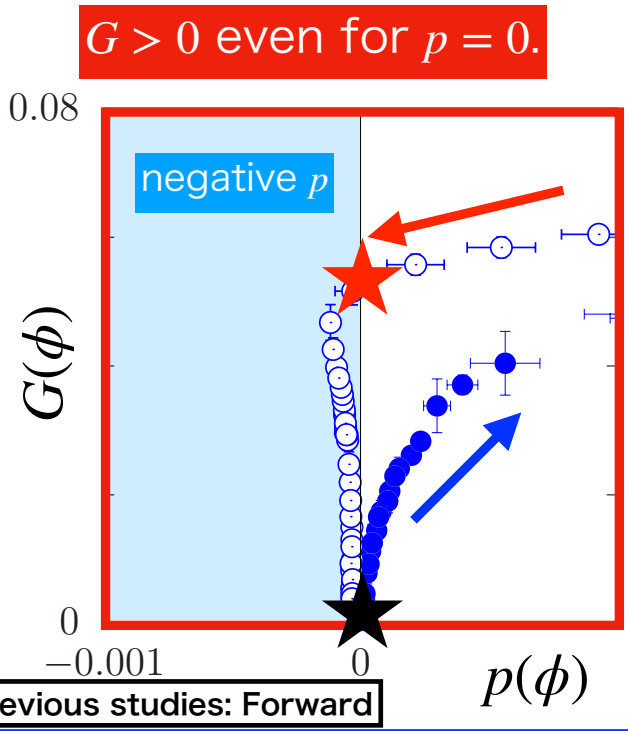


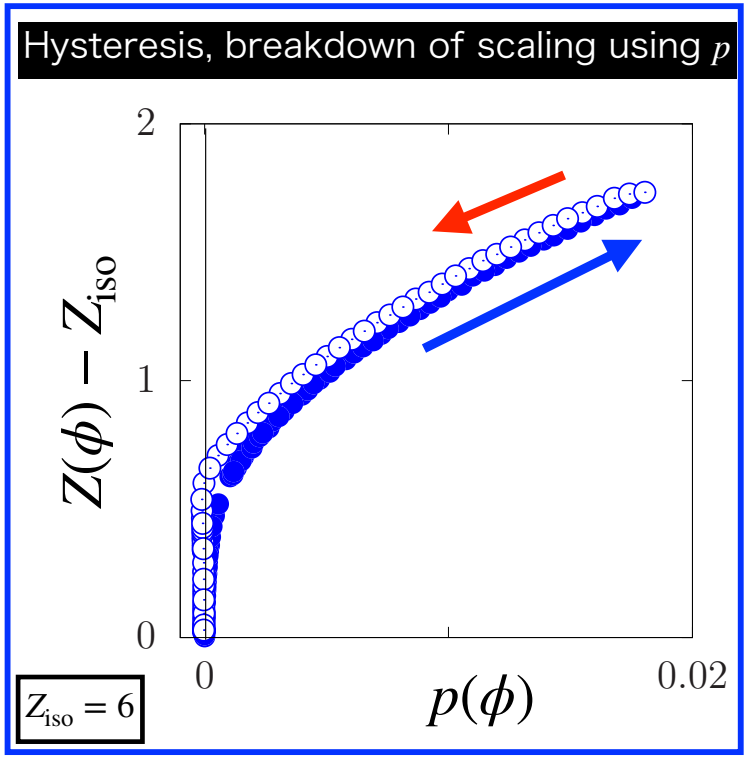




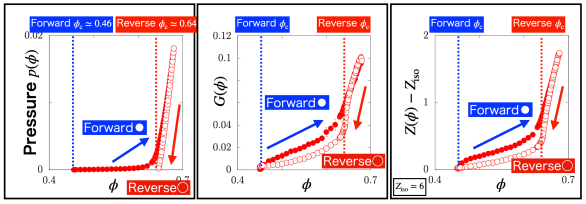
Scaling Plot using p for Cohesive Grains ($a = 10^{-3}$)



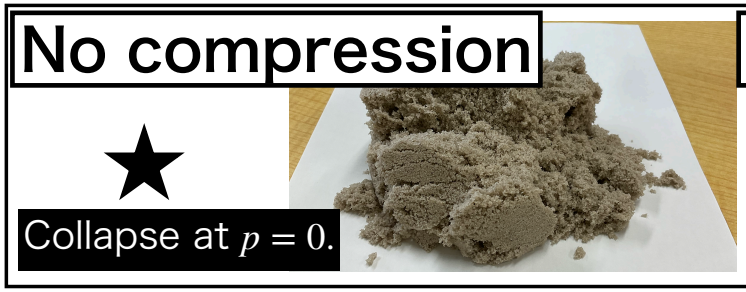




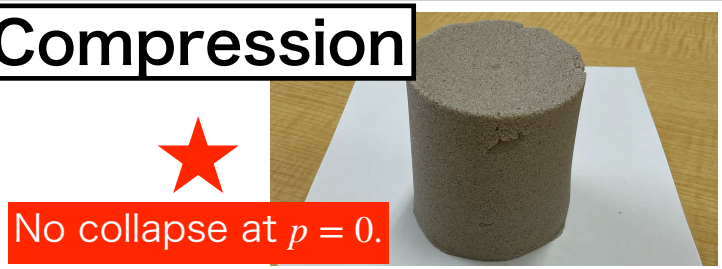
Original data



No compression



Compression



Summary and Outlook

- Topic: 3D-cohesive grains under cyclic compression
- Result: Hysteresis → Breakdown of scaling law
- Outlook: Yield stress, compaction in soil mechanics