Extrapolation to UV from IR physics

YITP, Kyoto University

Hiromasa Takaura

based on ongoing work in collaboration with

Wen Yin

(Tokyo Metropolitan U.)

@ mini workshop on new ideas in particle physics and cosmology

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What is the UV physics?

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Widely asked: BSM, Quantum gravity,...

We present a new idea that allows us to explore the UV physics *beyond the threshold of a given IR expansion.*

Background

Aiming to study low-energy dynamics of QCD(-like) theories,
I have proposed a method to bridge low-energy dynamics and high energy behavior in 2404.05589 HT:
low-energy limits of correlators can be extracted from systematically calculable high energy expansions

UV → IR

We discuss an application in the opposite direction: IR → UV.

Framework

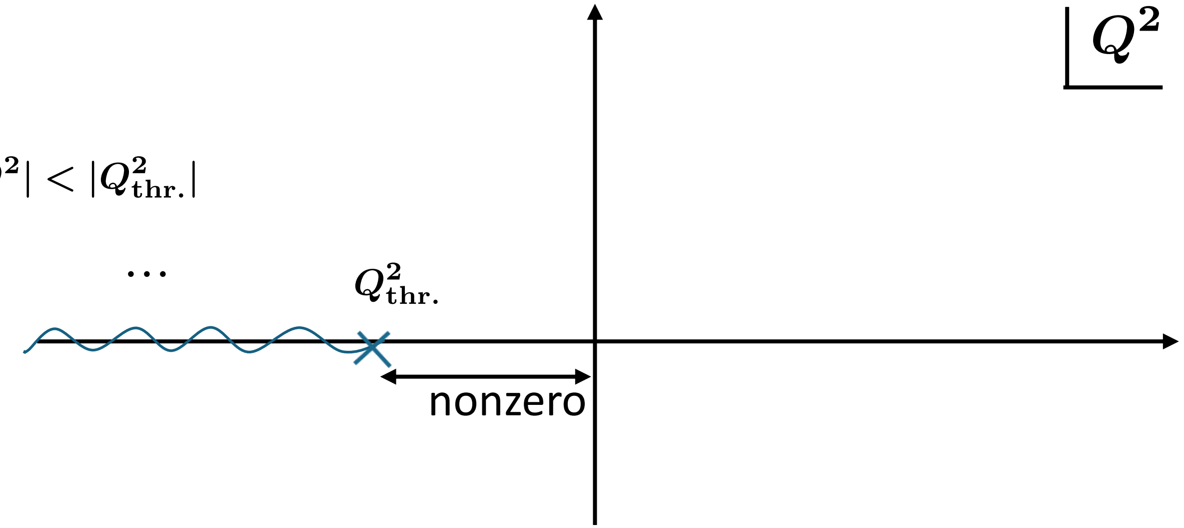
$Q^2 > 0$:Euclidean momentum

The assumption in this talk:

Consider an (analytic) physical quantity $f(Q^2)$ whose low-energy expansion has a finite radius of convergence (such as current-current correlator in a mass gapped theory).

⟷ The nearest singularity to the origin is located at $Q^2 = Q^2_{\mathrm{thr.}}$ with $|Q^2_{\mathrm{thr.}}| > 0$.

$$f(Q^2) = \sum_{n=0}^{\infty} c_n \left(\frac{Q^2}{Q^2_{\mathrm{thr.}}} \right)^n \quad \text{for } |Q^2| < |Q^2_{\mathrm{thr.}}|$$



In the following, we mainly consider the situation where the energy scale $Q_{\mathrm{thr.}}$ corresponds to the scale where heavy (unknown) degrees of freedom exist.

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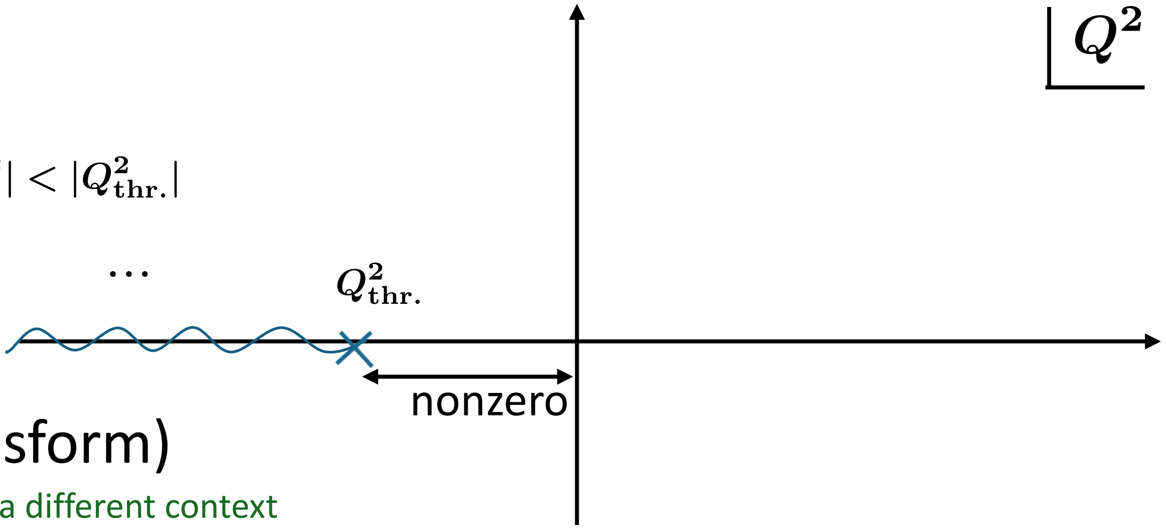


The inverse Laplace transform (Borel transform)

Originally introduced in 2303.16392 Hayashi, Mishima, Sumino, HT in a different context

$$\tilde{f}(\tau) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \frac{dr}{r} f(Q^2 = 1/r) e^{\tau r} = \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{\tau}{Q^2_{\rm thr.}} \right)^n$$

The convergence radius is infinity $(|c_n| \sim 1)$ and $\tilde{f}(\tau)$ is analytic anywhere in the τ-plane!

Key features (1)

If the IR expansion is available to sufficiently high orders,
one can construct a function valid up to higher energies $\tau \gg Q^2_{\mathrm{thr.}}$ (arbitrarily high energies, in principle).

$$f(Q^2) = \sum_{n=0}^{\infty} c_n \left(\frac{Q^2}{Q^2_{\mathrm{thr.}}} \right)^n \longrightarrow \tilde{f}(\tau) = \sum_{n=0}^{\infty} \frac{c_n}{n!} \left(\frac{\tau}{Q^2_{\mathrm{thr.}}} \right)^n$$

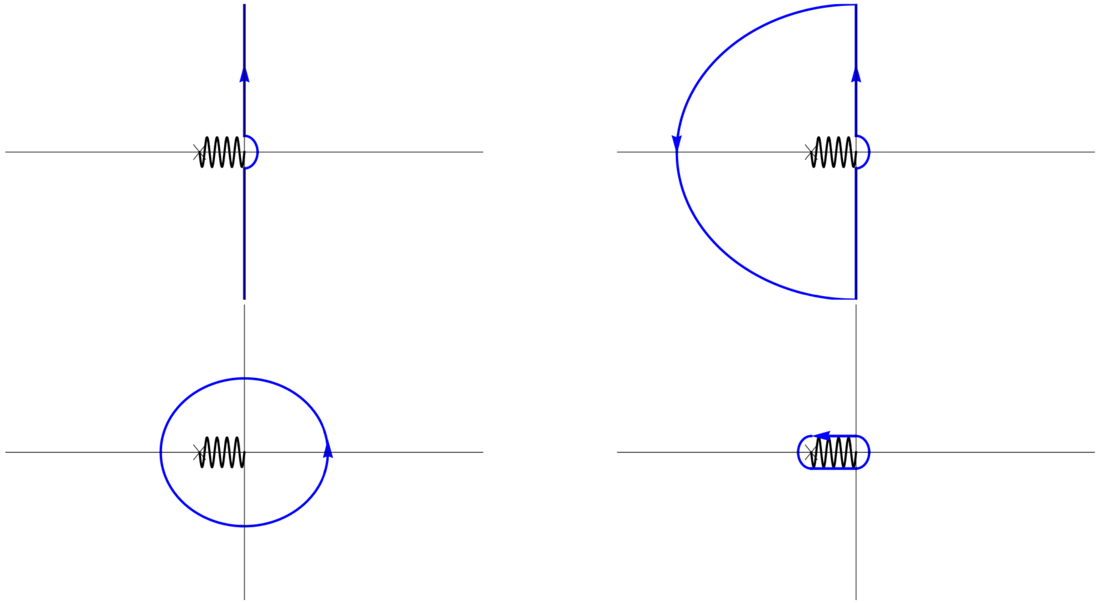
Key features (2)

1/Q²-plane

$$\tilde{f}(\tau) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{dr}{r} f(Q^2 = 1/r) e^{\tau r}$$

$$= \frac{1}{\pi}\int_0^{-1/Q^2_{\rm thr.}} \frac{dr}{r} \mathrm{Im}\, f(Q^2 = 1/(r+i0)) e^{\tau r}$$

$$(\tau > 0)$$



If $\mathrm{Im}\, f(-|Q^2| + i0)$ is positive, $\tilde{f}(\tau)$ is also positive for $\tau > 0$.

Key features (3)

Corrsponding to a high energy behavior $f(Q^2) \sim \left(\frac{Q^2_{\mathrm{thr.}}}{Q^2}\right)^a$,

the high energy behavior of $\tilde{f}(\tau)$ emerges as $f(\tau) \sim \left(\frac{Q^2_{\mathrm{thr.}}}{\tau^2}\right)^a$. More precisely,

$$\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{dr}{r}(Q^2_{\mathrm{thr.}}r)^a e^{\tau r} = \frac{1}{\pi}\left(\frac{Q^2_{\mathrm{thr.}}}{\tau}\right)^a \Gamma(a)\sin(\pi a)$$

For instance,

$$f(Q^2) \approx x + y\log\left(Q^2/\mu^2\right) + \mathcal{O}\left(Q^2_{\mathrm{thr.}}/Q^2\right)$$

$$\updownarrow$$

$$\tilde{f}(\tau) \approx x + y\log\left(\tau e^{\gamma_E}/\mu^2\right) + \mathcal{O}(Q^2_{\mathrm{thr.}}/\tau)$$

If the UV behavior is assumed to be governed by a weakly coupled gauge theory and $f(Q^2) \approx a_0\alpha(Q^2)$, $a_0 > 0$ follows if $\mathrm{Im}\, f(-|Q^2| + i0) > 0$ from the previous discussion.

In addition, since $f(Q^2) \approx a_0\alpha(Q^2) = a_0\alpha(\mu^2) - a_0 b_0\alpha^2(\mu^2)\log(\tau/\mu^2) + \mathcal{O}(\alpha^3)$ for $\mu^2\frac{d}{d\mu^2}\alpha(\mu^2) = -b_0\alpha^2 + \mathcal{O}(\alpha^3)$,

the sign of y tells us whether it is asymptotically free or not: $y < 0 \longleftrightarrow$ Asymptotically free

What we can do

IR expansion $f(Q^2 < Q^2_{\rm thr.}) = \sum_n c_n \left(\frac{Q^2}{Q^2_{\rm thr.}}\right)^n$

↓

The inverse Laplace transform $\tilde{f}(\tau) = \sum_n \frac{c_n}{n!}\left(\frac{\tau}{Q^2_{\rm thr.}}\right)^n \underset{(\tau \gg Q^2_{\rm thr.})}{\approx} x + y \log\left(\tau e^{\gamma_E}/\mu^2\right) + \mathcal{O}(Q^2_{\rm thr.}/\tau)$

↓

UV behavior $f(Q^2 \gg Q^2_{\rm thr.}) \approx x + y \log\left(Q^2/\mu^2\right) + \mathcal{O}(Q^2_{\rm thr.}/Q^2)$

We can reach the UV physics beyond the threshold of the original IR expansion.

Toy example

To examine if and how this framework works, we ask the following question:

Can we give QED predictions if we start from the Euler-Heisenberg theory?

Euler-Heisenberg theory: theory at $Q^2 < 4m_e^2 (= Q^2_{\text{thr.}})$ (the electron is decoupled)

No running gauge coupling

$$\mathcal{L}_{\text{E-H}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha^2}{m_e^4} \left\{ c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right\} + \mathcal{O}(m_e^{-6})$$

QED: gauge theory with a running coupling

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu(\partial_\mu + ieA_\mu) - m_e)\psi$$

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↓ *Is it possible to reach QED?*

QED: gauge theory with a running coupling

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu(\partial_\mu + ieA_\mu) - m_e)\psi$$

Starting point

Assume that we know the potential energy between an electron and an anti-electron at long distances $V_{\rm QED}(r \gg m_e)$ or its Fourier transform $\alpha_V(Q^2 \ll m_e^2) \approx \frac{\alpha(\mu^2)}{1-\Pi_{\rm QED}(Q^2)} \approx \alpha(\mu^2)(1+\Pi_{\rm QED}(Q^2))$

$$\alpha_V(Q^2 \ll m_e^2) = \sum_{n=0}^{\infty} c_n \left(\frac{Q^2}{Q^2_{\rm thr.}}\right)^n = \alpha + \frac{4\alpha^2}{15\pi}\frac{Q^2}{4m_e^2} - \frac{4\alpha^2}{35\pi}\left(\frac{Q^2}{4m_e^2}\right)^2 + \cdots$$

Since $\mathrm{Im}\,\Pi(-|Q^2| + i0) \propto \sigma(\gamma^* \to X) > 0$, positivity of $\tilde{\alpha}_V(\tau)$ follows.

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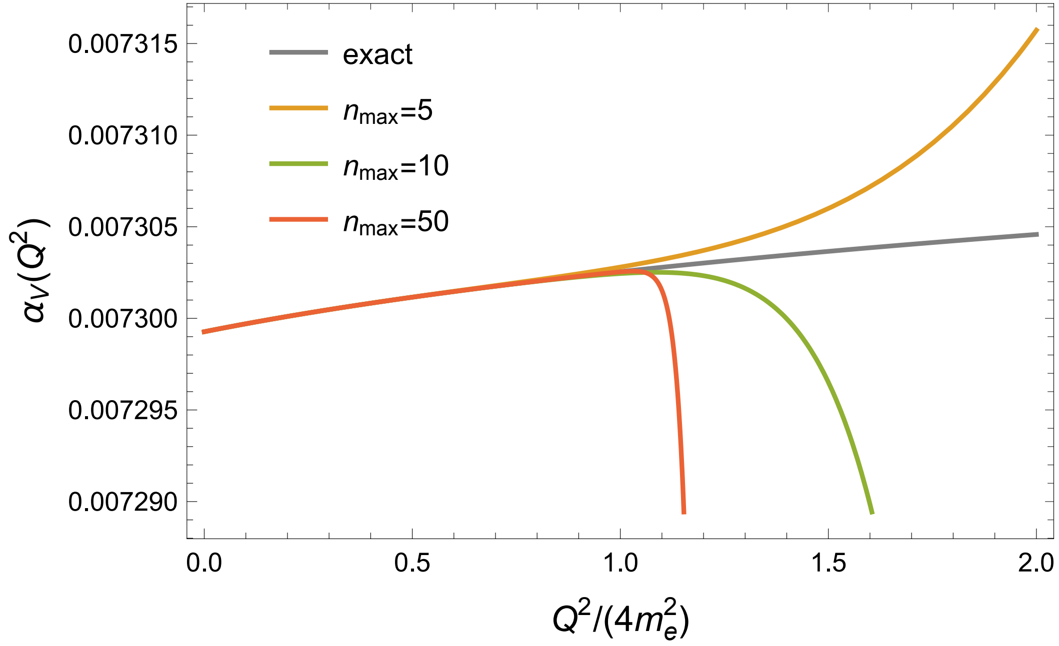
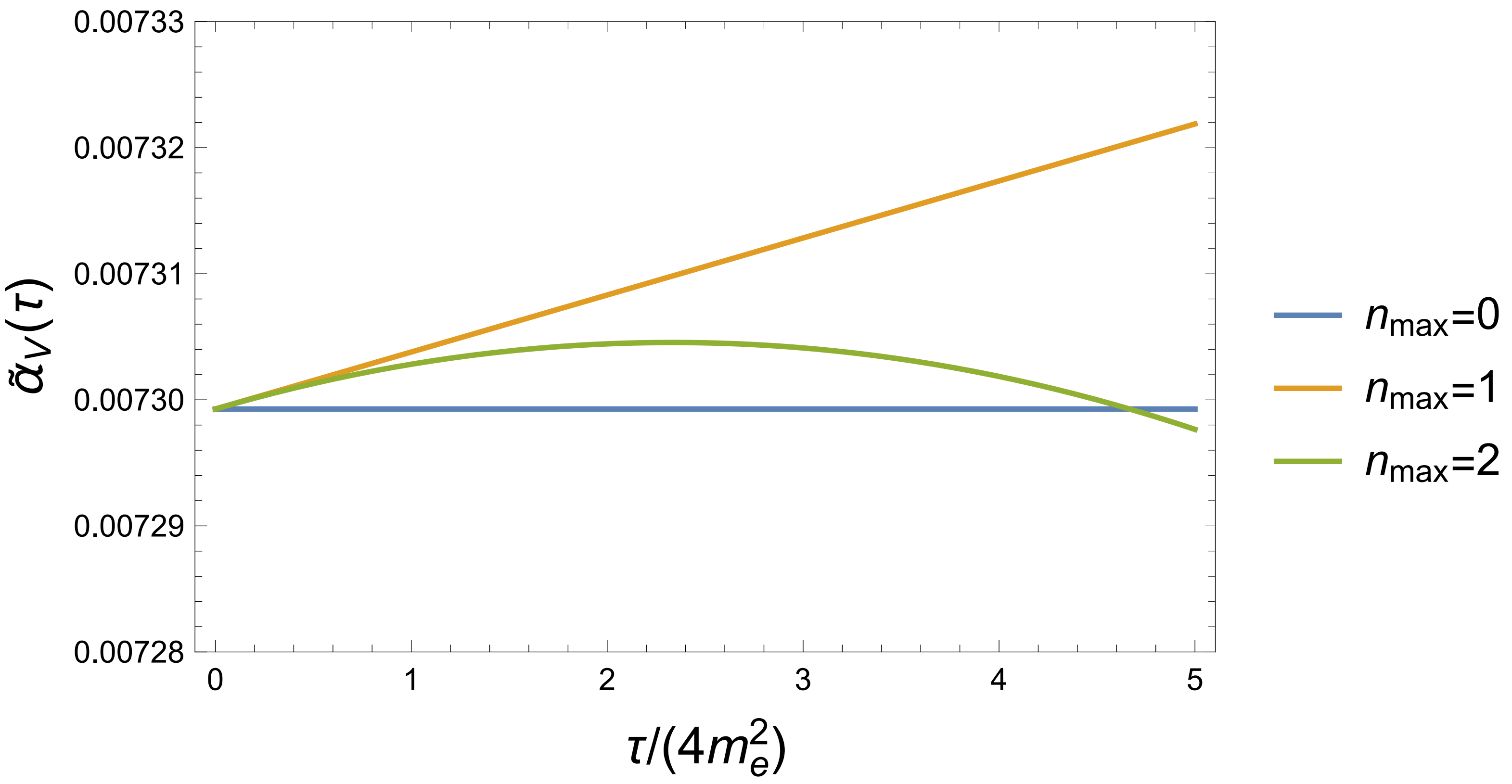
(We cannot go beyond the threshold in a straightforward method no matter how many terms we include.)



Inverse Laplace transform

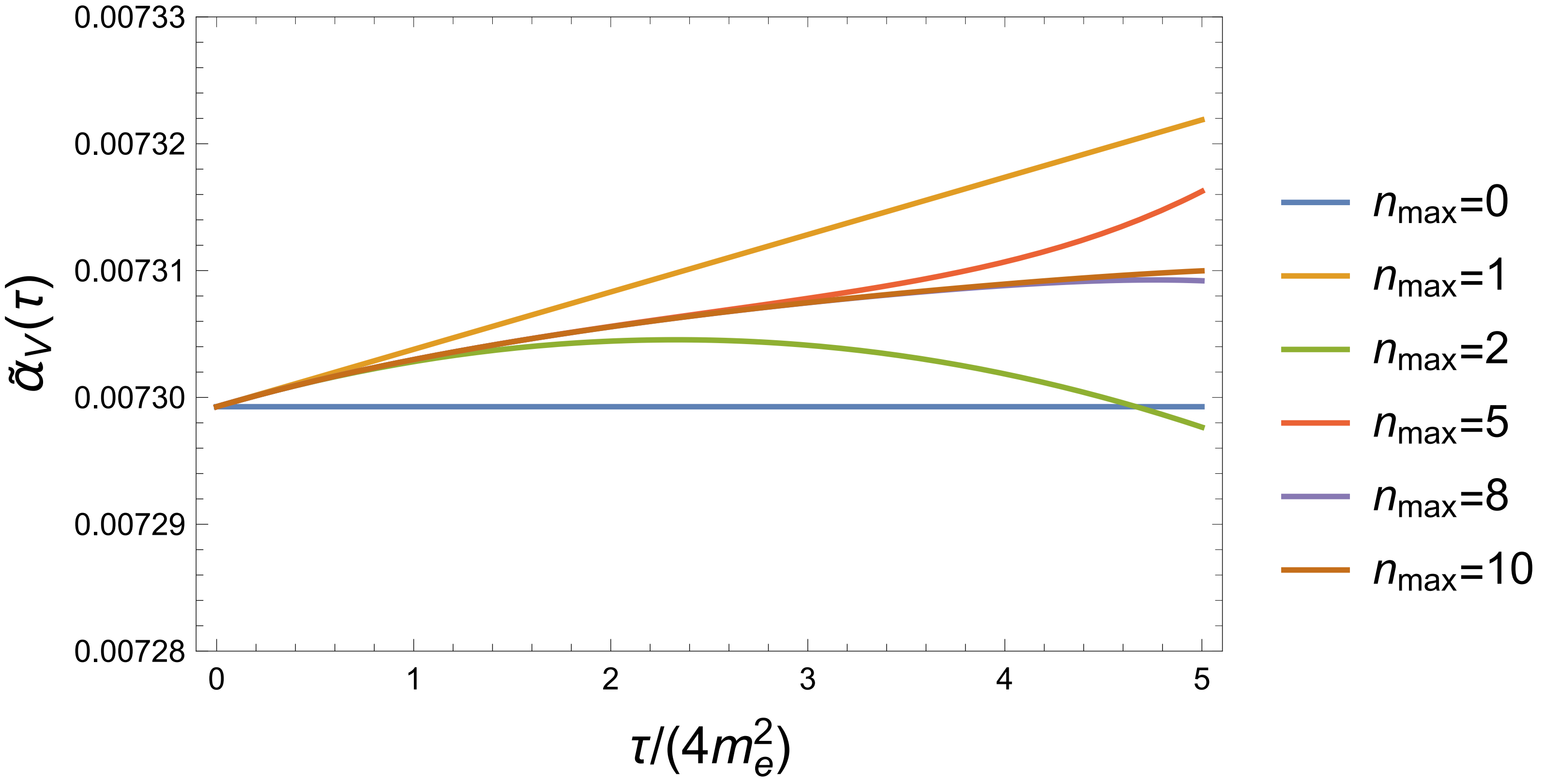
$$\tilde{\alpha}_V(\tau) \approx \sum_{n=0}^{n_{\max}} \frac{c_n}{n!} \left(\frac{\tau}{4m_e^2}\right)^n$$



Inverse Laplace transform

$$\tilde{\alpha}_V(\tau) \approx \sum_{n=0}^{n_{\max}} \frac{c_n}{n!} \left(\frac{\tau}{4m_e^2} \right)^n$$

Convergence in a wider range with more terms.



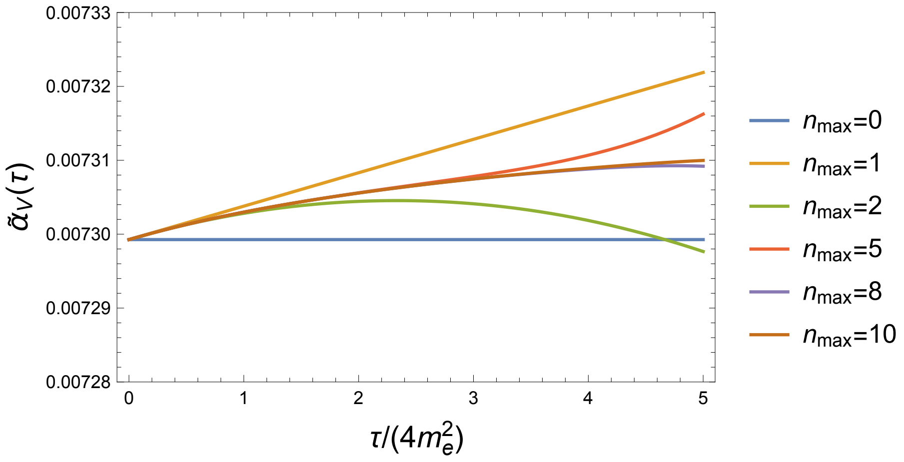
Fit

We fit the data given by $\tilde{\alpha}_V(\tau) \approx \sum_{n=0}^{5} \frac{c_n}{n!}\left(\frac{\tau}{4m_e^2}\right)^n$ for $2 \leq \tau/(4m_e^2) \leq 3$

with the ansatz $\tilde{\alpha}_V(\tau) \approx x + y \log\left(\tau e^{\gamma_E}/\mu^2\right) + \mathcal{O}(Q^2_{\rm thr.}/\tau)$ and taking μ=m_e.

We obtain

$$x = 0.00729, \; y = 5 \times 10^{-6}$$



The UV theory is suggested to be not asymptotically free.

UV behavior

$$f(Q^2) \approx x + y \log\left(Q^2/\mu^2\right) + \mathcal{O}\left(Q^2_{\rm thr.}/Q^2\right)$$

$$\updownarrow$$

$$\tilde{f}(\tau) \approx x + y \log\left(\tau e^{\gamma_E}/\mu^2\right) + \mathcal{O}(Q^2_{\rm thr.}/\tau)$$

From the prevous result, we infer the UV bahvior in the original space as

$$\alpha_V(Q^2) = 0.00729 + 5 \times 10^{-6} \log\left(Q^2/m_e^2\right) + \mathcal{O}(4m_e^2/Q^2)$$

UV behavior

$$f(Q^2) \approx x + y \log\left(Q^2/\mu^2\right) + \mathcal{O}\left(Q^2_{\rm thr.}/Q^2\right)$$
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Exact result from QED

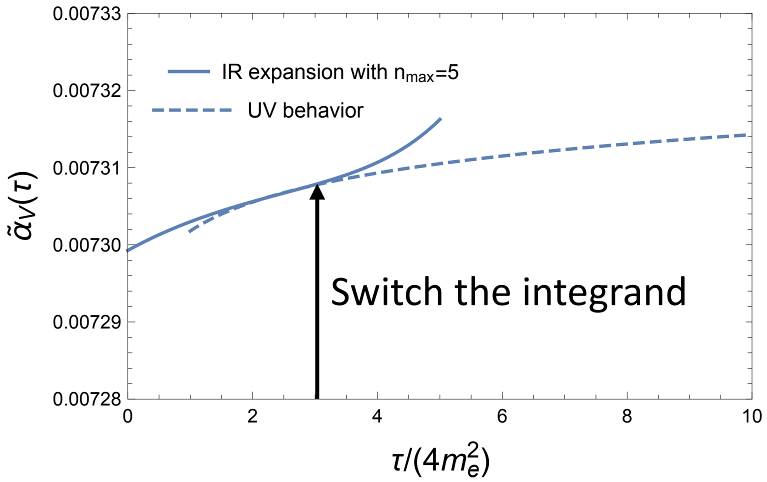
$$\alpha_V(Q^2) = 0.007298985 + 5.65311 \times 10^{-6} \log\left(Q^2/m_e^2\right) + \cdots$$

“Can we give QED predictions if we start from the Euler-Heisenberg theory?”

Yes, the UV behavior was successfully inferred from the IR expansion.

Behavior in the entire region

(Borel transform)+(extrapolation to UV)



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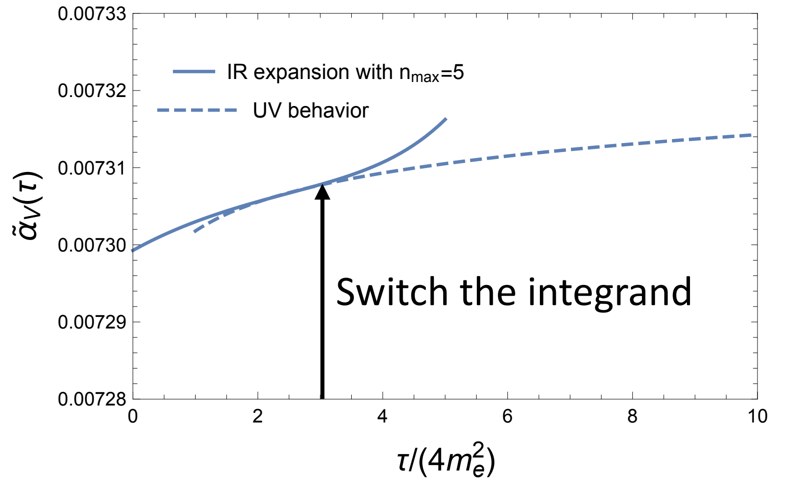
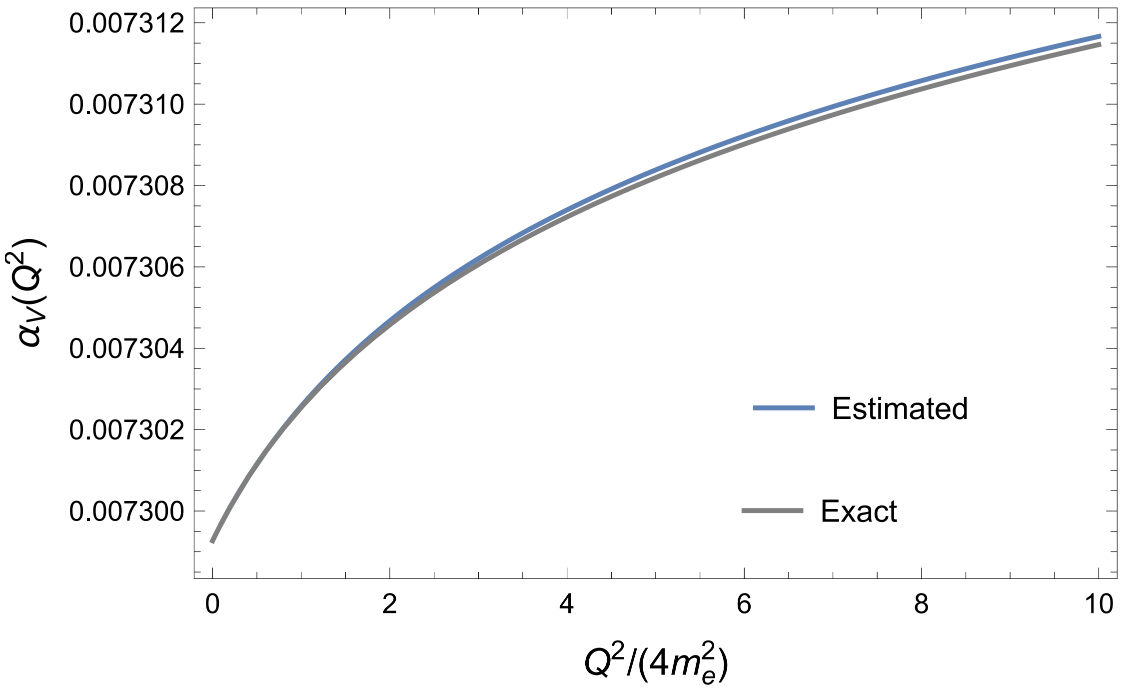


$$\alpha_V(Q^2) = \frac{1}{Q^2}\int_0^\infty d\tau\, \tilde{\alpha}_V(\tau) e^{-\tau/Q^2}$$



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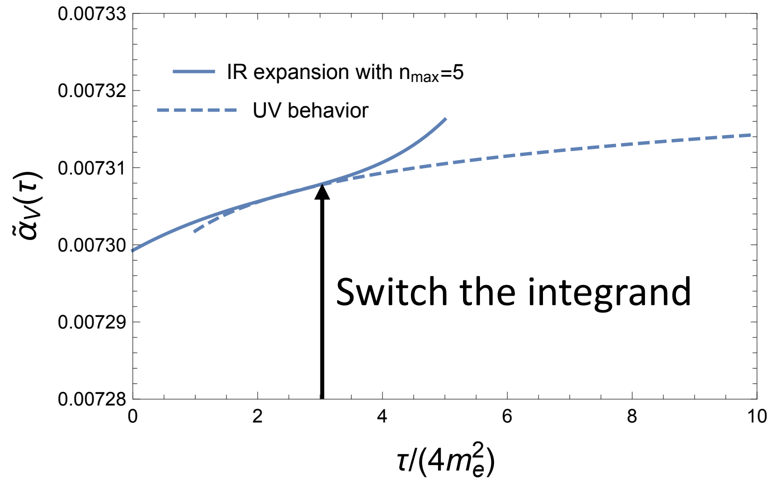

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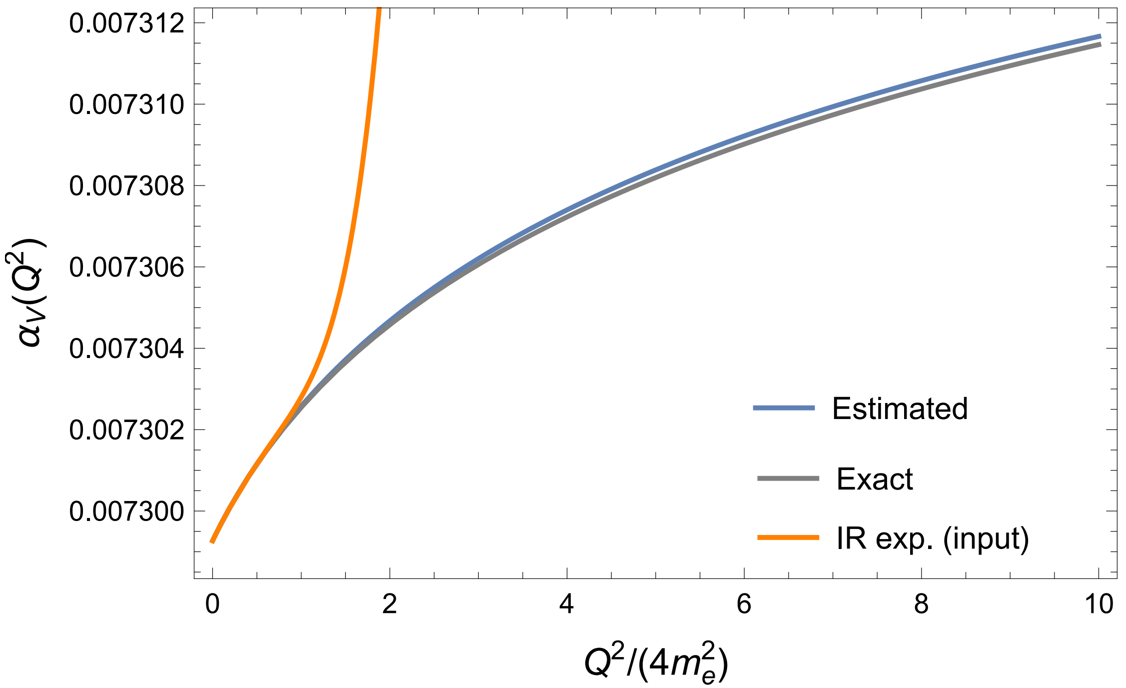

Summary

- We propose a method to explore the UV behavior from a given IR expansion *beyond the threshold*.

(IR expansion) ⟶ (Inverse Laplace transform) ⟶ (UV behavior)

$$f(Q^2 < Q^2_{\rm thr.}) = \sum_n c_n \left(\frac{Q^2}{Q^2_{\rm thr.}}\right)^n \qquad \tilde{f}(\tau) = \sum_n \frac{c_n}{n!}\left(\frac{\tau}{Q^2_{\rm thr.}}\right)^n \qquad f(Q^2 \gg Q^2_{\rm thr.})$$

Convergence raidus is infinity

- For QED as an example, we show that the running behavior, specific to the UV theory, was indeed extracted from its low energy effective theory.

Outlook

- We are working on another toy example where IR physics is fully nonperturbative, studying whether asymptotically free nature can be inferred from it.

Potential applications

- Bottom-up approach to BSM

SMEFT $\longrightarrow$ BSM predictions @ $Q > \Lambda_{\mathbf{SMEFT}}$

- Bottom-up approach to quantum gravity

The framework does not always require field theoretical descriptions.

- Many others…