# SQCDからQCDを探る

Dan Kondo (IPMU)

with Hitoshi Murayama, Bea Noether and Digvijay Roy Varier

9/2 素粒子物理学の進展 at YITP

arxiv: 2111.09690 (JHEP04(2025)152)
arxiv: 2505.18138

# 今日話す概要

カイラル対称性の破れた ASQCD の真空から

Chiral Lagrangian と WZW termを出した。

quark や gluonの凝縮を計算した。

mass spectrumを調べた。

**ASQCD が QCDの近似になり得る状況証拠を与えた。**

# 今日話そうと思う内容

1. Introduction/Motivation
2. SQCD+AMSB (ASQCD)の真空
3. Chiral Lagrangian+WZW term
4. Condensates
5. Mass spectrum

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目標：SQCD+AMSBでQCDの近似になり得る状況証拠を与えよう

・非摂動の物理は難しい

例：QCDのIR領域

# どういうときに IR理論は強結合になるのか?

⟹ $\Lambda$: **dynamical scale**

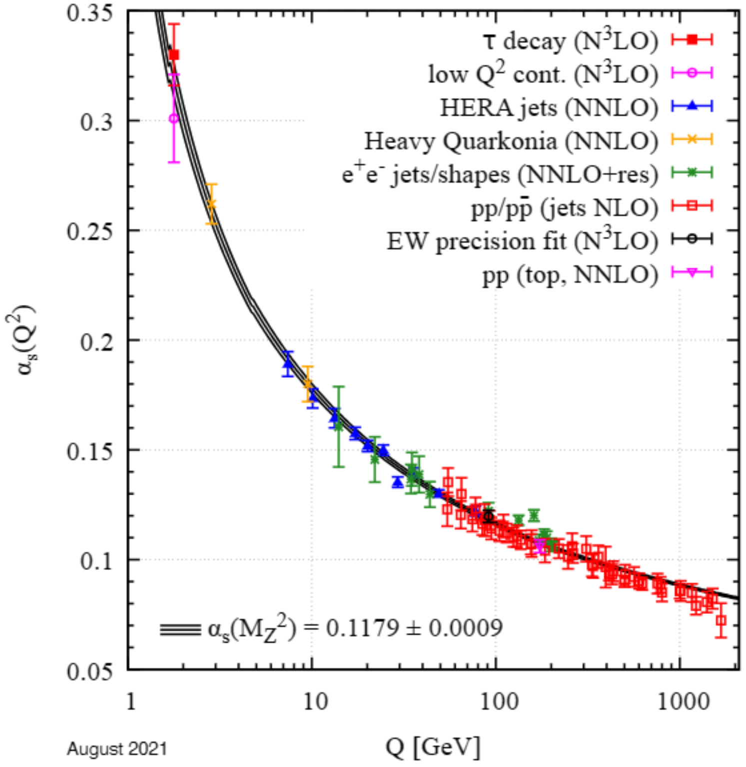
$\beta$ **function…** $\dfrac{dg}{d\log\mu} = \beta,\ \beta^{1loop} = -\dfrac{g^3}{16\pi^2} b_0$

$$\frac{1}{g(\mu)^2} = \frac{1}{g(\mu_0)^2} + \frac{b_0}{8\pi^2}\log\frac{\mu}{\mu_0}$$

**At dynamical scale** $\Lambda,\ g(\Lambda) \to \infty$

$$0 = \frac{1}{g(\mu_0)^2} + \frac{b_0}{8\pi^2}\log\frac{\Lambda}{\mu_0}$$

$$\Lambda^{b_0} = \mu^{b_0}\exp\left[-\frac{8\pi^2}{g(\mu)^2}\right]$$




***Ref: PDG QCD review***

# SUSY QCD (SQCD)

Super partners
Scalar quark $\tilde{q}$
Gluino $\lambda$

これらがdecoupleすると
QCDと同じ登場人物になる.

SQCD&AMSB (ASQCD)

SQCD　　SQCD+deformation　　$\Lambda$　　QCD?

Soft mass
$m$

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# $SU(N_c)$ flavor $N_f$ SQCD Lagrangian

``Quark superfield" $Q = Q + \theta\underline{\psi_Q} + \theta^2 F,\ \tilde{Q} = \tilde{Q} + \theta\underline{\tilde{\psi}_{\tilde{Q}}} + \theta^2 \tilde{F}$

$\psi_Q$ **Left-handed quark**

$i\sigma^2\tilde{\psi}_{\tilde{Q}}$ **Right-handed quark**

**ゲージ不変 かつ 超対称性のある**

$$L = tr\int d^4\theta\,\left(Q^\dagger e^V Q + \tilde{Q}e^{-V}\,\widetilde{Q^\dagger}\right) + \frac{1}{8\pi}Im\left[\tau\int d^2\theta W^{a\alpha}W^a_\alpha\right] + \int d^2\theta \underbrace{W(Q,\tilde{Q})}_{\text{Super potential}} + h.c$$

**より一般には超場$\Phi^i$に対して**

$$L = tr\int d^4\theta\ K\left(\Phi^i,\Phi^{\dagger j}\right) + \left[\int d^2\theta W\left(\Phi^i\right)\ + h.c\right]$$

# Weyl compensator…AMSBを簡便に計算する方法

$\Phi = 1 + \underbrace{\theta^2 m}_{\text{AMSB}}$ …SUSYを破る $m$ が取り入れられている。

$$L = \int d^4\theta\ \Phi^* \Phi K + \int d^2\theta\ \Phi^3 W + c.c$$

**Effects**

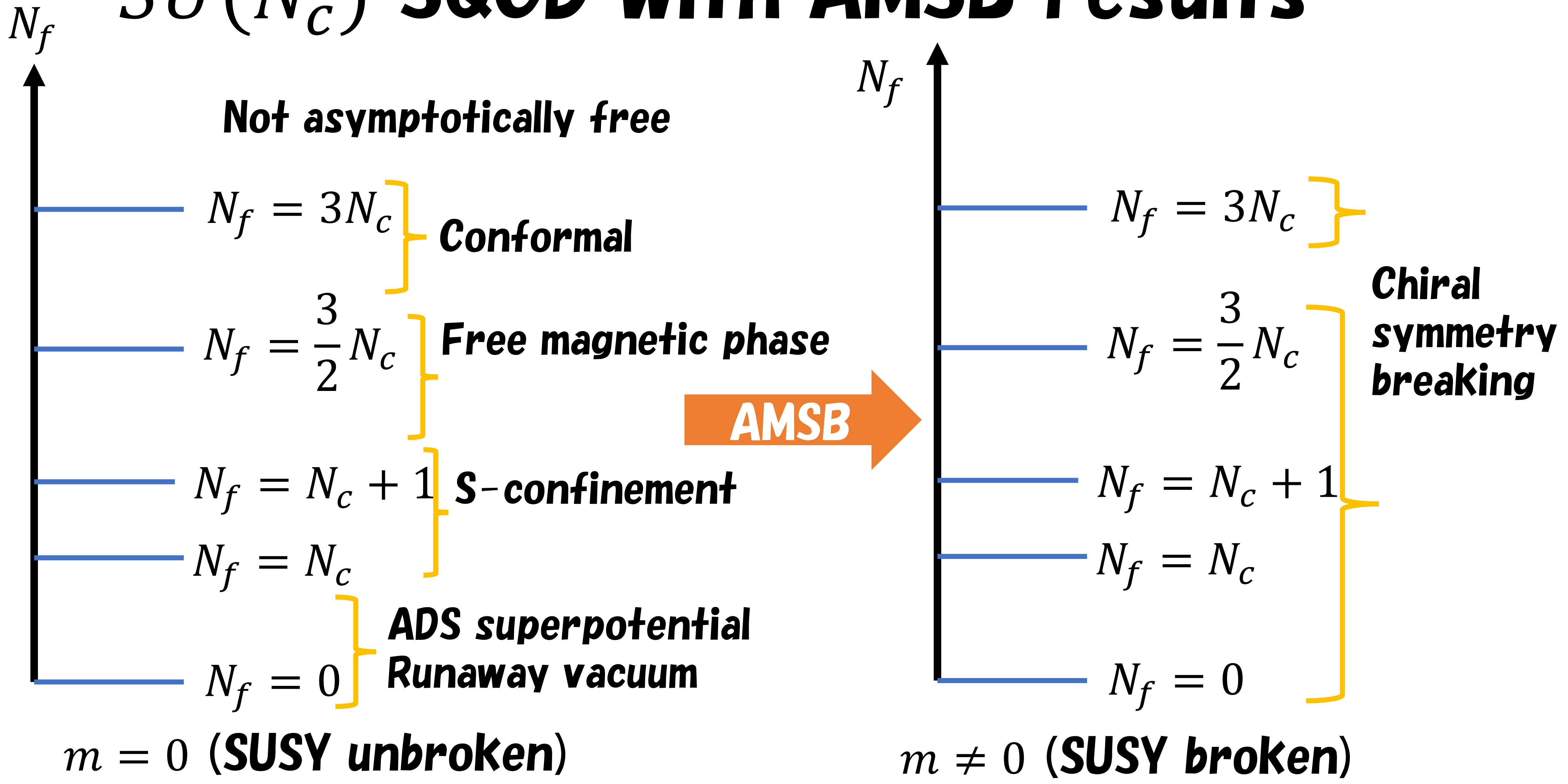
**Tree level** $\quad L_{tree} = -m\left(\phi_i \dfrac{\partial W}{\partial \phi_i} - 3W\right) + c.c$

**Induced mass** $\quad M_i = \dfrac{\beta_i(g^2)}{2g_i^2} m, \quad m_i^2 = -\dfrac{\dot{\gamma}_i}{4} m^2, \quad A_{ijk} = \dfrac{\gamma_i + \gamma_j + \gamma_k}{2} m$

AMSBは我々に興味のある低エネルギーの量にしかよらない。
つまりUV insensitive.

Ref: Pomarol et.al JHEP05(1999)013,arxiv:9903448

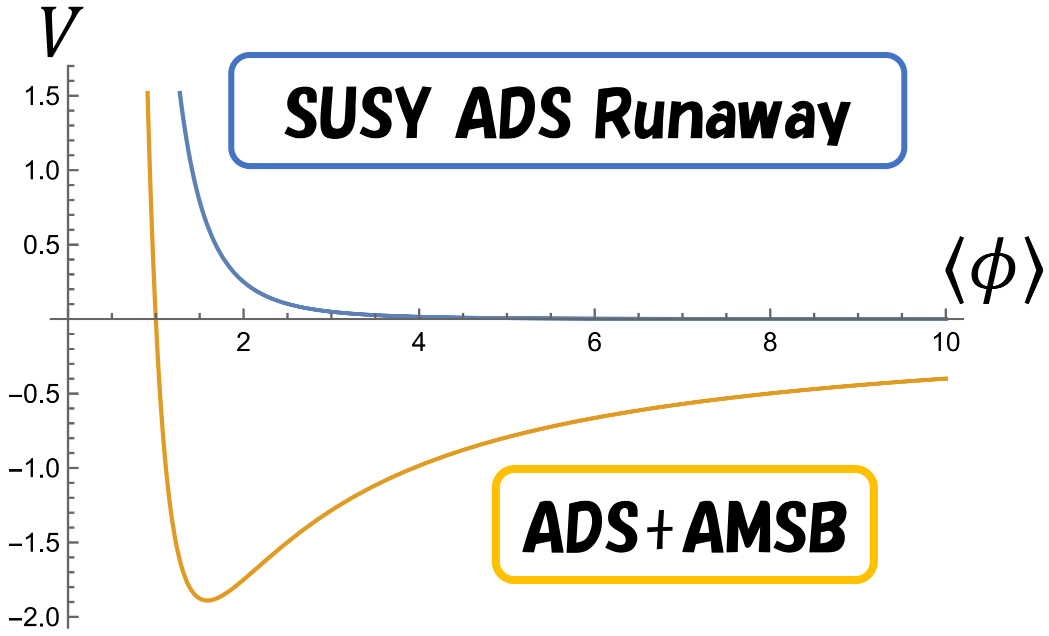
# $SU(N_c)$ SQCD with AMSB results



# 例として$0 \leq N_f < N_c$の場合

Refs: Affleck et.al
Phys.Rev.Lett.51,1026(1983)
Seiberg
Nucl.Phys.B435,129(1995);
Phys.Rev.D49,6857(1994)
Intriligator et.al
Nucl.Phys.B431,551(1994)

$$V = \underbrace{\frac{1}{2N_f}\left|2N_f\frac{1}{\phi}\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)}\right|^2}_{\text{ADS}} \underbrace{- (3N_c-N_f)m\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{\frac{1}{N_c-N_f}} + c.c.}_{\text{AMSB}}$$




$$M_{ij} = \underline{\delta_{ij}}\Lambda^2\left(\frac{4(N_c+N_f)}{3N_c-N_f}\frac{\Lambda}{m}\right)^{(N_c-N_f)/N_c}$$

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

…カイラル対称性が破れている

# Conformal window: $\mathrm{N_f}^{\mathrm{II}} < N_f < N_f^{\mathrm{I}}$

## Conformal window…漸近自由 かつ スケール不変

$\beta$-function… $\beta(g) = \frac{d}{d\log\mu^2}\left(\frac{\alpha_s}{\pi}\right) = -\left(\beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 + \beta_1\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots\right)$

上限 $(N_f^{\mathrm{I}})$ は漸近自由から決まる $\beta_0 > 0$ , $N_f^{\mathrm{I}} = 16.5$ in QCD $(N_c = 3)$ (摂動から)

一方, $\beta_1 < 0$ だと 非自明な固定点が存在する $\left(\frac{\alpha_s}{\pi}\right)_* = -\frac{\beta_0}{\beta_1}$ .

下限$(N_f^{\mathrm{II}})$…繰りこみ群で固定点に流れる (その間はスケール不変).

→この $N_f^{\mathrm{II}}$ を決めるのは難しい問題

Latticeによると SU(3) では $8 < N_f^{\mathrm{II}} < 13$ で $N_f = 12$ に関心が持たれている。

SU(2) では $N_f < 6$だとカイラル対称性の破れが実現しているらしい。

# SQCD Conformal… $\frac{3}{2}N_c \leq N_f \leq 3N_c$

上限について

NSVZ $\beta$-function (SQCD)… $\beta(g) = \frac{d}{d\log\mu}\left(\frac{8\pi^2}{g^2}\right) = -\frac{g^3}{16\pi^2}\frac{3N_c - N_f + N_f\gamma(g^2)}{1-\frac{g^2}{8\pi^2}N_c}$

Anomalous dimension…$\gamma(g^2) = -\frac{g^2}{8\pi^2}\frac{N_c^2-1}{N_c} + O(g^4)$ :one loop level

$x = \frac{N_f}{N_c} = 3 - \epsilon,\ \beta(g) \simeq -\frac{g^3}{16\pi^2}\left(N_c\epsilon - \frac{3g^2}{8\pi^2}N_c^2\right)$ を使うと

$\to \epsilon < 0\ \left(N_f > 3N_c\right)$のときに漸近自由でなくなる。

$\epsilon = \frac{3g_*^2}{8\pi^2}N_c$ または $g_*^2 N_c = \frac{8\pi^2}{3}\epsilon \ll 1$ で $\beta(g_*) = 0$.

Banks-Zaks 固定点

# 下限について… R対称性とunitarityから

## Scaling dimension $D$について

Meson field $\tilde{Q}Q$ $\left(\gamma = \frac{N_f - 3N_c}{N_f}\right)$ に適用すると

$$D = 2 + \gamma = 3\frac{N_f - N_c}{N_f}$$

Unitarityより $D \geq 1 \rightarrow N_f \geq \frac{3}{2}N_c$

SQCDではIR固定点は $\frac{3}{2}N_c \leq N_f \leq 3N_c$の範囲内にあるだろう。

# ASQCDに戻って $N_f \simeq 3N_c$ 付近でAMSBを適用する。

Seiberg duality の条件… 2回双対をとるともとの理論に戻る。

Superpotential $W = YN\left(Q\tilde{Q} - M\right)$

$\downarrow$ decouple (s)quark

$$W = N_c \Lambda^3 \left(\frac{\det(YN)}{\Lambda^{N_f}}\right) - Y\Lambda N M$$

$\downarrow$

繰りこみ群方程式 (RGE)

$$\frac{dg^2}{d\log\mu} = -g^4 \frac{3N_c - N_f - \frac{N_f}{2}\frac{1}{4\pi^2}\left(2g^2\frac{N_c^2-1}{2N_c} - N_f|Y|^2\right)}{8\pi^2 - N_c g^2}$$

$$\frac{d|Y|^2}{d\log\mu} = -|Y|^2\left[\frac{1}{4\pi^2}\left(2g^2\frac{N_c^2-1}{2N_c} - N_f|Y|^2\right) - \frac{N_c}{8\pi^2}|Y|^2\right]$$

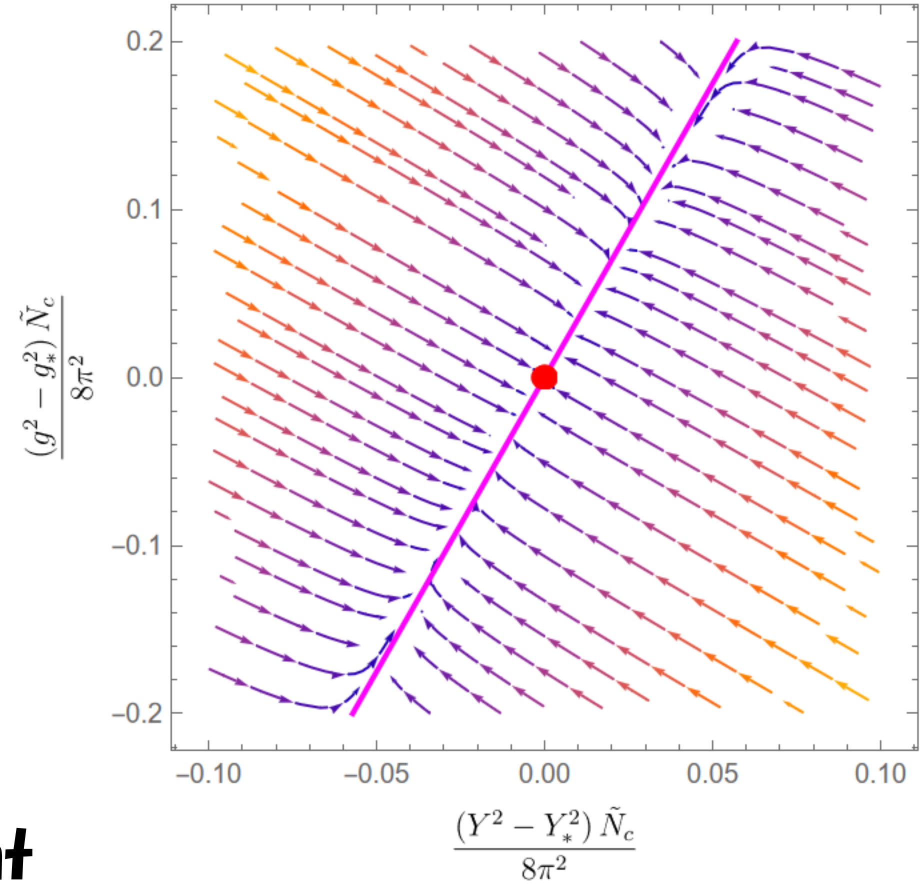
# 繰りこみ群方程式 (RGE)

$$\frac{dg^2}{d\log\mu} = -g^4 \frac{3N_c - N_f - \frac{N_f}{2}\frac{1}{4\pi^2}\left(2g^2\frac{N_c^2-1}{2N_c} - N_f|Y|^2\right)}{8\pi^2 - N_c g^2} \qquad \frac{d|Y|^2}{d\log\mu} = -|Y|^2\left[\frac{1}{4\pi^2}\left(2g^2\frac{N_c^2-1}{2N_c} - N_f|Y|^2\right) - \frac{N_c}{8\pi^2}|Y|^2\right]$$

**固定点は** $|Y_*|^2 = \frac{8\pi^2}{N_c}2\epsilon,\ g_*^2 = \frac{8\pi^2}{N_c}7\epsilon$

**固定点の周りで**

$$\frac{d}{d\log\mu}\begin{pmatrix}\delta|Y|^2\\ \delta g^2\end{pmatrix} = \begin{pmatrix}14\epsilon & -4\epsilon\\ -441\epsilon^2 & 147\epsilon^2\end{pmatrix}\begin{pmatrix}\delta|Y|^2\\ \delta g^2\end{pmatrix}$$




1. $O(\epsilon)$ **trajectory converges**
2. **approaches along** $O(\epsilon^2)$ **trajectories**

↓

$O(\epsilon)$**より遅い… AMSB effect is relevant**

# 実際にポテンシャルも計算できて、
# カイラル対称性の破れた真空が得られる。

**Superpotential** $W = YN(Q\tilde{Q} - M)$

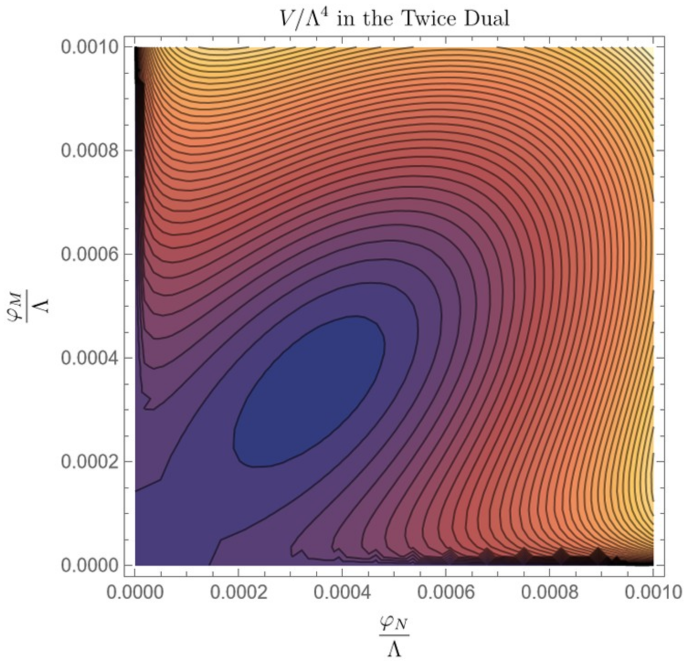
$\downarrow$ **decouple (s)quark**

$$W = N_c\Lambda^3\left(\frac{\det(YN)}{\Lambda^{N_f}}\right) - Y\Lambda NM$$

$\downarrow$ **with** $M = \phi\delta_{ij},\ N = \phi_N\delta_{ij}$

$$V \simeq \Lambda^4\left[\frac{Y^2}{c_M c_N}\left(\frac{\mu}{\Lambda}\right)^{2\epsilon}\left(\frac{\phi}{\Lambda}\right)^2 + \frac{Y^2}{c_M c_N}\left(\frac{\mu}{\Lambda}\right)^{2\epsilon}\left(\frac{\phi_N}{\Lambda}\right)^2 - \frac{2Y}{\sqrt{c_M c_N}}\frac{m}{\Lambda}\frac{\phi}{\Lambda}\frac{\phi_N}{\Lambda}\right]$$

$$\phi = \phi_N = \left(\frac{\sqrt{c_M c_N}}{Y}\frac{m}{\Lambda}\right)^{\frac{1}{\epsilon}}$$




$N_f^{\mathrm{II}} \geq 3N_c$だと予想

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# 真空周りのゆらぎをみる

$$Q = \begin{pmatrix} v\xi^T \\ 0 \end{pmatrix}, \qquad \tilde{Q} = \begin{pmatrix} v\xi \\ 0 \end{pmatrix} \qquad \xi \to h(\xi, g_L, g_R)\xi g_L^T, \xi^T \to h^*(\xi, g_L, g_R)\xi g_R^T, U = \xi^2$$

**Kinetic term** : $|D_\mu Q|^2 + \ |D_\mu \tilde{Q}|^2 = \frac{v^2}{2} Tr\left[\partial_\mu U^\dagger \partial^\mu U\right] + \frac{\mathrm{v}^2}{2}\mathrm{Tr}\left(2g\rho_\mu + i\left(-\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right)\right)^2$

$\longrightarrow$ **Decay constant** :

$$f_\pi^2 = 8\Lambda^2 \left(\frac{N_c + N_f}{3N_c - N_f}\frac{\Lambda}{m}\right)^{(N_c - N_f)/N_c}$$

$$F_\pi^2 \propto \Lambda_{\text{eff}}^2, \tag{3.8}$$

$$F_T \propto \Lambda_{\text{eff}}^3 \tag{3.9}$$

so that $m_\pi^2 \propto \Lambda_{\text{eff}} M_q$. Unfortunately, it is easy to check that Eqs. (3.8) and (3.9) do not have the same scaling behavior with $m_{\text{soft}}$ in the formal decoupling limit as the weak-coupling results Eqs. (3.3) and (3.4), no matter which of Eqs. (3.5), (3.6), or (3.7) applies. Furthermore, the large $N_c$ scaling of the large-$m_{\text{soft}}$ chiral Lagrangian does not conform with expectations from ordinary QCD. Nonsupersymmetric chiral perturbation theory implies [18] that $F_\pi^2 \propto N_c$ and $F_T \propto N_c$, but the formal decoupling limit of the weakly-coupled SQCD chiral Lagrangian scales as $F_\pi^2 \propto N_c^0$ and $F_T \propto N_c^0$.

$O(N_c^0)\cdots$ **glueball の寄与？**

**Martin & Wells**

**Phys.RevD58 (1998)115013**

# Rescaling anomaly N.Arkani-Hamed & H. Murayama Phys.RevD57(1998)6638-6648

$$\frac{8\pi^2}{g_h^2} = \frac{8\pi^2}{g_c^2} + \frac{N_c}{8\pi^2}\log g_c^2$$

**With t' Hooft coupling** $N_c g_c^2 = g_t^2$

**Scaling relation:**

$$\Lambda^{3N_c-N_f} = \mu^{3N_c-N_f} e^{-8\pi^2/g_h^2} = \mu^{3N_c-N_f} e^{-8\pi^2/g_c^2} (g_c^2)^{-N_c} = \mu^{3N_c-N_f} \underline{N_c^{N_c}} e^{-8\pi^2/g_t^2}\ (g_t^2)^{-N_c}$$

$$f_\pi^2 = 8\left(\frac{N_c+N_f}{3N_c-N_f}\ \frac{\Lambda^{3N_c-N_f}}{m^{N_c-N_f}}\right)^{\frac{1}{N_c}}$$

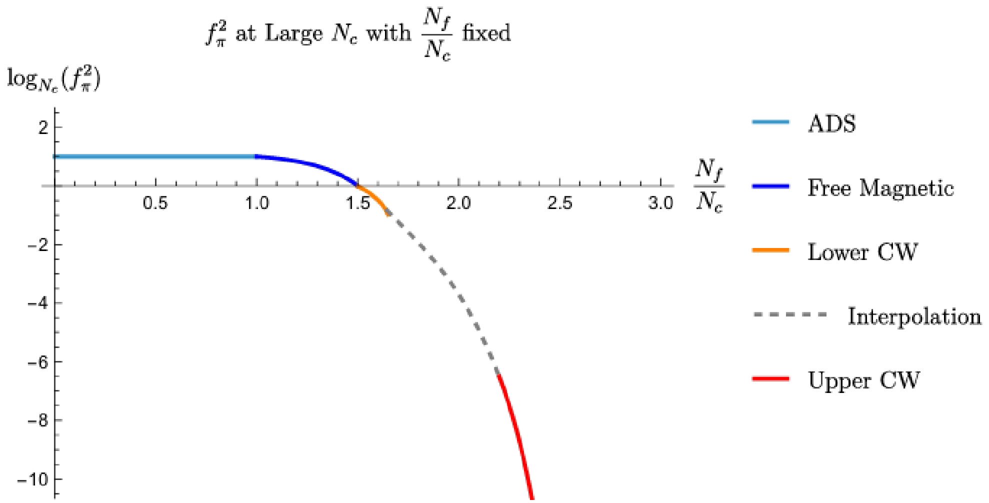
$$f_\pi^2 = O(N_c)$$

$O(N_c)$**である！**

# Scaling of $f_\pi^2 \cdots$ $x = \frac{N_f}{N_c}$ dependence

$$f_\pi^2 = N_c\left(1 + a_1\left(\frac{N_f}{N_c}\right) + a_2\left(\frac{N_f}{N_c}\right)^2 + \cdots\right)$$

## $f_\pi^2 \cdots$ power dependence $N_c^x$



# $\rho$ meson

Kawarabayashi and Suzuki Phys.Rev.Lett 16(1966)255
Riazuddin and Fayazuddin Phys.Rev. 147 (1966)1071
Bando et al Phys.Rev.Lett .54(1985) 1215
Bando et al Nucl.Phys.B259 (1985) 493

**Traditionally… ρ meson as dynamical gauge boson of Hidden Local symmetry (HLS)**

$G_{global} \times H_{local}$ **with** $G_{global} = SU(2)_L \times SU(2)_R$, $H_{local} = SU(2)_V$

$$L = L_A + aL_V$$

$L_V$ **is an auxiliary field and** $a$ **is arbitrary parameter.**

**If** $L_{kin}$ **is generated (quantum mechanically),** $L_{HLS} = L_V + aL_A + L_{kin}$
**successful phenomenological results can be derived (in particular with** $a = 2$**).**

**Universality of ρ meson coupling (ρ universality)** $g_{\rho\pi\pi} = g_{\rho NN} = \cdots$
**KSRF relation** $m_\rho^2 = a g_{\rho\pi\pi}^2 f_\pi^2$
**Vector meson dominance**

**Our case…**

**Kinetic term :** $|D_\mu Q|^2 + |D_\mu \tilde{Q}|^2 = \frac{v^2}{2} Tr\left[\partial_\mu U^\dagger \partial^\mu U\right] + \frac{v^2}{2} \mathrm{Tr}\left(2g\rho_\mu + i(-\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)\right)^2$

$\longrightarrow a = 1$

# Wess-Zumino-Witten (WZW) term

**Effective action** $W = -i\log\left[\int dQ\, d\tilde{Q}\, dV \exp\left[i\int d^4x\, L\right]\right]$

**Transformation:** $Q \to \xi^* Q,\ \tilde{Q} \to \xi^\dagger \tilde{Q}$

$\longrightarrow$ $DQD\tilde{Q} \to DQJ(\xi^T)D\tilde{Q}J(\xi)$

**Jacobians arise**

$$J(\xi) = \exp\left[i\int Tr\left(\xi^\dagger d\xi\right)^5\right] \qquad J(\xi^T) = \exp\left[-i\int Tr\left(\xi d\xi^\dagger\right)^5\right]$$

$$W' = W - i\log J(\xi) - i\log J(\xi^T)$$

$\longrightarrow$ $\int Tr\left[\left(\xi^\dagger d\xi\right)^5 - \left(\xi d\xi^\dagger\right)^5\right] = \int Tr\left[\left(\xi^\dagger d\xi - \xi d\xi^\dagger\right)^5\right] = \int Tr\left[\left(U^\dagger dU\right)^5\right]$

**(up to local counter term)**

Ref: D' Hoker & Farhi, NuclB248(1984)77-89

# Wess-Zumino-Witten (WZW) term

**Effective action** $W = -i \log\left[\int dQ\, d\tilde{Q}\, dV \exp\left[i \int d^4x\, L\right]\right]$

**Transformation:** $Q \to \xi^* Q,\ \tilde{Q} \to \xi^\dagger \tilde{Q}$

$\longrightarrow$ $DQD\tilde{Q} \to DQJ(\xi^T)D\tilde{Q}J(\xi)$

**Jacobians arise**

$$J(\xi) = \exp\left[iN_c \int Tr\left(\xi^\dagger d\xi\right)^5\right] \qquad J(\xi^T) = \exp\left[-iN_c \int Tr\left(\xi d\xi^\dagger\right)^5\right]$$

$$W' = W - i\log J(\xi) - i\log J(\xi^T)$$

$$\longrightarrow \quad N_c \int Tr\left[\left(\xi^\dagger d\xi\right)^5 - \left(\xi d\xi^\dagger\right)^5\right] = N_c \int Tr\left[\left(\xi^\dagger d\xi - \xi d\xi^\dagger\right)^5\right] = N_c \int Tr\left[\left(U^\dagger dU\right)^5\right]$$

**(up to local counter term)**

**$N_c$ comes from $N_f$ block of fermions and $N_c - N_f$ block of fermions**

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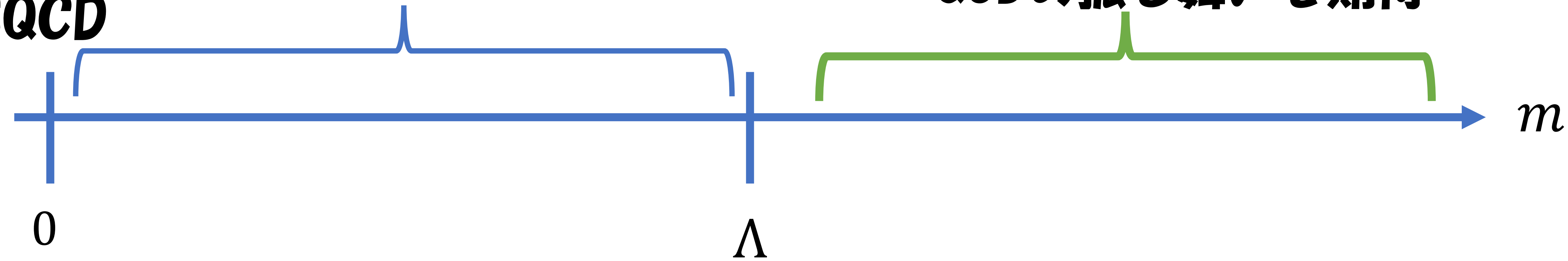
# $m \gg \Lambda$ ではQCDのようになっていると期待

$$\langle q\bar{q} \rangle \propto \Lambda_{QCD}^3$$
$$\langle GG \rangle \propto \Lambda_{QCD}^4$$

**ASQCD** → つながっているのだろうか?

**QCDの振る舞いを期待**

**SQCD**



# $m \ll \Lambda$ でどう計算するか？

Quark / Squark condensate

**As a chiral superfield**

$$\langle M \rangle = \langle \tilde{q}\tilde{q}^* \rangle + \theta^2 \langle q\bar{q} \rangle$$

**Potential minimum**

**運動方程式からF-componentを計算**

$$\langle q\bar{q} \rangle = \langle F_M \rangle = -\left\langle \frac{\partial W}{\partial M} \right\rangle$$

## Gluon/Gluino condensate

生成汎関数より

$$\log Z \supset \frac{1}{16\pi i}\int d^2\theta\ \tau WW$$

Promote to spurion, which sources $\langle GG\rangle$, F-component sources $\langle\lambda\lambda\rangle$

$$\langle GG\rangle = 16\pi i\frac{\partial}{\partial\tau}\log Z = 16\pi i\ \left\langle\frac{\partial V}{\partial\tau}\right\rangle$$

$$\langle\lambda\lambda\rangle = 16\pi i\frac{\partial}{\partial F_\tau}\log Z = 16\pi i\ \left\langle\frac{\partial W}{\partial\tau}\right\rangle$$

$$\langle GG \rangle = 16\pi i \frac{\partial}{\partial \tau} \log Z = 16\pi i \left\langle \frac{\partial V}{\partial \tau} \right\rangle$$

$$\langle \lambda\lambda \rangle = 16\pi i \frac{\partial}{\partial F_\tau} \log Z = 16\pi i \left\langle \frac{\partial W}{\partial \tau} \right\rangle$$

# Dynamical scaleを思い出すと

$$\Lambda = \mu \exp\left[\frac{2\pi i}{3N_c - N_f}\tau\right] \longrightarrow \frac{\partial}{\partial \tau} = \frac{2\pi i}{3N_c - N_f} \Lambda \frac{\partial}{\partial \Lambda}$$

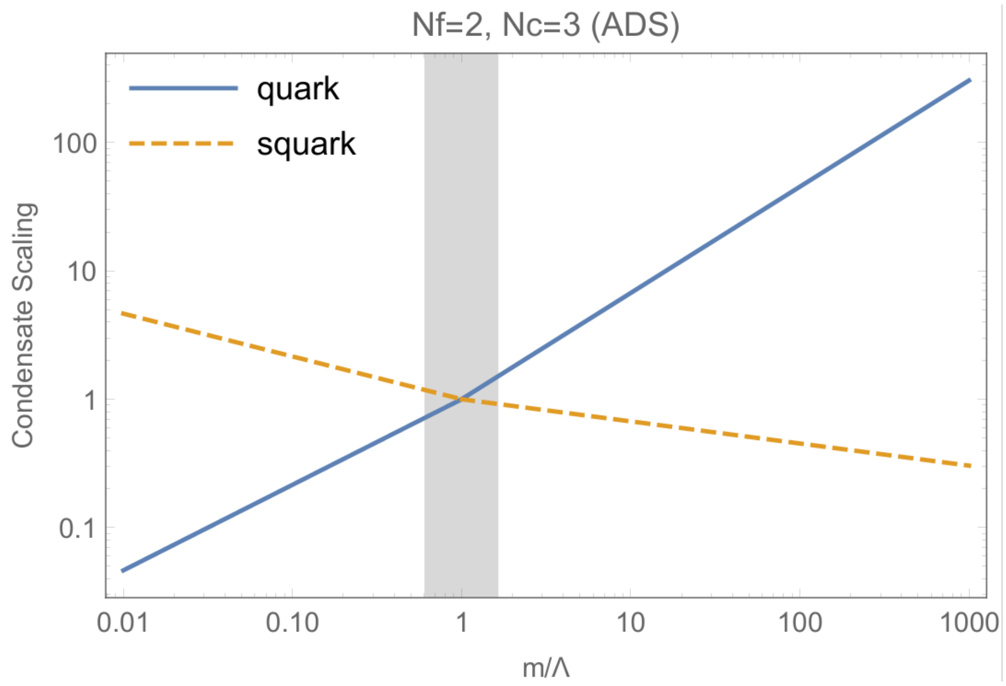
$$\boxed{\begin{aligned} \langle GG \rangle &= -\frac{32\pi^2}{3N_c - N_f} \Lambda \left\langle \frac{\partial V}{\partial \tau} \right\rangle \\ \langle \lambda\lambda \rangle &= -\frac{32\pi^2}{3N_c - N_f} \Lambda \left\langle \frac{\partial W}{\partial \tau} \right\rangle \end{aligned}}$$

# ADS $N_f < N_c$の場合

## Quark condensate

$$\langle \tilde{q}^* \tilde{q} \rangle = 4N_f \Lambda^2 \left( \frac{3N_c - N_f}{N_c + N_f} \frac{m}{\Lambda} \right)^{\frac{N_f}{N_c}}$$

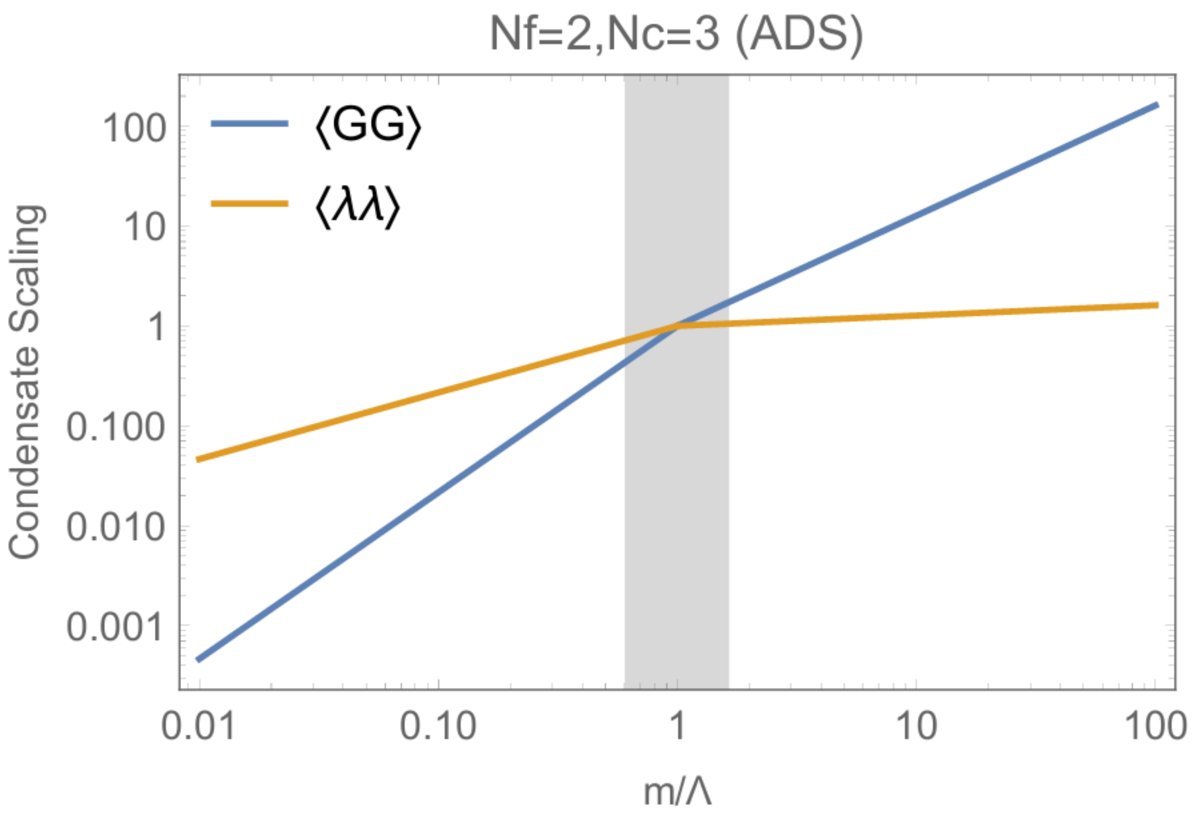
$$\langle \bar{q} q \rangle = 4N_f \Lambda^3 \left( \frac{3N_c - N_f}{N_c + N_f} \frac{m}{\Lambda} \right)^{\frac{N_f}{N_c}}$$



# Gluon condensate

$$\langle GG \rangle = 128\pi^2 N_f \Lambda^4 \left( \frac{3N_c - N_f}{N_c + N_f} \frac{m}{\Lambda} \right)^{\frac{N_f}{N_c}}$$

$$\langle \lambda\lambda \rangle = 32\pi^2 N_f \Lambda^3 \left( \frac{3N_c - N_f}{N_c + N_f} \frac{m}{\Lambda} \right)^{\frac{N_f}{N_c}}$$



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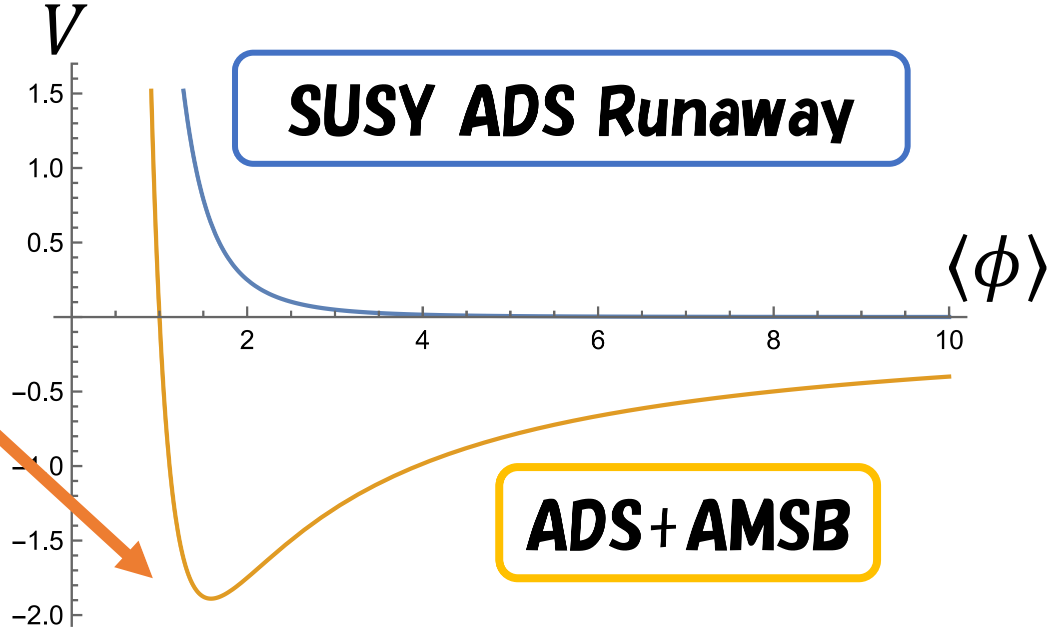
2. SQCD+AMSB (ASQCD)の真空
3. Chiral Lagrangian+WZW term
4. Condensates
5. Mass spectrum

# どう計算するか？

Potentialのminimumを知っている




ゆらぎを考える… $M = \phi_0 + \frac{1}{2}(\sigma + i\pi)$

質量行列は $\begin{pmatrix} \frac{\partial^2 V}{\partial \sigma^2} & \frac{\partial^2 V}{\partial \sigma \partial \pi} \\ \frac{\partial^2 V}{\partial \pi \partial \sigma} & \frac{\partial^2 V}{\partial \pi^2} \end{pmatrix} \Big|_{M=\phi_0}$

# ADS $N_f = 3, N_c = 4$の場合

## Quark mass $m_Q = 0$ のとき

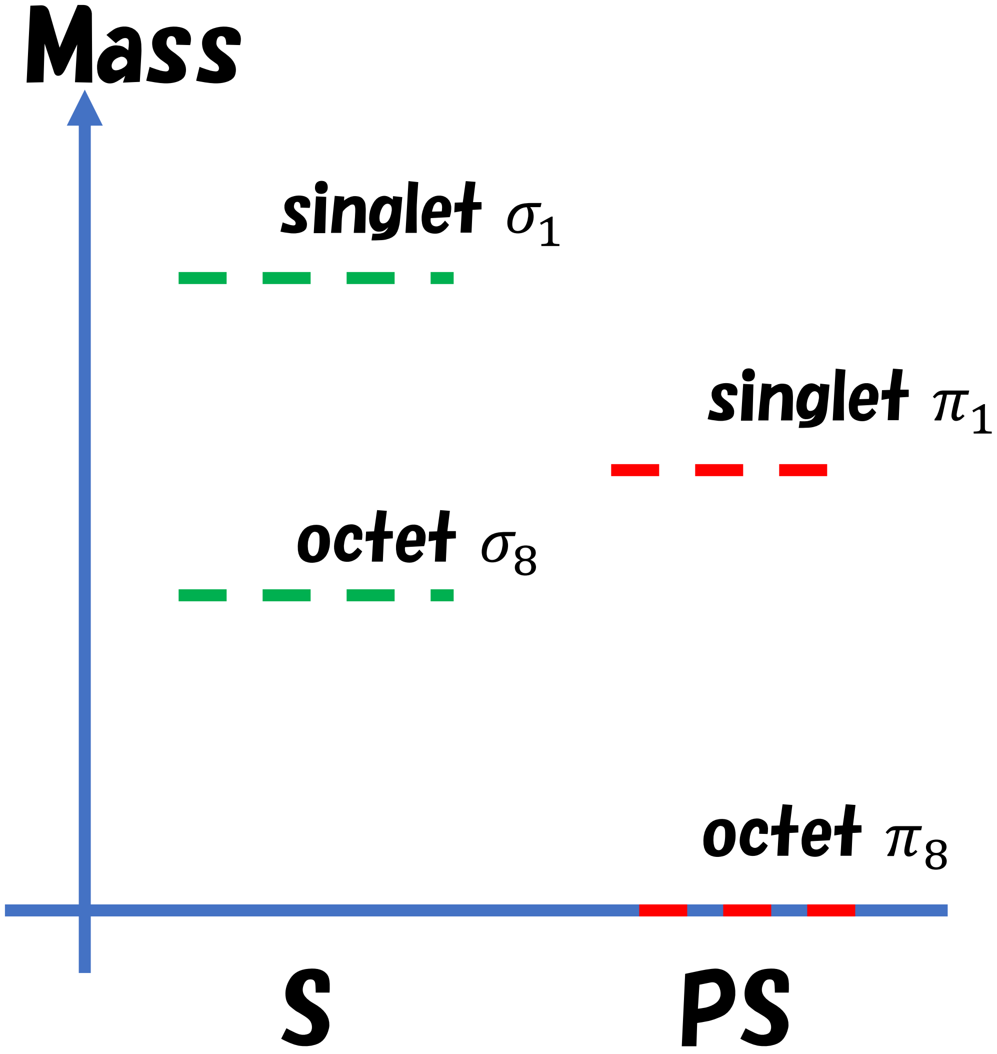
Singlet scalar : $\frac{648}{7} m^2$

Octet scalar : $\frac{162}{49} m^2$

Singlet pseudo-scalar : $\frac{486}{7} m^2$

Octet pseudo-scalar : $0$

Pion is Nambu-Goldstone boson



## quark massを取り込む

$$W \supset Tr\left[m_Q M\right]$$

**Singlet scalar :**

$$\frac{648}{7}m^2 + \frac{1496}{49}m\left(m_{q_1} + m_{q_2} + m_{q_3}\right)$$

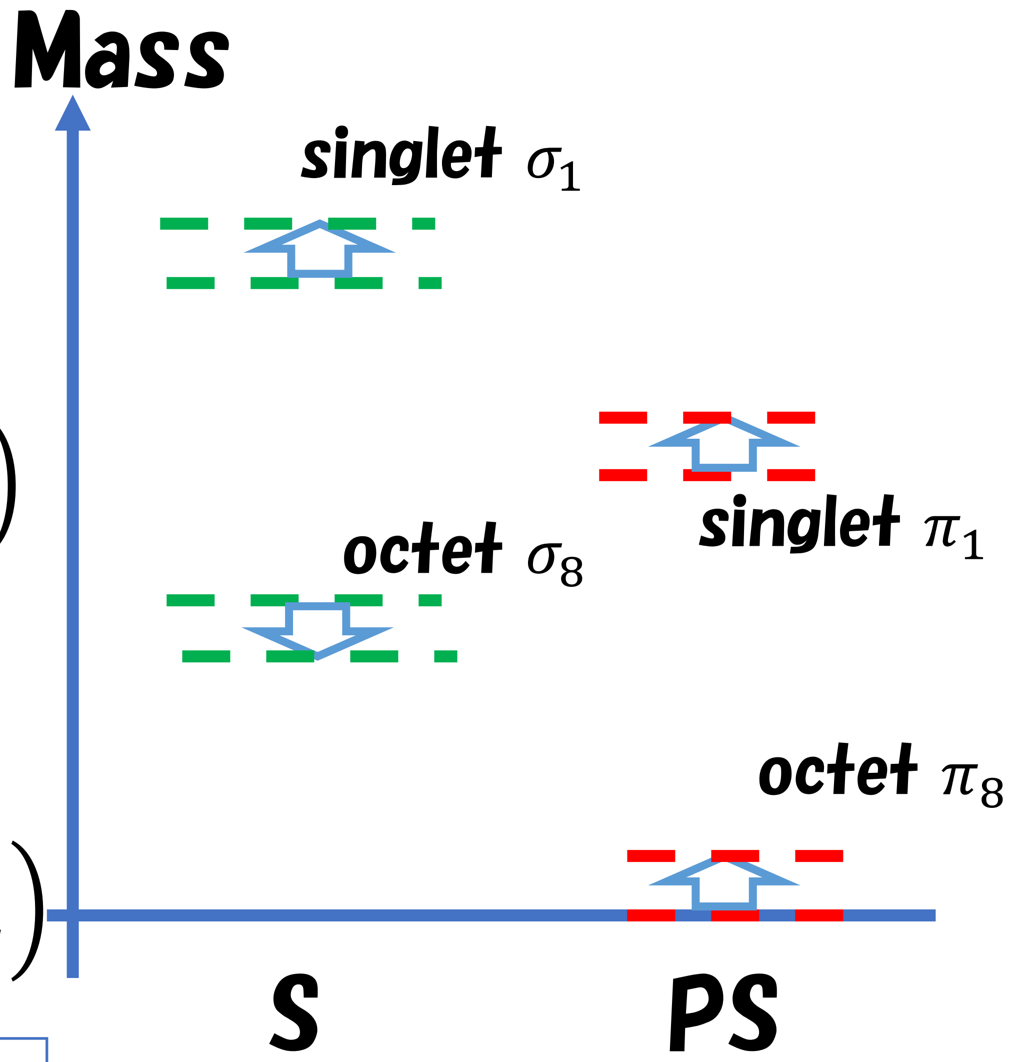
**Octet scalar :** $\frac{162}{49}m^2 - 3m\left(m_{q_i} + m_{q_j}\right)$

**Singlet pseudo-scalar:**

$$\frac{486}{7}m^2 + \frac{872}{21}m\left(m_{q_1} + m_{q_2} + m_{q_3}\right)$$

**Octet pseudo-scalar :** $\frac{25}{7}m\left(m_{q_i} + m_{q_j}\right)$

**Pion is pseudo-Nambu-Goldstone boson**



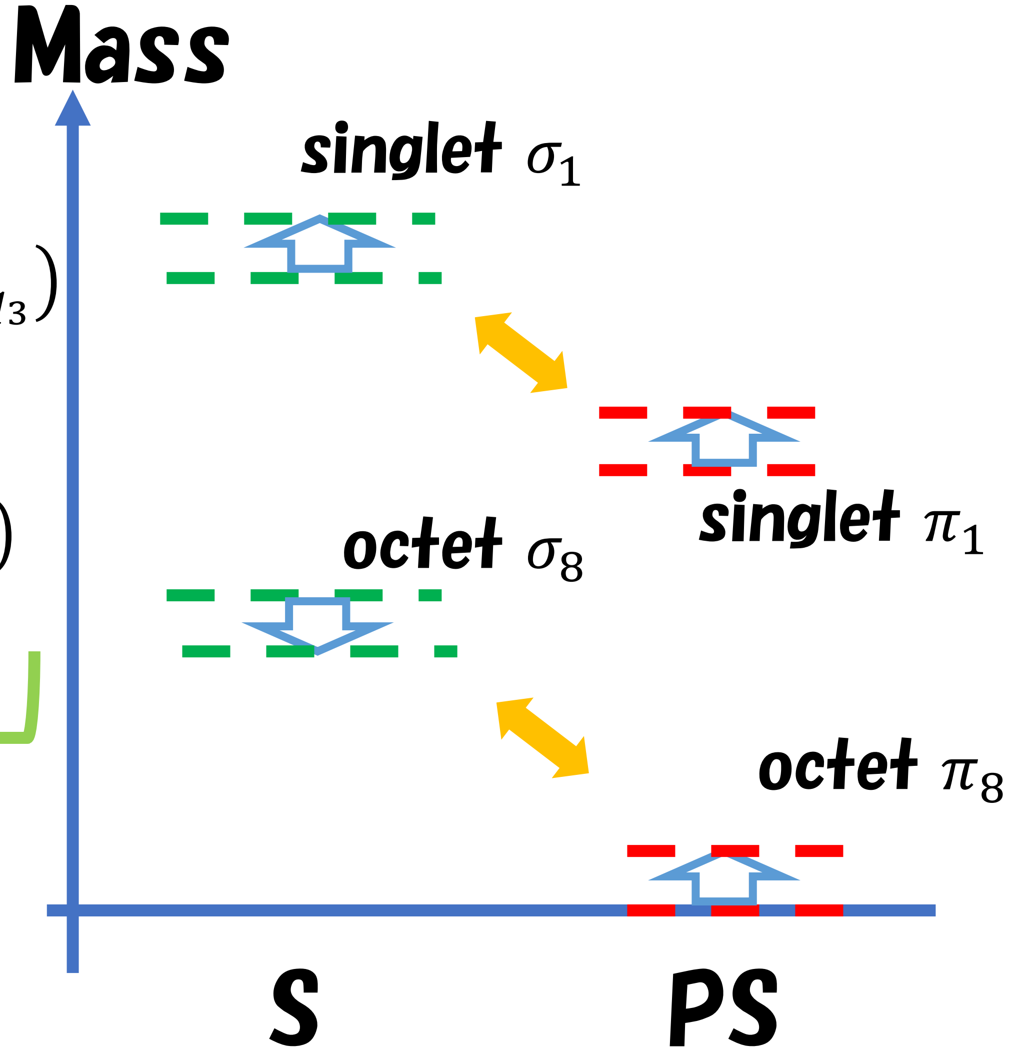
# ADS $N_f = 3, N_c = 4$の場合

## 比較すると

$$m_\sigma^2 + \frac{21}{25} m_\pi^2 = \begin{cases} \dfrac{20406}{175} m^2 + \dfrac{11904}{175} m\left(m_{q_1} + m_{q_2} + m_{q_3}\right) & \textbf{Singlet} \\ \dfrac{162}{49} m^2 + \dfrac{128}{49} m\left(m_{q_1} + m_{q_2} + m_{q_3}\right) & \textbf{Octet} \end{cases}$$

$\equiv m_0^2$ 質量の基準点とみなす

$$\Rightarrow \quad m_\sigma^2 = m_0^2 + c m_\pi^2\,;\ c < 0$$

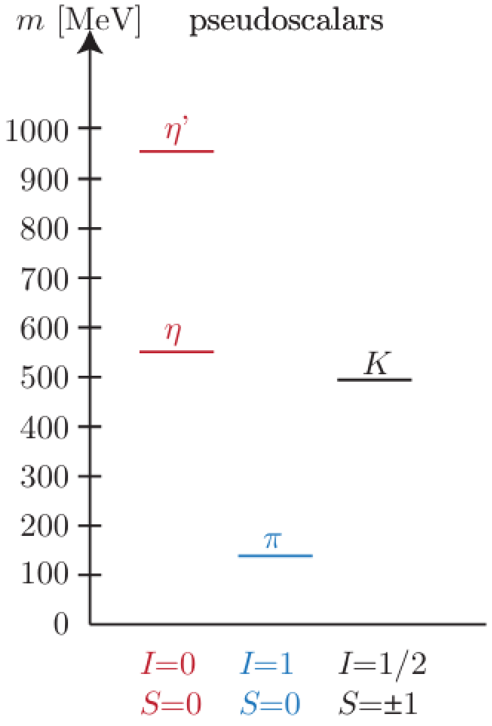


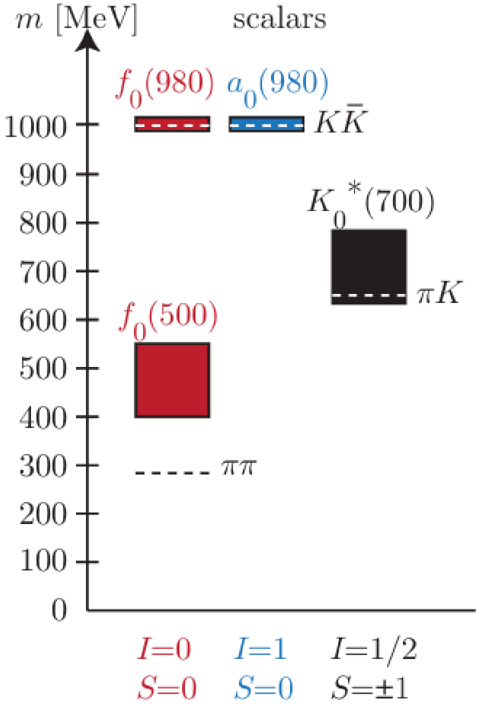
# Previous implication

$$m_\sigma^2 - m_0^2 = c\, m_\pi^2 \qquad c < 0$$

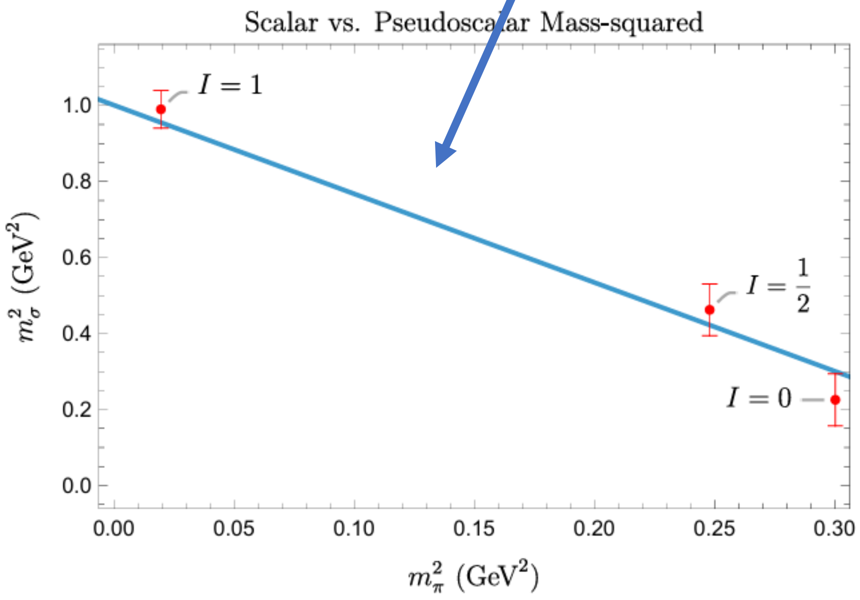
## SIDMを調べたときに…

PDG data (real)




$N_c = 3, N_f = 3$のとき$c = \frac{7}{3}$




Kondo et.al JHEP09(2022)041, arxiv:2205.08088

# 結論

カイラル対称性の破れた ASQCD の真空から

Chiral Lagrangian と WZW termを出した。

quark や gluonの凝縮を計算した。

mass spectrumを調べた。

**ASQCD が QCDの近似になり得る状況証拠を与えた。**