Dynamical determination of quark flavor structure from Casimir energy in an extra dimension

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Abstract

By minimizing the Casimir energy of a one-generation quark toy model on an interval-extra dimension with point interactions, we found that by generating two generations, we could dynamically produce the experimental measurements of quark masses $\{m_u, m_d, m_s, m_c\}$ and the Cabibbo angle $\sin\theta_{12}$.

1) Motivation

Mysteries of the Standard Model

- Chiral fermion : Why is the SM a chiral theory ?
- Generations : Who ordered the same packages ⋯ ?
- Mass Hierarchy : Why exponential differences are exist between the fermion's masses ⋯ ?
- Flavor mixing : What dynamics determines the flavor mixing ⋯ ?

2) Model setup

One-generation $SU(2)$ quark toy model in 5-dimension

- Total action

5-dim. $SU(2)$ gauge fields

$$S = S_G + S_F + S_Y + S_\Phi$$

- Fermion action

$$S_F = \int d^4x \int_0^L dy \Big[\bar{Q}(x,y)(i\Gamma^N D_N^{(Q)} + M_Q)Q(x,y)$$
$$+\bar{U}(x,y)(i\Gamma^N\partial_N + M_U)U(x,y) + \bar{D}(x,y)(i\Gamma^N\partial_N + M_D)D(x,y)$$
$$+\bar{\Psi}(x,y)(i\Gamma^N\partial_N + M_\Psi)\Psi(x,y)\Big]$$

5-dim. $SU(2)$ doublet fermion (One generation)

Bulk mass

5-dim. $SU(2)$ singlet up-type fermion (One generation)

5-dim. $SU(2)$ singlet down-type fermion (One generation)

5-dim. $SU(2)$ singlet *exotic* fermion (for flavor mixing)

- Yukawa interaction

$$S_Y = \int d^4x \int_0^L dy \Big[-Y_U\,\bar{Q}(x,y)(i\sigma_2\Phi^*(x,y))U(x,y)$$
$$-Y_D\,\bar{Q}(x,y)\Phi(x,y)D(x,y) + \text{(h.c.)}\Big]$$

Up-sector Yukawa interaction

Down-sector Yukawa interaction

- Scalar Action

$$S_\Phi = \int d^4x \int_0^L dy \left[\Phi^\dagger(x,y)(D^N D_N - M_\Phi^2)\Phi(x,y) - \frac{\lambda_\Phi}{2}(\Phi^\dagger(x,y)\Phi(x,y))^2\right]$$

5-dim. $SU(2)$ doublet scalar (Imitation of the Higgs)

Interval extra dimension + Point interactions

(Zero-width localized potential → Boundary conditions)

- Boundary conditions

Point interactions (felt only by fermions)

4d chirality

		$y=0$			$y=L$
gauge	W_μ^a	$\partial_y W_\mu^a(0)=0$			$\partial_y W_\mu^a(L)=0$
gauge	W_y^a	$W_y^a(0)=0$			$W_y^a(L)=0$
doublet	Q	$Q_R(0)=0$	$Q_R\left(\frac{L}{2}-\varepsilon\right)=0$	$Q_R\left(\frac{L}{2}+\varepsilon\right)=0$	$Q_R(L)=0$
singlet	U	$U_L(0)=0$	$U_L(L_1^{(U)}-\varepsilon)=0$	$U_L(L_1^{(U)}+\varepsilon)=0$	$U_L(L)=0$
exotic	Ψ	$\Psi_L(0)=0$	$\Psi_R(L_1^{(U)}-\varepsilon)=0$	$\Psi_R(L_1^{(U)}+\varepsilon)=0$	$\Psi_L(L)=0$
singlet	D	$D_L(0)=0$	$D_L\left(\frac{L}{2}-\varepsilon\right)=0$	$D_L\left(\frac{L}{2}+\varepsilon\right)=0$	$D_L(L)=0$
scalar	Φ	$\Phi(0)+L_+\partial_y\Phi(0)=0$			$\Phi(L)-L_-\partial_y\Phi(L)=0$

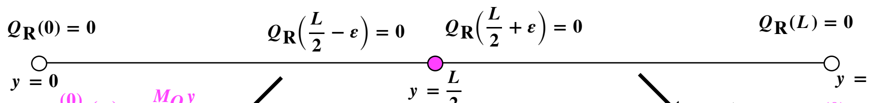
$y=\frac{L}{2}$, $y=L_1^{(U)}$, $y=\frac{L}{2}$; $(-\infty \le L_\pm \le +\infty)$

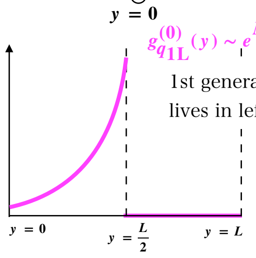
(N.B.) The boundary conditions imposed by introducing point interactions felt only by fermions consistent with 5d gauge invariance.

3) Dynamics

Point interactions —> Chiral fermions & Generations

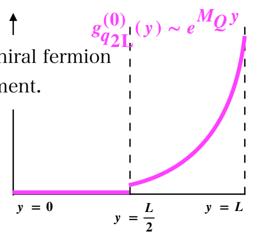
- 1st generation lives in left segment while 2nd generation lives in right segment.





1st generation chiral fermion lives in left segment.

2nd generation chiral fermion lives in right segment.



Chiral fermions & Generations

$$Q(x,y) = q_{1L}^{(0)}(x)g_{q1L}^{(0)}(y) + q_{2L}^{(0)}(x)g_{q2L}^{(0)}(y)$$
$$+\text{(KK massive modes)}$$

y-dependent VEV of scalar $\langle\Phi\rangle = \phi(y)$ —> Mass hierarchy

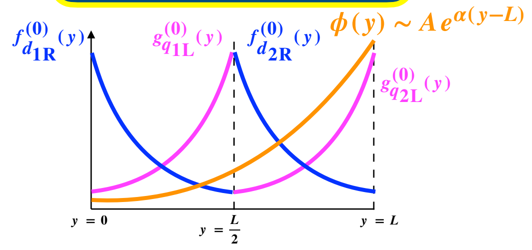
- y-dependent VEV via boundary conditions breaks translational invariance with respect to segments and leads to mass hierarchy.

$$m_{jk}^{(d)} = Y_D \int_0^L dy \left(g_{qjL}^{(0)}(y)\right)^* \phi(y) f_{dkR}^{(0)}(y)$$

4d quark masses = overlap integral of the mode functions with respect to the extra

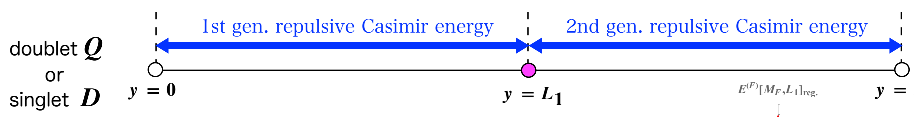
Mass hierarchy

$$m_{11}^{(d)} \ll m_{22}^{(d)}$$



Minimization of the Casimir energy —> Flavor mixing

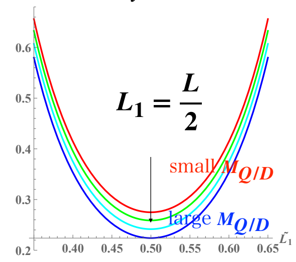
- Minimization of the Casimir energy determines the positions of point interactions.

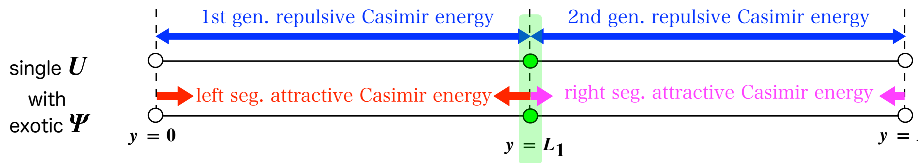


$$E(M_{Q/D}, L_1) = -\frac{|M_{Q/D}|^4}{4\pi^2}\int_0^\infty dx\, x^3 \ln\left(1 - e^{-2|M_{Q/D}|L_1\sqrt{1+x^2}}\right)$$
$$-\frac{|M_{Q/D}|^4}{4\pi^2}\int_0^\infty dx\, x^3 \ln\left(1 - e^{-2|M_{Q/D}|(L-L_1)\sqrt{1+x^2}}\right)$$

1st gen. Casimir energy Contribution

2nd gen. Casimir energy Contribution

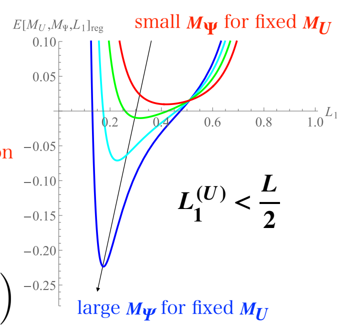




$$E(M_U, M_\Psi, L_1^{(U)}) = -\frac{|M_U|^4}{4\pi^2}\int_0^\infty dx\, x^3 \ln\left(1 - e^{-2|M_U|L_1^{(U)}\sqrt{1+x^2}}\right)$$
$$-\frac{|M_U|^4}{4\pi^2}\int_0^\infty dx\, x^3 \ln\left(1 - e^{-2|M_U|(L-L_1^{(U)})\sqrt{1+x^2}}\right)$$
$$-\frac{|M_\Psi|}{4\pi^2}\int_0^\infty dx\, x^3 \ln\left(1 + \frac{\sqrt{1+x^2}+\text{sgn}M_\Psi}{\sqrt{1+x^2}-\text{sgn}M_\Psi}e^{-2|M_\Psi|L_1^{(U)}\sqrt{1+x^2}}\right)$$
$$-\frac{|M_\Psi|}{4\pi^2}\int_0^\infty dx\, x^3 \ln\left(1 + \frac{\sqrt{1+x^2}-\text{sgn}M_\Psi}{\sqrt{1+x^2}+\text{sgn}M_\Psi}e^{-2|M_\Psi|(L-L_1^{(U)})\sqrt{1+x^2}}\right)$$

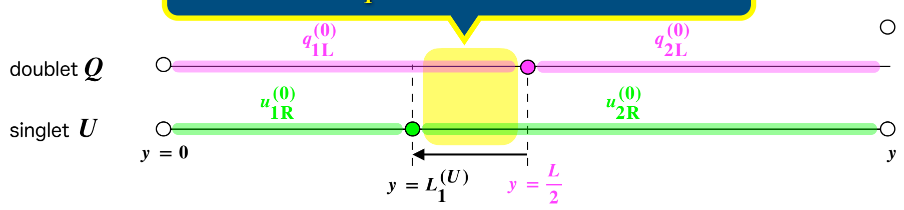
left seg. Casimir energy Contribution

right seg. Casimir energy Contribution



Flavor mixing

$$m_{12}^{(u)} = Y_U \int_{L_1^{(U)}}^{L/2} dy \left(g_{q1L}^{(0)}(y)\right)^* \phi(y) f_{u2R}^{(0)}(y)$$



4) Results & Future work

Model parameters assumptions

(※) The quantity with ~ represents the dimensionless quantity normalized by the size of the extra dimension L.

- $\tilde{M}_U = \tilde{M}_D = \tilde{M}_\Psi := \tilde{M},\quad \tilde{Y}_U = 1$ to $\{\tilde{M}_Q, \tilde{M}_U, \tilde{M}_\Psi, \tilde{M}_D, \tilde{Y}_U, \tilde{Y}_D, \tilde{\alpha}, \frac{1}{L}\tilde{A}\}$

Model Parameters	Values
$\tilde{M}_Q$	6.0
$\tilde{M}$	3.35
$\tilde{Y}_D$	0.018
$\tilde{\alpha}$	5.98
$\frac{1}{L}\tilde{A}$ [GeV]	31

➡

Quantity	Ratio $m^{(\text{model})}/m^{(\text{ref.})}$
m_u	0.984
m_d	1.17
m_s	1.17
m_c	0.973
$\lvert\sin\theta_{12}\rvert$	1.15

3-gen. realistic model, FCNC, Lepton-sector, ⋯ ???