9/1 (Mon.) @ PPP2025

Pseudo NG bosons from finite modular symmetry

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based on arXiv: 2402.02071 [JHEP] 2405.03996 [JHEP], 2409.19261 [JHEP]

in collaboration with

T.Higaki (Keio U.), T.H.Jung (IBS), T.Kobayashi (Hokkaido U.)

Pseudo Nambu-Goldstone [NG] boson

- "pseudo" NG boson

Goldstone theorem :

massless NG boson appears when **continuous symmetry is broken**

but if a symmetry is not exact…

- **pseudo** NG boson is not exactly massless, but **light**
- mass is induced by symmetry breaking effects

- Examples

	pion π	**axion a**	**majoron J**
symmetry	chiral $U(1)$	global $U(1)_{PQ}$	global $U(1)_{B-L}$
breaking	quark mass	QCD + ?	gravity ?

in SM **maybe dark mater [DM], this talk**

Axion

$\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$

- strong CP problem

$$\mathcal{L}_{QCD} \ni \theta \, F_{\mu\nu} \, \tilde{F}^{\mu\nu}$$

is not forbidden by sym., so $\theta \sim \mathcal{O}(1)$

but neutron EDM limit is $\theta < 10^{-10}$

Why is θ so small ?

- Axion solution …if global $U(1)_{PQ}$ has mixed anomaly with QCD

$\left(\theta + \frac{a}{f_a}\right) F_{\mu\nu} \, \tilde{F}^{\mu\nu}$, where a is axion as pNG of $U(1)_{PQ}$

→ $V(a) = \Lambda_{QCD}^4 \left[1 - \cos\left(\theta + \frac{a}{f_a}\right)\right]$ by QCD effect

→ axion is stabilized at $\left\langle \theta + \frac{a}{f_a} \right\rangle = 0$, thus no EDM

$U(1)_{PQ}$ symmetry ??

$U(1)_{PQ}$ has mixed QCD anomaly, so it is broken by QCD

- PQ (axion) quality
  - if QCD breaks $U(1)_{PQ}$, why not others ?
  - in general, global symmetry will be broken by gravity

$$V(a) = \Lambda_{QCD}^4 \left[1 - \cos\left(\theta + \frac{a}{f_a}\right)\right] + \textbf{[PQ violating terms]}$$

PQV effects should be so small that $\langle \theta + a/f_a \rangle < 10^{-10}$

→ **PQ quality problem**

This talk

finite modular symmetry Γ_N realizes pNG

➢ Finite modular axion and strong CP problem

- residual Z_N^T symmetry of Γ_N realizes **accidental** $U(1)_{PQ}$
- in KSVZ-like scenario, modulus is **stabilized by 1-loop potential**
- PQ quality is ensured by the Z_N^T symmetry

➢ Other examples

- majoron in type-I seesaw
- pNG modes in other stabilization

Outline

1. Introduction
2. **Brief review of finite modular symmetry Γ_N**
3. Finite modular axion and radiative stabilization
4. Other examples and summary

Finite modular symmetry

- **modular group Γ** $\Leftrightarrow$ special linear group SL(2,Z)

$$\Gamma := SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\middle|\; a, b, c, d \in \mathbb{Z}\,, ad - bc = 1 \right\}$$

we often consider $\bar{\Gamma} := \Gamma/\{\pm 1\} = PSL(2,\mathbb{Z})$

- **Finite** modular group $\Gamma_N := \bar{\Gamma}/\bar{\Gamma}(N)$

where $\bar{\Gamma}(N) := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \bar{\Gamma} \;\middle|\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \bmod N \right\}$

principal congruence group with level N

$(ST)^3 = S^2 = 1, T^N = 1$ with $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

→ isomorphic to **non-Abelian discrete symmetry**, e.g. $\mathbf{\Gamma_3 \simeq A_4}$

used for lepton/quark flavor

Action to modulus

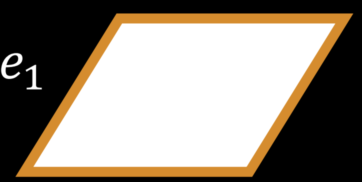
- **modulus $\boldsymbol{\tau}$** : complex scalar with $\mathrm{Im}\,\tau > 0$ $\quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$

$$\boldsymbol{\tau} \xrightarrow{\gamma} \frac{a\boldsymbol{\tau} + b}{c\boldsymbol{\tau} + d} \qquad \tau \xrightarrow{S} -1/\tau \qquad \tau \xrightarrow{T} \tau + 1$$

complex scalar field τ is transformed under Γ, so its VEV breaks Γ

T.Kobayashi, S.Nagamoto et.al. '17 '18 '20
J.Lauer, J.Mas, H.P.Nilless '89, '91, e.t.c.

- UV origin of modulus string theory predicts 6D extra dimension

T^2  $\sim$ 

$\sim \quad \tau \sim e_1/e_2$

- theory on torus has modular symmetry Γ
- it is broken by matters to finite modular sym. $\Gamma_N = \Gamma/\Gamma(N)$

Modular form of Γ_N

is a **holomorphic function of τ** transforms as

$$Y_r^{(k)} = Y_r^{(k)}(\tau) \to (c\tau + d)^k \rho(r)\, Y_r^{(k)}(\tau) \qquad \text{modulus } \tau \xrightarrow{\Gamma} \frac{a\tau+b}{c\tau+d}$$

$\rho(r)$: **representation matrix of r** under e.g. $A_4 \simeq \Gamma_3$

k : modular weight, positive integer valued

➢ What if Yukawa couplings are modular forms ?

Feruglio, "Are neutrino mass modular form ?" 17'

- Yukawa's are **holomorphic functions,** easily implemented in SUSY
- may couple to multiple flavors , so **more predictive**
- **only one modulus**, less than non-modular flavor symmetries w/ flavons

Residual $\mathbb{Z}_N^T$ symmetry Novichkov, Penedo, Petkov, 21'

$$(ST)^3 = S^2 = 1, \qquad \boldsymbol{T^N = 1}$$

- At $\tau \sim i\infty$

τ is insensitive to $\tau \xrightarrow{T} \tau + 1$ ➔ $\mathbb{Z}_N^T$ **symmetry** is unbroken

- Modular forms at $\mathrm{Im}\tau \gg 1$

$$\boldsymbol{q = \exp(2\pi i \tau)}$$

$$Y_3^{(2)}(\tau) \sim \begin{pmatrix} 1 \\ -6\, q^{1/3} \\ -18 q^{2/3} \end{pmatrix} \quad \begin{matrix} \mathbb{Z}_3^T\text{-charge} \\ 0 \\ 1 \\ 2 \end{matrix}$$

powers of $q^{1/3} = e^{2\pi i \tau/3} \ll 1$ is controlled by $\mathbb{Z}_3^T$ charge

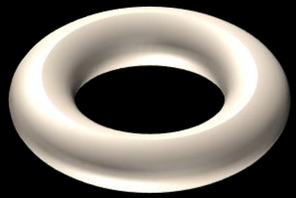
➔ Froggatt-Nielsen mechanism

$$\left(\frac{\langle \phi \rangle}{\Lambda}\right)^n \Leftrightarrow q^{n/3}$$

Γ_N for quark and lepton flavor hierarchies

Y.Abe, JK, T.Higaki, T.Kobayashi, 2301.07439, 2302.11183, 2307.01419
Tanimoto, Petkov '22 , S.Kikuchi, T.Kobayashi, K.Nasu et.al. '23, +

Summary of Γ_N

- Finite modular symmetry Γ_N $\sim$ 
  - is the quotient group $\bar{\Gamma}/\bar{\Gamma}(N)$
  - modular form transforms as

$$Y = Y(\tau) \to (c\tau + d)^k \rho(r) Y(\tau)$$

  - is found in string models T.Kobayashi, S.Nagamoto et.al. '17 '18 '20
    J.Lauer, J.Mas, H.P.Nilless '89, '91,
    S.Ferrara, D.Lust, S.Theisen, '89
    A.Baur, H.P.Nilles, A.Trautner, PKS.Vaudrevange S.Ramos-Sanches, '19, '20

- Applications for particle physics
  - quark/lepton **Yukawa couplings** (masses) **are modular forms**
  - residual symmetry can explain **flavor hierarchies**
  - value of modulus τ plays crucial role in those models

Outline

1. Introduction
2. Brief review of finite modular symmetry Γ_N
3. **Finite modular axion and radiative stabilization**
4. Other examples and summary

KSVZ axion model

'79 J.E.Kim, '80 M.A.Shifman, A.I.Vainshtein, V.I.Zakharov

$$\mathcal{L} = y\, P\overline{Q}Q$$

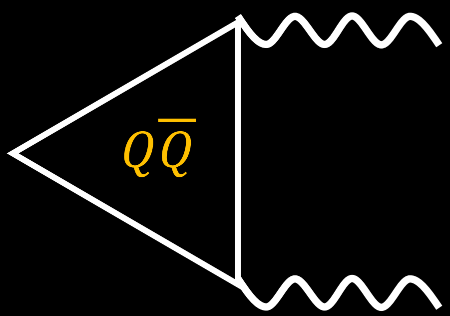
PQ-charge: $+1\ \ -1$

- complex scalar $\boldsymbol{P} \propto \boldsymbol{e^{ia/f_a}}$
- vector-like pair of non-SM quarks $(\boldsymbol{Q}, \overline{\boldsymbol{Q}})$

➢ PQ symmetry and QCD anomaly

PQ transformation: $a \to a + f_a\, \alpha\, ,\, Q\overline{Q} \to e^{-i\alpha} Q\overline{Q}\, ,\, \alpha \in \mathcal{R}$

→ axion a



QCD

→ $\left(\theta + \frac{a}{f_a}\right) F_{\mu\nu}\, \tilde{F}^{\mu\nu}$ with decay constant $f_a \sim \langle P \rangle \gg$ EW scale

Finite modular axion

2402.02071 T.Higaki, JK, T.Kobayashi

c.f. KSVZ axion

$$\mathcal{L} = \Lambda_Q Y_r^{(k)}(\tau)\overline{Q}Q \quad \leftrightarrow \quad \mathcal{L} = y\, P\overline{Q}Q$$

scalar P ∋ axion is replaced by a **modular form** $Y_r^{(k)}(\tau)$

- Accidental $U(1)_{PQ}$ assume $r = 1_t$ where t is a charge of $Z_N^T \subset \Gamma_N$

$$Y_{1_t}^{(k)}(\tau)\overline{Q}Q \sim \exp\left(\frac{2\pi i t\tau}{N}\right)(1+\mathcal{O}(q))\, Q\overline{Q} \quad \text{at } \mathrm{Im}\tau \gg 1$$

has **discrete** sym. $Z_N^T : \tau \to \tau + \mathbf{1},\ Q\overline{Q} \to \exp\left(-\frac{2\pi i t}{N}\right) Q\overline{Q}$

→ **accidental continuous** $U(1)_{PQ}: \tau \to \tau + \alpha,\ Q\overline{Q} \to \exp\left(-\frac{2\pi i t}{N}\alpha\right) Q\overline{Q}$

$\alpha \in \mathcal{R}$

A model for FM axion

2402.02071 T.Higaki, JK, T.Kobayashi

For concreteness, $\Gamma_3 \simeq A_4$ which has residual Z_3^T

- A_4 anomaly-free model with supersymmetry (SUSY)

$$W = \Lambda_{Q_1} \boldsymbol{Y}_{\mathbf{1}_1}^{(k)}(\boldsymbol{\tau}) \overline{Q}_1 Q_1 + \Lambda_{Q_2} \boldsymbol{Y}_{\mathbf{1}_2}^{(k)}(\boldsymbol{\tau}) \overline{Q}_2 Q_2$$

discrete anomaly $\mathcal{A}_{A_4 - SU(3)^2} = 1 + 2 \equiv \mathbf{0\ mod\ 3}$

- QCD θ-angle

$$\overline{\theta} = \theta_0 + \mathrm{Arg}\left(Y_{1_1}^{(k)} Y_{1_2}^{(k)}\right) \sim \theta_0 + \boldsymbol{\phi} + \mathcal{O}(|q|)$$

bare

effective QCD angle depends on $\phi = \mathbf{2\pi Re\tau}$, KSVZ-like axion

PQ quality

$$Y^{(k)}_{\mathbf{1}_t}(\tau)\overline{Q}_t Q_t \sim e^{-i\frac{t}{3}\phi}\overline{Q}_t Q_t$$

Z_3^T (=PQ)-charge = t = 1,2

non-linear realization of $U(1)_{PQ}$, accidentally by Z_3^T

- PQ-violation

$q = \exp(2\pi i\tau)$ is **invariant under Z_3^T**: $\tau \to \tau + 1$,

but not invariant under $U(1)_{PQ}$: $\tau \to \tau + \alpha, \quad \alpha \in \mathbb{R}$

→ PQ violation is accompanied with $|q| = e^{-2\pi \mathrm{Im}\tau}$

$$V = V_0 + V_1|q| + V_2|q|^2 + \cdots$$

Γ_N-invariant

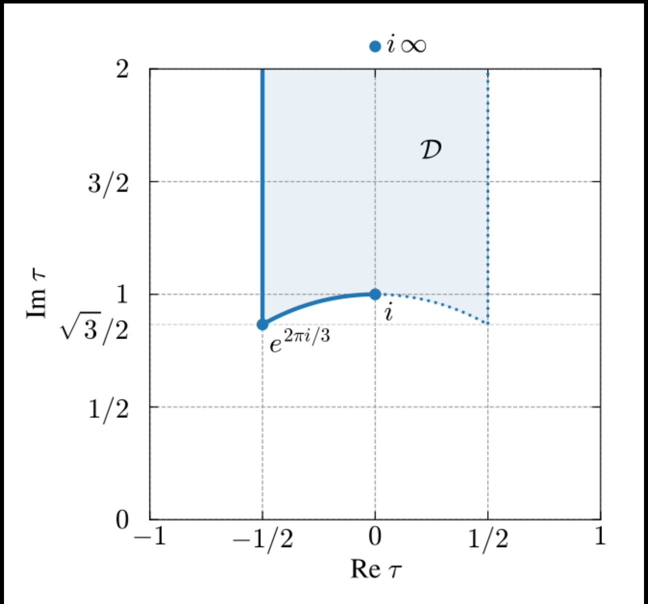
PQ-invariant PQ-violation

PQV is small if **$\mathrm{Im}\tau$ is large** and $|q| = e^{-2\pi \mathrm{Im}\tau} \ll 1$

Moduli stabilization

Where is the value of modulus τ ?

fundamental domain



2006.03058, P.Novichkov, J.Penedo, S.Petcov

- τ is stabilized at the **min. of potential**
- somewhere in the fundamental dom.

Radiative stabilization

2402.02071 T.Higaki, JK, T.Kobayashi

We do not need to extend the model,

$W = \Lambda_{Q_t} \boldsymbol{Y_{1_t}^{(k)}(\tau)} \overline{Q}_t Q_t$ generates modulus potential

➢ Coleman-Weinberg potential

$$V \sim \left(m_0^2 + m_Q^2(\tau)\right)^2 \left(\log \frac{m_0^2 + m_Q^2(\tau)}{\mu^2} - \frac{3}{2}\right) - \left(m_Q^2(\tau)\right)^2 \left(\log \frac{m_Q^2(\tau)}{\mu^2} - \frac{3}{2}\right)$$

from scalar quarks   from quarks

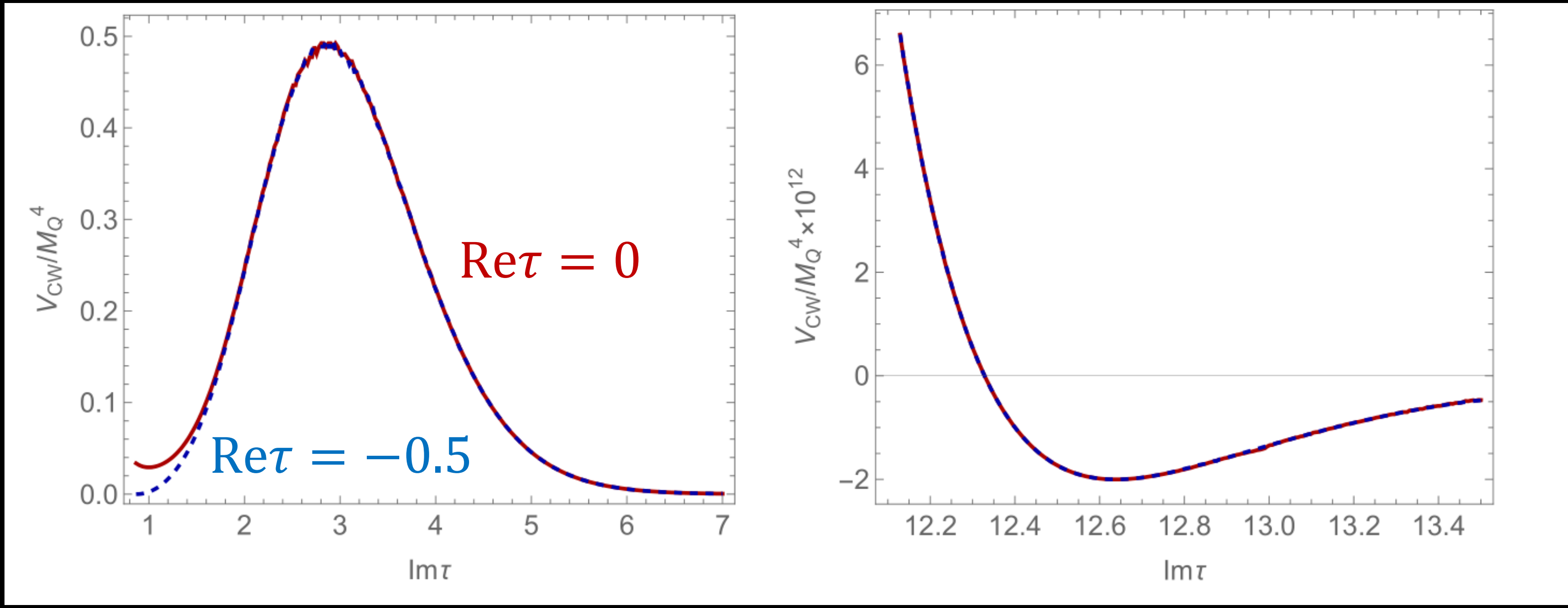
where VLQ mass $m_Q^2(\tau) = \Lambda_Q^2 (2\mathrm{Im}\tau)^k \left|Y_{1_t}^{(k)}(\tau)\right|^2$

m_0^2: soft SUSY breaking for squark, **assumed to be τ-independent**

μ: renormalization scale in $\overline{MS}$

Potential shape 2402.02071 T.Higaki, JK, T.Kobayashi

$$W = \Lambda_Q \, Y_{1_1}^{(12)} \overline{Q} Q \text{ under } \Gamma_3 \simeq A_4$$



$$m_0^2/M_Q^2 = 10^{-8}, \mu/\Lambda_Q = 0.01$$

- potential has **global minimum at $Im\,\tau \sim 13 \gg 1$**
- this minimum exists only for **non-trivial** singlet
- potential is **almost flat for $Re\tau$** around minimum

PQ quality in the model

from CW, PQ violation

$$V(\phi) \sim \Lambda_{QCD}^4 \cos(\theta_0 + \phi) + \frac{m_0^2 \Lambda_Q^2}{8\pi^2} c_0 c_1 (2\mathrm{Im}\tau_0)^k \, |q|^{7/3} \, \boldsymbol{\cos\phi}$$

$\Lambda_{QCD} \sim 100\ \mathrm{MeV}$

axion $\sim \phi = 2\pi\mathrm{Re}\tau$

solve strong CP

spoil strong CP

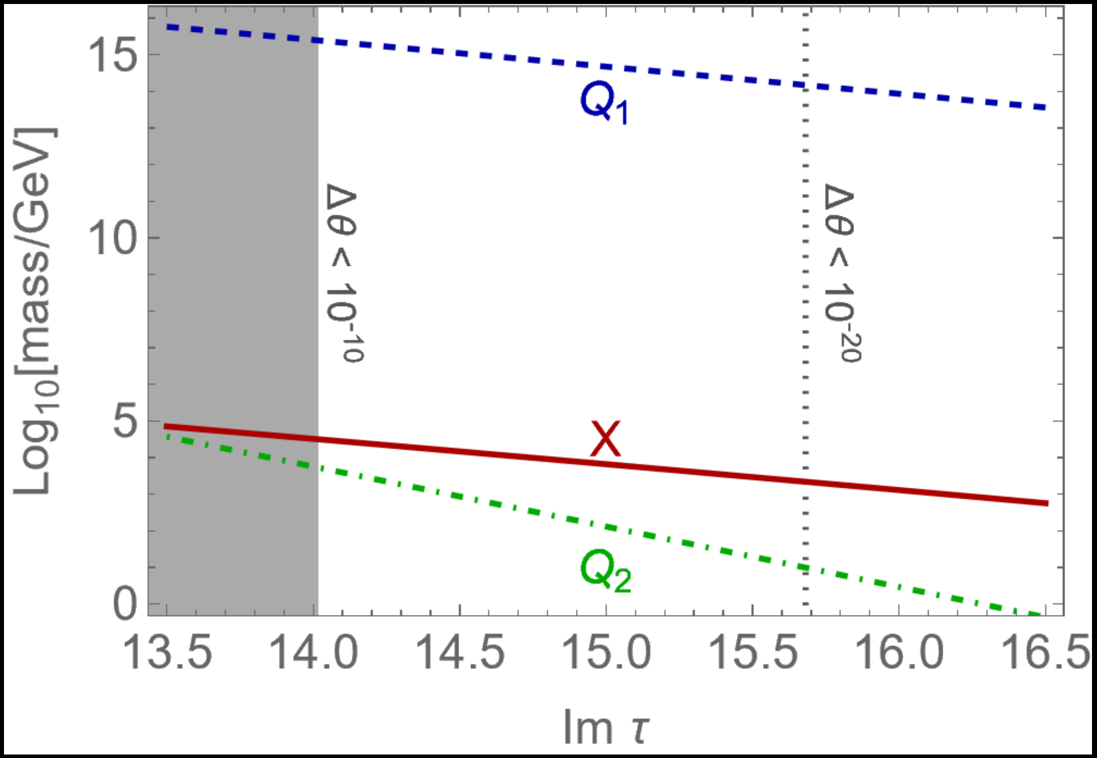
↓

- shift of $\langle\overline{\theta}\rangle$ from zero:

$$\Delta\theta \sim 10^{-10} \times \sin\theta_0 \left(\frac{m_0}{10^7\mathrm{GeV}}\right)^2 \left(\frac{\Lambda_Q}{10^{18}\mathrm{GeV}}\right)^2 \left(\frac{\mathrm{Im}\tau_0}{14}\right)^{12} \left(\frac{|q|^{1/3}}{10^{-12}}\right)^7$$

- **consistent with neutron EDM limit** for $|q|^{1/3} < \mathcal{O}(10^{-12})$
- can **solve the strong CP** problem for $\mathrm{Im}\tau_0 \gtrsim 14$

Modulus and vector-like quark masses



$\mathbf{14} \lesssim \mathrm{Im}\tau \lesssim \mathbf{15}$

Axion quality $\Delta\theta < 10^{-10}$

Q_2 heavier than TeV

saxion $\sim$ $\mathrm{Im}\,\tau$ heavier than 10 TeV

Summary of finite modular axion

- Summary
  - **accidental $U(1)_{PQ}$** is realized from residual Z_N^T in Γ_N
  - modulus can be **stabilized by CW** in KSVZ-type model
  - PQ quality is ensured by $\mathrm{Im}\,\tau \gg 1$
  - vector-like quark and modulus are at O(TeV) scale

- Discussions
  - other applications to **accidental U(1)**
  - **cosmological implications**, especially DM and modulus ?

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Finite modular majoron ‘24 JK and T.H.Jung

Finite modular symmetry can be used for the accidental $U(1)_{B-L}$

- Type-I seesaw model

c.f. FM axion

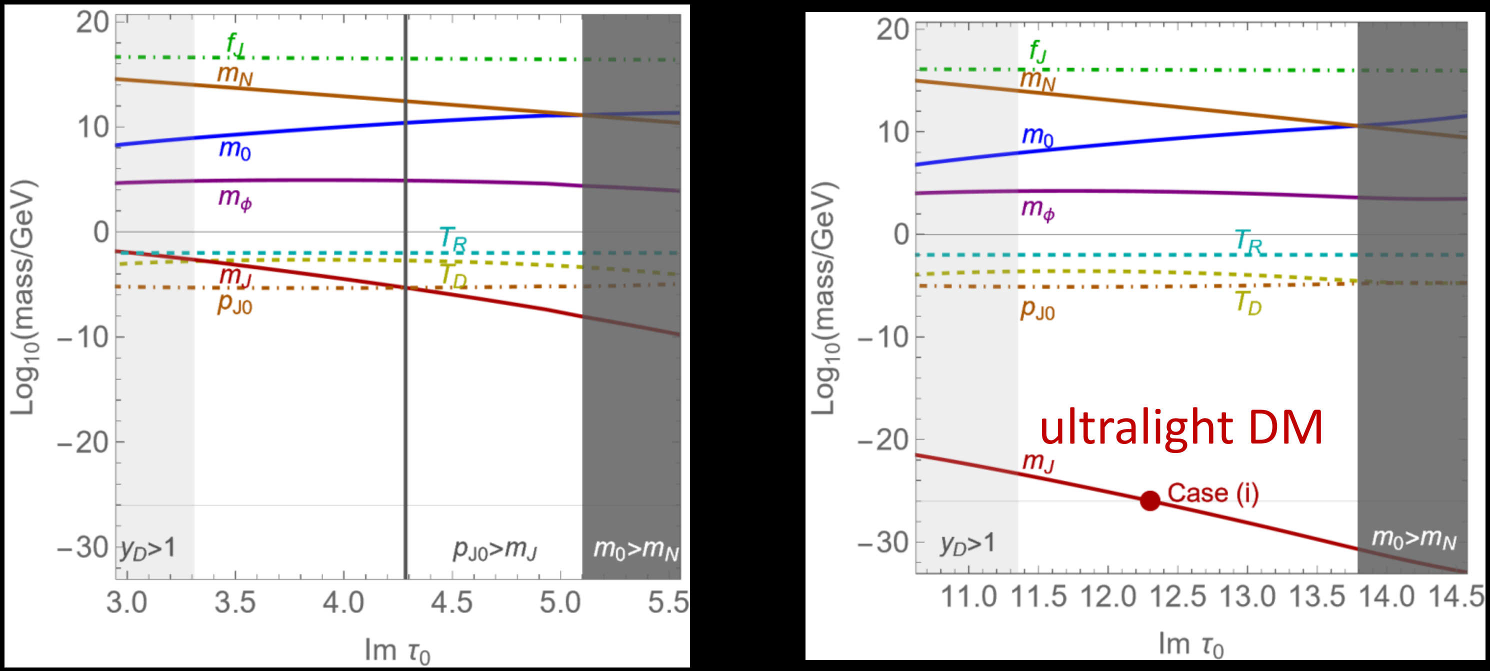
$$\mathcal{L} = \Lambda_N Y_r^{(k)}(\tau) N N \quad \longleftrightarrow \quad \mathcal{L} = \Lambda_Q Y_r^{(k)}(\tau) \overline{Q} Q$$

$J \sim \mathrm{Re}\,\tau$ is pNGB of B-L, so it is ***finite modular majoron***

- Majoron cosmology
  - majoron contributes both **dark matter** and **dark radiation**
  - modulus $\sim \mathrm{Im}\,\tau$ also play significant role
  - **additional matter domination** looks preferable

Masses when $\Delta N_{\mathrm{eff}} = 0.3$ * additional MD assumed

$k = 4$ $k = 24$



- RHN mass $m_N \sim 10^{10\sim14}$ GeV, soft mass $m_0 \sim 10^{8\sim10}$ GeV
- modulus mass is 10 TeV for $\Delta N_{\mathrm{eff}} \sim 0.3$
- majoron mass m_J can be in a **wide range** $m_J \in [10^{-30}, 1.]$ GeV

Other examples of pNG mode

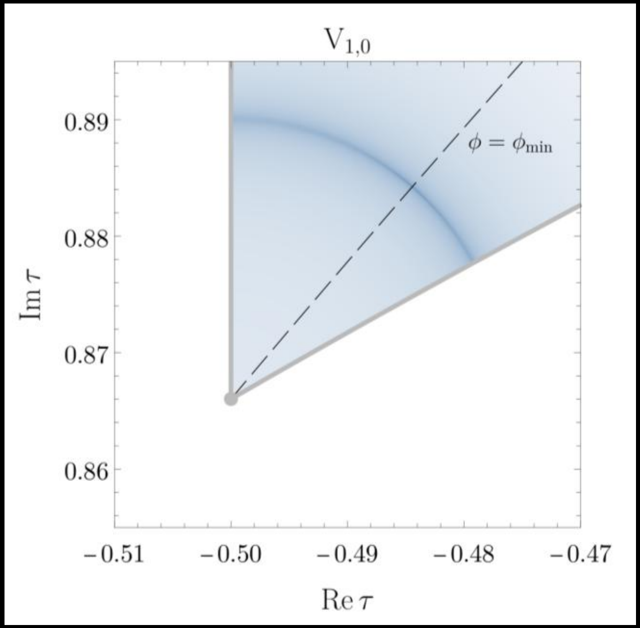
- Klein J function

  * J is modular-inv function

  '22 P.P.Novichkov, J.T. Penedo, S.T.Petcov

  '91 M. Cvetic, A. Font, L.E.Ibanez et.al.

$$W \sim (J(\tau)-1)^{m/2} J(\tau)^{n/3} P(J(\tau))$$

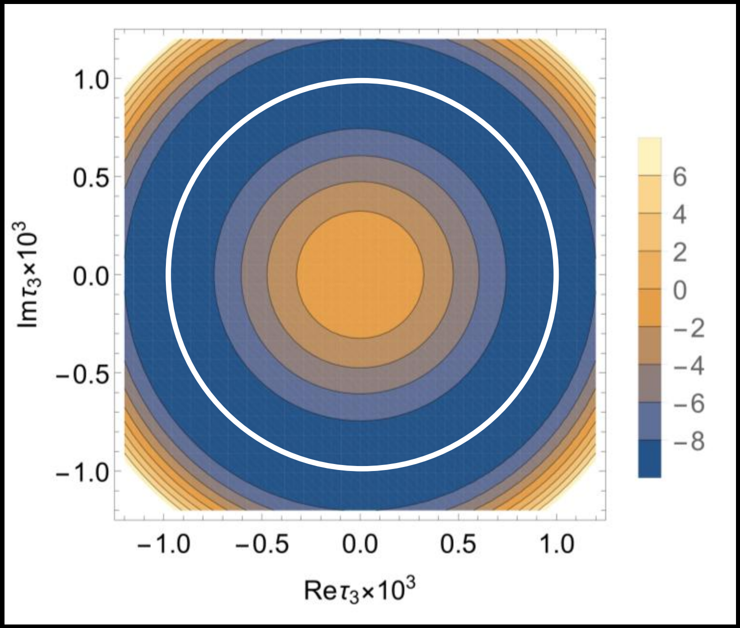


$$Z_3^{ST}: \tau \to -1/(\tau+1)$$

- Siegel group $Sp(4, Z)$

  '24 **JK**, H. Otsuka et.al.

$$W \sim Y(\tau_1,\tau_3)X + Y'(\tau_1,\tau_3)\bar{Q}Q$$

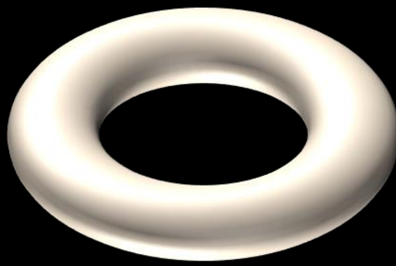


$$Z_2: \tau_3 \to -\tau_3$$

pseudo flat direction appears around a **fixed point**

Summary

extra dimension



$$\boldsymbol{\tau} \xrightarrow{\Gamma} \frac{a\boldsymbol{\tau}+b}{c\boldsymbol{\tau}+d}$$

$$Y = Y(\tau)$$

finite modular symmetry Γ_N

this talk

e.g. previous talk
by S. Nishimura

pNG mode ~ DM, inflaton, … ?

*posters by K. Goto (火), R. Yanagita (木)

Quark/Lepton flavor structure

flavored pNG mode

backups

Modular group

- **modular group Γ** ⟺ special linear group SL(2,Z)

$$\Gamma := SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\middle|\; a,b,c,d \in \mathbb{Z}\,, ad - bc = 1 \right\}$$

generators

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad TR = RT$$

- action to **modulus $\boldsymbol{\tau}$** : complex scalar with $\mathrm{Im}\,\tau > 0$

$$\boldsymbol{\tau} \xrightarrow{\Gamma} \frac{a\boldsymbol{\tau} + b}{c\boldsymbol{\tau} + d} \qquad \tau \xrightarrow{S} -1/\tau \qquad \tau \xrightarrow{T} \tau + 1 \qquad \boldsymbol{\tau} \xrightarrow{\boldsymbol{R}} \boldsymbol{\tau}$$

We often consider $\bar{\Gamma} := \Gamma / Z_2^R = PSL(2,\mathbb{Z})$

Minimum of the potential

2402.02071 T.Higaki, JK, T.Kobayashi

- For $\mathrm{Im}\tau \gg 1$ $\quad$ $\boldsymbol{t}$: Z_N^T-charge of modular form Y

$$Y_{1_t}^{(k)}(\tau) \sim q^{t/3}(c_0 + c_1 q + c_2 q^2 + \cdots) \qquad q = \exp(2\pi i\tau)$$

- Derivative along $x := 2\mathrm{Im}\tau$

$$\frac{\partial V}{\partial x} = \frac{c_0^2 m_0^2 \Lambda_Q^2}{16\pi^2} x^{k-1} e^{-2\pi t x/3} \left(k - \frac{2\pi t x}{3}\right)\left(\log\frac{c_0^2\Lambda_Q^2}{e\mu^2} + k\log x - \frac{2\pi t x}{3}\right)$$

maximum $\qquad$ minimum

$$x_{min} = \frac{3k}{2\pi t}\mathcal{W}\left(-\frac{2\pi t}{3k}\left(\frac{\mu}{c_0\Lambda_Q}\right)^{2/k}\right) > 1$$

minimum exists for $\boldsymbol{t > 0}$

$\leftrightarrow$ **non-trivial** singlet under Γ_N

* Lambert fct. $\mathcal{W}(z)e^{\mathcal{W}(z)} = z$

A modular form of $\Gamma_{N=3}$

- For $\boldsymbol{r=3}$, $\boldsymbol{k=2}$
  rep. weight

$$c_{ij} = \frac{i}{2\pi}\begin{pmatrix} 1 & 1 & 1 \\ -2 & -2w^2 & -2w \\ -2 & -2w & -2w^2 \end{pmatrix} \quad w = \exp\left(\frac{2\pi i}{3}\right)$$

$$Y_3^{(2)}(\tau) = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} \qquad Y_i(\tau) = \sum_{j=0}^{2} c_{ij} \frac{\eta'((\tau+j)/3)}{\eta((\tau+j)/3)} - 27\,\delta_{1i} \frac{\eta'(3\tau)}{\eta(3\tau)}$$

with Dedekind Eta function $\eta(\tau) := q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1-q^n)$

- q-expansion $q := e^{2\pi i \tau}$

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + \cdots \\ -6q^{1/3}(1 + 7q + \cdots) \\ -18q^{2/3}(1 + 2q + \cdots) \end{pmatrix}$$

* $|q| \ll 1$ for $\mathrm{Im}\tau \gg 1$

Finite modular group Γ_N

- Congruence group $\Gamma(N)$ level $N \in \mathbb{N}$

$$\bar{\Gamma}(N) := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \bar{\Gamma} := PSL(2,\mathbb{Z}) \;\middle|\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \bmod N \right\}$$

ex) $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \rightarrow T^N = \begin{pmatrix} 1 & N \\ 0 & 1 \end{pmatrix} \in \Gamma(N)$

- **Finite modular group** $\Gamma_N := \bar{\Gamma}/\bar{\Gamma}(N)$

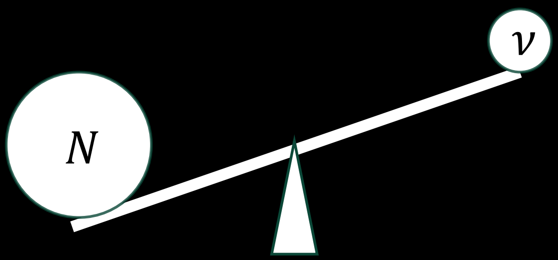
$$(ST)^3 = S^2 = 1, \qquad \boldsymbol{T^N = 1}$$

→ **isomorphic to non-Abelian discrete symmetries** for $N \leq 5$

$$\Gamma_2 \simeq S_3, \quad \boldsymbol{\Gamma_3 \simeq A_4}, \quad \boldsymbol{\Gamma_4 \simeq S_4}, \quad \Gamma_5 \simeq A_5$$

Type-I seesaw

$$\mathcal{L} = \frac{1}{2} m_N N N + H_u L^c Y_d N$$



$$\rightarrow \quad m_\nu \sim \frac{v_H^2 y_D^2}{m_N} \sim 0.1\ \mathrm{eV} \times \left(\frac{10^{14}\,\mathrm{GeV}}{m_N}\right) \left(\frac{y_D v_H}{100\ \mathrm{GeV}}\right)^2$$

- $U(1)_{B-L}$ symmetry
  - is the anomaly-free symmetry in the SM + RH neutrinos
  - **forbids Majorana mass**, since N has lepton number 1

→ Majorana mass m_N should be related to B-L breaking

$$m_N \sim v_{BL}$$

Majoron

’81 Y.Chikashige, R.N.Mohapatra, R.D.Peccei; ’81 G.B,Gelmini, M.Ronacadelli

If B-L symmetry is **global** (not gauged),

→ there exists a pseudo NG boson, named **majoron *J***

- Majoron DM

If majoron only couples to RH-neutrinos via $e^{iJ/f_J} m_N N N$

- decay width $\Gamma_J \sim m_\nu^2 m_J / f_J^2$ is small enough to be DM
- but it is a **decaying DM** particle

→ we may see neutrino flux from the DM decay

’17 C.Garcia-Cely, J.Heeck, ’23 K.Akita, M.Niibo

B-L quality ?

majoron is a **pseudo**-NG boson of $U(1)_{B-L}$ symmetry

→ majoron gets its mass from explicit $U(1)_{B-L}$ breaking

Ex) $$\mathcal{L}_{BLV} \sim \frac{\phi^n}{M_p^{n-4}} + h.c. \sim \frac{f_J^n}{M_p^{n-4}} \cos\left(n \frac{J}{f_J}\right)$$

- Question: How $U(1)_{B-L}$ is broken ?

need a mass term, while do not need interaction e.g. Jee

→ $U(1)_{B-L}$ should be **broken in a proper way** to be DM

B-L quality may matter, as for the PQ quality of axion

Finite modular majoron

'24 JK and T.H.Jung

Finite modular symmetry can be used for the accidental $U(1)_{B-L}$

- Model

$$\mathcal{L} = \Lambda_N Y_r^{(k)}(\tau) NN$$

c.f. FM axion

$$\longleftrightarrow \quad \mathcal{L} = \Lambda_Q Y_r^{(k)}(\tau)\overline{Q}Q$$

residual symmetry $Z_N^T : \tau \to \tau + 1, NN \to \exp\left(-\frac{2\pi t}{N}\right) NN$

→ **accidental continuous** $U(1)_{B-L}: \tau \to \tau + \alpha, NN \to \exp\left(-\frac{2\pi t}{N}\alpha\right) NN$

$\alpha \in \mathcal{R}$

$J \sim \mathrm{Re}\,\tau$ is pNGB of B-L, so it is ***finite modular majoron***

Radiative stabilization

the same **CW stabilization** works for the majoron

- Potential

$$V \sim \left(m_0^2 + M_N^2(\tau)\right)^2 \left(\log \frac{m_0^2 + M_N^2(\tau)}{\mu^2} - \frac{3}{2}\right) - \left(M_N^2(\tau)\right)^2 \left(\log \frac{M_N^2(\tau)}{\mu^2} - \frac{3}{2}\right)$$

where $M_N^2(\tau) = \Lambda_N^2 (2\mathrm{Im}\tau)^k \left|Y_{1_t}^{(k)}(\tau)\right|^2$

- Modulus $X \sim \mathrm{Im}\,\tau$

soft mass　　Majorana mass

mass　$m_X \sim \dfrac{m_0 m_N}{4\pi M_p} \sim 10\ \mathrm{TeV} \times \left(\dfrac{m_0}{10^{10}\mathrm{GeV}}\right)\left(\dfrac{m_N}{10^{14}\mathrm{GeV}}\right)$

decay　$\Gamma_X \sim \Gamma(X \to JJ) \sim (20\ s)^{-1} \times \left(\dfrac{m_\phi}{20\ \mathrm{TeV}}\right)^3$　via K$\ddot{a}$hler potential

both majoron and modulus are important for cosmology

$\sim \mathrm{Re}\,\tau$　　$\sim \mathrm{Im}\,\tau$

Majoron as dark matter

- Majoron potential from B-L breaking

$$V_{CW} \supset \frac{m_0^2 m_N^2}{16\pi^2} e^{-4\pi Im\tau}\left[1+\cos\left(\frac{J}{f_J}\right)\right]$$

where **decay constant** $f_J \sim \frac{M_p}{4\pi Im\tau} \sim \mathcal{O}(10^{16})$ **GeV**

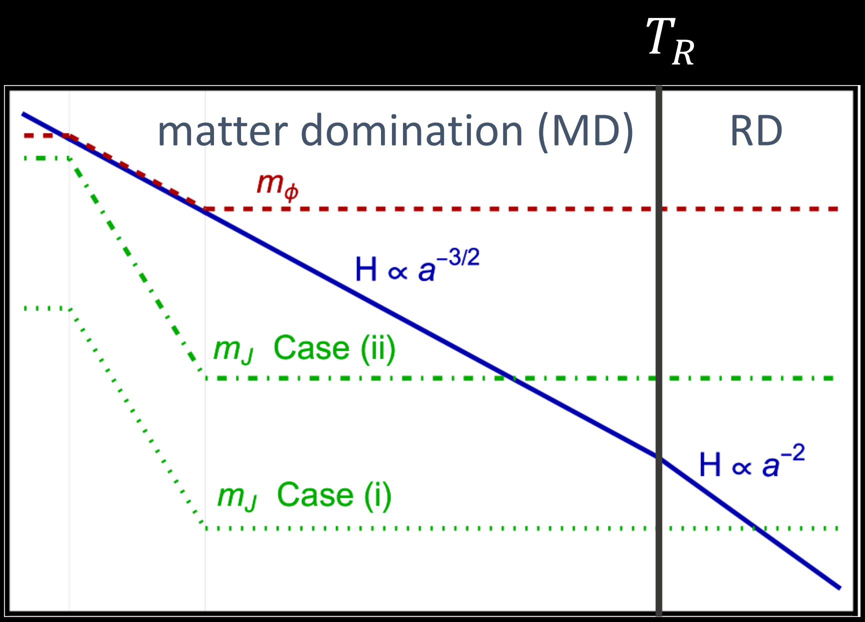
- Two scenarios for DM

1. oscillation starts **during RD**

   majoron mass is 10^{-17} eV, **ultralight DM**

2. oscillation starts **during MD**

   majoron mass is keV-GeV, **diluted by reheating** after MD



*RD= radiation domination

Majoron as dark radiation [DR]

- Modulus decay

modulus **decays $\mathbf{X} \to JJ$ after BBN** [Big Bang Nucleosynthesis]

→ produced majorons are **relativistic**, so contribute to DR

- Effective number of neutrinos, $N_{\text{eff}} \sim 3 + \Delta N_{\text{eff}}$

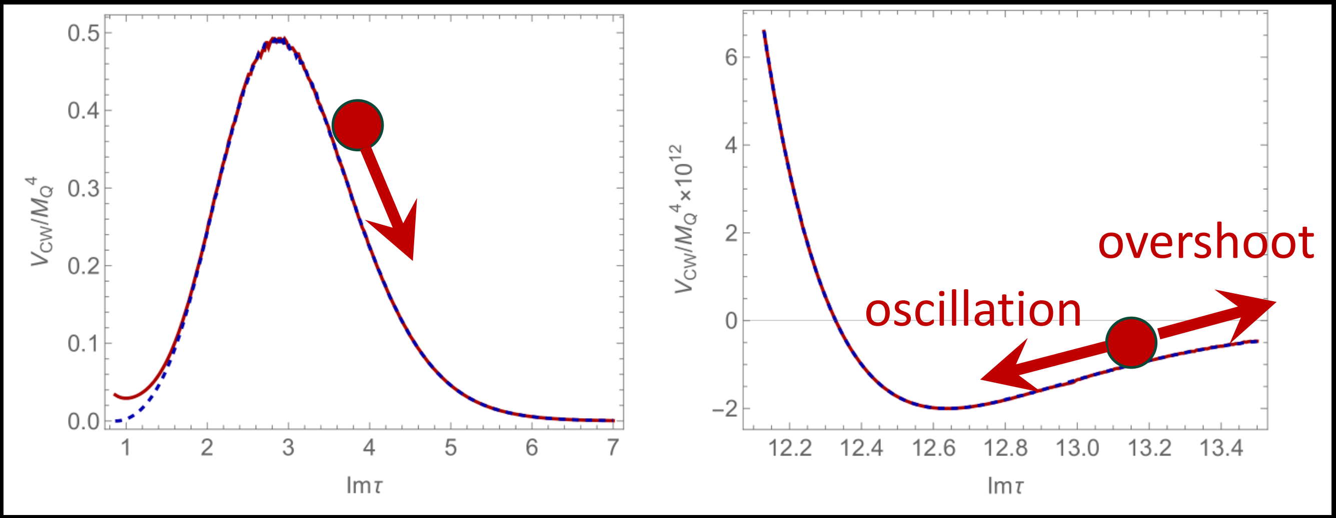
$$\Delta N_{\text{eff}} \sim 0.6 \times \left(\frac{\rho_X/\rho_{rad}}{10^{-3}}\right)\left(\frac{T_R}{10\text{ MeV}}\right)\left(\frac{10\text{ TeV}}{m_X}\right)^3$$

- assuming **MD by another particle χ**, reheating at T_R
- **modulus energy per radiation energy** should be small for $\Delta N_{\text{eff}} \sim \mathcal{O}(0.1)$
- limit from CMB is < 0.3, but **Hubble tension** may prefer $\Delta N_{\text{eff}} \sim 0.4$

'18 Planck J.L.Bernal et.al, 1607.05617, S.Vagnozzi, 1907.07569

Moduli dynamics and energy density

if no additional energy, modulus will **overshoot**



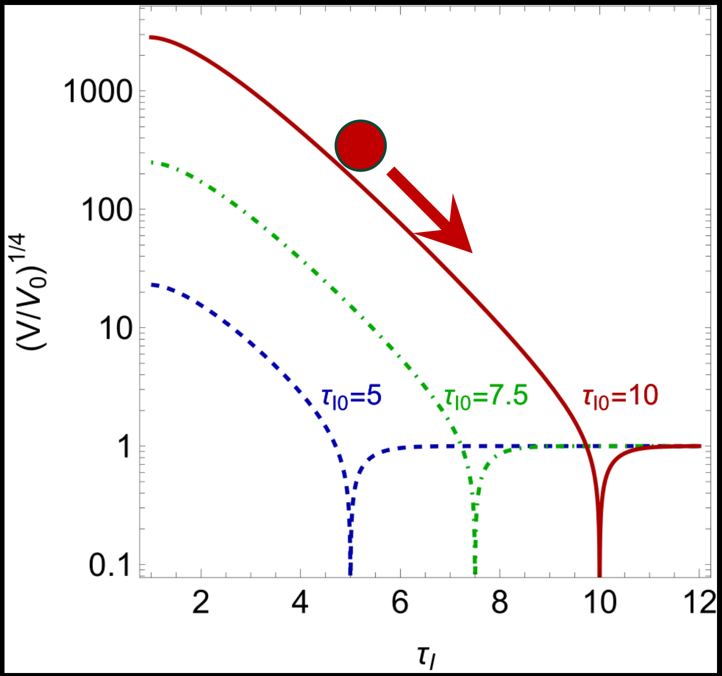
However, with the MD field χ, **Hubble friction** stops too fast rolling

EOM: $\ddot{X} + \boldsymbol{3H\dot{X}} + V_X = 0$

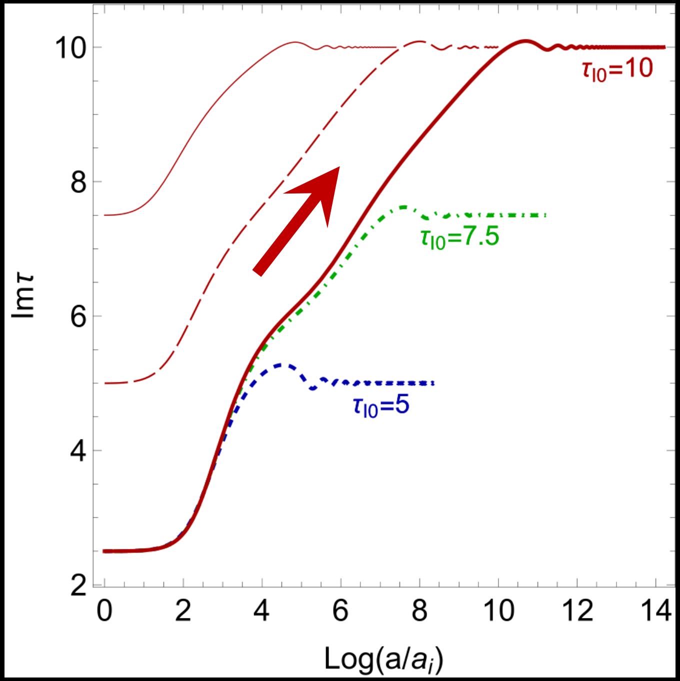
- then, modulus starts to oscillate around the minimum
- effective amplitude is small, so that $\boldsymbol{\rho_X/\rho_{rad} \sim \mathcal{O}(10^{-4})}$

Moduli dynamics, numerically

potential shape in log-scale



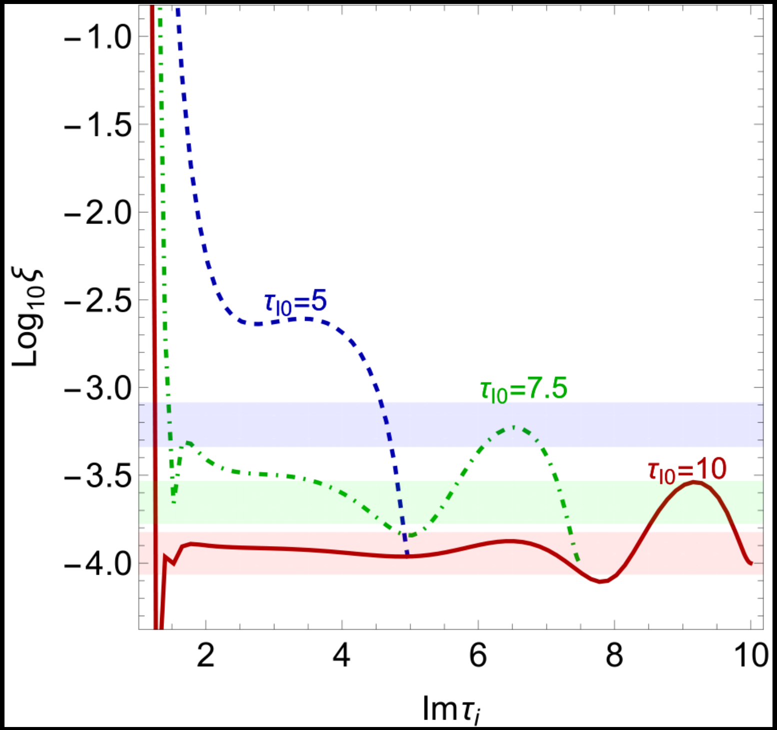
evolution of modulus



- **no overshoot**, if it starts from the **exponential slope**
- behavior of **oscillation** looks the **same for various cases**

Modulus energy density

Values of $\xi := \rho_X/\rho_\chi = \rho_X/\rho_{\rm rad}\,(T_R)$



- Analytical formula

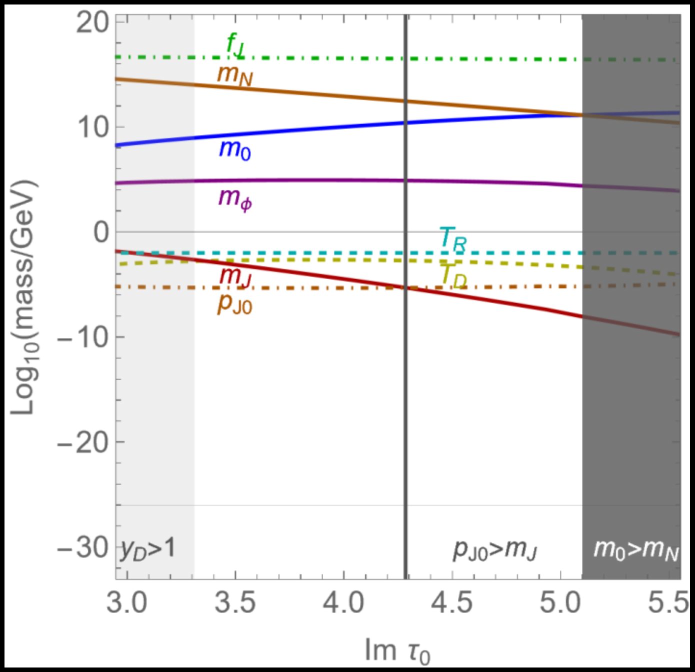
$$\xi \sim \frac{3}{2e^2}\left(k - \frac{4\pi t}{N}\mathrm{Im}\tau_{min}\right)^{-2}$$

$$\sim 10^{-4} \text{ for } \mathrm{Im}\tau_{min} \sim 0$$

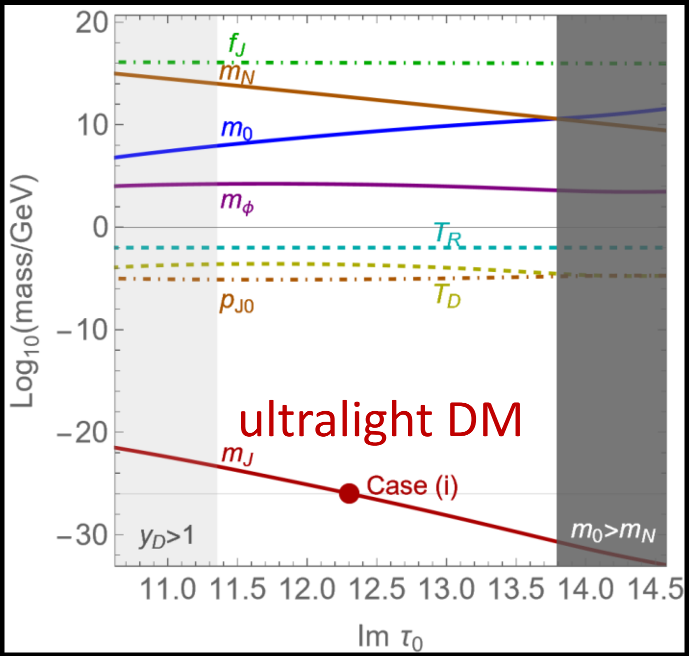
- **energy ratio is 10^{-4}** independent of initial position
- true for long slopes, and it can be larger for shorter slopes

Masses when $\Delta N_{\rm eff} = 0.3$

$k = 4$



$k = 24$



- RH neutrino is $10^{10\sim14}$ GeV, soft mass is $10^{8\sim10}$ GeV
- modulus mass is 10 TeV for $\Delta N_{\rm eff} \sim 0.3$
- majoron mass can be in a **wide range** $m_J \in [10^{-30}, 1.]$ GeV

Summary of finite modular majoron

- Summary
  - **accidental $U(1)_{B-L}$** is realized from residual Z_N^T in Γ_N
  - modulus can be **stabilized by CW** in type-I seesaw
  - modulus dost **not overshoot** because of Hubble friction
  - majoron contributes to both **DM and DR**

- Discussions
  - can we probe majoron with **keV-GeV mass** and $f_J \sim 10^{16}$ GeV ?
  - application to **flavor models** ?
  - relation to other cosmology, e.g. baryogenesis/inflation ?

Majoron limits

'23 K.Akita, N.Michiru

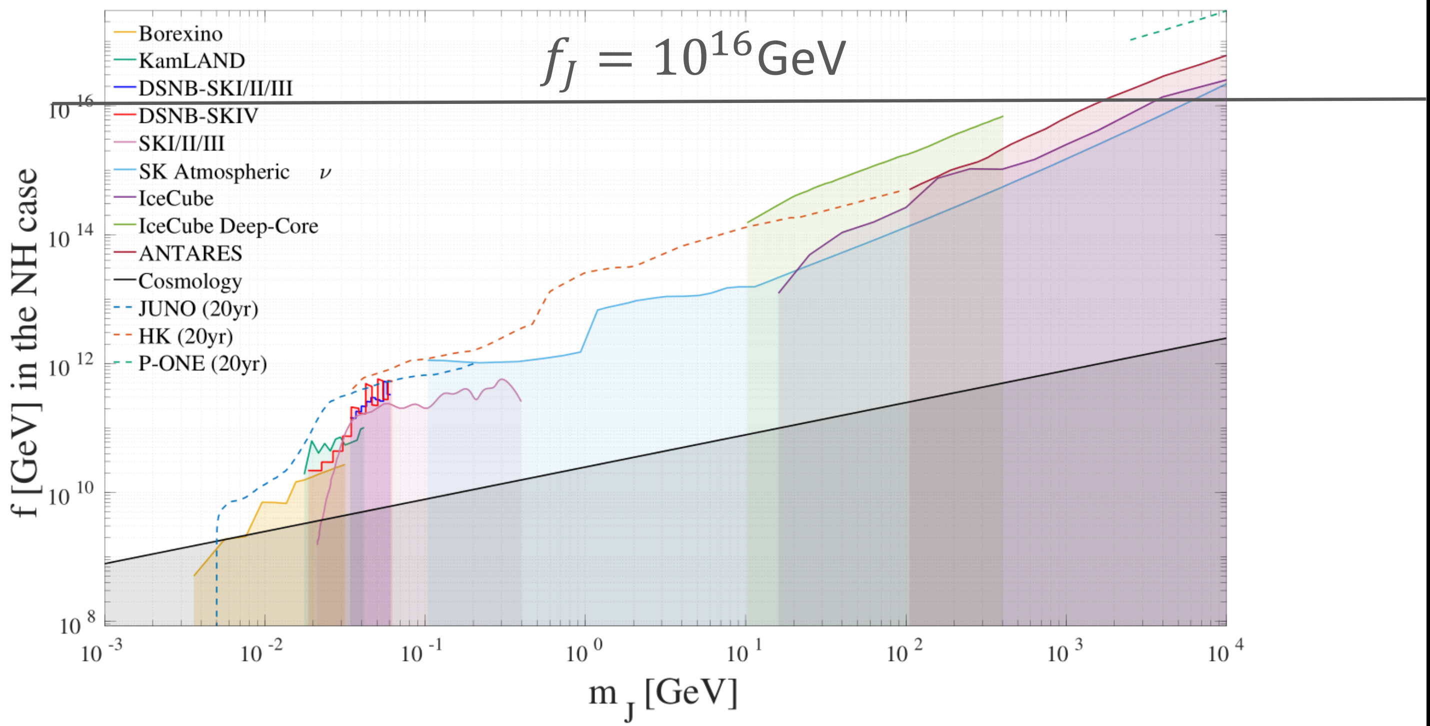


Figure 3: Lower bounds on the energy scale of the spontaneous lepton number symmetry breaking f in NH case, as a function of the majoron DM mass. The black region corresponds to the cosmological constraint on the DM lifetime comes from CMB+BAO analysis as $\leq$ 250 Gyr [58–62]. The other colored regions with solid curves describe the current constraints from Borexino [53] (yellow), KamLAND [54] (green), Super-Kamiokande [31–33, 55, 56] (red, blue, pink, light-blue), IceCube [35, 36] (light-green, purple), and ANTARES [38, 57], and the dashed curves describe the expected sensitivities of future neutrino detectors, JUNO [40] (blue), HK [39] (orange) and P-ONE [38, 44] (green).

Known mechanisms for stabilization

- SL(2,Z) invariant ‘91 M.Cvetic, A.Font, L.E.Ibanez, D.Lust, F.Quevedo
  2006.03058, P.Novichkov, J.Penedo, S.Petcov

$$W \sim (j(\tau) - 1728)^{\frac{m}{2}} j(\tau)^{\frac{n}{3}} \mathcal{P}(j(\tau))$$

$j(\tau)$: Klein function

non-perturbative effects

- Γ_N invariant potential 1909.05139, 1910.11553
  T.Kobayashi, Y.Shimizu, K.Takagi, M.Tanimoto, T.Tatsuishi, H.Uchida

$$W \sim X_1^{(k_X)} \left(Y_1^{(k_Y)}\right)^p$$

X has **non-zero weight**, additional field

- 3-form flux potential 2011.09154, 2206.04313 K.Ishiguro H.Okada, T.Kobayashi, H.Otsuka

$$W \sim \sum_{n=0}^{3} c_n \tau^n$$

Polynomial of τ, **coefficients transform under SL(2,Z)**

Canonical normalization

- Modular invariant kinetic term

kinetic term $\frac{\bar{Q}Q}{(-i\tau+i\bar{\tau})^{k_q}}$ → $\bar{Q}Q$

canonical basis

Yukawa coup. $Y^{(k_Y)}(\tau)$ → $(2Im\tau)^{k_Y/2}\,Y^{(k_Y)}$

- When $\epsilon(\tau) \ll 1$

$\epsilon(\tau) \sim 0.05$ → $t \coloneqq 2\mathrm{Im}\,\tau \sim 5$ gives additional structure

another FN-like mechanism controlled by **modular weights**

Axion decay constant

$$K = -h\, log(-i\tau + i\tau^*) \longrightarrow \mathcal{L}_{kin} = \frac{h\, M_p^2}{(2Im\tau)^2}\partial_\mu \tau^* \partial^\mu \tau$$

$M_p \simeq 2 \times 10^{18}$ GeV : reduced Planck mass

- After canonical normalization,

$$f_\phi = \frac{\sqrt{h} M_p}{4\pi \mathrm{Im}\tau} \sim 2 \times 10^{16}\ \mathrm{GeV} \times \left(\frac{h}{3}\right)^{\frac{1}{2}} \left(\frac{14}{\mathrm{Im}\tau}\right)$$

need entropy production maybe by saxion X and/or fine-tuned initial condition

$$m_X = \frac{c_0 m_0 M_Q}{2\sqrt{2h}\pi M_p^2}(2\mathrm{Im}\tau)^k e^{-\frac{2\pi \mathrm{Im}\tau}{3}} \left| k - \frac{4\pi \mathrm{Im}\tau}{3} \right|$$

Modular forms of A4

$$Y^{(12)}_{1_1}(\tau) = (Y_1^2 + 2\,Y_2\,Y_3)^2(Y_3^2 + 2\,Y_1\,Y_2)$$

$$Y^{(12)}_{1_2}(\tau) = (Y_1^2 + 2\,Y_2\,Y_3)(Y_3^2 + 2\,Y_1\,Y_2)^2$$

where

$$\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left[\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - 27\frac{\eta'(3\tau)}{\eta(3\tau)}\right], \\
Y_2(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'(\tau/3)}{\eta(\tau/3)} + w^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + w\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right], \\
Y_3(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'(\tau/3)}{\eta(\tau/3)} + w\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + w^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right],
\end{aligned}$$

- q-expansion

$$Y^{(12)}_{1_1} = -12q^{1/3}\left(1 + 472q + \mathcal{O}\left(q^2\right)\right), \quad Y^{(12)}_{1_2} = 144q^{2/3}\left(1 + 224q + \mathcal{O}\left(q^2\right)\right).$$