素粒子模型に基づく宇宙ひもからの粒子放出率評価

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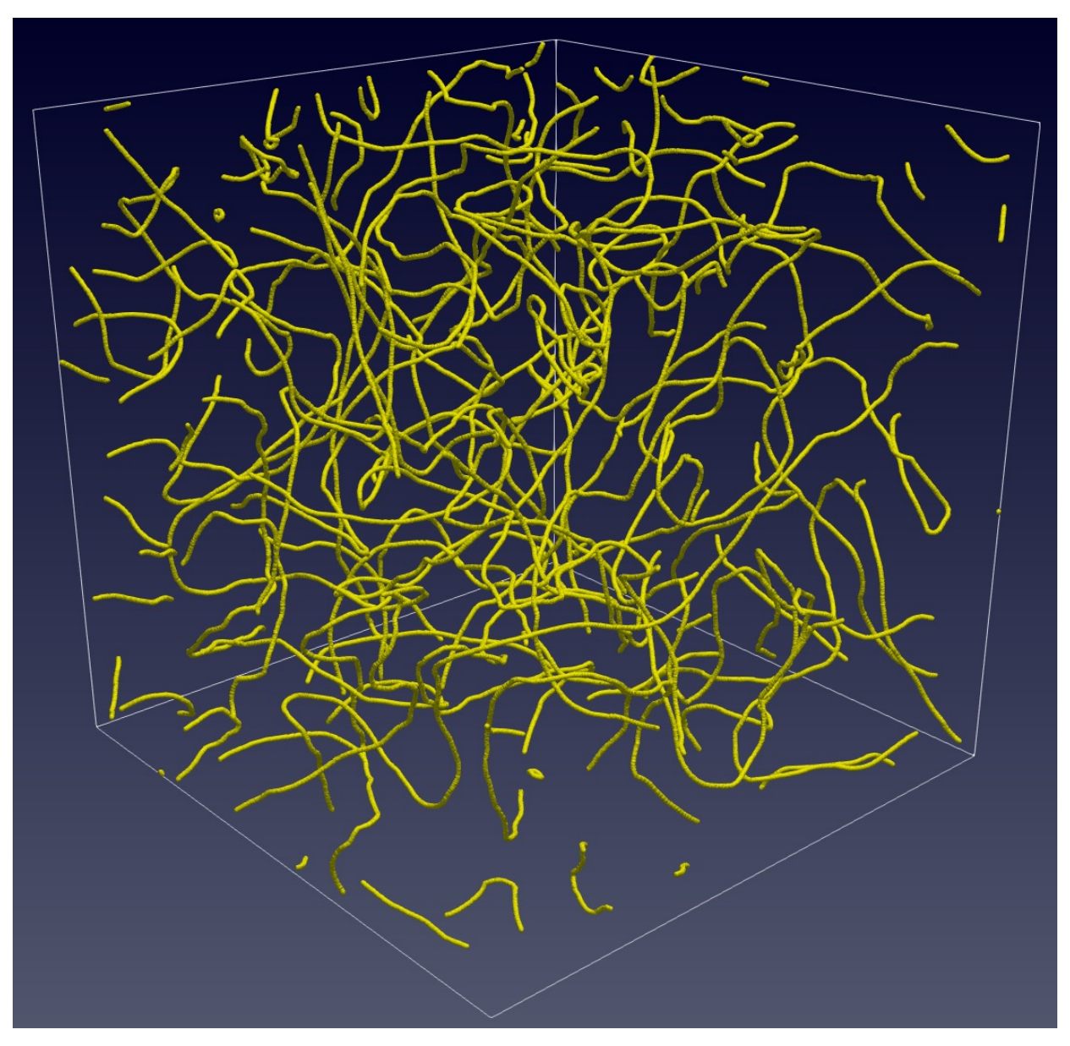
Cosmic strings are line-like objects produced after a phase transition in which $U(1)$ symmetry is broken. If they interact with light particles, the strings can emit them at cusps. Previous analytical calculation methods for particle emission rates considered only specific interaction terms. We generalize the methods to evaluate the emission rates of all light particles in general particle models. As an application, we study the Abelian-Higgs (AH) model coupled to the Standard Model (SM) through a gauge kinetic mixing term. We find that particle emission is enhanced when the string width increases, *i.e.* when the gauge coupling of the AH model becomes small. We further investigate the impact on the gravitational-wave spectrum from cosmic strings. We show that it can be consistent with the NANOGrav 2023 results while avoiding the LIGO–Virgo-KAGRA's (LVK) O3 constraints in certain parameter regions.

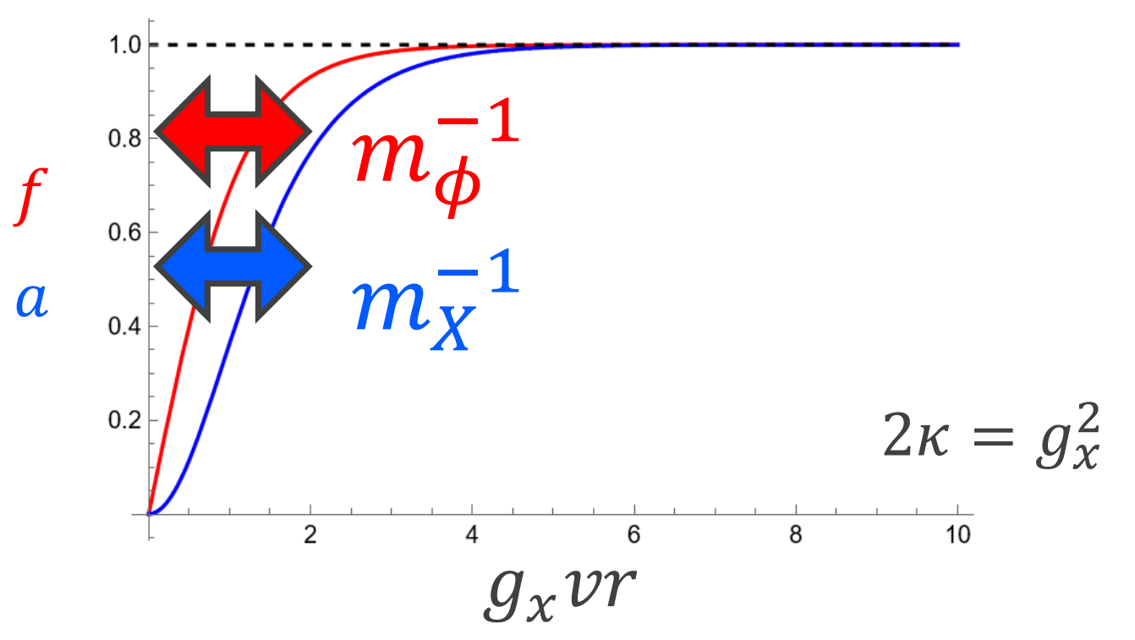


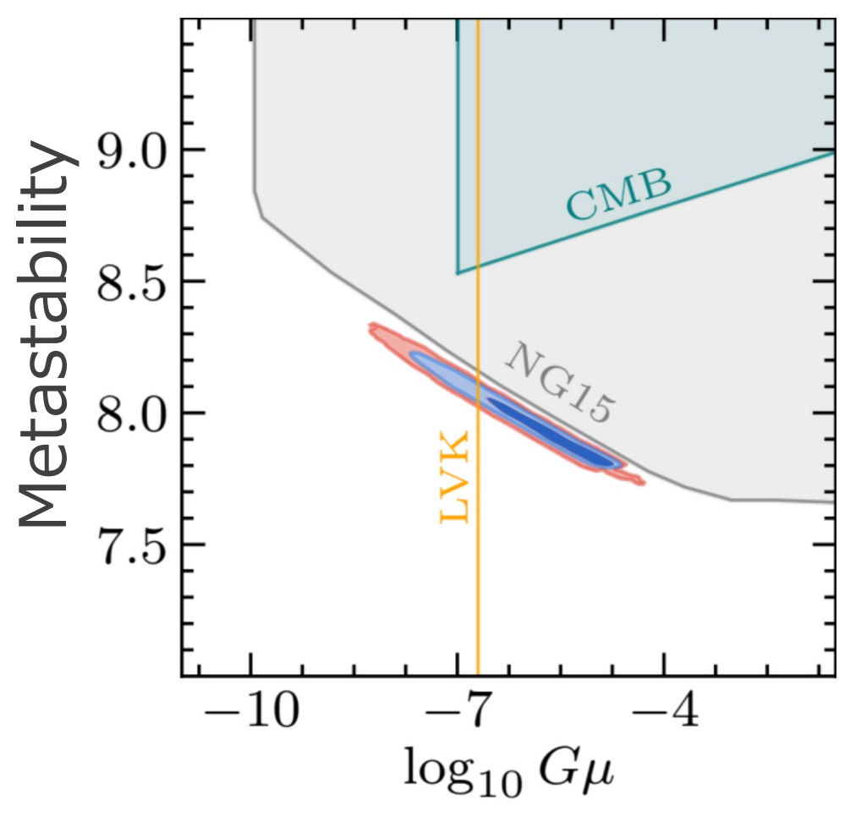
1. Cosmic string [Kibble (1976), Vilenkin, Shellard (2000)]

AH model: $\mathcal{L}_{AH} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \left|(\partial_\mu - ig_x X_\mu)\phi\right|^2 - \kappa(|\phi|^2 - v^2)^2$

String solution [Nielsen, Olesen (1973)]

$$\phi_s(x) = f(r)ve^{i\theta}, \qquad X_{s,\mu}(x) = \frac{a(r)}{g_x r}$$



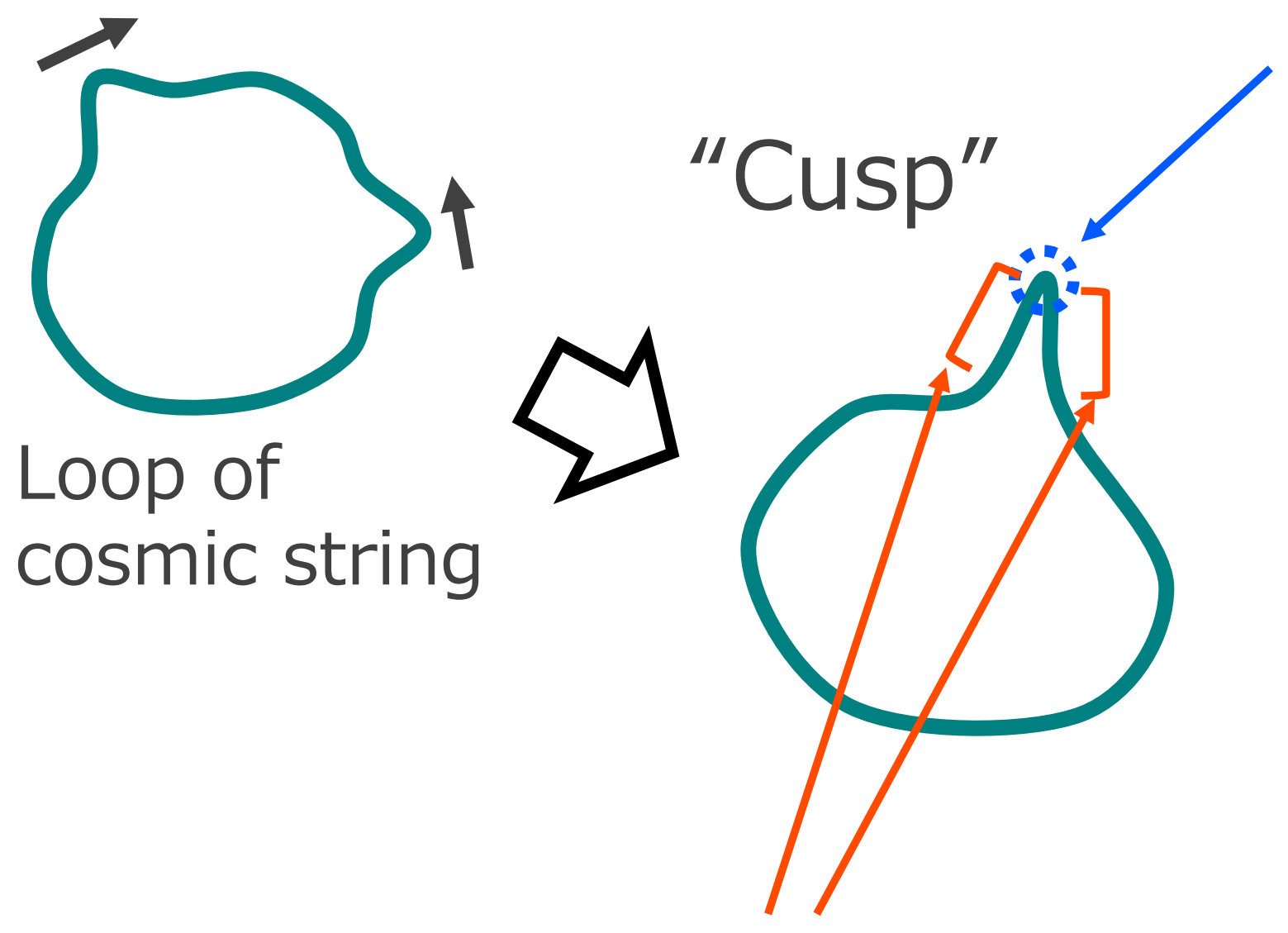


Cosmic strings radiate the gravitational waves (GW).

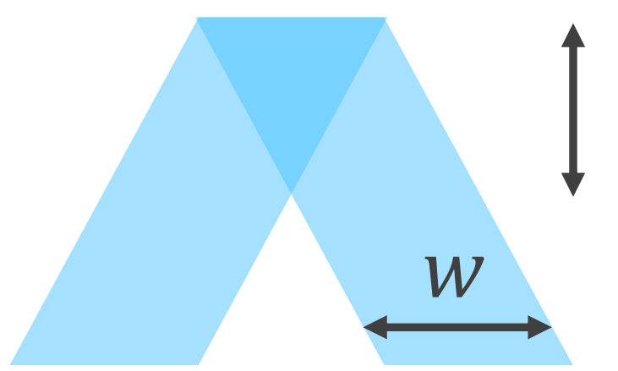
The NANOGrav's 2023 results prefer $10^{15} < v < 10^{17}$ GeV (for metastable string) at the 68% credible level, but that region is excluded by LVK O3 results.

[LVK collaborations (2021), NANOGrav collaboration (2023)]

2. Particle emission (Previous works)



Annihilation [Olum, Blanco-Pillado (1998)]



$\sigma \sim \sqrt{wL}$

(L: Loop length)

Power radiated per unit time is

$$P \sim \frac{\mu\sigma}{L} \sim \frac{\mu}{\sqrt{w^{-1}L}} \xrightarrow{w^{-1}\sim v} \frac{\mu}{\sqrt{vL}}$$

Light particle emission

- Emission from ϕ_s [Srednicki, Theisen (1986)]

In the thin-string limit,

$\mathbb{X}$: String trajectory
ζ^a: Worldsheet coordinates

$$\langle S|\tilde{\phi}^n(x)|S\rangle \simeq v^n w^2 \int d^2\zeta\,\sqrt{-\gamma}\,\delta^{(4)}\left(x - \mathbb{X}(\zeta)\right) \qquad (\tilde{\phi} \equiv v - \phi)$$

If $\alpha s^2|\phi|^2$ exists in the Lagrangian, (s: light particle ($m_s \ll v$))

$$\alpha s^2|\phi|^2 \supset \alpha s^2\left(\tilde{\phi}^2 - 2v\tilde{\phi}\right) \equiv \mathcal{L}_{int,\phi}$$

The S-matrix element for two s emission from a string loop is

$$\langle k_1, k_2; S|-i\textstyle\int d^4x\mathcal{L}_{int}|S\rangle = i\alpha\displaystyle\int d^4x\, e^{i(k_1+k_2)x}\langle S|(\tilde{\phi}^2 - 2v\tilde{\phi})|S\rangle$$
$$= -i\alpha v^2 w^2 \int d^2\zeta\sqrt{-\gamma}\, e^{i(k_1+k_2)\mathbb{X}(\zeta)}$$

$$\therefore P = \frac{2}{L}\int \frac{d^3k_1}{(2\pi)^3 2\omega_1}\frac{d^3k_2}{(2\pi)^3 2\omega_2}(\omega_1+\omega_2)\left|\langle k_1, k_2; S|-i\textstyle\int d^4x\mathcal{L}_{int}|S\rangle\right|^2$$
$$\sim \mathcal{O}(0.01)\times\alpha^2\frac{w^{-2}}{\sqrt{w^{-1}L}} \xrightarrow{w^{-1}\sim v} \mathcal{O}(0.01)\times\alpha^2\frac{v^2}{\sqrt{vL}} \qquad \left(\begin{matrix} m_s\sqrt{m_s L} < |\vec{k}| < v\sqrt{vL} \\ |\theta| < \left(|\vec{k}|L\right)^{-1/3}\end{matrix}\right)$$

- Emission from $X_{s,\mu}$ [Alford, Wilczek (1988)]

$$\langle S|X_{s,\mu}(x)|S\rangle \simeq \frac{\pi}{g_x}\int\frac{d^4k}{(2\pi)^4}\frac{ik^\nu}{k^2}\int d^2\zeta\,\epsilon_{\mu\nu\rho\sigma}\,\epsilon^{ab}\frac{\partial^\rho\mathbb{X}}{\partial\zeta^a}\frac{\partial^\sigma\mathbb{X}}{\partial\zeta^b}\,e^{-ik(x-\mathbb{X})}$$

($\because$magnetic field $\langle S|X_{s,\mu\nu}(x)|S\rangle \simeq \frac{\pi}{g_x}\int d^2\zeta\,\epsilon_{\mu\nu\rho\sigma}\,\epsilon^{ab}\frac{\partial^\rho\mathbb{X}}{\partial\zeta^a}\frac{\partial^\sigma\mathbb{X}}{\partial\zeta^b}\delta^{(4)}(x-\mathbb{X})$)

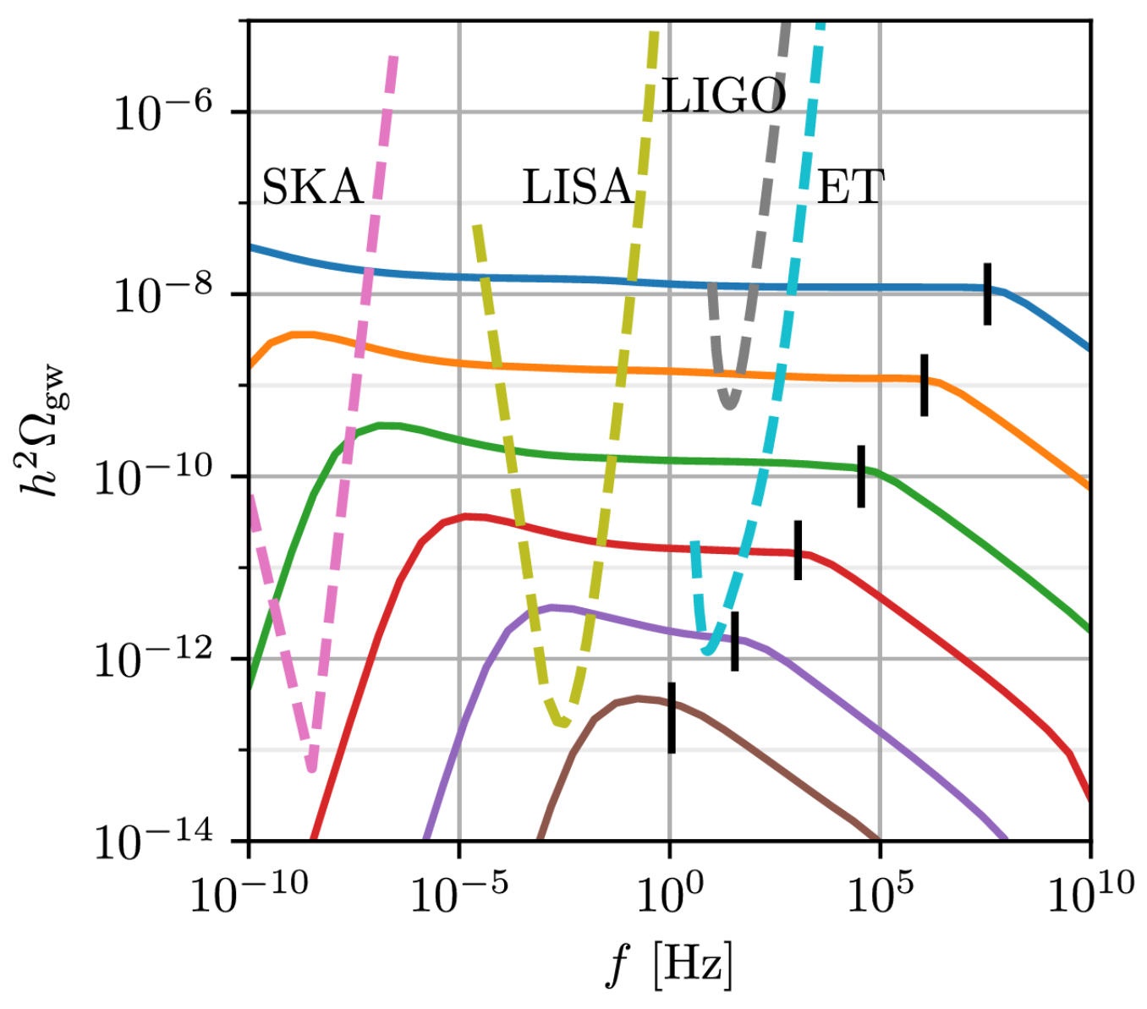
$$\mathcal{L} \supset g_x q X_\mu J^\mu \quad \Rightarrow \quad P \sim \mathcal{O}(0.1)\times q^2\frac{w^{-2}}{\sqrt{w^{-1}L}} \xrightarrow{w^{-1}\sim v} \mathcal{O}(0.1)\times q^2\frac{v^2}{\sqrt{vL}}$$

★ GW radiation vs. Particle emission

$$P_{GW} \simeq 50\times G\mu^2 \qquad\qquad P_{PE} \propto \frac{v^2}{\sqrt{vL}}$$

Particle emission affects the GW spectrum at very high frequencies only.
(Ex. Suppression at $f > 10^8$Hz for $v\sim 10^{16}$GeV)

[Auclair, Steer, Vachaspati (2019)]



3. How to derive all the interaction terms

We derive all the interaction terms by expanding the fields in the action around the string solutions.

$$S[\phi_{s,i} + \delta\phi_i] = S[\phi_{s,i}] + \frac{\delta S}{\delta\phi_i}[\phi_{s,i}]\delta\phi_i + \frac{1}{2}\frac{\delta^2 S}{\delta\phi_i\delta\phi_j}[\phi_{s,i}]\delta\phi_i\delta\phi_j + \cdots$$

This reduces to the Nambu-Goto action in the thin-string limit.
[Nielsen, Olesen (1973), Vilenkin, Shellard (2000)]

Vanish
($\because$ classical solution)

All the interaction terms between the string and light particles, including

- Emission from ϕ_s [Srednicki, Theisen (1986)]
- Emission from $X_{s,\mu}$ [Alford, Wilczek (1988)]

4. Application for gauge kinetic mixing

$$\mathcal{L} = \mathcal{L}_{AH} + \mathcal{L}_{SM} + \frac{\epsilon}{2}X_{\mu\nu}Y^{\mu\nu} \qquad (Y_\mu\text{: } U(1)_Y \text{ gauge field})$$

Previous work: [Hyde, Long, Vachaspati (2013)], but not sufficient.

Derive interaction terms based on our method

Z_μ: Z boson, h: SM Higgs, ψ_j: SM fermion ($\eta = 246$ GeV)

$$\mathcal{L}_{int} \simeq \frac{g_1^2 g_z^2\epsilon^2}{g_x^2}\frac{\eta^2}{v^2}\left(\tilde{\phi}^2 - 2v\tilde{\phi}\right)Z_\mu Z^\mu + g_1 q_{Y,j}\epsilon X_{s,\mu}\bar{\psi}_j\gamma^\mu P_{L,R}\psi_j$$
$$+\frac{g_1^2\epsilon^2}{8}X_{s,\mu}X_s^\mu h^2 + \frac{g_1 g_z\epsilon}{4}X_{s,\mu}Z^\mu h^2 + \cdots$$

New types of interactions !

g_1: $U(1)_Y$ gauge coupling
g_z: coupling between Z_μ and h
$q_{Y,j}$: $U(1)_Y$ charge of ψ_j

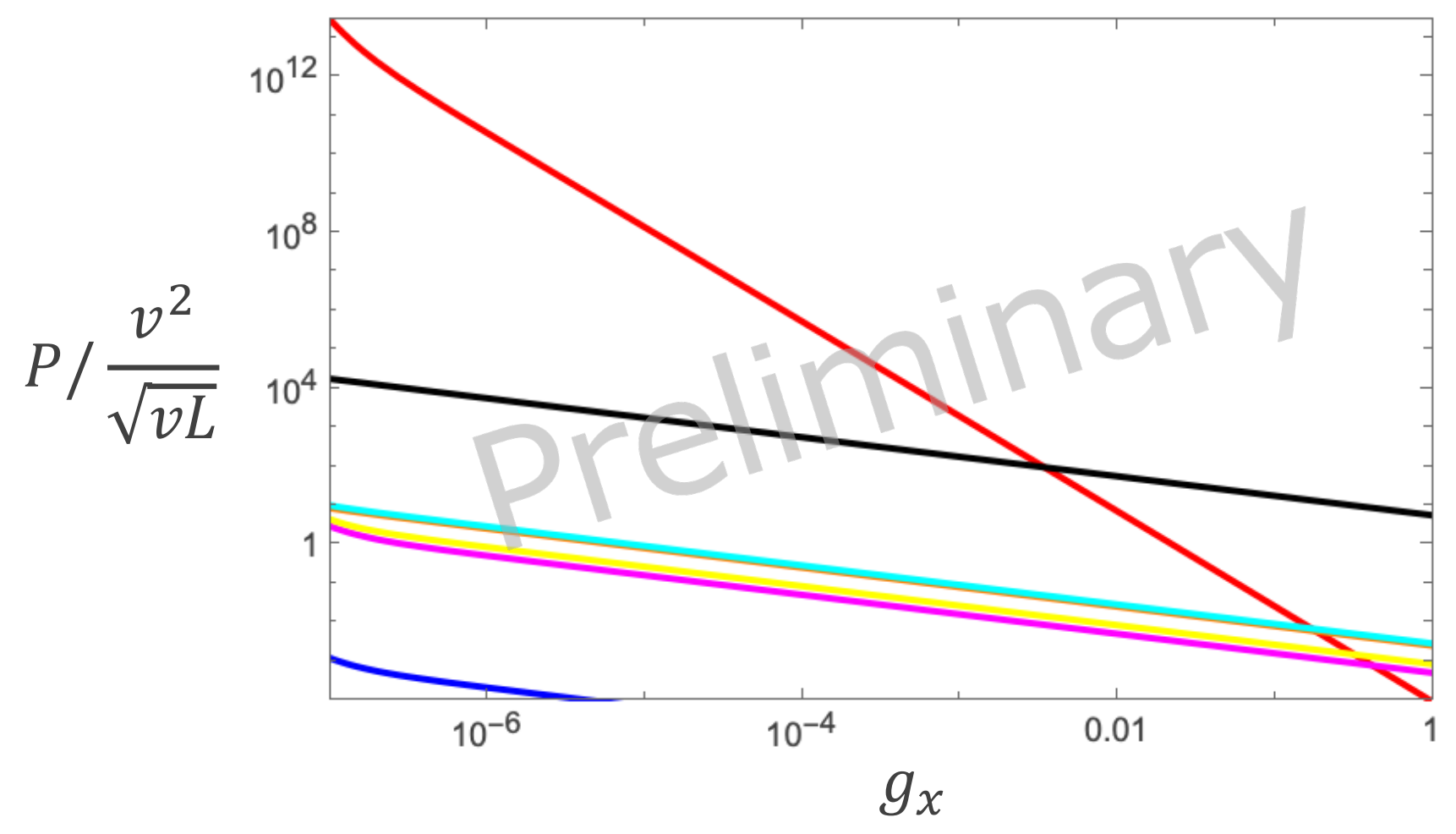
Moreover, we investigate the g_x-dependence of each P.

$(w \sim (g_x v)^{-1}$ but $2\kappa = g_x^2)$

Annihilation: $P_{ann} \propto w^{-\frac{1}{2}} \propto g_x^{-\frac{1}{2}}$

Fermions: $P_{\psi\bar{\psi}} \propto g_x^{-2}w^{\frac{3}{2}} \propto g_x^{-\frac{1}{2}}$ ⬅ Cannot exceed P_{ann}

Two SM Higgs: $P_{hh} \propto g_x^{-4}w^{\frac{3}{2}} \propto g_x^{-\frac{5}{2}}$ ⬅ More enhanced at low g_x



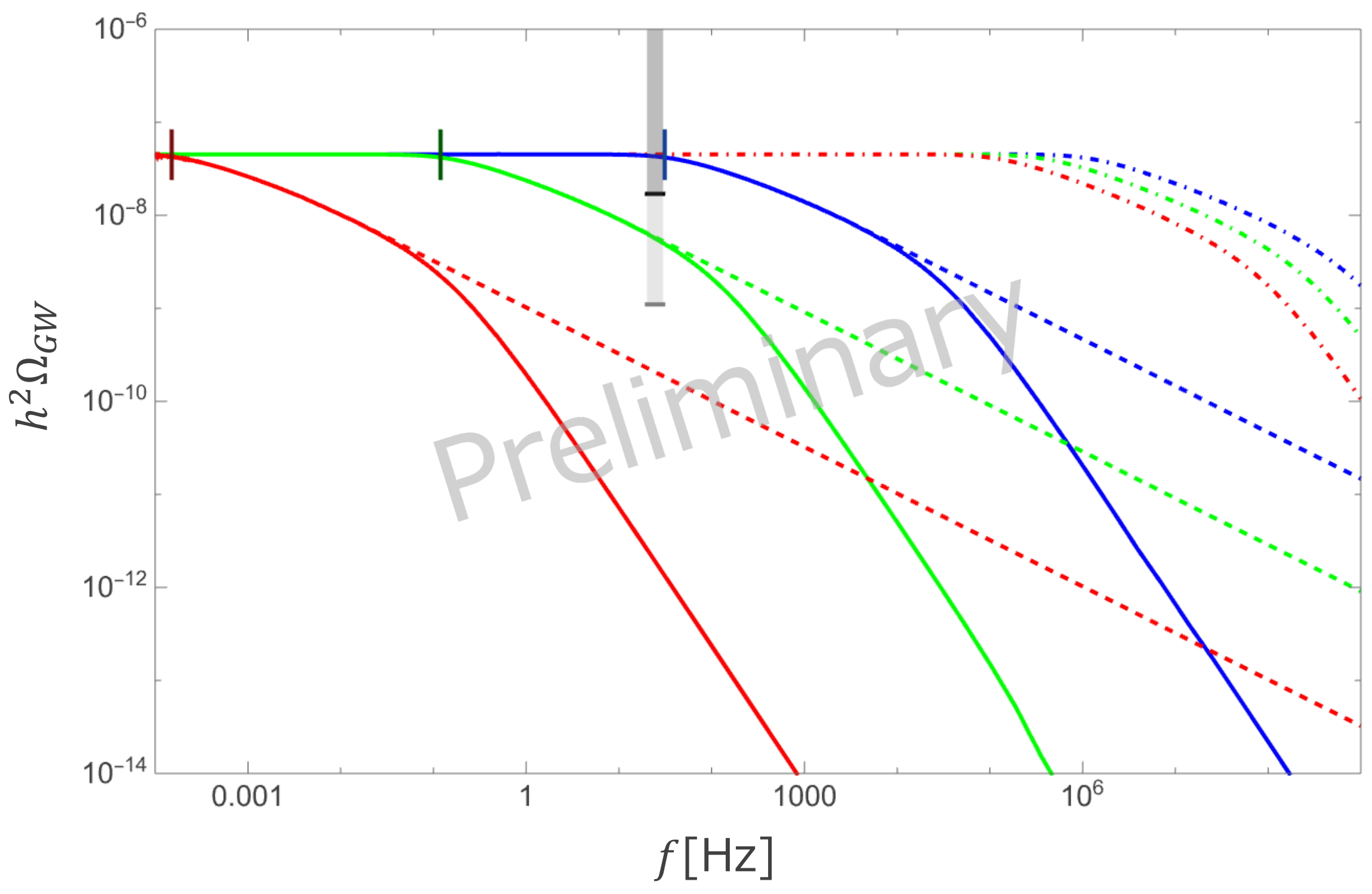
P vs. g_x at $v = 10^{16}$ GeV, $\epsilon = 0.1$

— : Annihilation
— : hh
— : hhZ_μ
— : Fermions

$P_{hh} > P_{ann}$ in $g_x < 3\times 10^{-3}$

5. Impact on the GW spectrum

The GW spectrum at $v = 10^{16}$ GeV, $\epsilon = 0.1$ $(G\mu \simeq 4.2\times 10^{-6})$



— : $g_x = 10^{-5}$
— : $g_x = 10^{-6}$
— : $g_x = 10^{-7}$
Dashed: Extrapolation ($\propto f^{-1/2}$)
Dot-dashed: Only annihilation
▩ : LVK O3 constraints
☐ : LVK design sensitivity

f_{cut} decreases as g_x decreases.
$(g_x, f_{cut}[\mathrm{Hz}]) = (10^{-5}, 3.1\times 10), (10^{-6}, 1.2\times 10^{-1}), (10^{-7}, 1.5\times 10^{-4})$

➡ New mechanism for avoiding the LVK O3 constraints!