Bias with a Timer: Axion Domain Wall Decay and Dark Matter

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1. Motivation

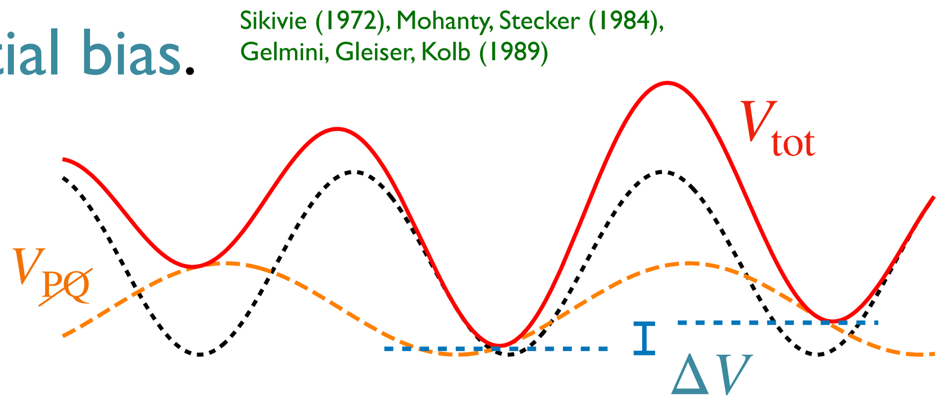
QCD axion can explain the strong CP problem and dark matter (DM) simultaneously. Peccei and Quinn (1977) Weinberg (1978), Wilczek (1978)

However, when the SSB of $\mathrm{U}(1)_{\mathrm{PQ}}$ after inflation, domain walls (DW) form and might dominate the Universe.

A possible solution is potential bias. Sikivie (1972), Mohanty, Stecker (1984), Gelmini, Gleiser, Kolb (1989)

$\Delta V \sim \sigma_{\mathrm{wall}}/t_{\mathrm{dec}}$

→ DW annihilation



One problem is that a potential bias can spoil the Peccei-Quinn (PQ) mechanism at the same time.

Can we turn off the bias term after DW collapse?

2. PQ mechanism with a light scalar

Hao, **SN**, Nakai, Suzuki (2025)

We introduce a light complex scalar field S mixed with the PQ scalar P: Ibe, Kobayashi, Suzuki, Yanagida (2020)

$$V_{\mathrm{PQ}}(P,S) \supset m_S^2|S|^2 + \frac{1}{(n!)^2}\frac{\lambda_S^2}{M_{\mathrm{Pl}}^{2n-4}}|S|^{2n} + \left(\frac{\lambda}{m!\ell! M_{\mathrm{Pl}}^{m+\ell-4}} S^m P^\ell + \mathrm{h.c.}\right)$$

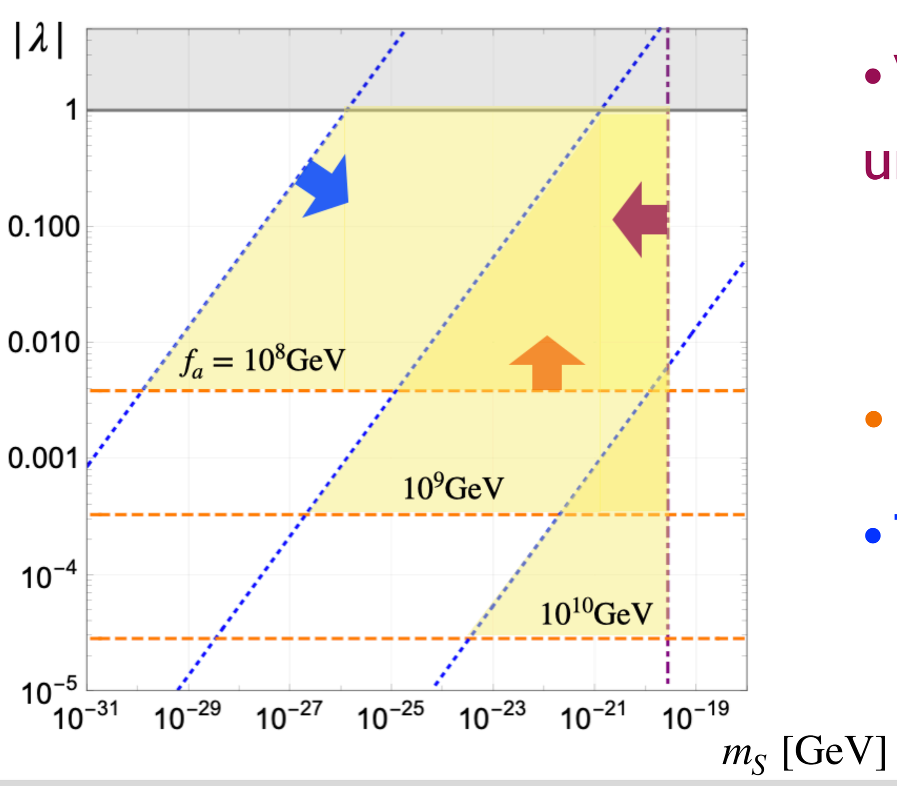
l, m, n: integers

$$|\ddot{S}| + 3H|\dot{S}| + \frac{n\lambda_S^2}{(n!)^2 M_{\mathrm{Pl}}^{2n-4}}|S|^{2n-1} = 0 \;\rightarrow\; \langle |S| \rangle \propto \left(\frac{H}{M_{\mathrm{Pl}}}\right)^{\frac{1}{n-1}} M_{\mathrm{Pl}}$$

An effective PQ breaking potential is induced.

$$V_{\not{PQ}} \simeq -\frac{1}{\ell^2} m_{\not{PQ}}^2 v_{\mathrm{PQ}}^2 \cos\left(\ell\frac{a}{v_{\mathrm{PQ}}} + m\frac{b}{\chi} + \delta\right) \qquad m_{\not{PQ}}^2(T) \simeq \frac{|\lambda|\ell^2}{2^{\ell/2-1}m!\ell!}\frac{\langle S\rangle^m v_{\mathrm{PQ}}^{\ell-2}}{M_{\mathrm{Pl}}^{m+\ell-4}}$$

When $H \sim m_S$, S starts to oscillate around the origin ($S \sim 0$), and the effective potential $V_{\not{PQ}}$ disappears.



- We assume $V_{\not{PQ}}$ remains at least until QCD scale.

$$m_S \lesssim \sqrt{\frac{\pi^2 g_*}{90}}\frac{\Lambda_{\mathrm{QCD}}^2}{M_{\mathrm{Pl}}} \simeq 3\times10^{-11}\mathrm{eV}$$

- $T_{\mathrm{osc}} > T_{\mathrm{osc}}^{(\mathrm{conv})}$
- To avoid backreaction

$$\frac{1}{(n!)^2}\frac{\lambda_S^2}{M_{\mathrm{Pl}}^{2n-4}}|S|^{2n} > \frac{|\lambda|}{m!l!M_{\mathrm{Pl}}^{m+l-4}}|S|^m v_{\mathrm{PQ}}^l$$

3. Evolution of string-wall system

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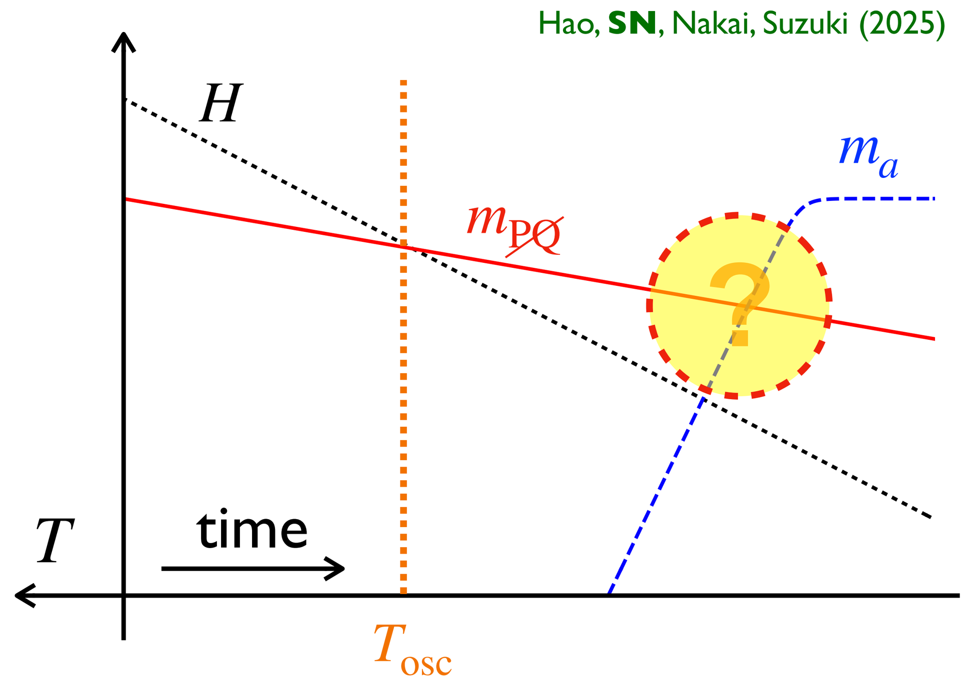
At $T < T_{\mathrm{osc}}$, the l walls are attached with the string.

$$V_{\not{PQ}} = -\frac{1}{\ell^2} m_{\not{PQ}}^2 v_{\mathrm{PQ}}^2 \cos\left(\ell\frac{a}{v_{\mathrm{PQ}}} + \delta'\right)$$

DW for $V_{\not{PQ}}$



Consider how this system can collapse from the following aspects:

(i) Volume pressure

(ii) Structural instability

(i) Volume pressure

The potential difference induces the volume pressure on the domain wall, which makes the system unstable when $p_V \sim p_T$.

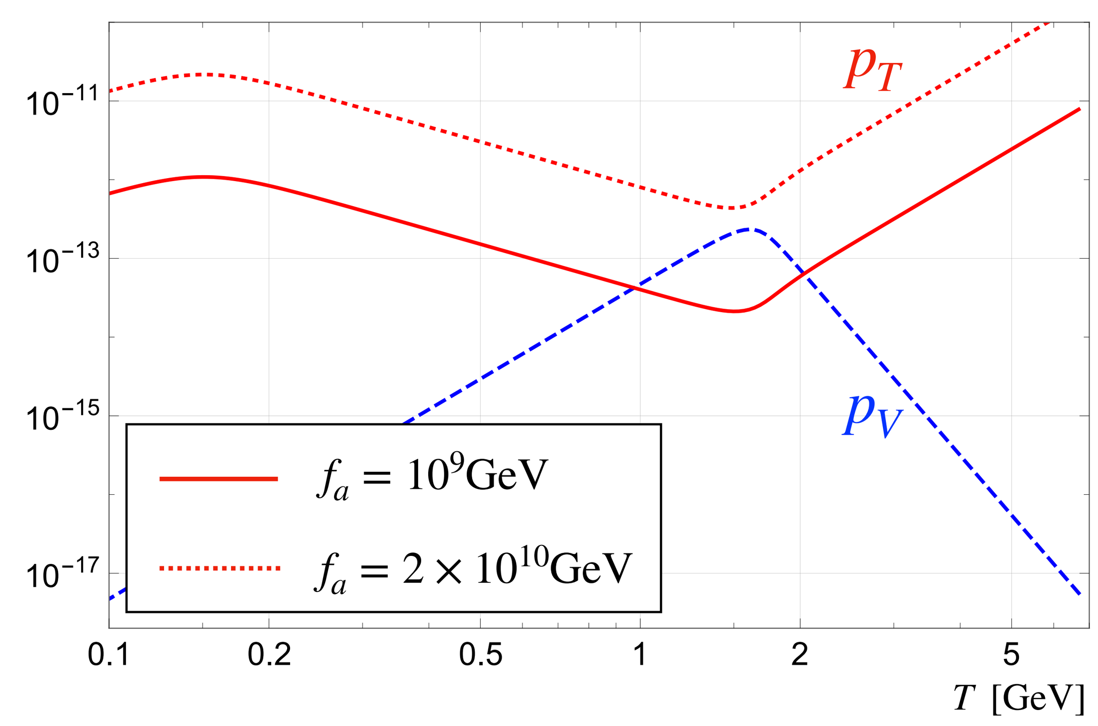
Volume pressure

$p_V \sim \Delta V$

Tension force

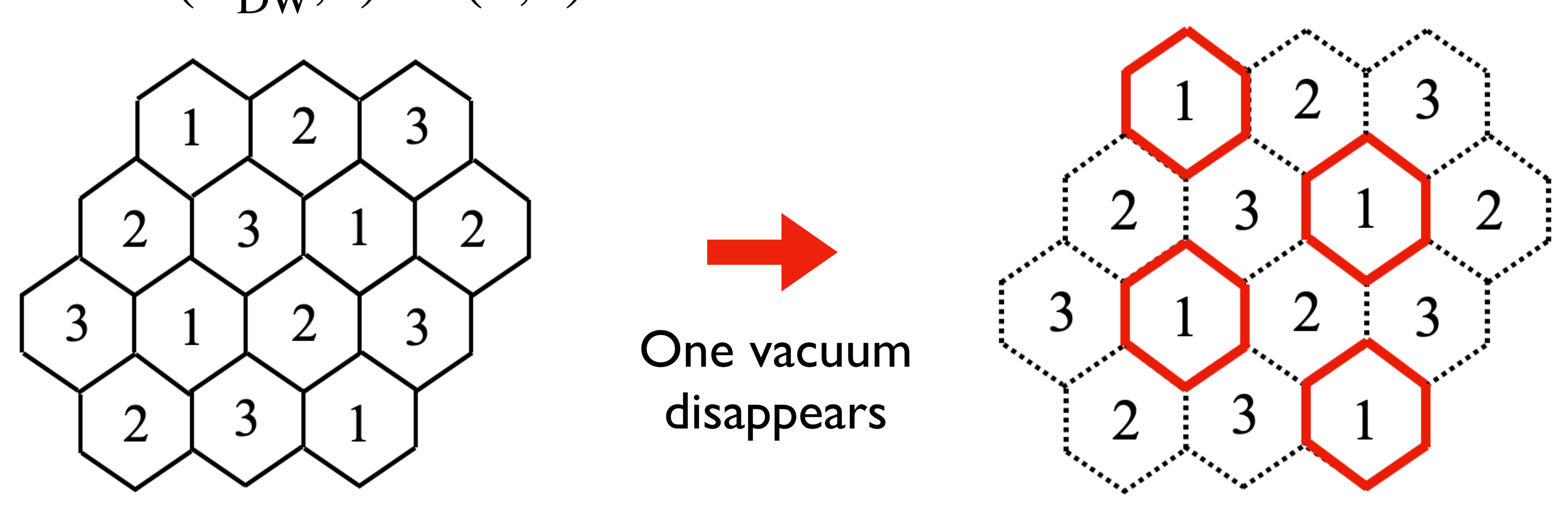
$p_T \sim \sigma_{\mathrm{wall}} H$

$f_a \lesssim 10^9$GeV is required for the system collapse due to p_V.



(ii) Structural instability

Consider $(N_{\mathrm{DW}}, l) = (2,3)$.



In addition, the axion distribution is biased at the QCD scale. As a result, such systems may be broken soon. Kitajima, Lee, Takahashi, Yin (2023)

When $|V_{\not{PQ}}| \sim |V_{\mathrm{QCD}}|$, the system seems to be most unstable.

$$m_{\not{PQ}}(T_{\mathrm{tr}}) \sim \frac{\ell}{N_{\mathrm{DW}}} m_a(T_{\mathrm{tr}}) \;\leftrightarrow\; T_{\mathrm{tr}} \simeq 1.6\ \mathrm{GeV}\left(\frac{|\lambda|}{0.01}\right)^{-\alpha}\left(\frac{v_{PQ}}{2\times10^9\ \mathrm{GeV}}\right)^{-\ell\alpha}$$

We assume that the annihilation occurs at $T_{\mathrm{ann}} = \kappa T_{\mathrm{tr}}$.

$$\Omega_{a,\mathrm{dec}}h^2 \simeq 0.12\frac{1}{\sqrt{1+\epsilon_a^2}}\left(\frac{\kappa}{0.1}\right)^{-1}\left(\frac{|\lambda|}{2\times10^{-4}}\right)^{\alpha}\left(\frac{N_{\mathrm{DW}}}{2}\right)^{\ell\alpha}\left(\frac{f_a}{2.4\times10^{10}\ \mathrm{GeV}}\right)^{1+\ell\alpha}$$

4. Viable parameter spaces

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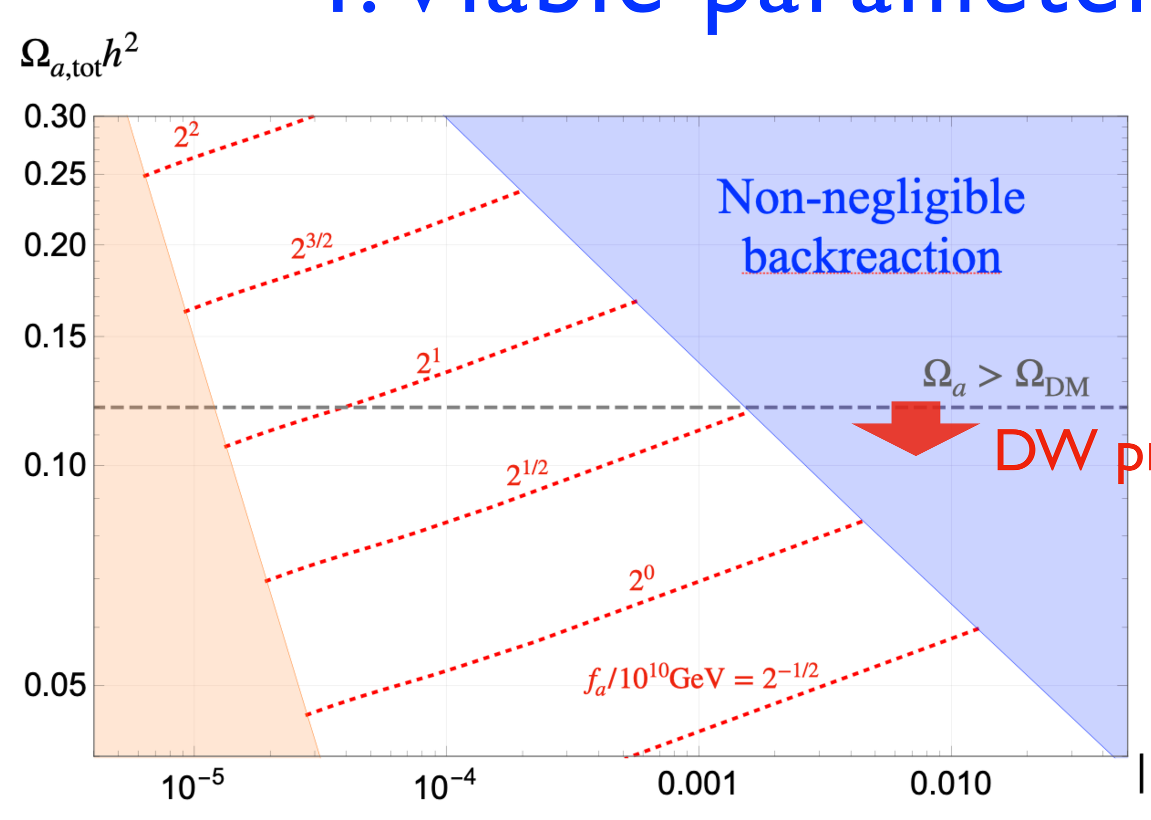

$(N_{\mathrm{DW}}, l, m, n) = (2,3,9,6)$ $\delta' = 1$ $\kappa = 0.1$ $(T_{\mathrm{ann}} = \kappa T_{\mathrm{tr}})$

$$\Omega_{a,\mathrm{tot}} = \Omega_{a,\mathrm{dec}} + \Omega_{a,\mathrm{mis}}$$

DW problem can be solved.

$f_a \lesssim 2\times10^{10}$GeV

Summary

- We consider the DW problem by introducing a mixing coupling between the PQ scalar and a light scalar.
- The mixing coupling induces a time-dependent bias potential, which makes the string-DW system unstable.
- In addition of misalignment contribution, we show that the overproduction can be avoided for $f_a \lesssim 10^{10}$GeV, even in the presence of small volume pressure.