Bootstrap for Cosmology & Gravity

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Kitano-san gave me a title “S-matrix with gravity.”

Katsuki told me that this workshop should be for brainstorming.

So, I decided to talk about what I want to work on right now:

Toward S-matrix bootstrap in de Sitter spacetime.

I will also mention some on flat space S-matrix w/gravity in this context.

Contents

The history of particle physics says

that fundamental principles (ex. unitarity, analyticity) are useful

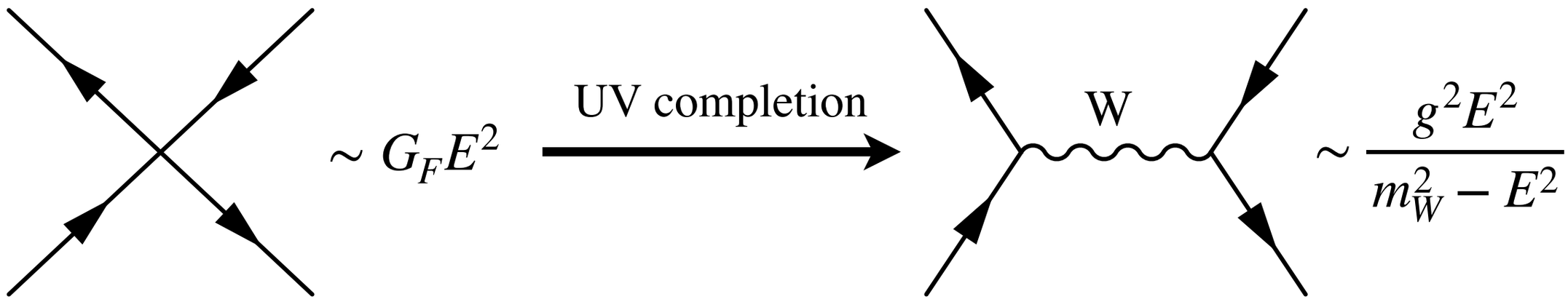
to predict new physics required for UV completion

Unitarity and New Physics

Unitarity bounds on scattering amplitudes ($0 \leq$ probability ≤ 1):

For 4pt scattering in 4D, $|\mathscr{M}| \lesssim E^0$ at high energy $E \to \infty$.

ex. Weak bosons unitarize Fermi interactions

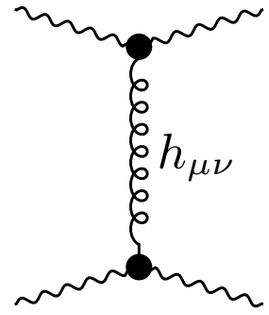


ex. Higgs boson unitarizes weak boson scattering

Analyticity makes it more powerful: Birth of String Theory

Strings from S-matrix (1/2)

4pt scattering of massless scalars in Einstein gravity



$$M_{\rm GR}(s,t) = \frac{1}{2M_{\rm Pl}^2}\frac{s^4+t^4+u^4}{stu}\,.$$

- massless spin-2 pole: $M_{\rm GR}(s,t) = \frac{1}{M_{\rm Pl}^2}\frac{-t^2}{s} + \mathcal{O}(s^0),\ (s+t+u=0).$

- high energy behavior

1) Regge limit (fixed momentum transfer t)

$M_{\rm GR}(s,t) \sim \frac{1}{2M_{\rm Pl}^2}\frac{-s^2}{t}$, violating the Froissart-Martin bound ($|M| < s^2$).

2) hard scattering limit (fixed scattering angle θ)

$M_{\rm GR}(s,t) \sim \frac{E^2}{M_{\rm Pl}^2}$, violating the unitarity bound ($|M| < 1$).

Q. Can we improve UV behavior respecting unitarity & analyticity?

Strings from S-matrix (2/2)

4pt scattering of massless scalars in type II superstring

$$M_{\text{string}}(s,t) = \frac{1}{2M_{\text{Pl}}^2}\frac{s^4+t^4+u^4}{stu} \times \frac{\Gamma(1-\frac{\alpha' s}{4})\Gamma(1-\frac{\alpha' t}{4})\Gamma(1-\frac{\alpha' u}{4})}{\Gamma(1+\frac{\alpha' s}{4})\Gamma(1+\frac{\alpha' t}{4})\Gamma(1+\frac{\alpha' u}{4})}.$$

- reproduce GR amplitude at IR ($|\alpha' s|, |\alpha' t| \ll 1$).

- analytic on the complex s-plane except s, u-poles on the real axis.

※ each pole contains *a finite number* of higher spin resonances

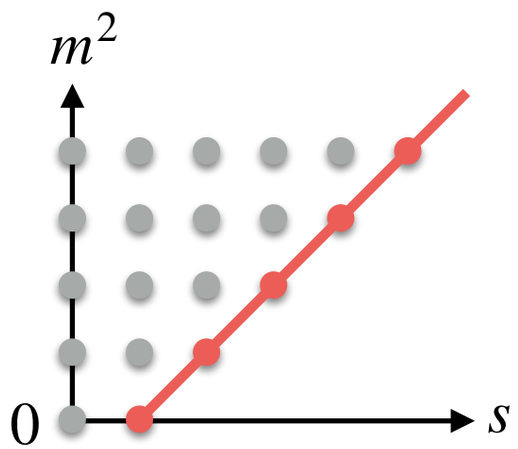
identified w/excitation modes of the string. → string theory!

- mild UV behavior

(1) Regge: $M_{\text{string}}(s,t) \sim s^{2+\frac{\alpha'}{2}t}$ (compatible w/FM bound)

(2) hard: $M_{\text{string}}(s,t) \sim \exp\left[-\frac{\alpha'}{2}(s\ln s + t\ln t + u\ln u)\right]$

$\sim \exp\left[-(\text{wolrdsheet area})\right]$



Regge trajectory

In this manner,

unitarity & analyticity of scattering amplitudes were used to connect UV & IR.

※ They are historical examples for **S-matrix bootstrap** in high-energy physics.

※ More recent progress is mentioned in the next section.

Contents

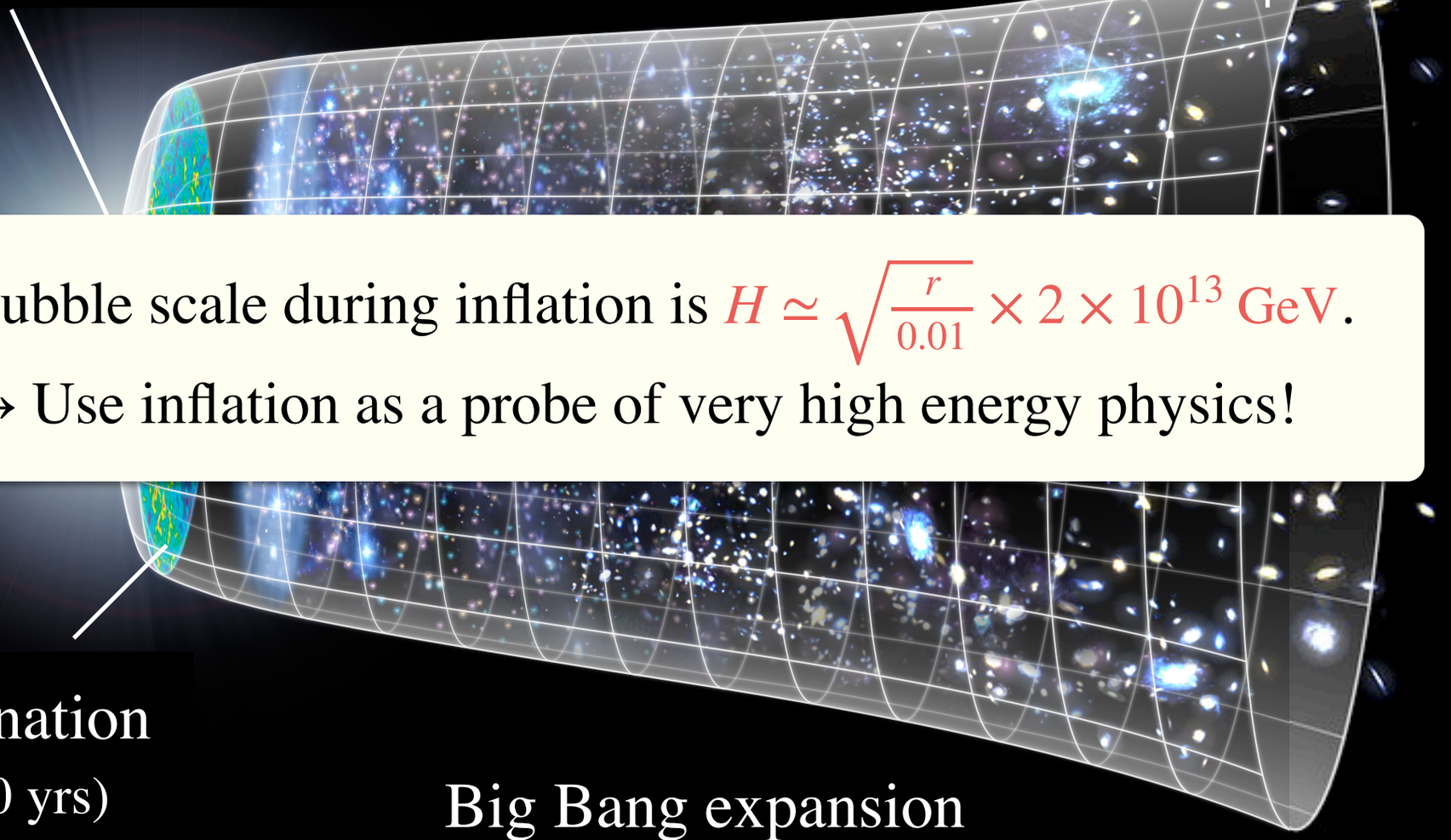
Two motivations for de Sitter spacetime:

- Inflation as a probe of high-energy physics

- de Sitter spacetime vs quantum gravity

Cosmic Inflation:
accelerated expansion of the early universe

accelerated expansion
driven by dark energy



Hubble scale during inflation is $H \simeq \sqrt{\frac{r}{0.01}} \times 2 \times 10^{13}\ \mathrm{GeV}$.

→ Use inflation as a probe of very high energy physics!

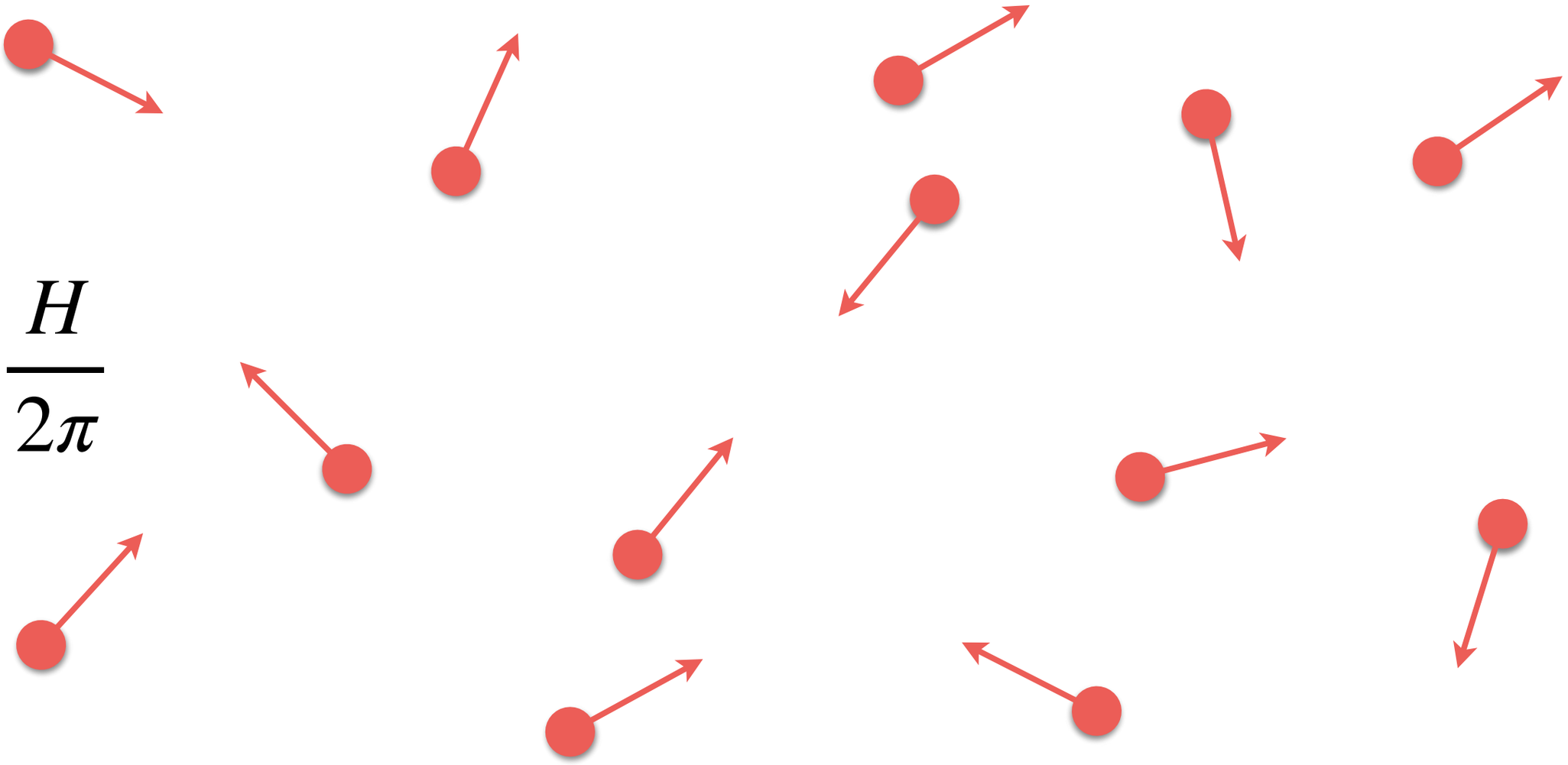
recombination
(400,000 yrs)

Big Bang expansion
(14 billions yrs)

past → now

Inflationary Universe = Thermal Bath

$$T = \frac{H}{2\pi}$$



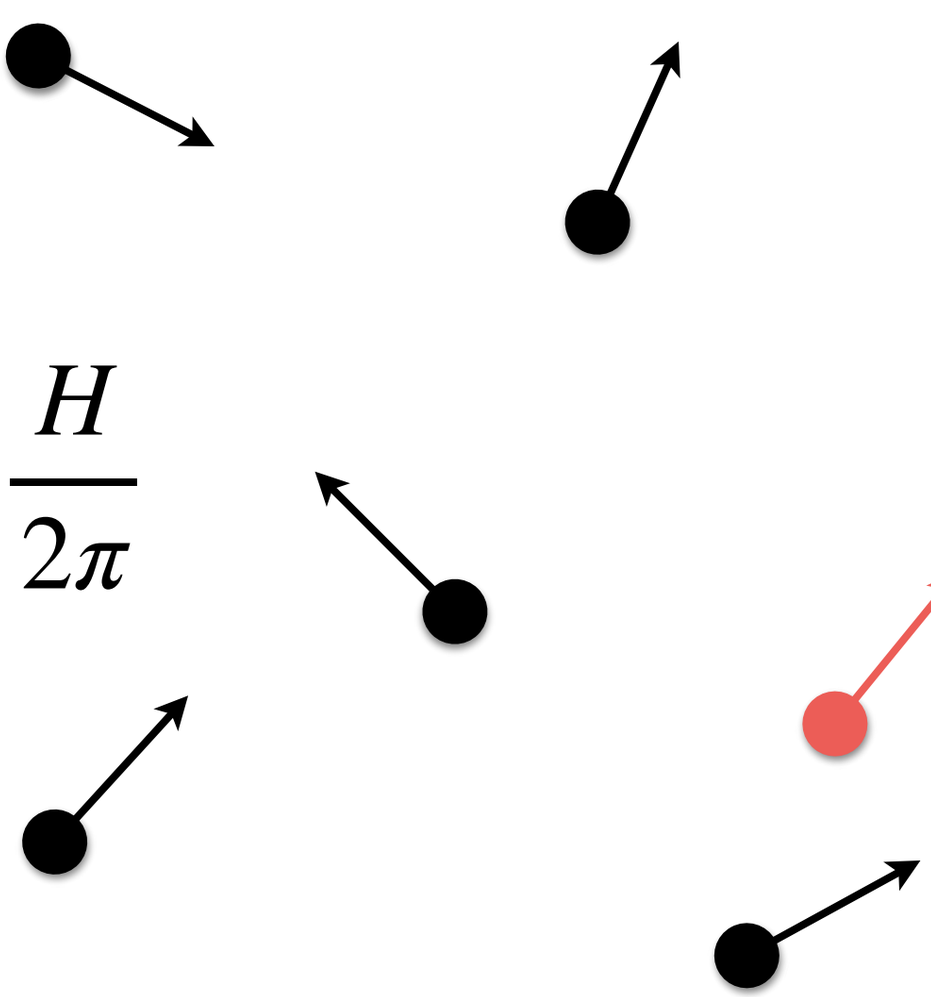
The inflationary universe is full of very high-energy particles!

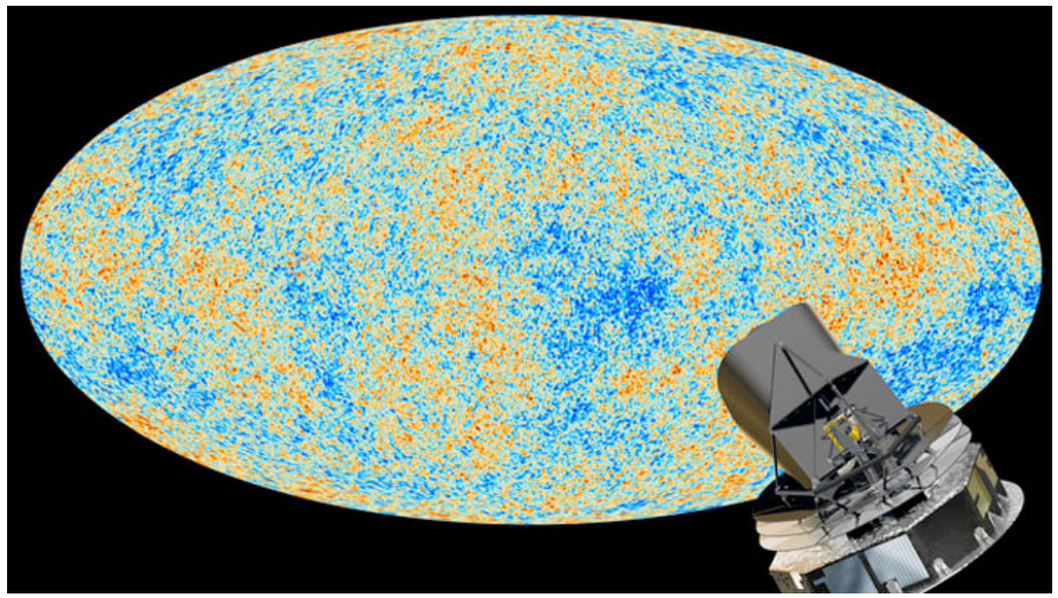
※ Recall $H \simeq \sqrt{\frac{r}{0.01}} \times 2 \times 10^{13}$ GeV for early universe inflation.

※ Primordial fluctuations are nothing but the thermal fluctuation.

Cosmological Collider Program

$$T = \frac{H}{2\pi}$$





Inflation is a natural particle collider "**Cosmological Collider**"!

→ Imprints of new particles on cosmological correlators.

[Chen-Wang '10, Baumann-Green '12, **TN**-Yamaguchi-Yokoyama '13, ArkaniHamed-Maldacena '15, …]

In the past 10 years+, inflation as a probe of HEP has been studied based on **analogy w/collider experiments & scattering amplitudes**.

So far, it has been well studied
how to explore new particles near the Hubble scale (e.g., in SUGRA)
by analogy with resonance signals at ordinary particle colliders.

Q. Can we probe physics above the Hubble scale? (ex. GUT, extra dim)

cf. weak bosons from fermi interactions

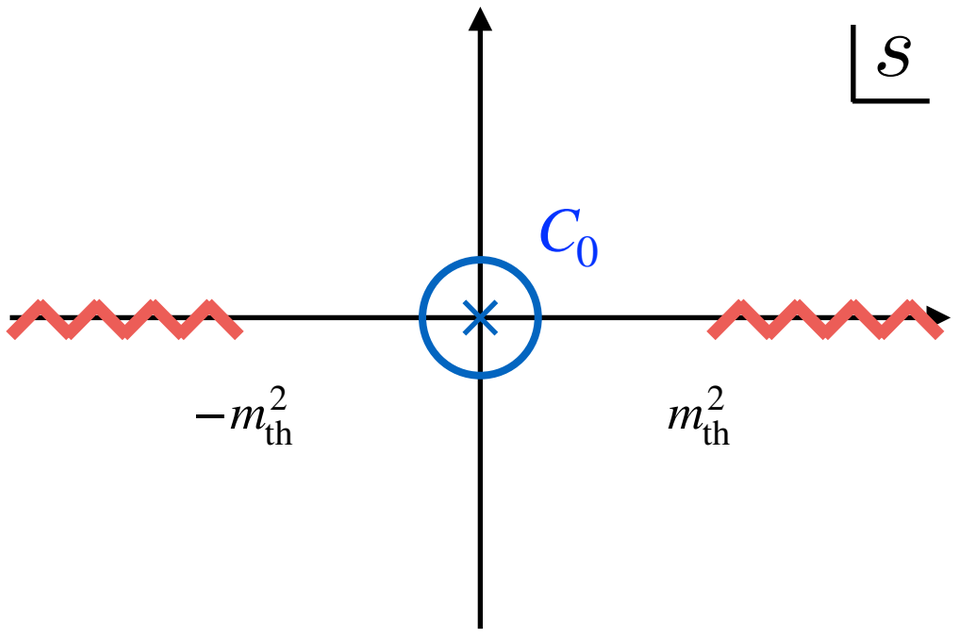
$$H \simeq \sqrt{\frac{r}{0.01}} \times 2 \times 10^{13}\ \mathrm{GeV}$$

On flat spacetime,

dispersion relations that follow from unitarity & analyticity can be used to obtain UV information from IR effective theory.

Warmup: Dispersion relations of forward amplitudes

Consider an s-u symmetric scattering amplitude $\mathscr{M}(s,t)$ in the forward limit.



analytic structure of $\mathscr{M}(s,t=0)$

IR expansion in the forward limit:

$$\mathscr{M}(s,t=0) = \sum_n a_{2n}\, s^{2n},$$

$$a_{2n} = \oint_{C_0} \frac{ds}{2\pi i} \frac{\mathscr{M}(s,t=0)}{s^{2n+1}}.$$

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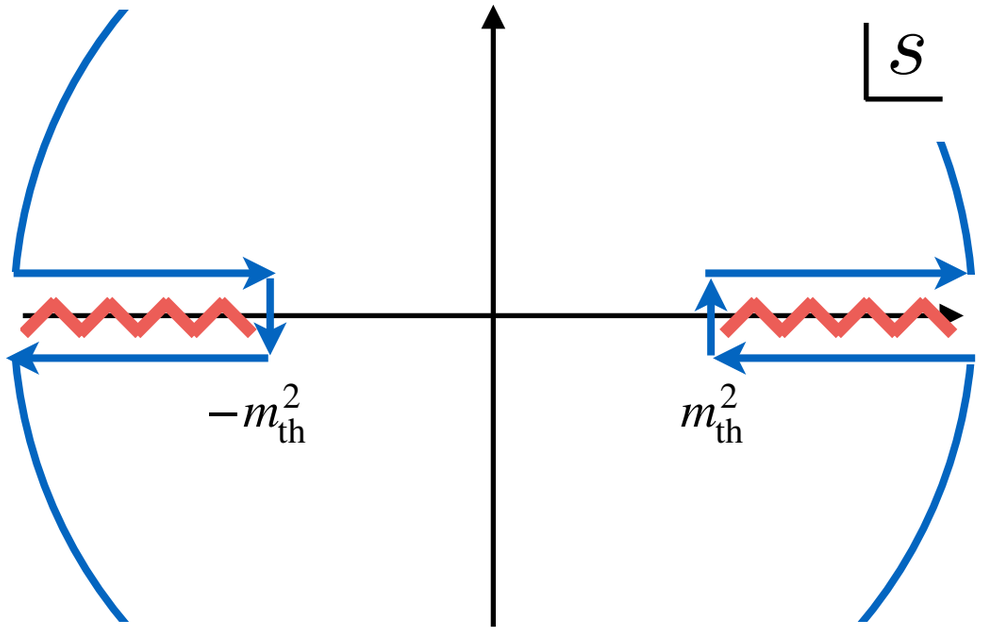

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Deform the integration contour to rewrite it in the UV language:

$$a_{2n}=\frac{2}{\pi}\int_{m_{\text{th}}^2}^{\infty}ds\frac{\text{Im}\mathscr{M}(s,t=0)}{s^{2n+1}}+\oint_{C_\infty}\frac{ds}{2\pi i}\frac{\mathscr{M}(s,t=0)}{s^{2n+1}}.$$

※ used the s-u symmetry and $\text{Disc}\,\mathscr{M}(s,t=0)=2i\,\text{Im}\,\mathscr{M}(s,t=0)$

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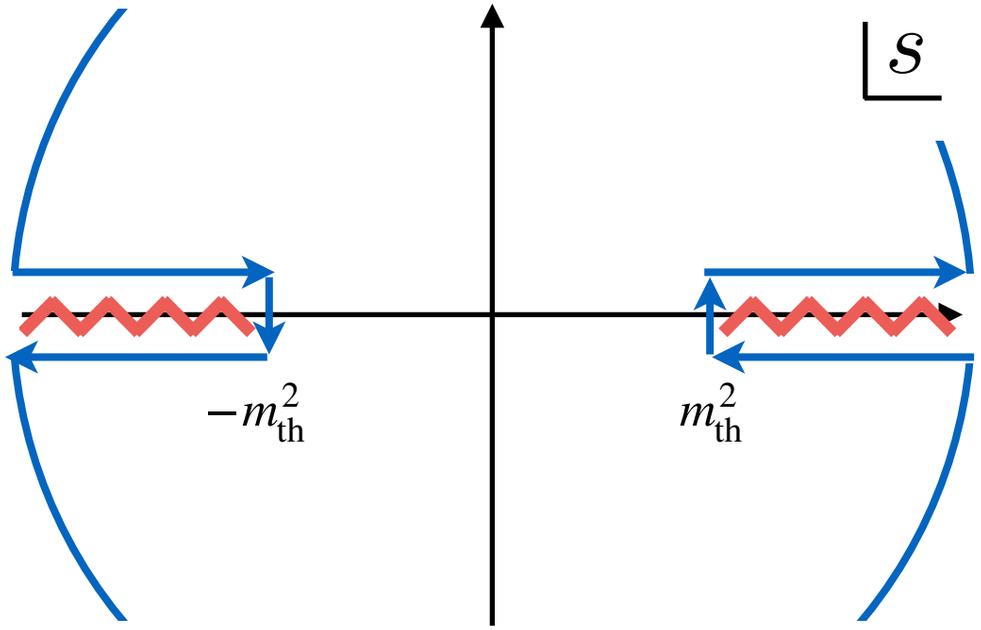

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If $|\mathscr{M}(s,t=0)|<|s|^{2n}$

Deform the integration contour to rewrite it in the UV language:

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※ used the s-u symmetry and Disc $\mathscr{M}(s,t=0)=2i\,{\rm Im}\,\mathscr{M}(s,t=0)$

In local gapped theories, unitarity implies $|\mathscr{M}(s,t=0)| < s\ln^2 s \quad (s\to\infty)$.

(Froissart bound)

This leads to the following dispersion relation and universal positivity [Adams et al '06]:

$$a_{2n} = \frac{2}{\pi}\int_{m_{\rm th}^2}^{\infty} ds \frac{{\rm Im}\mathscr{M}(s,t=0)}{s^{2n+1}} \geq 0 \qquad \text{for} \qquad 2n = 2,4,\ldots$$

Positive because of unitarity!

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha\left(\partial_\mu\phi\partial^\mu\phi\right)^2 + \cdots$$

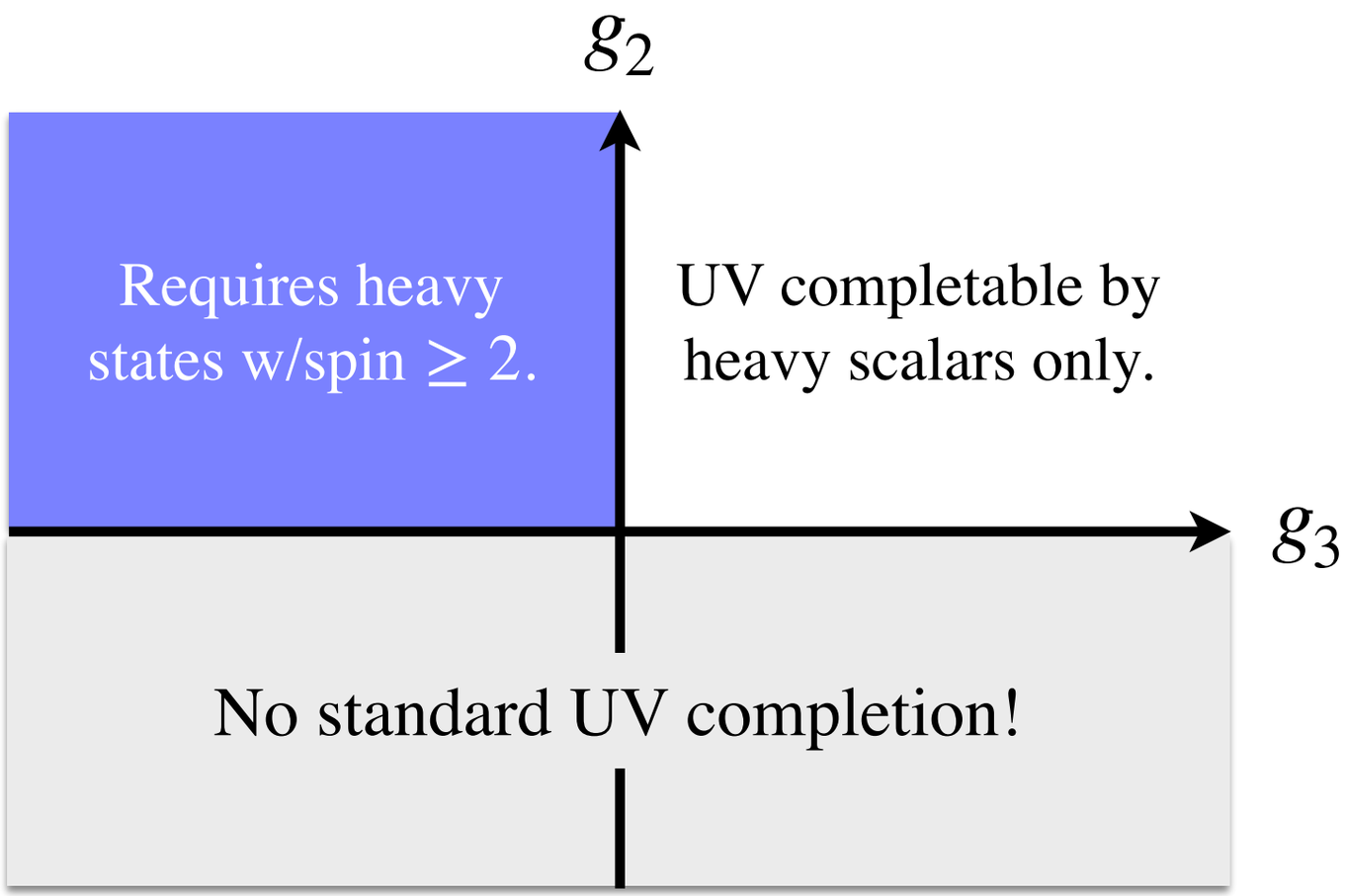
$$\alpha, \alpha_1, \alpha_2 \geq 0$$

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_1\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \alpha_2\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2 + \ldots$$

Extension to non-forward amplitude can be used to read off **spins** of UV spectra

[Kim-Takeuchi-**TN**-Zhou ’19]

Extension to non-forward amplitudes



ex. IR expansion of 4pt scattering of identical massless scalars

$$\mathscr{M}(s,t) = g_2\,(s^2+t^2+u^2) + g_3\,stu + g_4\,(s^2+t^2+u^2)^2 + \cdots$$

Dispersion relations imply that g_2, g_4 are universally positive, whereas sign of g_3 depends on spins of intermediate UV states.

→ **spins of UV states from signs of IR coefficients!**

Potential application to EFT of inflation:

KK graviton vs heavy scalars in GUT from sign of non-Gaussianities!

※ Extension to QFT on de Sitter spacetime is required.

※ Extension to Lorentz symmetry broken phase is required for $f_{\rm NL} \gtrsim 1$.

Two motivations for de Sitter spacetime:

- Inflation as a probe of high-energy physics

- de Sitter spacetime vs quantum gravity

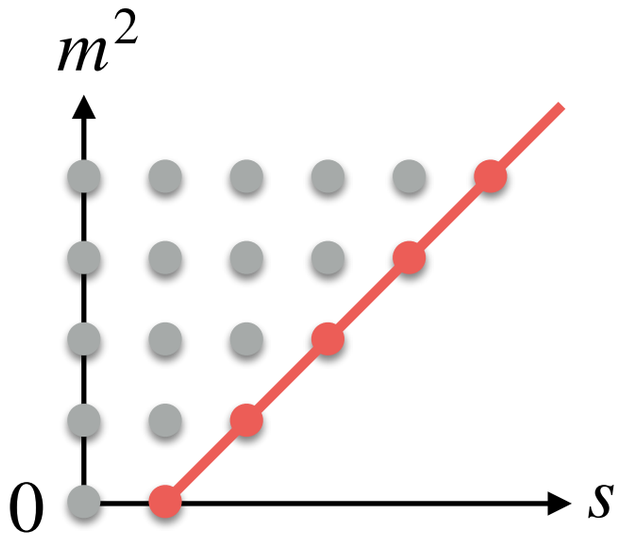
Q. Can we discover “string theory” on de Sitter spacetime similarly to the history of “Strings from S-matrix”?

See, e.g., a nice review [Danielsson-Van Riet ’18] for proposals & debates on dS construction in string theory.

To motivate this question, let me advertise my old work [**TN**-Takeuchi-Zhou '19]

on **string Regge trajectories on de Sitter spacetime**.

Regge trajectory on flat spacetime



string theory on flat spacetime:

- higher spin Regge states are crucial for UV completion

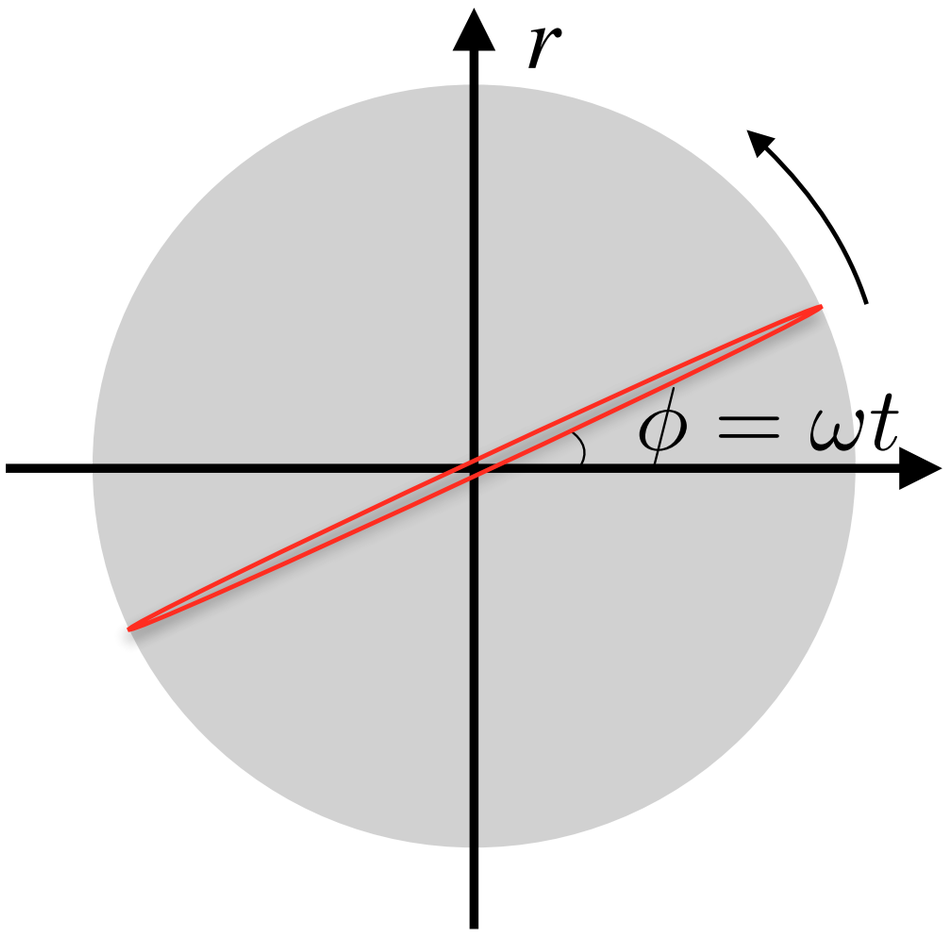
- leading Regge trajectory is linear: $\alpha' m^2 = s - 2$

For large spin, the spectrum on the leading Regge trajectory

can be studied by classical rotating folded strings! → next slide

Rotating folded string on flat spacetime

[ex. Zwiebach's textbook]



radius: $r = \omega^{-1}$

Boundaries of the Nambu-Goto string propagate with the speed of light.

- string tension vs centrifugal force
- causality implies string speed ≤ 1
- bound is saturated at boundaries

energy: $E = \dfrac{r}{\alpha'}$, angular momentum: $S = \dfrac{r^2}{2\alpha'}$

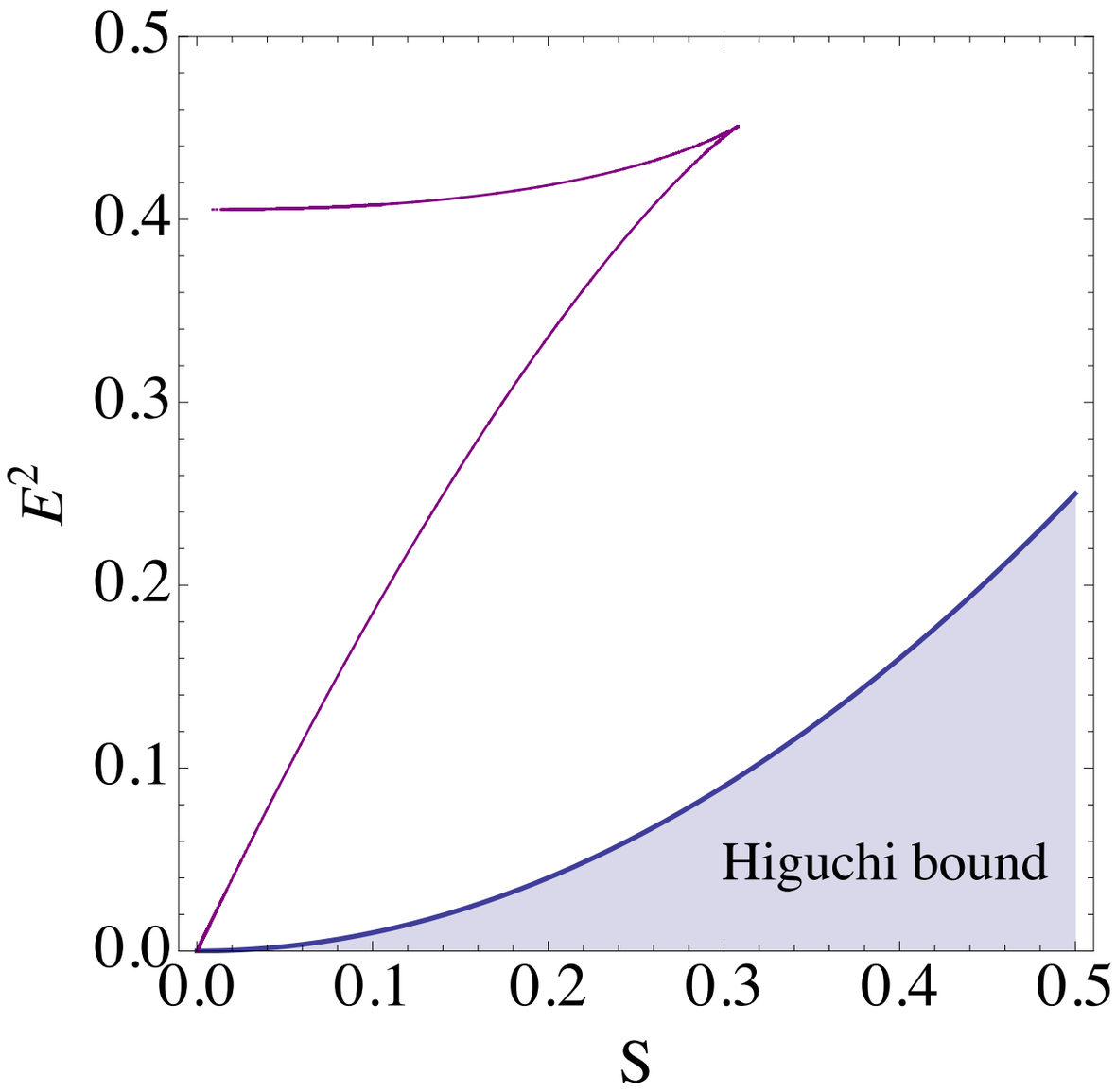
$\rightarrow$ linear Regge trajectory $E^2 = \dfrac{2}{\alpha'} S$

In [**TN**-Takeuchi-Zhou '19],

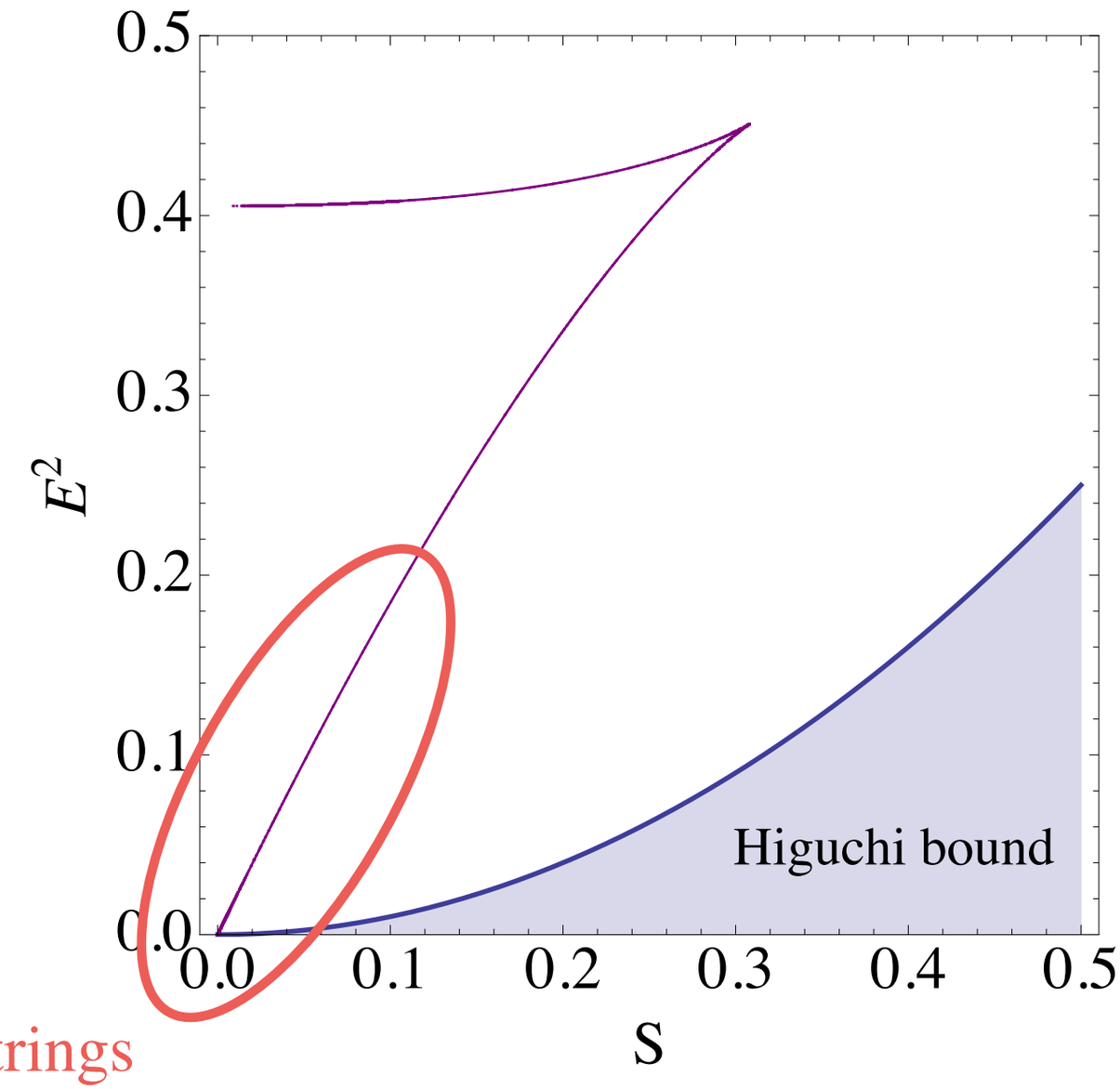
we studied the Regge trajectory of folded strings on de Sitter spacetime,

extending analysis in AdS spacetime (called GKP string in AdS/CFT context).

Regge trajectory on de Sitter spacetime



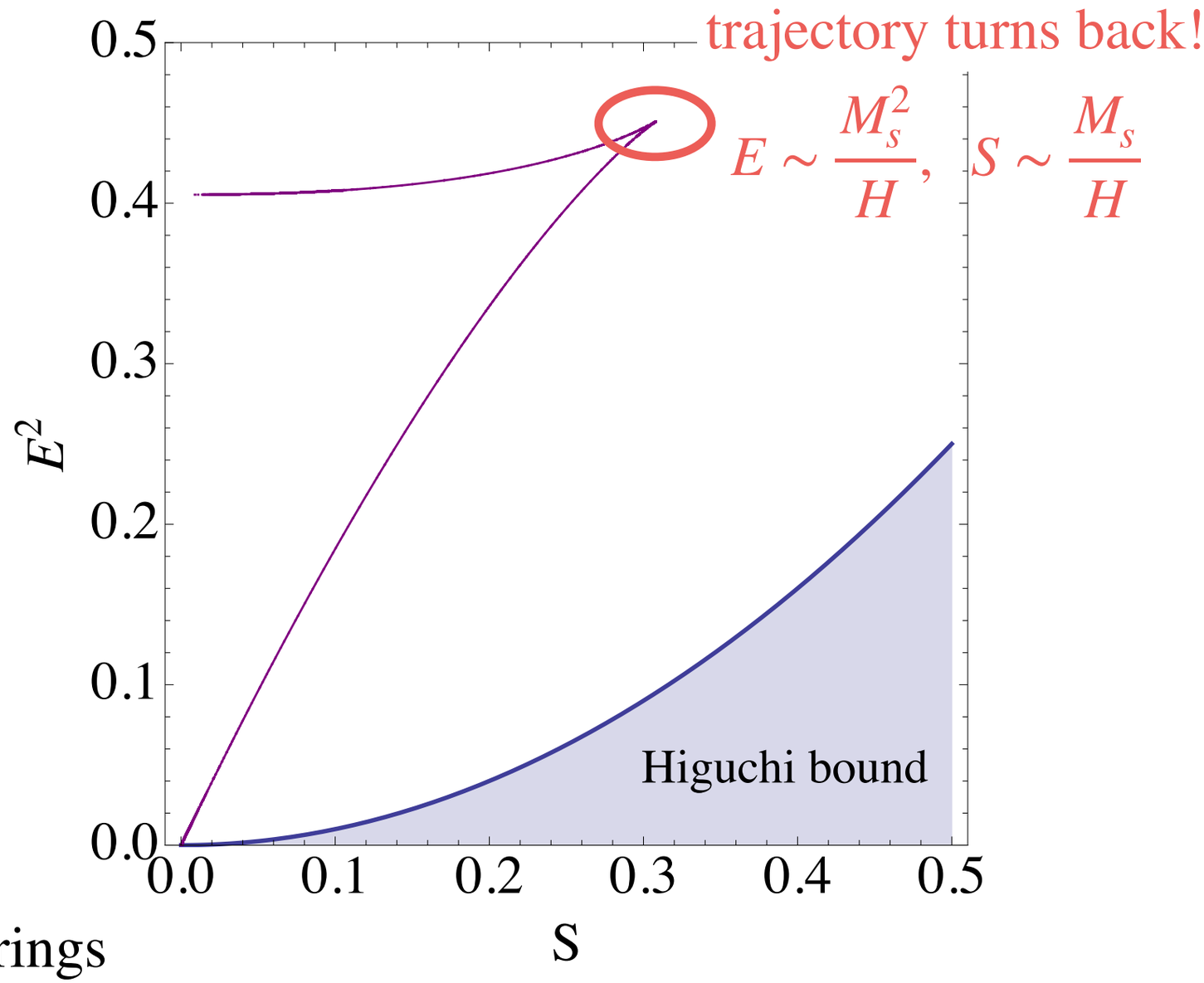
Regge trajectory on de Sitter spacetime



large ω = short strings

→ linear Regge trajectory of flat space

Regge trajectory on de Sitter spacetime



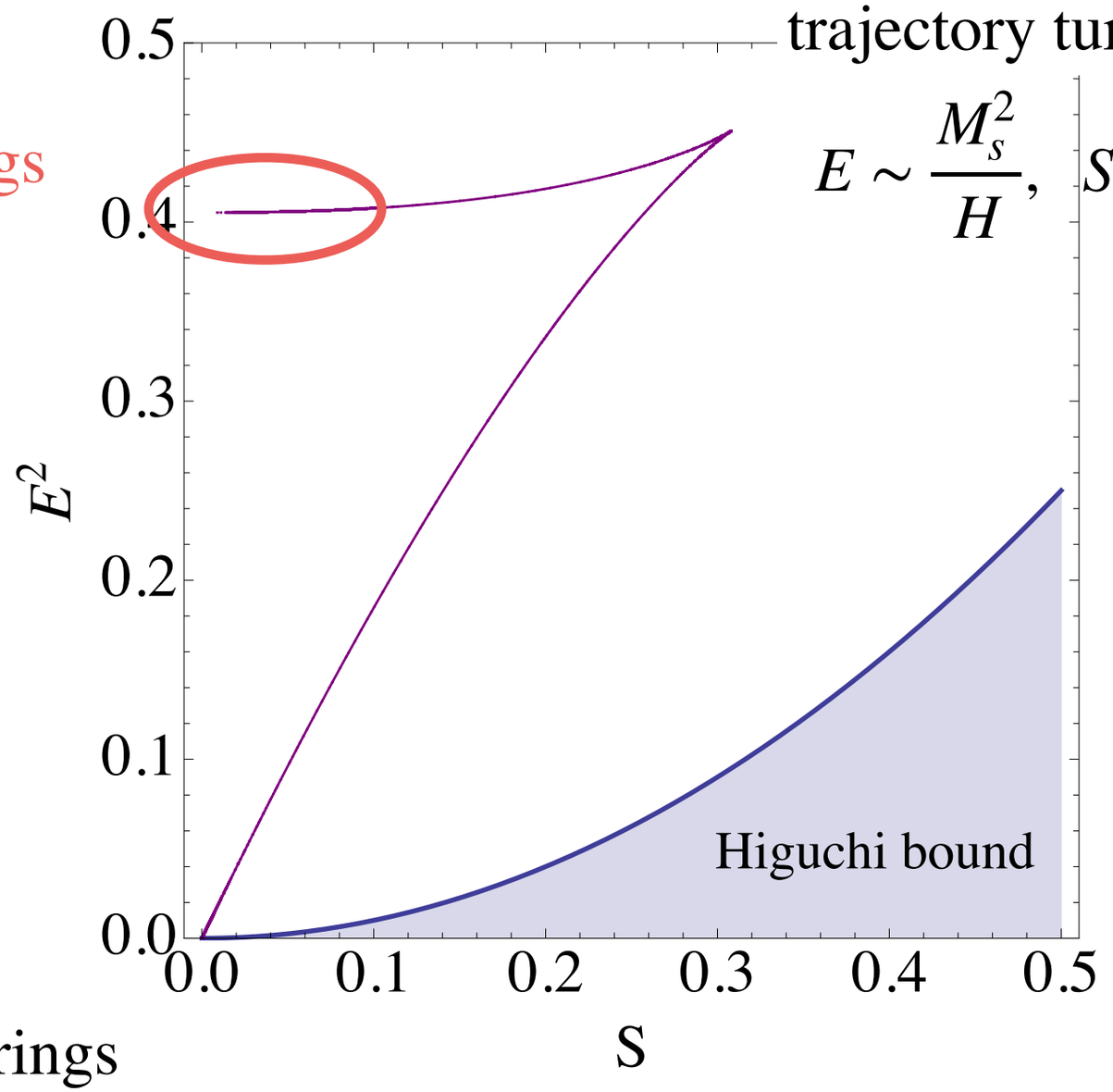
large ω = short strings

→ linear Regge trajectory of flat space

Regge trajectory on de Sitter spacetime

small ω = long strings

E = finite, $S = 0$



trajectory turns back!

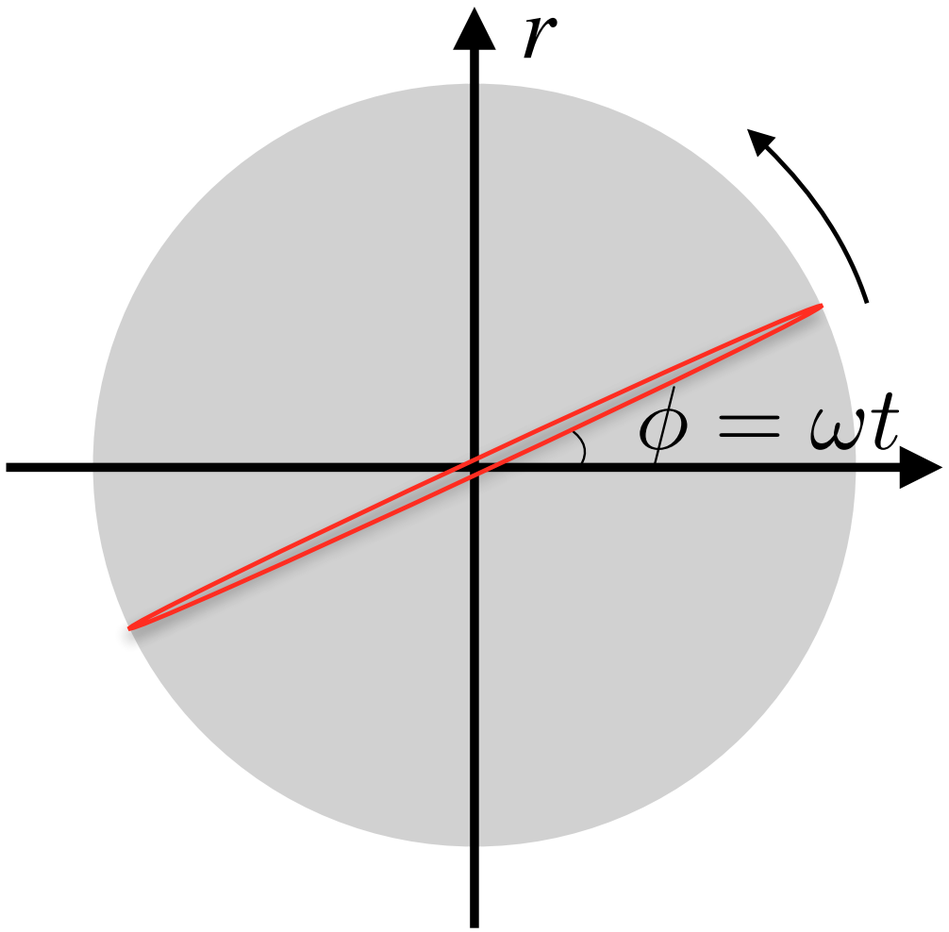
$$E \sim \frac{M_s^2}{H}, \quad S \sim \frac{M_s}{H}$$

large ω = short strings

→ linear Regge trajectory of flat space

small ω region: horizon and causality

small ω region: horizon and causality



in the small ω limit, we find $r \to H^{-1}$

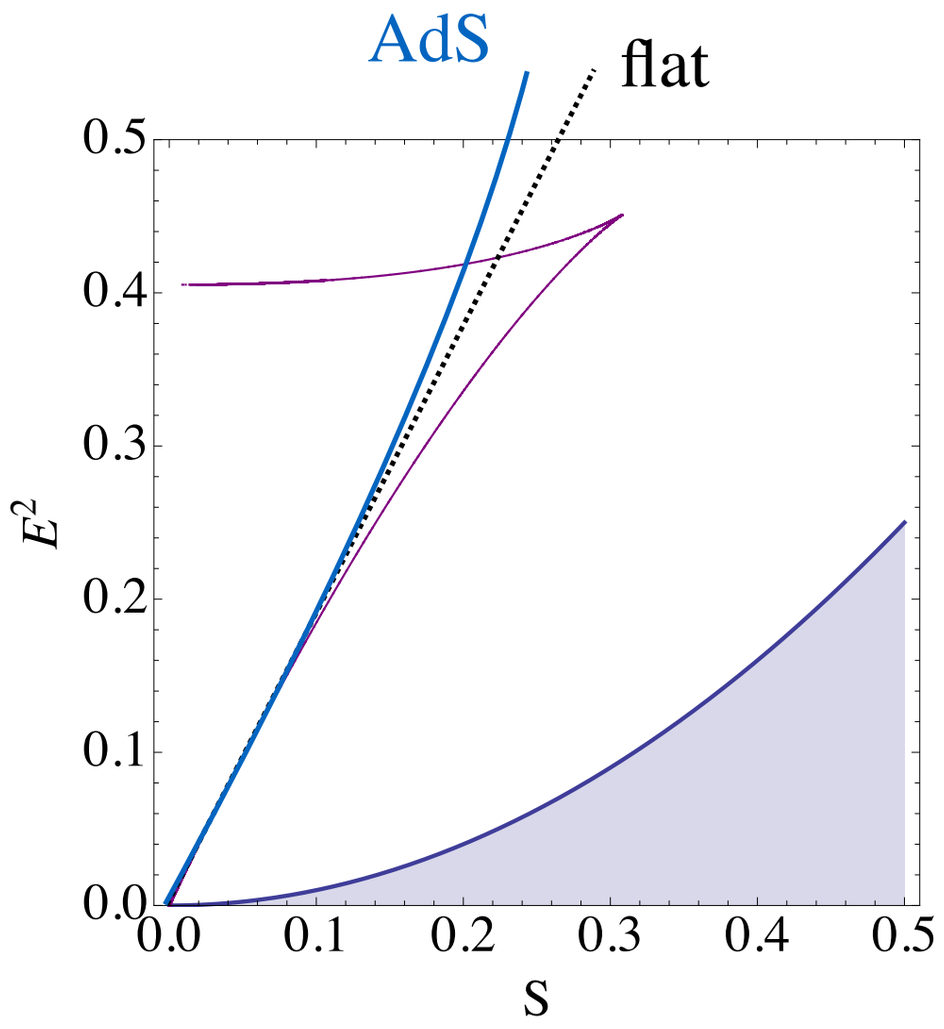
※ boundaries touch the horizon!

when boundaries touching the horizon,

causality requires $\omega = 0$ to avoid superluminal propagation

$\to E =$ finite, $\quad S = 0$. This is an origin of the maximal spin.

How UV completion of gravity is achieved in dS?



Turning point appears below the 4D Planck scale,

if the condition $M_s^4 \lesssim M_{\rm Pl}^2 H^2 \sim V_{\rm inf}$ is satisfied.

cf. $V_{\rm inf} \simeq \frac{r}{0.01} \times \left(10^{16}\,{\rm GeV}\right)^4$ for inflation.

Regge trajectory on dS is qualitatively different from flat & AdS ($E^2 \sim S^2$).

- Is the stringy UV completion achieved in the standard manner?

- Are there any constraints on dS in string theory/quantum gravity?

cf. conservative bound: dS temp < Hagedorn temp $M_s < H$.

Q. What are consistent “stringy scattering amplitudes” in dS?

cf. progress in S-matrix bootstrap on flat space

- finite energy sum rule for gravitational Regge amplitudes

[de Rham-Jaitly-Tolley ’22, **TN**-Tokuda ’22]

- bootstrap Regge trajectory in large N QCD

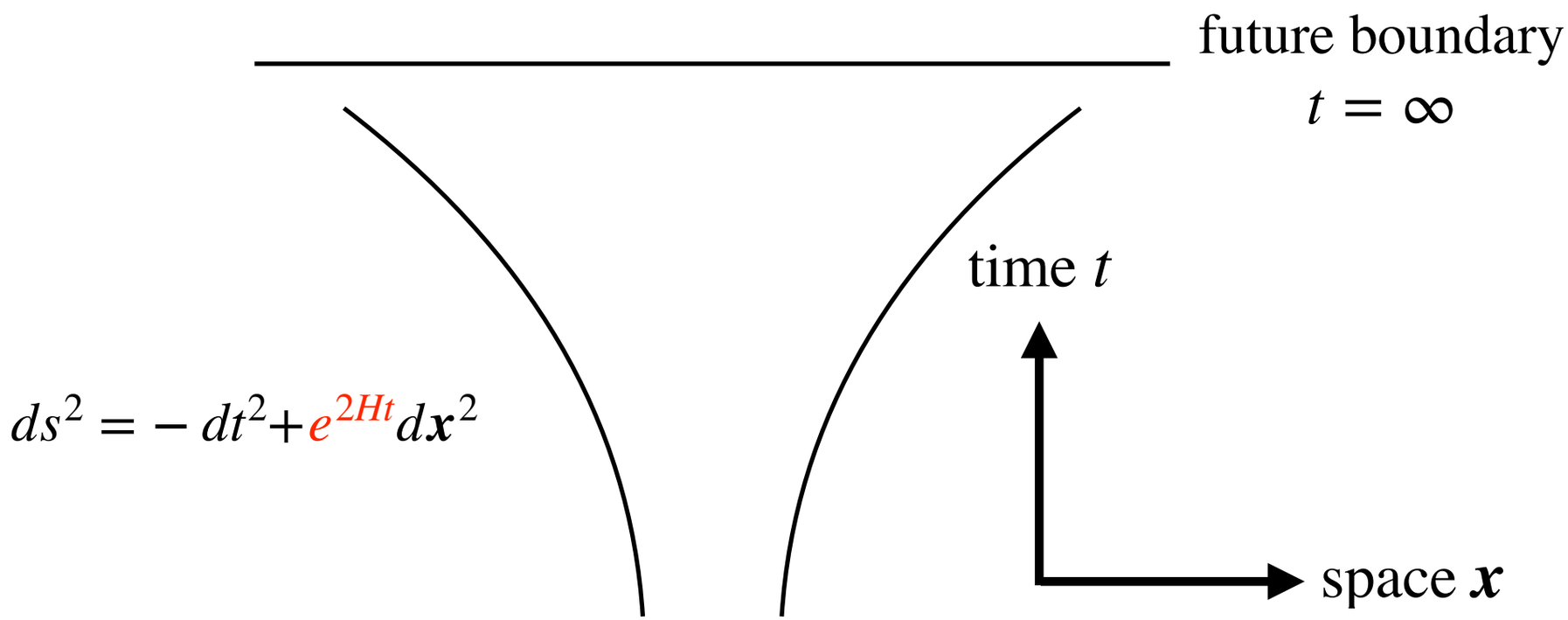
[Albert-Henrikssonf-Rastelli-Vichi ’23]

Contents

There appeared interesting papers (not mine!) by Melville & Pimentel.

[Melville-Pimentel ’23, ’24]

Ordinary Cosmological Correlators for Inflation



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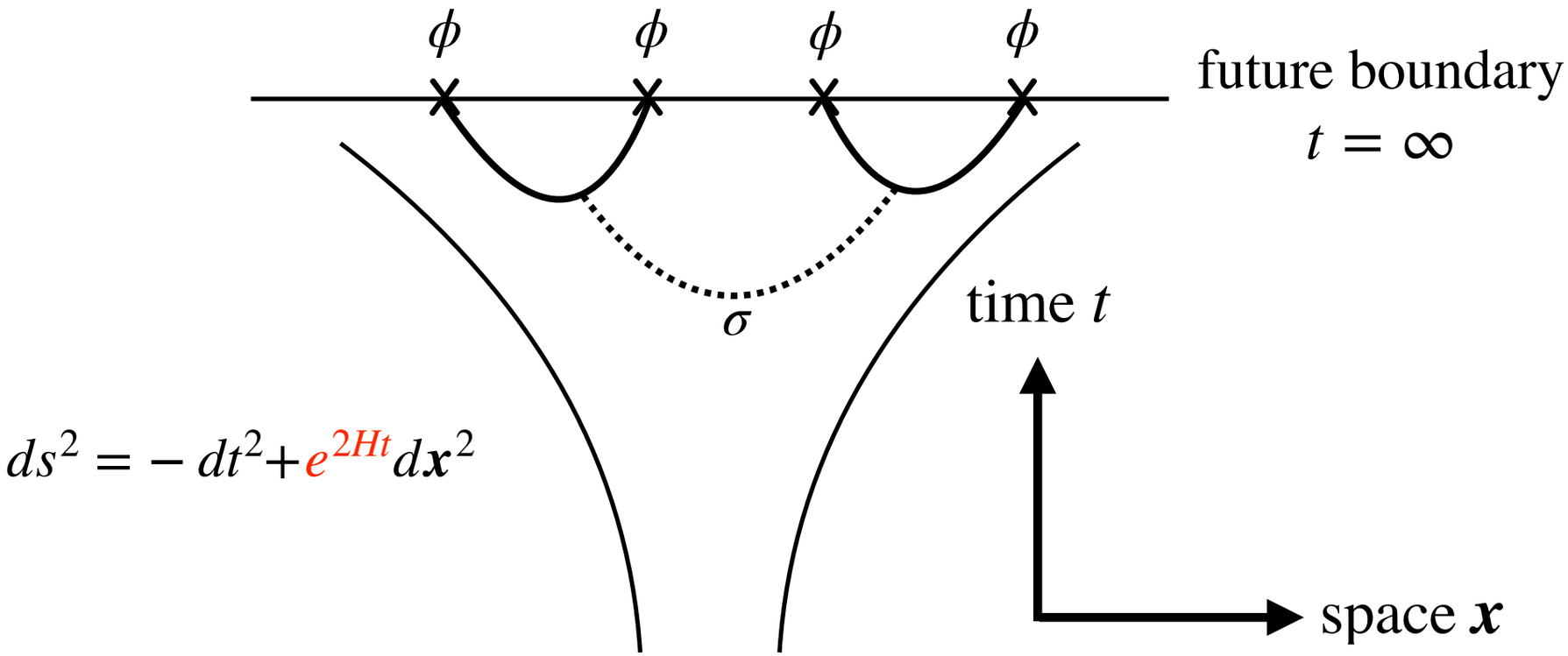


$$ds^2 = -dt^2 + e^{2Ht} d\boldsymbol{x}^2$$

- In the context of the early universe inflation,

  we are interested in **correlation functions at the end of inflation**,

  which specify the initial conditions of Big Bang Cosmology.

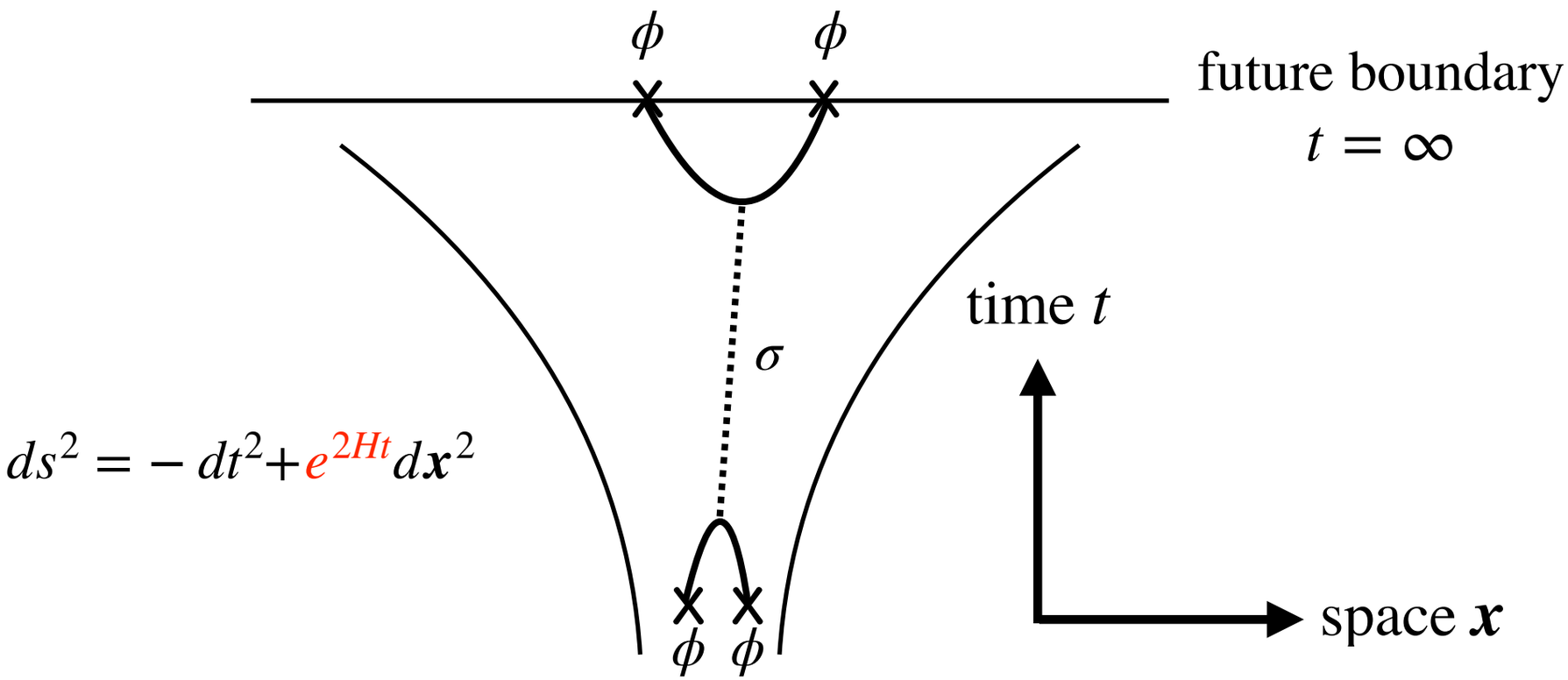
- More concretely, we compute

$$\left\langle \prod_i \phi_{\boldsymbol{k}_i}(t=\infty) \right\rangle = \langle 0 | \prod_i \phi_{\boldsymbol{k}_i}(t=\infty) | 0 \rangle .$$

※ $|0\rangle$: vacuum @ $t = -\infty$ (Bunch-Davies vacuum)

A proposal for S-matrix in de Sitter spacetime

[Melville-Pimentel '23, '24]



- Define in/out states by excitation of BD vacuum in the free theory:

  ex. $|\boldsymbol{k}_1, \boldsymbol{k}_2\rangle_{\text{in}} = a^\dagger_{\boldsymbol{k}_1} a^\dagger_{\boldsymbol{k}_1} |0\rangle_{\text{in}}, \quad |\boldsymbol{k}_1, \boldsymbol{k}_2\rangle_{\text{out}} = a^\dagger_{\boldsymbol{k}_1} a^\dagger_{\boldsymbol{k}_1} |0\rangle_{\text{out}}.$

- Define S-matrix based on the adiabatic hypothesis

  ex. $S_{2\to 2} = {}_{\text{out}}\langle \boldsymbol{k}_3, \boldsymbol{k}_4 | \, U(t_+, t_-) \, | \boldsymbol{k}_1, \boldsymbol{k}_2 \rangle_{\text{in}}$ (interaction picture).

※ $t_\pm$: modes $\boldsymbol{k}_i$ are sufficiently super/sub-horizon

※ Well motivated at least in IR-finite theories.

S-matrix defined in this manner

- satisfies standard properties such as LSZ, crossing and optical theorem.
- violates energy conservation due to cosmic expansion.

→ A framework suitable for S-matrix theory on de Sitter spacetime??

Long to-do list, now becoming tractable!

- partial wave unitarity & unitarity bound

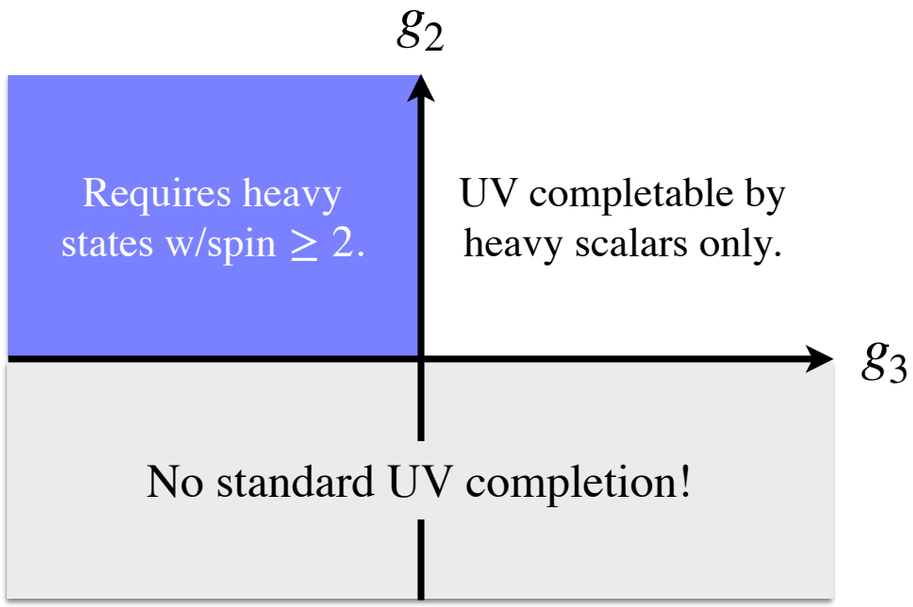
- analyticity in “Mandelstam variables”

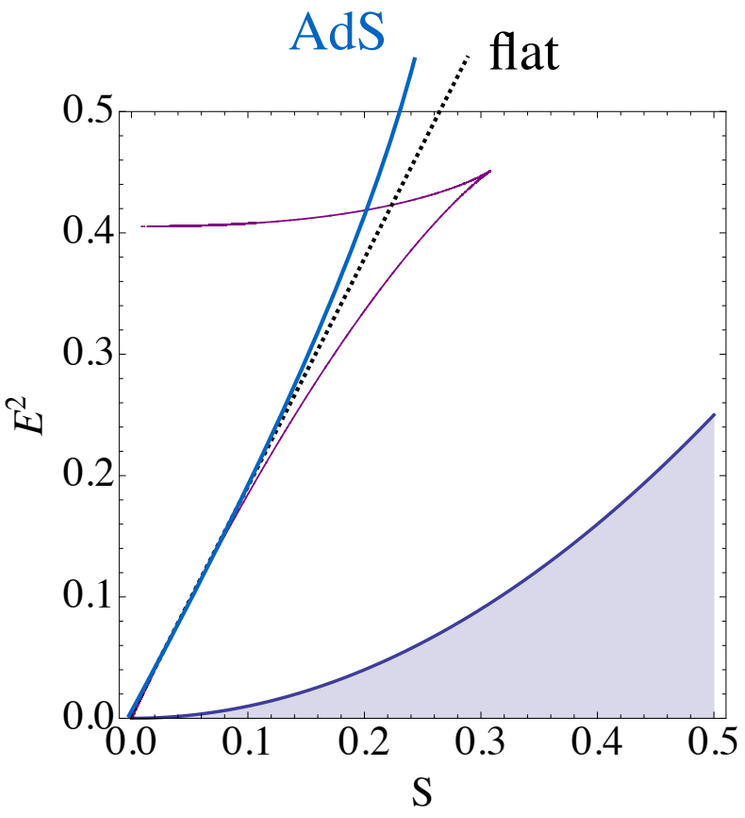
- dispersion relations connecting UV & IR

- analyticity in spin & Regge theory

- cosmological Veneziano amplitudes, …

Summary and prospects





S-matrix bootstrap: connect UV & IR by unitarity, analyticity etc.

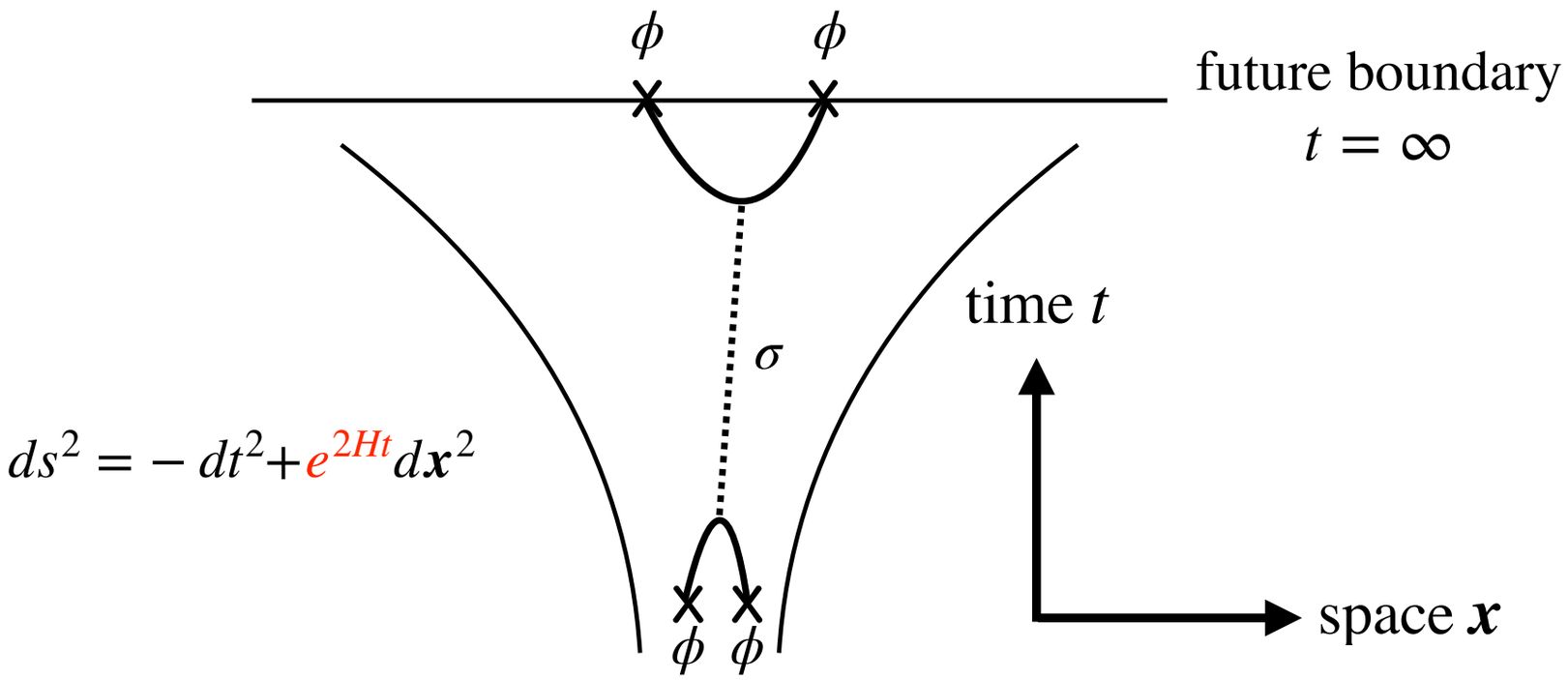
Its dS extension would make it possible

- to explore physics above inflation scale through EFT of inflation

ex. KK graviton vs heavy scalars in GUT

- to explore “string theory on dS” similarly to history on flat space

cf. Regge trajectories on dS are qualitatively different from flat/AdS.



A proposal for dS S-matrix [Melville-Pimentel '23, '24]

- a natural framework for bootstrap in dS
- shares various similarities with flat space S-matrix
- violates energy conservation due to cosmic expansion
- there are a lot more to explore along this line!

Thank you!