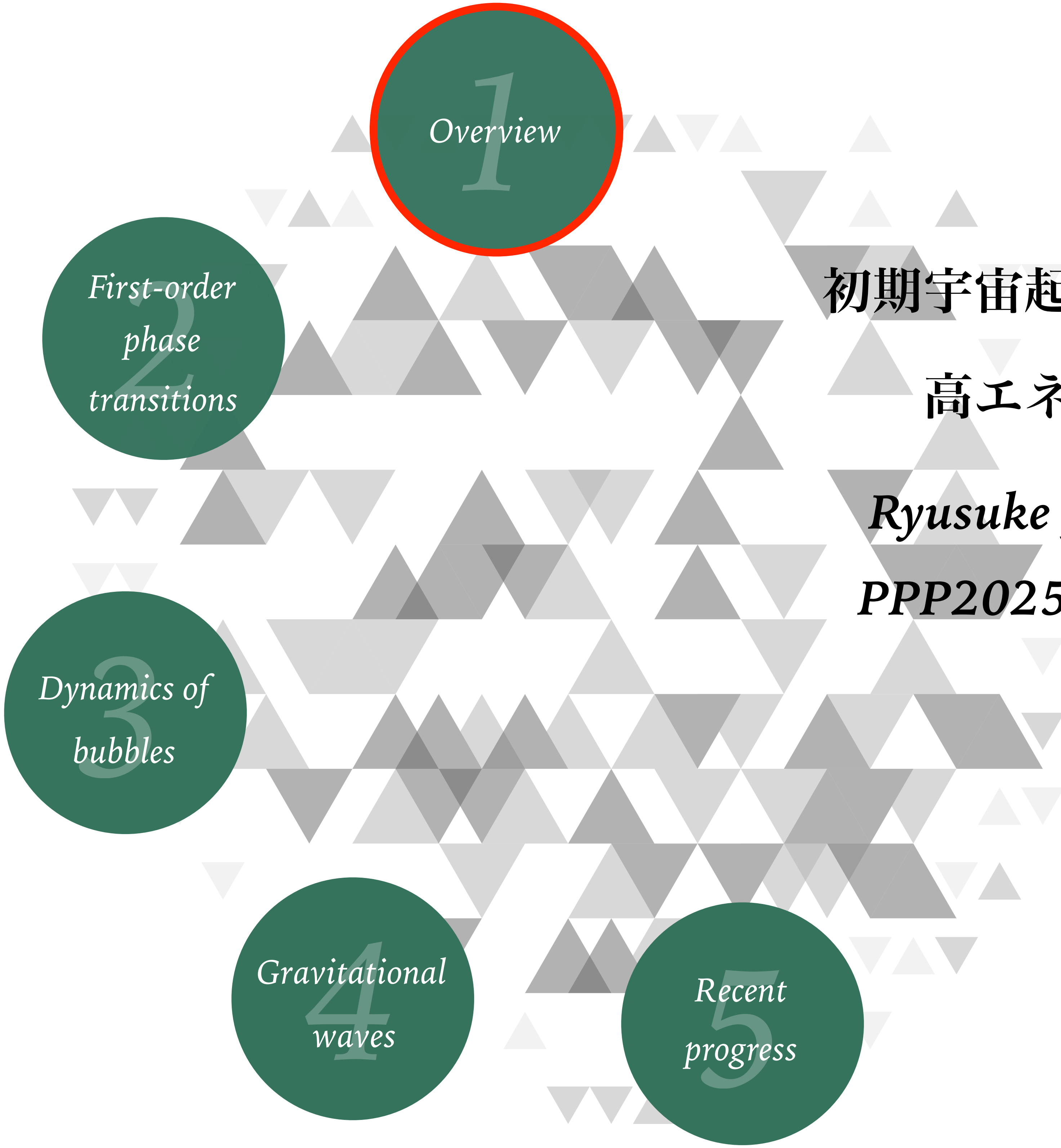
初期宇宙起源の重力波を用いた高エネルギー物理探査

Ryusuke Jinno (Kobe Univ.)

PPP2025@Kyoto, 2025/9/4

KOBE UNIVERSITY



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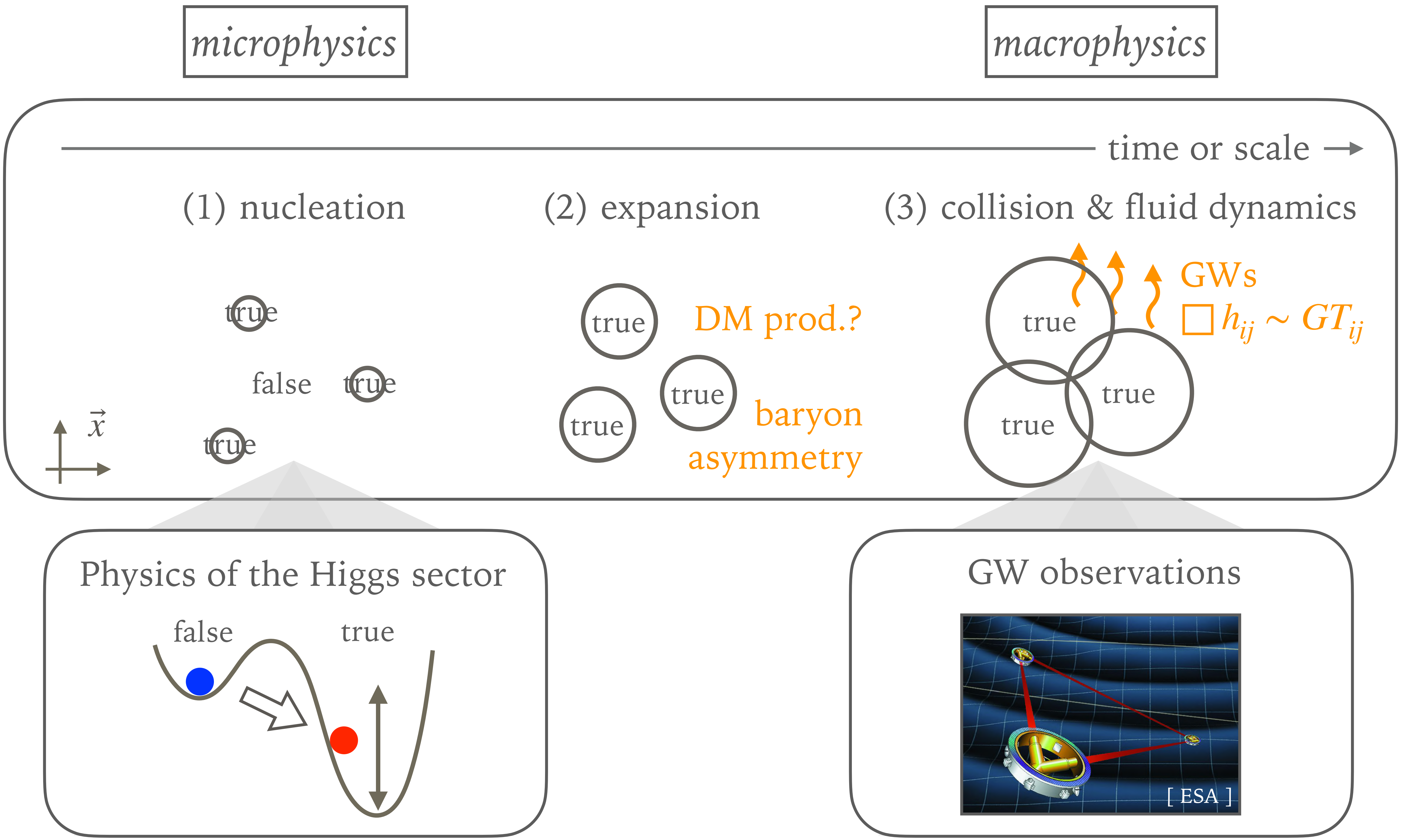
FIRST-ORDER PHASE TRANSITIONS IN THE EARLY UNIVERSE



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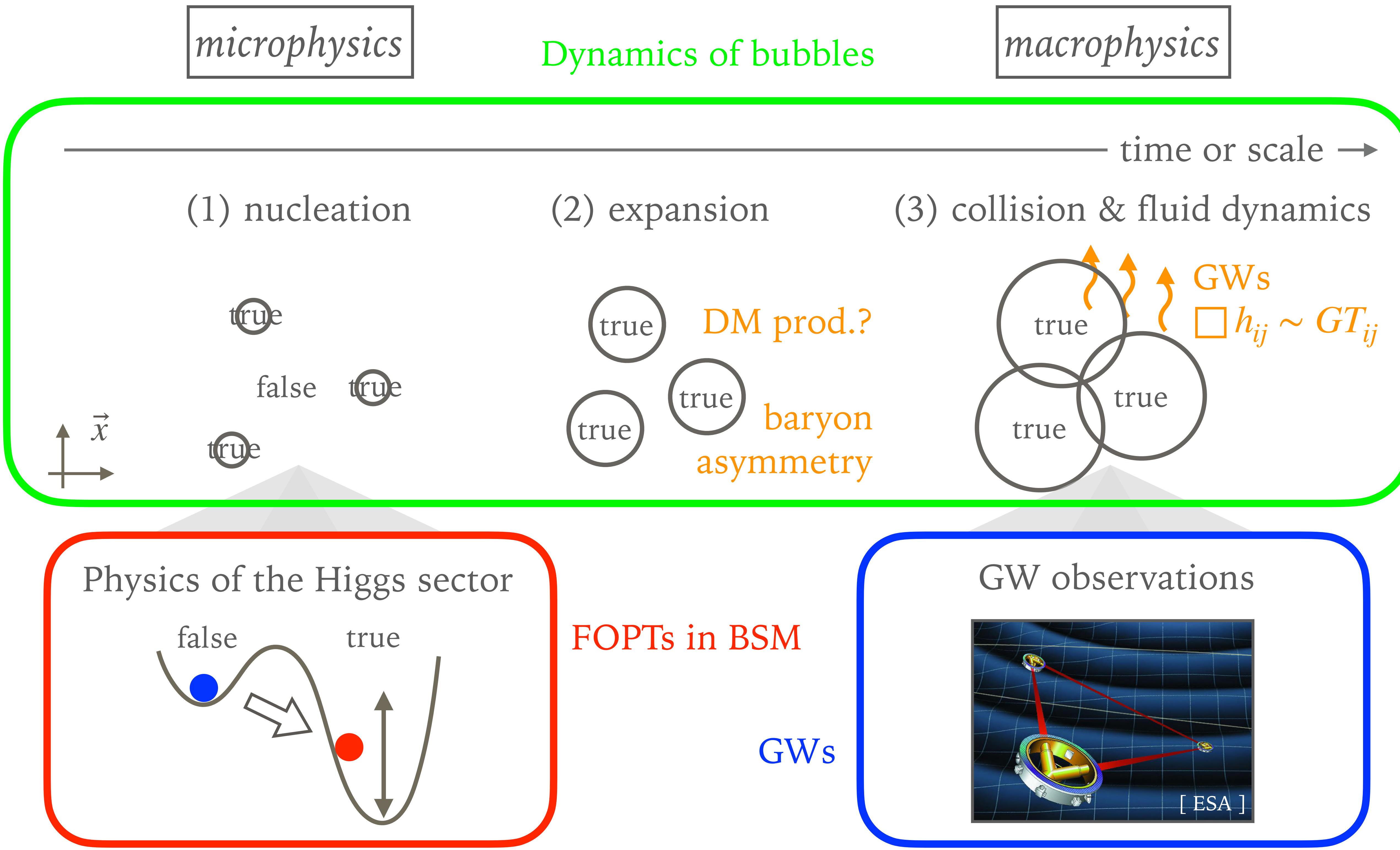
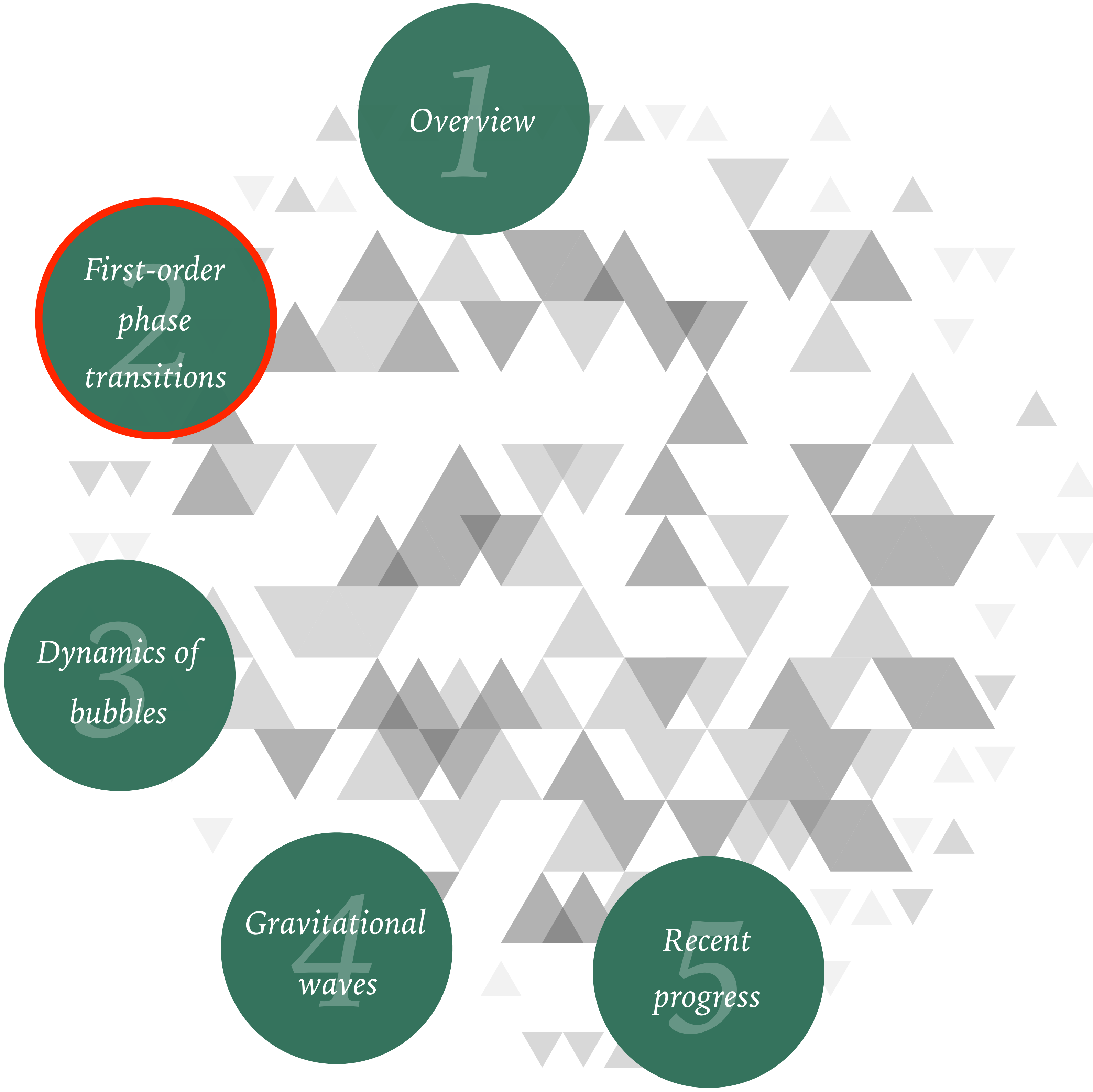


TALK PLAN

~20min
- 2. First-order phase transitions in beyond the Standard Model
- 3. Dynamics of bubbles
- 4. Gravitational wave production & observational prospects

~25min
- 5. Recent progress


1
Overview
2
First-order phase transitions
3
Dynamics of bubbles
4
Gravitational waves
5
Recent progress

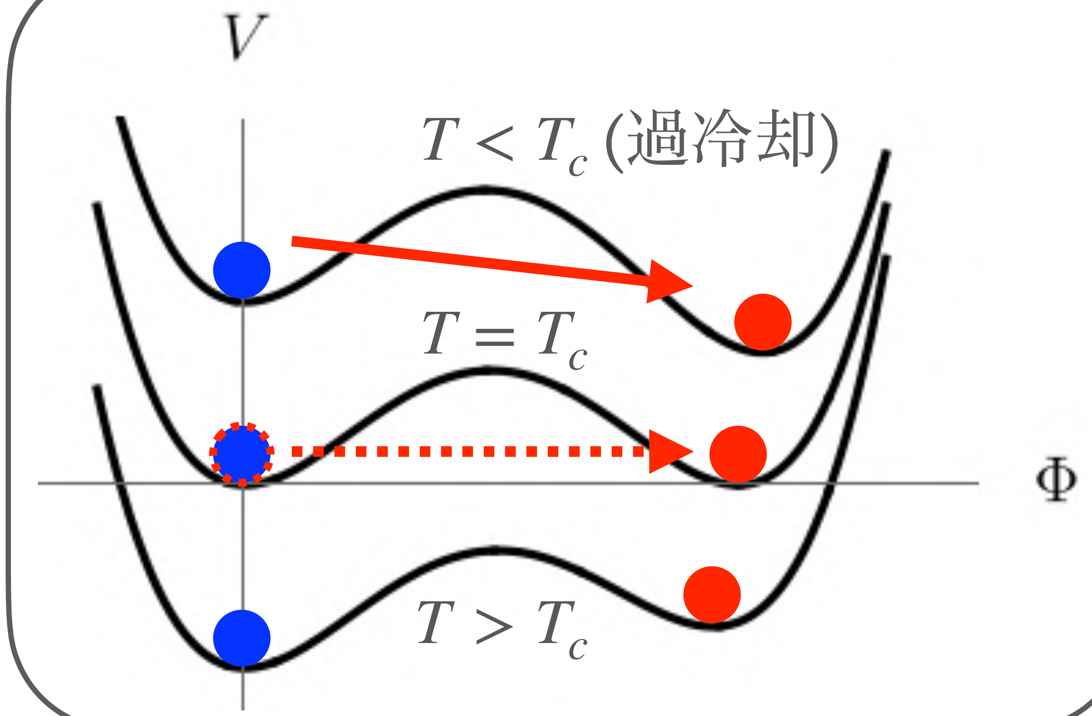
FIRST-ORDER PHASE TRANSITIONS

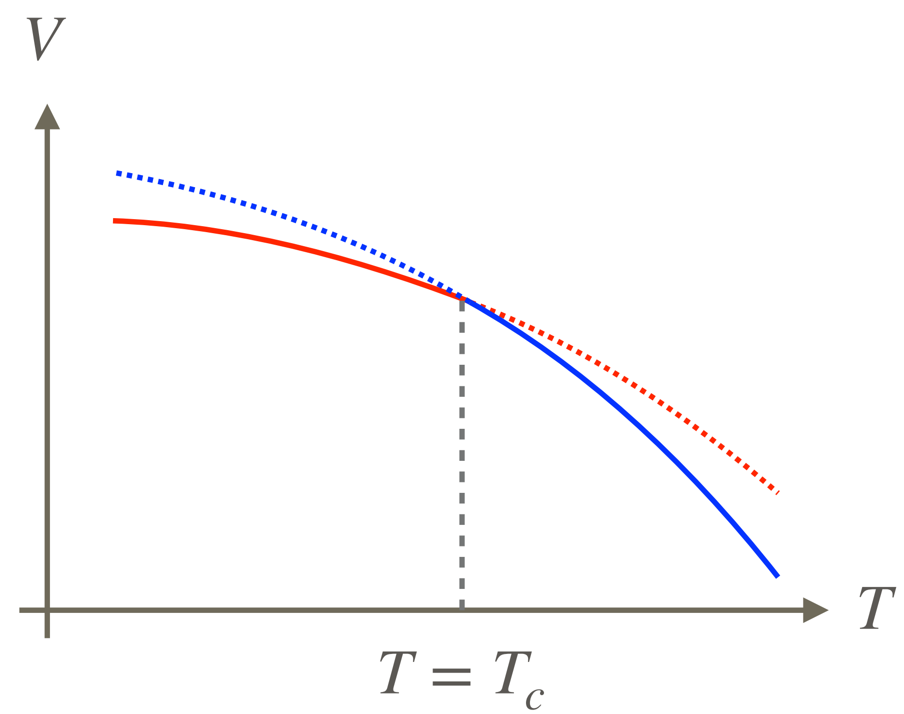
一次相転移

完全な熱力学関数がその自然な変数に関して

1階微分不可能なとき、一次相転移と言う

- 典型的な例：Φ (秩序パラメータ) と T (温度) に対する自由エネルギーの振る舞い

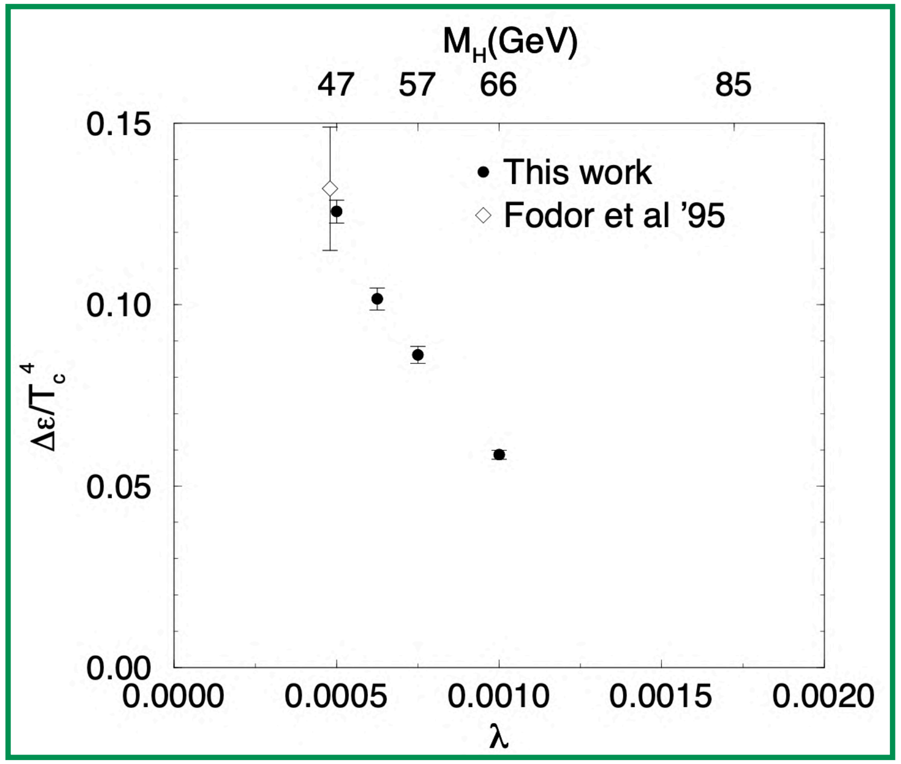




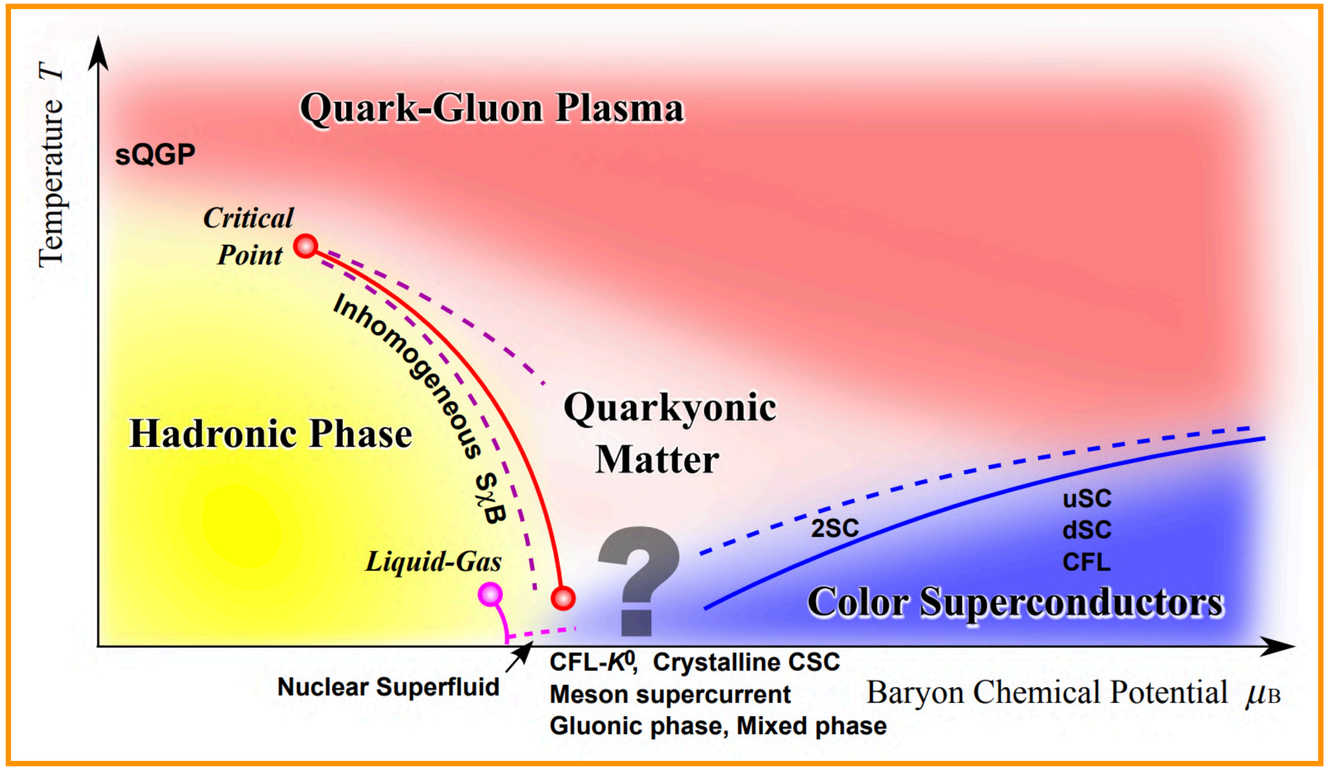
THERMAL HISTORY OF THE UNIVERSE

- Two candidates for FOPTs in the Standard Model (SM)

Electroweak "phase transition" & QCD "phase transition"



[Aoki '97]
see also
[Kajantie, Laine, Rummukainen, Shaposhnikov '96]
[Karsch, Neuhaus, Patkós, Rank '97]



[Fukushima, Hatsuda '11]

→ Unfortunately, both are crossover, meaning they are not even phase transitions

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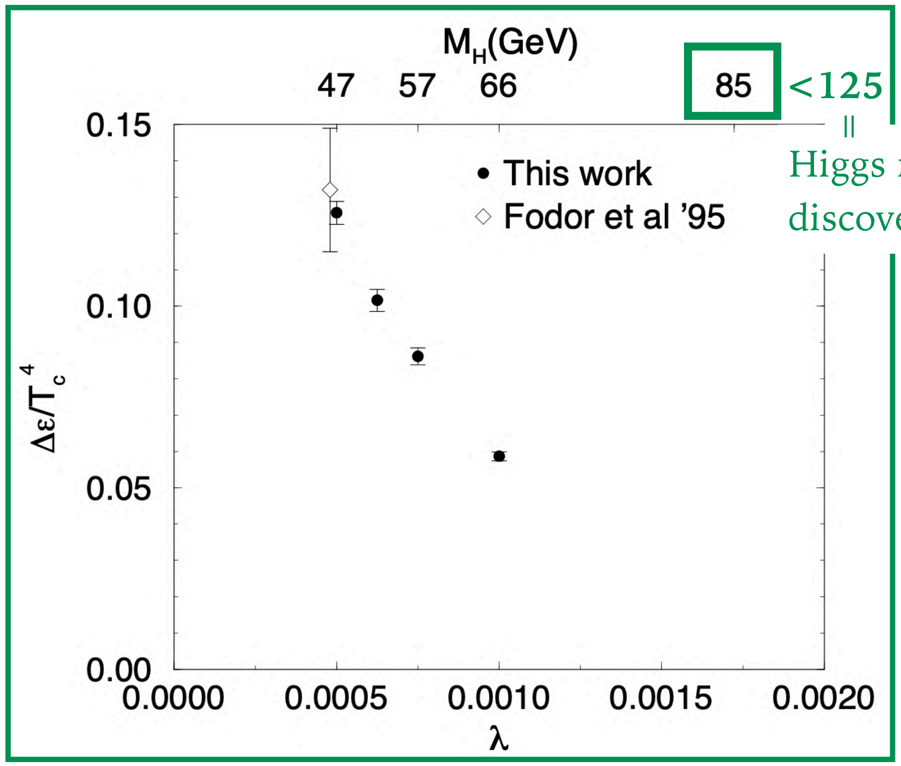

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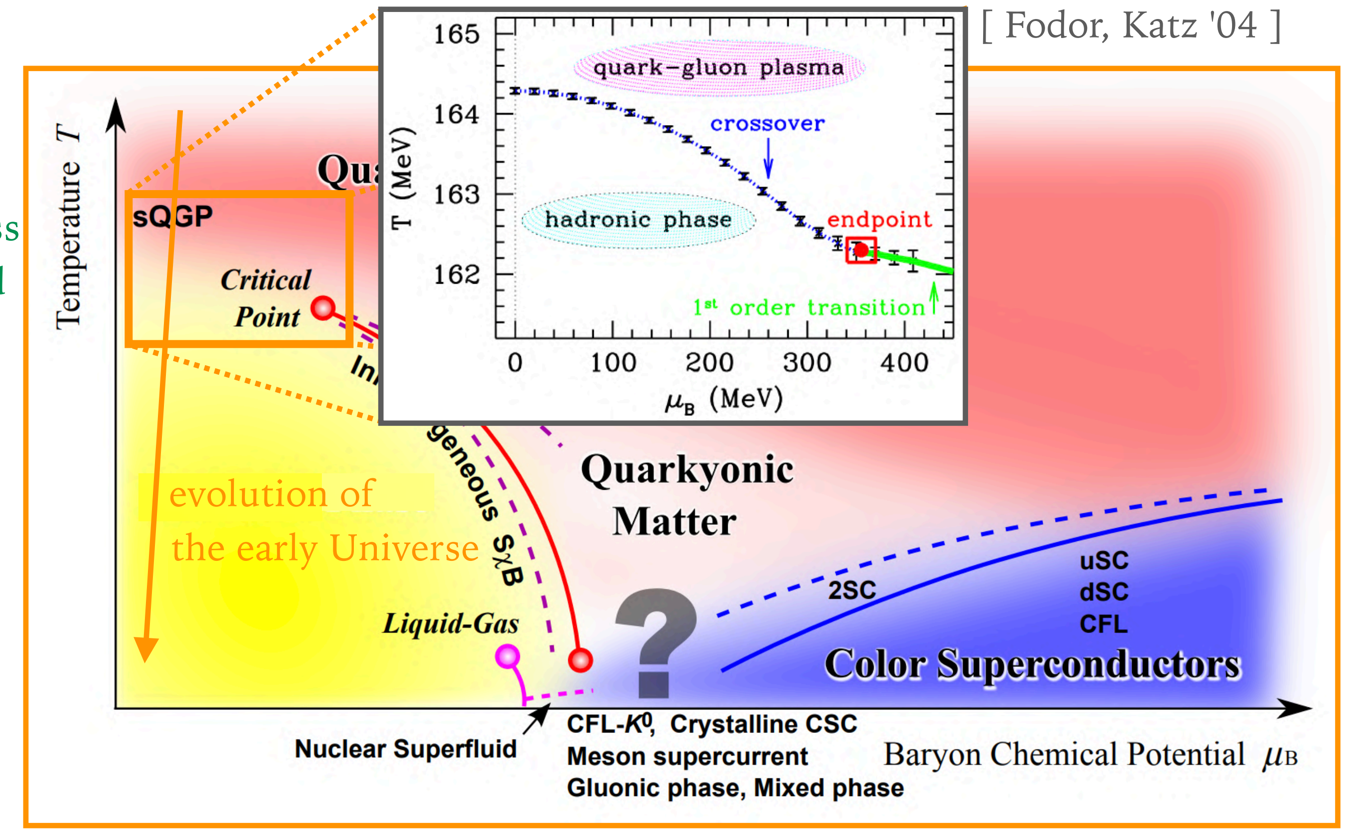

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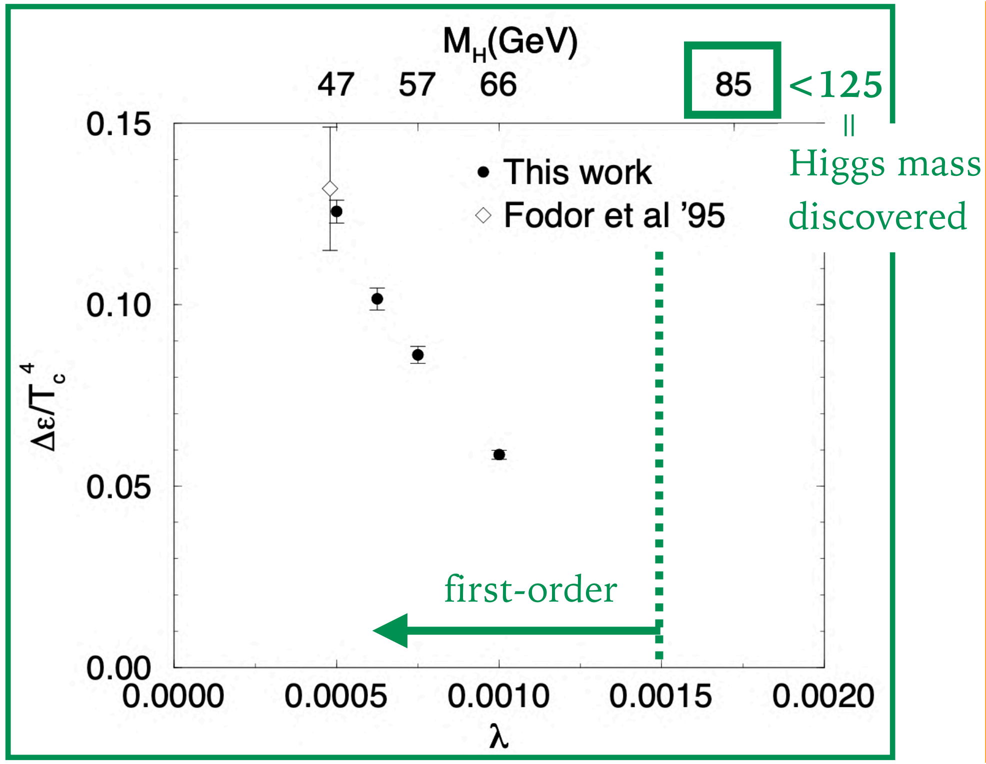

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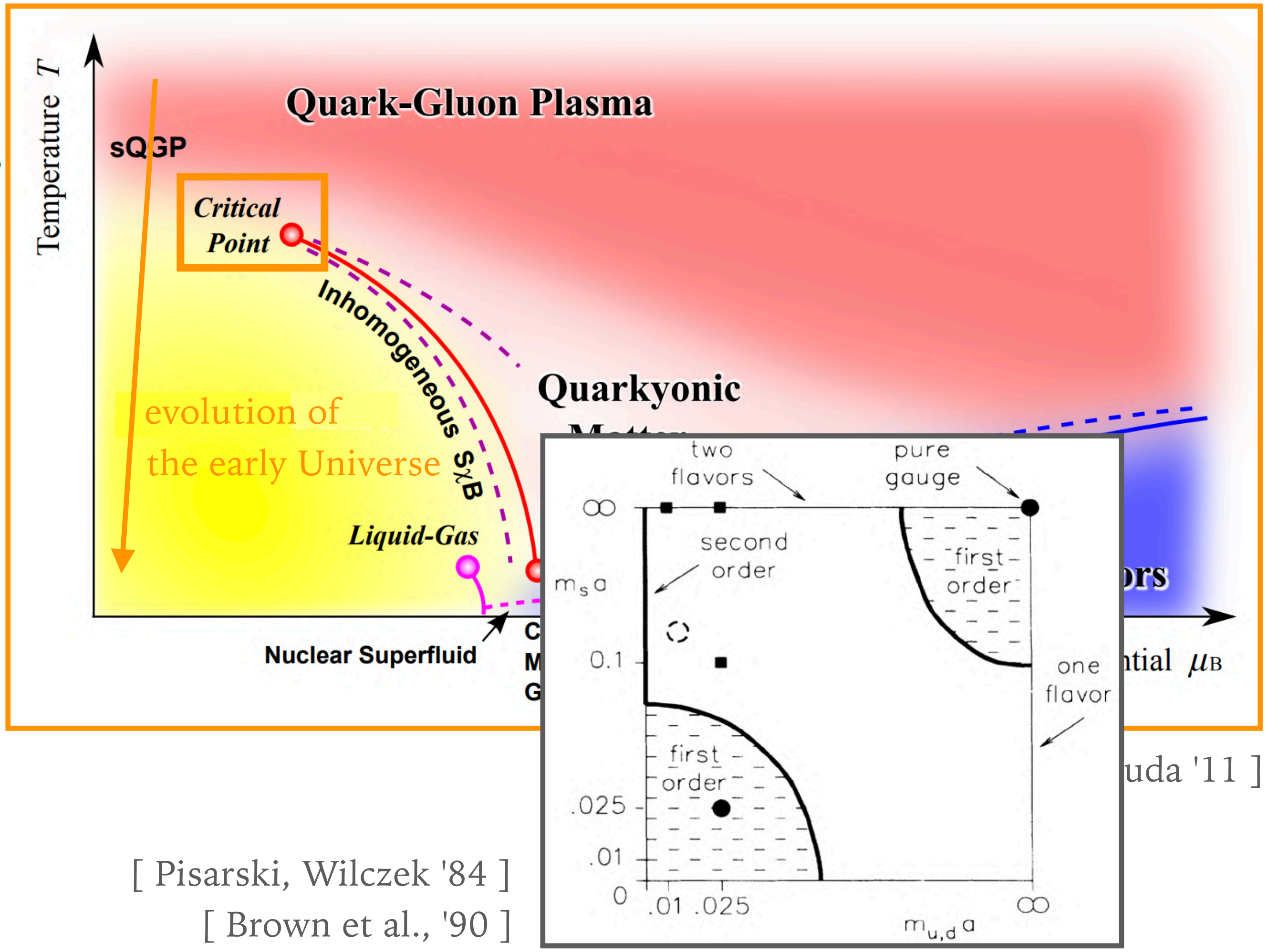


[Pisarski, Wilczek '84]
[Brown et al., '90]
…uda '11]

→ Unfortunately, both are crossover, meaning they are not even phase transitions

MOTIVATIONS FOR FIRST-ORDER PHASE TRANSITIONS

- The vast energy scale the Universe might have experienced from inflation ($\lesssim 10^{15}\text{GeV}$) down to the present ($\sim 10^{-4}\text{eV}$)

- Spontaneous symmetry breaking that might have happened

  - Breaking of the GUT group (→ GUT)

  - Breaking of Peccei-Quinn symmetry $U(1)_{\rm PQ}$ (→ strong CP)

  - Breaking of B-L symmetry $U(1)_{\rm B-L}$ (→ neutrino masses)

  - Breaking of dark groups

- Testability of the process in the coming 10-20 yrs with GWs

TRADITIONAL MOTIVATION FOR FIRST-ORDER PHASE TRANSITIONS

➤ Baryon asymmetry of the Universe (BAU)

= Why more baryons than antibaryons?

Galaxy

Antigalaxy

➤ 3 conditions to generate baryon asymmetry (Sakharov's conditions)

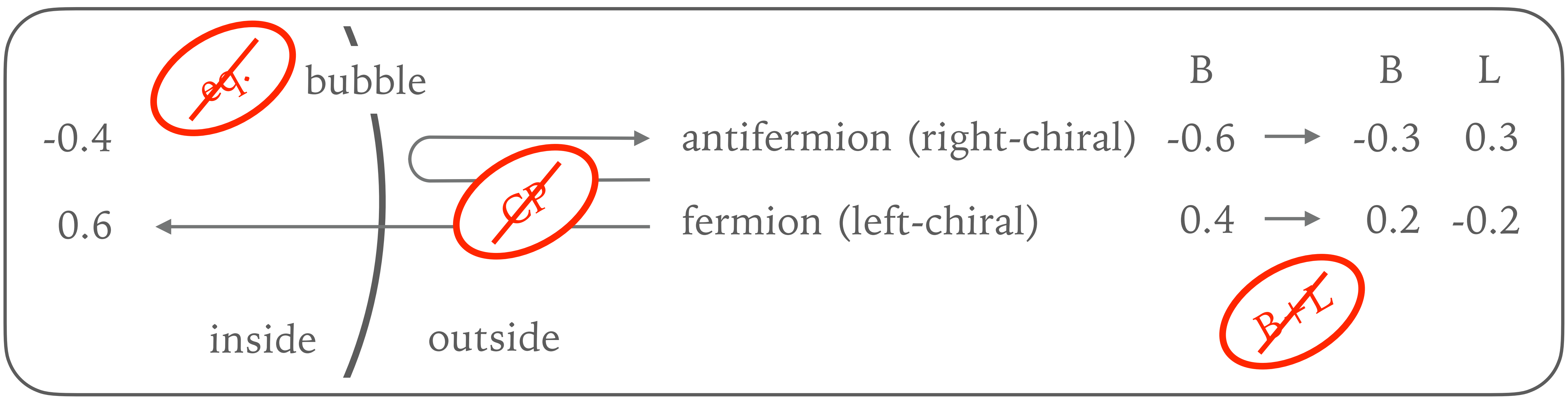
[Sakharov '67]

1) B violation   2) C&CP violation   3) Interactions out of thermal equilibrium

TRADITIONAL MOTIVATION FOR FIRST-ORDER PHASE TRANSITIONS

- Part of Sakharov's conditions are satisfied if an FOPT occurs

  (called electroweak baryogenesis) [Kuzmin, Rubakov, Shaposhnikov '85]



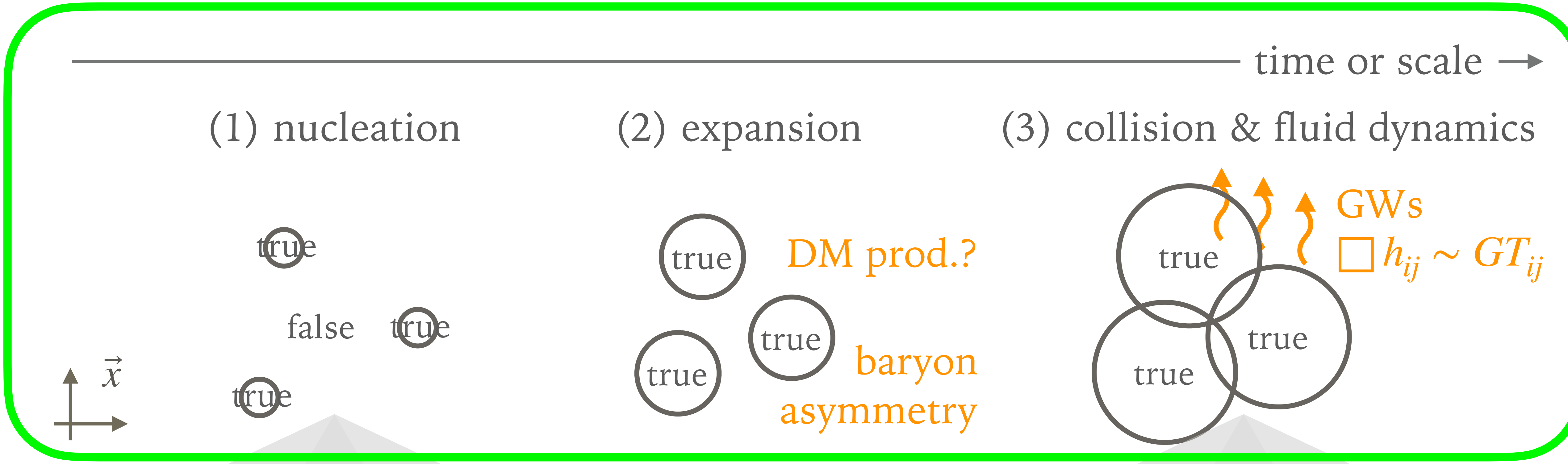
- However, electric dipole moments put stringent constraints

FIRST-ORDER PHASE TRANSITIONS IN THE EARLY UNIVERSE

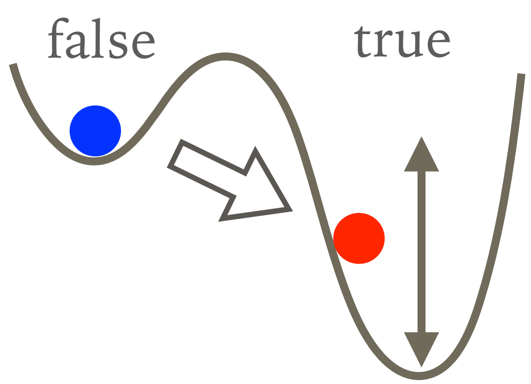
microphysics

Dynamics of bubbles

macrophysics



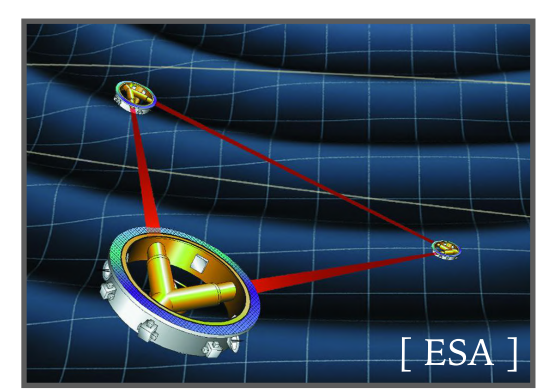
Physics of the Higgs sector



FOPTs in BSM

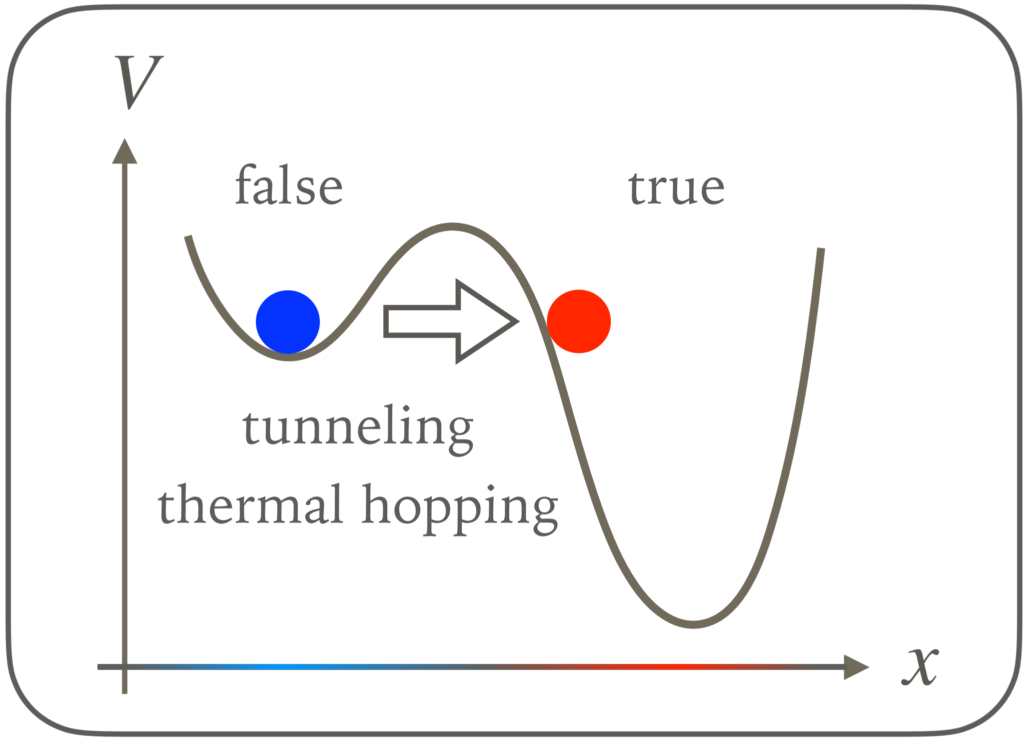
GWs

GW observations

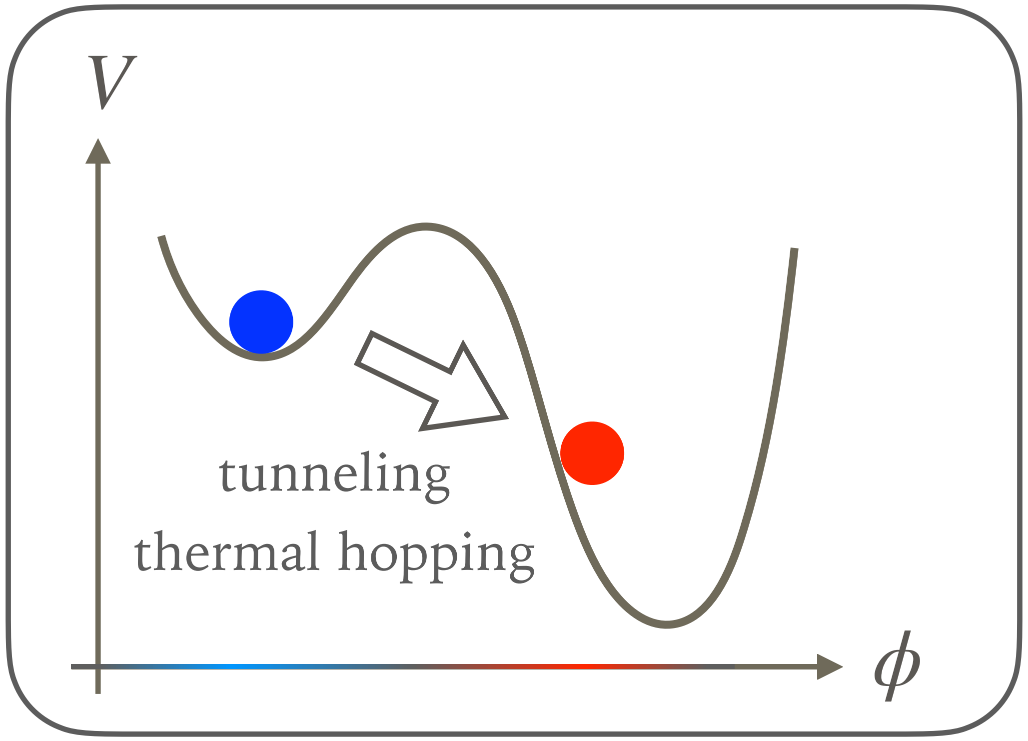


TUNNELING IN QUANTUM MECHANICS AND QFT

Quantum mechanics



Quantum field theory



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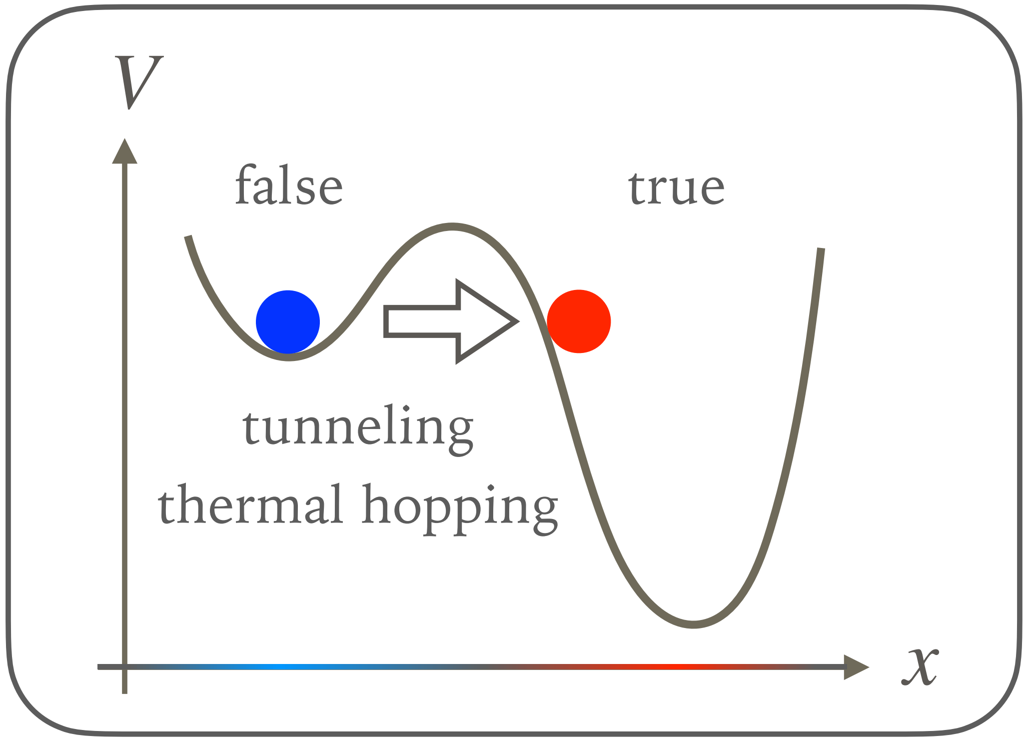

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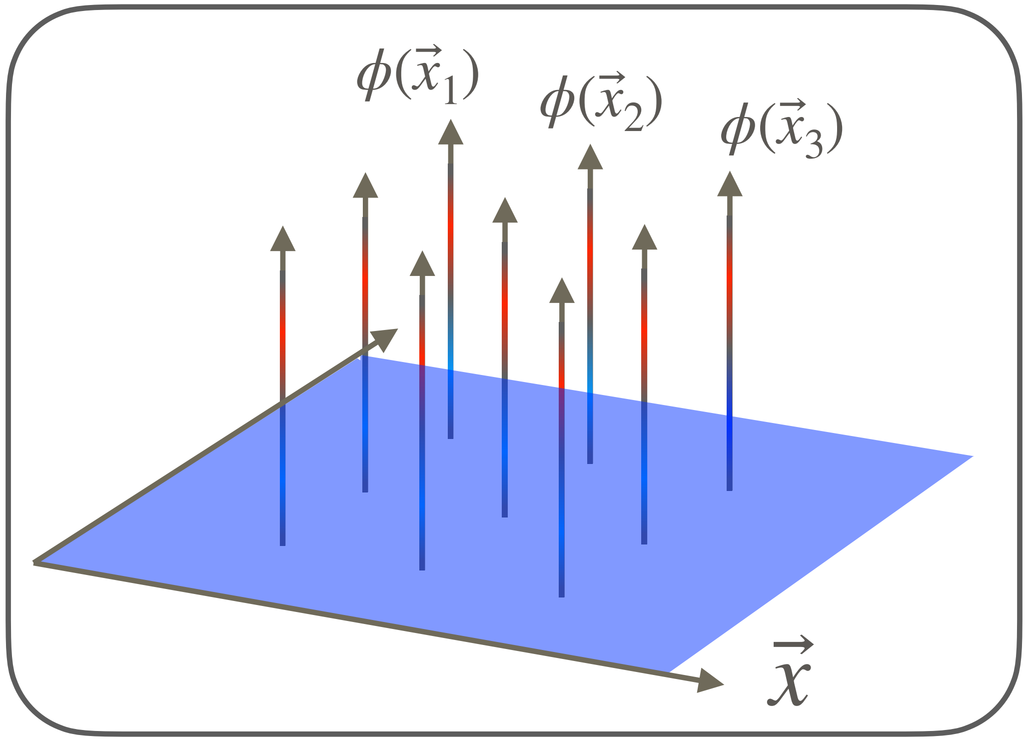

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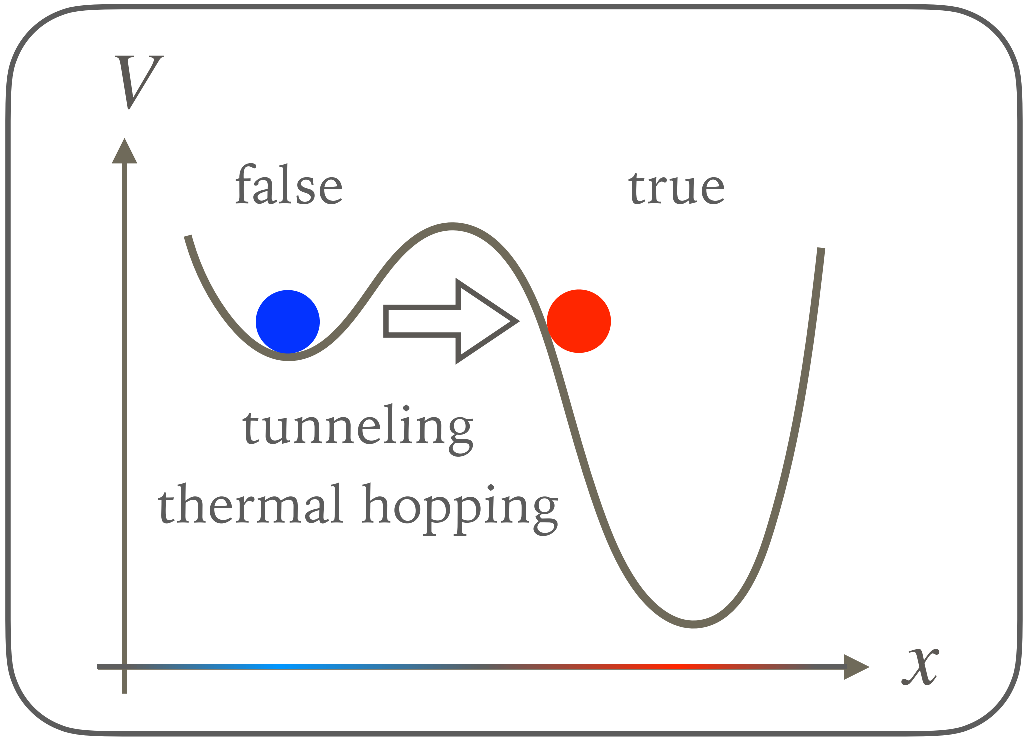

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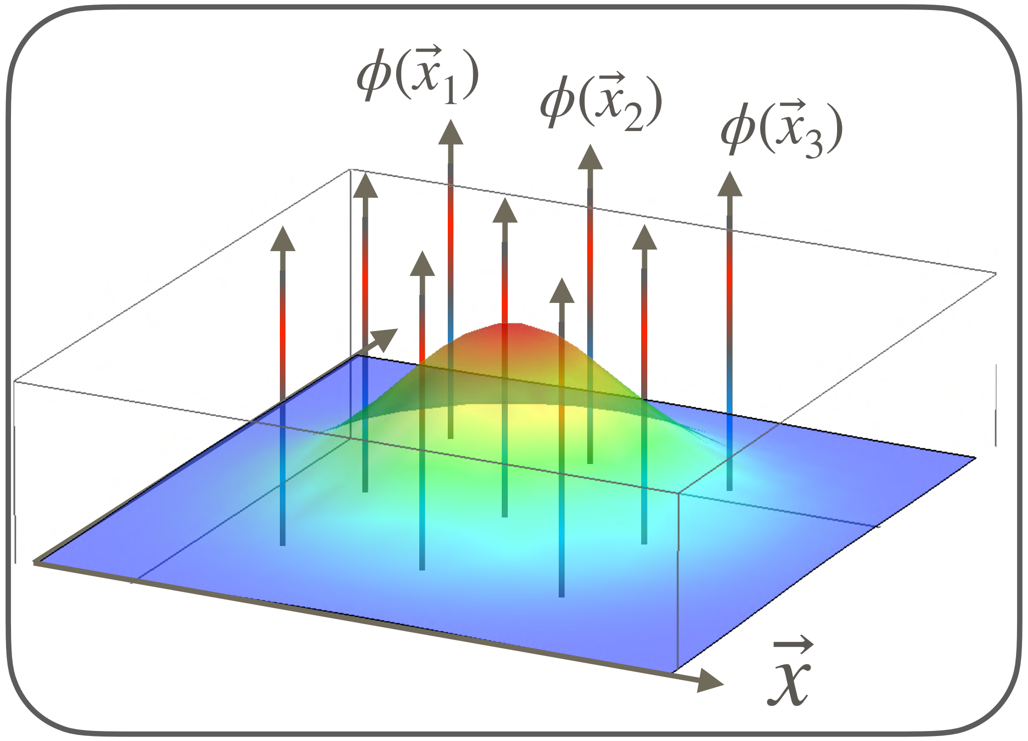
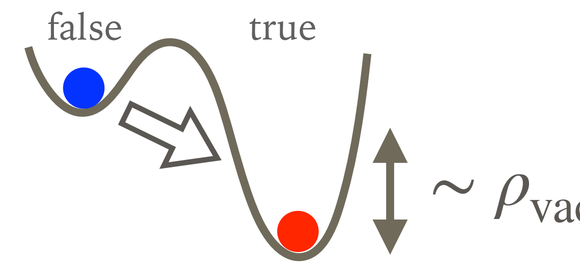


nucleation (核生成)

BUBBLE EXPANSION



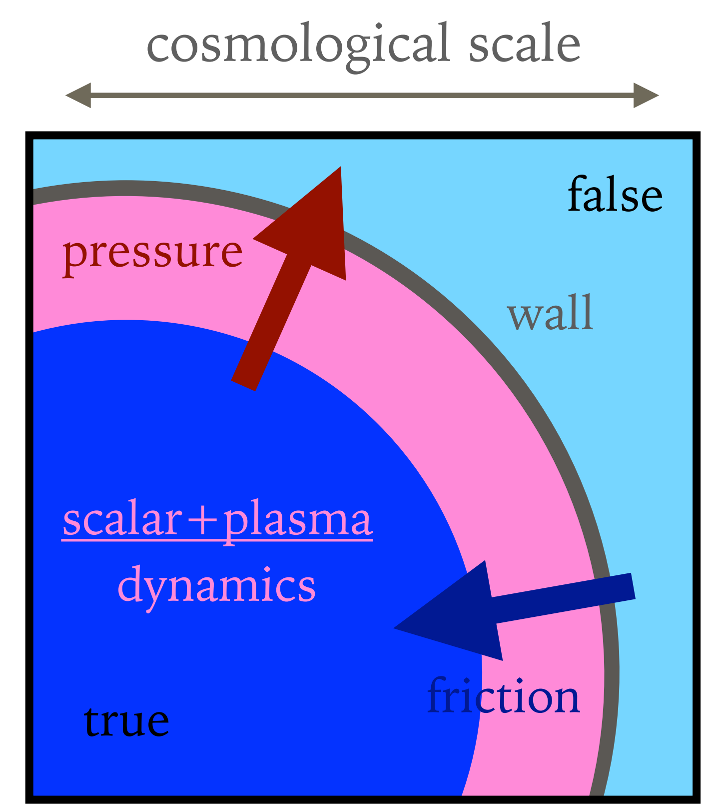
➤ "Pressure vs. Friction" determines the behavior:

(1) Pressure: wall is pushed by the released energy

Determined by $\alpha \equiv \rho_{\rm vac}/\rho_{\rm plasma}$

see e.g. [Espinosa et al. '10, Hindmarsh et al. '15, Giese et al. '20]

(2) Friction: wall is pushed back by plasma particles



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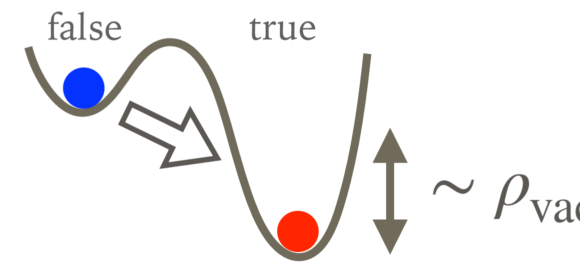

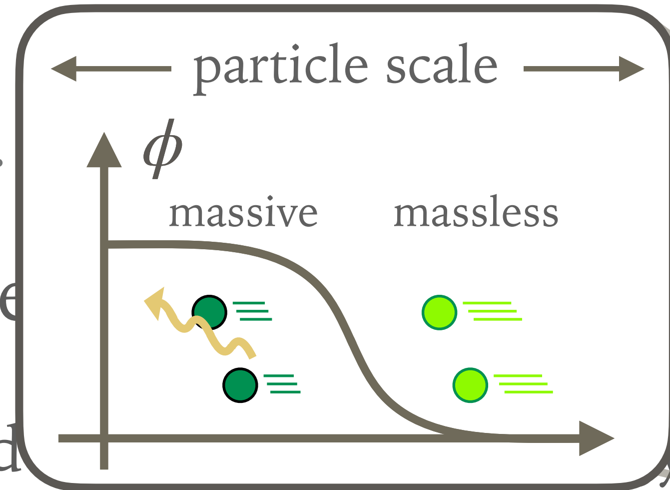

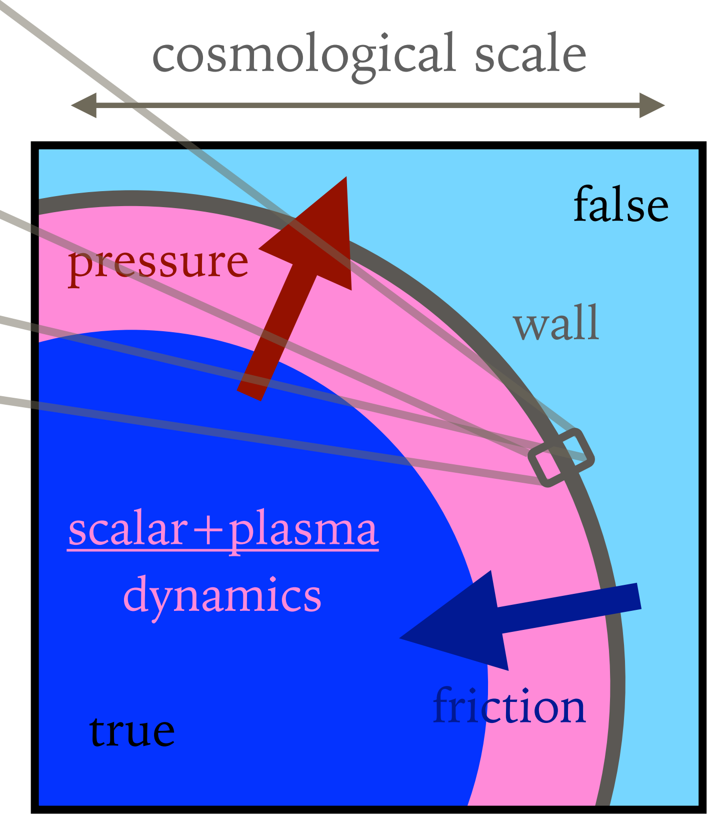

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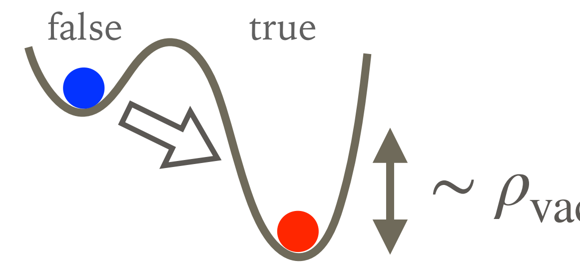

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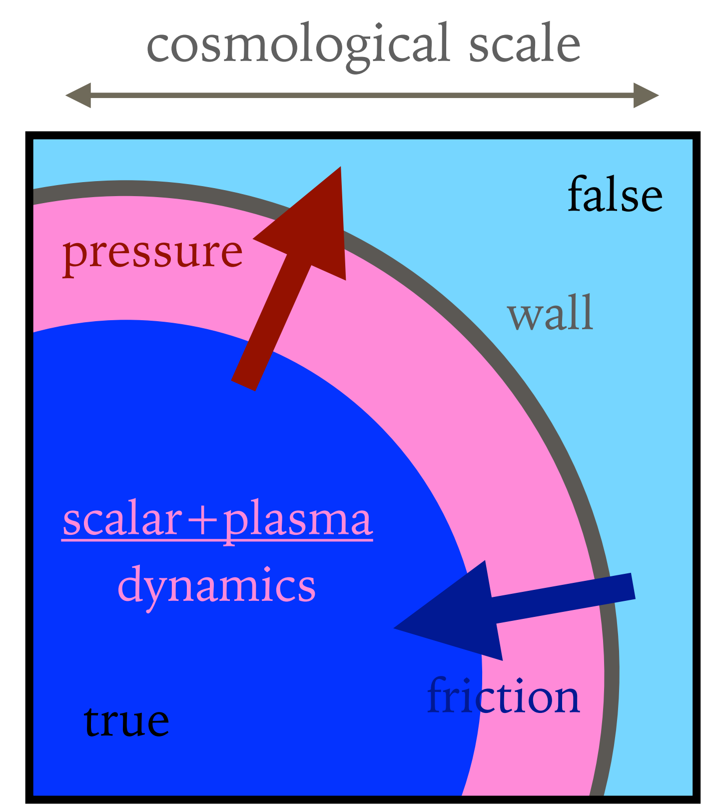
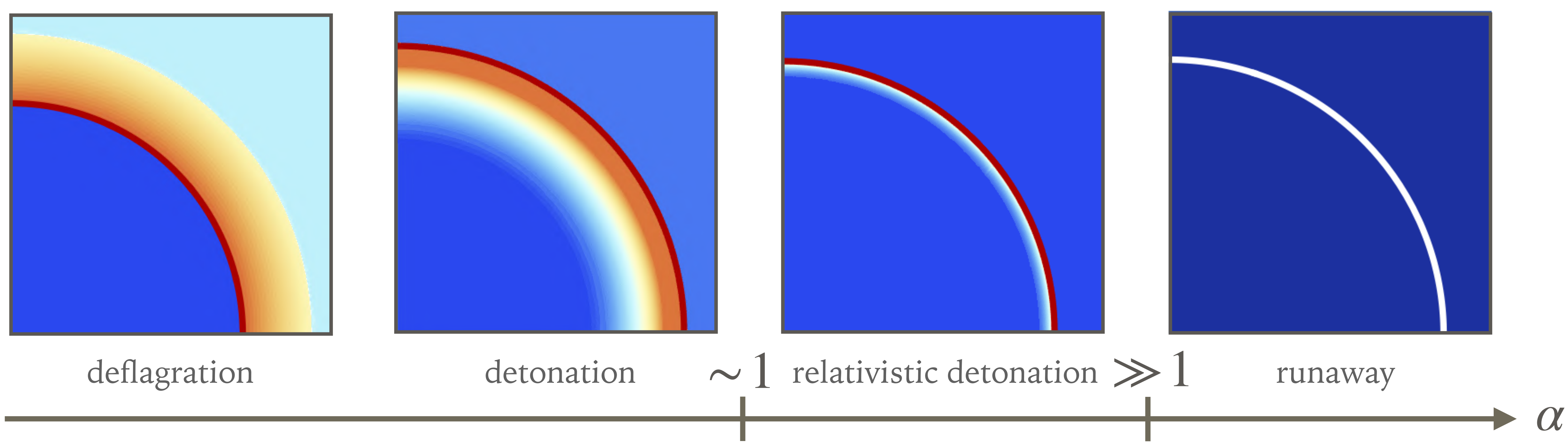


- Different types of bubble expansion



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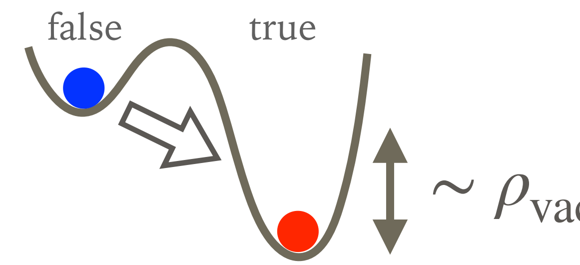
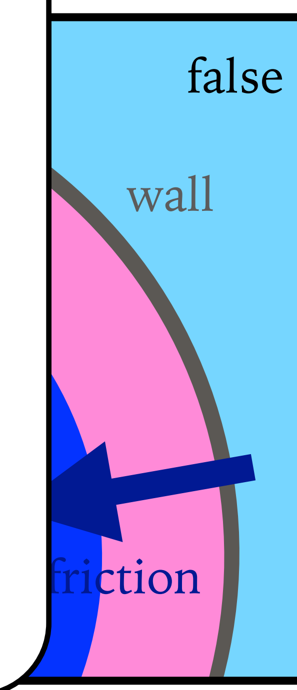


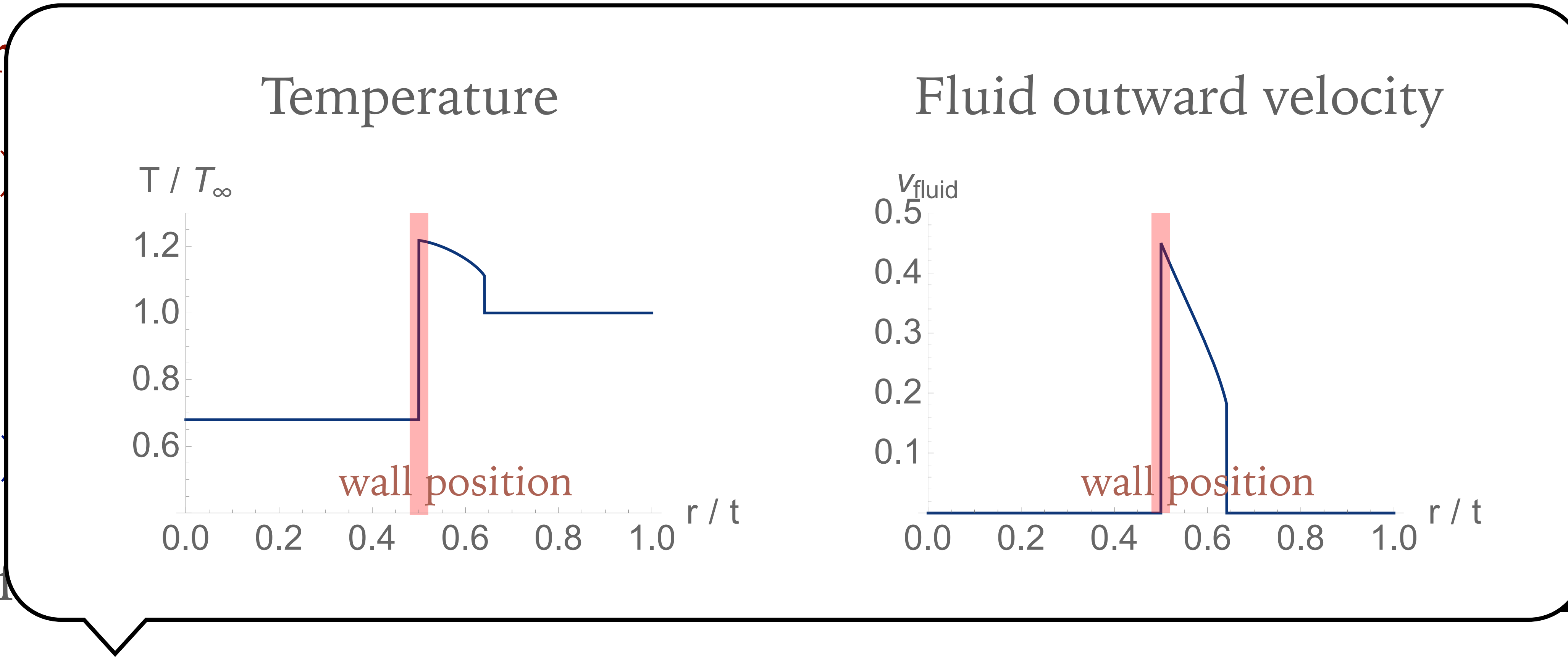
➤ "Pr

(1)

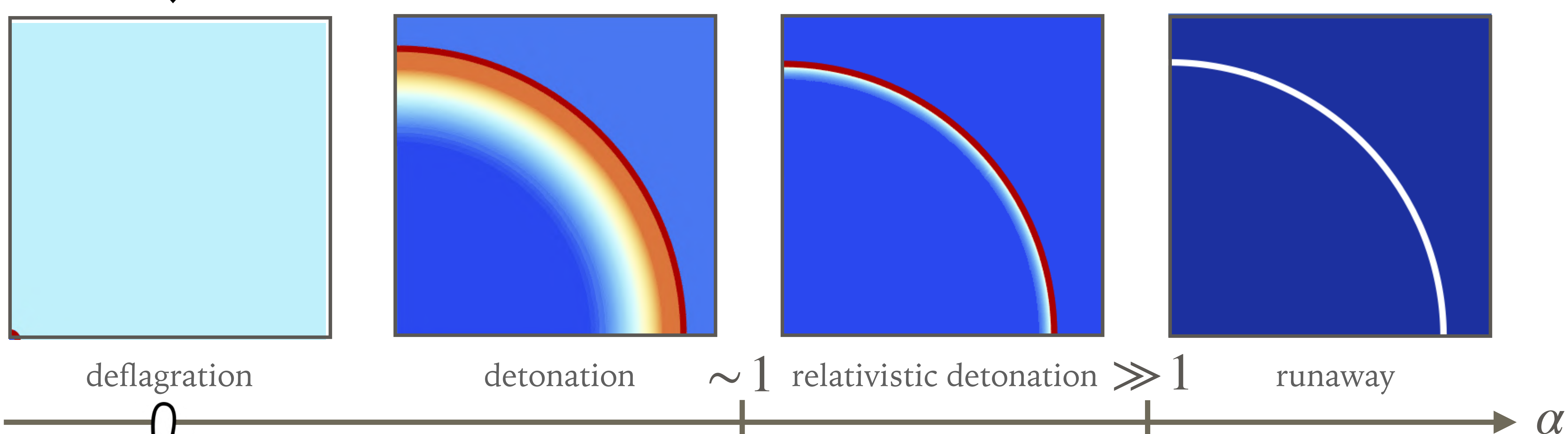
(2)

l scale





➤ Di



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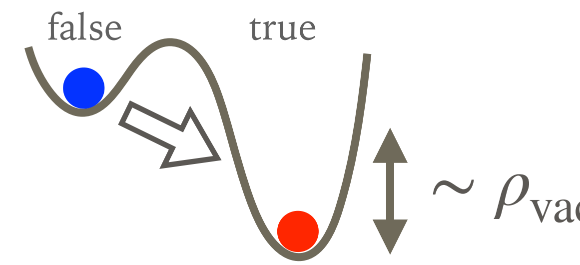

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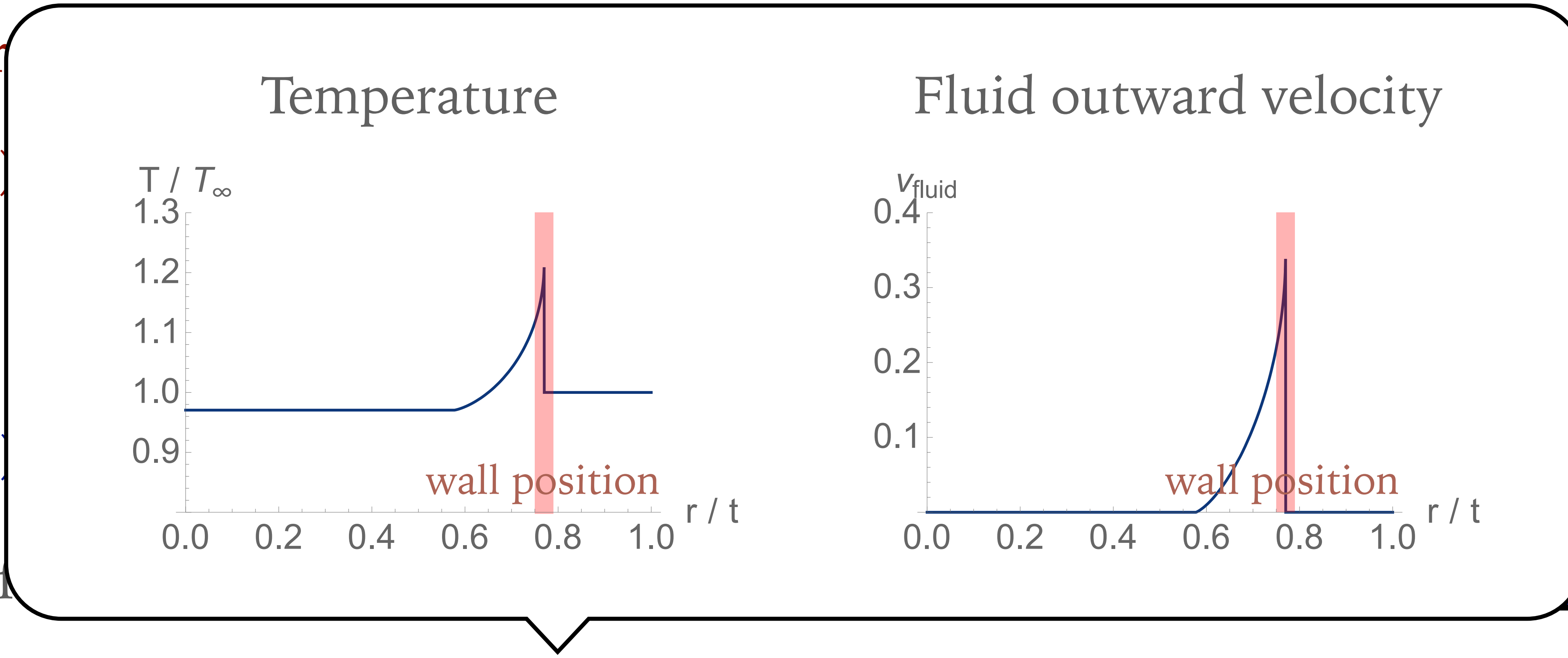

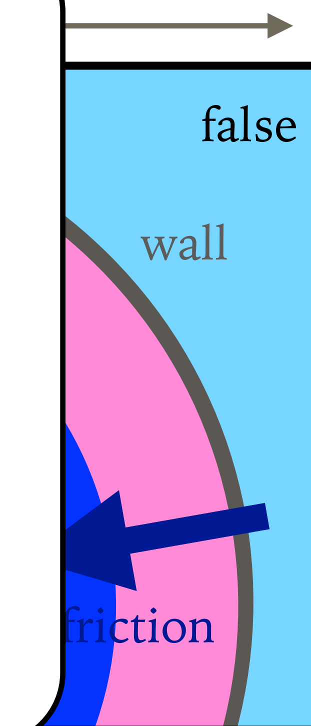

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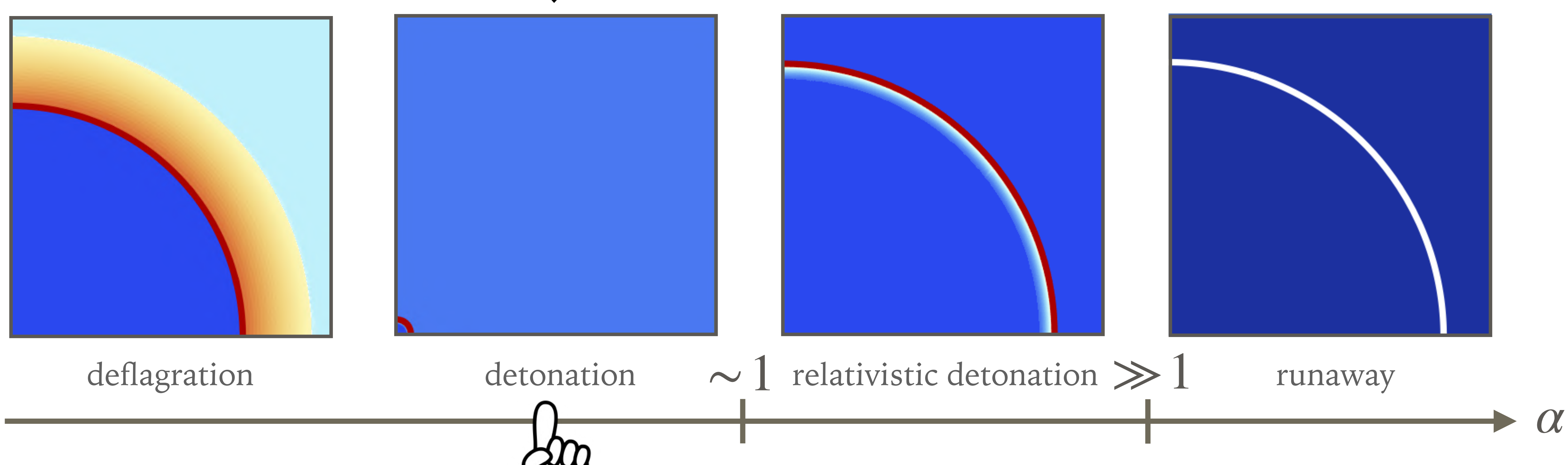

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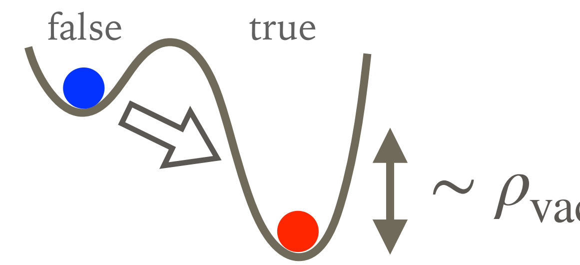

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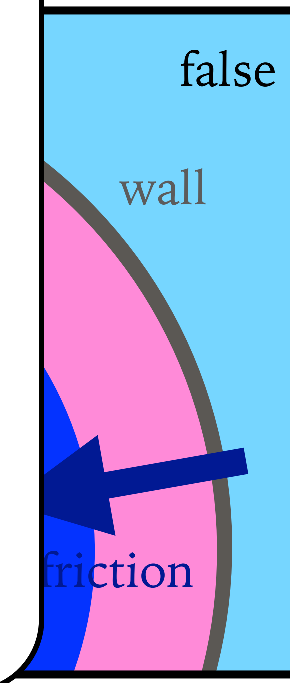

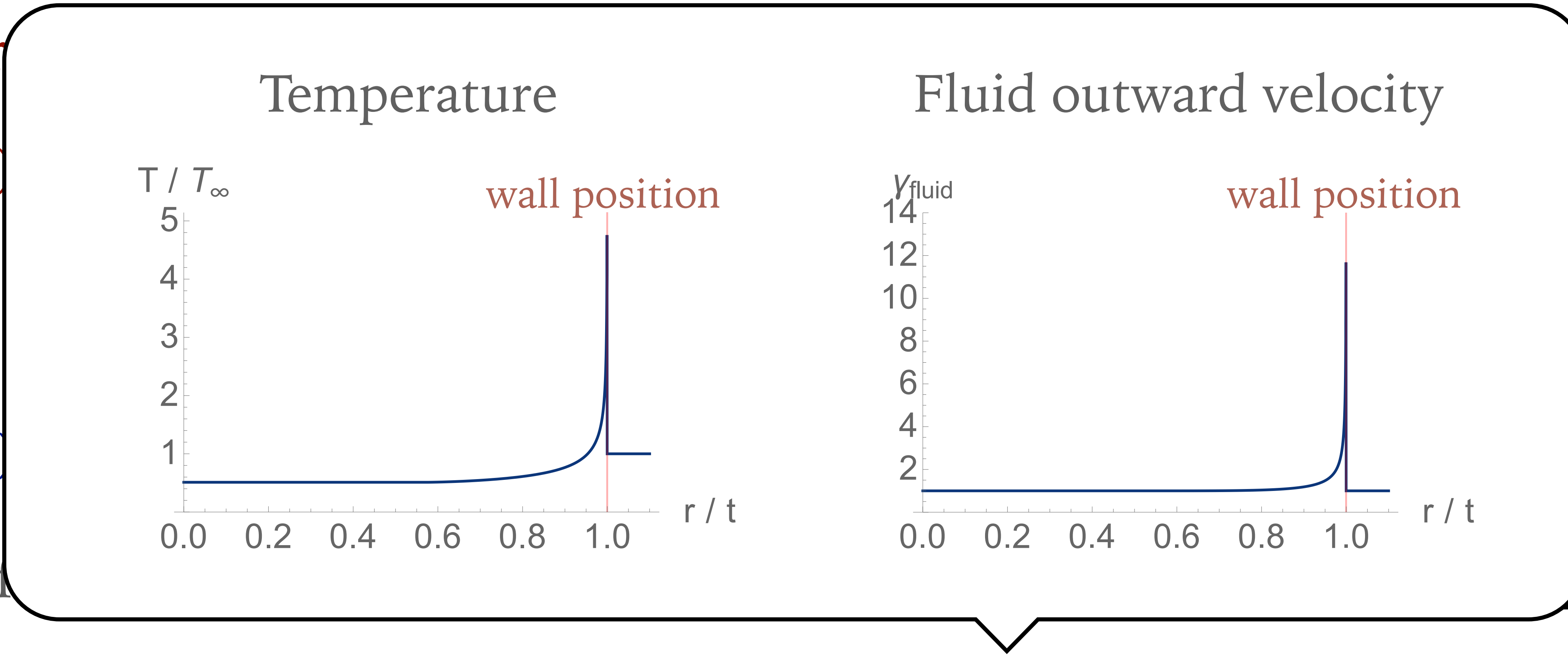

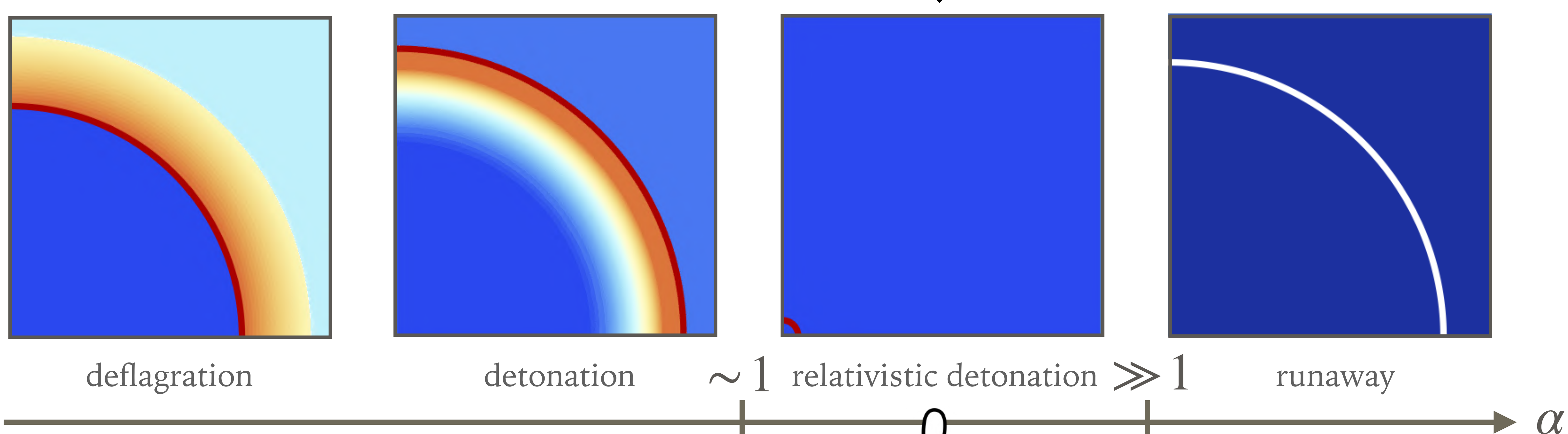

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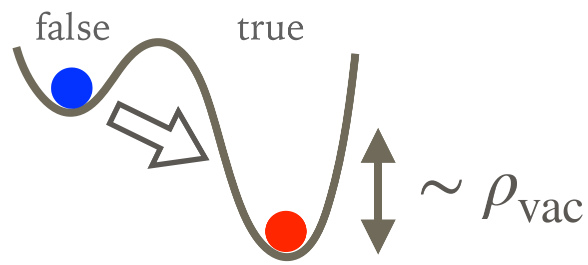


➤ "Pr

(1)

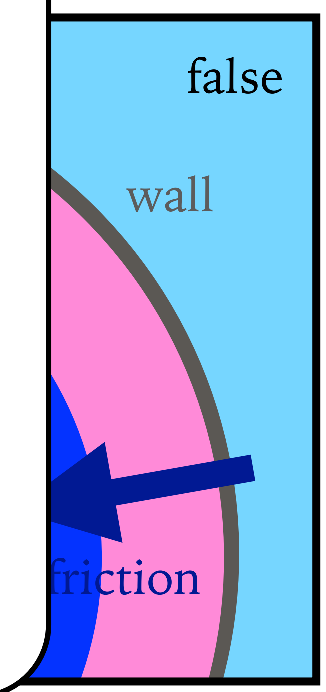
(2)

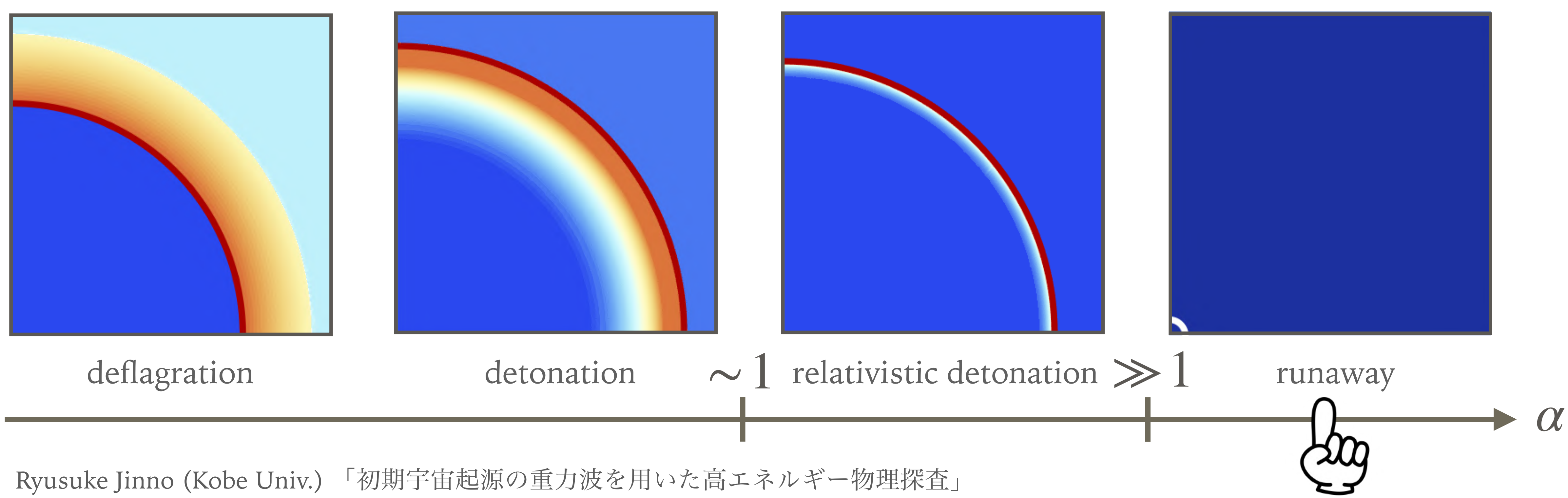
➤ Di

Plasma particles cannot stop the acceleration of the walls:

walls continue to accelerate until they collide with others

[Bodeker & Moore '09, '17]

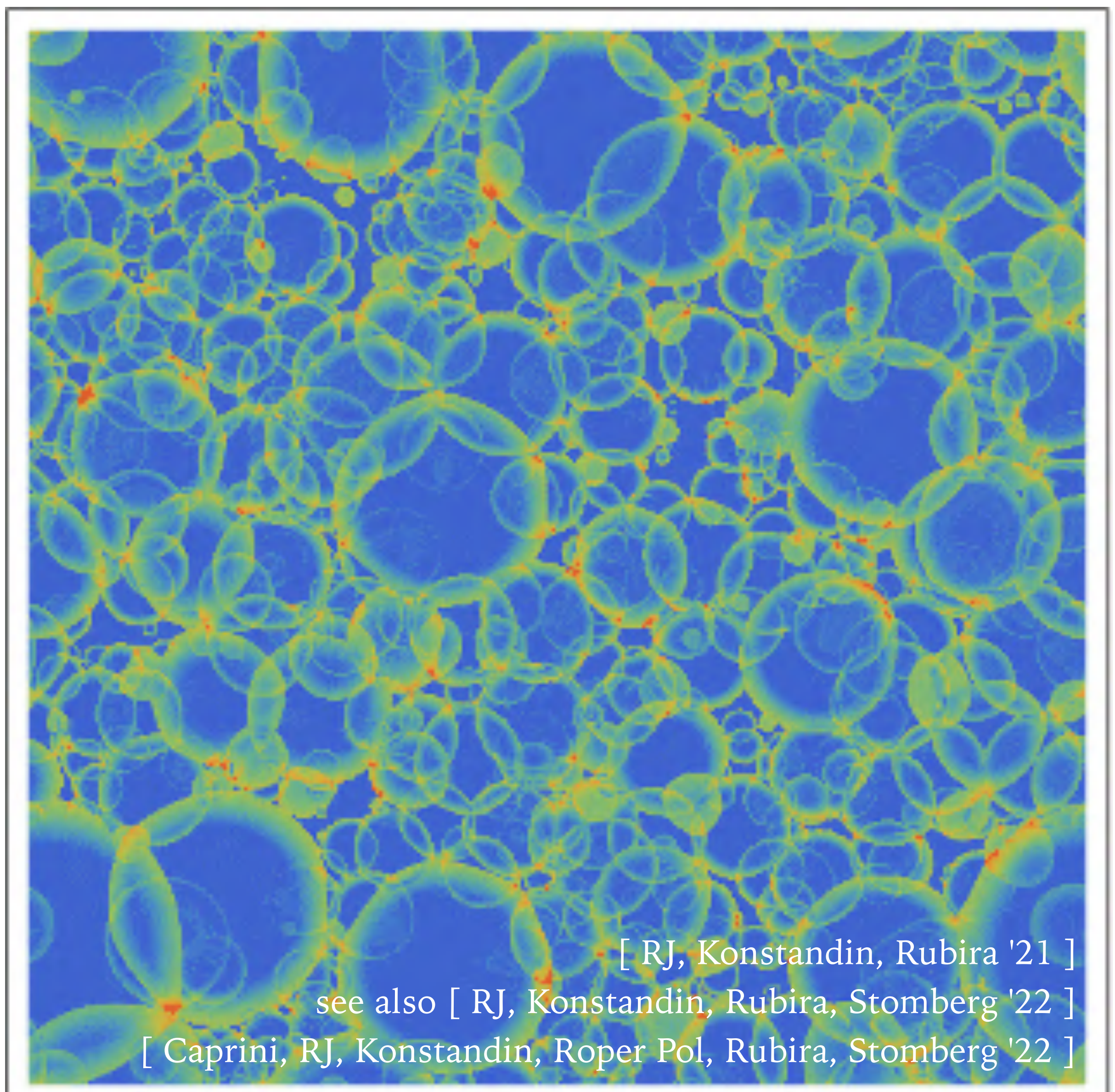





Youtube "Explosions: 100 ton test detonation"

BUBBLE COLLISION & FLUID DYNAMICS

- Bubbles collide, and fluid dynamics sets in (example for )



TRANSITION (≒ THERMODYNAMIC) PARAMETERS

- Remind the spirit of thermodynamics

  - Only a few parameters determine macroscopic properties

TRANSITION (≒ THERMODYNAMIC) PARAMETERS

- Remind the spirit of thermodynamics

  - Only a few parameters determine macroscopic properties

- What are parameters that describe the present macroscopic system?

Particle physics → Transition parameters → Prediction on macroscopic outcome

Lagrangian $\mathscr{L}$

α : transition strength

β : nucleation increase rate

v_w : wall velocity

T_* : transition temperature

GW spectrum $\Omega_{\rm GW}$

Baryon asymmetry

PBHs

TRANSITION (≒ THERMODYNAMIC) PARAMETERS

see e.g. [Caprini et al. '16]
[Caprini et al. '20]

- ➤ Transition strength $\alpha \equiv \rho_{\rm vac}/\rho_{\rm plasma}$

  - How much energy (= latent heat) is released, compared to the plasma energy

  - The numerator $\rho_{\rm vac} = \rho_{\rm vac,false} - \rho_{\rm vac,true}$ is calculated from the Helmholtz free energy, through the relation $U = F + TS = F - T\left(\frac{\partial F}{\partial T}\right)_V$ as

$$\rho_{\rm vac,true} = V_{\rm eff}(\phi_{\rm true}, T) - T\left(\frac{\partial V_{\rm eff}(\phi_{\rm true}, T)}{\partial T}\right)$$

$$\rho_{\rm vac,false} = V_{\rm eff}(\phi_{\rm false}, T) - T\left(\frac{\partial V_{\rm eff}(\phi_{\rm false}, T)}{\partial T}\right)$$

  - One might use trace anomaly instead as parametrization

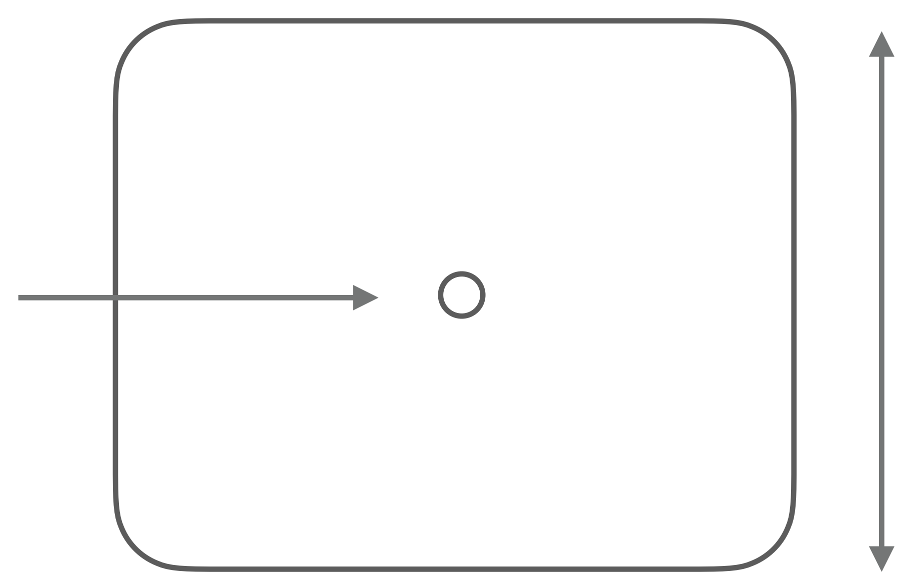
TRANSITION (≒ THERMODYNAMIC) PARAMETERS

see e.g. [Caprini et al. '16]
[Caprini et al. '20]

➤ Nucleation rate increase β : $\Gamma(t) \propto e^{\beta(t-t_*)+\cdots}$

- Calculate $\Gamma(T)$ as a function of temperature, using thermal field theory
- Translate $\Gamma(T)$ into $\Gamma(t)$ using (cosmological temperature) ⇔ (cosmological time)
- Taylor-expand the exponent around the typical transition time $t = t_*$

The first bubble
in the Hubble patch



Hubble radius

$$H^{-1} = \left(\frac{\dot{a}}{a}\right)^{-1}$$

TRANSITION (≒ THERMODYNAMIC) PARAMETERS

see e.g. [Caprini et al. '16]
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Thermal field theory | Cosmology

$$\Gamma(T) \sim T^4 e^{-S_3/T} \quad = \quad H^4 \sim (T^2/M_P)^4$$

TRANSITION (≒ THERMODYNAMIC) PARAMETERS

see e.g. [Caprini et al. '16]
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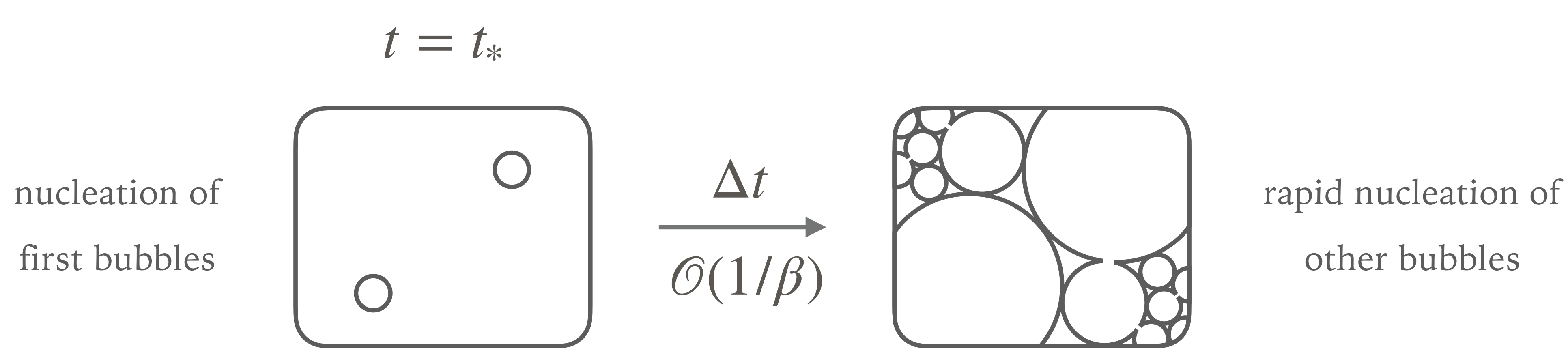
- Taylor-expand the exponent around the typical transition time $t = t_*$
- Interesting property: v_w/β gives the typical bubble size at the time of collision



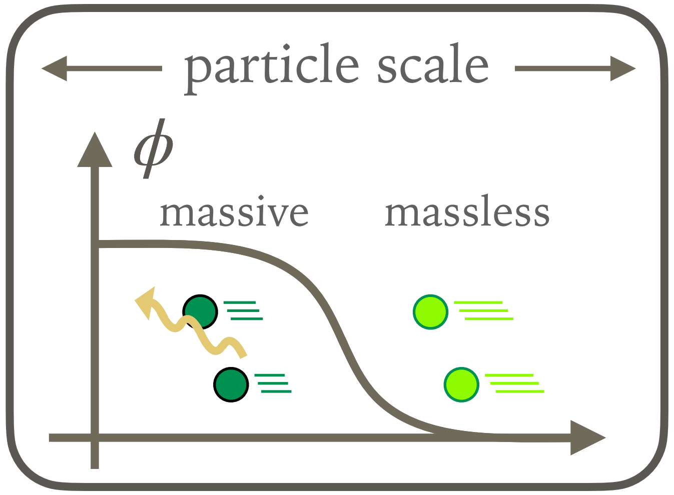
TRANSITION (≒ THERMODYNAMIC) PARAMETERS

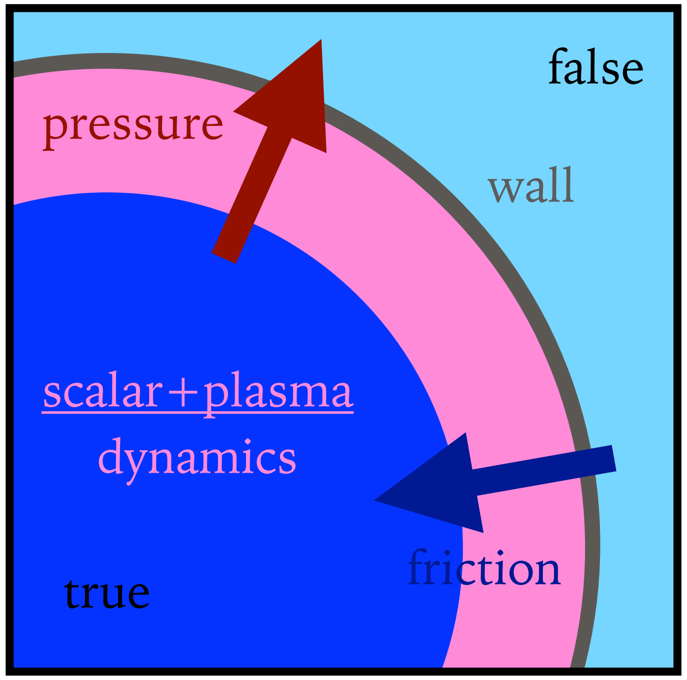
see e.g. [Caprini et al. '16]
[Caprini et al. '20]

➤ Wall velocity v_w

- Determined from "pressure vs. friction"

- In principle one should solve Boltzmann eq.,
  but people often put by hand
  (regarded as trade-off btwn. coupling ⇔ velocity)





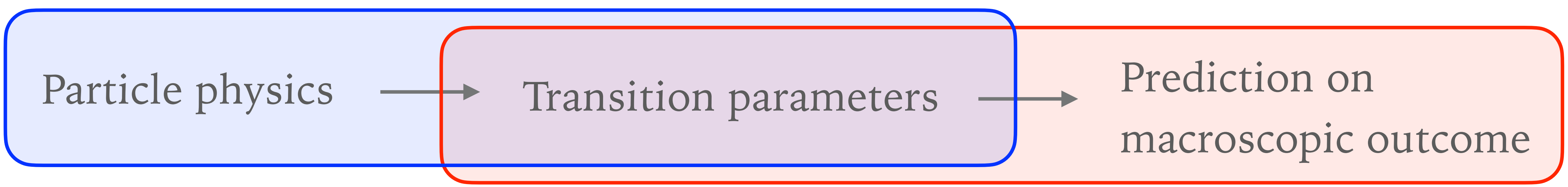
➤ Transition temperature T_*

- Determined from your microphysical theory

TRANSITION (≒ THERMODYNAMIC) PARAMETERS

- ➤ Remind the spirit of thermodynamics
  - Only a few parameters determine macroscopic properties

- ➤ What are parameters that describe the present macroscopic system?



Lagrangian $\mathscr{L}$

α : transition strength

β : nucleation increase rate

v_w : wall velocity

T_* : transition temperature

GW spectrum $\Omega_{\rm GW}$

Baryon asymmetry

PBHs

1. Overview
2. First-order phase transitions
3. Dynamics of bubbles
4. Gravitational waves
5. Recent progress

GRAVITATIONAL WAVES: A NEW PROBE TO THE UNIVERSE

- Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

"Spacetime tells matter how to move. Matter tells spacetime how to curve."

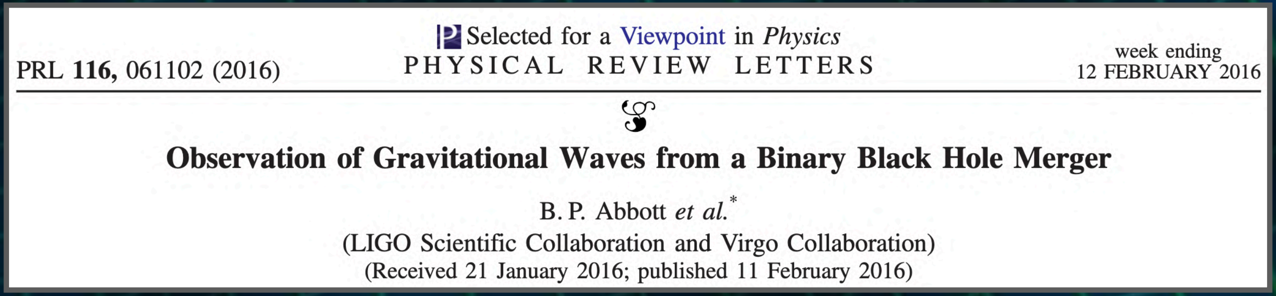
- Gravitational waves: transverse-traceless part of the metric

$$ds^2 = -dt^2 + a^2(\delta_{ij}+h_{ij})dx^i dx^j \qquad \partial_i h_{ij} = h_{ii} = 0$$

- After expanding the Einstein equation, GWs obey a wave equation sourced by the energy-momentum tensor of the system

$$\Box h_{ij} = 16\pi G \Lambda_{ij,kl} T_{kl}$$

- LIGO/Virgo detected GWs from binary black holes for the first time in 2015



Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]

(LIGO Scientific Collaboration and Virgo Collaboration)

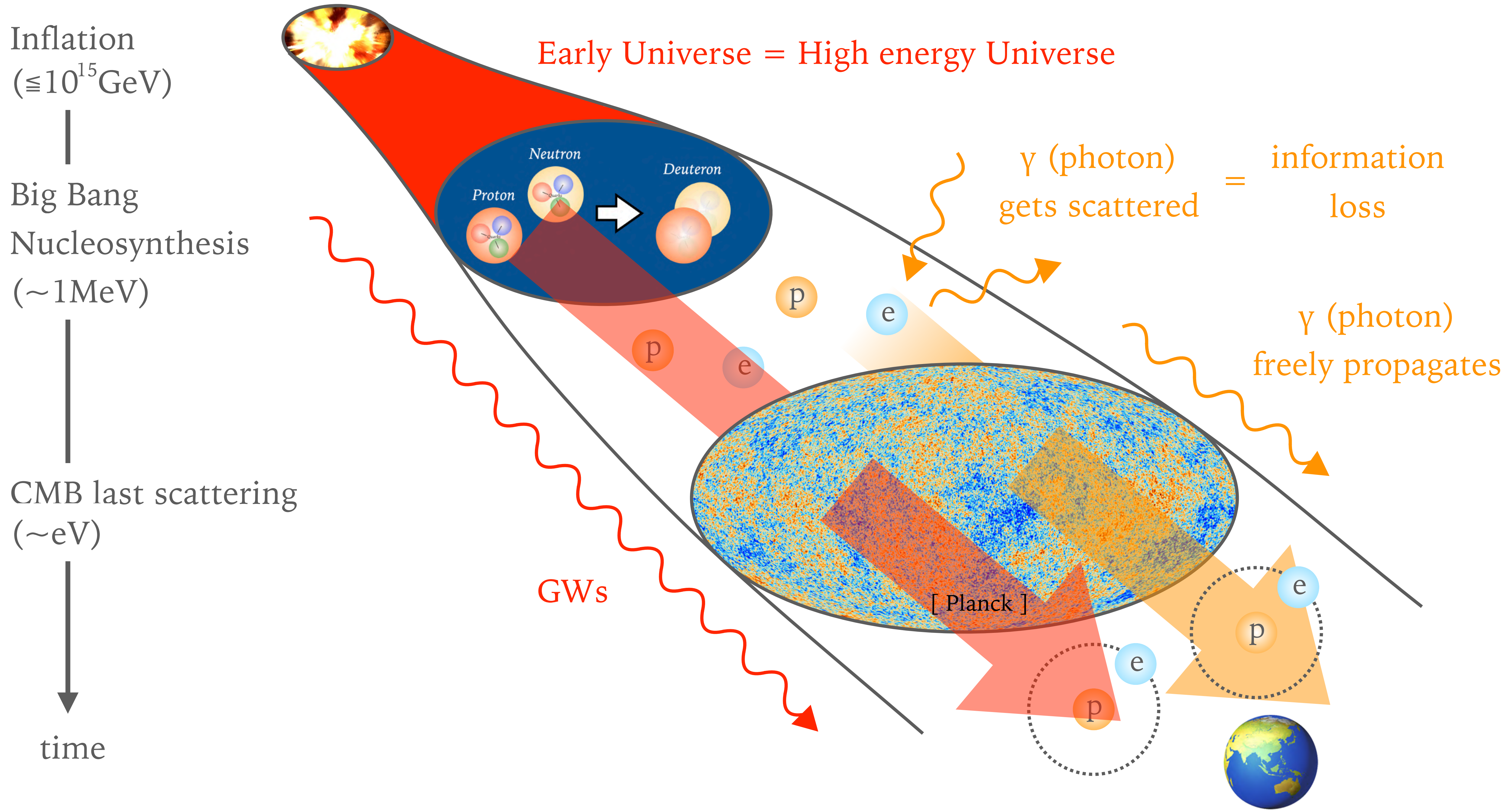
(Received 21 January 2016; published 11 February 2016)

$$36M_\odot + 29M_\odot \rightarrow 62M_\odot + 3M_\odot \text{ (GWs)}$$

[R. Hurt/Caltech-JPL]

GWS AS A PROBE OF THE EARLY UNIVERSE

➤ Comparison between CMB and GWs



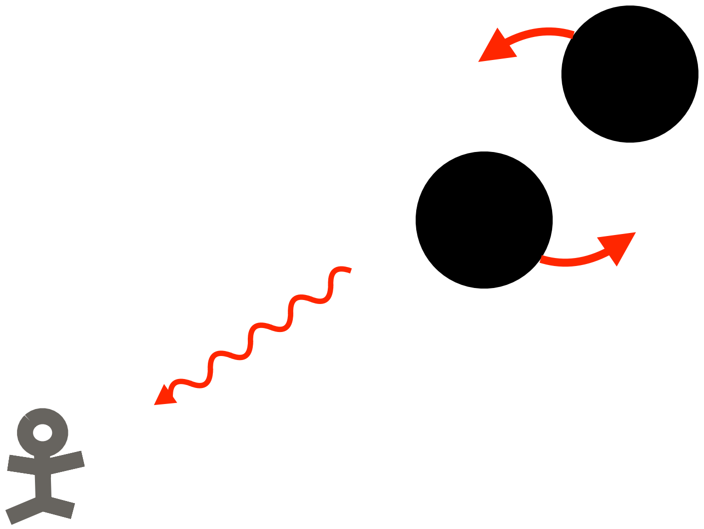
天文学的重力波 VS. 宇宙論的重力波

- 天文学的重力波 = 点源からの重力波

  - 決まった方向

  - 狭い周波数帯

  - 検出器がそれぞれのソースを
    特定できないときに限り、stochasitcに見える see e.g. [Romano, Cornish '17] for further discussion

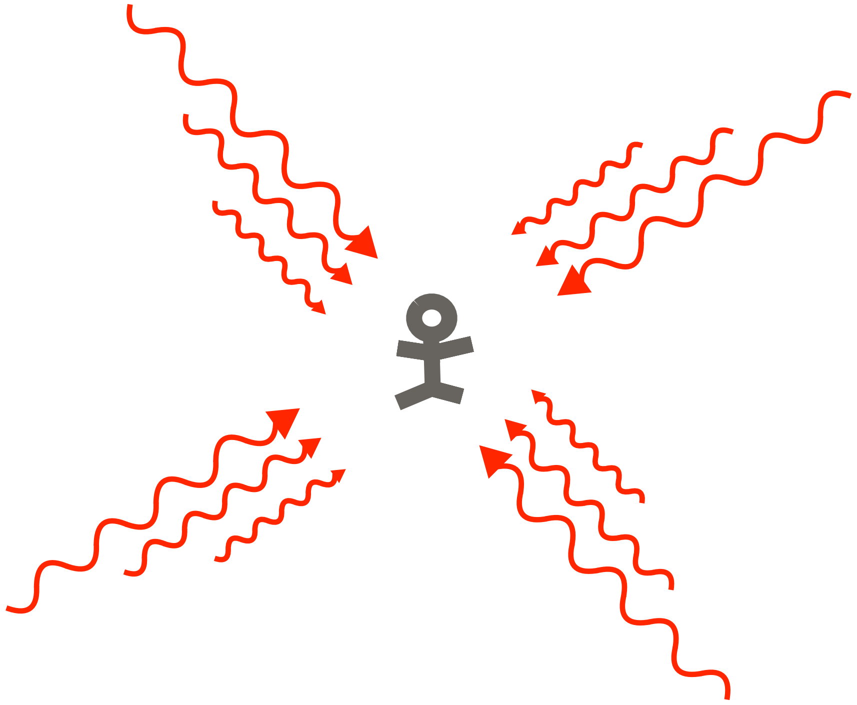


- 宇宙論的重力波 = stochastic

  - ランダムな方向

  - 幅広い周波数帯

  - パワースペクトルにより特徴付けられる



PRESENT & FUTURE OBSERVATIONS

Pulsar timing arrays

$\sim 10^{-8}$ Hz

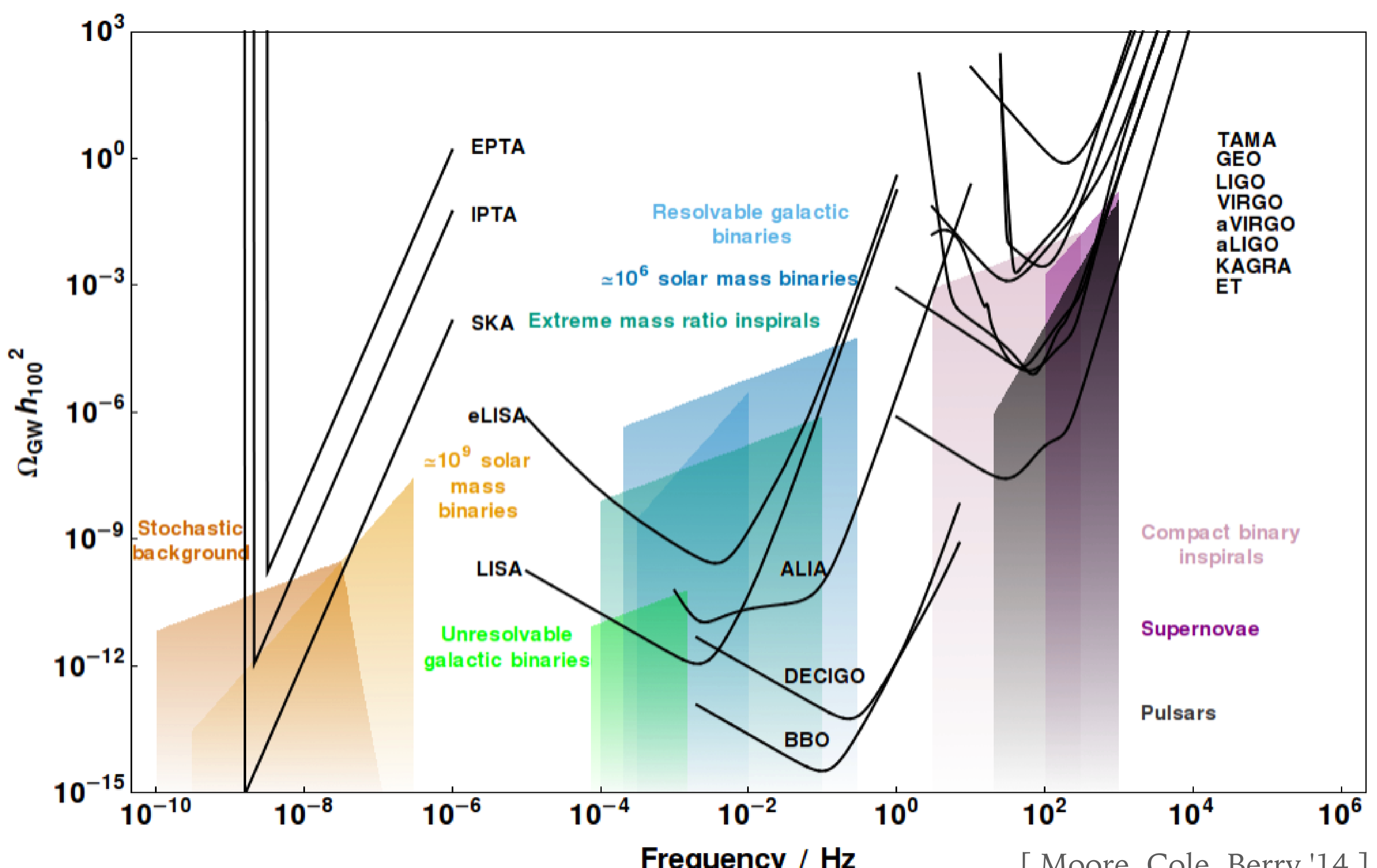
Space-borne interferometers

$\sim$ mHz – Hz

Ground-based interferometers

~ 100 Hz

$\frac{\text{GW energy density per unit log freq.}}{\text{total energy density of the Universe}}$

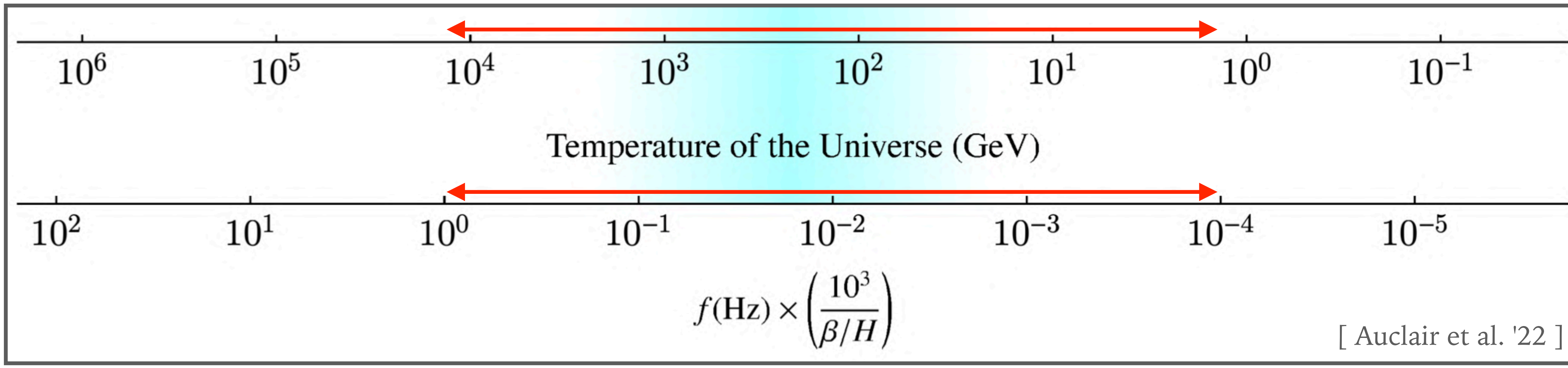


[Moore, Cole, Berry '14]

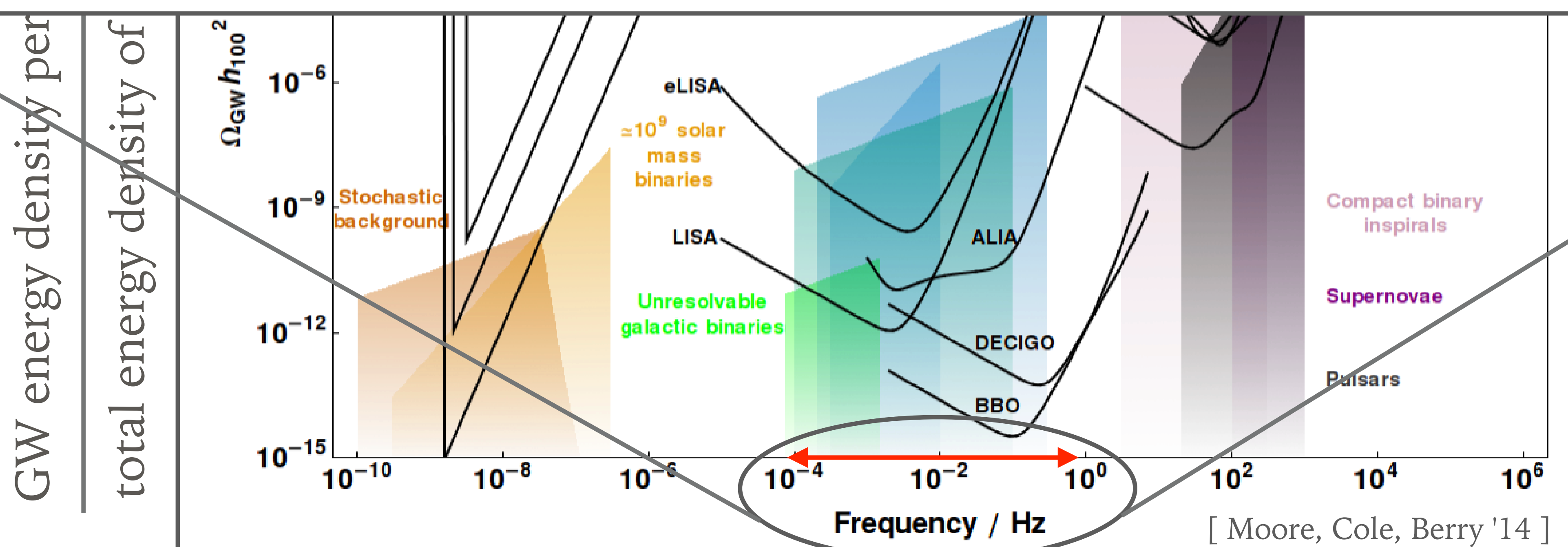
Present frequency of cosmological GWs

$\propto$ Energy scale (temperature) at the time of production

TeV scale physics

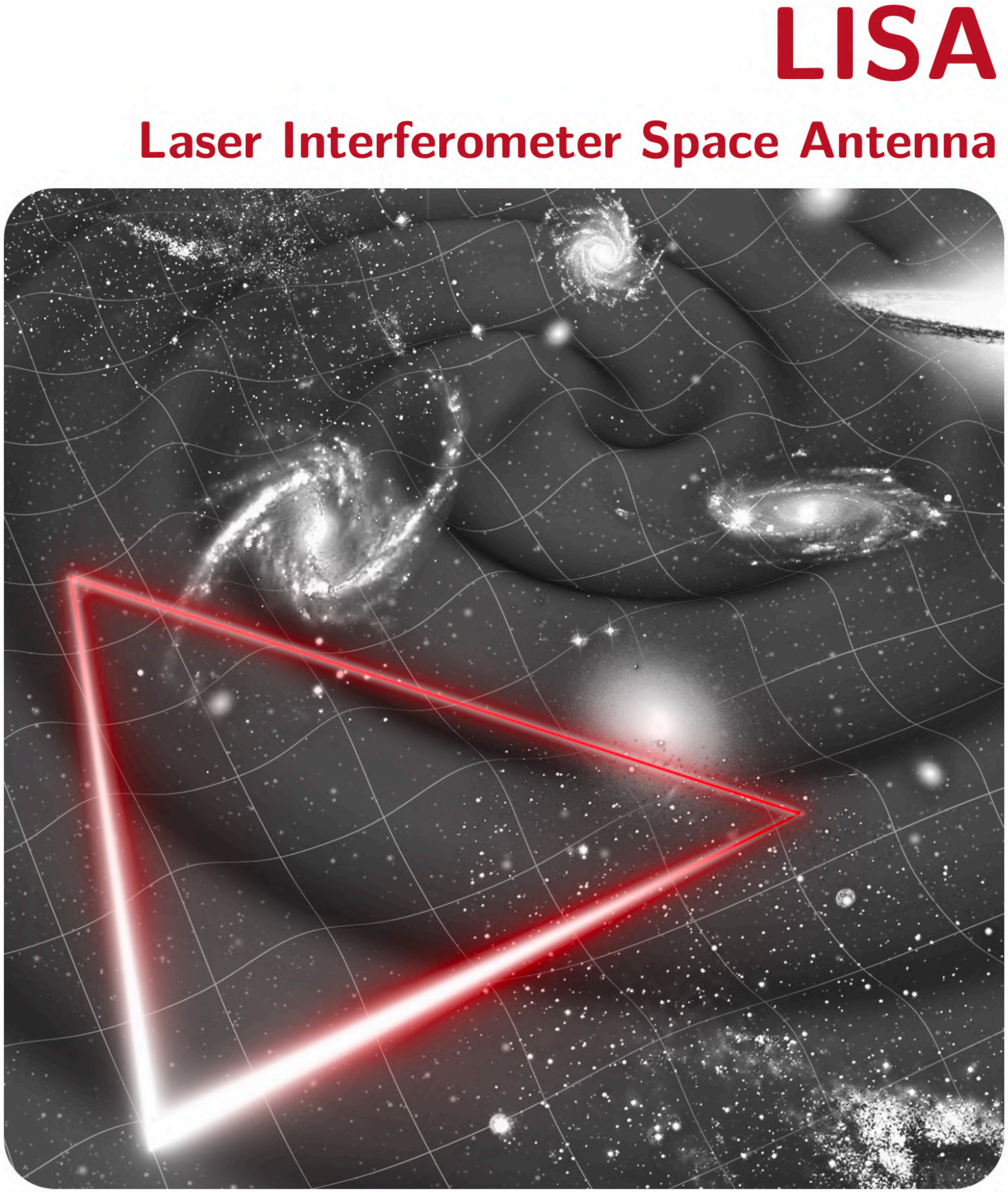


[Auclair et al. '22]



[Moore, Cole, Berry '14]

LISA MISSION



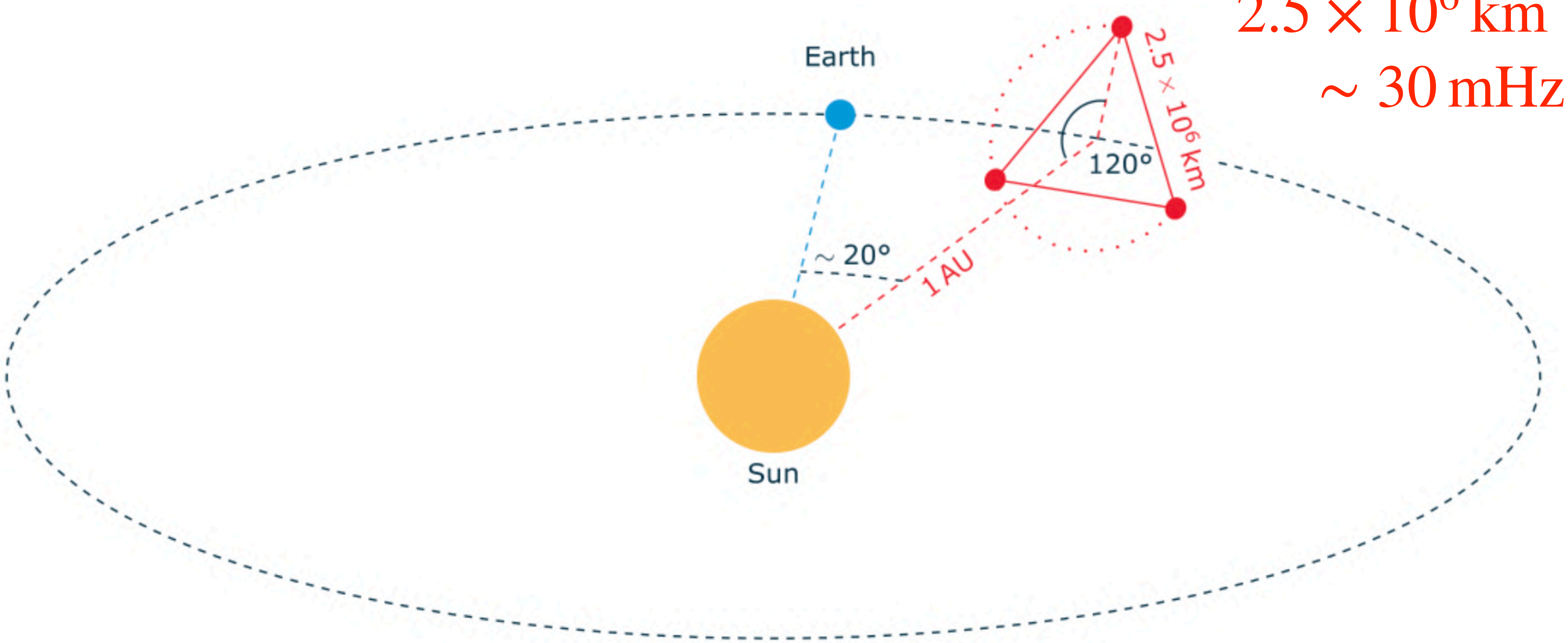
LISA MISSION

LISA MISSION SUMMARY

Science Objectives

- Study the formation and evolution of **compact binary stars** and the structure of the Milky Way Galaxy
- Trace the origins, growth and merger histories of **massive Black Holes** across cosmic epochs
- Probe the properties and immediate environments of Black Holes in the local Universe using **extreme mass-ratio inspirals** and **intermediate mass-ratio inspirals**
- Understand the astrophysics of **stellar-mass Black Holes**
- Explore the **fundamental nature of gravity** and Black Holes
- Probe the rate of **expansion of the Universe** with standard sirens
- Understand **stochastic gravitational wave backgrounds** and their implications for the early Universe and TeV-scale particle physics
- Search for gravitational wave bursts and **unforeseen sources**

LISA MISSION: OUTLINE



2.5×10^6 km に決定

~ 30 mHz

Figure 2.3: Schematic depiction of the LISA orbit, not to scale. The three satellites are arranged in a equilateral triangle, the constellation barycentre follows a heliocentric orbit lagging or leading approximately 20°, or about 50×10^6 km, behind Earth. The plane of the constellation (marked with the dotted line) is inclined at 60° with respect to the Ecliptic and the triangular array undergoes an annual rotation within the plane. See Chapter 6 for further details.

LISA MISSION: OUTLINE

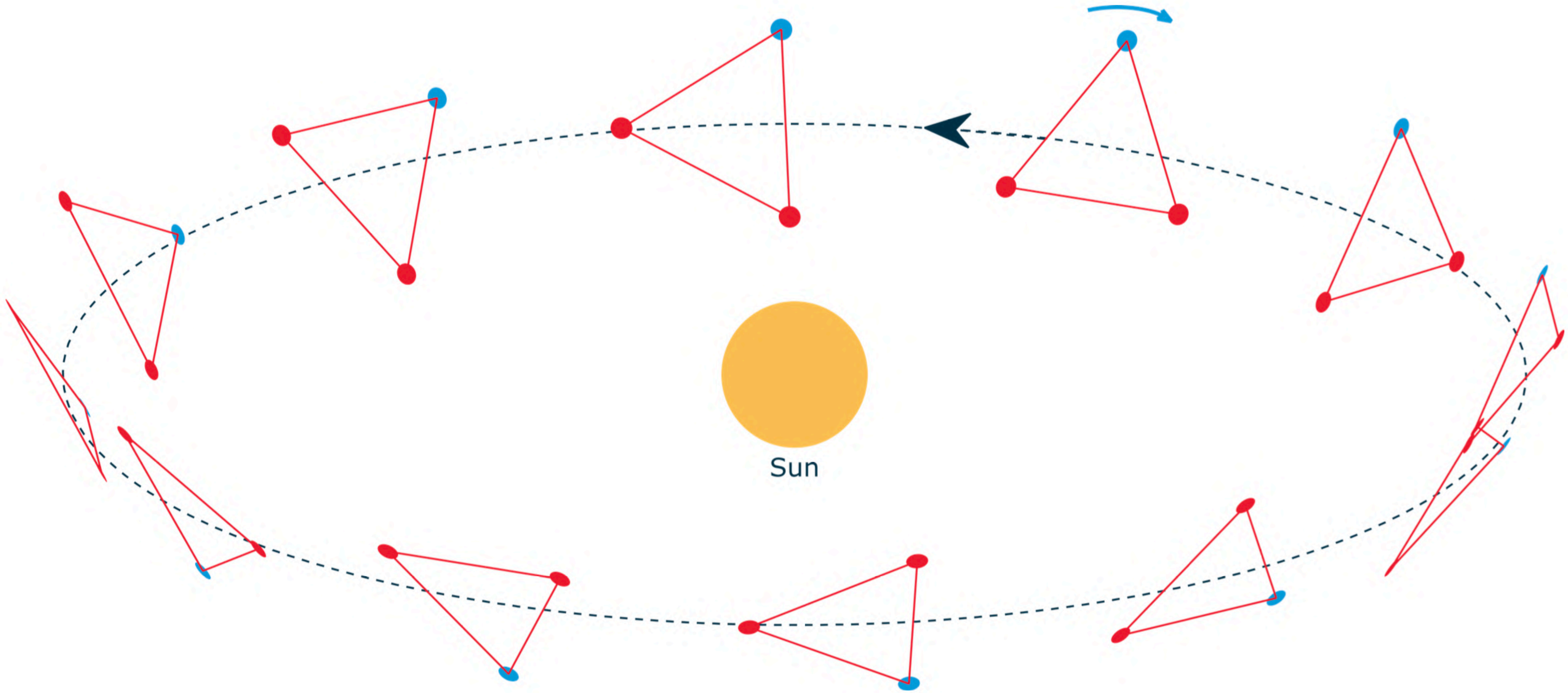
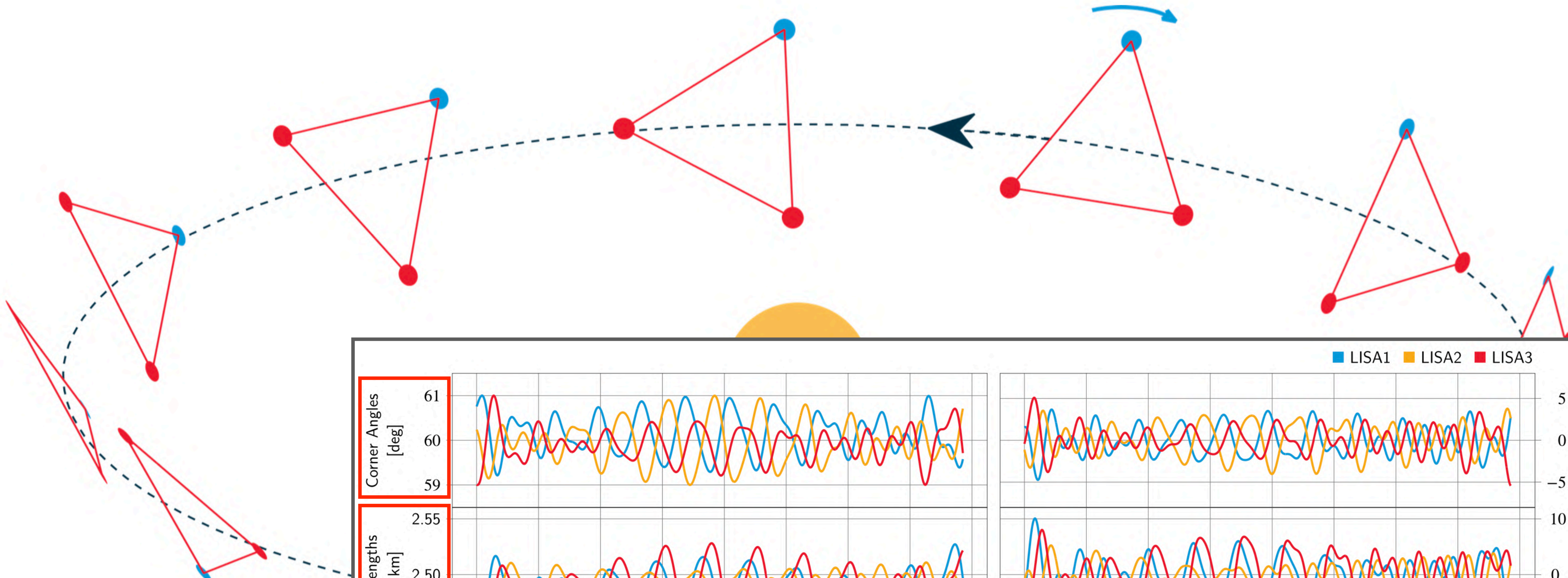


Figure 6.3: LISA orbits for the three individual spacecraft. The centre of mass of the constellation follows a heliocentric, circular orbit with no inclination. The orbits of the spacecraft have a small eccentricity $e = \eta/(2\sqrt{3}$ and an inclination of $\iota = \arcsin(\eta/2)$ where $\eta = L/1\,\mathrm{AU} \approx 0.017\,(L/2.5 \times 10^6\,\mathrm{km})$ is the armlength measured in AU. The 60° inclination of the constellation results in a backwards rotation of the constellation (blue arrow).

LISA MISSION: OUTLINE



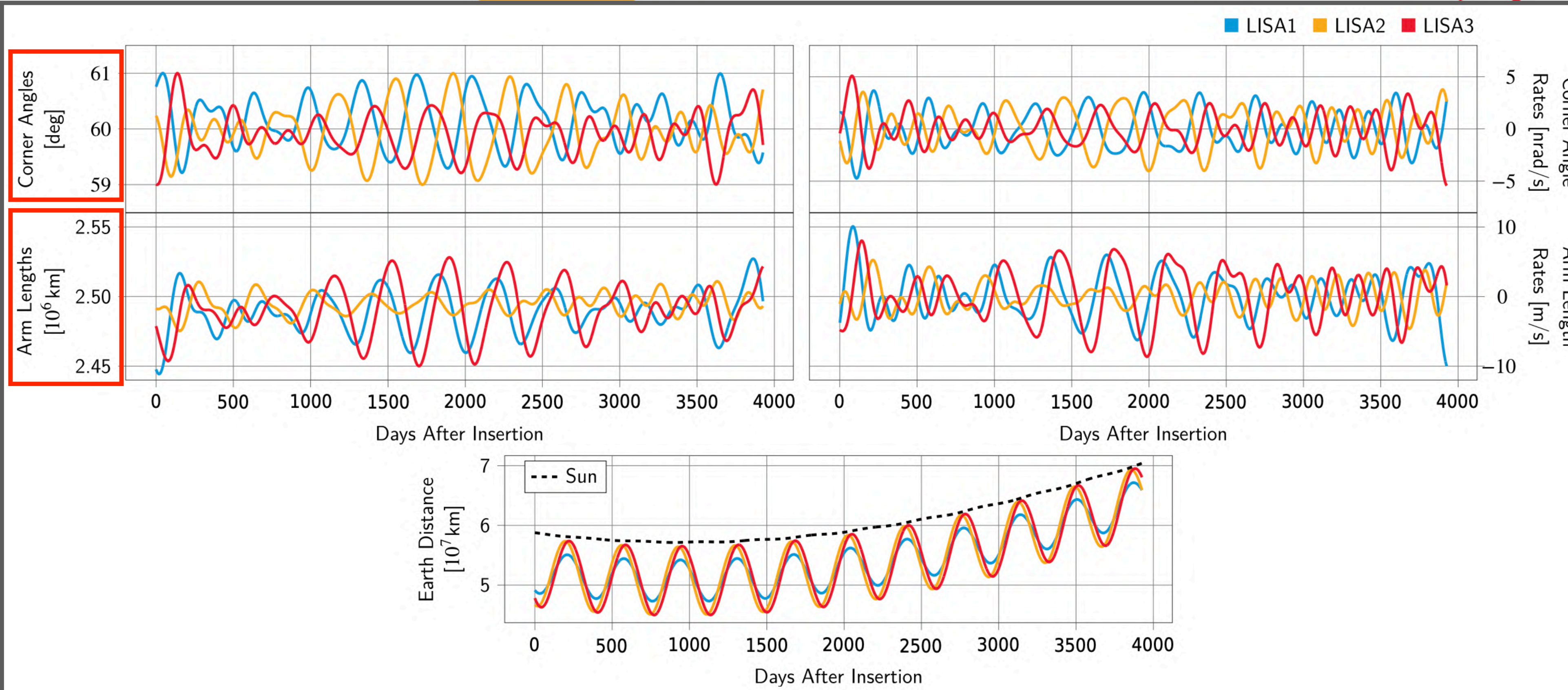
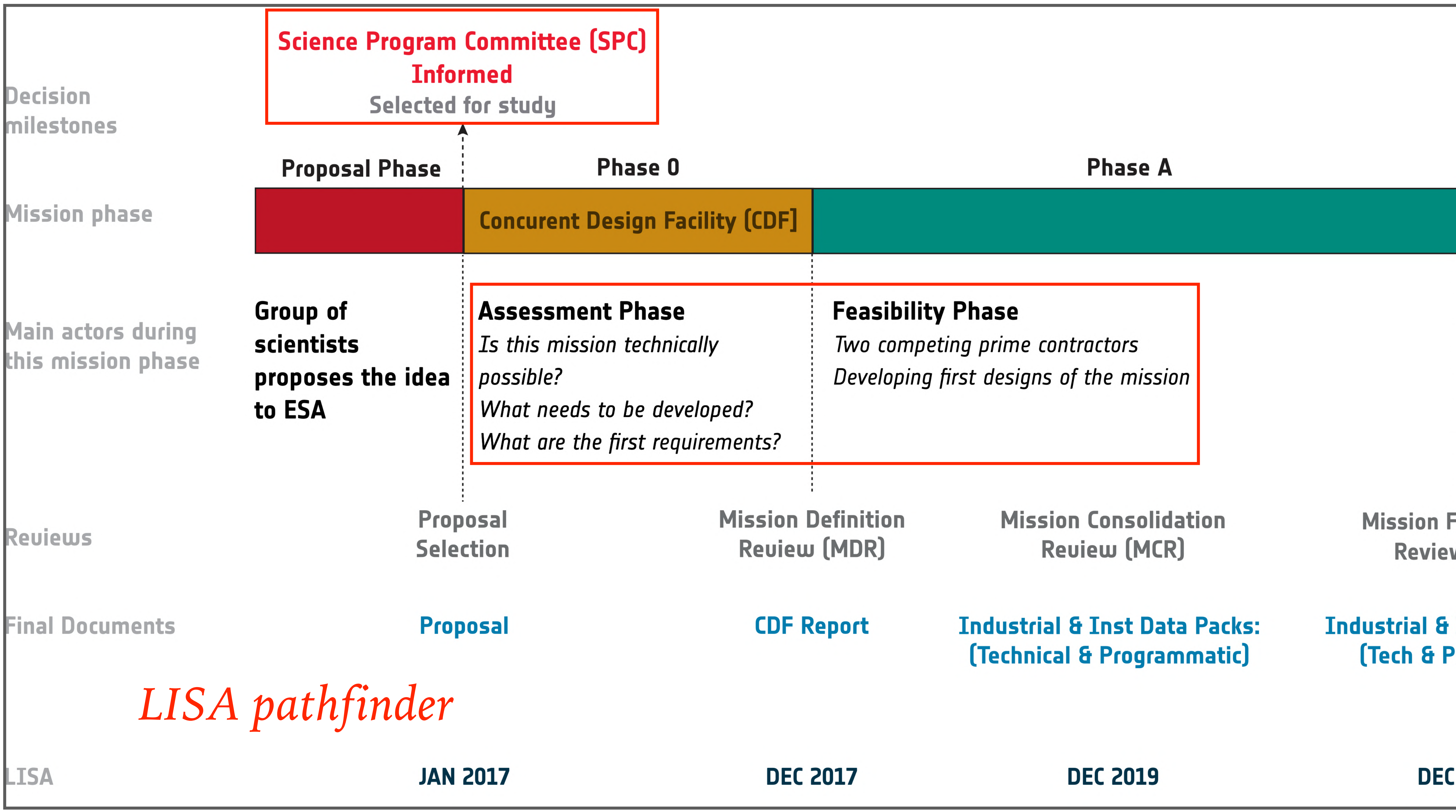


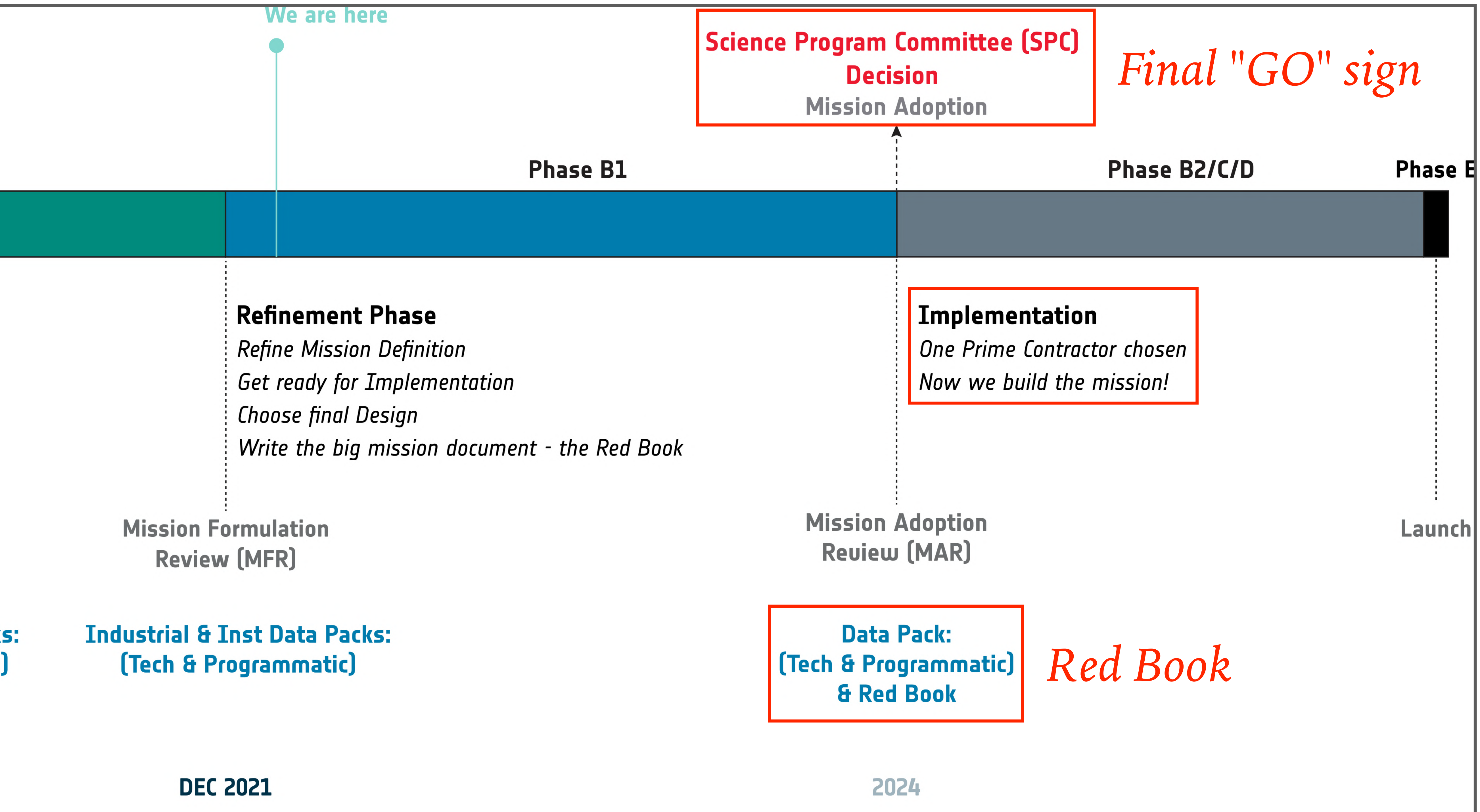
Figure 6.2: Overview of the orbital parameters for the baseline orbit.

Figure 6.3: LISA orbits
heliocentric, circular orbit
an inclination of ι = arcsi
60° inclination of the cons

LISA MISSION: TIMELINE



LISA MISSION: TIMELINE



LISA MISSION: TARGETS

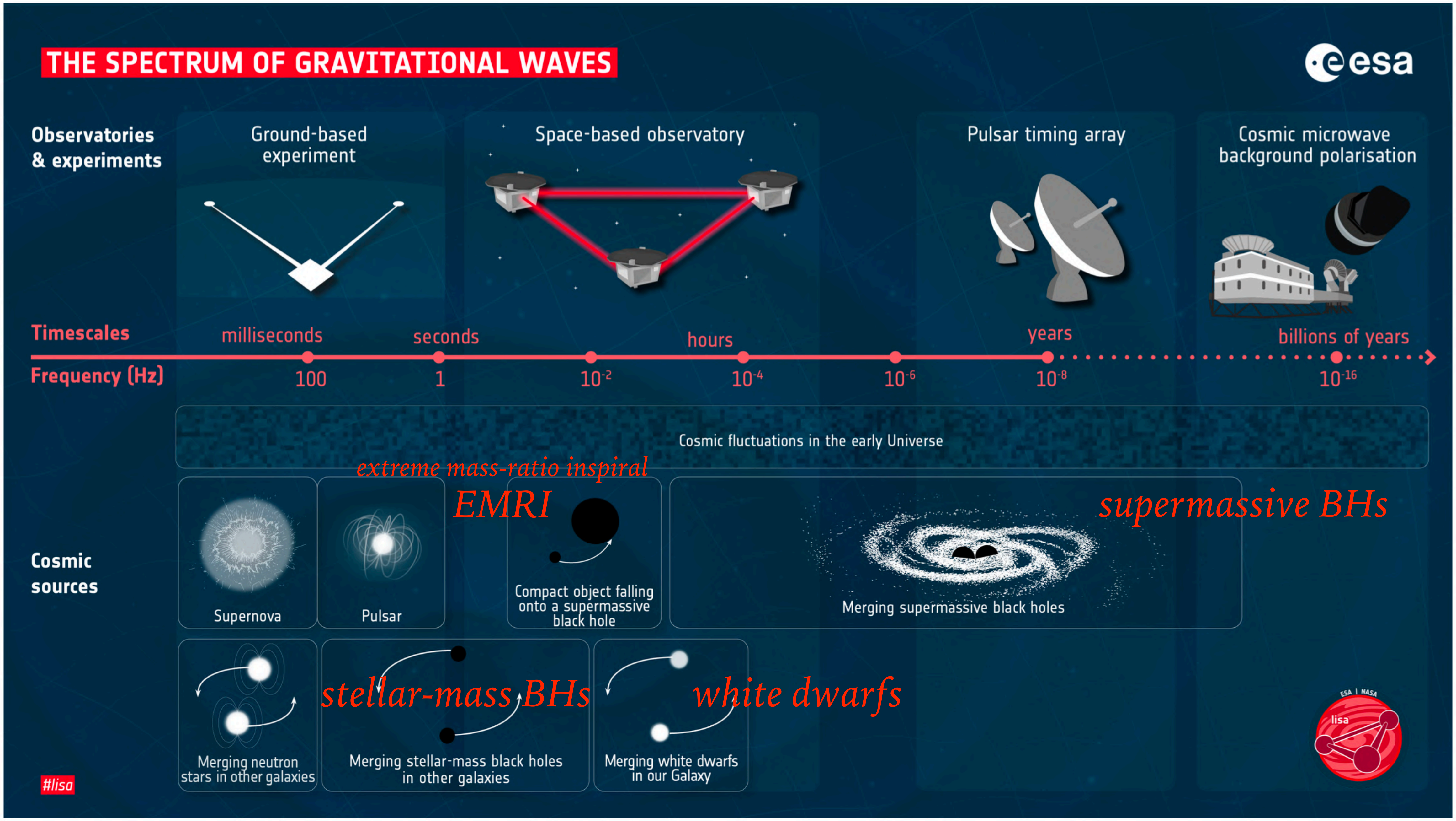


Figure 2.1: LISA targets the millihertz band of gravitational waves, lying between the nanohertz regime probed by pulsar timing arrays and the decahertz regime accessible to ground-based detectors. Several types of astrophysical sources produce gravitational waves (GWs) in this band.

LISA MISSION: PRIMARY SOURCES

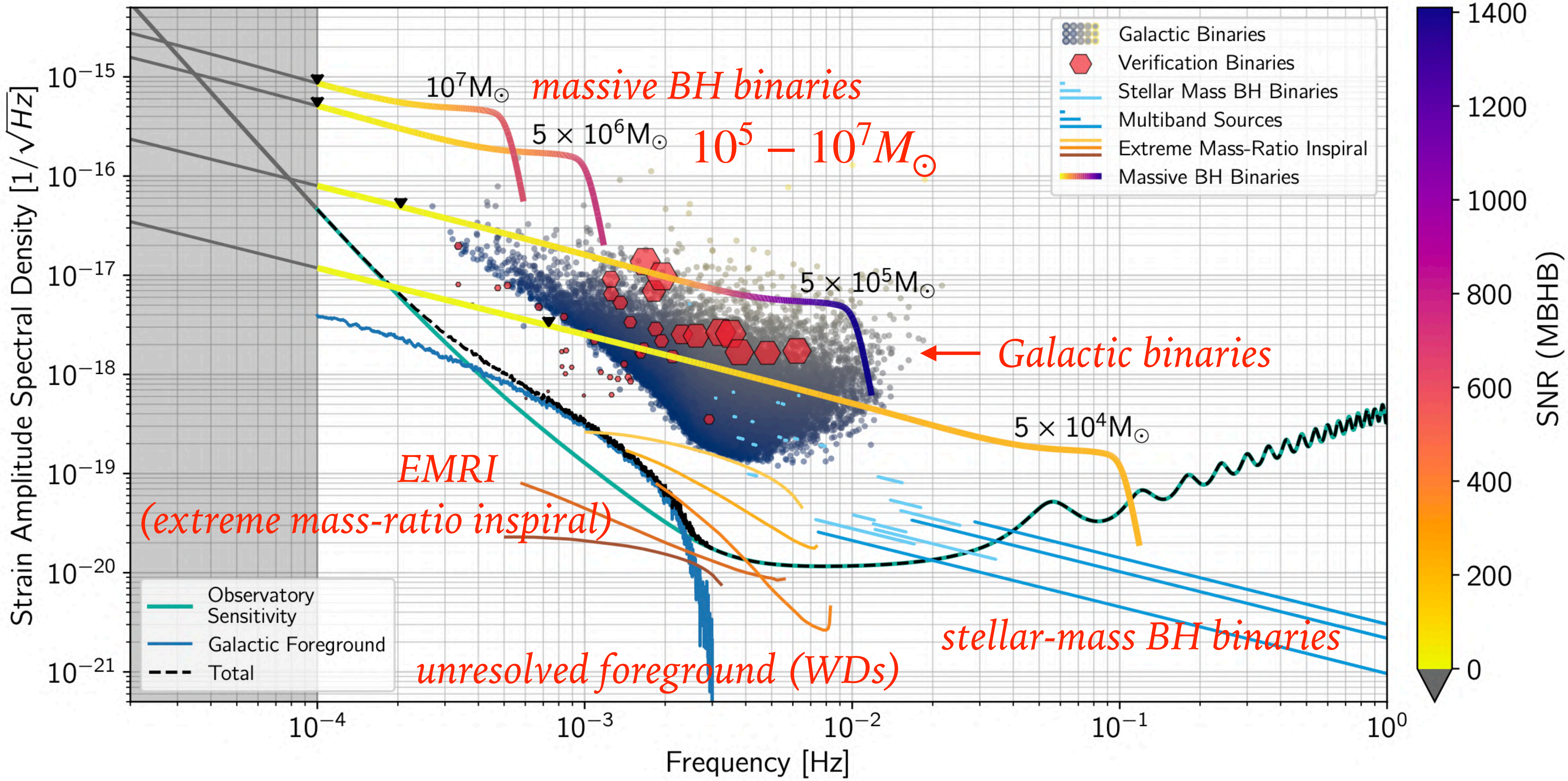


Figure 2.2: Illustration of the primary LISA source classes in the gravitational wave (GW) frequency-amplitude plane. Included are merging massive Black Hole binaries (MBHBs) and an extreme mass-ratio inspiral (EMRI) at moderate redshift; stellar-mass Black Holes (sBHs), including potential multiband sources, at low redshift; and Galactic binaries (GBs), including verification binaries (VBs), in the Milky Way.

LISA MISSION: PRIMARY SOURCES

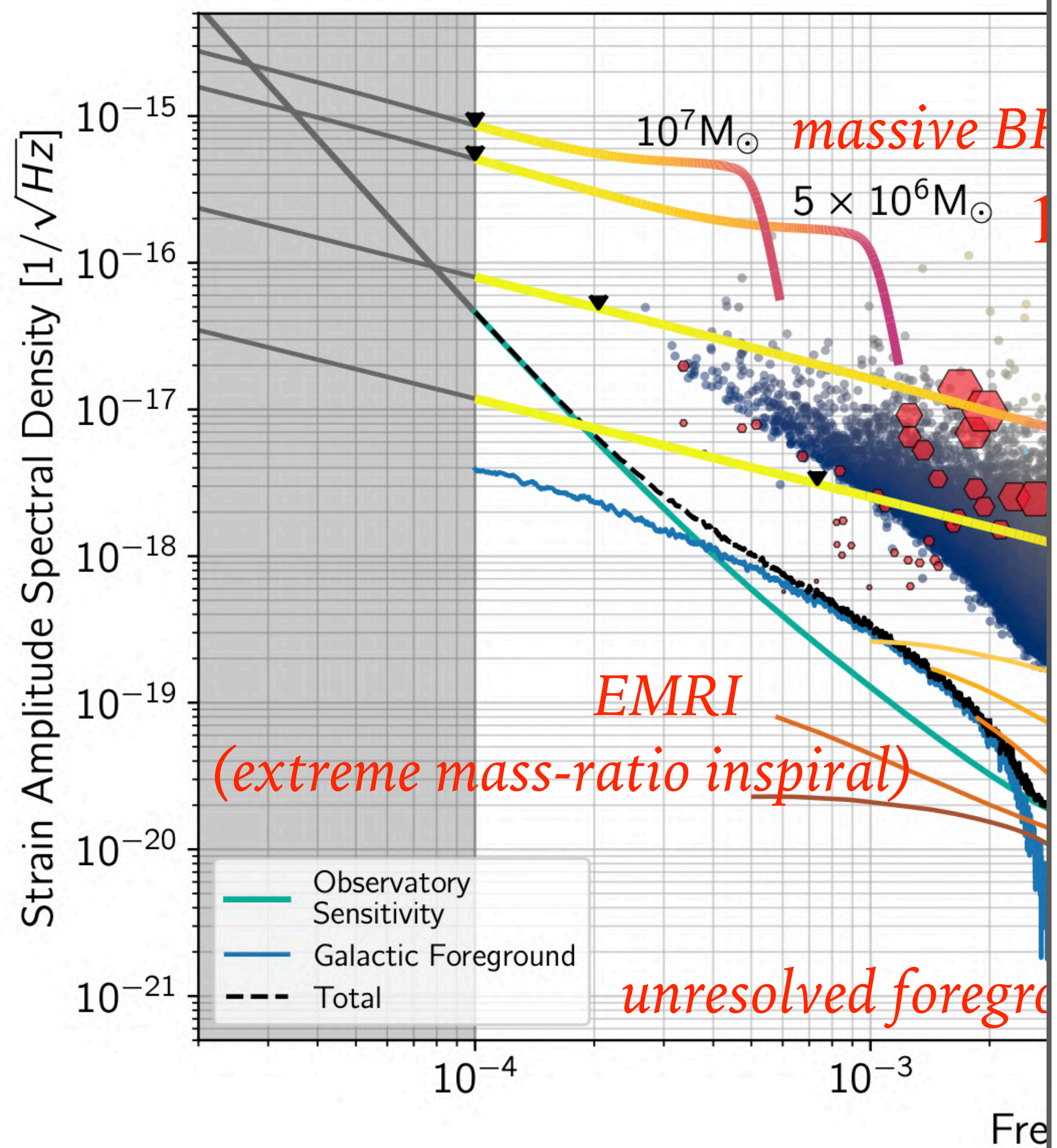


Figure 2.2: Illustration of the primary LISA source c
Included are merging massive Black Hole binaries (M
redshift; stellar-mass Black Holes (sBHs), including p
(GBs), including verification binaries (VBs), in the

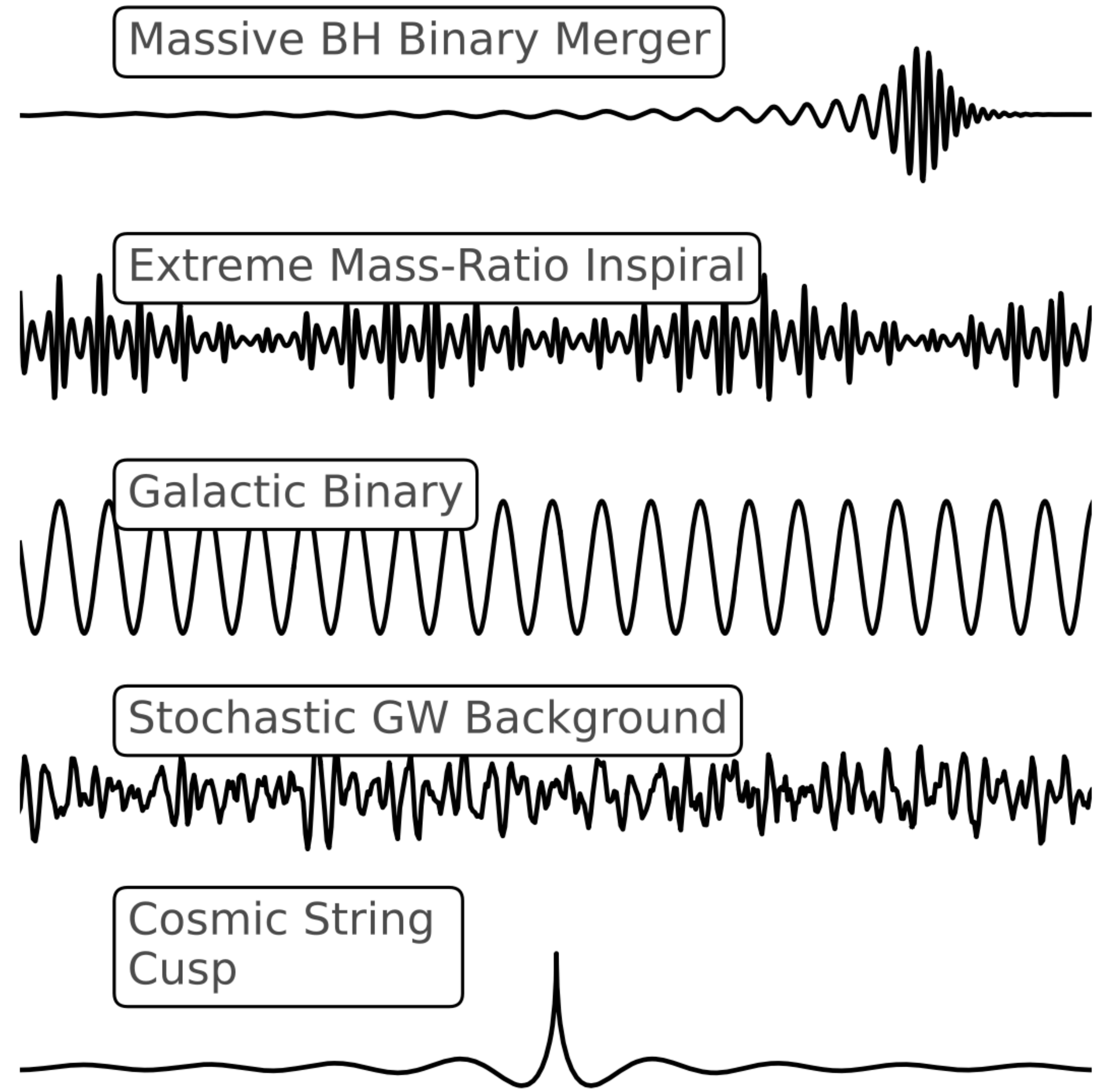
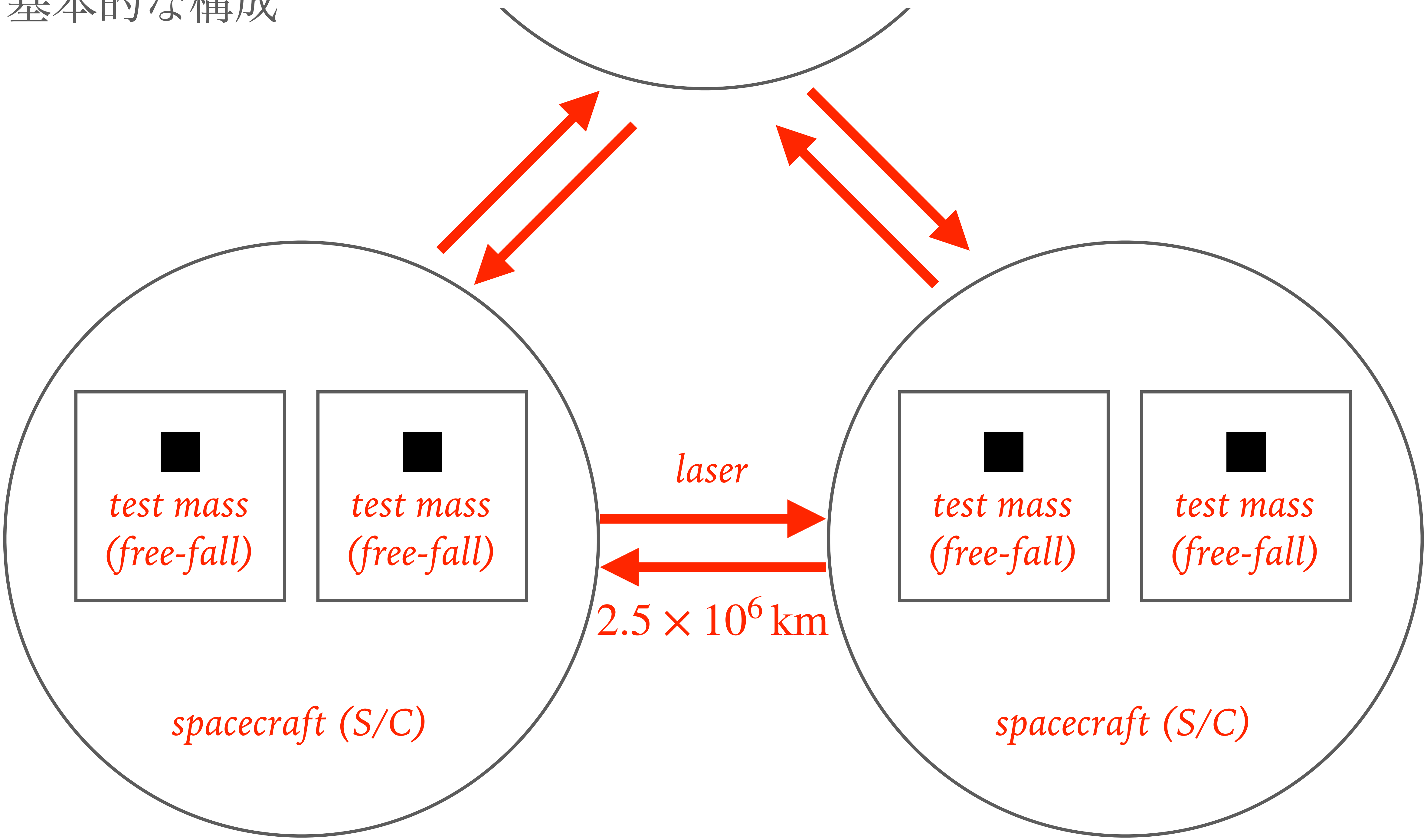


Figure 8.3: Shapes of the waveforms corresponding to the GW emission of (from top to bottom): Massive BH binary mergers; Extreme-mass-ratio inspirals; a single Galactic binary; a typical stochastic process; and a cosmic string cusp.

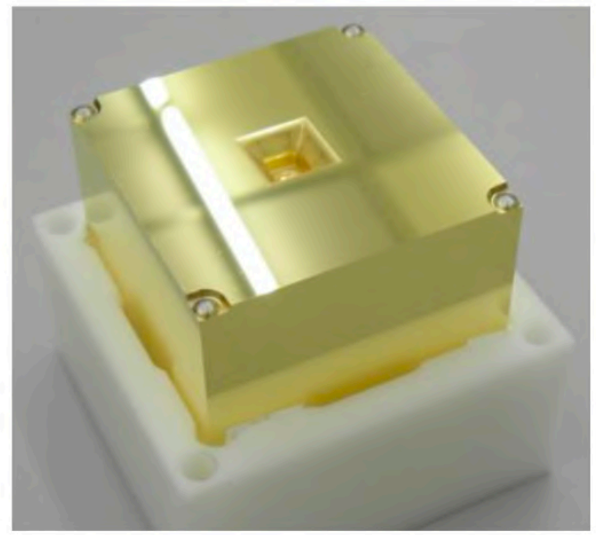
LISA MISSION: STRUCTURE OF THE SATELLITES

- 基本的な構成

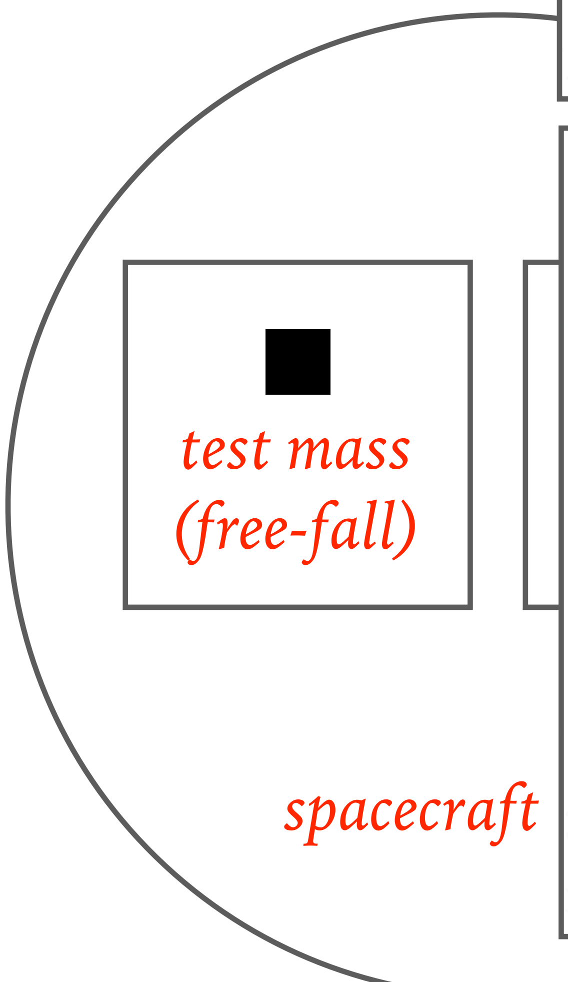


LISA MISSION: STRUCTURE OF THE SATELLITES

- 基本的な構成



46mm cube of Au-Pt



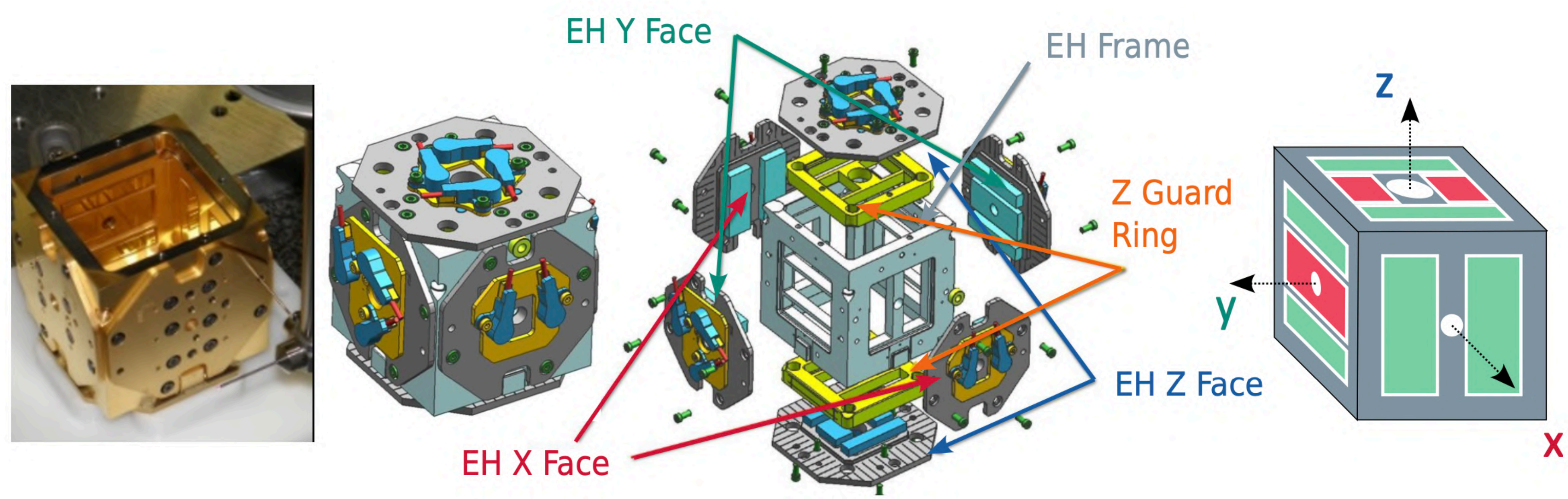


Figure 5.5: From left to right: Photo of the test mass electrode housing taken during integration for LISA Pathfinder (LPF); CAD rendering of the mechanical implementation of the electrode housing, in molybdenum (Mo) with sapphire insulating spacers, with shielded coaxial cable connectors; conceptual design with sensing (green), injection (red), and guard ring (grey) surfaces. CAD and photo images courtesy of OHB-Italia.

LISA MISSION: STRUCTURE OF THE SATELLITES

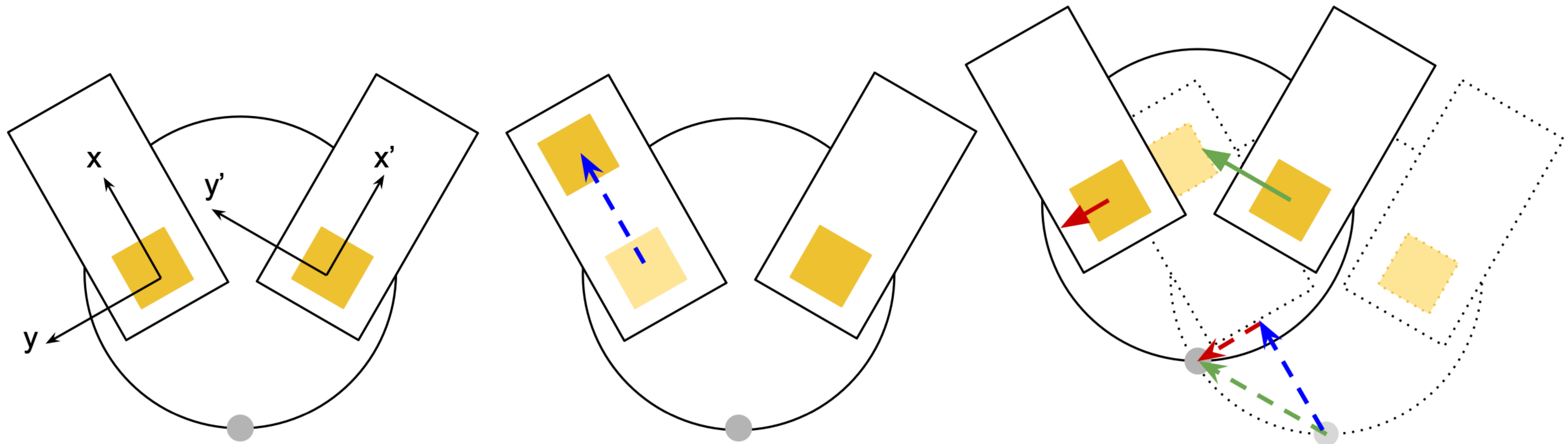
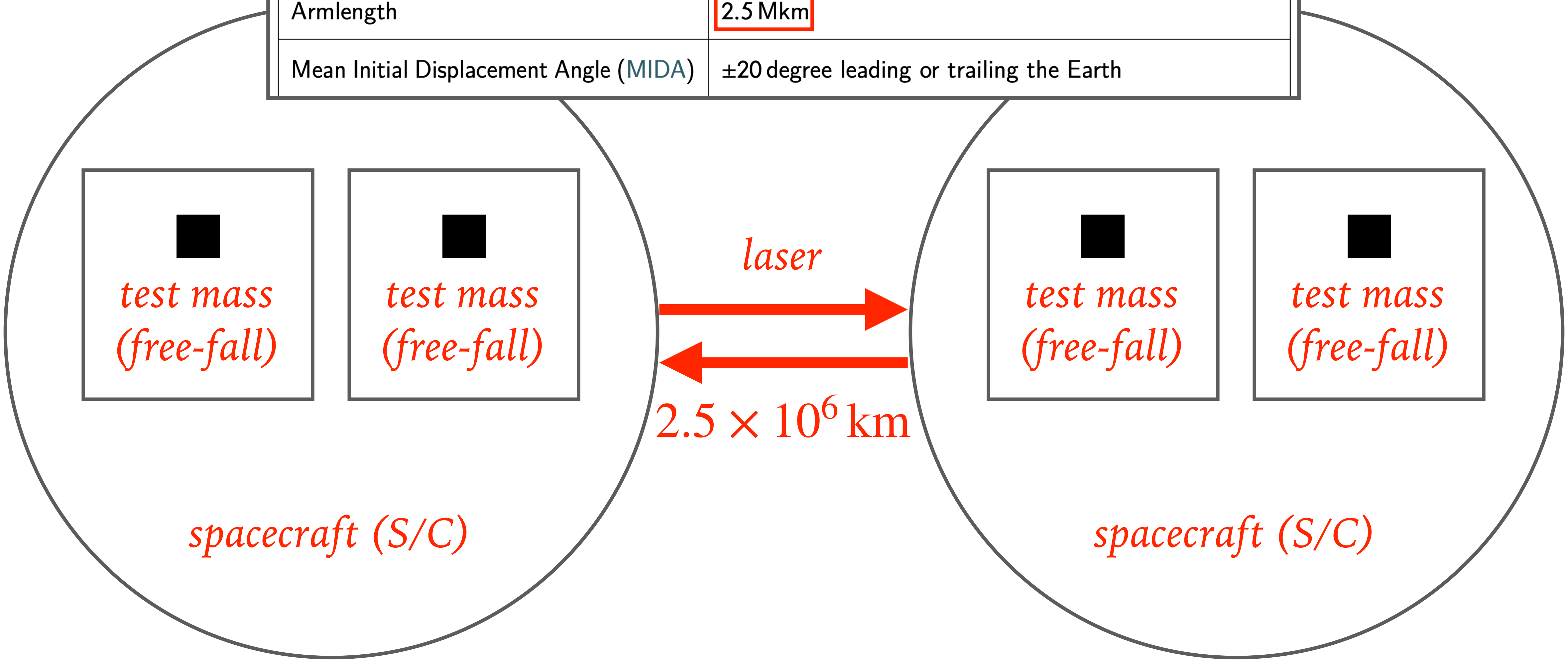


Figure 2.4: Schematic description of the approach employed by the Drag-Free Attitude Control System (DFACS) to enable free-fall of the two test masses in each LISA spacecraft (S/C) along the respective sensitive axes. Note that all movements are overemphasised for clarity. First panel: The LISA S/C is represented by the circle with the grey dot serving as a visual reference for displacements and rotations. The two Moving Optical Sub-Assemblies (MOSAs) are indicated by the black rectangles with the gold cubes representing the test masses and the sensitive axes are denoted as x and x'. Second panel: the left test mass follows a different orbit and moves along the sensitive axis x (blue dashed vector) with respect to the S/C as the movement along the y axis is restricted by the test mass-control. Third panel: the DFACS algorithm compensates by moving the spacecraft in the direction of the insensitive axis of the other test mass (y', green dashed vector) so that the displacement of the first test mass is compensated (blue dashed vector), simultaneously moving in the direction of the sensitive axis x of the first test mass, applying forces to the test masses only along the insensitive axes y' and y (green and red solid vectors, respectively). This results in each test mass remaining in free-fall along its corresponding sensitive axis.

LISA MISSI

➤ 基本的な構

Spacecraft Overview	
Key Parameters	
Space Segment	3 nearly identical spacecraft and launch adaptor
Launch Mass	8212 kg
Power	2300 W
Mission Lifetime	6.25 years
Nominal Science Phase	4.5 years
Constellation Orbit	
Orbit Type	Heliocentric 1 AU orbits
Armlength	2.5 Mkm
Mean Initial Displacement Angle (MIDA)	±20 degree leading or trailing the Earth



LISA MISSION: LISA PATHFINDER

THE EUROPEAN SPACE AGENCY

esa

LISA factsheet

ESA / Science & Exploration / Space Science

Overview of the LISA mission.

Name: Laser Interferometer Space Antenna (LISA)

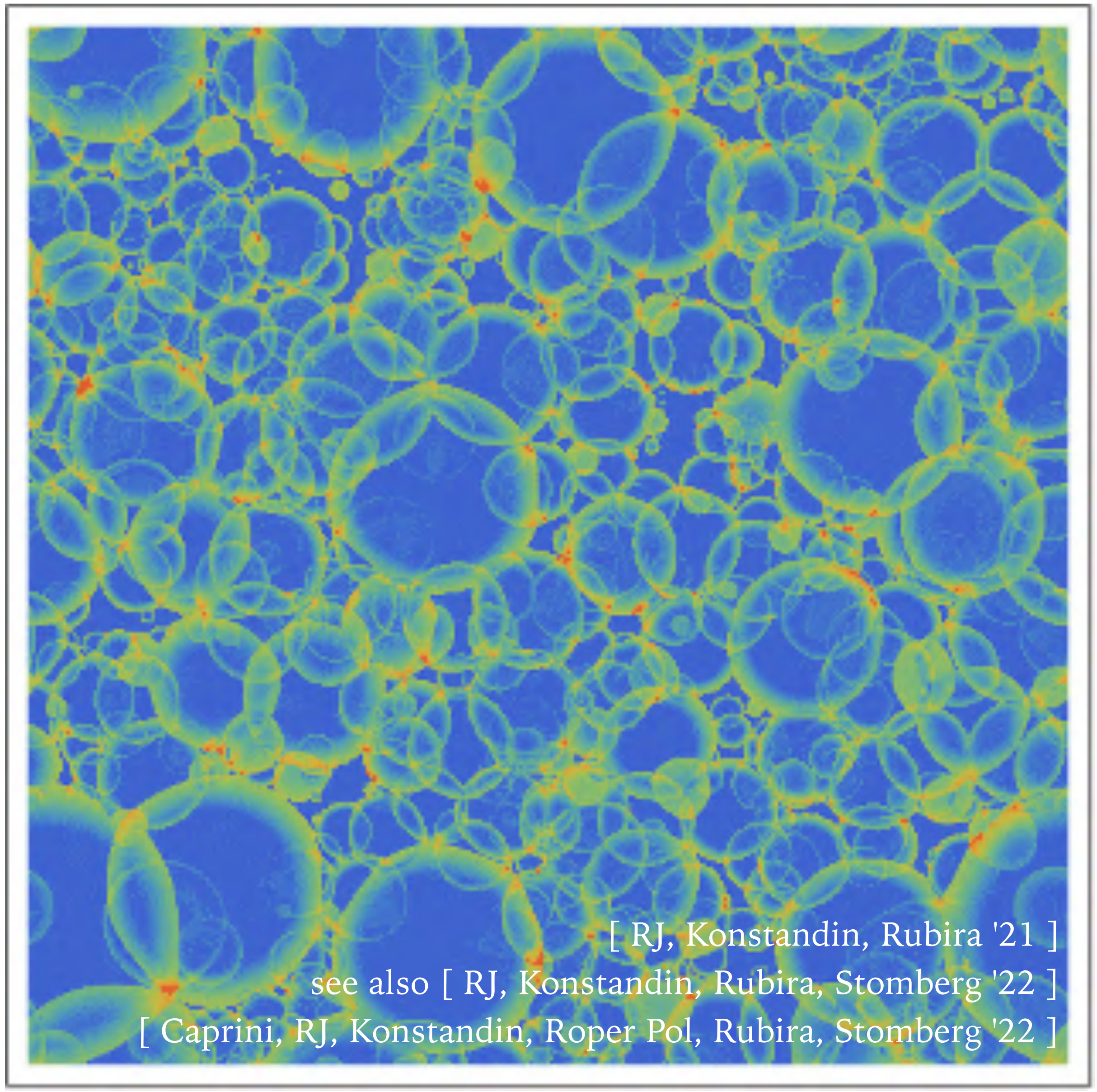
Planned launch: ~2035

Mission: First gravitational wave detector in space.

Status: On 20 June 2017, LISA was selected as the third large-class mission, L3, under ESA's Cosmic Vision 2015-2025. It was then adopted on 25 January 2024. Construction will begin in January 2025 after a prime contractor has been chosen.

BUBBLE COLLISION & FLUID DYNAMICS

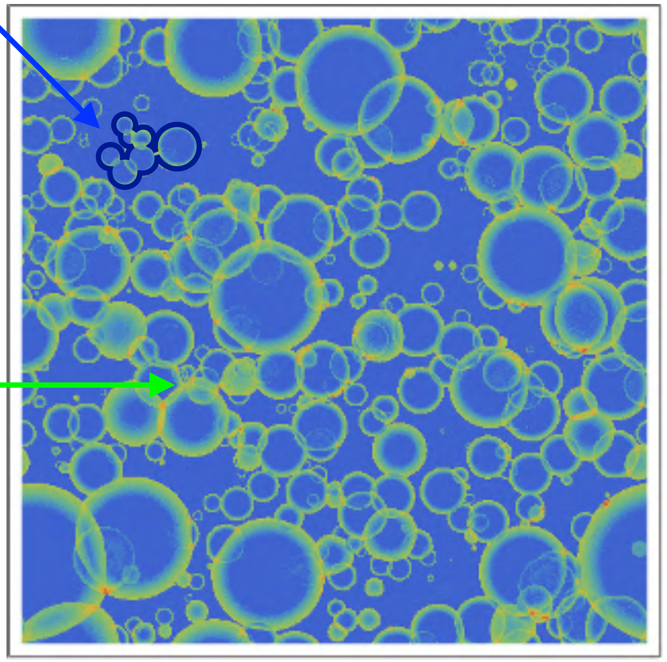
➤ Bubbles collide, and fluid dynamics sets in (example for )



GRAVITATIONAL WAVE SOURCES

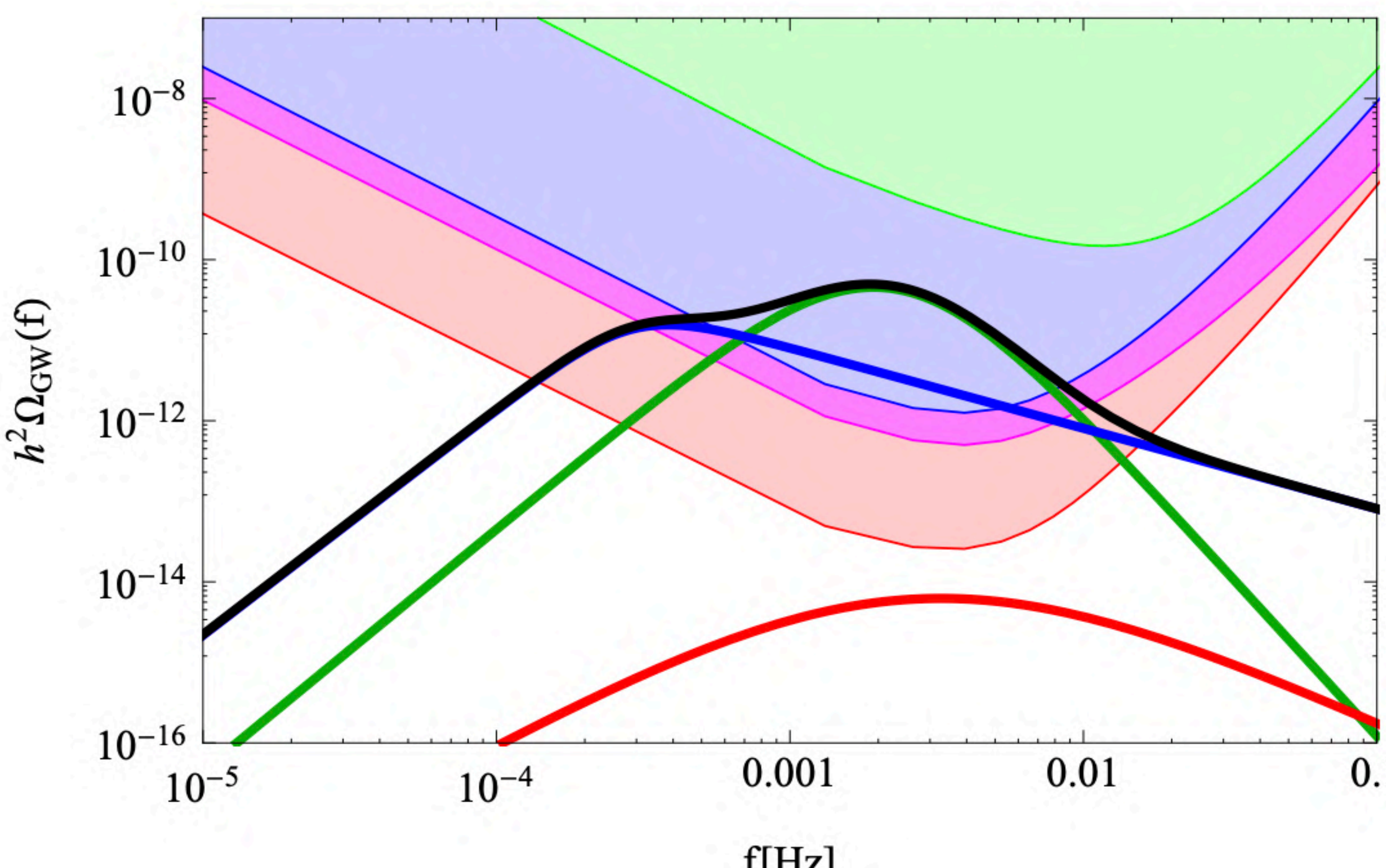
[Kosowsky, Turner, Watkins '92]
[Kosowsky, Turner '92]
[Kamionkowski, Kosowsky, Turner '93]
and e.g. [Caprini et al. '16] [Caprini et al. '20]

- ➤ Bubble collision
  - Kinetic & gradient energy of the scalar field (= order parameter field)
  - Dominant when the transition is extremely strong and the walls runaway
- ➤ Sound waves
  - Compression mode of the fluid motion
  - Dominant unless the transition is extremely strong
- ➤ Turbulence
  - Turbulent motion caused by fluid nonlinearity
  - Expected to develop at a later stage



important at later stage

GRAVITATIONAL WAVE SPECTRUM

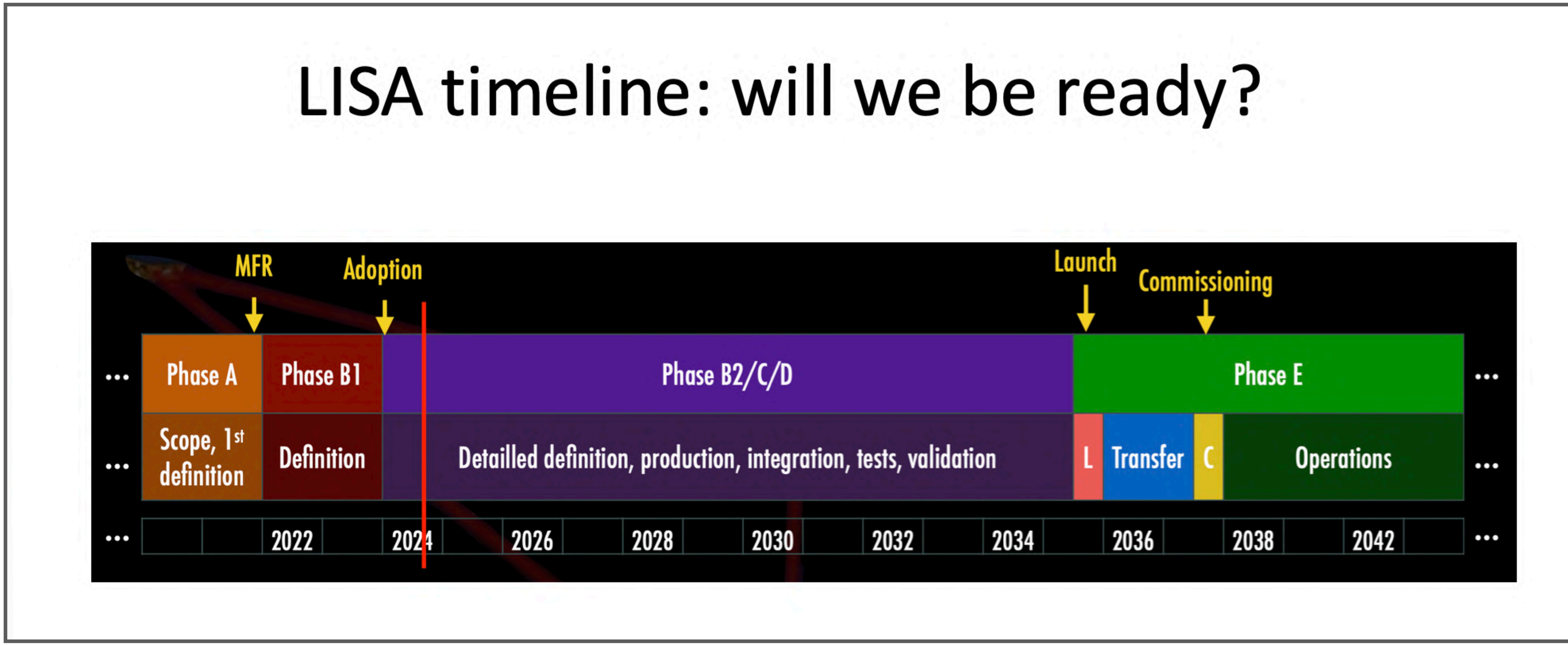


[Caprini et al. '16]

1. Overview
2. First-order phase transitions
3. Dynamics of bubbles
4. Gravitational waves
5. Recent progress

RECENT PROGRESS

ARE WE READY?



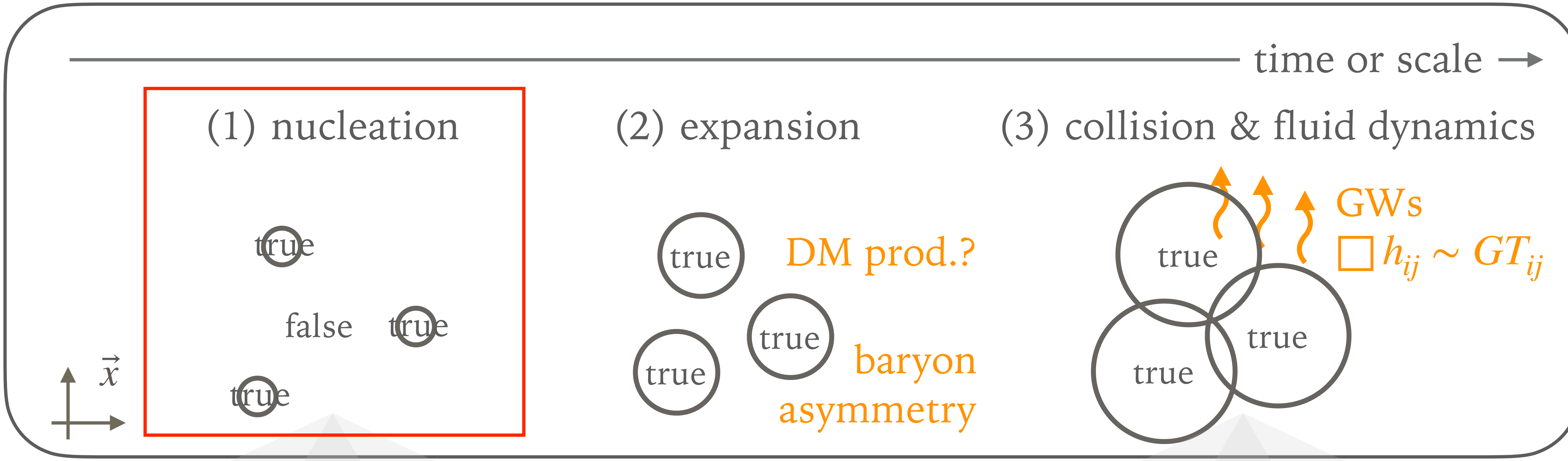
[from M. Hindmarsh's slides]

Particle physics → Transition parameters → Prediction on macroscopic outcome

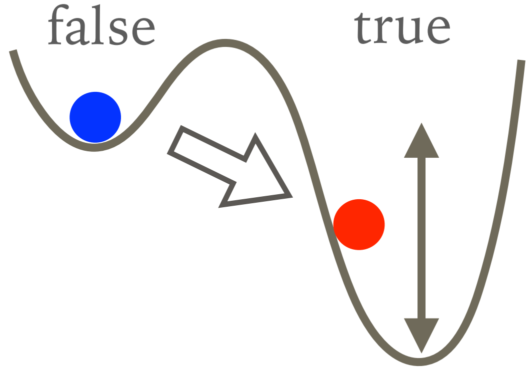
FIRST-ORDER PHASE TRANSITIONS IN THE EARLY UNIVERSE

microphysics

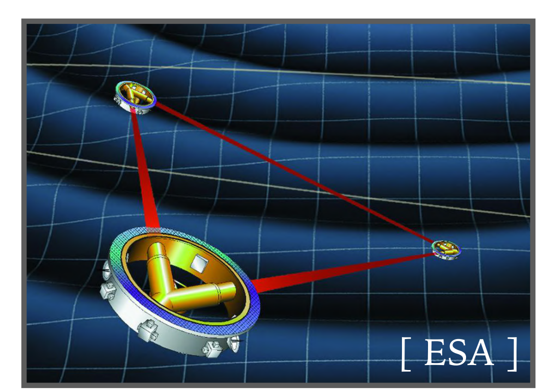
macrophysics



Physics of the Higgs sector

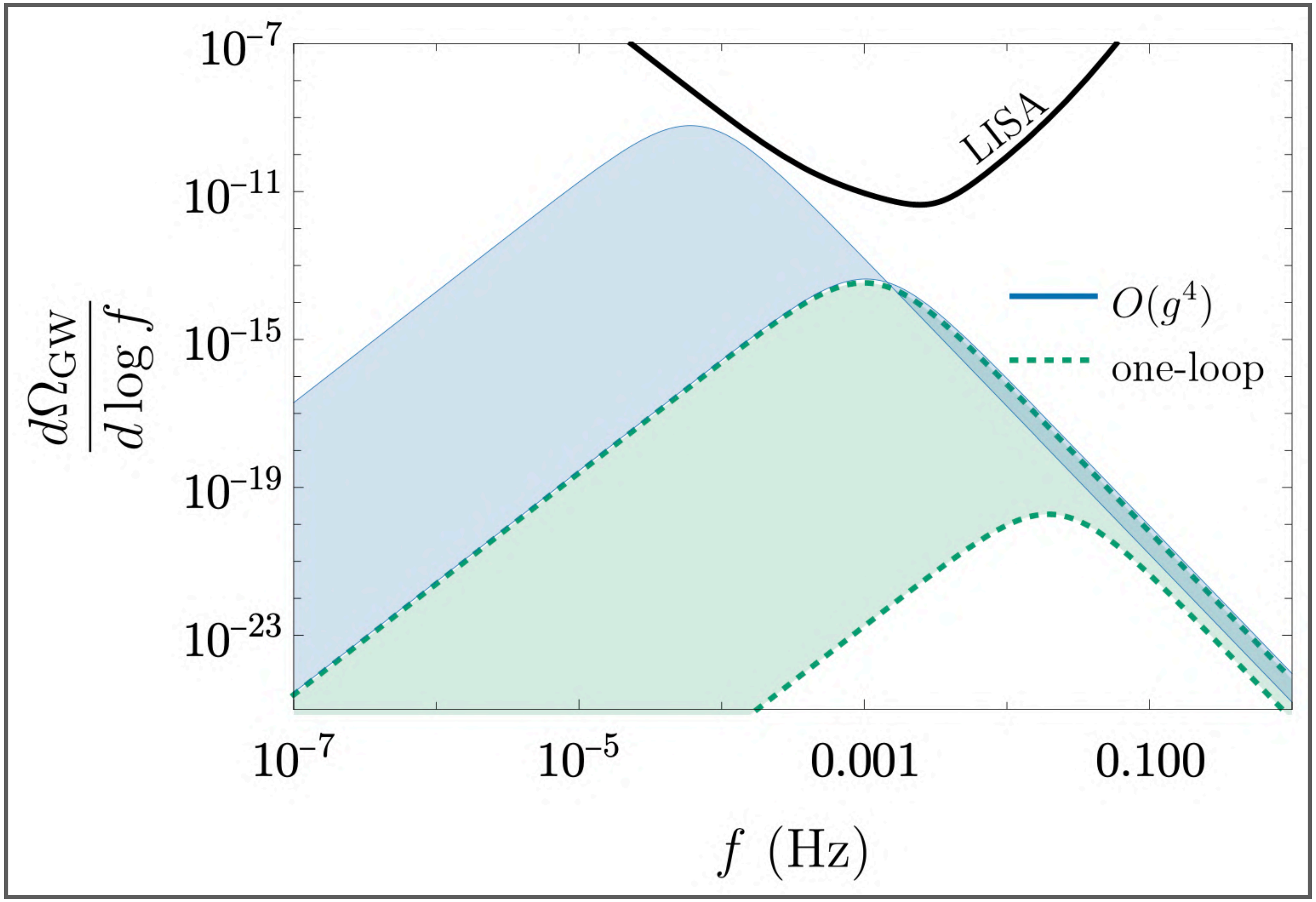


GW observations



MICROSCOPIC SCALES

- Nucleation is important



[Gould, Tenkanen '21]

MICROSCOPIC-1: LANGER VS. LINDE

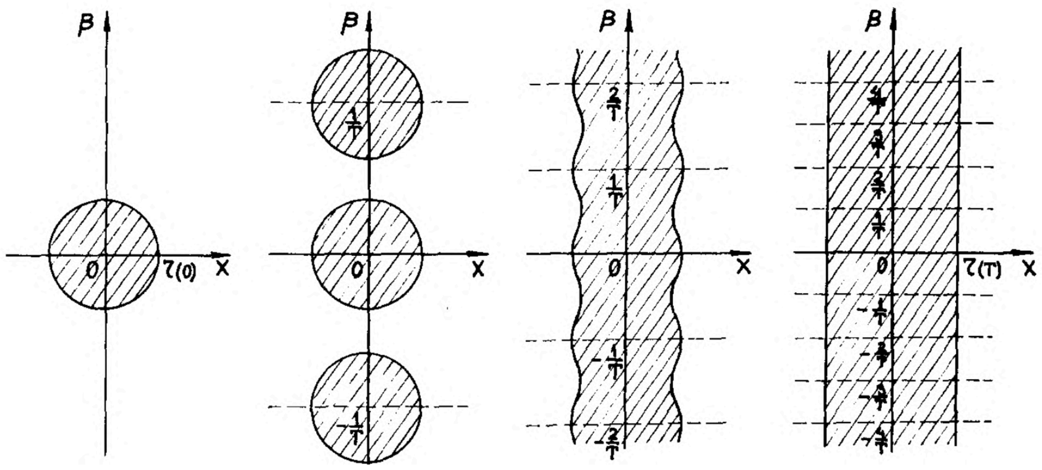
➤ Nucleation rate: Langer vs. Linde

- Nucleation in vacuum: Coleman's method

$$\Gamma/V = A \exp\left[-S_4(\varphi)\right]\,, \quad A = \left(\frac{S_4(\varphi)}{2\pi}\right)^2 \left(\frac{\det'\left[-\Box + V''(\varphi)\right]}{\det\left[-\Box + V''(0)\right]}\right)^{-1/2}$$

- Natural extension to finite tenperature: Linde's nucleation rate

$$\frac{\Gamma}{V} = T\left(\frac{S_3(\varphi)}{2\pi T}\right)^{3/2} \left(\frac{\det'\left[-\Delta + V''(\varphi, T)\right]}{\det\left[-\Delta + V''(0, T)\right]}\right)^{-1/2} \exp\left[-S_3(\varphi)/T\right]$$



[Linde '81]

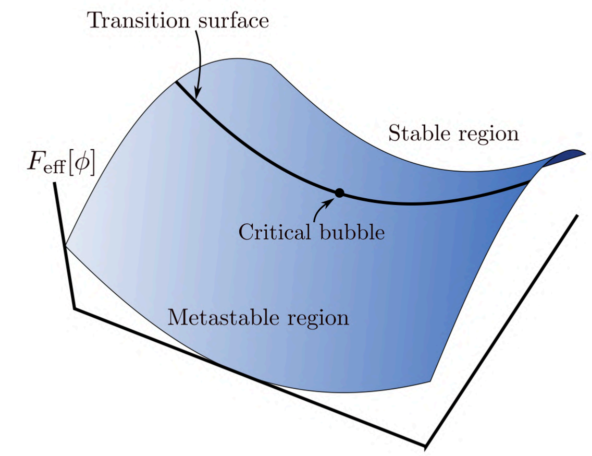
MICROSCOPIC-1: LANGER VS. LINDE

➤ Linde's nucleation rate

- Linde's nucleation rate is based on quantities in equilibrium

$$F_{\rm eff}[\phi_{\rm CB} + \varphi] \approx F_{\rm eff}[\phi_{\rm CB}] + \frac{1}{2}\int_{\mathbf{x}}\int_{\mathbf{y}} \varphi(\mathbf{x}) F''_{\rm eff}[\phi_{\rm CB}](\mathbf{x},\mathbf{y})\varphi(\mathbf{y}),$$

$$F''_{\rm eff}[\phi_{\rm CB}](\mathbf{x},\mathbf{y}) = \left.\frac{\delta^2 F_{\rm eff}}{\delta\phi(\mathbf{x})\delta\phi(\mathbf{y})}\right|_{\phi=\phi_{\rm CB}}$$



[Gould, Hirvonen '21]

→ where's the information on "the system is initially on the metastable side"?

➤ Langer's nucleation rate [Langer '67]

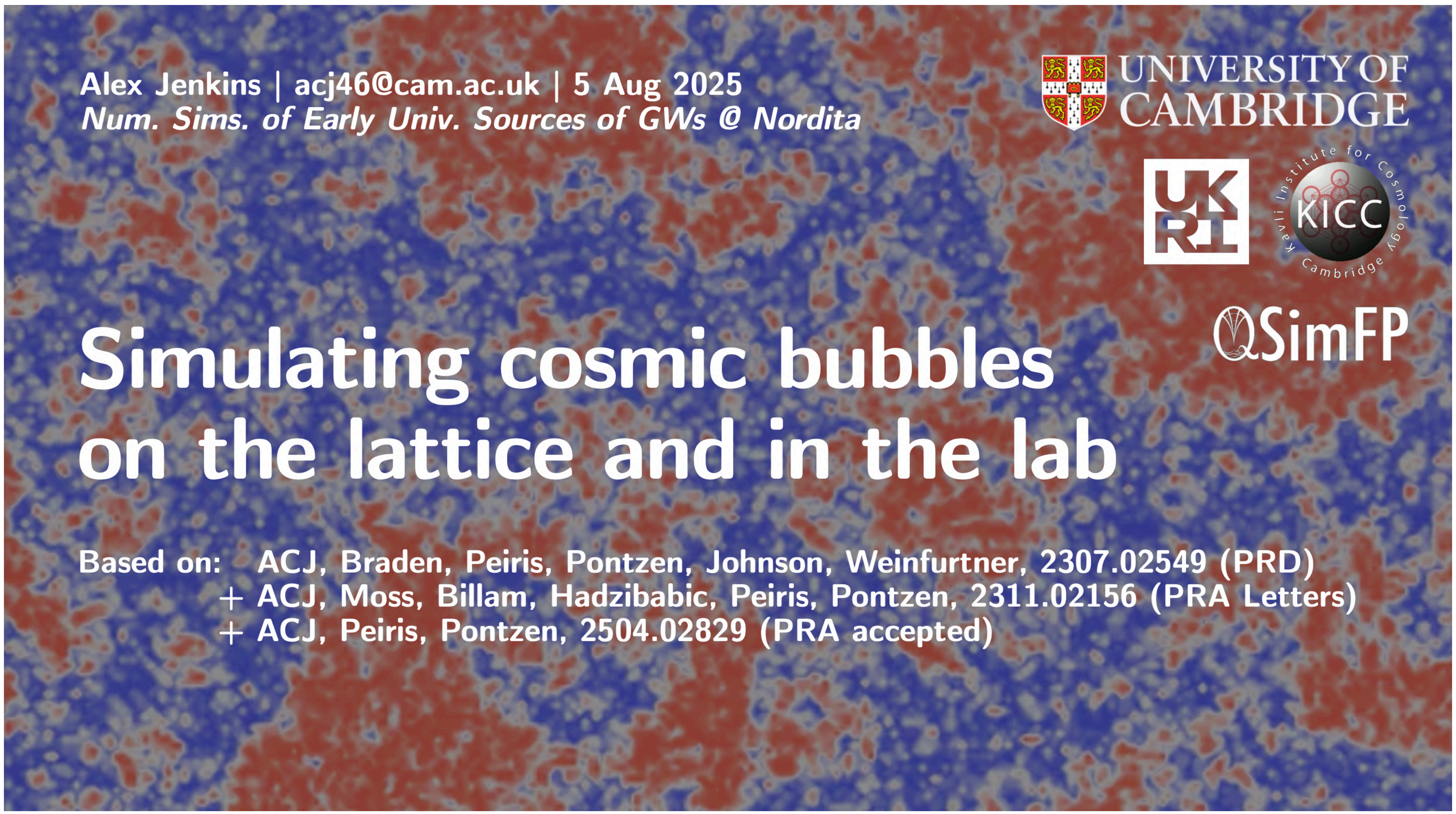
- Known in statistical mechanics from 60's

- Difference: dynamical prefactor, in addition to Linde's nucleation rate

$$\Gamma = \boxed{\frac{\kappa}{2\pi}}\Sigma\,, \quad \Sigma = \mathcal{V}\sqrt{\left|\frac{\det(\beta F''_{\rm eff}[\phi_{\rm meta}]/2\pi)}{\det'(\beta F''_{\rm eff}[\phi_{\rm CB}]/2\pi)}\right|}\, e^{-\beta\Delta F_{\rm eff}}$$

MICROSCOPIC-2: ANALOGUE TUNNELING

➤ Tunneling in laboratory



[from A. Jenkins's slides]

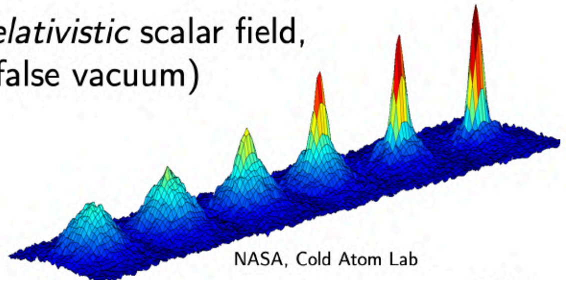
MICROSCOPIC-2: ANALOGUE TUNNELING

Mean-field theory of cold atoms

- At ultra-low temperatures, bosonic atoms form a Bose-Einstein condensate
- Resulting many-body wavefunction ψ obeys a nonlinear Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2M} + V_{ext}(x) + g|\psi|^2\right)\psi$$

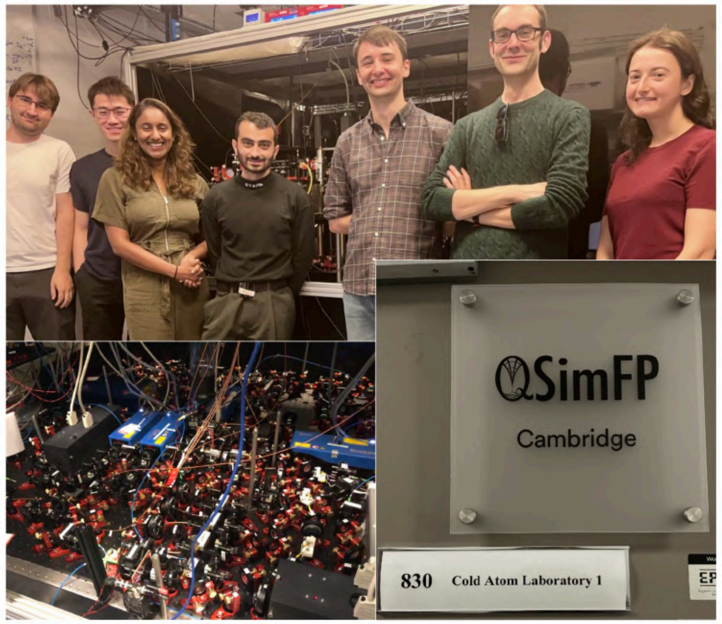
- This is the equation of motion for a *non-relativistic* scalar field, with a $|\psi|^4$ self-interaction potential (no false vacuum)
- How do we get cosmology from this?



NASA, Cold Atom Lab

The experiment

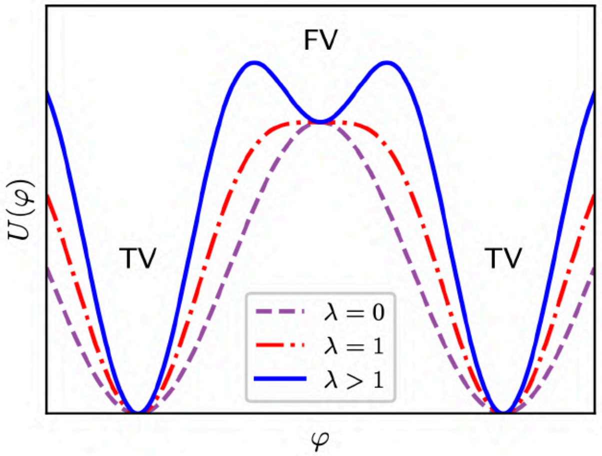
- ...is being developed right now by Zoran Hadzibabic's group in Cambridge
- Relies only on pre-existing techniques + technologies, just applied to a new problem
- Many key technical requirements already achieved, first science data in the coming months



How do we get a false vacuum?

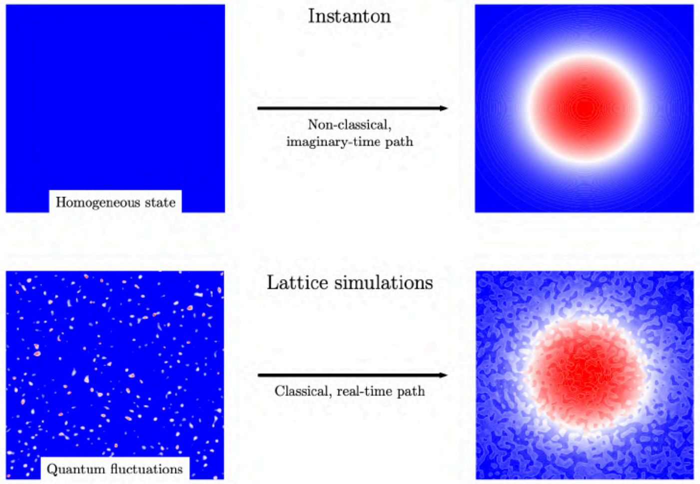
- Rapid oscillations of the Rabi coupling stabilise the false vacuum
- Analogous to a pendulum with an oscillating pivot point

Fialko+, 1408.1163, 1607.01460



Semiclassical lattice simulations

- In the Euclidean picture, bubbles appear from a "classical" initial state via *non-classical* paths
- Opposite approach: sample initial state (inc. quantum fluctuations), then evolve forward *classically:* "truncated Wigner approximation"
- First applied to vacuum decay in Braden+, 1806.06069, opening up a new real-time approach to studying bubble nucleation

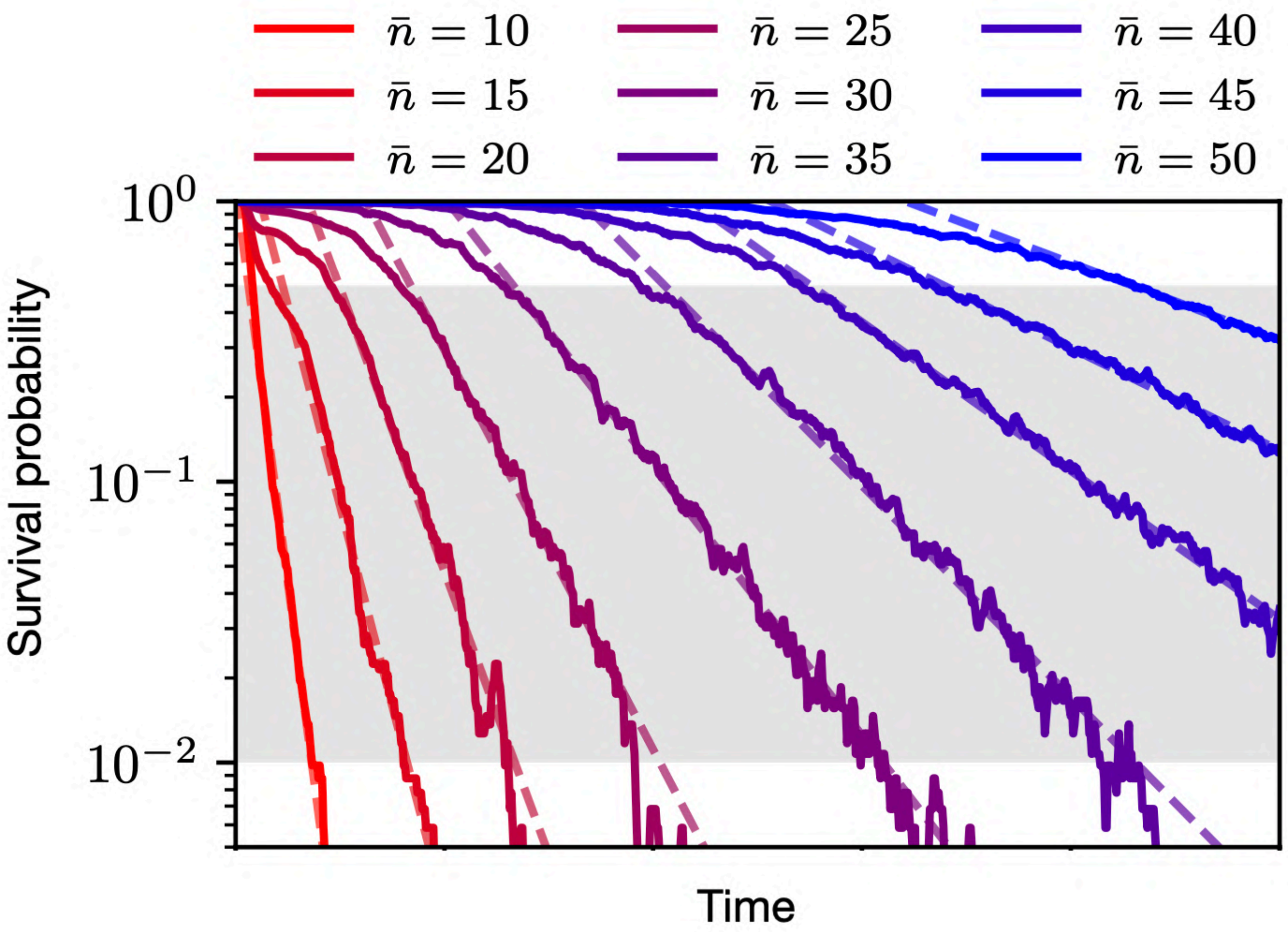


[from A. Jenkins's slides]

Bubble nucleation rates

- Run large ensemble of sims, count how many have decayed as a function of time
- As expected, we find exponential decay at rate $\Gamma \sim -\log \bar{n}$, where $\bar{n}$ is the condensate number density
- Normalisation of Γ is significantly larger than the Euclidean prediction — potentially resolved by accounting for *renormalisation*

(Braden+, 2204.11867)



MICROSCOPIC-3: TUNNELING POTENTIAL

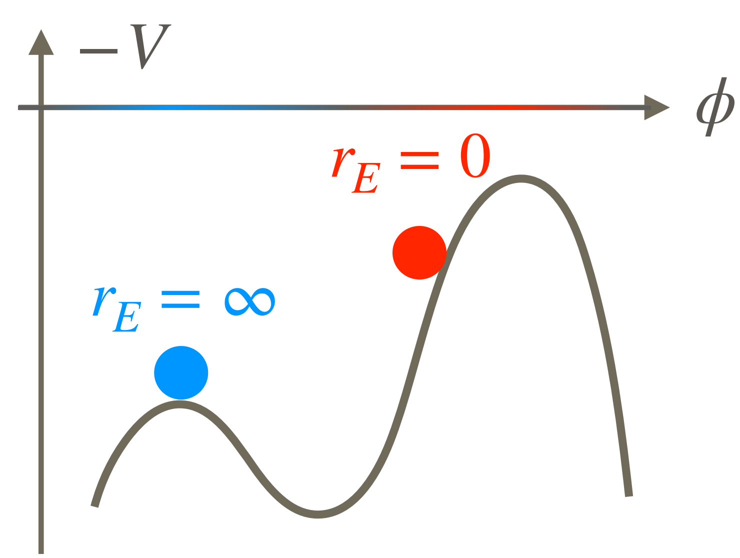
[Coleman '77]
[Callan, Coleman '77]

- Tunneling rate Γ is estimated from the Euclidean action $S[\phi]$

$$S[\phi] = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right] \overset{\text{O(4) sym.}}{=} \int 2\pi^2 r_E^3 \, dr_E \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial r_E}\right)^2 + V(\phi)\right]$$

with ϕ evaluated at the bounce solution $\bar{\phi}$

$$\frac{\partial^2\bar{\phi}}{\partial r_E^2} + \frac{3}{r_E}\frac{\partial\bar{\phi}}{\partial r_E} - \frac{\partial V}{\partial\bar{\phi}} = 0 \quad \text{with} \quad \left.\frac{\partial\bar{\phi}}{\partial r_E}\right|_{r_E=0} = 0, \quad \bar{\phi}\Big|_{r_E=\infty} = 0$$



MICROSCOPIC-3: TUNNELING POTENTIAL

- Calculation of the tunneling action in ϕ-space has been proposed

  [Espinosa '18]

- Dynamical variable is the tunneling potential $V_t(\phi)$

- So far formulated in the O(4) case, with interesting properties:

  - The action is ***manifestly positive definite***: $S[V_t] = \int_0 d\phi \, \frac{54\pi(V(\phi) - V_t(\phi))^2}{(-\dot{V}_t(\phi))^3}$

    dot is ϕ derivative hereafter

  - Solution of the EOM is ***a minimum, not a saddle point***

  - Once one takes gravity into account, ***both CdL and HM actions are obtained in a unified way***

$$S[V_t] = \int_0 d\phi \, \frac{6\pi^2 M_P^2 (D + \dot{V}_t)^2}{D V_t^2} \quad \text{with} \quad D = \sqrt{\dot{V}_t^2 + \frac{6(V - V_t)V_t}{M_P^2}}$$

MICROSCOPIC-3: TUNNELING POTENTIAL

- Systematic derivation with canonical tf. was found in [Espinosa, RJ, Konstandin '22] both for the cases w/o and w/ gravity in the O(4) case

- We generalize it to less symmetric cases in 3 steps [+Matake, Miyachi]

Step 1: Translate $\phi(t,\vec{x})$ into $t(\phi,\vec{x})$ (with some assumption about monotonicity)

$$S[\phi] = \int dt \int d^3x \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + V \right] = \int d\phi \int d^3x \left[\frac{1+(\nabla t)^2}{2\dot{t}} + \dot{t}V \right]$$

dot is again ϕ derivative

Step 2: Move to Hamiltonian

$$p = \frac{\partial\mathscr{L}}{\partial\dot{t}} = -\frac{1+(\nabla t)^2}{2\dot{t}^2} + V \quad \rightarrow \quad \mathscr{H} = p\dot{t} - \mathscr{L} = -\sqrt{2(1+(\nabla t)^2)(V-p)}$$

MICROSCOPIC-3: TUNNELING POTENTIAL

- Systematic derivation with canonical tf. was found in [Espinosa, RJ, Konstandin '22] both for the cases w/o and w/ gravity in the O(4) case
- We generalize it to less symmetric cases in 3 steps [+Matake, Miyachi]

Step 3: Move back to Lagrangian, but integrate t out instead of p

$$\mathscr{L} = p\dot{t} - \mathscr{H} = p\dot{t} + \sqrt{2(1+(\nabla t)^2)(V-p)}\,\Big|_t$$

To integrate t out, find a vector $\boldsymbol{p}$ such that $p = \nabla \cdot \boldsymbol{p}$ (guaranteed) and then

$$\mathscr{L} \sim \dot{\boldsymbol{p}}\,(\nabla t) + \sqrt{2(1+(\nabla t)^2)(V-\nabla\cdot\boldsymbol{p})}\,\Big|_t = 2\sqrt{2(V-\nabla\cdot\boldsymbol{p}) - \dot{\boldsymbol{p}}^2}$$

up to surface term

magic factor 2 (from going forth/back of the bounce)

MICROSCOPIC-3: TUNNELING POTENTIAL

- So, we obtained the action $S[\boldsymbol{p}] = 2\int d\phi \int d^3x \sqrt{2(V - \nabla \cdot \boldsymbol{p}) - \dot{\boldsymbol{p}}^2}$

- How is it related to the tunneling potential?

  Consider O(3) case, then $\boldsymbol{p} = \frac{\boldsymbol{r}}{3} V_t(\phi, r)$ and the action becomes

$$S[V_t] = \int d\phi \int dr \sqrt{128\pi^2 r^4 \left[V - V_t - \frac{r}{3}V_t' - \frac{r^2}{18}\dot{V}_t^2\right]}$$

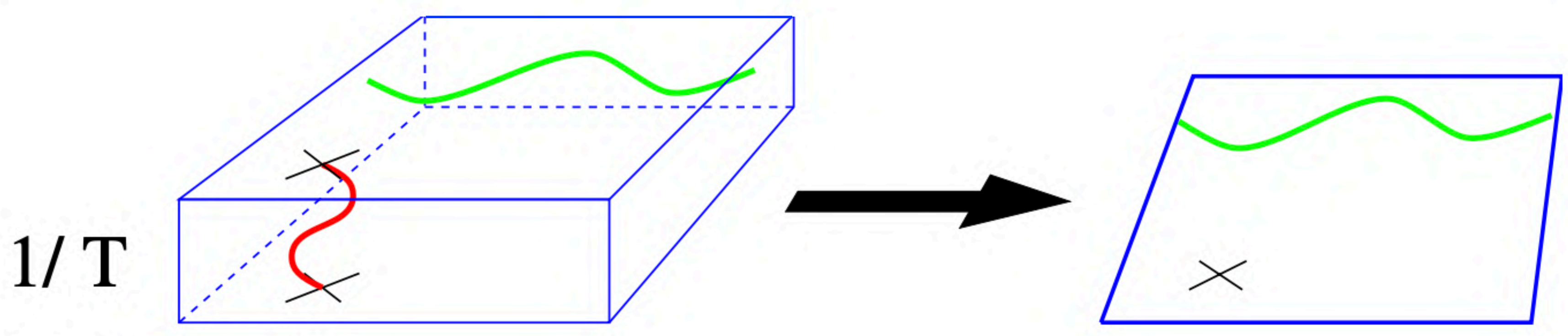
  prime is r derivative

  which actually reduces to the known tunneling action in the O(4) limit

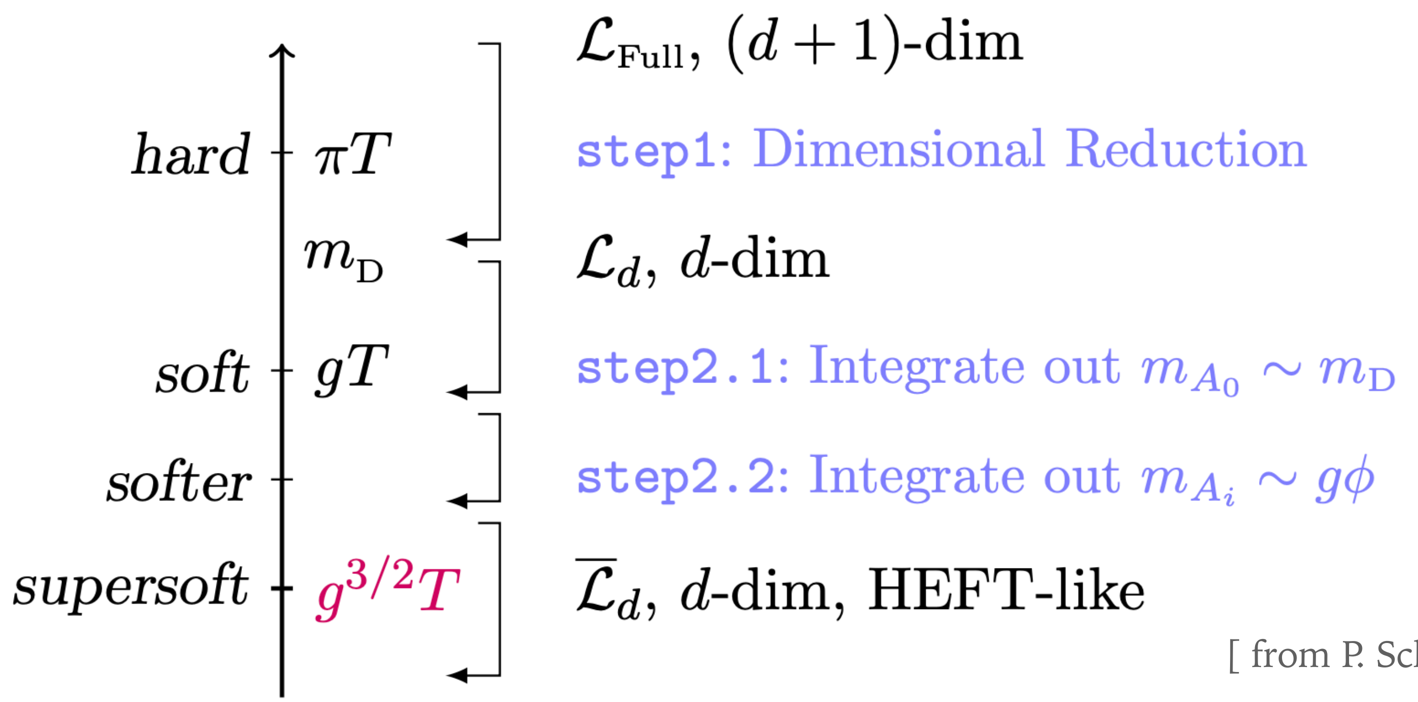
- This formulation might be useful in estimating tunneling rate around impurities, $V = V(\phi, r)$

MICROSCOPIC-4 : DIMENSIONAL REDUCTION & LATTICE SIMULATION

- Demensional reduction by integrating out heavy Matsubara modes



[from K. Rumukainen's slides]



[from P. Schicho's slides]

MICROSCOPIC-4 : DIMENSIONAL REDUCTION & LATTICE SIMULATION

[Gould, Kormu, Weir 2404.01876]

- Calculation of the bubble nucleation rate

  - Map your 4d theory to 3d by dimensional reduction, e.g.

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) - J_1\varphi - J_2\varphi^2$$

$$V(\varphi) = \sigma\varphi + \frac{1}{2}m^2\varphi^2 + \frac{1}{3!}g\varphi^3 + \frac{1}{4!}\lambda\varphi^4$$

$$\longrightarrow$$

$$S_{\rm lat} = \sum_x a^3\Big[-\frac{1}{2}Z_\phi\phi_x(\nabla^2_{\rm lat}\phi)_x + \sigma_{\rm lat}\phi_x + \frac{1}{2}Z_\phi Z_m m^2_{\rm lat}\phi_x^2 + \frac{1}{4!}Z_\phi^2\lambda_{\rm lat}\phi_x^4\Big],$$

  - Simulate probability distribution $P(\theta_{\rm op})$, and calculate the prob. of critical bubbles in $\theta_{\rm op} \in [\theta_c - \epsilon/2, \theta_c + \epsilon/2]$ realative to the metastable phase

  - Draw separatrix configurations from $\theta_{\rm op} \in [\theta_c - \epsilon/2, \theta_c + \epsilon/2]$

  - Evolve trajectories backwards and forwards in time to calculate to calculate the tunneling fraction

MICROSCOPIC-4 : DIMENSIONAL REDUCTION & LATTICE SIMULATION

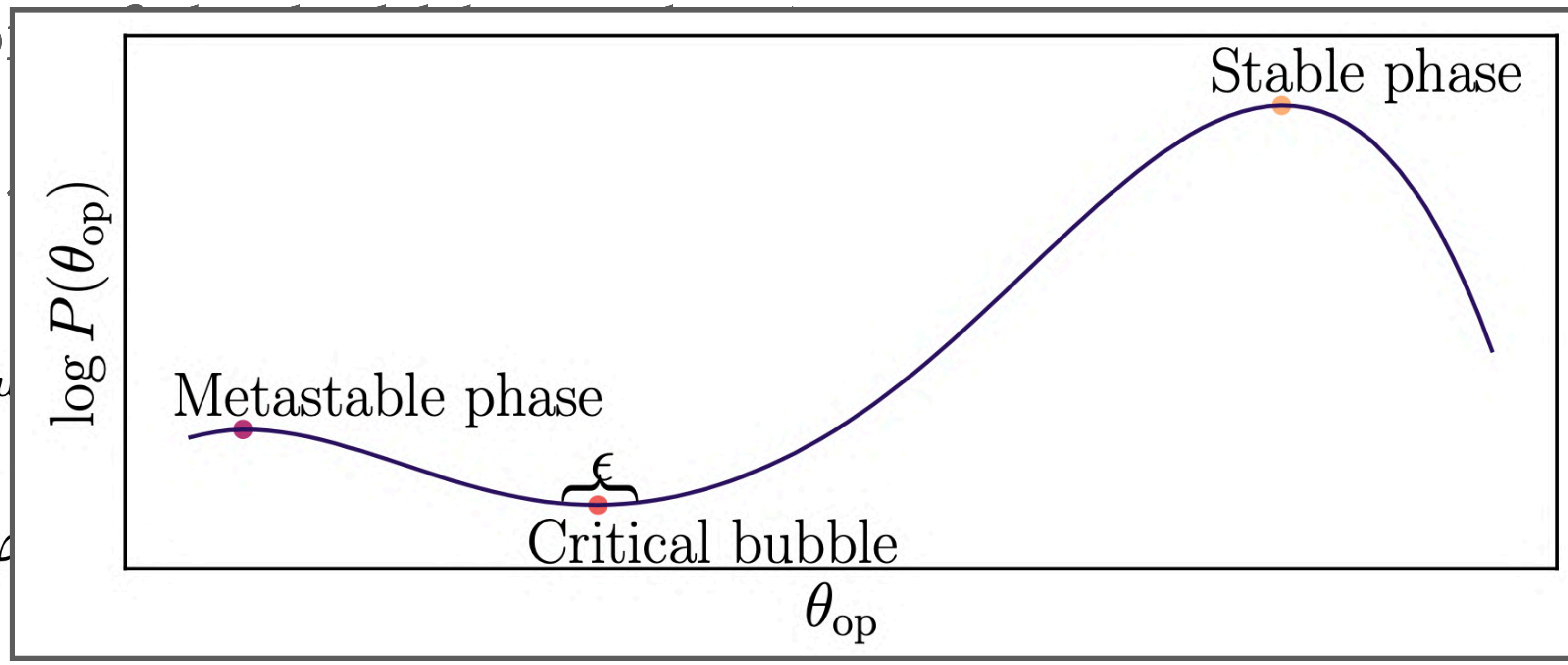
[Gould, Kormu, Weir 2404.01876]

➤ Calculatio

- Map your

$$\mathscr{L} = -\frac{1}{2}\partial_\mu \quad \ldots \quad \nabla^2_{\rm lat}\phi)_x + \sigma_{\rm lat}\phi_x$$

$$V(\varphi) = \sigma\varphi \quad \ldots \quad \frac{1}{4!}Z_\phi^2\lambda_{\rm lat}\phi_x^4\Big],$$



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MICROSCOPIC-4 … TICE SIMULATION

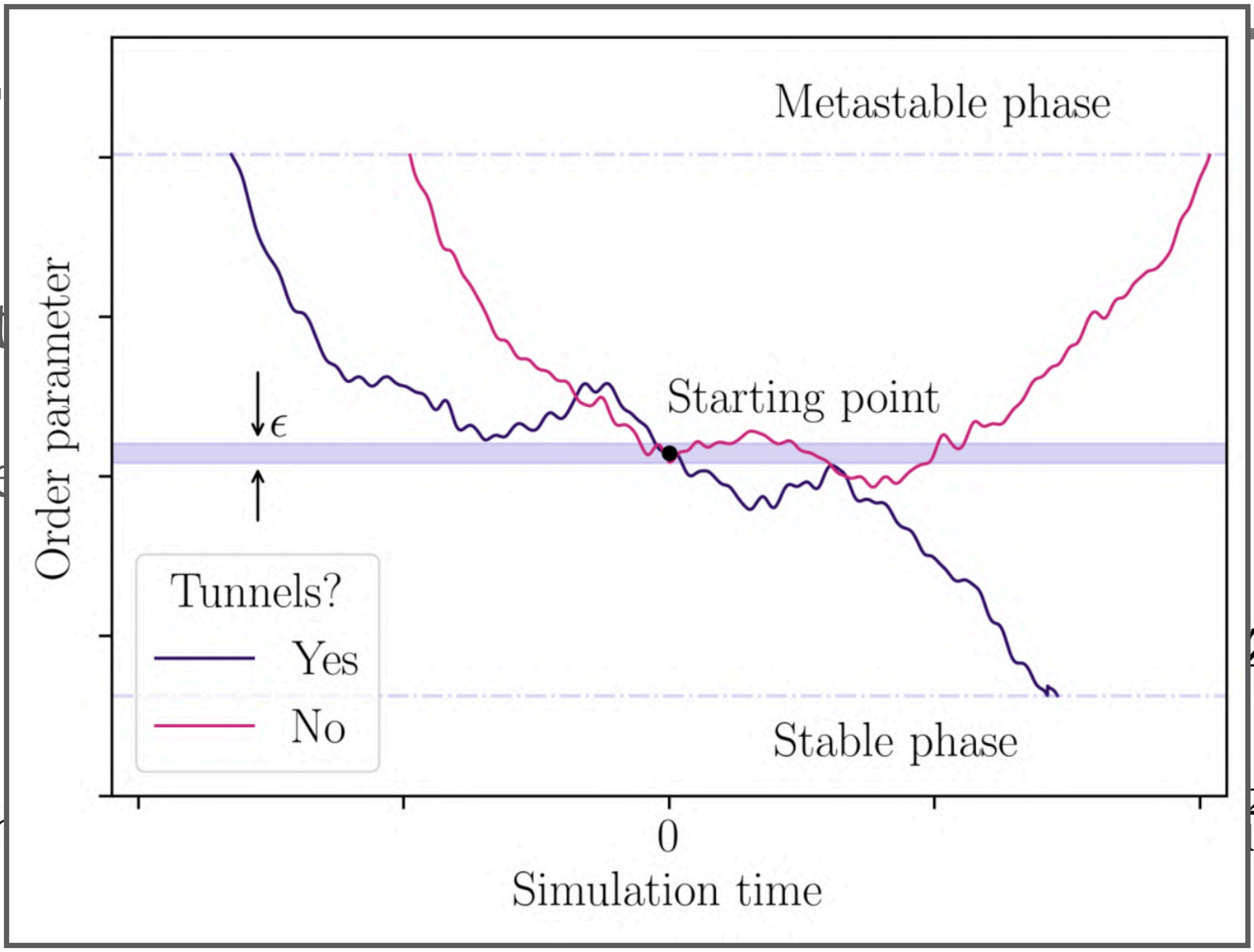
[Gould, Kormu, Weir 2404.01876]

➤ Calculation of t…

- Map your 4d the…

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi \quad \ldots \quad Z_\phi\phi_x(\nabla^2_{\rm lat}\phi)_x + \sigma_{\rm lat}\phi_x$$

$$V(\varphi) = \sigma\varphi + \frac{1}{2}m \quad \ldots \quad {}^2_{\rm lat}\phi_x^2 + \frac{1}{4!}Z_\phi^2\lambda_{\rm lat}\phi_x^4\Big],$$



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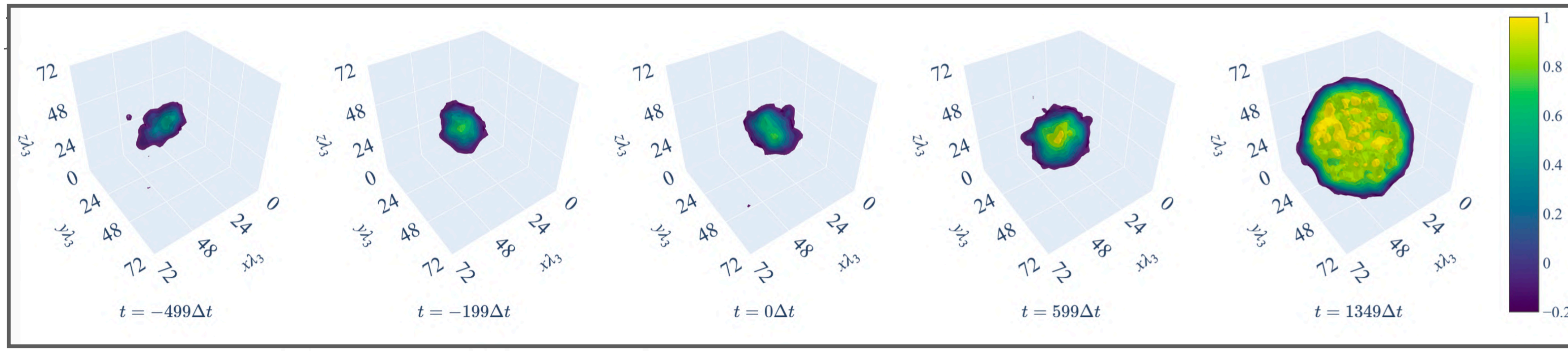
[Gould, Kormu, Weir 2404.01876]

➤ Calculation of the bubble nucleation rate



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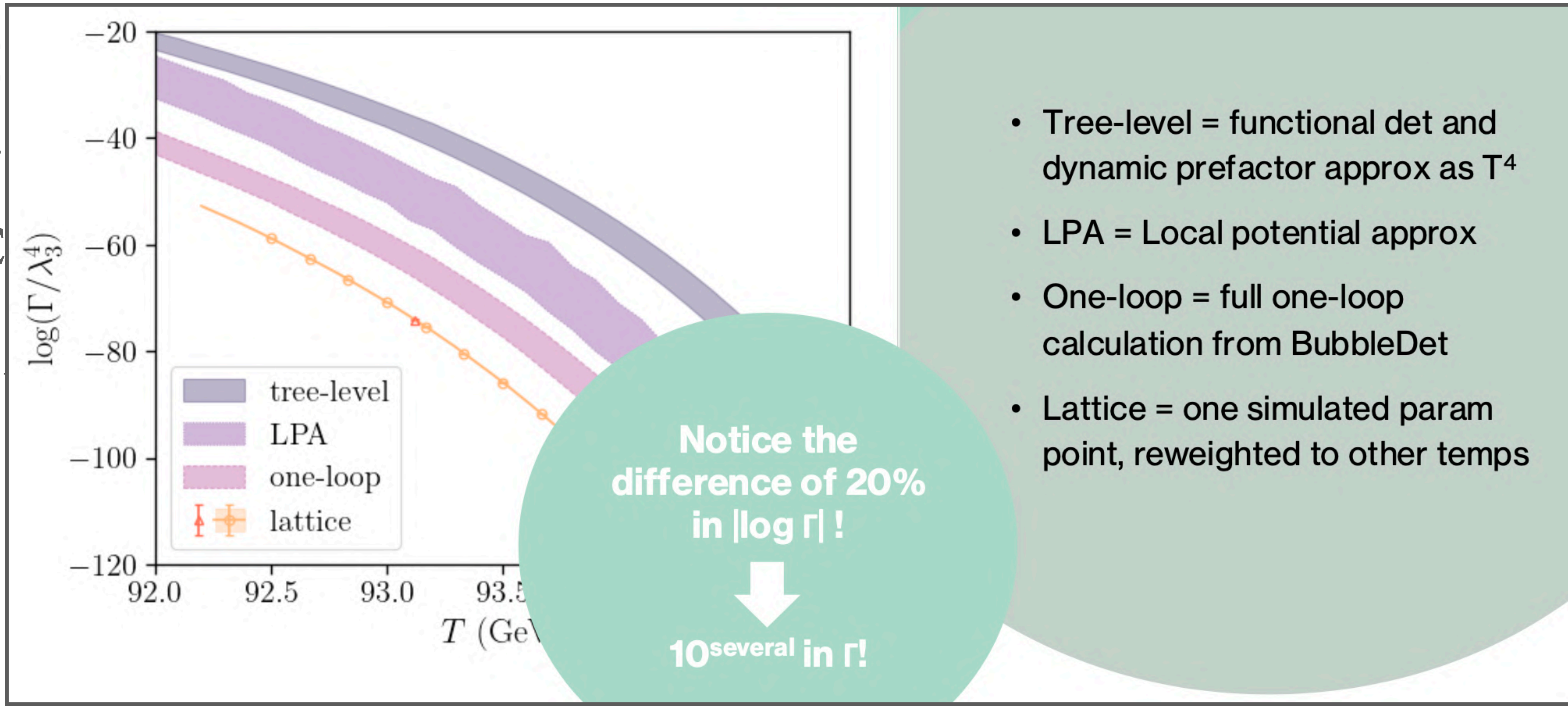

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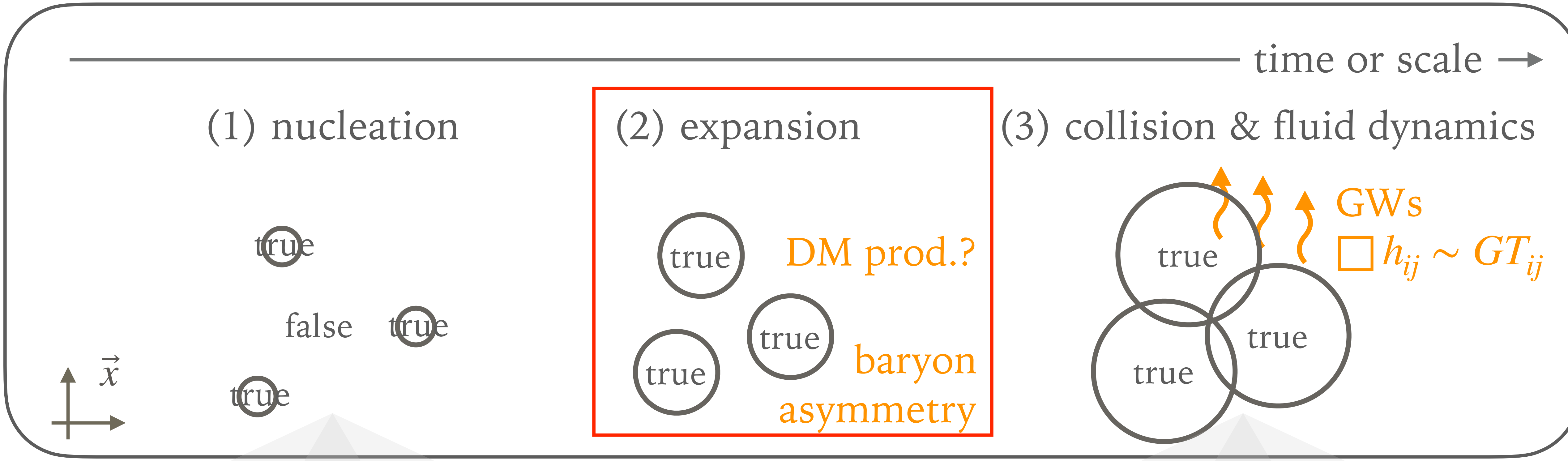
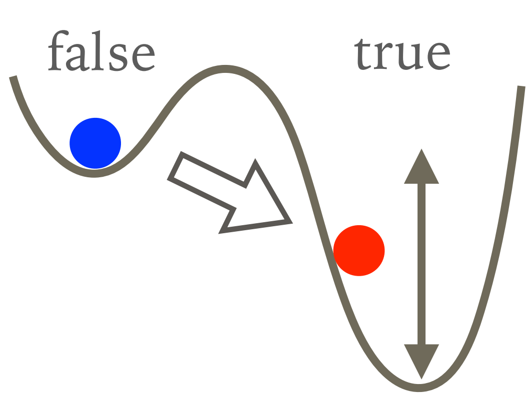
FIRST-ORDER PHASE TRANSITIONS IN THE EARLY UNIVERSE

microphysics

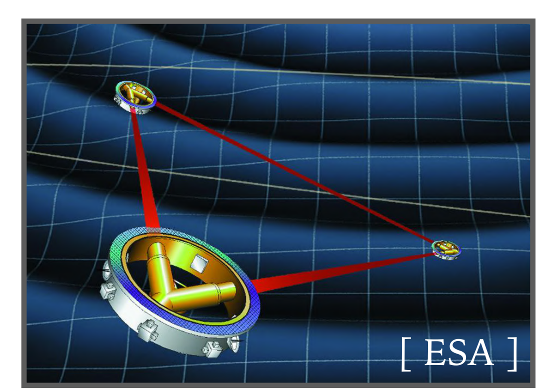
macrophysics



Physics of the Higgs sector



GW observations



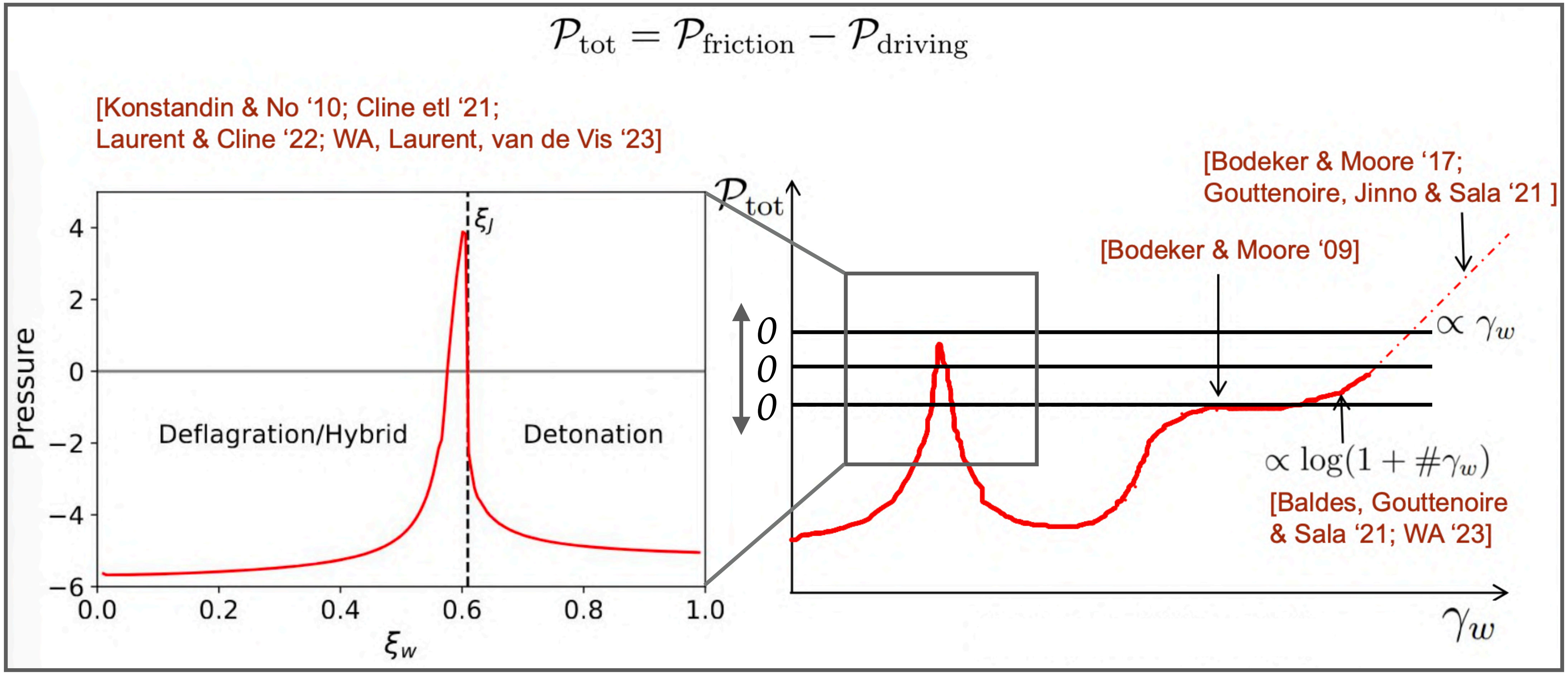
INTERMIDIATE SCALES

➤ Wall velocity is important

- GWs from sound waves $\Omega_{\rm GW} \sim v_w^5$ (for slow walls)

- Efficiency of EWBG, as well as other possible relics, depend crucially on v_w

内向き摩擦(壁の速度による) ↘ ↙ 外向き圧力(モデルを決めれば定数)



[from W. Y. Ai's slides]

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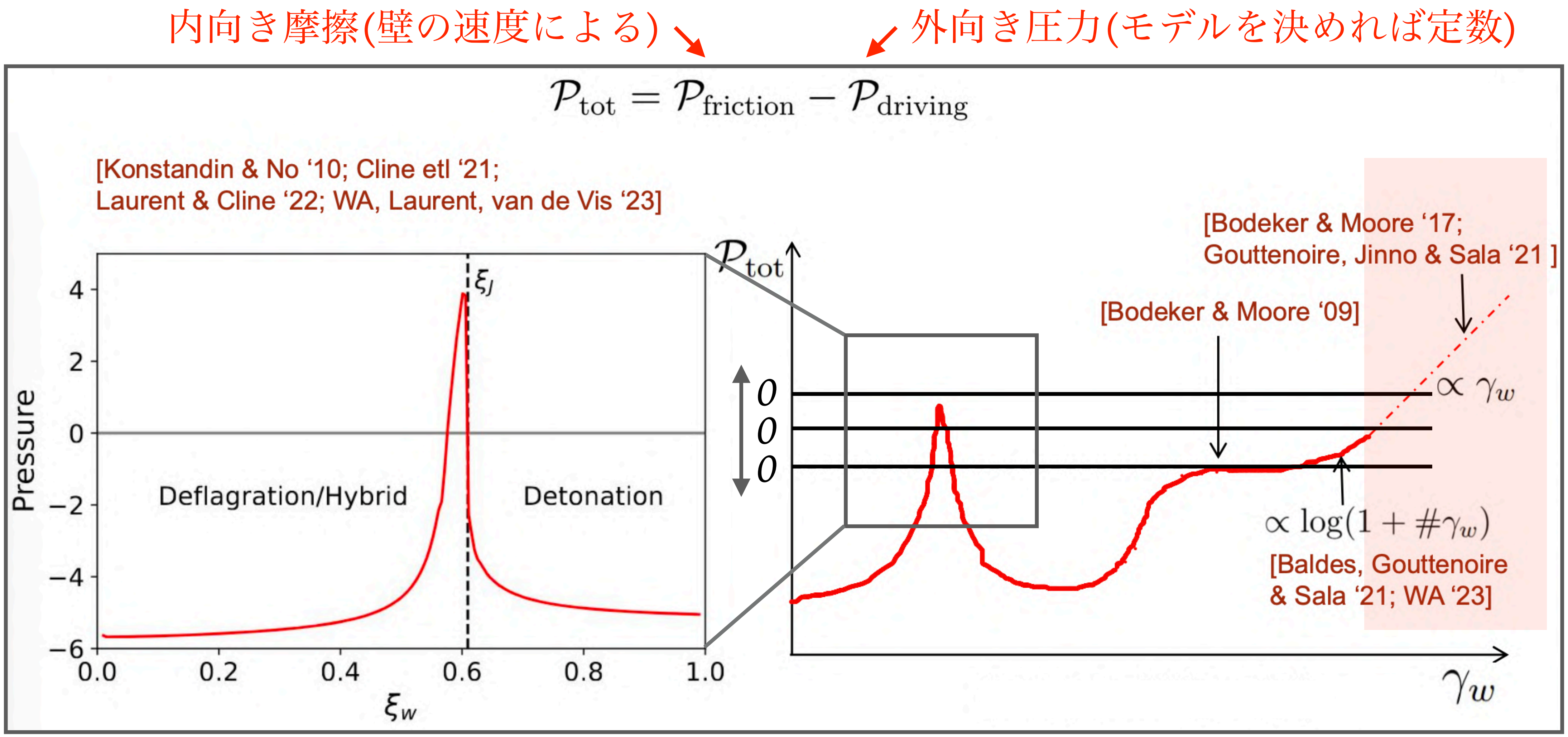
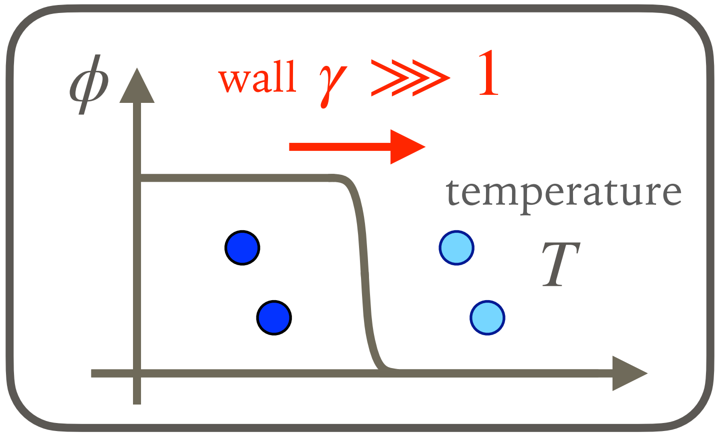


[from W. Y. Ai's slides]

INTERMIDIATE-1: FRICTION ON FAST WALLS

➤ Leading order friction

Plasma frame

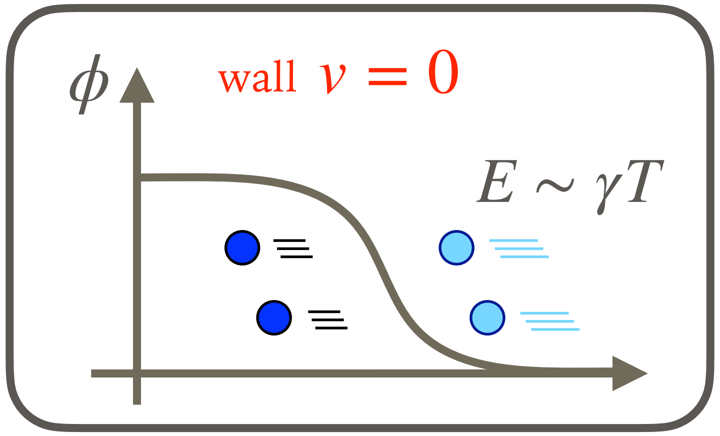


- Consider a wall moving with Lorentz factor $\gamma \gg 1$ in the plasma frame

INTERMIDIATE-1: FRICTION ON FAST WALLS

- Leading order friction

Rest frame of the wall



- Consider a wall moving with Lorentz factor $\gamma \gg 1$ in the plasma frame

- In the wall frame, each particle has huge energy $(\gg T, \langle \phi \rangle)$

- Upon impinging, each particle gives momentum $\Delta p_z = E - \sqrt{E^2 - m^2} \simeq \dfrac{m^2}{2E}$ to the wall

- After integrating over the phase space, the friction (= force/area = dim.-4 quantity) is

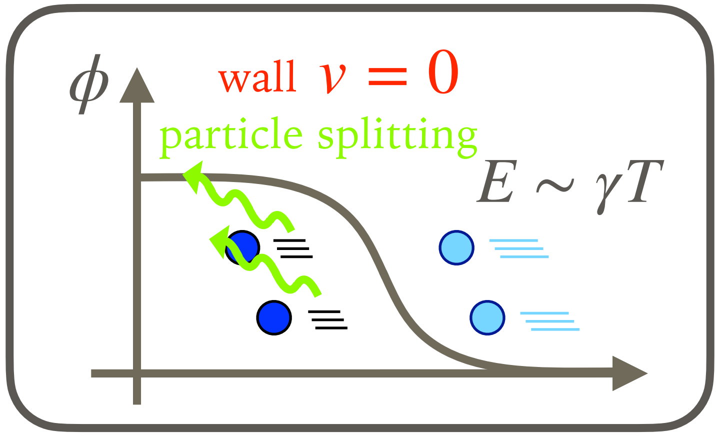
$$\mathscr{P} = \int \frac{d^3p}{(2\pi)^3} f_{\text{wall}}(p) \frac{m^2}{2E} \quad \text{(wall frame)} \quad = \int \frac{d^3p}{(2\pi)^3} f_{\text{thermal}}(p) \frac{m^2}{2E} \quad \text{(plasma frame)} \quad \sim m^2 T^2$$

INTERMIDIATE-1: FRICTION ON FAST WALLS

[Bodeker & Moore '17]

➤ Next-leading order friction (= particle splitting, transition splitting)

Rest frame of the wall



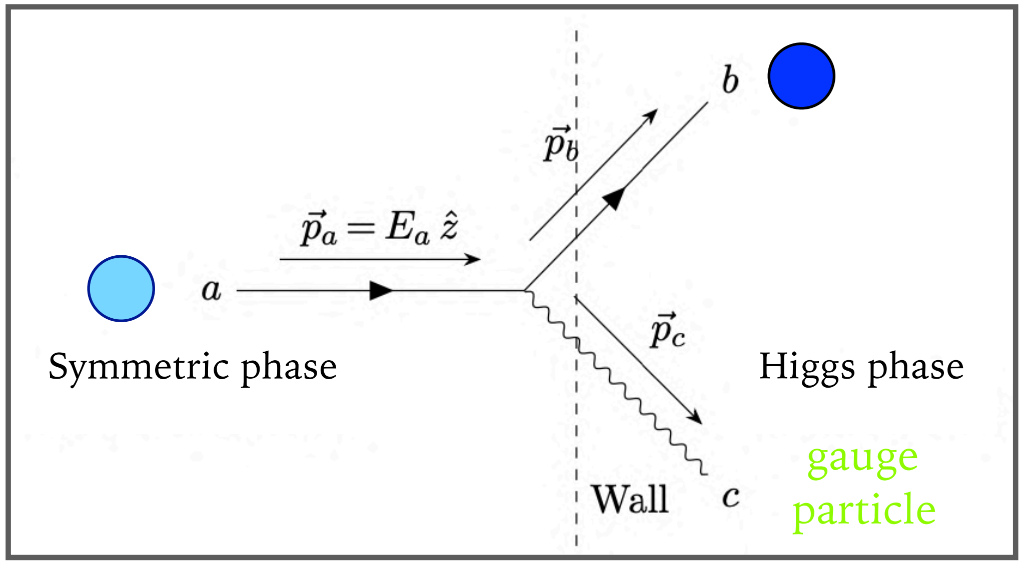
- Occurs when the impinging particles are gauge charged. The process is

$$a \to bc \qquad c : \text{gauge boson}$$

- The splitting probability involves the gauge coupling g , thus the name "next-leading"

- [B&M '17] showed that NLO friction dominates LO when the wall is moving fast.

INTERMIDIATE-1



- Results reported from several groups

  (1) [Bodeker & Moore '17]

  $a \to bc$ process gives friction proportional to the wall γ factor: $\mathscr{P} \sim \gamma m_c T^3$

  (2) [Hoeche, Kozaczuk, Long, Turner, Wang '20]

  multiple splitting $a \to bccc\cdots$ gives even stronger friction: $\mathscr{P} \sim \gamma^2 T^4$

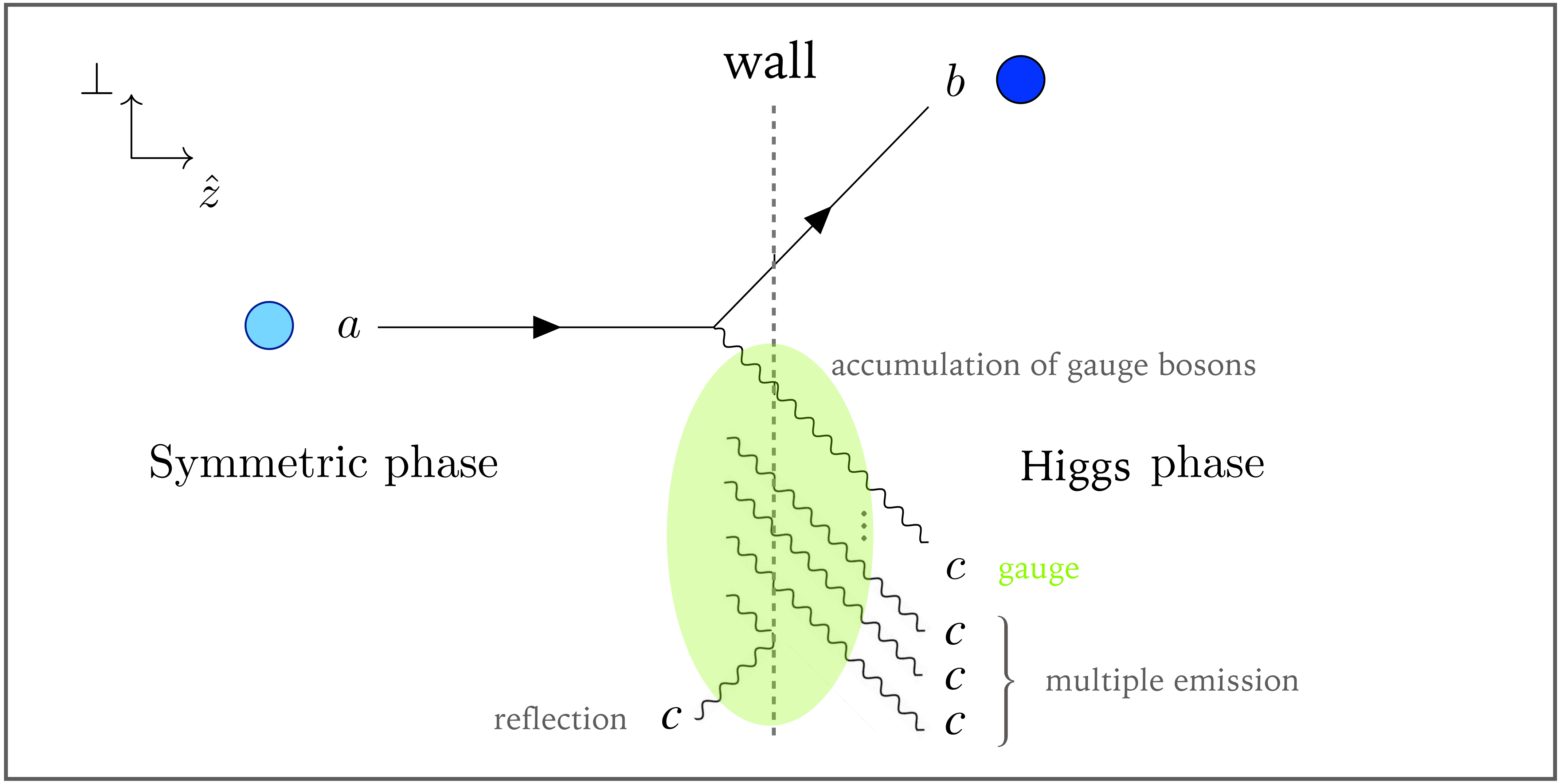
  (3) [Gouttenoire, Jinno, Sala '21]

  multiple splitting $a \to bccc\cdots$ rather gives $\mathscr{P} \sim \gamma m_c T^3$

- Now people basically agree on the scaling $\mathscr{P} \sim \gamma m_c T^3$

INTERMIDIATE-1: FRICTION ON FAST WALLS

- Ultimate goal



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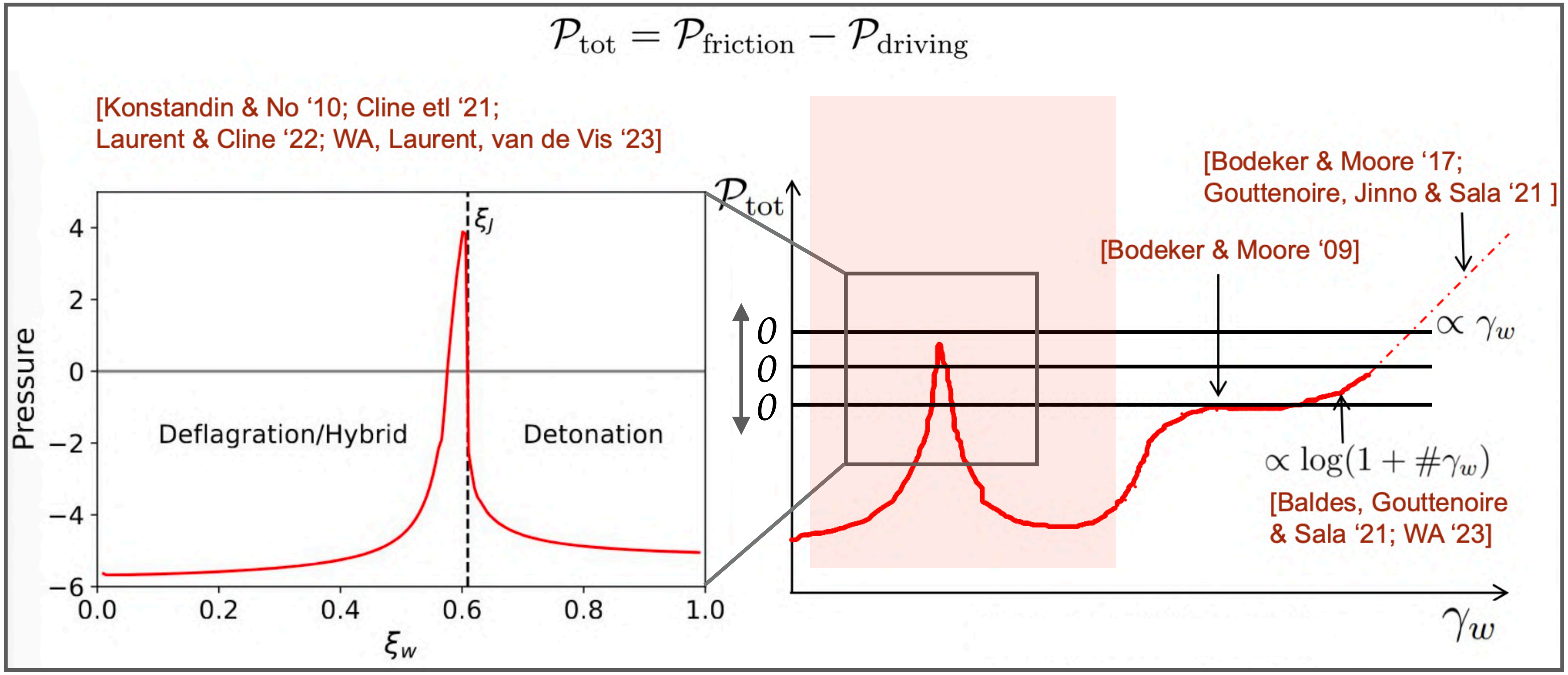


[from W. Y. Ai's slides]

INTERMIDIATE-2: WALL VELOCITY FROM ENTROPY CONSERVATION

➤ Wall velocity from entropy conservation (fluid picture from now on)

In general, we have 5 unknown quantities:

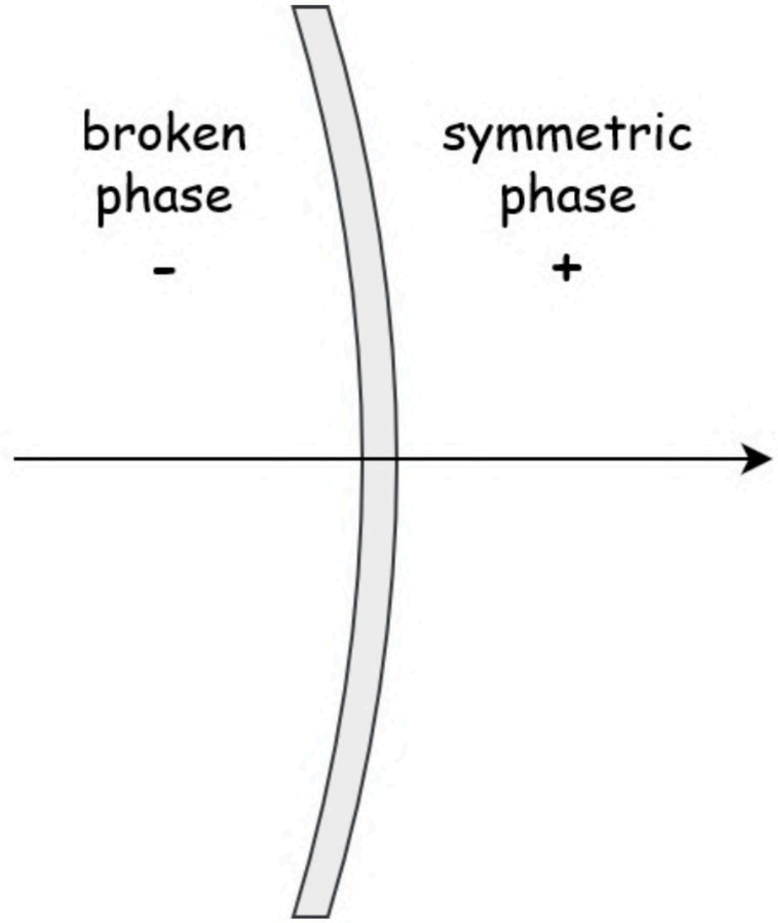
- Plasma temperatures $T_\pm$,
- Plasma velocities $v_\pm$,
- Wall velocity v_w.

2 are known from **boundary conditions**, and 2 can be obtained from **conservation of energy-momentum**

$$\partial_\mu T^{\mu\nu} = 0$$

$$\Rightarrow w_+ \gamma_+^2 v_+ = w_- \gamma_-^2 v_-$$

$$w_+ \gamma_+^2 v_+^2 + p_+ = w_- \gamma_-^2 v_-^2 + p_-$$



W. Ai, BL and J. van de Vis (2303.10171)

We still need one final equation!

[from B. Laurent's slides]

INTERMIDIATE-2: WALL VELOCITY FROM ENTROPY CONSERVATION

➤ Proposal to use entropy conservation as the final equation

Local thermal equilibrium

When $\Gamma \to \infty$, $f(z) \to f_{\rm eq}(T(z), v_{\rm pl}(z))$ and **entropy is conserved**:

$$\partial_\mu (u^\mu_{\rm pl} s) = 0,$$

$$\Rightarrow s_+ \gamma_+ v_+ = s_- \gamma_- v_- .$$

This third conservation equation can be used to determine all the unknown quantities $T_\pm$, $v_\pm$ and v_w.

[from B. Laurent's slides]

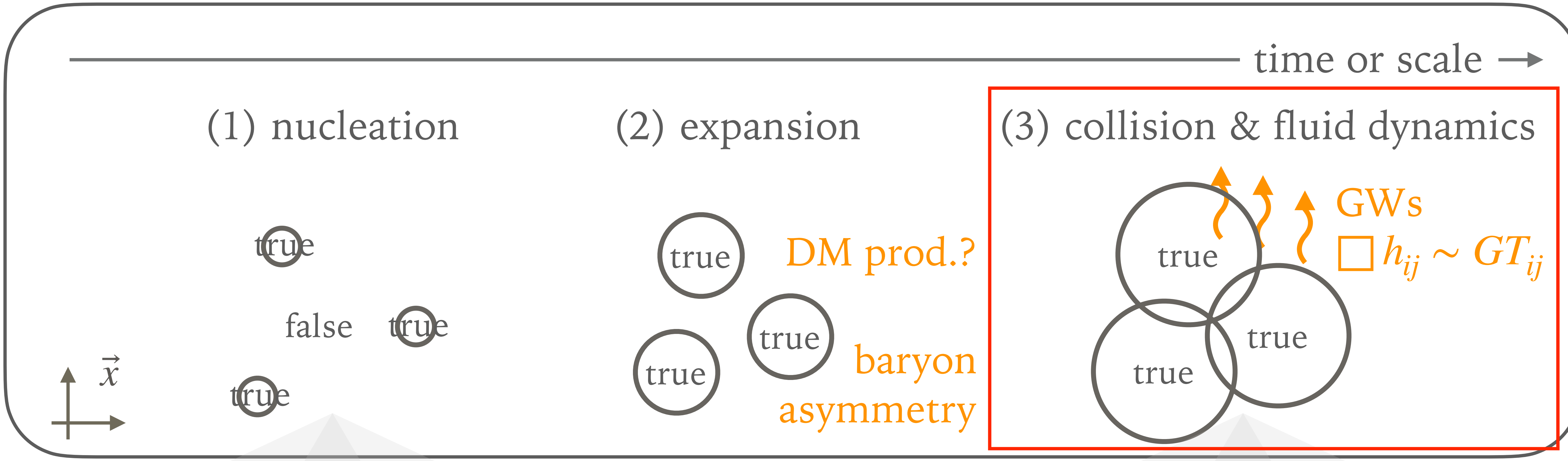
➤ However, to what extent is entropy conserved in bubble expansion?

- See e.g. [M. Laine 2507.07755]

FIRST-ORDER PHASE TRANSITIONS IN THE EARLY UNIVERSE

microphysics

macrophysics



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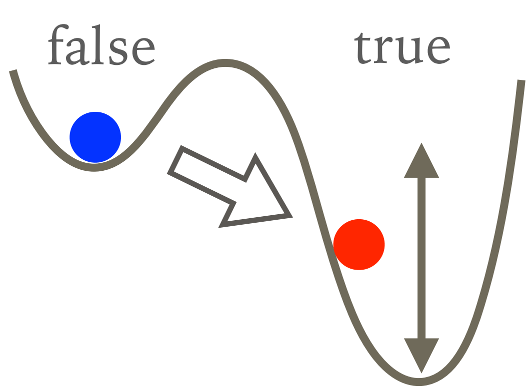
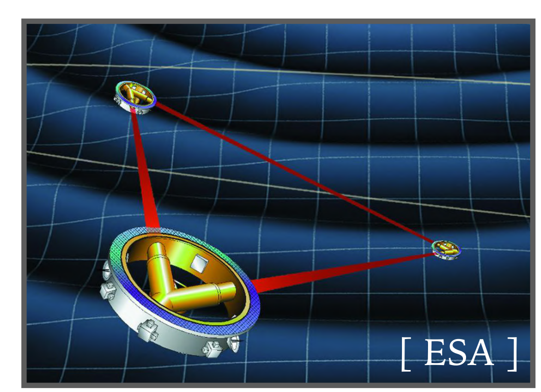


GW observations



MACROSCOPIC-1: HIGGSLESS SIMULATION

[Kosowsky, Turner, Watkins '92]
[Kosowsky, Turner '92]
[Kamionkowski, Kosowsky, Turner '93]
and e.g. [Caprini et al. '16] [Caprini et al. '20]

➤ Bubble collision

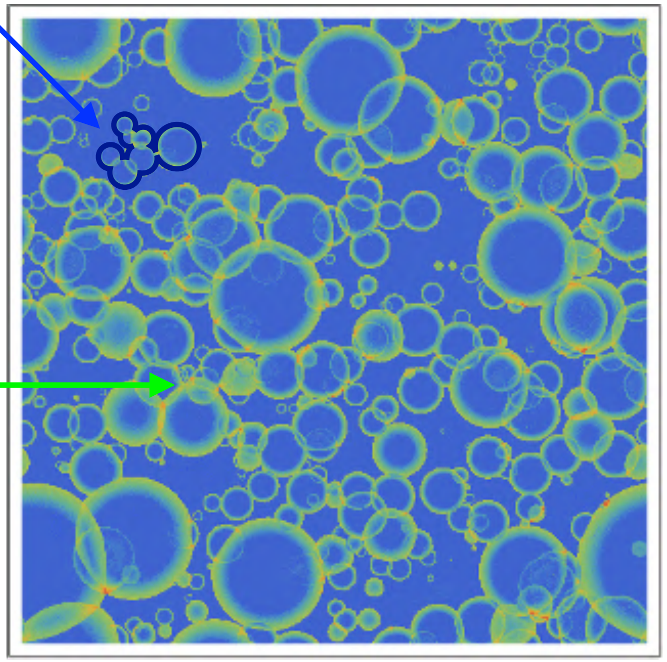
- Kinetic & gradient energy of the scalar field (= order parameter field)
- Dominant when the transition is extremely strong and the walls runaway

➤ Sound waves

- Compression mode of the fluid motion
- Dominant unless the transition is extremely strong

➤ Turbulence

- Turbulent motion caused by fluid nonlinearity
- Expected to develop at a later stage



important at later stage

MACROSCOPIC-1: HIGGSLESS SIMULATION

- Fluid 3d simulation is harder than you might imagine:

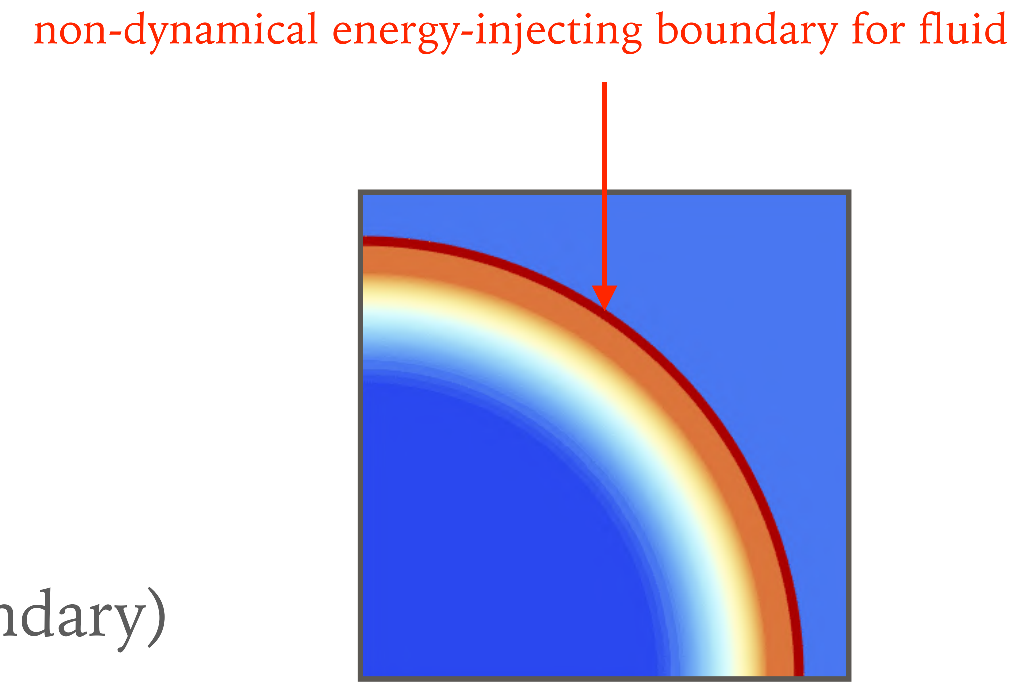
  - Shock waves
  - Numerical viscosity → currently only 2 groups, working on sound wave simulations
  - Computational resources

- Our proposal: Higgsless scheme

  [RJ, Konstandin, Rubira '21] [RJ, Konstandin, Rubira, Stomberg '22]

  [Caprini, RJ, Konstandin, Roper Pol, Rubira, Stomberg '24]

  - We do ***not*** solve both the scalar field and fluid but rather "integrate out" the scalar field (= treat the scalar field as non-dynamical boundary)



MACROSCOPIC-1: HIGGSLESS SIMULATION

- The fluid evolution is determined from

  ① Energy-momentum conservation of the fluid $\partial_\mu T^{\mu\nu} = 0$

  ② Energy injection at the wall, parametrized by $\epsilon_{\rm vac} = \begin{cases} \epsilon_f & \text{(false vac.)} \\ \epsilon_t & \text{(true vac.)} \end{cases}$

- How can we implement these in simulations?

  ① Assume relativistic perfect fluid (for simplicity), $T^{\mu\nu} = wu^\mu u^\nu - g^{\mu\nu} p$

  ② Define $K^\mu \equiv T^{\mu 0}$, then $\partial_\mu T^{\mu\nu} = 0$ reduces to $\begin{cases} \partial_0 K^0 + \partial_i K^i = 0 \\ \partial_0 K^i + \partial_j T^{ij}(K^0, K^i) = 0 \end{cases}$

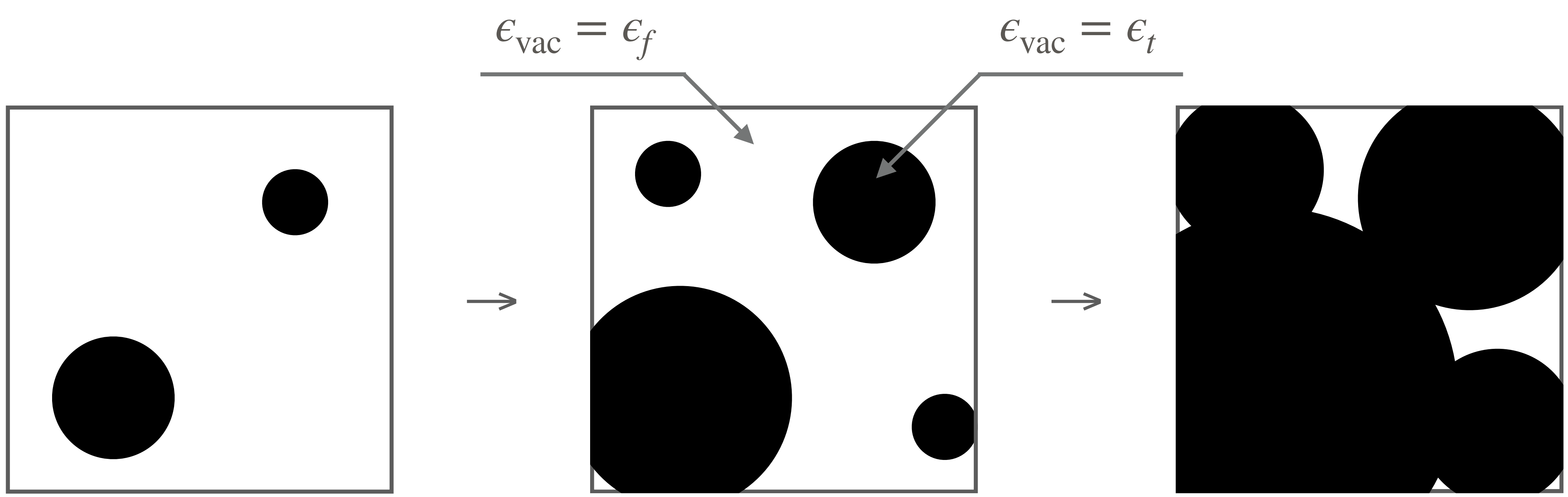
  ③ Where does the energy injection enter? Answer: in $T^{ij}(K^0, K^i)$

$$T^{ij}(K^0, K^i) = \frac{3}{2} \frac{K^i K^j}{(K^0 - \epsilon_{\rm vac}) + \sqrt{(K^0 - \epsilon_{\rm vac})^2 - \frac{3}{4} K^i K^i}}$$

MACROSCOPIC-1: HIGGSLESS SIMULATION

- We first numerically generate nucleation points,

  and determine the false-true boundary of the bubbles



- We then evolve the fluid in this box according to $\begin{cases} \partial_0 K^0 + \partial_i K^i = 0 \\ \partial_0 K^i + \partial_j T^{ij}(K^0, K^i) = 0 \end{cases}$

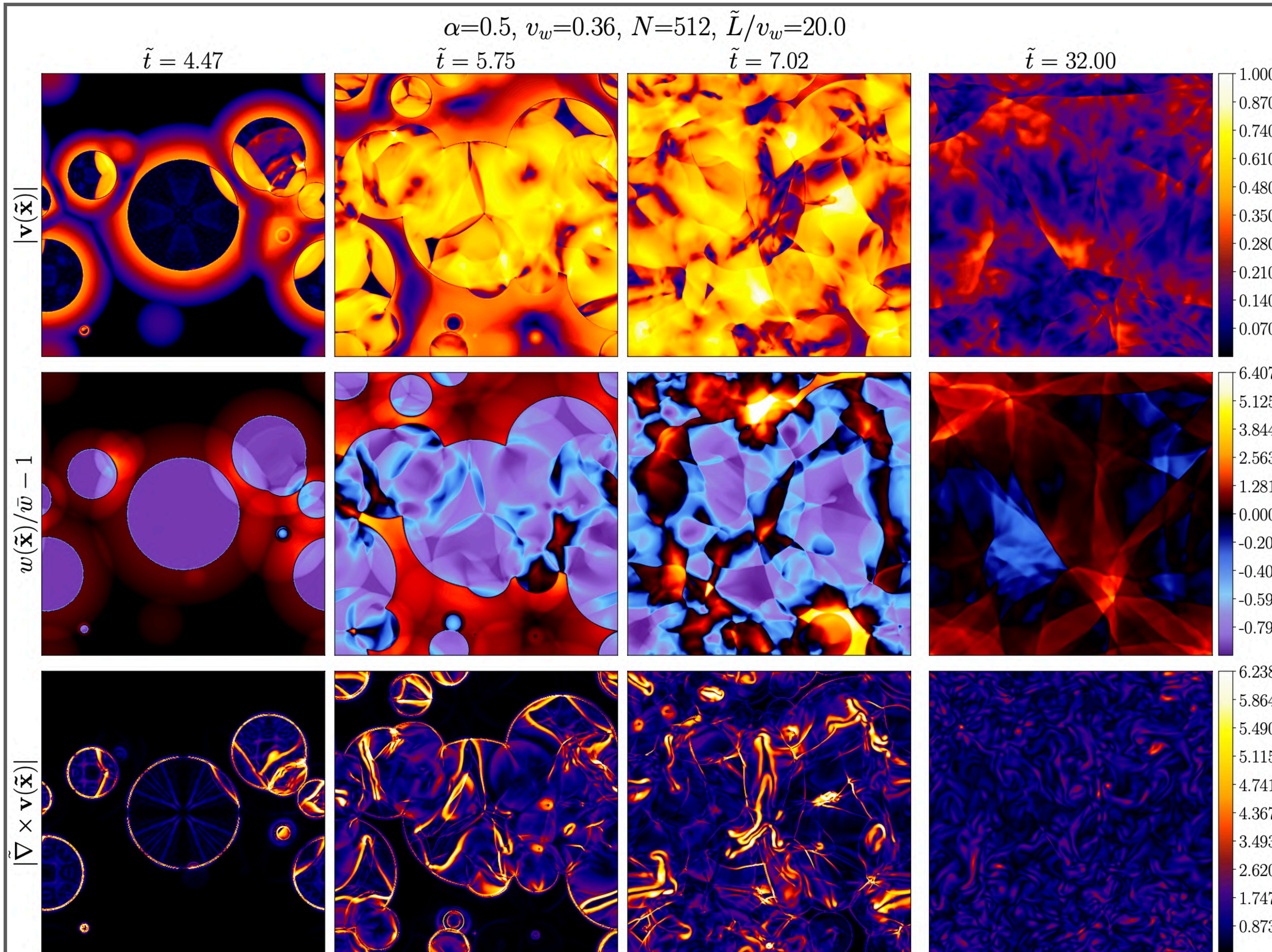
  $\rightarrow$ Fluid automatically develops profiles

MACROSCOPIC-1: HIGGSLESS SIMULATION

fluid velocity

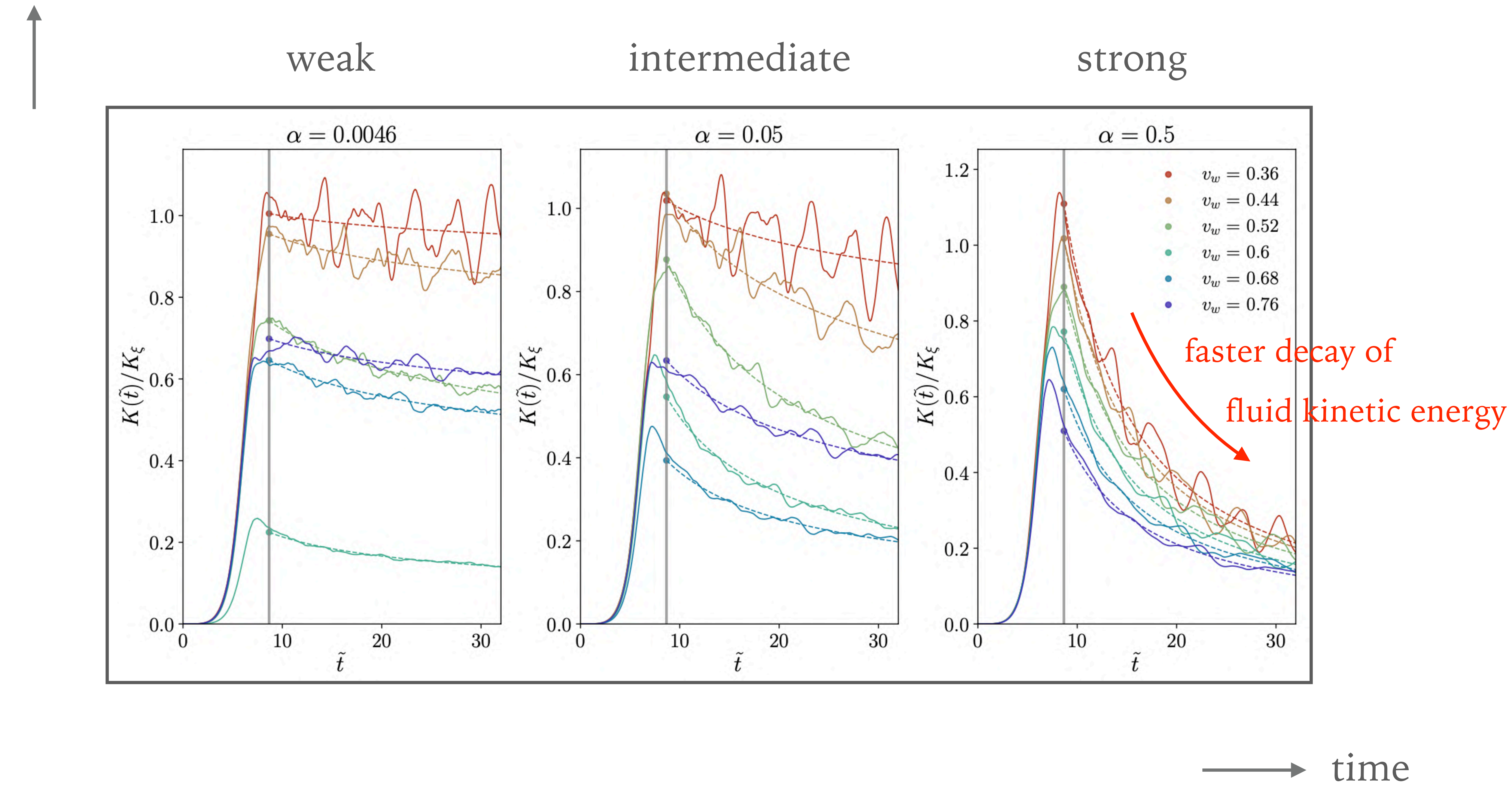
enthalpy

vorticity



MACROSCOPIC-1: HIGGSLESS SIMULATION

fluid kinetic energy

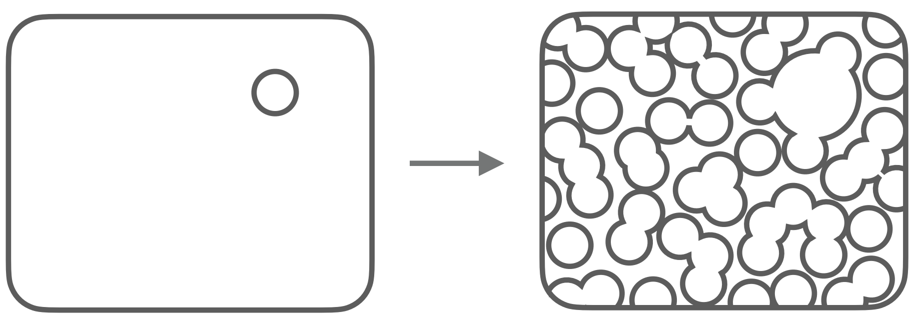


MACROSCOPIC-2: STRONGEST TYPE OF FOPT

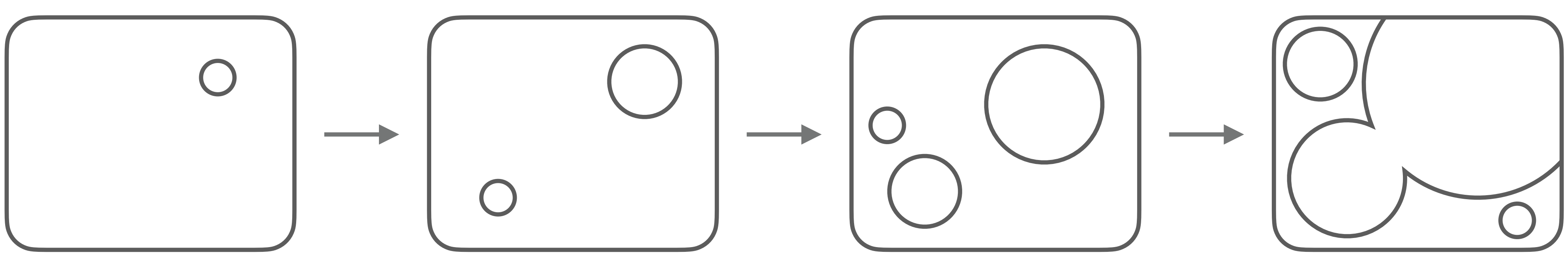
[Randall, Servant '07]
[Espinosa, Konstandin, No, Quiros '08]

- If the microphysics model is nearly scale invariant, bubbles grow big, resulting in huge GW production

Typical models



Nearly scale-invariant models



the system looks almost the same at different temperatures → gradual nucleation of bubbles

time ↗
temperature ↘

$T = T_{\rm initial}$ $\qquad$ $T \ll T_{\rm initial}$

MACROSCOPIC-2: STRONGEST TYPE OF FOPT

[Iso, Okada, Orikasa '09]
[RJ, Takimoto '16]
[Iso, Serpico, Shimada '17]

➤ One example: Classically conformal B-L model

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	**3**	**2**	+1/6	+1/3
u_R^i	**3**	**1**	+2/3	+1/3
d_R^i	**3**	**1**	-1/3	+1/3
l_L^i	**1**	**2**	+1/6	-1
e_R^i	**1**	**1**	-1	-1
ν_R^i	**1**	**1**	0	-1
H	**1**	**2**	-1/2	0
Φ	**1**	**1**	0	+2

TABLE I: Matter contents of the classically conformal $B-L$ model. In addition to the standard model matters, three generations of right-handed neutrinos ν_R^i and a $B-L$ charged complex scalar field Φ are introduced.

MACROSCOPIC-2: STRONGEST TYPE OF FOPT

[Iso, Okada, Orikasa '09]
[RJ, Takimoto '16]
[Iso, Serpico, Shimada '17]

➤ Assumption: absence of mass scales

$$V_0 = \lambda_H |H|^4 + \lambda |\Phi|^4 - \lambda' |\Phi|^2 |H|^2$$

- Only quartic terms in the potential: no parameters with a finite mass dimension

- Scale dependence enters only through running of couplings

- Phase transition in Φ direction can be extremely strong

MACROSCOPIC-2: STRONGEST TYPE OF FOPT

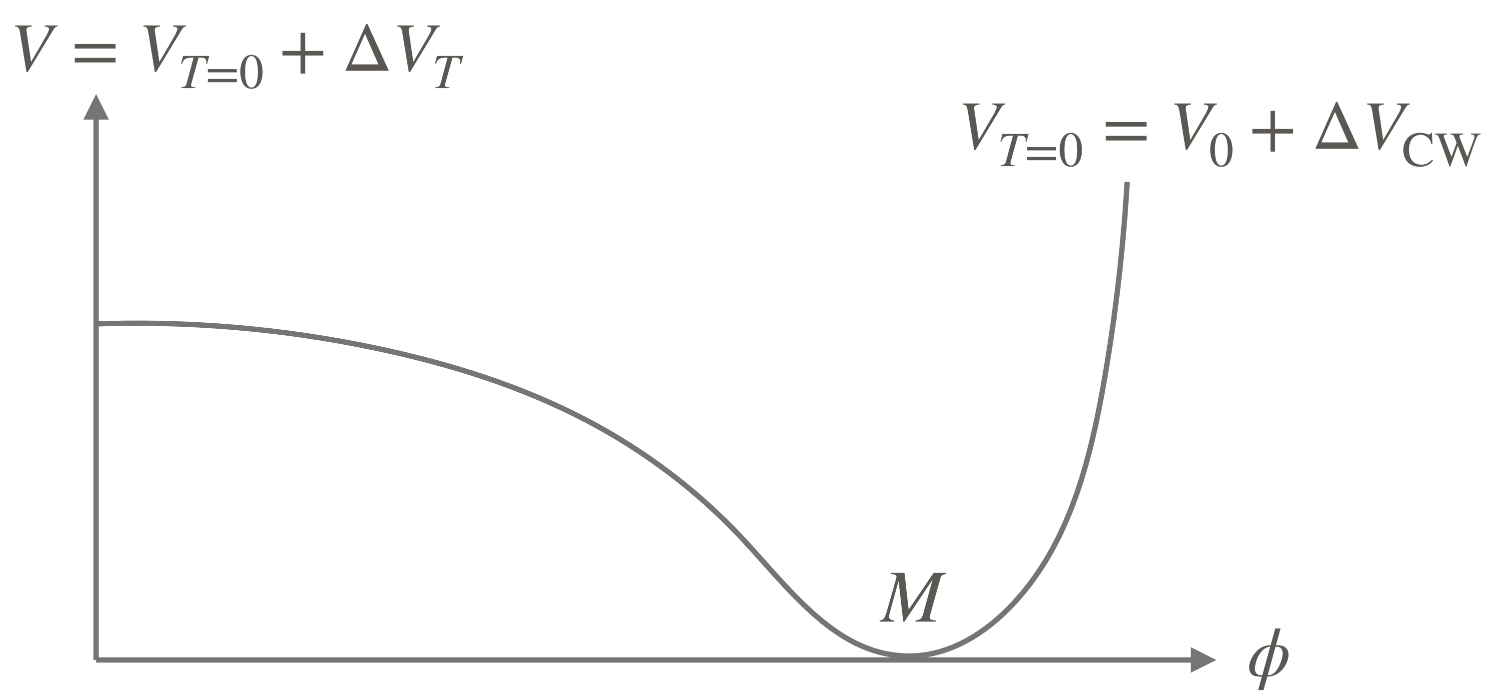
➤ Zero-temperature behavior

- The zero-temperature potential including loop corrections behaves as

  $\phi^4 \times$ (running coupling constant)

$$V(\phi) \sim \lambda(\phi)\phi^4$$

- The B-L scale M is generated a la Coleman-Weinberg

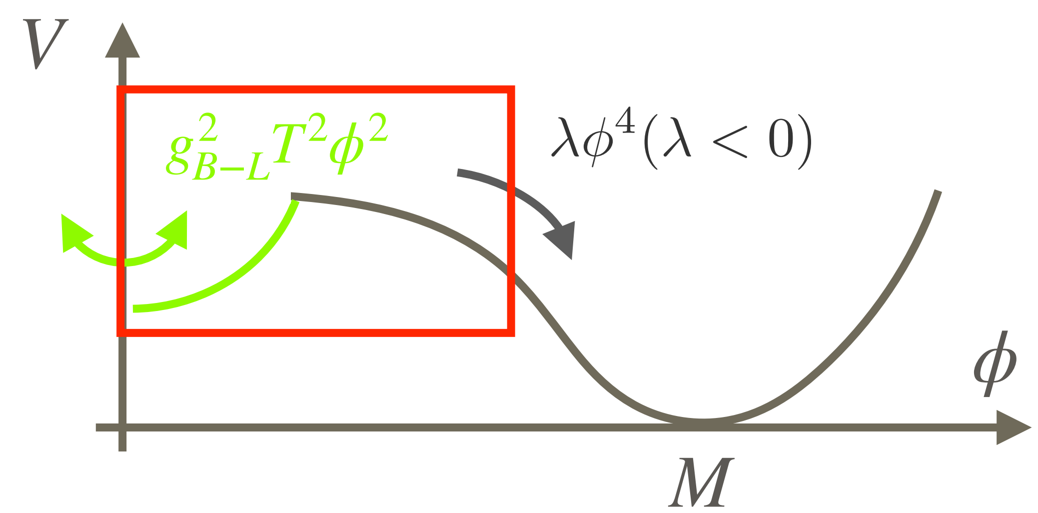


MACROSCOPIC-2: STRONGEST TYPE OF FOPT

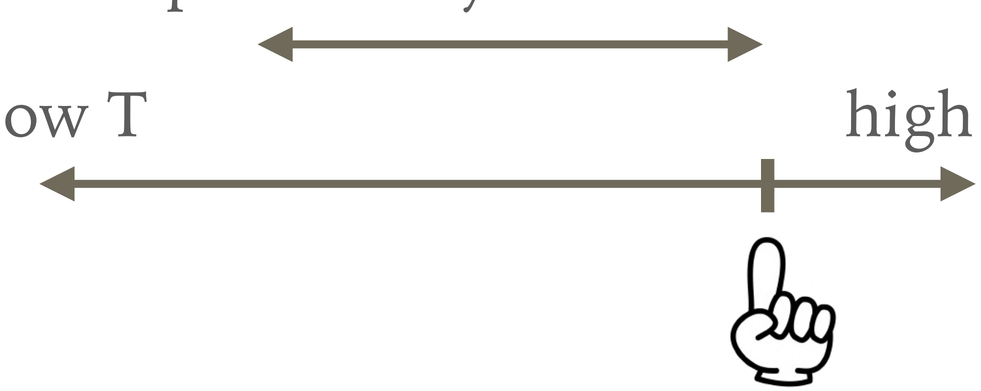
➤ Finite-temperature behavior

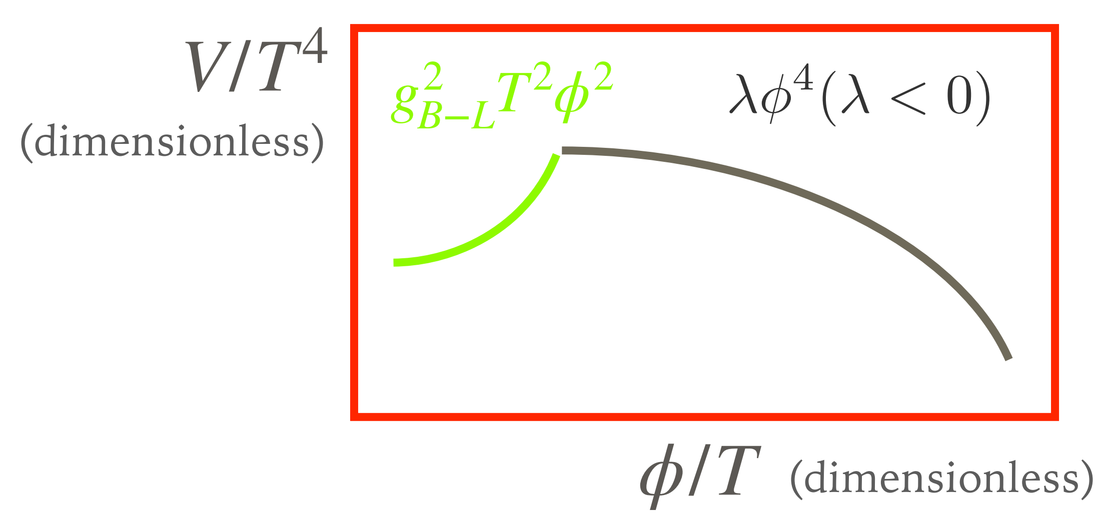
- Thermal corrections create a quadratic trap, which persists down to small T

$$V \sim g_{B-L}^2 T^2 \phi^2 + \lambda(\max(T,\phi))\phi^4$$



- Behavior of the potential as the temperature decreases





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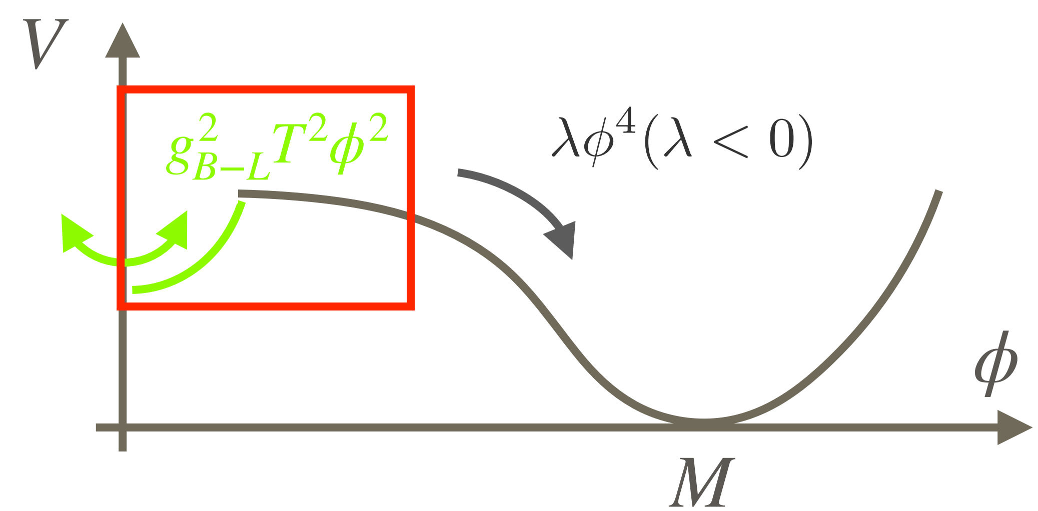

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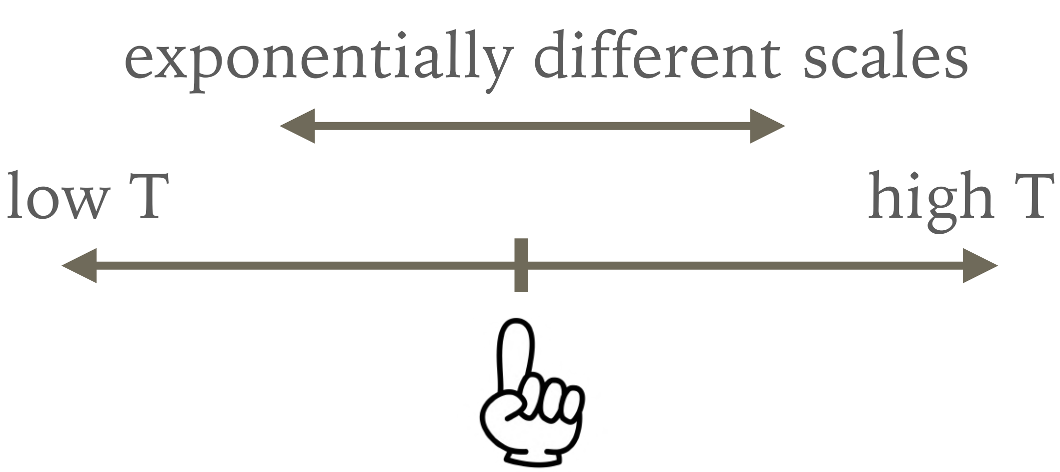

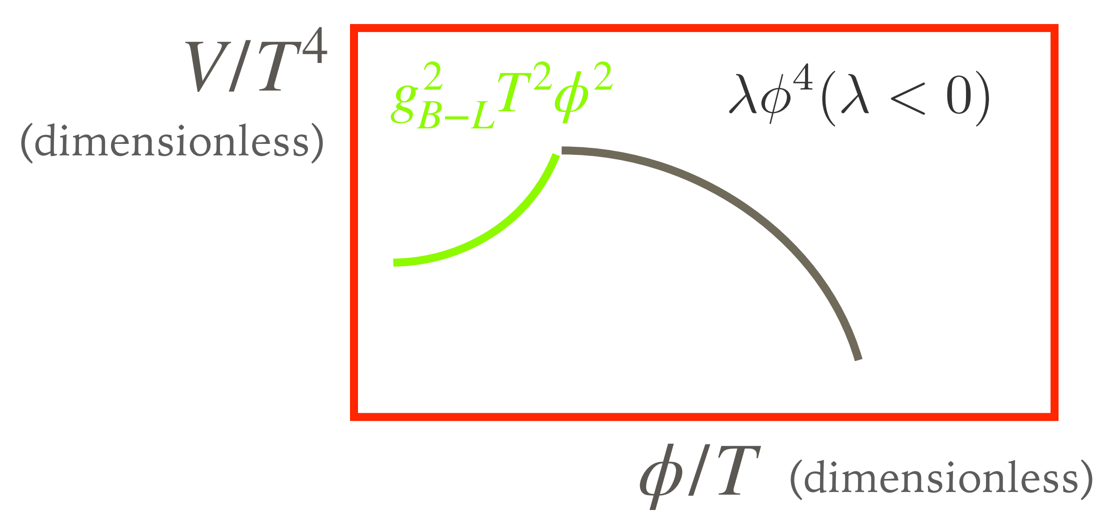

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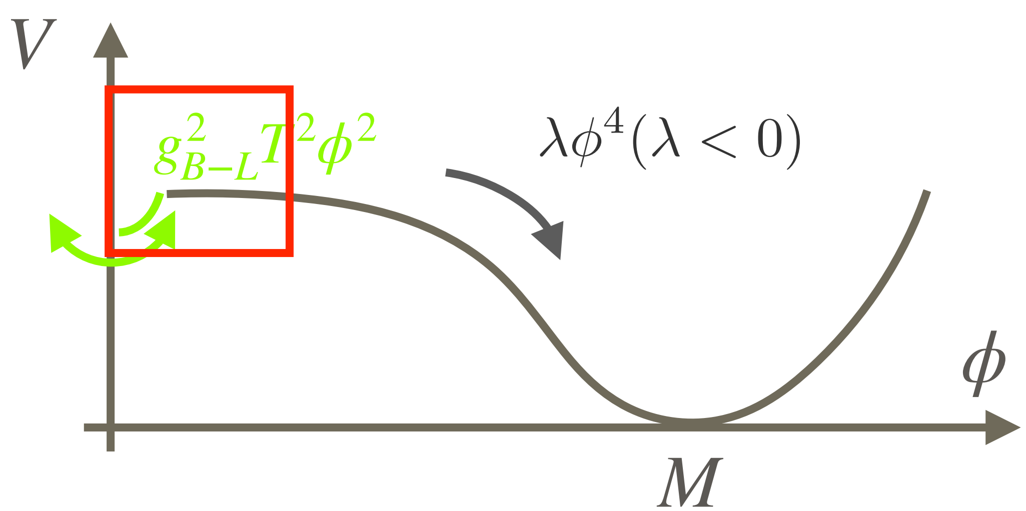

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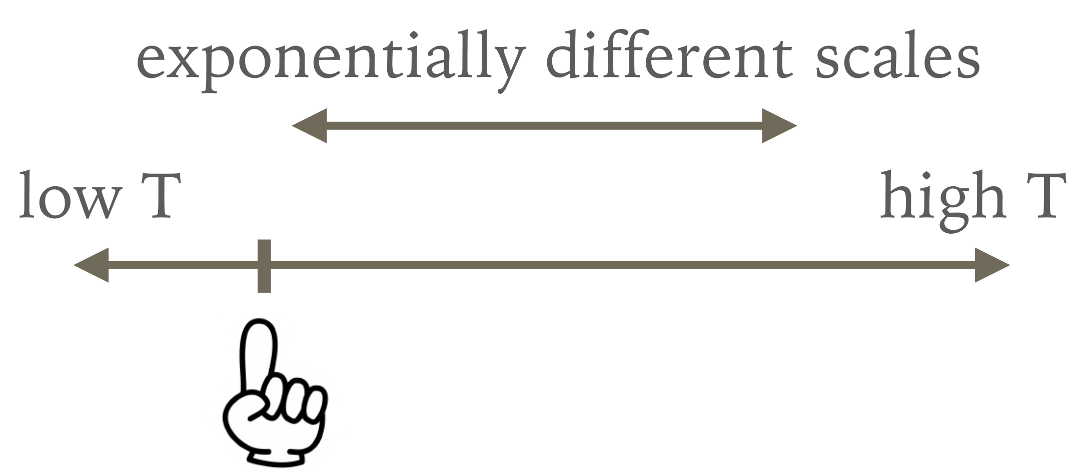
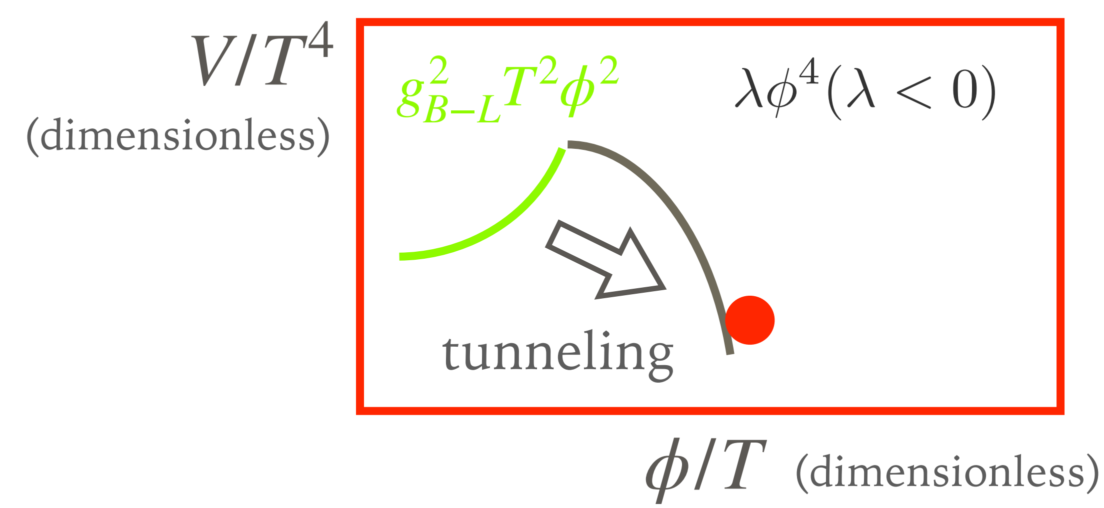
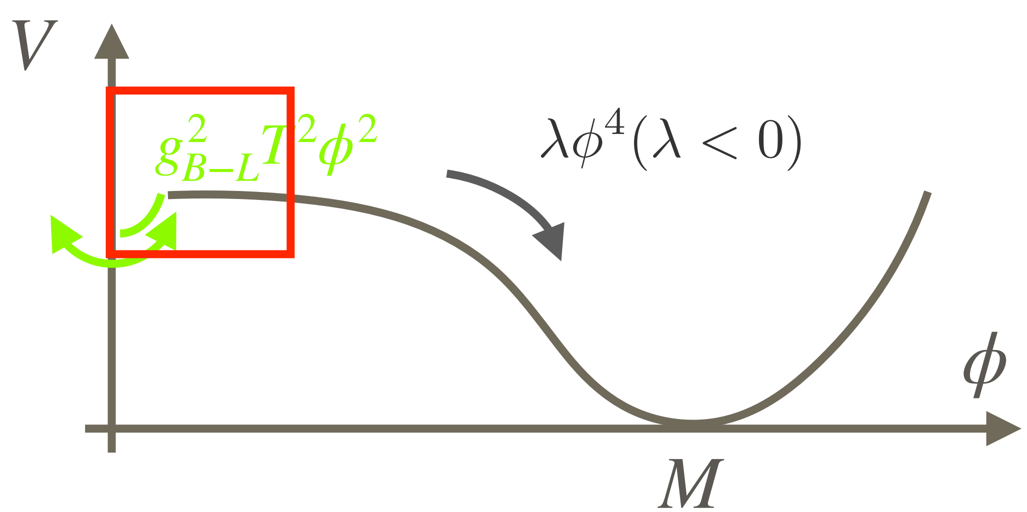




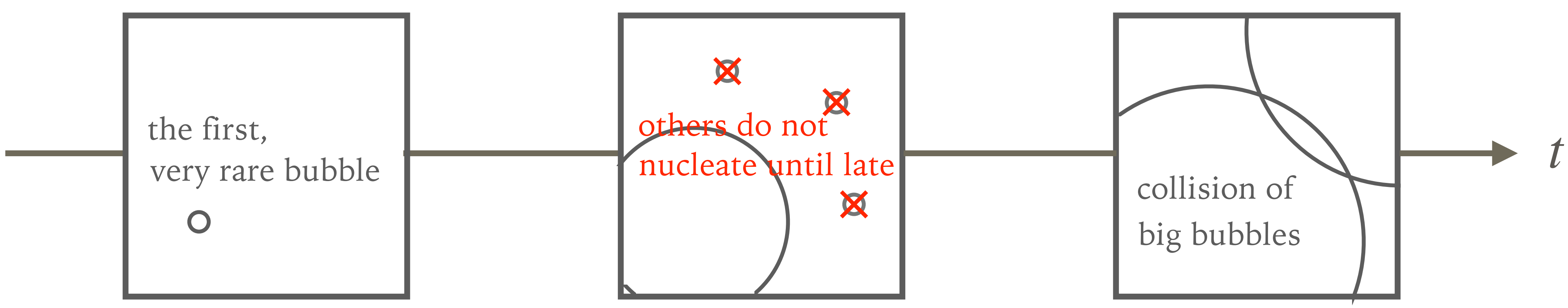
MACROSCOPIC-2: STRONGEST TYPE OF FOPT

- When the B-L field tunnels and settles down, the energy release is huge



$\rightarrow$ large $\alpha \equiv \frac{\rho_{\rm vac}}{\rho_{\rm rad}}$

- The system changes only logarithmically, so bubble nucleation is slow

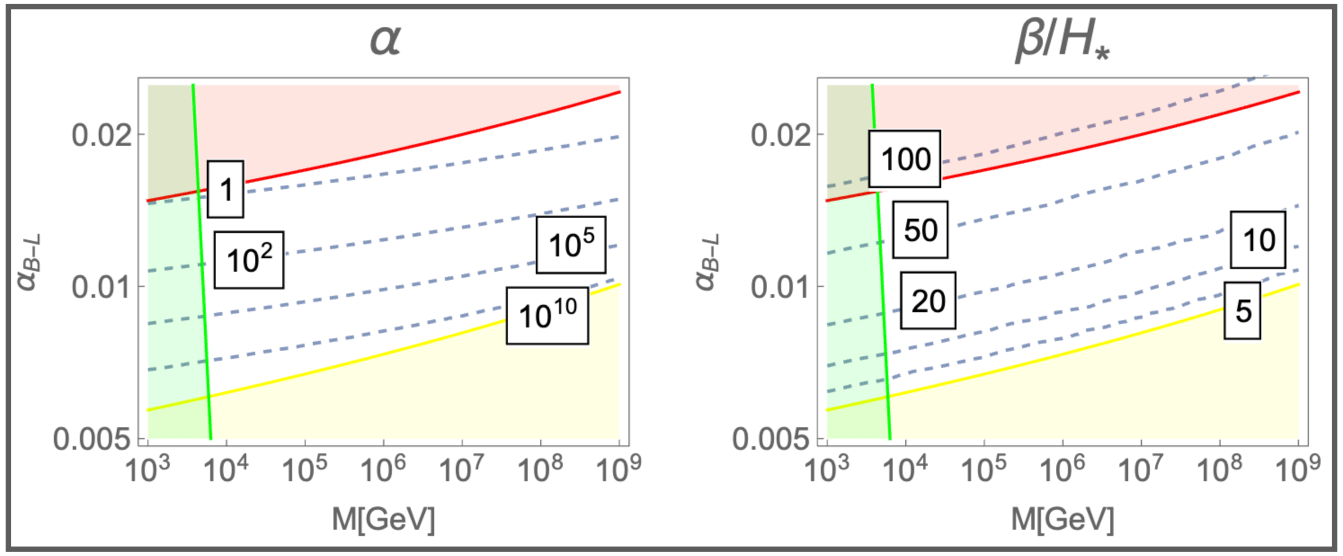


$\rightarrow$ collision of big bubbles (small β/H_*)

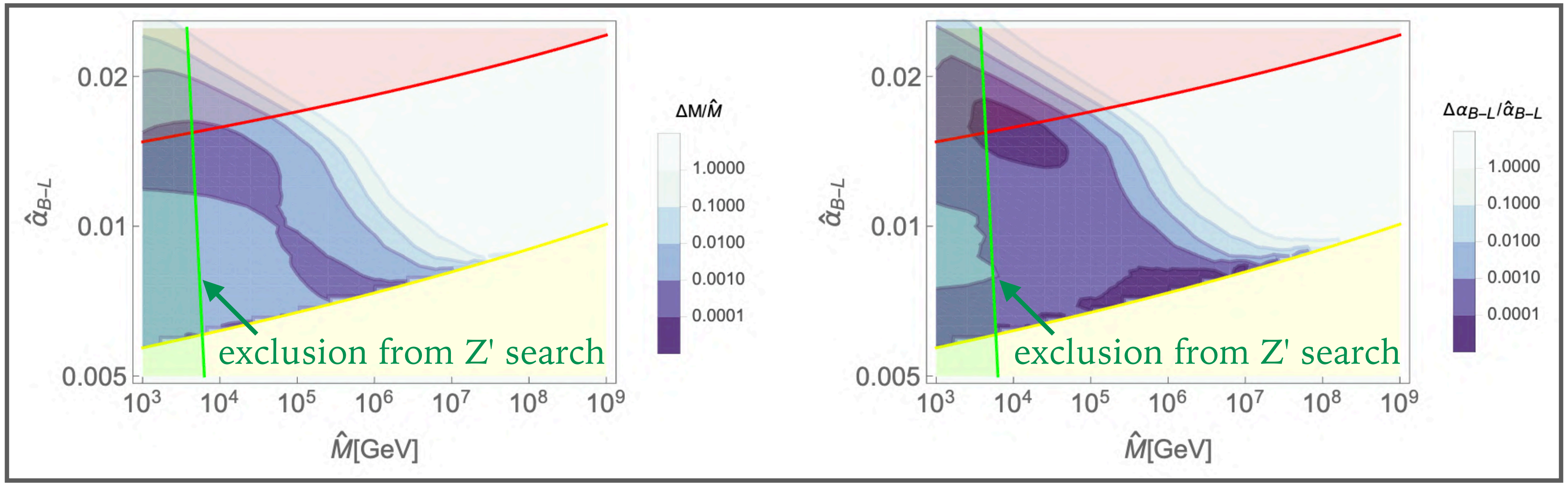
MACROSCOPIC-2: STRONGEST TYPE OF FOPT

[RJ, Takimoto '16]
[Hashino, RJ, Kanemura, Kakizaki, Takahashi, Takimoto '16]

➤ Transition parameters

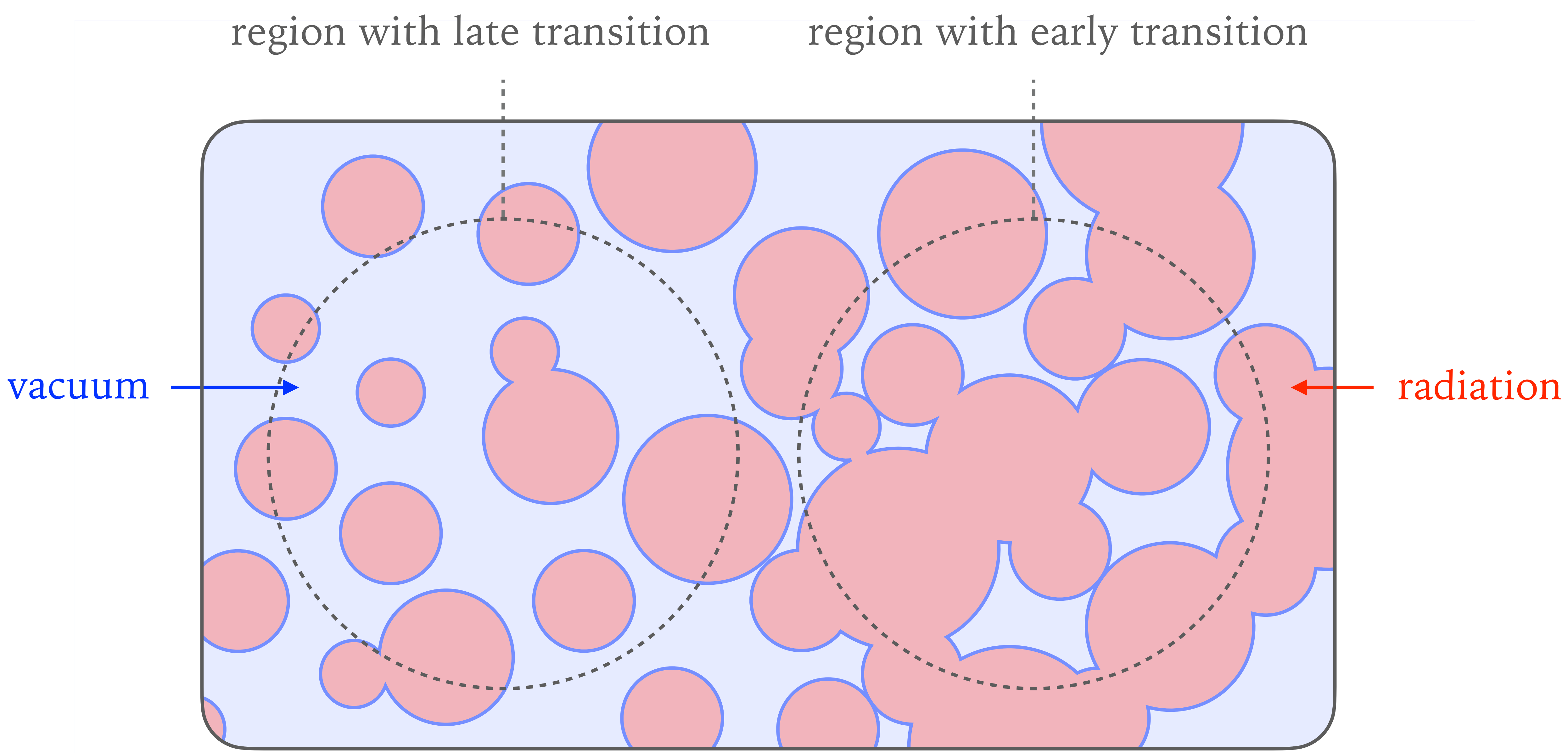


➤ Synergy between collider and GW experiments



MACROSCOPIC-3: PBH FORMATION

- How large can the curvature perturbation be? ($\to$ PBHs? GWs?)

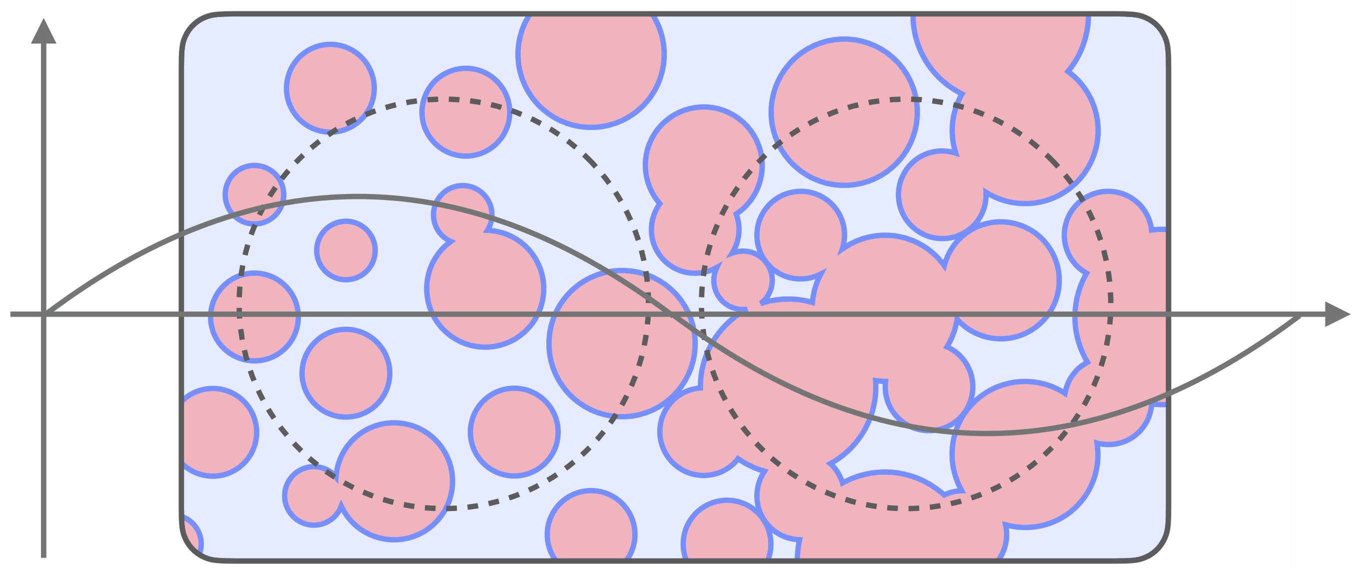


MACROSCOPIC-3: PBH FORMATION

- Can PBHs form from curvature perturbation generated by small β/H (but still $\gtrsim$ a few) FOPTs?

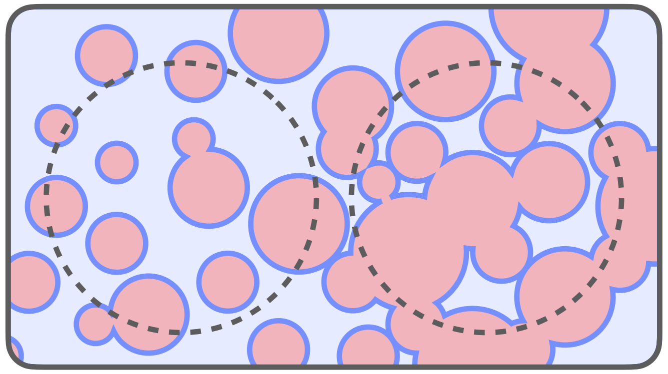
Intuitively

$$\delta \left(= \frac{\delta\rho}{\rho} \right)$$



- With a careful treatment of gauges (in cosmological perturbations), we answered to this question in the negative

MACROSCOPIC-3: PBH FORMATION



➤ Setup & findings of [Lewicki, Troczek, Vaskonen '24]

① Background

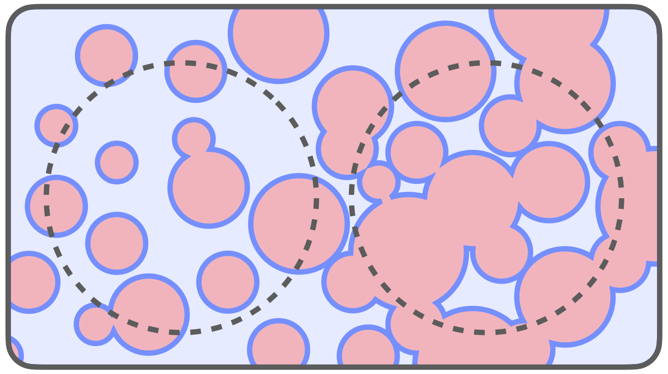
- Radiation & vacuum energy $\bar{\rho}_r' + 4\mathscr{H}\bar{\rho}_r = -\bar{\rho}_V'$
- Initially the universe is vacuum energy dominated $\bar{\rho}_V(t = -\infty) = \Delta V$, and then radiation takes over
- Vacuum energy decays with the exponential nucleation of bubbles

$$\Gamma(t) = H_*^4 e^{\beta(t - t_*)}$$

meaning that $\bar{\rho}_V$ decreases with the average false vacuum fraction $\bar{F}(t)$ as

$$\bar{\rho}_V = \bar{F}(t) \times \Delta V \qquad \bar{F}(t) = \exp\left[-\frac{4\pi}{3}\int_{-\infty}^{t} dt_n \, \Gamma(t_n)\, a(t_n)^3 \left(\int_{t_n}^{t} \frac{d\tilde{t}}{a(\tilde{t})}\right)^3\right]$$

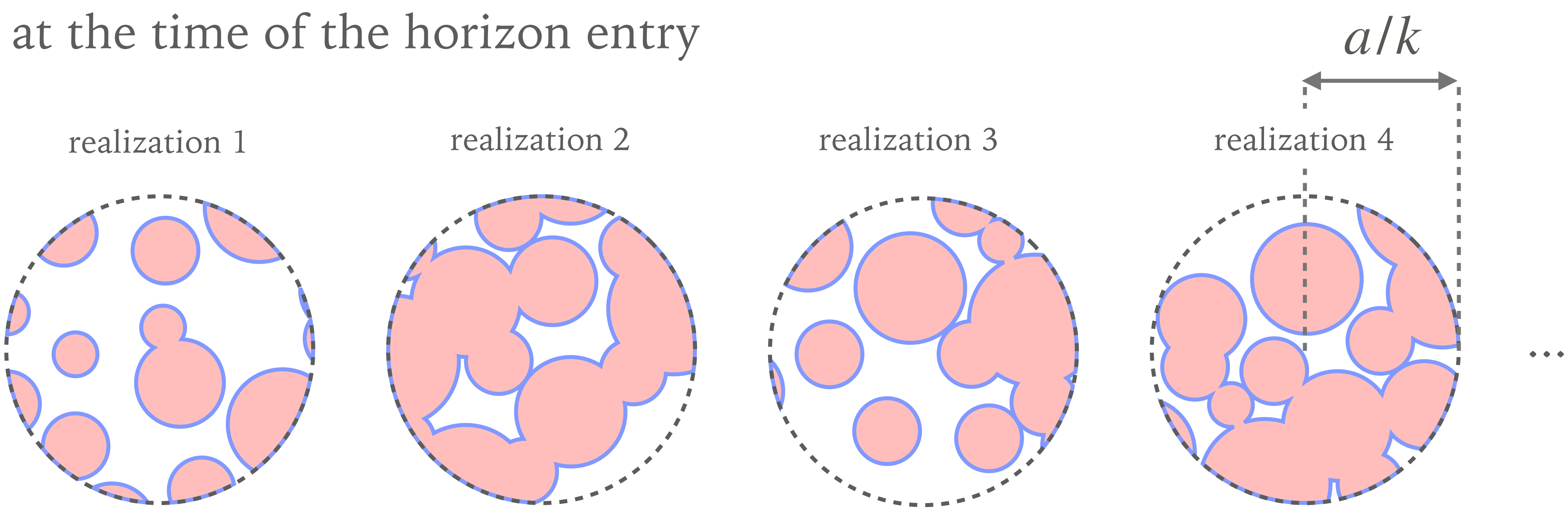
MACROSCOPIC-3: PBH FORMATION



➤ Setup & findings of [Lewicki, Troczek, Vaskonen '24]

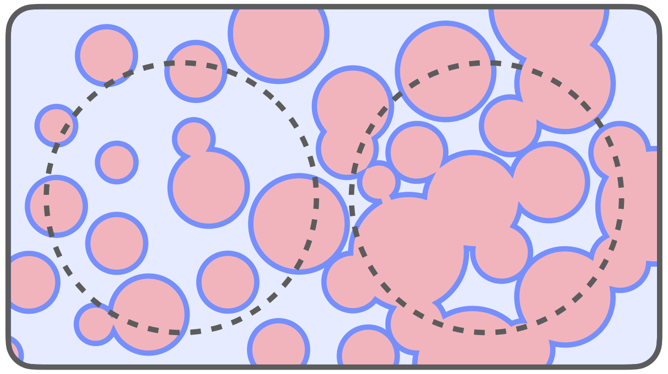
② Perturbation

- Stochastic process of bubble nucleation induces density fluctuations
- For a fixed comoving wavenumber k, consider a sphere of comoving radius $1/k$, and numerically calculate the PDF of the density contrast of this region at the time of the horizon entry



These pictures are just for illustration: they develop a much more efficient algorithm than naively generating bubbles

MACROSCOPIC-3: PBH FORMATION



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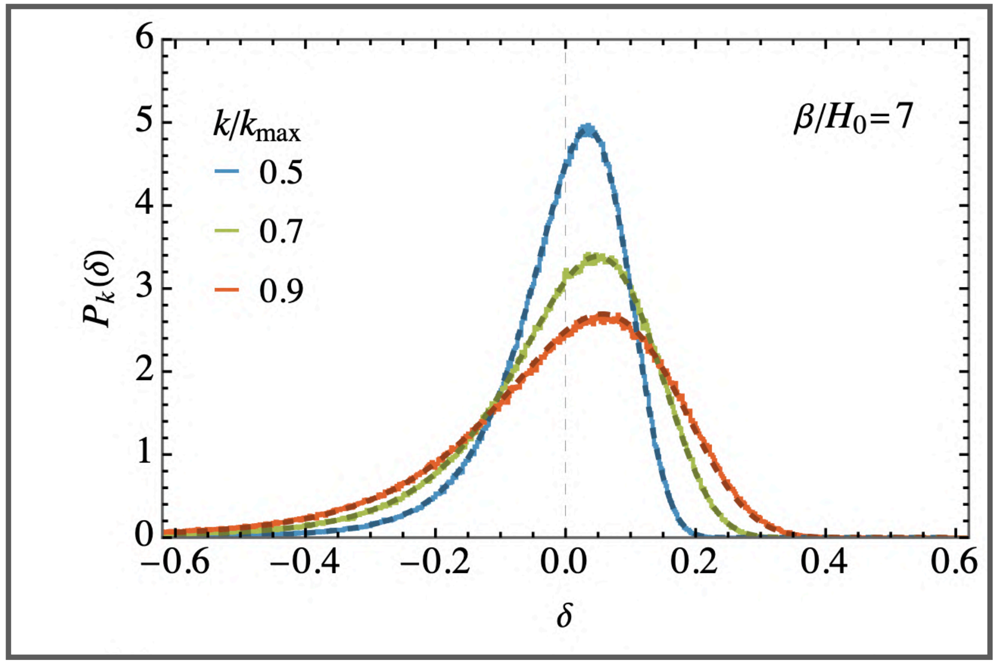
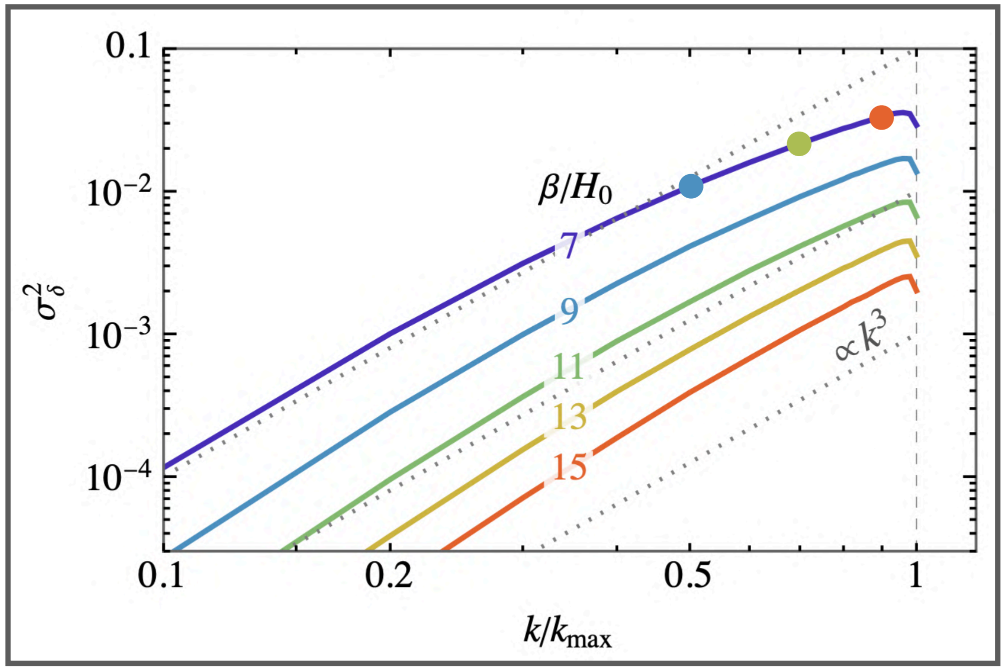
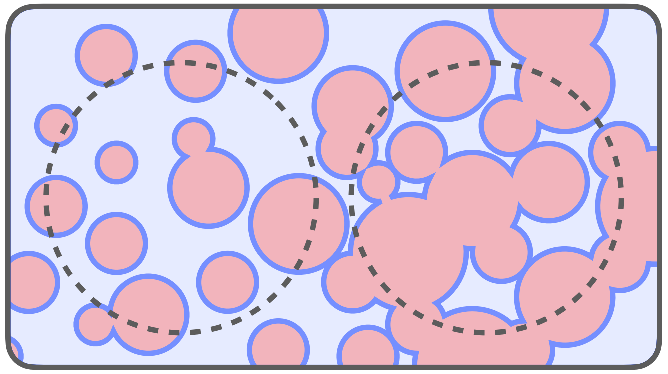


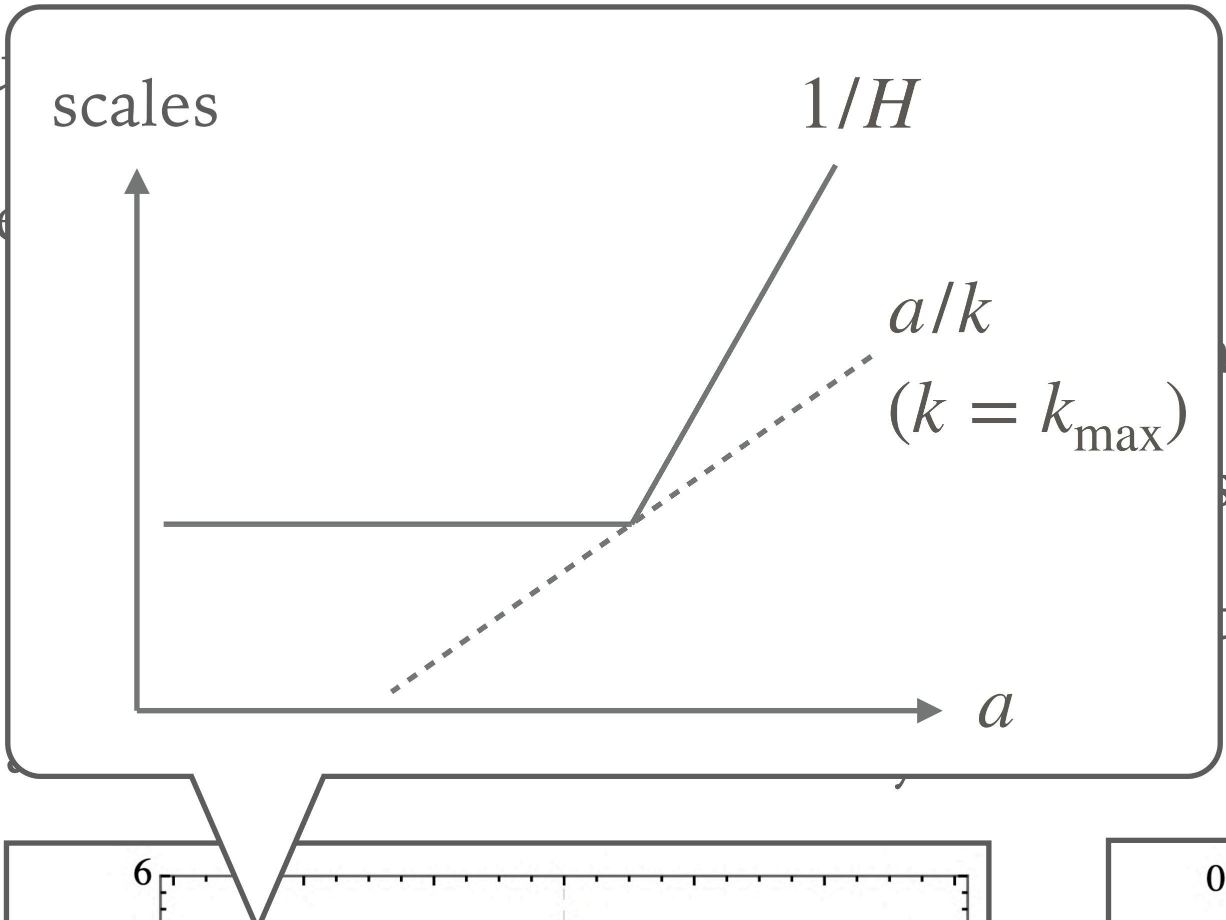


MACROSCOPIC-3: PBH FORMATION



➤ Setu

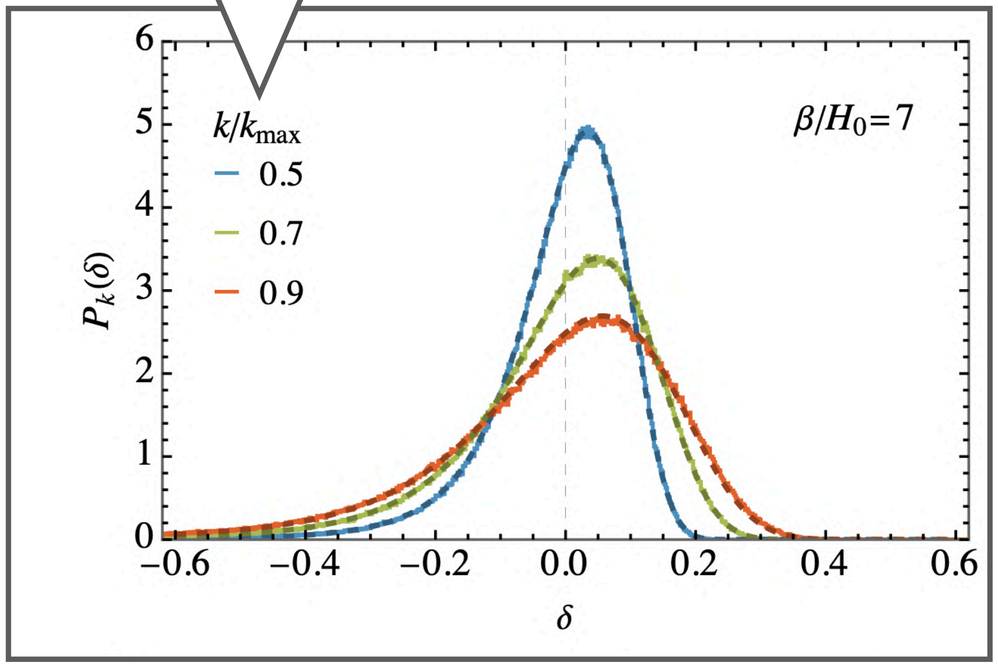
② Pe

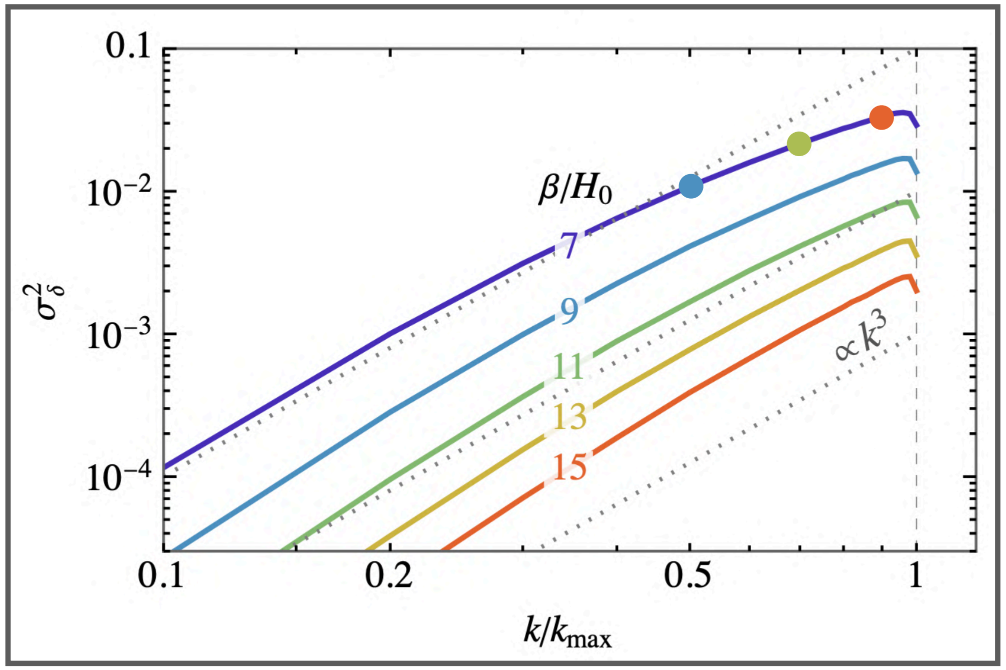


- ...nduces density fluctuations

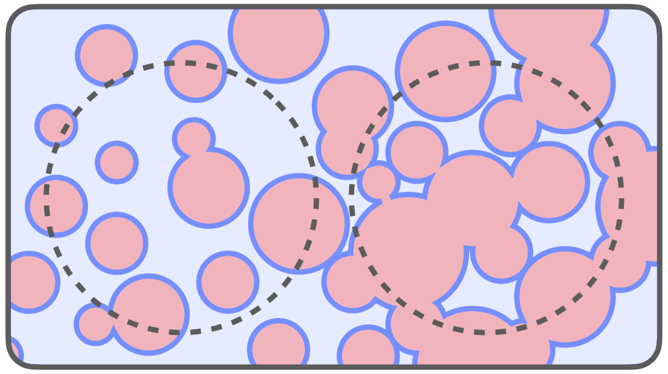
- ...sider a sphere of comoving radius $1/k$,

...density contrast of this region





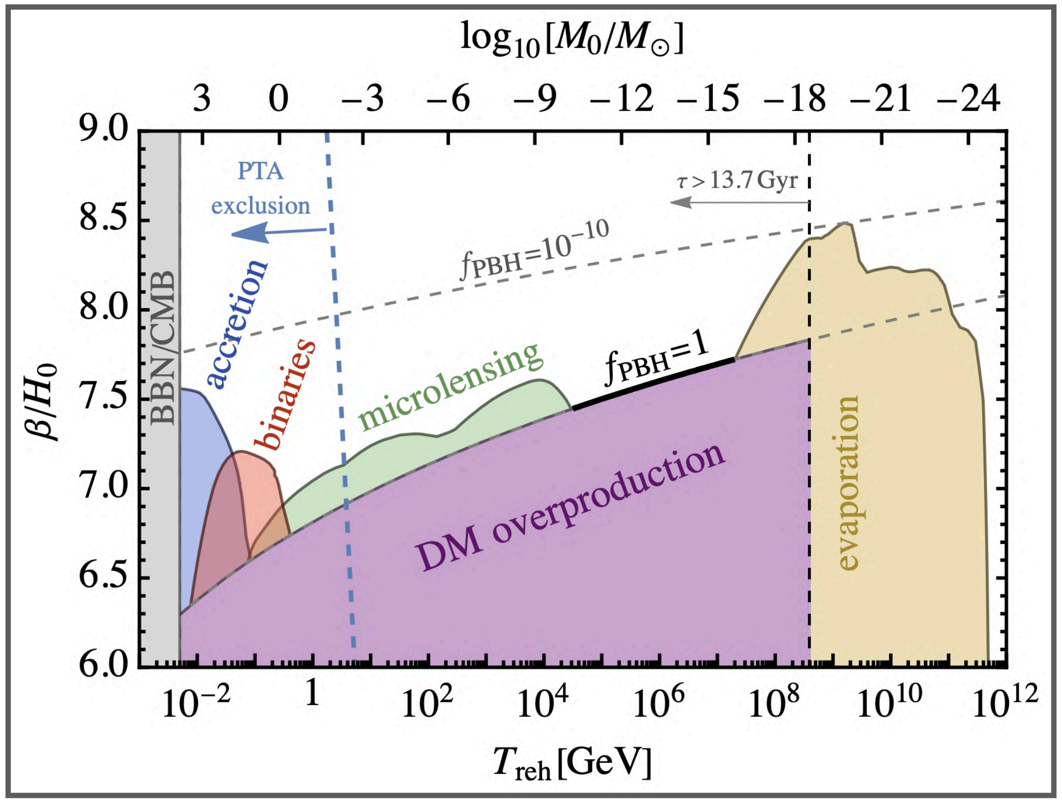
MACROSCOPIC-3: PBH FORMATION



➤ Setup & findings of [Lewicki, Troczek, Vaskonen '24]

② Perturbation

- For $\beta/H_* \lesssim 7$ the variance of the density contrast is so large that the density contrast δ exceeds the threshold for PBH formation $\delta_c = 0.55$ frequently enough to explain the whole DM by PBHs



MACROSCOPIC-3: PBH FORMATION

[Franciolini, RJ, Gouttenoire 2503.01962]

- δ is the density contrast, but in which gauge?

- Our point: δ should be interpreted as the density contrast in the flat gauge $\delta^{(F)}$, since in the algorithm of [Lewicki, Troczek, Vaskonen '24] the density contrast is computed in a ***flat*** FLRW universe

- On the other hand, the threshold $\delta_c \sim 0.5$ is estimated in the comoving gauge

- How would the conclusion change if we use the gauge consistently?

MACROSCOPIC-3: PBH FORMATION

[Franciolini, RJ, Gouttenoire 2503.01962]

- ➤ Perturbation equations we solve

encodes the false-vacuum fraction

$$\delta_{\boldsymbol{k}}^{(F)\prime} + 3\mathscr{H}(c_s^2 - w)\delta_{\boldsymbol{k}}^{(F)} = (1+w)\mathscr{V}_{\boldsymbol{k}} - 3\mathscr{H}\underline{\delta_{p,\mathrm{nad},\boldsymbol{k}}}$$

$$\Phi_{\boldsymbol{k}}'' + 3(1+c_s^2)\mathscr{H}\Phi_{\boldsymbol{k}}' + \left[3(c_s^2 - w)\mathscr{H}^2 + c_s^2 k^2\right]\Phi_{\boldsymbol{k}} = \frac{3}{2}\mathscr{H}\underline{\delta_{p,\mathrm{nad},\boldsymbol{k}}}$$

$$\mathscr{V}_{\boldsymbol{k}} = -\frac{2}{3(1+w)}\frac{\Phi_{\boldsymbol{k}}' + \mathscr{H}\Phi_{\boldsymbol{k}}}{\mathscr{H}}$$

- Equation of state $w = \bar{p}/\bar{\rho}$ & sound speed $c_s^2 = \bar{p}'/\bar{\rho}'$

- Gauge-invariant Newtonian potential Φ & scalar velocity $\mathscr{V}$

- Gauge-invariant non-adiabatic pressure $\delta_{p,\mathrm{nad}} = \dfrac{\delta p_{\mathrm{nad}}}{\bar{\rho}}$, $\delta p_{\mathrm{nad}} = \delta p^{(F)} - c_s^2 \delta\rho^{(F)}$

- In the present case $\delta p_{\mathrm{nad}} = \dfrac{1-3c_s^2}{3}\bar{\rho}\delta^{(F)} + \dfrac{4}{3}\Delta V\,\underline{\delta F^{(F)}}$

fluctuation in the false-vacuum fraction

MACROSCOPIC-3: PBH FORMATION

[Francioloni, RJ, Gouttenoire 2503.01962]

- We use the (very efficient) code developed in [Lewicki, Troczek, Vaskonen '24] to calculate the distribution of the fluctuation $\delta F^{(F)}$

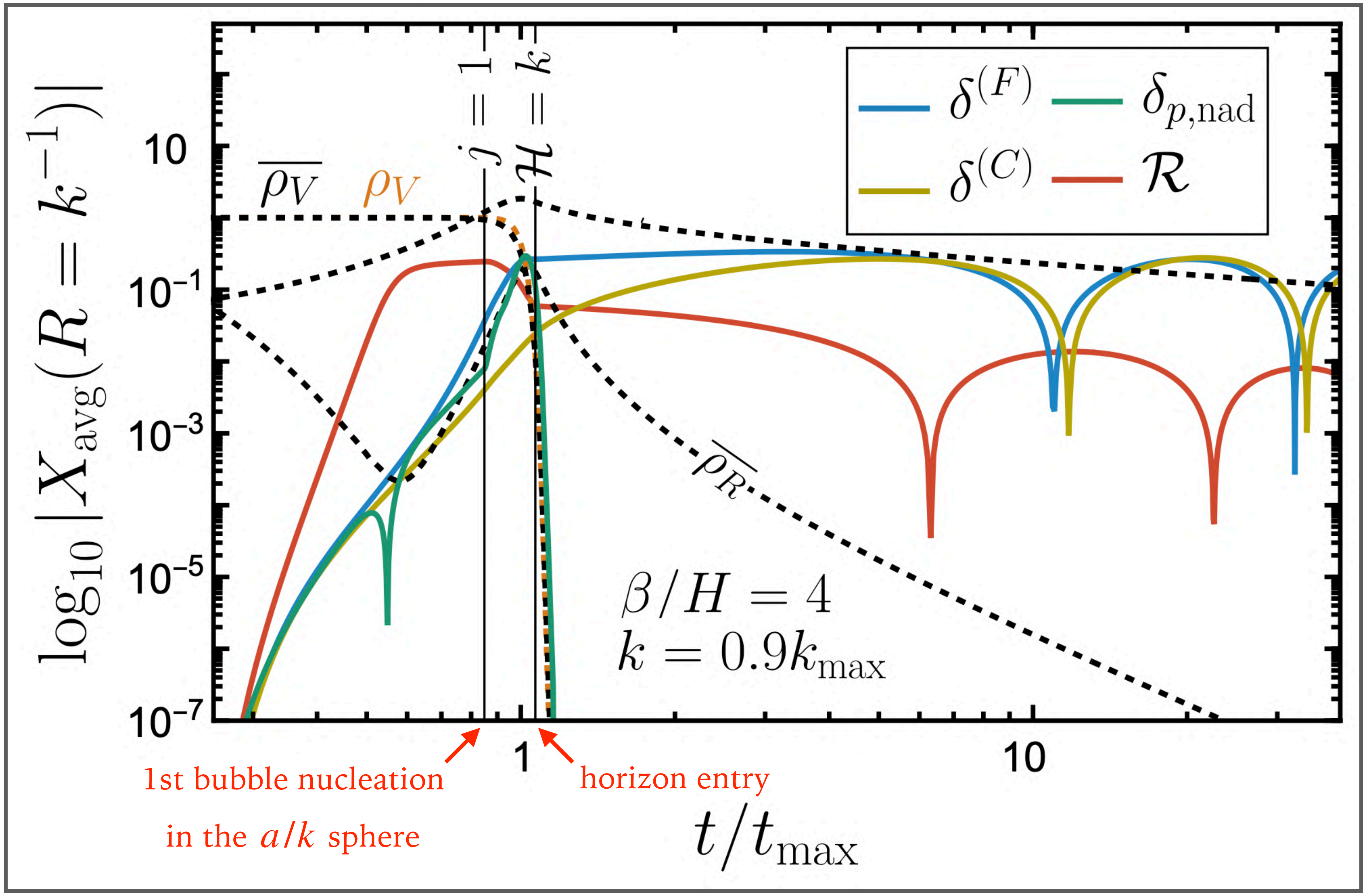
- The only difference is we identify it as the quantity in the flat gauge

- Once the perturbation equations are solved, we also estimate $\delta_{\boldsymbol{k}}^{(C)}$ with

$$\delta_{\boldsymbol{k}}^{(C)} = \delta_{\boldsymbol{k}}^{(F)} + (5 + 3w)\Phi_{\boldsymbol{k}} + \frac{2\Phi'_{\boldsymbol{k}}}{\mathscr{H}}$$

MACROSCOPIC-3: PBH FORMATION

- Point: difference between $\delta_{\boldsymbol{k}}^{(F)}$ and $\delta_{\boldsymbol{k}}^{(C)}$ around the horizon entry



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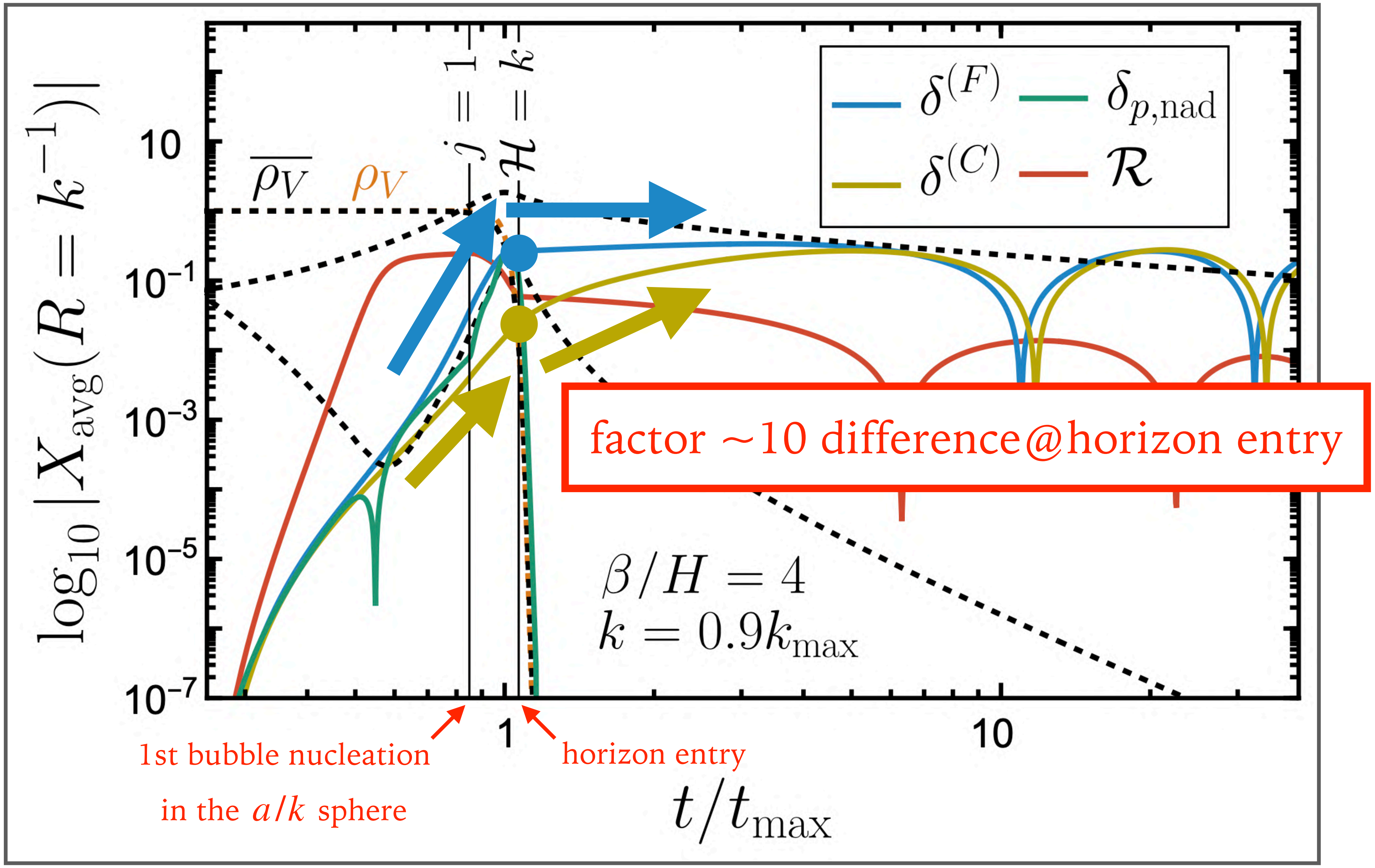

MACROSCOPIC-3: PBH FORMATION