Mass spectrum and symmetry breaking of gauge fields in non-Abelian gauge theory with extra dimensions of S^2

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Introduction

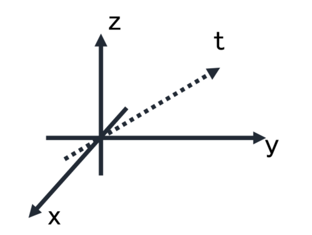
1. Problems of the Standard Model

- Unification of the gauge group
- The origin of the Higgs boson
- Yukawa coupling constants

A clear understanding of **gauge symmetry breaking** is essential.

2. the higher-dimensional theory

Minkowski spacetime

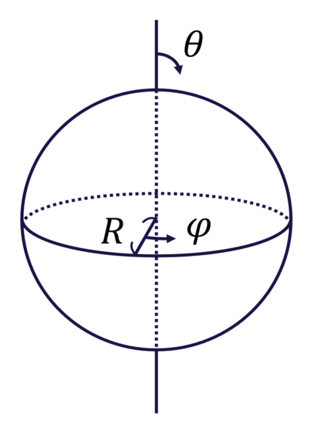


× (Extra-dimensional space)

i. Extra-dimensional component appear as **Lorentz scalars**.
ii. Extra-dimensional momenta generate **a mass**.
iii. Extra-dimensional geometry could cause **symmetry breaking**.

Our study: **Gauge symmetry breaking** by extra-S^2 curvature.

Set up



- Metric: $G_{MN} = \mathrm{diag}\left(1,\ -1,\ -1,\ -1,\ -R^2,\ -R^2\sin^2\theta\right)$
- Coordinates: $X^M = (x^\mu, y^{\hat\alpha}) = (x^\mu, \theta, \varphi)$
- Euler-Lagrange equation of YM sector:

$$\nabla_{\hat\beta}F^{\hat\beta\hat\alpha} - ig\left[A_{\hat\beta}, F^{\hat\beta\hat\alpha}\right] = 0$$

$A_\varphi \propto \cos\theta$ is a **non-trivial classical solution.**

➡ Introduce a background field $\langle A_\varphi\rangle = \mu\cos\theta$.

$\mu \equiv \mu^i H_i$ (constant) chosen along Cartan.

- Redefined gauge fields:

$$A_M(X) \to A_M(X) + \langle A_M\rangle = \left(A_\mu(X),\ A_\theta(X),\ A_\varphi(X) + \mu\cos\theta\right)$$

(Cartan-Weyl Basis: $A_M = \sum_{i:\ \text{all Cartans}} A_M^i H_i + \sum_{\alpha:\ \text{all roots}} A_M^\alpha E_\alpha$)

- Lagrangian:

$$S = \int d^4x\, d\theta\, d\varphi\, R^2\sin\theta\left\{-\frac{1}{2}\mathrm{tr}[F_{MN}F^{MN}]\right\}$$

$$= \int d^4x\, d\theta\, d\varphi\, R^2\sin\theta\left\{-\frac{1}{2}\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] - \mathrm{tr}[F_{\mu\hat\alpha}F^{\mu\hat\alpha}] - \mathrm{tr}[F_{\theta\varphi}F^{\theta\varphi}]\right\}$$

$$\left(F_{MN} = \partial_M(A_N + \langle A_N\rangle) - \partial_N(A_M + \langle A_M\rangle) - ig[A_M + \langle A_M\rangle, A_N + \langle A_N\rangle]\right)$$

Gauge symmetry breaking with S^2

6. Quadratic terms of four-dimensional gauge fields A_μ

$$\mathcal{L}^{\text{quadratic}}_{\text{gauge}} = \frac{1}{2}R^2\sin\theta\left[A^i_\mu\left(\Box + \frac{1}{R^2}\boldsymbol{L}^2_{★}\right)A^{\mu i} + A^{-\alpha}_\mu\left\{\Box + \frac{1}{R^2}\tilde{\boldsymbol{J}}^{(\alpha)2}_{★}\right\}A^{\mu\alpha}\right],$$

Kaluza-Klein expansion

$$A^i_\mu(x,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{\sqrt{2}}{R}A^i_{\mu,lm}(x)\,Y_{lm}(\theta,\varphi)\ ,$$

$$A^\alpha_\mu(x,\theta,\varphi) = \sum_{j=|k_\alpha|}^{\infty}\sum_{m=-j}^{j}\frac{\sqrt{2}}{R}A^\alpha_{\mu,jm}(x)\mathcal{Y}_{k_\alpha jm}(z,\varphi)\ ,$$

$k_\alpha \equiv g\,(\mu\cdot\alpha)$

$$S^{\text{quadratic}}_{\text{gauge}} = \int d^4x\left[\sum_{l=0}^{\infty}\sum_{m=-l}^{l}A^i_{\mu,lm}\left\{\Box + \frac{l(l+1)}{R^2}\right\}A^{\mu,i}_{lm} + \sum_{j=|k_\alpha|}^{\infty}\sum_{m=-j}^{j}A^{-\alpha}_{\mu,jm}\left\{\Box + \frac{j(j+1)-k_\alpha^2}{R^2}\right\}A^{\mu,\alpha}_{jm}\right].$$

(Here, we omit the gauge-fixing term.)

Kaluza-Klein masses

- Cartan components,
- root components with $k_\alpha = 0$,

$\dfrac{l(l+1)}{R^2}$ massless mode exist ! ($l = 0$)

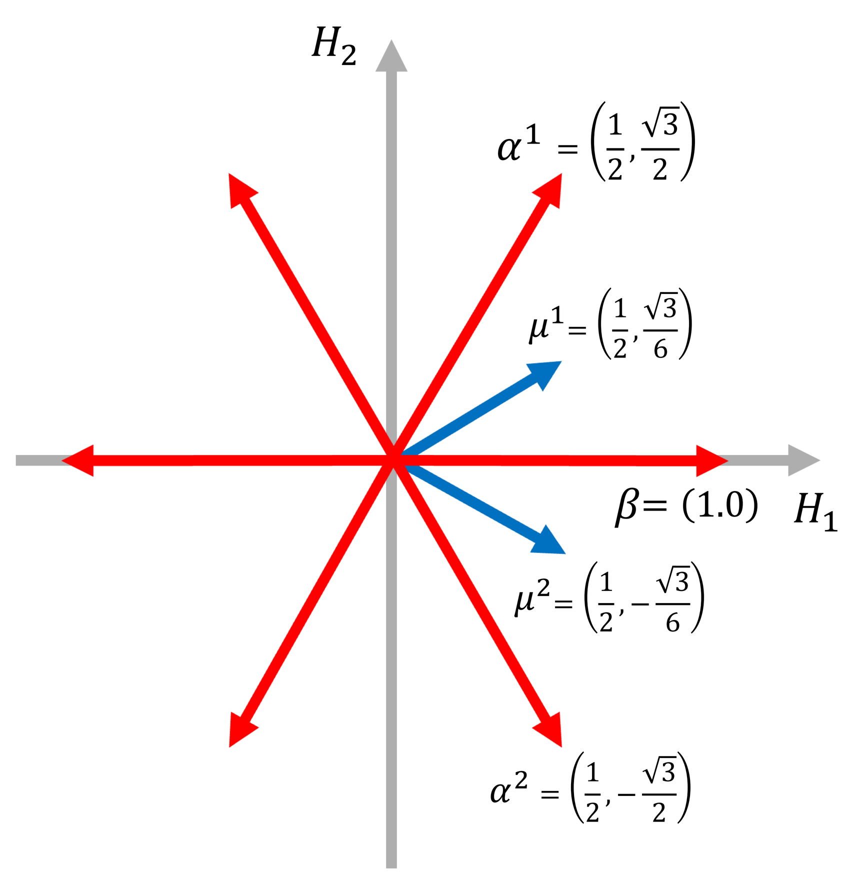
- For root components with $k_\alpha \neq 0$,

$\dfrac{j(j+1)-k_\alpha^2}{R^2}$ No massless modes…

Result

When $k_\alpha = 0$, that is, **commuting with background fields**, the gauge symmetry in the α-direction is **unbroken** in four dimensions.

7. Simple example: gauge group = SU(3)



Ex. 1) for background $\mu^1 + \mu^2$,

$k_{\alpha^1,\alpha^2,\beta} \neq 0$ (for all root)

➡ $SU(3) \to U(1)\times U(1)$.

Ex. 2) for background $\mu^1 - \mu^2$,

$k_{\alpha^1,\alpha^2} \neq 0$, $k_\beta = 0$

➡ $SU(3) \to SU(2)\times U(1)$.

H_1, H_2 : Cartan generators of SU(3)
α^1, α^2 : simple roots
β : root ($\equiv \alpha^1 + \alpha^2$)
μ^1, μ^2 : fundamental weights $\left(\mu^i\cdot\alpha^j = \frac{1}{2}\delta^{ij}\right)$

8. Quadratic terms of extra-dimensional gauge fields A_θ, A_φ

A_θ, A_φ are interpreted as **scalar fields**:

$$A_\theta = -\frac{1}{\sin\theta}D_\varphi\phi + D_\theta\chi$$

$$A_\varphi = \sin\theta D_\theta\phi + D_\varphi\chi$$

$$\left(D_M \equiv \partial_M - ig\left[\langle A_M\rangle\, ,\ \bullet\,\right]\right)$$

ϕ as a physical scalar boson, χ as a Nambu-Goldstone boson.

$$\mathcal{L}^{\text{quadratic}}_{\text{extra gauge}} = -\frac{1}{2}\sin\theta\Big\{\phi_i\Box\left(\boldsymbol{L}^2_{★}\phi^i\right) + \chi_i\Box\left(\boldsymbol{L}^2\chi^i\right) + \frac{1}{R^2}\left(\hat{\boldsymbol{L}}^2\phi_i\right)\left(\hat{\boldsymbol{L}}^2\phi^i\right) + \frac{\xi}{R^2}\left(\hat{\boldsymbol{L}}^2\chi_i\right)\left(\hat{\boldsymbol{L}}^2\chi^i\right)$$
$$+ \phi^{-\alpha}\Box\left(\tilde{\boldsymbol{J}}^{(\alpha)2}_{★}\phi^\alpha\right) + \chi^{-\alpha}\Box\left(\tilde{\boldsymbol{J}}^{(\alpha)2}\chi^\alpha\right) - 2ik_\alpha\phi^{-\alpha}\Box\chi^\alpha$$
$$+ \frac{1}{R^2}\left(\tilde{\boldsymbol{J}}^{(-\alpha)2}\phi^{-\alpha}\right)\left(\tilde{\boldsymbol{J}}^{(\alpha)2}\phi^\alpha\right) - \frac{1}{R^2}k_\alpha^2\phi^{-\alpha}\phi^\alpha + \frac{\xi}{R^2}\left(\tilde{\boldsymbol{J}}^{(-\alpha)2}\chi^{-\alpha}\right)\left(\tilde{\boldsymbol{J}}^{(\alpha)2}\chi^\alpha\right)\Big\},$$

(ξ: gauge-fixing parameter)

Kaluza-Klein expansion

$$\phi^i(\chi^i)(x,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-j}^{j}\frac{\sqrt{2}}{\sqrt{l(l+1)}}\phi^i_{lm}(\chi^i_{lm})(x)\,Y_{lm}(\theta,\varphi),$$

$$\phi^\alpha(\chi^\alpha)(x,\theta,\varphi) = \sum_{j=|k_\alpha|}^{\infty}\sum_{m=-j}^{j}\frac{\sqrt{2}}{\sqrt{j(j+1)-k_\alpha^2}}\phi^\alpha_{jm}(\chi^\alpha_{jm})(x)\mathcal{Y}_{k_\alpha jm}(z,\varphi)\ ,$$

$$S^{\text{quadratic}}_{\text{gauge}} = \int d^4x\Bigg(\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left[-\phi^i_{lm}\left\{\Box + \frac{l(l+1)}{R^2}\right\}\phi^i_{lm} - \chi^i_{lm}\left\{\Box + \xi\frac{l(l+1)}{R^2}\right\}\chi^i_{lm}\right]$$
$$+ \sum_{j=|k_\alpha|}^{\infty}\sum_{m=-j}^{j}\Bigg[-\phi^{-\alpha}_{1,jm}\left\{\Box + \frac{\left(j(j+1)-k_\alpha^2\right)^2 - k_\alpha^2}{R^2\left(j(j+1)-k_\alpha^2\right)}\right\}\phi^\alpha_{1,jm}$$
$$- \chi^{-\alpha}_{jm}\left\{\Box + \xi\frac{j(j+1)-k_\alpha^2}{R^2}\right\}\chi^\alpha_{jm} + 2ik_\alpha\frac{1}{j(j+1)-k_\alpha^2}\phi^{-\alpha}_{jm}\Box\chi^\alpha_{jm}\Bigg]\Bigg).$$

(We treat these cross terms perturbatively.)

★ 9. Details of SO(3) operators ★

$$\boldsymbol{L}^2_{★} \equiv -\frac{1}{\sin\theta}\partial_\theta\sin\theta\partial_\theta - \frac{1}{\sin^2\theta}\partial_\varphi^2$$

(the angular momentum operator)

$$\tilde{\boldsymbol{J}}^{(\alpha)2}_{★} \equiv \boldsymbol{L}^2 + 2\frac{\cos\theta}{\sin^2\theta}k_\alpha i\partial_\varphi + \frac{\cos^2\theta}{\sin^2\theta}k_\alpha^2$$

$\partial_\varphi \to \partial_\varphi - ik_\alpha\cos\theta$

Consider covariant derivative !

k_α is defined by $g\left[\langle A_\varphi\rangle, E_\alpha\right] = g\,(\mu\cdot\alpha)\,E_\alpha \equiv k_\alpha\ E_\alpha$.

Eigenfunctions / Eigenvalues

$$\boldsymbol{L}^2Y_{lm}(\theta,\varphi) = l(l+1)\ Y_{lm}(\theta,\varphi)$$

$l = 0, 1, 2, \cdots$
$m = -l, -l+1, \cdots, l$

$$\left(\tilde{\boldsymbol{J}}^{(\alpha)2} + k_\alpha^2\right)\mathcal{Y}_{k_\alpha jm}(\theta,\varphi) = j(j+1)\ \mathcal{Y}_{k_\alpha jm}(\theta,\varphi)$$

$|k_\alpha| = 0, 1, 2, \cdots$
$j = |k_\alpha|, |k_\alpha|+1, |k_\alpha|+2, \cdots$
$m = -j, -j+1, \cdots, j$

k_α: restricted by vanishing surface terms.

Summery

- We obtained gauge symmetry breaking in a higher-dimensional theory with S^2 as the extra dimensions. This breaking was induced by a background field originating from the curvature of S^2.