Revisiting Brans-Dicke Gravity

Scale invariance + Gravity ?

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Based on 2503.18648, 2506.01543
in collaboration w/ M.Hong & T.T.Yanagida

Framework of Standard Model + GR

- Quantum Field Theory ← SRelativity + Quantum

Speed of light $[\mathrm{L\,T^{-1}}]$

$c = 299792458\,\mathrm{m/s}$

Planck Constant $[\mathrm{M\,L^2\,T^{-1}}]$

$\hbar = 197.3269804\,\mathrm{MeV\,fm}\,c^{-1}$

- Gauge symmetry

$\mathrm{U}(1)_Y \times \mathrm{SU}(2)_W \times \mathrm{SU}(3)_c$

photon Z boson g'

$W^{\pm}$ boson g

gluon g_s

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Newton Constant $[\mathrm{M^{-1}\,L^3\,T^{-2}}]$

$G_\mathrm{N} = 6.70883 \times 10^{-39}\mathrm{GeV}^{-2}\,\hbar c$

- Gauge symmetry

$\mathrm{U}(1)_Y$	$\times$	$\mathrm{SU}(2)_W$	$\times$	$\mathrm{SU}(3)_c$	$\times$	Diffeo.
photon	Z boson	g'		gluon g_s		graviton G_N
$W^\pm$ boson		g				

Newton Constant

- Only the **ratio** matters

Physical quantity

$$\frac{m_\bullet}{M_{\rm Pl}} \quad \text{where} \quad M_{\rm Pl} \equiv \sqrt{\frac{\hbar c}{4\pi G_{\rm N}}}$$

e.g., Newton's gravity

$$F = G_{\rm N}\frac{m_1 m_2}{r^2} \propto \frac{m_1}{M_{\rm Pl}}\frac{m_2}{M_{\rm Pl}}$$

- Redundancy (or Symmetry?)

$$m_\bullet \mapsto \omega m_\bullet \quad M_{\rm Pl} \mapsto \omega M_{\rm Pl}$$

$$\boldsymbol{x} \mapsto \omega^{-1}\boldsymbol{x}$$

($* \ \hbar = c = 1$)

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(* $\hbar = c = 1$)

$$m_\bullet \equiv \lambda_\bullet \phi \quad M_{\rm Pl} \equiv \xi^{1/2}\phi$$

$$m_\bullet / M_{\rm pl} = \lambda_\bullet / \xi^{1/2}$$

Symmetry

$$\phi \mapsto \omega\phi \quad g_{\mu\nu} \mapsto \omega^{-2} g_{\mu\nu}$$

Field Dependent Newton Constant

- Detour: Brans–Dicke Gravity

$$\mathscr{L}_{\rm EH} = \frac{1}{2} M_{\rm Pl}^2 R$$

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Global Weyl symmetry

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Einstein frame $g_{\mu\nu} = \Omega^{-2} g^{\mathrm{E}}_{\mu\nu}, \quad \Omega^2 = \frac{\xi\phi^2}{M_*^2} = e^{\sqrt{\frac{2}{3}\frac{6\xi}{6\xi+1}}\frac{\chi}{M_*}}$ **excl.** $\boldsymbol{\phi = 0}$

$$\mathscr{L}_{\mathrm{BD,E}} = \frac{1}{2}M_*^2 R_{\mathrm{E}} + \frac{1}{2}g_{\mathrm{E}}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{\lambda_{\mathrm{cc}}}{4\xi}M_*^4$$

Shift symmetry

$$\chi \mapsto \chi + \mathrm{const}\,.$$

Dilaton: χ + **CC**: $\lambda_\Lambda/(4\xi)$

Fifth Force Constraints on Matter Coupling

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[Brans, Dicke 1961]

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- **Long-range force** from shift coupling (= global Weyl coupling)

  e.g., Higgs mass

$$m_{H,\mathrm{J}}^2 \left|H\right|^2 \xrightarrow{\text{E-frame}} m_{H,\mathrm{J}}^2\,\Omega^{-2}\left|H_{\mathrm{E}}\right|^2$$

$$g_{\mu\nu} = \Omega^{-2}g_{\mu\nu}^{\mathrm{E}}$$

$$H = \Omega\, H_{\mathrm{E}}$$

$$\Omega^2 = e^{\sqrt{\frac{2}{3}\frac{6\xi}{6\xi+1}}\frac{\chi}{M_*}}$$

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mixing

$g_{\mu\nu} = \Omega^{-2} g_{\mu\nu}^{\mathrm{E}}$

$H = \Omega \, H_{\mathrm{E}}$

$\Omega^2 = e^{\sqrt{\frac{2}{3}\frac{6\xi}{6\xi+1}}\frac{\chi}{M_*}}$

$\langle H_E^2 \rangle = v_*^2/2$

$\langle \chi \rangle = \chi_0$

$m_{H*}^2 = m_{H,\mathrm{J}}^2 \, e^{-\sqrt{\frac{2}{3}\frac{6\xi}{6\xi+1}}\frac{\chi_0}{M_*}}$

χ_0 must be tuned s.t. $\dfrac{m_{H*}}{M_*} = \dfrac{m_H}{M_{\mathrm{Pl}}}$

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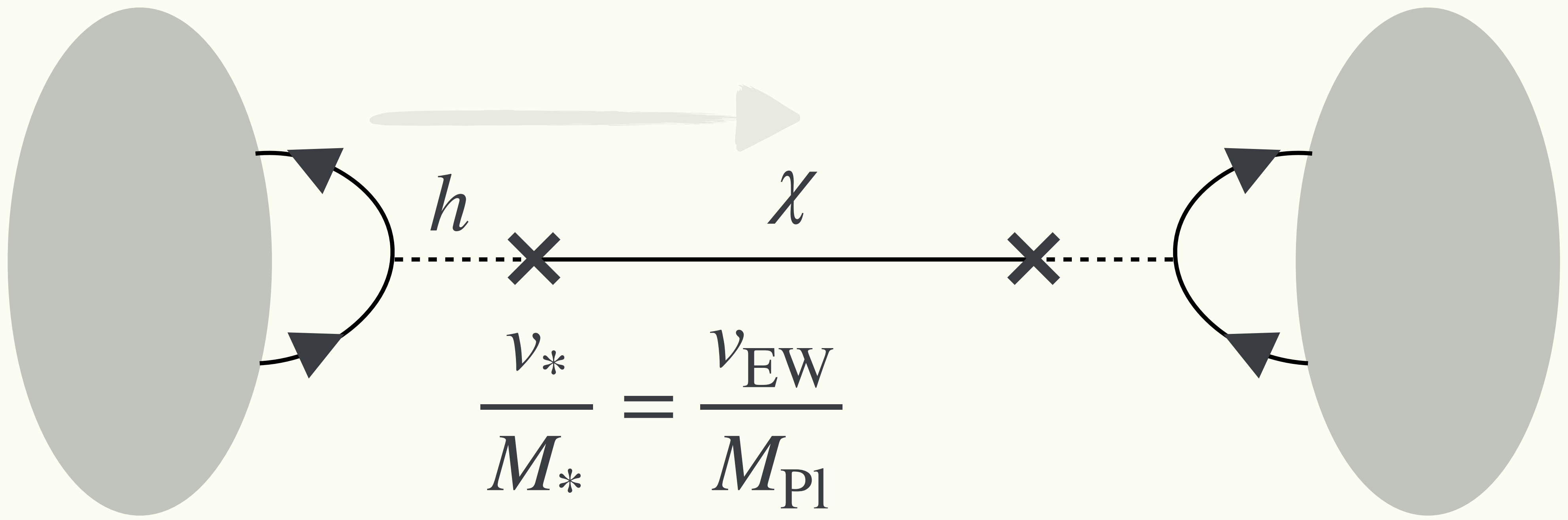

$$\propto \xi \frac{v_{\mathrm{EW}}^2}{M_{\mathrm{Pl}}^2}\frac{1}{r}$$

Fifth force constraint

solar system tests [Will '14; Adelberger+ '09]

$$\xi \lesssim 10^{-4}$$

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Scale Invariance + Brans–Dicke Gravity

- **Classical** scale invariance

[e.g., Strumia+ 2015]

$$\mathscr{L}_{\mathrm{BD,J}} = \frac{1}{2}\xi\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{4}\lambda_{\mathrm{cc}}\phi^4 + \mathscr{L}_{\mathrm{mat}}\Big|_{m_\bullet \to \lambda_{m_\bullet}\phi}$$

Global Weyl symmetry

$\Phi \mapsto \omega^{n_\Phi}\Phi$ for **all** Φ

- All mass parameters ($m_\bullet$) of $\mathscr{L}_{\mathrm{mat}}$ replaced with ϕ

Higgs mass

$$m^2_{H,\mathrm{J}}\,|H|^2 \dashrightarrow \lambda^2_{m_H}\phi^2\,|H|^2$$

$n_\Phi = 2\times 1 \quad 2\times 1$

Majorana RHN mass

$$M_i\bar{N}_i N_i \dashrightarrow \lambda_M\,\phi\,\bar{N}_i N_i$$

$n_\Phi = 1 \quad 2\times 3/2$

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- Quantum-ness introduces additional **mass-scales**

  From the perspective of **cutoff (Λ)** e.g. [Hamada+ '16, Hong, **KM**, Yanagida '25]

dimensionless

$$S_{\mathrm{cl}}\left[\{\Phi\};\{\lambda_\bullet\}\right] \longrightarrow S_{\mathrm{q}}\left[\{\Phi\};\{\lambda_\bullet\},\Lambda\right]$$

inv. under $\Phi \mapsto \omega^{n_\Phi}\Phi$

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$$\Omega = e^{\sqrt{\frac{\xi}{6\xi+1}}\frac{\chi}{M_*}}$$

Beta func.

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e.g., gauge coupling

$$\frac{\chi}{M_*}\left(\Lambda\frac{\partial S_{\mathrm{q}}}{\partial\Lambda}\right)_{\lambda_\bullet^{\mathrm{ren}}} \supset \frac{\chi}{M_*}\frac{\beta_g}{2g}F^a_{\mu\nu}F^{a\mu\nu} + \cdots$$

e.g., Top Yukawa coupling

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[Hong, **KM**, Yanagida '25]

From the perspective of **dim. reg.** (μ)

$S_{\mathrm{q}}\left[\{\Phi\};\{\lambda_\bullet\mu^{n_\bullet\epsilon}\}\right]$ w/ $\epsilon = \dfrac{4-d}{2}$

~~Global Weyl~~

[Englert+ 1976 ;Shaposhnikov, Zenhausern '09; Kugo '20; ...]

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- Resolution to **Quantum** scale invariance [Englert+ 1976 ;Shaposhnikov, Zenhausern '09]

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Global Weyl symmetry

$\Phi \mapsto \omega^{n_\Phi}\Phi$ for **all** Φ

- All mass-scales, **including**, e.g., **cutoff** Λ, replaced with ϕ

From the perspective of **cutoff (Λ)** e.g. [Hamada+ '16, Hong, **KM**, Yanagida '25]

$$S_{\mathrm{q}}\left[\{\Phi\};\{\lambda_\bullet\},\Lambda\right] \xrightarrow{\Lambda \to \lambda_\Lambda\phi} S_{\mathrm{q}}\left[\{\Phi\};\{\lambda_\bullet\},\lambda_\Lambda\phi\right]$$

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Cutoff scale

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[Englert+ 1976 ;Shaposhnikov, Zenhausern '09; Kugo '20; ...]

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No-scale Brans–Dicke Gravity

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Einstein frame $\quad g_{\mu\nu} = \Omega^{-2} g^{\mathrm{E}}_{\mu\nu}, \quad \Omega^2 = \frac{\xi\phi^2}{M_*^2} = e^{\sqrt{\frac{2}{3}\frac{6\xi}{6\xi+1}}\frac{\chi}{M_*}}$ **excl.** $\boldsymbol{\phi = 0}$

$$\mathscr{L}_{\mathrm{BD,E}} = \frac{1}{2}M_*^2 R_{\mathrm{E}} + \frac{1}{2}g_{\mathrm{E}}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{\lambda_{\mathrm{cc}}}{4\xi}M_*^4 + \mathscr{L}_{\mathrm{mat}}\Big|_{m_\bullet \to \lambda_{m_\bullet}\xi^{-1/2}M_*}$$

Cutoff scale $\Lambda \to \lambda_\Lambda \xi^{-1/2} M_*$

Shift symmetry

$\chi \mapsto \chi + \mathrm{const}\,.$

[Hong, **KM**, Yanagida '25]

No-scale Brans–Dicke Gravity

- Cosmological implications
  - No-scale BD gravity + SM + RHN

$$\mathscr{L}_{\rm BD,\,J} = \alpha R^2 + \frac{\xi\phi^2 + 2\xi_H |H|^2}{2} R + \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + g^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H + \frac{1}{2}\bar{N}_i i\partial\cdot\gamma N_i$$
$$- \frac{\lambda_H}{4}\left(|H|^2 - \lambda_v^2\phi^2\right)^2 - \frac{1}{4}\lambda_{\rm cc}\phi^4 - \frac{1}{2}\lambda_M\phi\bar{N}_i N_i + \mathscr{L}_{\rm SM'}$$

Inflation & Stability of EW vacuum

Dark Energy

Leptogenesis & **Dark Matter**

Dark Radiation

No-scale Brans–Dicke Gravity

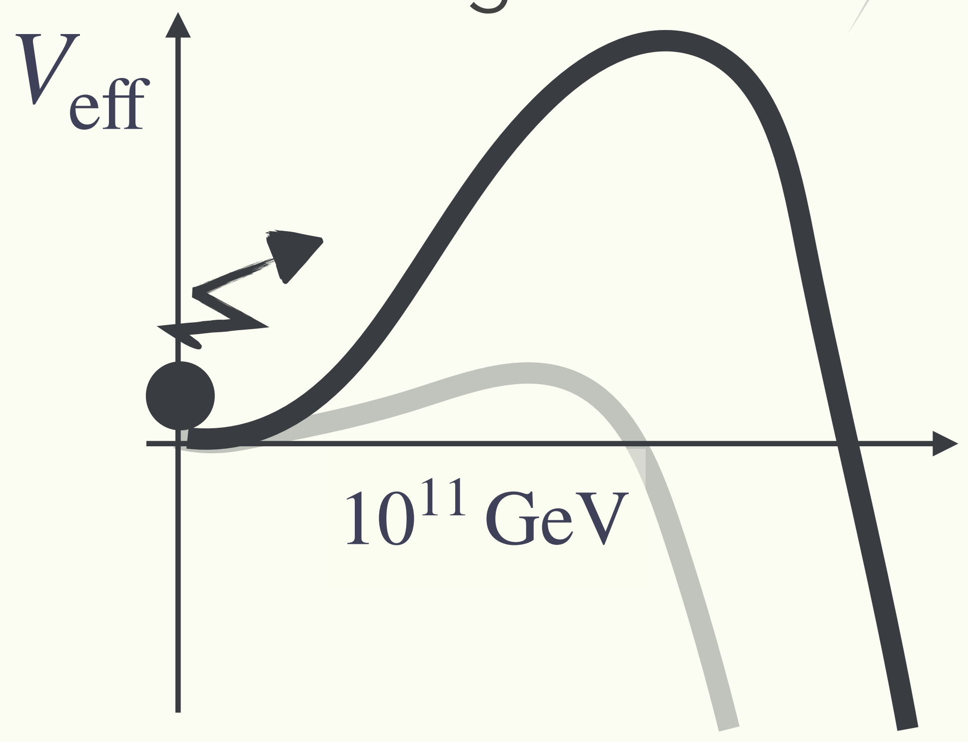
- Cosmological implications – **Inflation**
  - **Starobinsky-like** inflation

$$\mathscr{L}_{\mathrm{BD,J}} \supset \alpha R^2 + \frac{\xi\phi^2 + 2\xi_H |H|^2}{2} R + \frac{1}{2}(\partial\phi)^2 + |\partial H|^2 - \frac{\lambda_H}{4}\left(|H|^2 - \lambda_v^2\phi^2\right)^2 + \mathscr{L}_{\mathrm{SM}'}$$

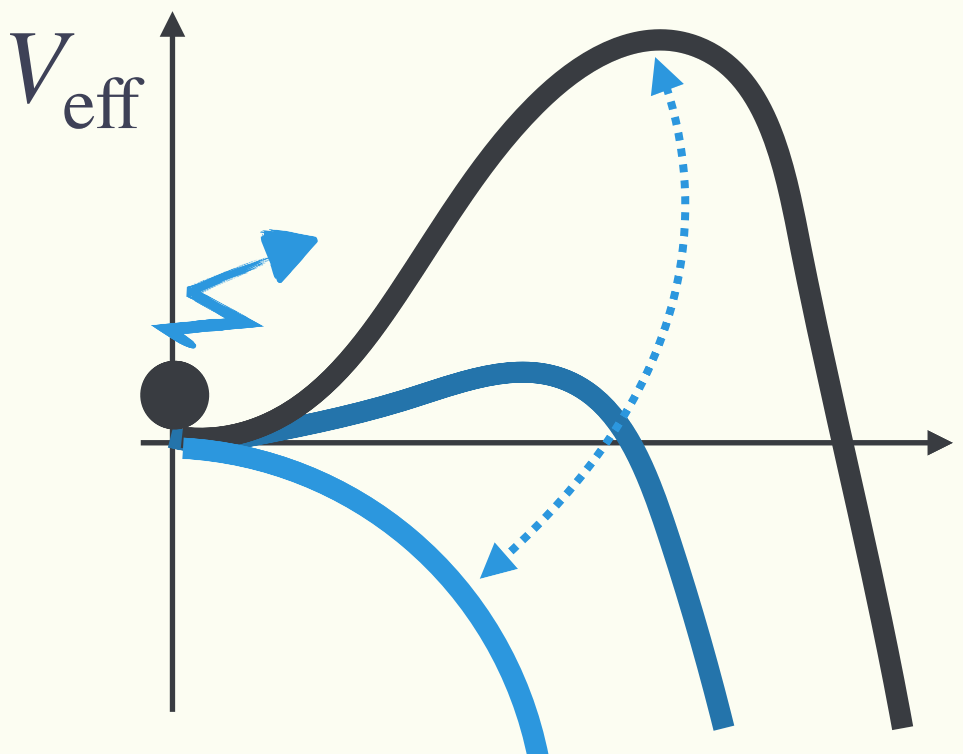
Stabilize EW vacuum: $-0.1 \gtrsim \xi_H \gtrsim -1.7$

[**KM**+ '16; Moroi+ '22]

During inflation



After inflation



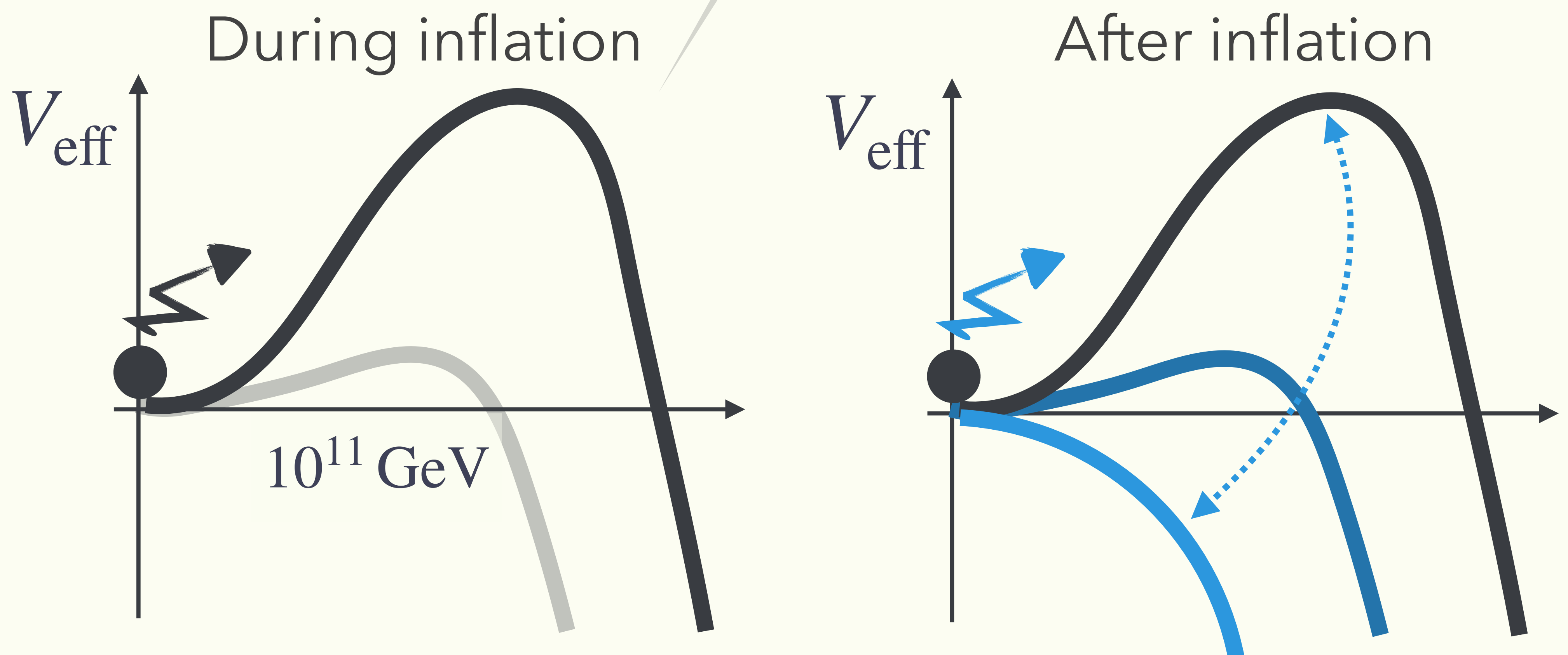
No-scale Brans–Dicke Gravity

- Cosmological implications – **Inflation**
  - **Starobinsky-like** inflation

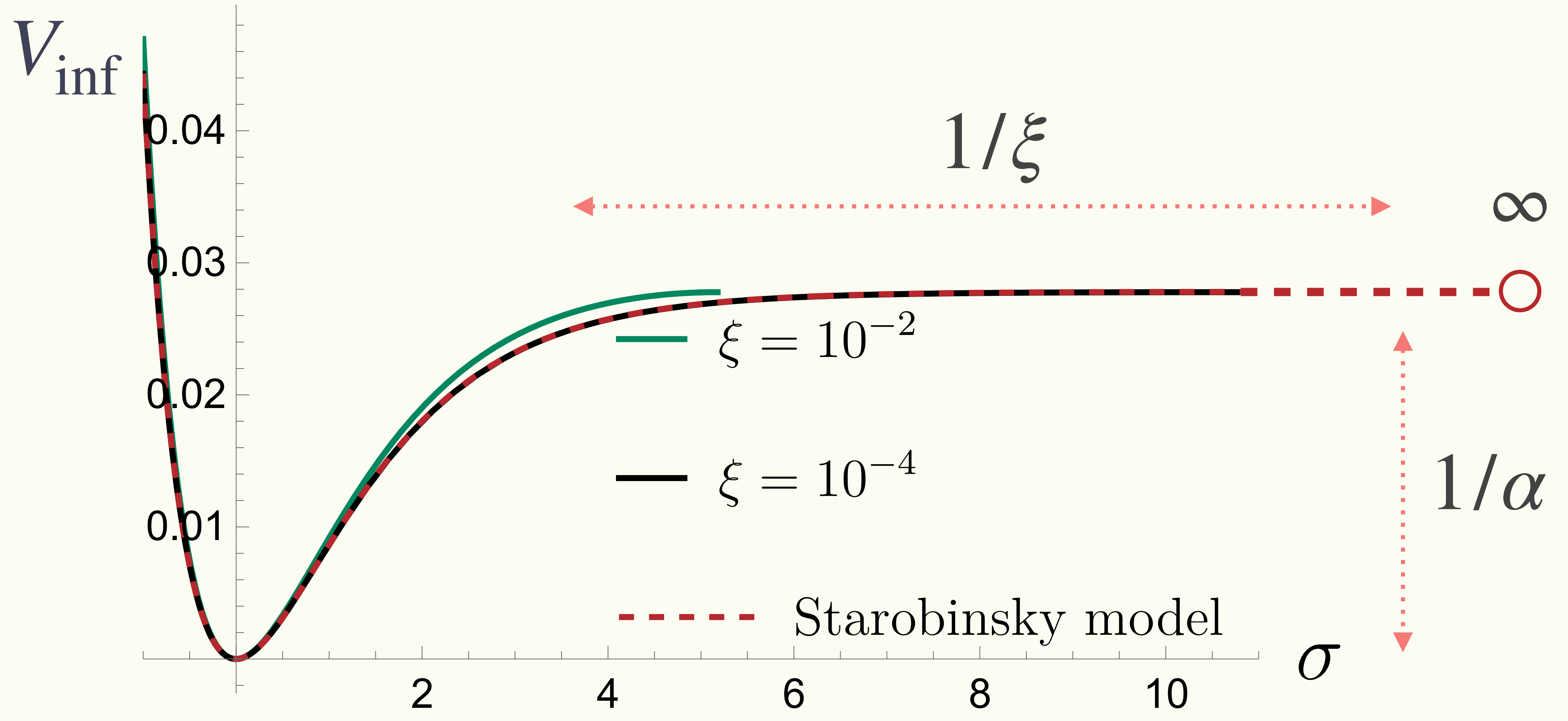
$$\mathscr{L}_{\mathrm{BD,J}} \supset \alpha R^2 + \frac{\xi\phi^2 + 2\xi_H |H|^2}{2} R + \frac{1}{2}(\partial\phi)^2 + |\partial H|^2 - \frac{\lambda_H}{4}\left(|H|^2 - \lambda_v^2\phi^2\right)^2 + \mathscr{L}_{\mathrm{SM}'}$$

Stabilize EW vacuum: $-0.1 \gtrsim \xi_H \gtrsim -1.7$

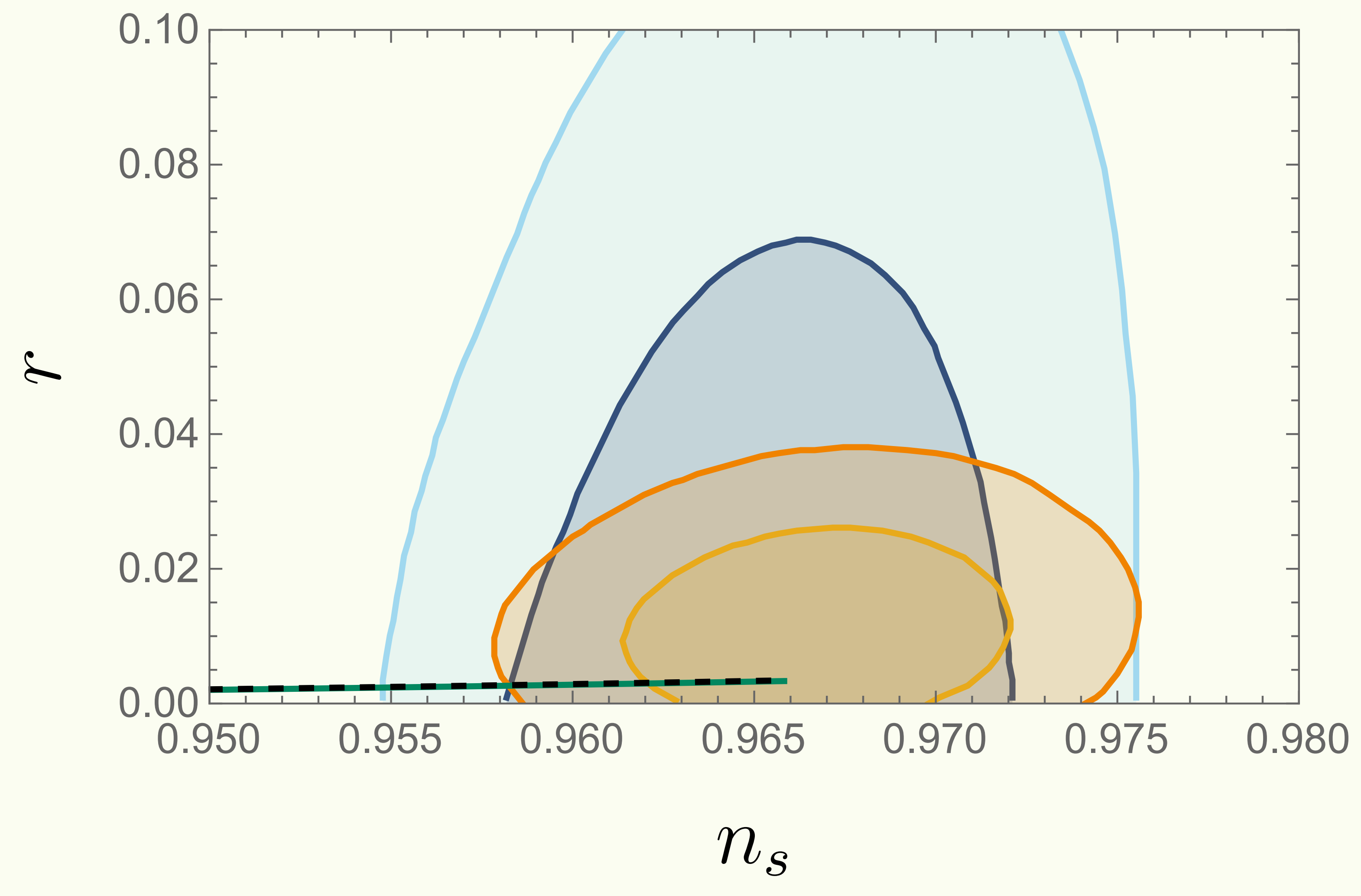
[**KM**+ '16; Moroi+ '22]

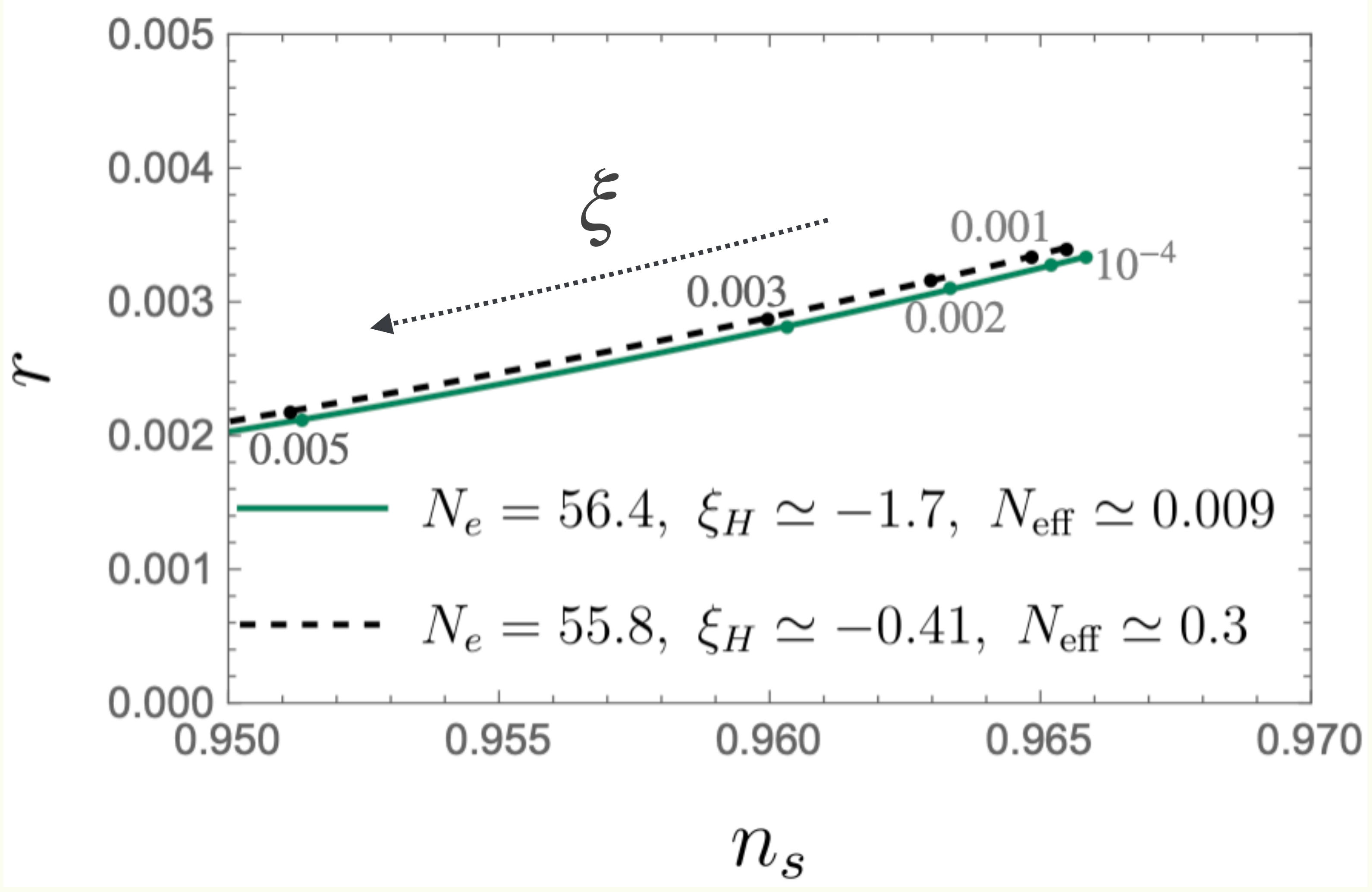


Starobinsky-like inflation: $\xi \lesssim 10^{-3}$

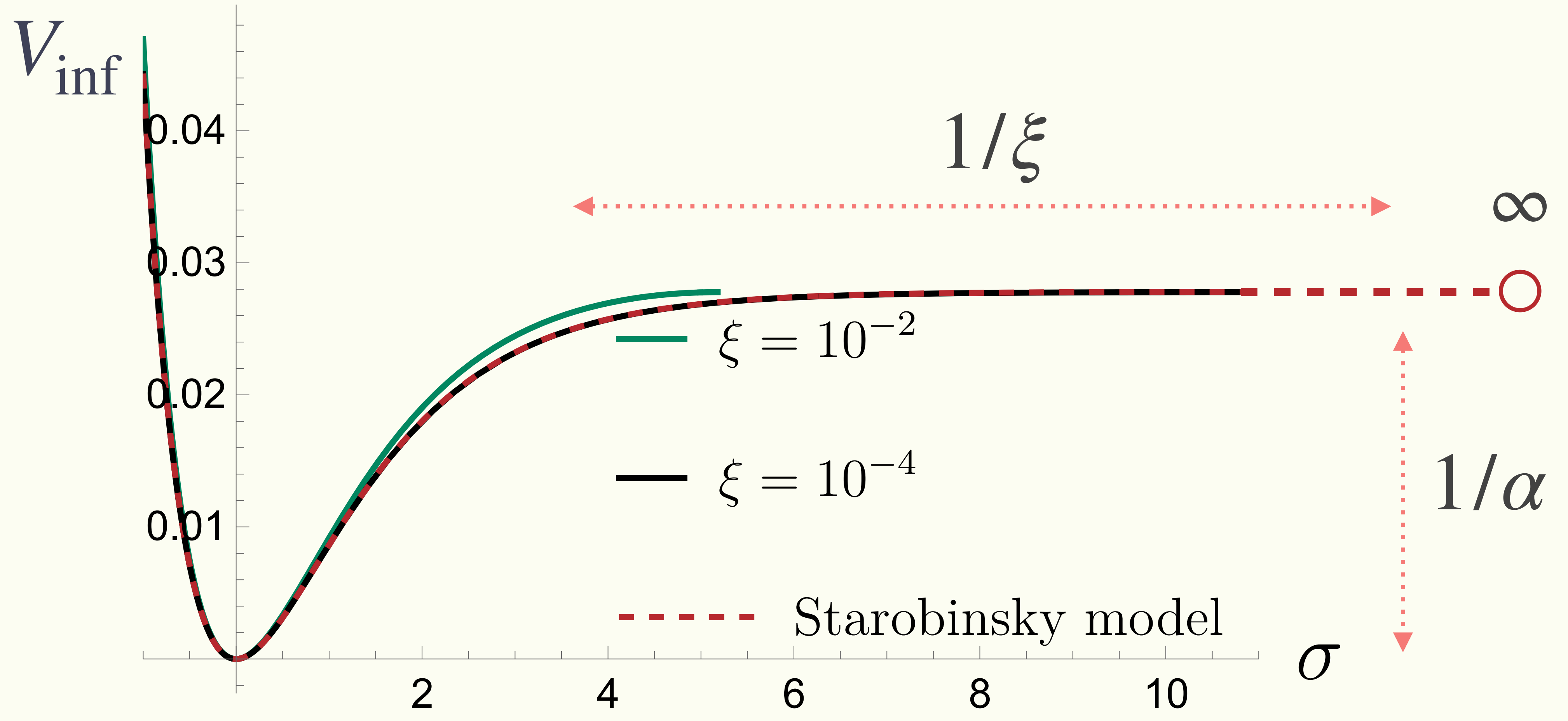


No-scale Brans–Dicke Gravity





Starobinsky-like inflation: $\xi \lesssim 10^{-3}$



No-scale Brans–Dicke Gravity

- Cosmological implications – Inflation, **Reheating & DM**
  - Reheating after Starobinsky-like inflation

$$\mathscr{L}_{\rm BD,J} \supset \alpha R^2 + \frac{\xi\phi^2 + 2\xi_H |H|^2}{2} R + \frac{1}{2}(\partial\phi)^2 + |\partial H|^2 - \frac{1}{2}\lambda_M \phi \bar{N}_i N_i + \cdots$$

Inflaton to **Higgs**

$$\Gamma_{\sigma\to HH^\dagger} \simeq (6\xi_H + 1)^2 \frac{m_\sigma^3}{48\pi M_{\rm Pl}^2}$$

Reheating temperature

$$T_{\rm R} \simeq 3\times 10^{10}{\rm GeV} \left|\xi_H + \frac{1}{6}\right|$$

Inflaton to **RHN**

$$\Gamma_{\sigma\to N_i N_i} \simeq \frac{1}{96\pi}\frac{m_\sigma^3}{M_{\rm Pl}^2}\left(\frac{M_i^2}{m_\sigma^2}\right)\left(1 - \frac{4M_i^2}{m_\sigma^2}\right)^{3/2}$$

Sterile Neutrino **Dark Matter**

$$\Omega_{N_1} h^2 \simeq 0.16 \left|\xi_H + \frac{1}{6}\right|^{-1}\left(\frac{M_1}{10^{-6} m_\sigma}\right)^3$$

***Stability of DM** → $m_{\nu_1} \simeq 0$

Inflaton to **Dilaton**

$$\Gamma_{\sigma\to\chi\chi} \simeq \frac{m_\sigma^3}{192\pi M_{\rm Pl}^2}$$

Dark Radiation

$$\Delta N_{\rm eff} \simeq 0.02 \left(\xi_H + \frac{1}{6}\right)^{-2}$$

[Hong, **KM**, Yanagida '25]

No-scale Brans–Dicke Gravity

- Cosmological implications – Inflation, Reheating & DM, **Leptogenesis**
  - Stability of EW vacuum & **Leptogenesis**

$$\mathscr{L}_{\mathrm{BD,J}} \supset \alpha R^2 + \frac{\xi\phi^2 + 2\xi_H |H|^2}{2} R + \frac{1}{2}(\partial\phi)^2 + |\partial H|^2 - \frac{1}{2}\lambda_M \phi \bar{N}_i N_i + \cdots$$

Inflaton to **Higgs**

$$\Gamma_{\sigma\to HH^\dagger} \simeq (6\xi_H + 1)^2 \frac{m_\sigma^3}{48\pi M_{\mathrm{Pl}}^2}$$

Reheating temperature

$$T_{\mathrm{R}} \simeq 3\times 10^{10}\mathrm{GeV}\left|\xi_H + \frac{1}{6}\right|$$

Stabilize EW vacuum

$$-0.1 \gtrsim \xi_H \gtrsim -1.7$$

Thermal Leptogenesis via N_2 & N_3

$$\frac{n_B}{s} \simeq 10^{-10}\left(\frac{\kappa}{0.1}\right)\left(\frac{M_2}{10^{10}\mathrm{GeV}}\right)\left(\frac{m_{\nu_3}}{0.05\mathrm{eV}}\right)\delta_{\mathrm{eff}}$$

(*Inverted-Hierarchy → $\delta_{\mathrm{eff}} \leqslant \frac{\Delta m^2_{\mathrm{sol}}}{2\Delta m^2_{\mathrm{atm}}} \simeq 0.015$

→ resonant, non-thermal LG,…)

Summary

- Newton constant may depend on field (**BD gravity**)
- Long-range fifth force is absent w/ scale symmetry
- Scale symmetry (J) = Shift symmetry (E)
- **No-scale BD gravity** = Quantum scale invariance + gravity
- **SM** + **RHN** based on **no-scale BD gravity** is presented