Semiclassical approaches to confinement: monopole and center vortex

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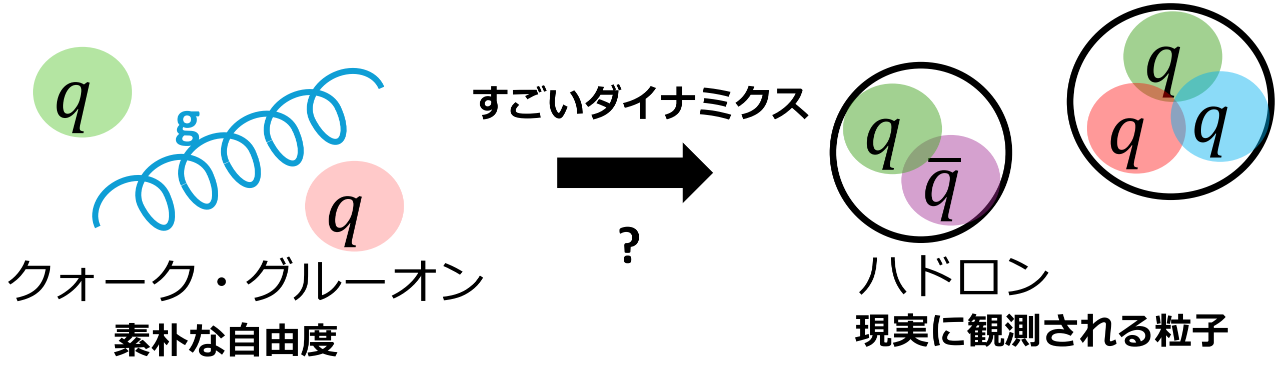
(partially) based on:
PRL **133**, 171902 (2024) [arXiv:2405.12402 [hep-th]] with Yuya Tanizaki (YITP)
JHEP **05**, 194 (2025) [arXiv:2410.21392 [hep-th]] with Tatsuhiro Misumi (Kindai U.) and Yuya Tanizaki (YITP)   (special thanks to Mithat Ünsal(NCSU))

はなすこと



- **動機:** (定性的にでも) **閉じ込めを知りたい**

- **方針**: 定性的に近い「解ける」極限から攻める

(これはよくやる: SUSY, lattice strong coupling, large-N,…)

- **このトーク:**
  (工夫した) S^1 or T^2 コンパクト化による「弱結合閉じ込め」

半古典的に解析できる

4d SU(N) Yang-Mills
(strongly coupled, hard problem)

→

Deformed theory
(weakly coupled, easy problem)

しりたい

Deformation (preserving confinement)

Idea: adiabatic continuity

- We consider S^1 or T^2 compactification
- With the naïve compactification, there is a deconfinement transition somewhere

Deconfinement transition (SSB of center symmetry)

$\mathbb{R}^3 \times S^1$: add Polyakov-loop potential
$\mathbb{R}^2 \times T^2$: 't Hooft-twisted boundary condition

- By adding "**center-stabilizing deformation**" (eliminating the deconfinement transition), we can expect the **adiabatic continuity**.

Lattice works

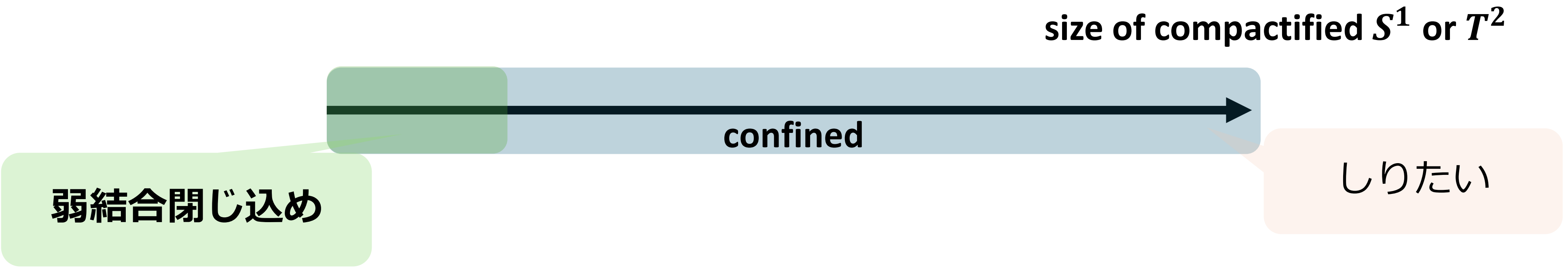
- deformed YM on $\mathbb{R}^3 \times S^1$ [Bonati-Cardinali-D'Elia-Mazziotti '19] [Athenodorou-Cardinali-D'Elia '21]
- YM on $\mathbb{R}^2 \times T^2$ w/ 't Hooft twist [Bergner – González-Arroyo – Soler '25]

Message

$\mathbb{R}^3 \times S^1$: add Polyakov-loop potential
$\mathbb{R}^2 \times T^2$: ‘t Hooft-twisted boundary condition

「**工夫したS^1 or T^2 コンパクト化**」をすると
取り扱える閉じ込めを示す理論になる！

- 期待: コンパクト化前と相転移なくつながる (“adiabatic continuity”)



Summary

Motto: deforming SU(N) YM to **weakly-coupled** theory with **keeping confinement**.

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with “center-stabilizing deformation”

⇒ **confinement by 3d monopole gas**

2d center-vortex semiclassics

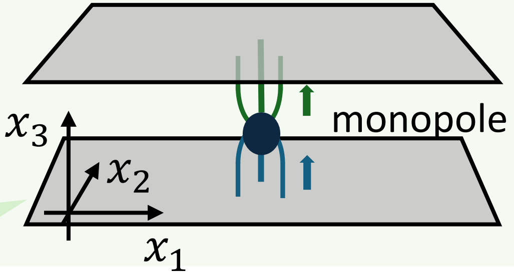
[Tanizaki-Ünsal ’22, …]

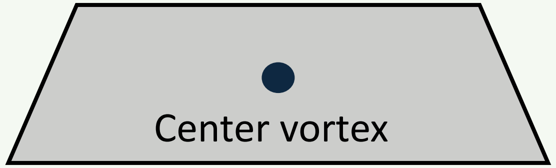
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with ‘t Hooft flux

⇒ **confinement by 2d center-vortex gas**

Unifying two methods [YH-Tanizaki ‘24]: an interpolating setup on $(\mathbb{R}^2 \times \boldsymbol{S}^1) \times \boldsymbol{S}^1$

Monopole in $\mathbb{R}^2 \times \boldsymbol{S}^1$ = **Center vortex in 2d**





“monopole as junction of center vortices”

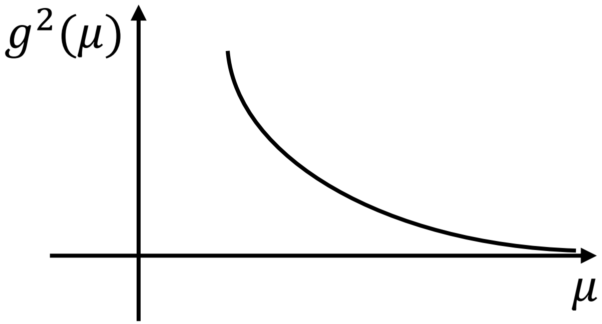
Outline

1. Introduction (4 slides)
2. Monopole semiclassics and center-vortex semiclassics (12 slides)
    - 3d monopole semiclassics
    - 2d center-vortex semiclassics
3. Monopole-vortex continuity (7 slides)
4. Summary

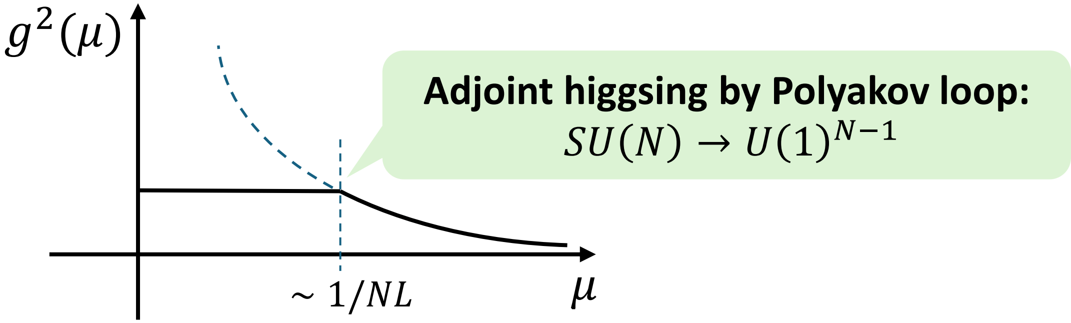
2-1. 3d monopole semiclassics: (deformed-)Yang-Mills theory on $\mathbb{R}^3 \times \boldsymbol{S}^1$

SYM: [Davies-Hollowood-Khoze-Mattis ‘99, …]
QCD(adj.), (deformed-)YM: [Ünsal ’07, Ünsal-Yaffe ’08,…]
(Many works by Anber, Cherman, Schäfer, Shifman, Sulejmanpasic, Poppitz, Ünsal, …)

Review: [Poppitz ‘21]



S^1 compactification
+ center stabilization



IR: 3d $U(1)^{N-1}$ gauge + monopoles

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…] (cf. [Davies-Hollowood-Khoze-Mattis ’99,…] for SYM)

- **SU(N) Yang-Mills on $\mathbb{R}^3 \times S^1$ with “center-stabilizing deformation”** [Ünsal-Yaffe ’08]:

$$S = S_{YM} + \int d^3x \sum_{n=1}^{[N/2]} a_n \, |\mathrm{tr}\,(P^n)|^2$$

Add a potential for Polyakov loop (by hand) to keep center symmetry

⇒ Center symmetry is kept for **small S^1** (, realizing weak-coupling confinement)

- **3d effective theory on $\mathbb{R}^3$**

3d EFT: 3d SU(N) gauge + adjoint scalar (Polyakov loop)

At the center symmetric vacuum, “$\langle P \rangle \sim C$" (up to gauge)

⇒ adjoint higgsing $SU(N) \to U(1)^{N-1}$

e.g.) clock matrix for $N = 3$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

3d effective theory = 3d $U(1)^{N-1}$ gauge theory + monopoles

- Polyakov confinement by dilute gas of monopoles (in 3d Abelian gauge theory) [Polyakov ’77]

3d monopole-instantons

- **Monopoles in adjoint higgsing** $SU(N) \to U(1)^{N-1}$:

Adjoint higgsing by "$\langle P_4 \rangle$"$=$ diag $\left(1, e^{\frac{2\pi i}{N}}, e^{\frac{4\pi i}{N}}, \cdots, e^{\frac{2\pi i}{N}(N-1)}\right)$

Monopole can be constructed by embedding SU(2) monopole (selecting 2 components)

Lightest monopoles: neighboring 2 components $e^{\frac{2\pi i}{N}(k-1)} \leftrightarrow e^{\frac{2\pi i}{N}k}$ (magnetic charge $\vec{\alpha}_k$)

- **N kinds of monopoles**: $Q_{top} = 1/N$ fractional instantons

"compactness of adjoint higgs"
[Kraan-van Baal '98] [Lee-Lu '98][Lee-Yi '97]

(N-1) BPS monopoles + KK monopole

Magnetic charge: $\vec{\alpha}_1$ $\vec{\alpha}_2$ .... $\vec{\alpha}_{N-1}$ $\vec{\alpha}_N (= -\vec{\alpha}_1 - \cdots - \vec{\alpha}_{N-1})$
(in $U(1)^{N-1}$ gauge)

3d effective theory with monopoles

- **Abelian duality (to describe monopole with locality)**

3d U(1) gauge field → 3d U(1)-valued compact boson ($d\sigma = * f$)

$$\int_{U(1)} Da\, e^{-\frac{1}{2e^2}\int |da|^2} = \int_{\mathbb{R}} Df \int_{U(1)} D\sigma\ e^{-\frac{1}{2e^2}\int |f|^2 + \frac{\mathrm{i}}{2\pi}\int d\sigma \wedge f} = \int_{U(1)} D\sigma\ e^{-\frac{e^2}{2}\int |d\sigma|^2}$$

Wilson loop ⇔ monodromy defect of σ : $\sigma \sim \sigma + 2\pi$

Monopole (source of magnetic flux) ⇔ operator $e^{i\sigma}$

- **3d effective theory**

3d abelian duality: $U(1)^{N-1}$ gauge field→ $U(1)^{N-1}$-valued compact boson $\vec{\sigma}$ ($d\vec{\sigma} = * \vec{f}$)

By dilute gas approximation of monopoles, the 3d effective theory is,

Monopole amplitude

$$[\mathcal{M}_i] \sim e^{-\frac{8\pi^2}{Ng^2}}\ \mathrm{e}^{i\vec{\alpha}_i \cdot \vec{\sigma} + i\,\theta/N}$$

$$S = \int d^3x \left[\frac{\# g^2}{L} |d\vec{\sigma}|^2 - \#\ e^{-\frac{8\pi^2}{Ng^2}} \sum_{i=1,\cdots,N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$

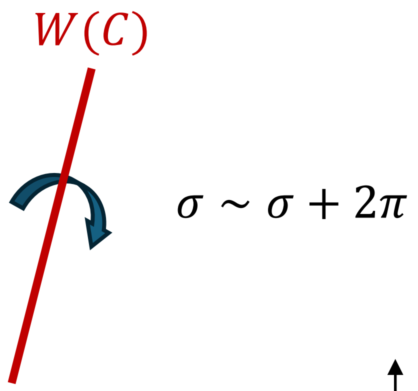
3d effective theory with monopoles

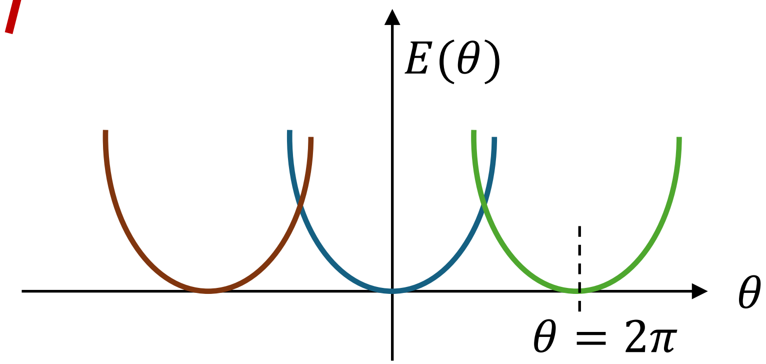
- **3d effective theory**

$$S = \int d^3x \left[\frac{\# g^2}{L} |d\vec{\sigma}|^2 - \#\, e^{-\frac{8\pi^2}{Ng^2}} \sum_{i=1,\cdots,N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$

$\Rightarrow$

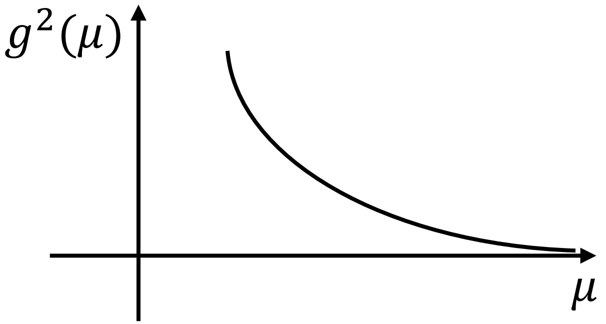
- **Gapped system**
- **Confinement (of Wilson loop on $\mathbb{R}^3$):**
  Wilson loop $\Leftrightarrow$ monodromy defect of σ : $\sigma \sim \sigma + 2\pi$
  Needs kink of σ spanning the surface $\Rightarrow$ Area law!
- **Multi-branch structure**
  extrema of the monopole potential:
  $\vec{\sigma} = \vec{\sigma}_k = \frac{2\pi k}{N}(\vec{\mu}_1 + \cdots + \vec{\mu}_{N-1}), (\mathrm{k} = 0,1,\cdots, N-1)$



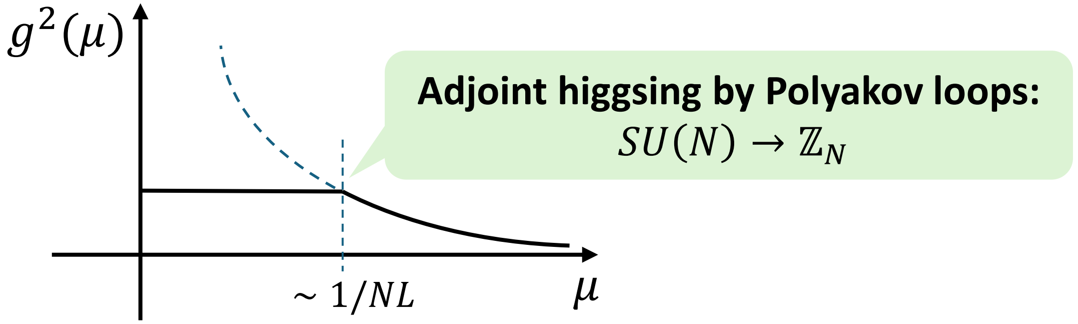


2-2. 2d center-vortex semiclassics: Yang-Mills theory on $\mathbb{R}^2 \times \boldsymbol{T}^2$ w/ ’t Hooft flux

[Tanizaki-Ünsal ‘22]
(cf. [Yamazaki-Yonekura ‘17] [Cox-Poppitz-Wandler ’21; Poppitz-Wandler ‘22])
Further development: [Tanizaki-Ünsal ’22; YH-Tanizaki-Watanabe ’23,’24; YH-Tanizaki ’24]



T^2 compactification
+ ‘t Hooft flux

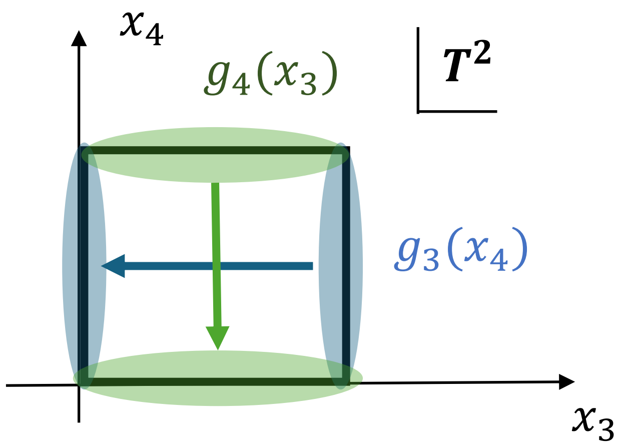


IR: 2d $\mathbb{Z}_N$ gauge + center vortices

SU(N) YM on $\mathbb{R}^2 \times T^2$ with 't Hooft flux (1)

[Tanizaki-Ünsal '22, ……] (cf. [Yamazaki-Yonekura '17])

- **'t Hooft flux for T^2 (or $\mathbb{Z}_N^{[1]}$ background)**



A unit 't Hooft flux ⇔ choose $g_3(0)g_4(L)g_3^\dagger(L)g_4^\dagger(0) = e^{\frac{2\pi\mathrm{i}}{N}}$.

($g_3(x_4), g_4(x_3)$: transition functions on T^2)

$$\begin{cases} a(\vec{x}, x_3 + L, x_4) = g_3^\dagger a g_3 - \mathrm{i}\, g_3^\dagger \mathrm{d} g_3 \\ a(\vec{x}, x_3, x_4 + L) = g_4^\dagger a g_4 - \mathrm{i}\, g_4^\dagger \mathrm{d} g_4 \end{cases}$$

Up to gauge, we can take $g_3 = S,\ g_4 = C$ (shift and clock matrices of $SU(N)$).

⇒ In this gauge, inserting 't Hooft flux ⇔ twisted boundary condition:

$$\begin{cases} a(\vec{x}, x_3 + L, x_4) = S^\dagger\, a\, S \\ a(\vec{x}, x_3, x_4 + L) = C^\dagger\, a\, C \end{cases}$$

e.g.) $N = 3$

$$S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},\ C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

SU(N) YM on $\mathbb{R}^2 \times T^2$ with ‘t Hooft flux (2)

- **Consequences from ‘t Hooft-twisted compactification**

✓**Center symmetry is kept at small T^2**

Classically, $P_3 = S$ and $P_4 = C \Rightarrow \langle \mathrm{tr}\, P_3 \rangle = \langle \mathrm{tr}\, P_4 \rangle = 0.$

✓**Perturbatively gapped gluons:**

$$\begin{cases} a(\vec{x}, x_3 + L, x_4) = S^\dagger\, a\, S \\ a(\vec{x}, x_3, x_4 + L) = C^\dagger\, a\, C \end{cases}$$

∼**adjoint higgsing by Polyakov loops P_3, P_4:** $SU(N) \to \mathbb{Z}_N$

⇒ no zeromode; O(1/NL) KK mass

For confinement on $\mathbb{R}^2$:

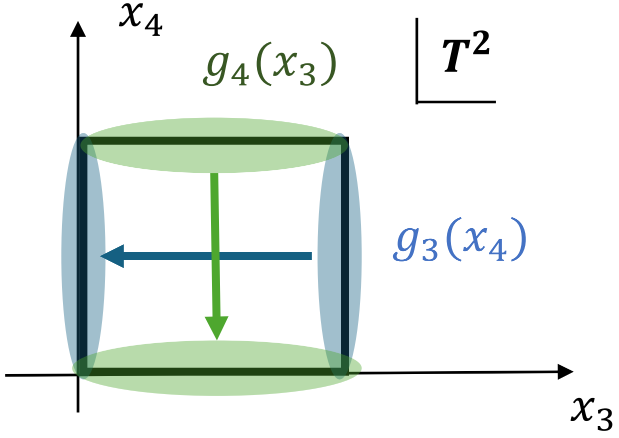
✓**Numerical evidence for ∃“center vortex” (fractional instanton)**

with $S = \frac{8\pi^2}{Ng^2}$, $Q_{top} = 1/N$ as a “local solution” (scale ∼ Size(T^2))

[Gonzalez-Arroyo–Montero ’98, Montero ’99 ‘00]

(cf. [García Pérez–Gonzalez-Arroyo–Soderberg ’90; Itou ‘18] for $\mathbb{R} \times T^3$)

Note) fractional topological charge: it cannot exist alone when $\mathbb{R}^2$ is compactified with periodic BC.



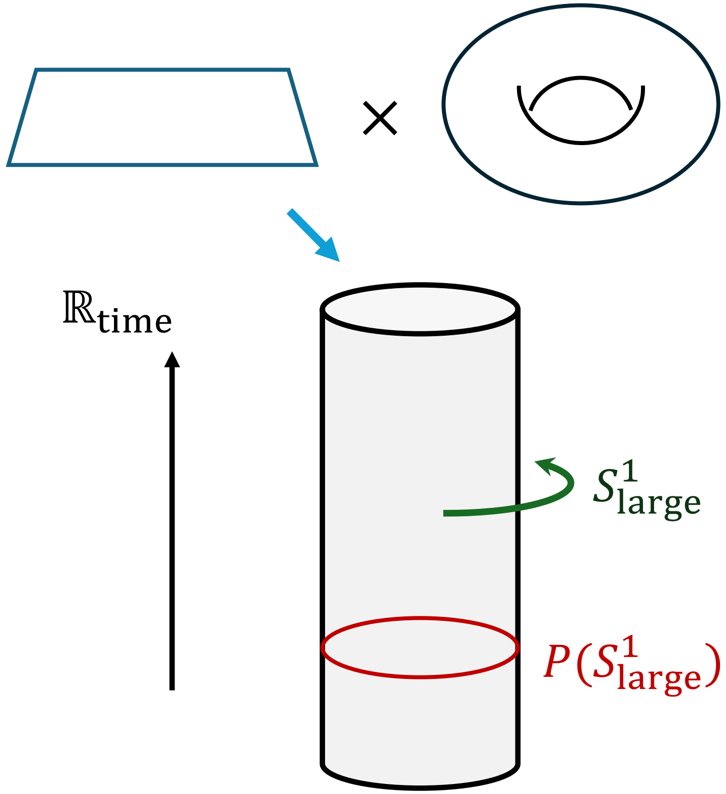
$g_3 = S,\ g_4 = C$

e.g.) $N = 3$

$$S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

Fractional instanton in QM



- <u>Reduction to QM</u> (cf. [Yamazaki-Yonekura '17])

Further spatial compactification:

$\mathbb{R}^2 \times T^2 \to \mathbb{R}_{\text{time}} \times S^1_{\text{large}} \times T^2$

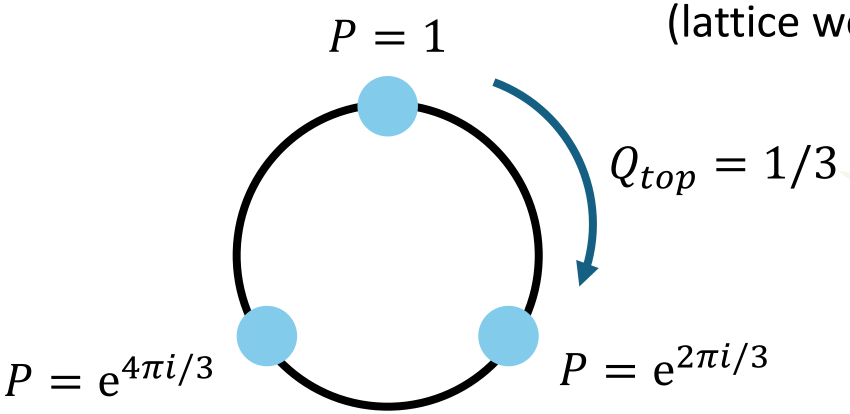
with $\text{Size}(T^2) \ll \text{Size}(S^1_{\text{large}}) \ll \Lambda^{-1}$

2d EFT: $(SU(N) \to)\ \mathbb{Z}_N$ gauge

⇒ **N classical vacua** with $P(S^1_{\text{large}}) = \mathrm{e}^{2\pi k i/N}\mathbf{1}$ $(k = 0, \cdots, N-1)$

- "Fractional instanton = tunneling event"

(lattice work: [Garcia Perez–Gonzalez-Arroyo–Soderberg '90; Itou '18])



- does not globally exist if the periodic BC is imposed
- can exist globally under the twisted BC

2d center-vortex semiclassics [Tanizaki-Ünsal '22]

- **Dilute gas of center vortices**

For calculating partition function, we compactify $\mathbb{R}^2$ without 't Hooft flux.
⇒ total topological charge is constrained $Q_{top} \in \mathbb{Z}$

The center-vortex and anti-center-vortex vertices are:

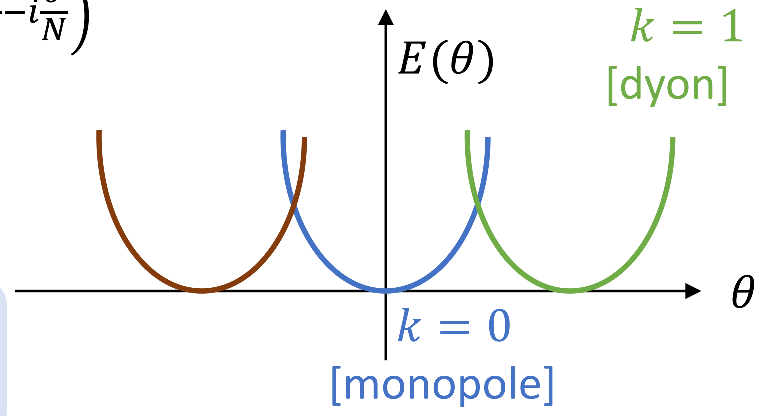
$$[\mathcal{V}] = K e^{-\frac{8\pi^2}{Ng^2} + i\frac{\theta}{N}}, \qquad [\bar{\mathcal{V}}] = K e^{-\frac{8\pi^2}{Ng^2} - i\,\theta/N}$$

with a dimensionful constant K.

Then, the dilute gas approximation yields, (only configurations with $Q_{top} \in \mathbb{Z}$ are admitted)

$$Z_{2d} = \sum_{n,\bar{n}\geq 0} \frac{1}{n!\,\bar{n}!} \delta_{n-\bar{n}\in N\mathbb{Z}} \left(VKe^{-\frac{8\pi^2}{Ng^2} + i\frac{\theta}{N}} \right)^{n} \left(VKe^{-\frac{8\pi^2}{Ng^2} - i\frac{\theta}{N}} \right)^{\bar{n}}$$

$$= \sum_{k\in\mathbb{Z}_N} \exp\left[-V \left(-2Ke^{-\frac{8\pi^2}{Ng^2}} \cos\left(\frac{\theta - 2\pi k}{N} \right) \right) \right]$$



N semiclassical vacua

Energy density of k-th vacuum
→multibranch structure!

✅ One can also derive area-law falloff of the Wilson loop from the dilute gas of center vortices.

おまけ: QCD on $\mathbb{R}^2 \times T^2$ [Tanizaki-Ünsal '22; YH-Tanizaki '24]

- In the presence of fundamental fermions, we introduce both 't Hooft flux and baryon magnetic flux (for consistency). [We cannot introduce 't Hooft flux alone in this case]
- 2d effective theory of QCD on $\mathbb{R}^2 \times T^2$:
  "2d analog of $U(N_f)$ chiral Lagrangian" (by bosonization).
- The η' potential is generated by center vortices,
  η' mass term: $(\det U)^{1/N}$ (U: $U(N_f)$ meson field)
  (fractional-instanton vertex) $\sim e^{i\theta/N}$
  (with periodicity-extension for well-definedness ⇔ in the limit where Chern-Simons DW is decoupled)
- This is a finite-N version of log-det vertex at large N!
  (rather than commonly used Kobayashi-Maskawa-'t Hooft instanton vertex)

※ The 2d semiclassics unexpectedly works well, compared to 3d semiclassics [Cherman-Schäfer-Ünsal '16]

Summary of Backgrounds / Question

Motto: deforming SU(N) YM to **weakly-coupled** one with **keeping confinement**.

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with "center-stabilizing deformation"

⇒ **3d** $U(1)^{N-1}$ **gauge theory**

+ monopole gas

⟷

2d center vortex semiclassics

[Tanizaki-Ünsal ’22, …]

SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with ‘t Hooft flux

⇒ **confinement by 2d center-vortex gas**

Question: Relation between them?

How monopole transmutes to center vortex?

Outline

Interpolating setup [YH, Tanizaki '24] (cf. [Güvendik-Schäfer-Ünsal '24])

Interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \underbrace{\overbrace{(\boldsymbol{S^1})_3 \times (\boldsymbol{S^1})_4}^{\text{'t Hooft flux}}}_{}$

(L_4: always small)

$(\boldsymbol{S^1})_4$: **center-stabilizing deformation**

$L_3 \to \infty$ ↙ ↘ $L_3 \to L_4$

<u>3d monopole semiclassics</u>

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S^1}$ with center-stabilizing deformation

<u>2d center vortex semiclassics</u>

SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with 't Hooft flux

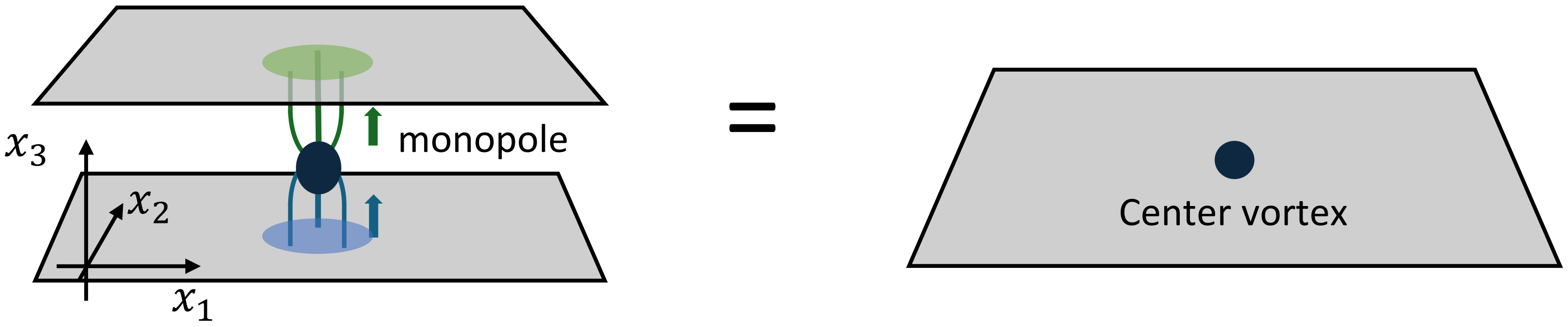
What we will see:

setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{(S^1)_3 \times \underbrace{(S^1)_4}_{\text{center-stabilizing deformation}}}^{\text{'t Hooft flux}}$

1. 3d effective theory on $\mathbb{R}^2 \times (S^1)_3$ $\Rightarrow$ 2d center-vortex gas on $\mathbb{R}^2$

2. BPS/KK monopole in $\mathbb{R}^2 \times (S^1)_3$ (3d monopole-instanton)

$\Rightarrow$ center vortex on $\mathbb{R}^2$ (2d center-vortex-instanton)



3d effective theory on $\mathbb{R}^2 \times (S^1)_3$

‘t Hooft flux

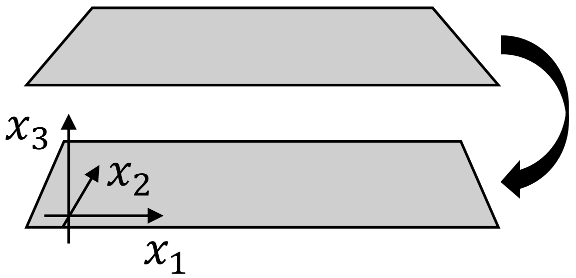
Interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times (S^1)_3 \times (S^1)_4$

(L_4: always small)

center-stabilizing deformation

small L_4, adjoint higgsing by P_4

3d $U(1)^{N-1}$ gauge theory + monopoles on $\mathbb{R}^2 \times (S^1)_3$
with “shift-twisted” boundary conditions



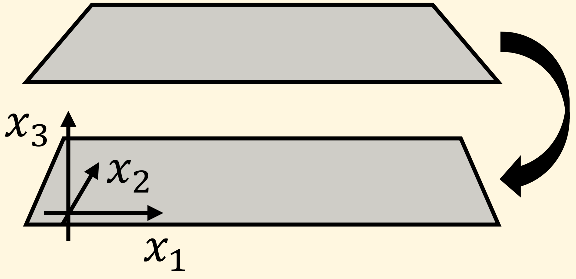
In the gauge $\langle P_4 \rangle = C$ (clock matrix), the transition function for $(S^1)_3$ is $g_3 = S$ (shift matrix).

or, ‘t Hooft flux ⇒ $\left(\mathbb{Z}_N^{[0]}\right)_{3\mathrm{d}}$-twisted boundary condition (∼ **Weyl permutation** for dual photon $\vec{\sigma}(\boldsymbol{x}, x_3)$)

From 3d monopole gas to 2d center-vortex gas

3d $U(1)^{N-1}$ gauge theory + monopoles on $\mathbb{R}^2 \times (S^1)_3$
with "shift-twisted" boundary conditions

$$S_{3d}[\vec{\sigma}] = \int d^3x \left[\frac{\# g^2}{L_4} |d\vec{\sigma}|^2 - \#\, e^{-\frac{8\pi^2}{Ng^2}} \sum_{i=1,\cdots,N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$



$\vec{\sigma}\,(\boldsymbol{x}, x_3 + L_3) = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$

$L_3 \ll \Lambda^{-1}$: **restricted to** $\vec{\sigma} = S^{-1}\vec{\sigma}$

N vacua: $\vec{\sigma} = \vec{\sigma}_k = \frac{2\pi k}{N}(\vec{\mu}_1 + \cdots + \vec{\mu}_{N-1})$

$(k = 0, \cdots, N-1)$

identical to extrema of the monopole potential (**3d-2d adiabatic continuity**)

2d center-vortex gas

$$Z_{\mathbb{R}^2\times(S^1)_3} = \int_{\substack{\vec{\sigma}\,(\boldsymbol{x},x_3+L_3) \\ = S^{-1}\vec{\sigma}(\boldsymbol{x},x_3)}} \mathcal{D}\vec{\sigma}\ e^{-S_{3d}[\vec{\sigma}]} \approx \sum_{\substack{\vec{\sigma}=\vec{\sigma}_k \\ k\in\mathbb{Z}_N}} e^{-S_{3d}[\vec{\sigma}]} = \sum_{k\in\mathbb{Z}_N} e^{\#\, V_{2d}\, e^{-\frac{8\pi^2}{Ng^2}} \cos\left(\frac{\theta+2\pi k}{N}\right)} = Z_{2d}$$

How monopole looks like in $\mathbb{R}^2 \times (S^1)_3$

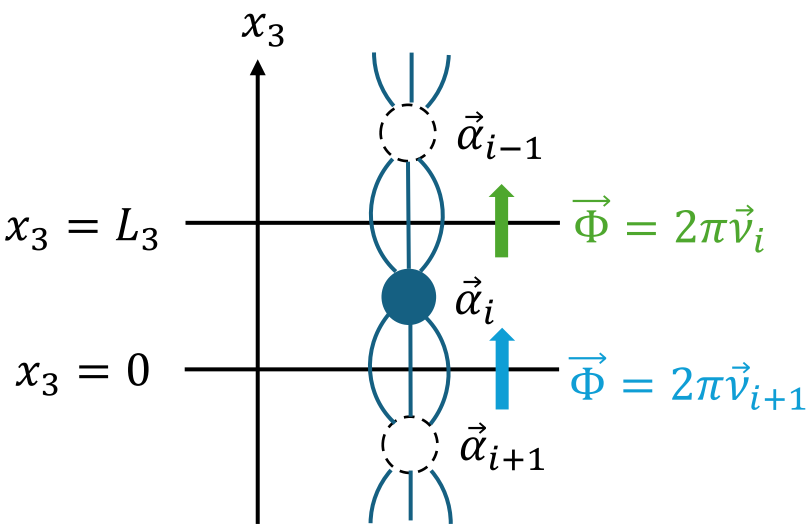
- BPS/KK monopole in 3d effective theory:

magnetic charge $\vec{\alpha}_i \Rightarrow \nabla^2 \vec{\sigma} \sim 2\pi\vec{\alpha}_i\, \delta^{(3)}(x - x_*)$

boundary condition: $\vec{\sigma}\,(\boldsymbol{x}, x_3 + L_3) = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$

$\Rightarrow$ "mirror image": infinite chain of BPS/KK monopoles

$$\vec{\sigma} \sim \text{"}\sum_{n\in\mathbb{Z}} \frac{\vec{\alpha}_{i-n\ (\mathrm{mod\ N})}}{|x - x_* - nL_3\,\hat{x}_3\,|}\text{"}$$



- A proper expression (with good convergence):

$$\vec{\sigma} \sim \sum_{k\in\mathbb{Z}} \left[\sum_{\ell\in\mathbb{Z}_N} \vec{v}_{i-\ell\ (\mathrm{mod\ N})} \left\{ \frac{1}{|x - x_* - (Nk + \ell)L_3\,\hat{x}_3|} - \frac{1}{|x - x_* - (Nk + \ell + 1)L_3\,\hat{x}_3|} \right\} \right]$$

$\vec{v}_i$: weight vector of fundamental representation
$\vec{\alpha}_i = \vec{v}_i - \vec{v}_{i+1}$

outgoing magnetic flux
$\vec{\Phi} = 2\pi\vec{v}_i$

incoming magnetic flux
$\vec{\Phi} = 2\pi\vec{v}_{i+1}$

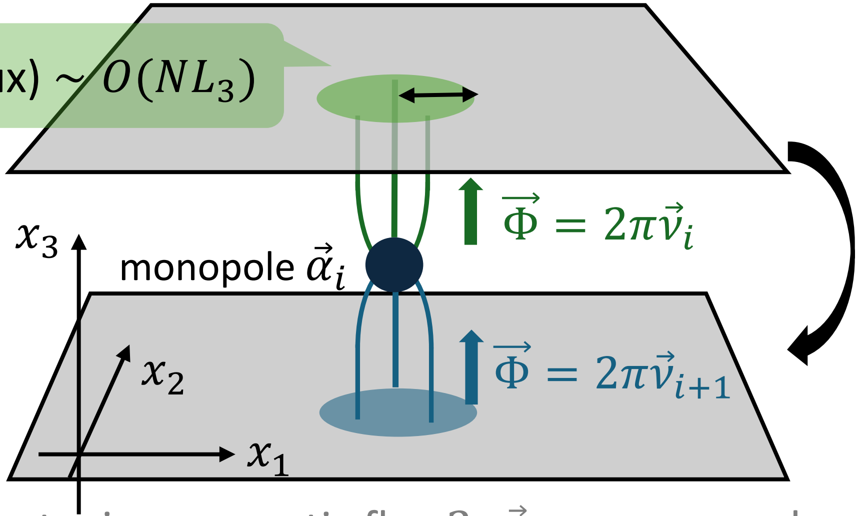
How monopole looks like in $\mathbb{R}^2 \times (S^1)_3$

This solution explains:

The $\vec{\alpha}_i$-monopole emits magnetic flux $2\pi\vec{\alpha}_i = 2\pi\vec{v}_i - 2\pi\vec{v}_{i+1}$

$\vec{v}_i$: weight vector of fundamental representation

(size of magnetic flux) $\sim O(NL_3)$



$\vec{\sigma}\,(\boldsymbol{x}, x_3 + L_3) = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$

Magnetic flux $2\pi\vec{v}_i \mapsto 2\pi\vec{v}_{i+1}$

Suppose that the outgoing magnetic flux $2\pi\vec{v}_i$ goes upward

⇒ Shift-twisted boundary condition: $2\pi\vec{v}_i \mapsto 2\pi\vec{v}_{i+1}$ ⇒ The incoming flux $2\pi\vec{v}_{i+1}$ comes from bottom

The magnetic flux is localized in 2d ($\sim O(NL_3)$) .

N Species of monopole (BPS/KK) can be included in extended moduli $x_3 \in [0, NL_3)$.

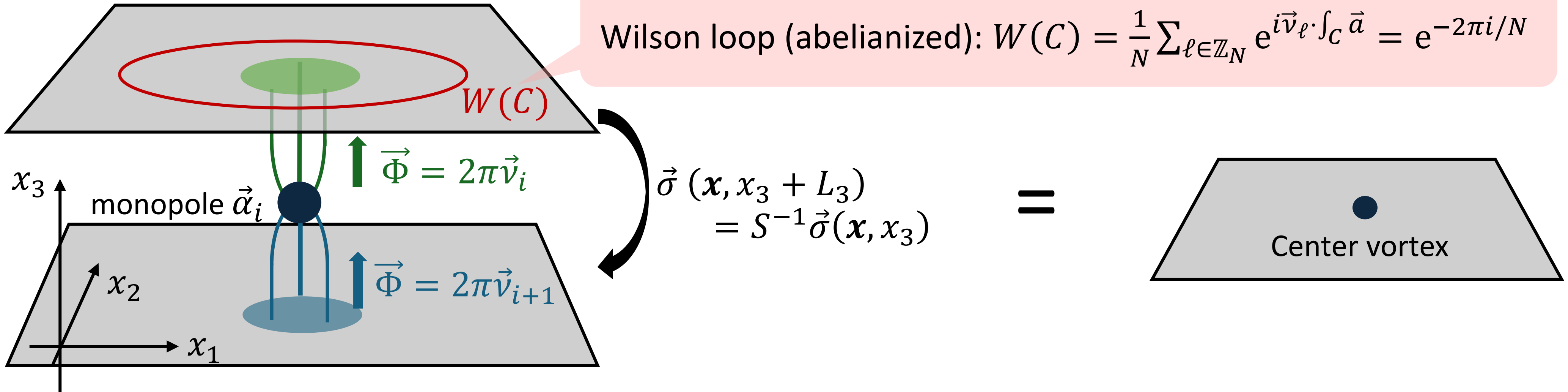
3d BPS/KK monopoles become 2d center vortex

- The magnetic flux (of size $O(NL_3)$) is indeed center vortex: Wilson loop acquires $\mathrm{e}^{-2\pi i/N}$ phase.
- **3d BPS/KK monopole-instanton = 2d center-vortex-instanton**:

The 3d/2d semiclassical confinement mechanisms are essentially same!

- **“monopole as junction of center vortex” (realizing the old expectation!)**

Wilson loop (abelianized): $W(C) = \frac{1}{N}\sum_{\ell\in\mathbb{Z}_N} \mathrm{e}^{i\vec{\nu}_\ell\cdot\int_C \vec{a}} = \mathrm{e}^{-2\pi i/N}$



Outline

Summary

Motto: deforming SU(N) YM to **weakly-coupled** theory with **keeping confinement**.

compactification

3d monopole semiclassics

[Ünsal '07, Ünsal-Yaffe '08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with "center-stabilizing deformation"

⇒ **confinement by 3d monopole gas**

2d center-vortex semiclassics

[Tanizaki-Ünsal '22, …]

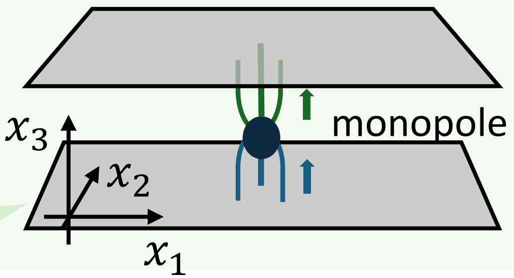
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with 't Hooft flux

⇒ **confinement by 2d center-vortex gas**

Unifying two methods [YH-Tanizaki '24]: an interpolating setup on $(\mathbb{R}^2 \times \boldsymbol{S}^1) \times \boldsymbol{S}^1$

Monopole in $\mathbb{R}^2 \times \boldsymbol{S}^1$

"monopole as junction of center vortices"



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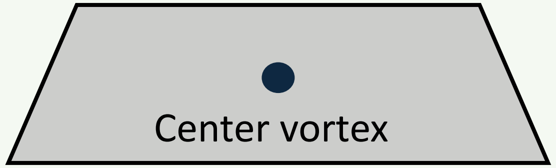
Center vortex in 2d

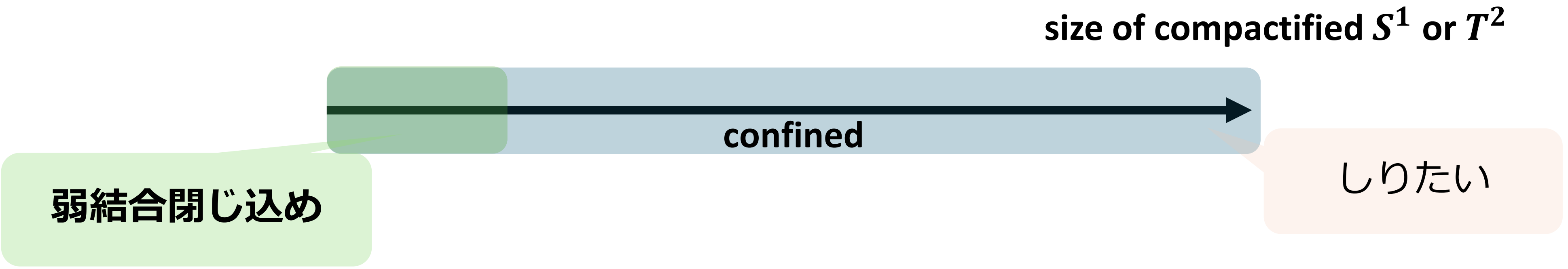


Message

$\mathbb{R}^3 \times S^1$: add Polyakov-loop potential
$\mathbb{R}^2 \times T^2$: ‘t Hooft-twisted boundary condition

「**工夫したS^1 or T^2 コンパクト化**」をすると
取り扱える閉じ込めを示す理論になる！

- 期待: コンパクト化前と相転移なくつながる (“adiabatic continuity”)



Further developments / future directions

- Interplay between 3d/2d semiclassics
  - $\mathcal{N} = 1$ SYM, QCD(adj) [YH-Misumi-Tanizaki '24]:

    3d semiclassics is well developed, but 2d semiclassics was somewhat puzzling

    Perimeter-law in 2d ($\Leftarrow$ 3d double string picture), fate of bion mechanism…
  - QCD(F) (2d semiclassics unexpectedly works well, why?)
  - Other gauge groups…
  - …
- Monopole-vortex complex as soliton (in Higgs phase)

  In Higgs phase, monopole is confined in vortex [YH-Misumi-Nitta-Ohashi-Tanizaki '25, and ongoing]
- Center-vortex semiclassics with a non-minimal twist

  What if 't Hooft twist is not minimal? (related to large-N stuff (center stability, twisted Eguchi-Kawai…)) [YH-Tanizaki-Ünsal '25]

backup

Example: SU(2) case

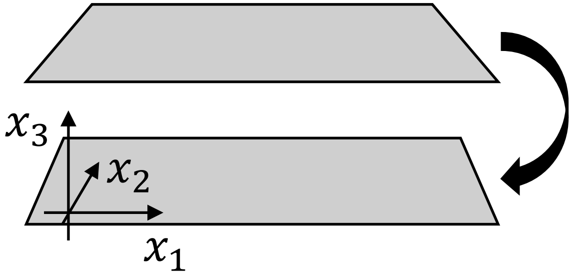
- Adjoint higgsing by $P_4(\propto \sigma_3$, up to gauge$): SU(2) \to U(1)$ $\Rightarrow$ one compact scalar $\sigma \sim \sigma + 2\pi$

$$S_{3d}[\sigma] = \int d^3x \left[\frac{\# g^2}{L_4} |d\sigma|^2 - \#\, e^{-\frac{8\pi^2}{Ng^2}} \left(\cos\left(\sigma + \frac{\theta}{2}\right) + \cos\left(-\sigma + \frac{\theta}{2}\right)\right)\right]$$

- the transition function for $(S^1)_3$ is $g_3 \propto \sigma_1$(shift matrix):

$\Rightarrow$ flipping the basis $P_4 \mapsto -P_4$, equivalent to $\sigma(\boldsymbol{x}, x_3 + L_3) = -\sigma(\boldsymbol{x}, x_3)$

3d $U(1)$ gauge theory + monopoles on $\mathbb{R}^2 \times (S^1)_3$
with "shift-twisted" boundary conditions



$\sigma(\boldsymbol{x}, x_3 + L_3)$
$= -\sigma(\boldsymbol{x}, x_3)$

$L_3 \ll \Lambda^{-1}$: restricted to "zeromode": $\sigma = -\sigma$

$\Rightarrow$ 2 vacua: $\sigma = 0, \pi$

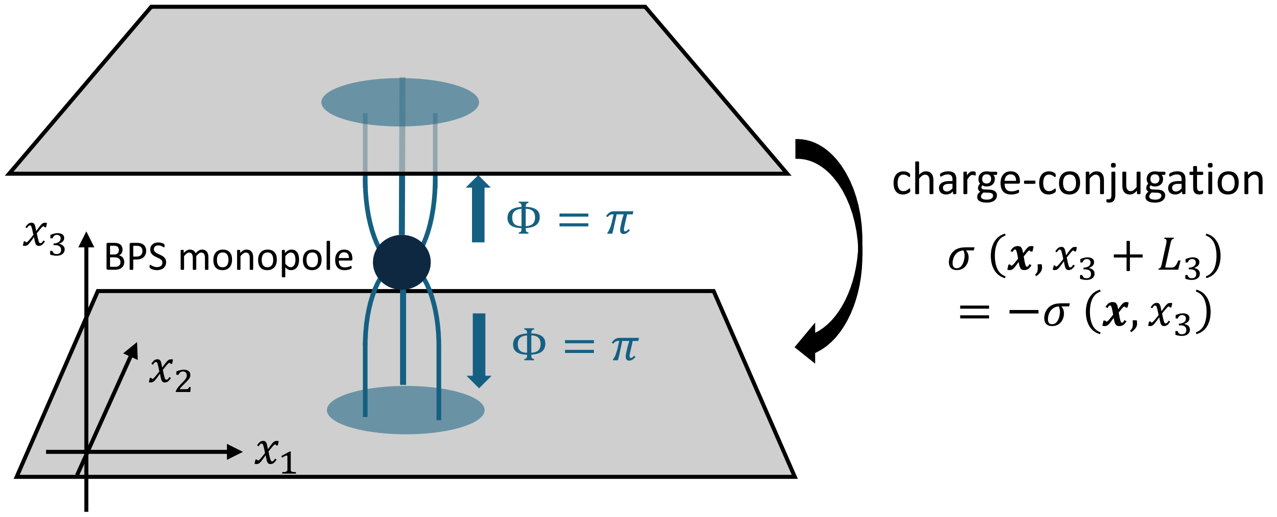
identical to extrema of the monopole potential
(3d-2d adiabatic continuity)

2d center-vortex gas

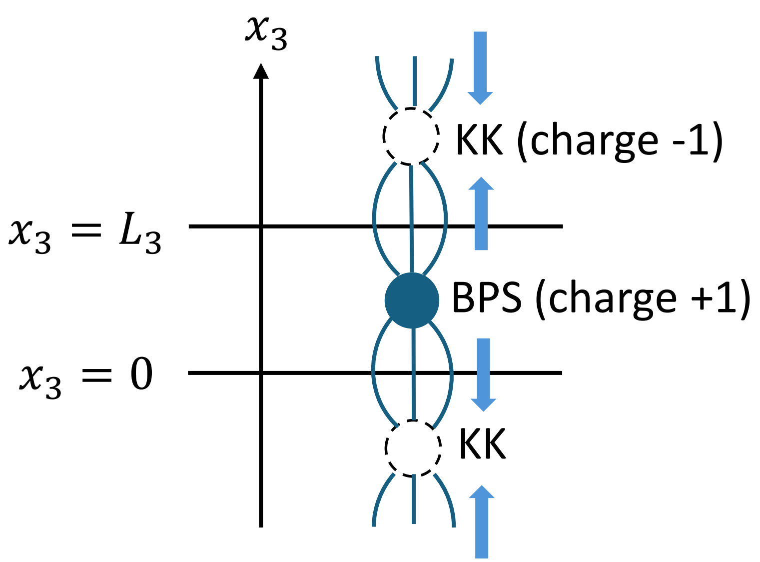
$$Z_{\mathbb{R}^2 \times (S^1)_3} \approx \sum_{k \in \mathbb{Z}_2} \mathrm{e}^{\# V_{2d} \mathrm{e}^{-\frac{8\pi^2}{2g^2}} \cos\left(\frac{\theta + 2\pi k}{2}\right)} = Z_{\text{2d vortex gas}}$$

Example: SU(2) case

- One compact scalar $\sigma \sim \sigma + 2\pi$
- BPS monopole: magnetic charge +1, KK monopole: magnetic charge -1
- boundary condition: $\sigma\,(\boldsymbol{x}, x_3 + L_3) = -\sigma(\boldsymbol{x}, x_3)$



"mirror image" solution:



"Flux Fractionalization":

1/N fractional magnetic flux, rotating the Wilson loop by a center element (-1)