

Implications of Nonlinear-Supersymmetric General Relativity

- Unification of space, time and matter -

Kazunari Shima
Saitama Institute of Technology

OUTLINE

1. Motivation
2. **Nonlinear-supersymmetric general relativity theory(NLSGR)**
3. **Evolution of NLSGR (Big Collapse and Superon-Graviton Model)**
4. **Linearization of NLSUSY and the vacuum of $L_{SGM}(e, \psi)$ (NL/L SUSY relation)**
5. **Low energy particle physics of NLSGR**
6. **Cosmological Implications of NLSGR.**
7. **Summary**

1. Motivation

@ The success of **GR** and **SM(GWS) model**,

but **many unsolved fundamental problems are left**, e. g.,

- Unification of two SMs,
- Space-time dimension **four**,
- Three-generations structure of quarks and leptons,
- C-asymmetry of Nature,
- Tiny Neutrino mass M_ν , proton stability in GUT
- Dark Matter, Dark Energy; $\rho_{D.E.} \sim (M_\nu)^4 \Leftrightarrow \Lambda CDM$, Inflation?

\Rightarrow **SUSY, SUGRA**,

- Origin of SUSY breaking ?, \dots etc.

@ GR describes geometry of space-time.

However, **unpleasant differences** between **GR** and **SUGRA**:

- **GR** \Leftrightarrow Geometry of Riemann space-time (**Physical**: $[x^\mu]$, $GL(4, R)$)
- **SUGRA** \Leftrightarrow Geometry of superspace (**Mathematical**: $[x^\mu, \theta_\alpha]$, sPoicaré)

\Rightarrow New **SUSY** paradigm on **physical space-time!**.

@ As for the **three-generations structure**

among $SO(N)$ LSUSY sP, **only $N = 10$ gives SM with just 3 generations:**, where

• The decomposition of 10 supercharges $Q^I, (I = 1, 2, \dots, 10)$ are:

$$\underline{10}_{SO(10)} = \underline{5}_{SU(5)} + \underline{5}^*_{SU(5)}$$

$$\underline{5}_{SU(5)} = [\{\underline{3}^{*c}, \underline{1}^{ew}, (\frac{e}{3}, \frac{e}{3}, \frac{e}{3}) : Q_a (a = 1, 2, 3)\}, \{\underline{1}^c, \underline{2}^{ew}, (0, -e) : Q_m (m = 4, 5)\}].$$

\Leftrightarrow Supercharge $\underline{5}_{SU(5)}$ has the same quantum numbers as **5** of SU(5) GUT.

• Massless helicity states of **gravity supermultiplet** of $SO(10)$ sP with CPT conjugation

are specified by the helicity $h = (2 - \frac{n}{2})$ and the dimension $\underline{d}_{[n]} = \frac{10!}{n!(10-n)!}$:

$|h \rangle = Q^n Q^{n-1} \dots Q^2 Q^1 |2 \rangle, Q^n (n = a, m, a^*, m^*):$ supercharge

| $ h $ | 3 | $\frac{5}{2}$ | 2 | $\frac{3}{2}$ | 1 | $\frac{1}{2}$ | 0 |
|-----------------------|------------------------|------------------------|------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| | | | $\underline{1}_{[0]}$ | $\underline{10}_{[1]}$ | $\underline{45}_{[2]}$ | $\underline{120}_{[3]}$ | $\underline{210}_{[4]}$ |
| $\underline{d}_{[n]}$ | $\underline{1}_{[10]}$ | $\underline{10}_{[9]}$ | $\underline{45}_{[8]}$ | $\underline{120}_{[7]}$ | $\underline{210}_{[6]}$ | $\underline{252}_{[5]}$ | $\underline{210}_{[4]}$ |

@ Dirac particle survivors after tentative superHiggs-like mechanism

| $SU(3)$ | Q_e | $SU(2) \otimes U(1)$ |
|----------|----------------------------|---|
| <u>1</u> | 0 -1 -2 | $\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} (N)$ (E) |
| <u>3</u> | 5/3 2/3 -1/3 -4/3 | $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} h \\ o \end{pmatrix} \begin{pmatrix} a \\ f \end{pmatrix} \begin{pmatrix} g \\ m \end{pmatrix} \begin{pmatrix} r \\ i \\ n \end{pmatrix}$ |
| <u>6</u> | 4/3 1/3 -2/3 | $\begin{pmatrix} P \\ Q \\ R \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ |
| <u>8</u> | 0 -1 | $\begin{pmatrix} N_1 \\ E_1 \end{pmatrix} \begin{pmatrix} N_2 \\ E_2 \end{pmatrix}$ |

@ SM Higgs-doublet state survives in $h = 0$ state.

- $N > 10$ case is unrealistic and **excluded**.

@ How to construct N=10 SUSY with gravity despite the **No-Go** theorem in **S-matrix** ?

- To circumvent the No-Go theorem degeneracy of space-time is essential.



We show in this talk:

- N=10 SUSY with gravity is obtained by the geometric description of General Relativity principle on specific unstable *physical* (Riemann) space-time whose tangent space possesses global NLSUSY structure.

A quick review of NLSUSY:

- Take (flat) space-time specified by x^a for $SO(1,3)$ and ψ_α for $SL(2,C)$.
- Consider one form $\omega^a = dx^a + \frac{\kappa^2}{2i}(\bar{\psi}\gamma^a d\psi - d\bar{\psi}\gamma^a\psi)$,
 κ is an **arbitrary** constant with the dimension l^{+2} .

- $\delta\omega^a = 0$ under $\delta x^a = \frac{i\kappa^2}{2}(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)$ and $\delta\psi = \zeta$ with a **global** spinor parameter ζ .

- An invariant action(\sim invariant volume) is obtained:

$$S = -\frac{1}{2\kappa^2} \int \omega^0 \wedge \omega^1 \wedge \omega^2 \wedge \omega^3 = \int d^4x L_{VA},$$

L_{VA} is **N=1 Volkov-Akulov model of NLSUSY** given by

$$L_{VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2} [1 - t^a_a + \frac{1}{2}(t^a_a t^b_b - t^a_b t^b_a) + \dots],$$

$$|w_{VA}| = \det w^a_b = \det(\delta^a_b + t^a_b), \quad t^a_b = -i\kappa^2(\bar{\psi}\gamma^a\partial_b\psi - \partial_b\bar{\psi}\gamma^a\psi),$$

which is invariant under **N=1 NLSUSY** transformation:

$$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)\partial_a\psi, \quad [\delta_1, \delta_2] = \delta_P.$$

- ψ is **Nambu-Goldstone(NG) fermion** for $\frac{\text{supertranslation}}{\text{translation}}$.
- ψ is quantized canonically in compatible with **SUSY algebra**.

2. Nonlinear-Supersymmetric General Relativity (NLSGR)

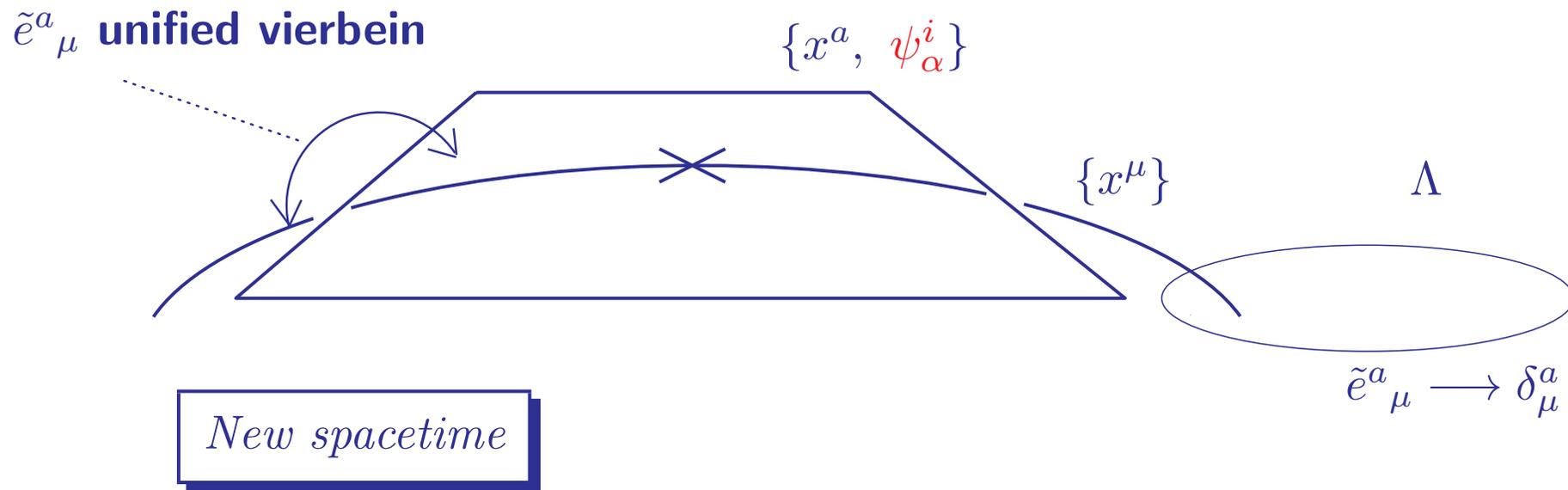
2.1. New Space-time as Ultimate Shape of Nature

We consider **new (unstable) physical space-time** inspired by **nonlinear(NL) SUSY:**

The tangent space of new space-time is specified by **Grassmann coordinates** ψ_α for $SL(2,C)$ besides the ordinary Minkowski coordinates x^a for $SO(1,3)$,

i.e.,
the coordinate ψ_α of the coset space $\frac{superGL(4,R)}{GL(4,R)}$ turning to the NLSUSY NG fermion (called *superon* hereafter) are attached besides x^a at every curved space-time point.

- New unstable space-time(ultimate shape of nature):



(Groups $SO(1,3)$ and $SL(2,C)$ for space-time symmetry are analogous to $SO(3)$ and $SU(2)$ for gauge symmetry of 't Hooft-Polyakov monopole.)

- Note that $SO(1,3) \cong SL(2,C)$.

4 dimensional space-time is singled out.

2. 2 Nonlinear-Supersymmetric General Relativity (NLSGR)

The geometrical arguments of Einstein general relativity (GR) can be extended to **new (unstable) space-time**.

- Unified vierbein $\tilde{e}^a{}_\mu(x)$ of new space-time:

$$\tilde{e}^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi) \equiv w^a{}_b e^b{}_\mu, \quad w^a{}_b = \delta^a{}_b + t^a{}_b,$$

$$t^a{}_b(\psi) = \frac{\kappa^2}{2i} (\bar{\psi}^I \gamma^a \partial_b \psi^I - \partial_b \bar{\psi}^I \gamma^a \psi^I), \quad (I = 1, 2, \dots, N)$$

$$\tilde{e}^\mu{}_a(x) = e^\mu{}_a - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a + \dots \equiv e^\mu{}_b w^b{}_a,$$

$$\tilde{e}^a{}_\mu(x) \tilde{e}^\mu{}_b(x) = \delta^a{}_b \quad \tilde{e}^a{}_\mu(x) \tilde{e}^\nu{}_a(x) = \delta^\nu{}_\mu$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i} (\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), \quad (I = 1, 2, \dots, N)$$

(Note that Grassmann *odd* d.o.f. induces the imaginary part of $\tilde{e}^a{}_\mu(x)$.)

- **N -extended NLSGR action of Einstein-Hilbert (EH)-type for new space-time. \implies**

N -extended NLSGR action of EH-type for new space-time:

Phys.Lett.B501,237(2001), B507,260(2001)

$$L_{\text{NLSGR}}(\tilde{e}) = -\frac{c^4}{16\pi G} |\tilde{e}| \{ \tilde{R}(\tilde{e}) + \Lambda \}, \quad (1)$$

$$|\tilde{e}| = \det \tilde{e}^a_{\mu} = \det(e^a_{\mu} + t^a_{\mu}(\psi)), \quad (2)$$

$$t^a_{\mu}(\psi) = \frac{\kappa^2}{2i} (\bar{\psi}^I \gamma^a \partial_{\mu} \psi^I - \partial_{\mu} \bar{\psi}^I \gamma^a \psi^I), \quad (I = 1, 2, \dots, N) \quad (3)$$

- $\tilde{e}^a_{\mu}(x) (= e^a_{\mu} + t^a_{\mu}(\psi))$: the unified vierbein of new space-time
- $e^a_{\mu}(x)$: the ordinary vierbein for the local $\text{SO}(1,3)$ d.o.f. of GR,
- $t^a_{\mu}(\psi(x))$: the mimic vierbein for the local $\text{SL}(2, \mathbb{C})$ d.o.f. composed of the stress-energy-momentum of NG fermion $\psi(x)^I$ (called *superons*),
- $\tilde{R}(\tilde{e})$: Ricci scalar curvature of new space-time computed in terms of \tilde{e}^a_{μ} ,
- $\tilde{g}_{\mu\nu} \equiv \tilde{e}^a_{\mu} \eta_{ab} \tilde{e}^b_{\nu}$, $\tilde{g}^{\mu\nu}(x) \equiv \tilde{e}^{\mu}_a(x) \eta^{ab} \tilde{e}^{\nu}_a(x)$: metric tensors of new space-time.
- G : the Newton gravitational constant.
- $\Lambda > 0$: cosmological constant required by the **correct** NLSUSY structure of tangent space.

- NLSGR scenario fixes the arbitrary constant κ^2 to

$$\kappa^2 = \left(\frac{c^4 \Lambda}{8\pi G}\right)^{-1},$$

with the dimension $(length)^4 \sim (energy)^{-4}$.

- $\Lambda > 0$ in the action L_{NLSGR} allows **negative dark energy density interpretation of $\frac{\Lambda}{G}$** in the Einstein equation. \rightarrow Sec.5.
- No-go theorem for $N > 8$ SUSY has been circumvented **by the global NLSUSY, i.e. by the vacuum(flat space) degeneracy.**
- Note that $SO(1, D - 1) \cong SL(d, C)$, i.e. $\frac{D(D-1)}{2} = 2(d^2 - 1)$ holds **for only $D = 4, d = 2$.**

NLSGR scenario predicts 4 dimensional space-time.

2.3. Symmetries of NLSGR $L_{\text{NLSGR}}(w)$ (N-extended action)

- **Space-time symmetries** ($\sim sP$):

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4, \mathbb{R})] \otimes [\text{local Lorentz}] \quad (4)$$

- **Internal symmetries** for N-extended NLSUSY GR (N-superons ψ^I ($I = 1, 2, \dots, N$)):

$$[\text{global SO}(N)] \otimes [\text{local U}(1)^N] \otimes [\text{chiral}]. \quad (5)$$

For example:

- Invariance under the new NLSUSY transformation;

$$\delta_\zeta \psi^I = \frac{1}{\kappa} \zeta^I - i\kappa \bar{\zeta}^J \gamma^\rho \psi^J \partial_\rho \psi^I, \quad \delta_\zeta e^a{}_\mu = i\kappa \bar{\zeta}^J \gamma^\rho \psi^J \partial_{[\mu} e^a{}_{\rho]}. \quad (6)$$

induces **GL(4,R) transformations** on $\tilde{e}^a{}_\mu$ and the unified metric $\tilde{g}_{\mu\nu}$

$$\delta_\zeta \tilde{e}^a{}_\mu = \xi^\nu \partial_\nu \tilde{e}^a{}_\mu + \partial_\mu \xi^\nu \tilde{e}^a{}_\nu, \quad \delta_\zeta \tilde{g}_{\mu\nu} = \xi^\kappa \partial_\kappa \tilde{g}_{\mu\nu} + \partial_\mu \xi^\kappa \tilde{g}_{\kappa\nu} + \partial_\nu \xi^\kappa \tilde{g}_{\mu\kappa}, \quad (7)$$

where ζ is a constant spinor parameter, $\partial_{[\rho} e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$ and $\xi^\rho = -i\kappa \bar{\zeta}^I \gamma^\rho \psi^I$.

- Commutators of two new NLSUSY transformations (??) on ψ^I and $e^a{}_\mu$ close to **GL(4,R)**,

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] \psi^I = \Xi^\mu \partial_\mu \psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}] e^a{}_\mu = \Xi^\rho \partial_\rho e^a{}_\mu + e^a{}_\rho \partial_\mu \Xi^\rho, \quad (8)$$

where $\Xi^\mu = 2i\bar{\zeta}_1^I \gamma^\mu \zeta_2^I - \xi_1^\rho \xi_2^\sigma e_a{}^\mu \partial_{[\rho} e^a{}_{\sigma]}$.
q.e.d.

- New NLSUSY (??) is the square-root of GL(4,R);

$$[\delta_1, \delta_2] = \delta_{\text{GL}(4,\mathbb{R})}, \quad \text{i.e.} \quad \delta \sim \sqrt{\delta_{\text{GL}(4,\mathbb{R})}}.$$

c.f. SUGRA (LSUSY)

$$[\delta_1, \delta_2] = \delta_P + \delta_L + \delta_g$$

- The ordinary local GL(4,R) invariance is manifest by the construction.

- Invariance under new local Lorentz transformation;

$$\delta_L \psi^I = -\frac{i}{2} \epsilon_{ab} \sigma^{ab} \psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}^I \gamma_5 \gamma_d \psi^I (\partial_\mu \epsilon_{bc}) \quad (9)$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$.

(??) induce the familiar local Lorentz transformation on $\tilde{e}^a{}_\mu$:

$$\delta_L \tilde{e}^a{}_\mu = \epsilon^a{}_b \tilde{e}^b{}_\mu \quad (10)$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

The local Lorentz transformation forms a closed algebra,
e.g., **the new form** on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}] e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}^j \gamma_5 \gamma_d \psi^j (\partial_\mu \beta_{bc}), \quad (11)$$

where $\beta_{ab} = -\beta_{ba}$ is given by $\beta_{ab} = \epsilon_{2ac} \epsilon_1^c{}_b - \epsilon_{2bc} \epsilon_1^c{}_a$.
q.e.d.

3. Evolution of NLSGR (Big Collapse to Superon-Graviton Model)

3.1. Big Collapse of ultimate space-time toward the vacuum configuration

@ $L_{\text{NLSGR}}(\tilde{e})$ breaks down spontaneously (Big Collapse) to ordinary Riemann space-time (graviton) and NG fermion (superon): $L_{\text{SGM}}(e, \psi)$.

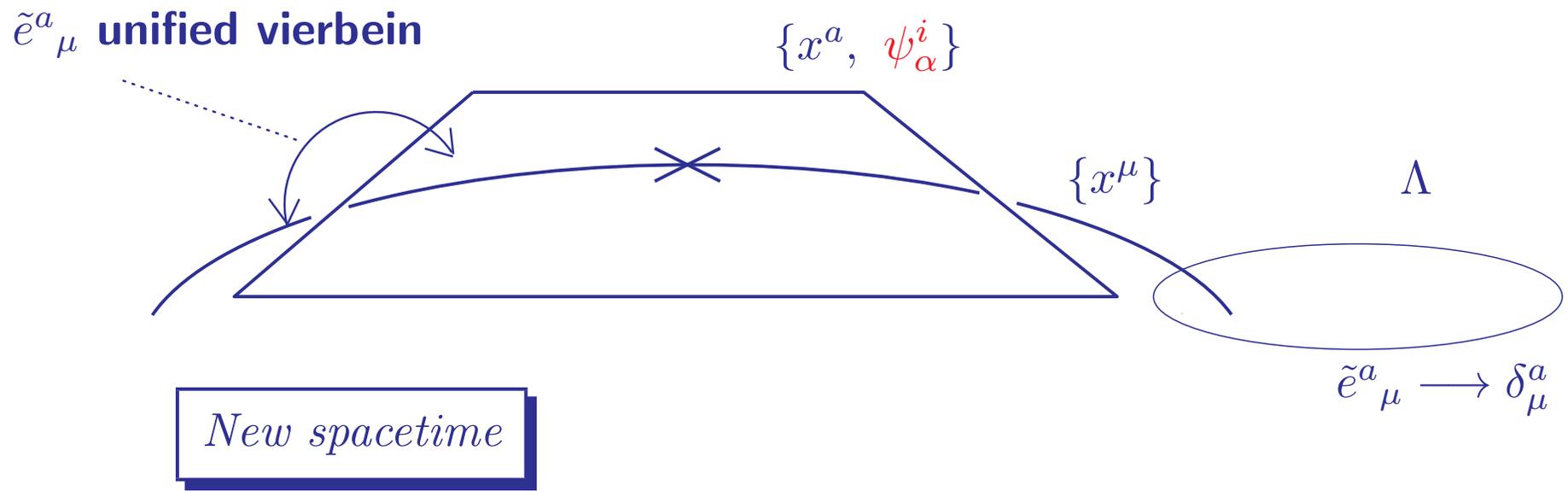
$$L_{\text{NLSGR}}(\tilde{e}) = L_{\text{SGM}}(e, \psi) = -\frac{c^4}{16\pi G} |e| \{R(e) + |w(\psi^I)|\Lambda + \tilde{T}(e, \psi^I)\}. \quad (12)$$

Superon-Graviton Model (SGM) before Big Bang

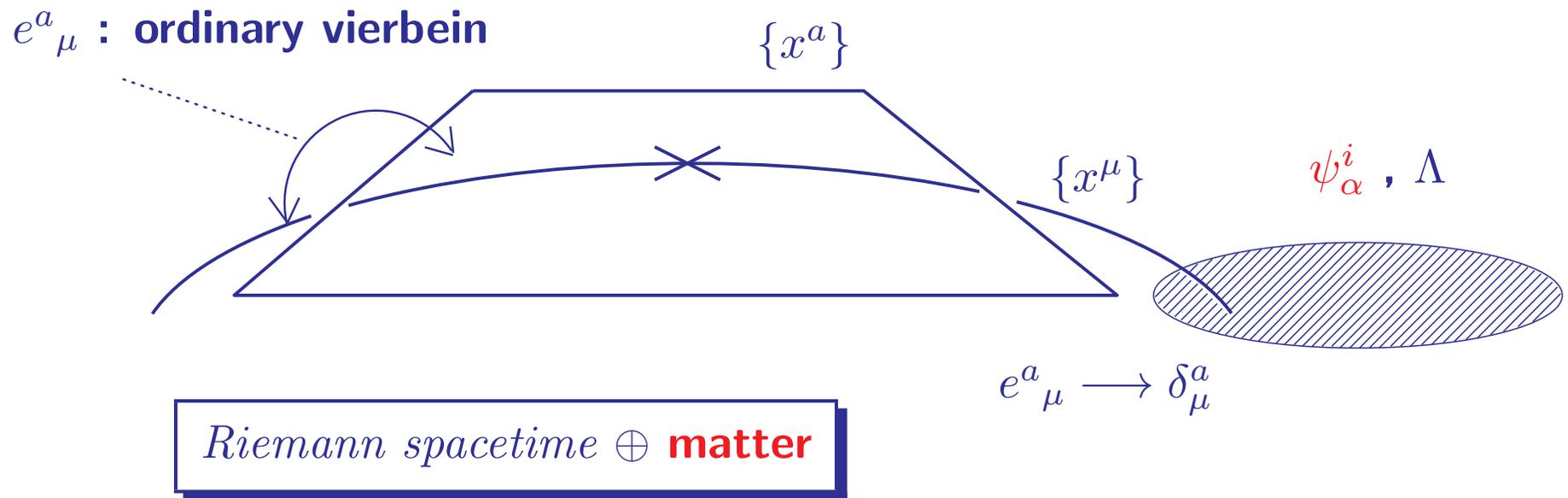
- $R(e)$: the Ricci scalar curvature of ordinary Riemann space-time
- ψ^j : NG fermion, $\langle \delta\psi^j \rangle = \kappa^{-1}\zeta^j$
- Λ : the cosmological constant
- $|w(\psi^I)| = \det w^a_b = \det \{\delta^a_b + t^a_b(\psi^I)\}$: NLSUSY action for superon
- $\tilde{T}(e, \psi^I)$: the gravitational interaction of superon

@ Big Collapse $L_{\text{SGM}}(e, \psi^I)$ induces :
the instantaneous expansion of space-time by the Pauli principle → Sec. 5

@ $L_{\text{SGM}}(e, \psi^I)$ evolves toward the true vacuum by constituting
gravitational composite (massless) eigenstates of broken LSUSY $\text{SO}(N)$ sP,
which corresponds to the **ignition of the Big Bang SM scenario.**



⇓ **Big Collapse**



Ignition of the Big Bang

- The Noether's theorem gives the conserved supercurrent:

$$S^{I\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}}\gamma^\mu\psi^I + \dots \quad (13)$$

- The supercurrent couples the superon(NG fermion) to the vacuum

$$\langle \psi_\beta^J | S_\alpha^{I\mu} | 0 \rangle = i\sqrt{\frac{c^4\Lambda}{8\pi G}}\delta^{IJ}(\gamma^\mu)_{\alpha\beta} \quad (14)$$

with the strength $g_{sv} \equiv \sqrt{\frac{c^4\Lambda}{8\pi G}} = \kappa^{-1}$.

The vacuum structure of $L_{SGM(e,\psi)}$ determines the evolution.

4. Linearization of NLSUSY (NL/L susy relation)

The graviton is the universal attractive force and creates all possible gravitational composites of superons,
which are helicity eigenstates of LSUSY sP algebra of space-time symmetry

- By linearizing NLSUSY we anticipate that we will obtain the (SUGRA-analogue) **broken global LSUSY theory expressed in terms of the composite of superon $\psi(x)$ and $e^a{}_\mu$:**

$$L_{\text{LSUSY}}(e^a{}_\mu, \psi_\nu(e, \psi), M(e, \psi), N(e, \psi), \dots)$$

which satisfies:

NL/L SUSY relation (equivalence):

$$L_{\text{NLSGR}}(w) = L_{\text{SGM}}(e, \psi) = L_{\text{LSUSY}}(e^a{}_\mu, \psi_\nu(e, \psi), M(e, \psi), N(e, \psi), \dots).$$

4. 1. NL/L SUSY relation (equivalence)

We demonstrate **NL/L SUSY relation for $N=2$ SUSY** in *flat space*:

@ $N = 2$ SGM in Riemann-flat ($e^a{}_\mu \rightarrow \delta^a{}_\mu$) space reduces to
 $N = 2$ NLSUSY (i.e., Λ term of $N = 2$ NLSGR):

We consider the linearization of Λ term of $N = 2$ NLSUSY
and show explicitly NL/L SUSY relation.

@ **$N=2, d=2$ NLSUSY theory:**

$$L_{N=2NLSUSY} = -\frac{1}{2\kappa^2} |\tilde{e}^a{}_\mu| = -\frac{1}{2\kappa^2} \left[1 + t^a{}_a + \frac{1}{2} (t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots \right], \quad (15)$$

$$|\tilde{e}^a{}_\mu| = \det w^a{}_b = \det(\delta_b^a + t^a{}_b), \quad t^a{}_b = -i\kappa^2 (\bar{\psi}_j \gamma^a \partial_b \psi^j - \bar{\psi}_j \gamma^a \partial_b \psi^j), \quad (j = 1, 2),$$

which is invariant under $N=2$ NLSUSY transformation,

$$\delta_\zeta \psi^j = \frac{1}{\kappa} \zeta^j - i\kappa (\bar{\zeta}_k \gamma^a \psi^k - \bar{\zeta}_k \gamma^a \psi^k) \partial_a \psi^j, \quad (j = 1, 2).$$

@ N=2, d=2 LSUSY Theory (SUSY QED):

- Helicity states of N=2 vector supermultiplet:

$$\left(\begin{array}{c} +1 \\ +\frac{1}{2}, +\frac{1}{2} \\ 0 \end{array} \right) + [\text{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY off-shell **minimal** vector supermultiplet: $(v^a, \lambda^i, A, \phi, D; i=1,2)$ in WZ gauge. (A and ϕ are two singlets, 0^+ and 0^- , scalar fields.)

- Helicity states of N=2 scalar supermultiplet:

$$\left(\begin{array}{c} +\frac{1}{2} \\ 0, 0 \\ -\frac{1}{2} \end{array} \right) + [\text{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY two scalar supermultiplets: $(\chi, B^i, \nu, F^i; i = 1, 2)$, B^i and F^i are complex.

- The most general $N = 2, d = 2$ SUSYQED action ($m = 0$ case) :

$$L_{N=2\text{SUSYQED}} = L_{V0} + L'_{\Phi0} + L_e + L_{Vf}, \quad (16)$$

$$L_{V0} = -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \not{\partial} \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D,$$

$$L'_{\Phi0} = \frac{i}{2}\bar{\chi} \not{\partial} \chi + \frac{1}{2}|\partial_a B^i|^2 + \frac{i}{2}\bar{\nu} \not{\partial} \nu + \frac{1}{2}|F^i|^2,$$

$$L_e = e \left\{ i v_a \bar{\chi} \gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \frac{1}{2} A (\bar{\chi} \chi + \bar{\nu} \nu) - \phi \bar{\chi} \gamma_5 \nu \right. \\ \left. + B^i (\bar{\lambda}^i \chi - \epsilon^{ij} \bar{\lambda}^j \nu) - \frac{1}{2} |B^i|^2 D \right\} + \{h.c.\} + \frac{1}{2} e^2 (v_a^2 - A^2 - \phi^2) |B^i|^2,$$

$$L_{Vf} = f \{ A \bar{\lambda}^i \lambda^i + \epsilon^{ij} \phi \bar{\lambda}^i \gamma_5 \lambda^j + (A^2 - \phi^2) D - \epsilon^{ab} A \phi F_{ab} \} \quad (17)$$

- Note that

$J = 0$ states in the vector multiplet for $N \geq 2$ SUSY induce Yukawa coupling.

$L_{N=2\text{SUSYQED}}$ is invariant under $N = 2$ **LSUSY** transformation:

- For the **minimal** vector off-shell supermultiplet:

$$\begin{aligned}
 \delta_\zeta v^a &= -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j, \\
 \delta_\zeta\lambda^i &= (D - i\cancel{\partial}A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\cancel{\partial}\phi\zeta^j, \\
 \delta_\zeta A &= \bar{\zeta}^i\lambda^i, \\
 \delta_\zeta\phi &= -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j, \\
 \delta_\zeta D &= -i\bar{\zeta}^i\cancel{\partial}\lambda^i.
 \end{aligned} \tag{18}$$

$$[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{19}$$

where $\zeta^i, i = 1, 2$ are constant spinors and $\delta_g(\theta)$ is the $U(1)$ gauge transformation for only v^a with $\Xi^\mu = 2i\bar{\zeta}_1^I\gamma^\mu\zeta_2^I$, $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- For the two scalar off-shell supermultiplets:

$$\begin{aligned}
\delta_\zeta \chi &= (F^i - i\partial B^i)\zeta^i - e\epsilon^{ij}V^i B^j, \\
\delta_\zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij}\bar{\zeta}^j \nu, \\
\delta_\zeta \nu &= \epsilon^{ij}(F^i + i\partial B^i)\zeta^j + eV^i B^i, \\
\delta_\zeta F^i &= -i\bar{\zeta}^i \partial \chi - i\epsilon^{ij}\bar{\zeta}^j \partial \nu \\
&\quad - e\{\epsilon^{ij}\bar{V}^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i)B^j - \bar{\zeta}^j \lambda^j B^i\}, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}] \chi &= \Xi^a \partial_a \chi - e\theta \nu, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}] B^i &= \Xi^a \partial_a B^i - e\epsilon^{ij}\theta B^j, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}] \nu &= \Xi^a \partial_a \nu + e\theta \chi, \\
[\delta_{\zeta_1}, \delta_{\zeta_2}] F^i &= \Xi^a \partial_a F^i + e\epsilon^{ij}\theta F^j,
\end{aligned} \tag{20}$$

with $V^i = iv_a \gamma^a \zeta^i - \epsilon^{ij} A \zeta^j - \phi \gamma_5 \zeta^i$ and the **U(1)** gauge parameter θ .

$N = 2$ NL/L SUSY relation(equivalence):

$$L_{\mathbf{N}=2\text{SUSYQED}} = L_{V0} + L'_{\Phi0} + L_e + L_{Vf} = L_{\mathbf{N}=2\text{NLSUSY}} + [\text{surface terms}], \quad (21)$$

is obtained by

(I) commutator-based (heuristic) linearization

or

(II) superfield-based (systematic) linearization.

(I) Commutator-based linearization:

@The product of **Lorentz tensors composed of ψ^i** multiplied by $|w\rangle$, $(\psi)^n \equiv 0, n > 4$ play a basic role.

- For such Lorentz tensors (of current) of ψ^i multiplied by $|w\rangle$,

$$b^i_A{}^{jk}{}_B{}^{l\dots m}{}_C{}^n \left((\psi^i)^{2(n-1)} |w\rangle \right) = \kappa^{2n-3} \bar{\psi}^i \gamma_A \psi^j \bar{\psi}^k \gamma_B \psi^l \dots \bar{\psi}^m \gamma_C \psi^n |w\rangle, \quad (22)$$

$$f^{ij}{}_A{}^{kl}{}_B{}^{m\dots n}{}_C{}^p \left((\psi^i)^{2n-1} |w\rangle \right) = \kappa^{2(n-1)} \psi^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p |w\rangle, \quad (23)$$

the variations under the NLSUSY transformations become

$$\begin{aligned} \delta_\zeta b^i_A{}^{jk}{}_B{}^{l\dots m}{}_C{}^n &= \kappa^{2(n-1)} \left[\left\{ (\bar{\zeta}^i \gamma_A \psi^j + \bar{\psi}^i \gamma_A \zeta^j) \bar{\psi}^k \gamma_B \psi^l \dots \bar{\psi}^m \gamma_C \psi^n + \dots \right\} |w\rangle \right. \\ &\quad \left. + \kappa \partial_a \left(\xi^a \bar{\psi}^i \gamma_A \psi^j \bar{\psi}^k \gamma_B \psi^l \dots \bar{\psi}^m \gamma_C \psi^n |w\rangle \right) \right], \end{aligned} \quad (24)$$

$$\begin{aligned} \delta_\zeta f^{ij}{}_A{}^{kl}{}_B{}^{m\dots n}{}_C{}^p &= \kappa^{2n-1} \left[\left\{ \zeta^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p \right. \right. \\ &\quad \left. \left. + \psi^i (\bar{\zeta}^j \gamma_A \psi^k + \bar{\psi}^j \gamma_A \zeta^k) \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p + \dots \right\} |w\rangle \right. \\ &\quad \left. + \kappa \partial_a \left(\xi^a \psi^i \bar{\psi}^j \gamma_A \psi^k \bar{\psi}^l \gamma_B \psi^m \dots \bar{\psi}^n \gamma_C \psi^p |w\rangle \right) \right], \end{aligned} \quad (25)$$

where $\xi^a = i\kappa\bar{\zeta}^i\gamma^a\psi^i$.

- These show that the tables of products of Lorentz tensors (of currents) of ψ^i multiplied by $|w|$ give a **finite representation of NLSUSY algebra**.
- The self-contained part of Lorentz-tensor table corresponds to smaller LSUSY multiplet.
- They satisfy the basic commutator under NLSGR tr.

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v),$$

where $\delta_P(v)$ is a translation with a parameter $v^a = 2i(\bar{\zeta}_{1L}^i\gamma^a\zeta_{2L}^i - \bar{\zeta}_{1R}^i\gamma^a\zeta_{2R}^i)$

- These results show that the commutator-based linearization closes on the all possible Lorentz tensors composed of ψ^i and gives a finite dimensional representation of sP algebra.
- Assign each composite Lorentz tensor to the component field of the LSUSY supermultiplet including the auxiliary field (**susy compositeness**), which reproduces the familiar LSUSY transformation among the supermultiplet under the NLSUSY transformations of constituents ψ^i .
- Substituting SUSY compositeness into $L_{N=2LSUSYQED}$, we obtain $L_{N=2NLSUSY}$, i .e. the **NL/L SUSY relation(equivalence)**.

- **SUSY compositeness** for the vector off-shell **minimal** supermultiplet:

$$v^a = -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|,$$

$$\lambda^i = \xi\psi^i|w|,$$

$$A = \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|,$$

$$\phi = -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|,$$

$$D = \frac{\xi}{\kappa}|w|, \tag{26}$$

where ξ is a VEV factor of the auxiliary field D .

- Note that ψ^i is the low energy leading term of the supercharge Q^i .

- **SUSY compositeness** for scalar off-shell **minimal** supermultiplets:

$$\begin{aligned}
\chi &= \xi^i \left[\psi^i |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j |w| \} \right] \\
B^i &= -\kappa \left(\frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|, \\
\nu &= \xi^i \epsilon^{ij} \left[\psi^j |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k |w| \} \right], \\
F^i &= \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8} \kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|) \right\} - i \kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|) \\
&\quad - \frac{1}{4} e \kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|. \tag{27}
\end{aligned}$$

- The quartic fermion self-interaction term in F^i realizes the local $U(1)$ gauge symmetry of LSUSY.
- ξ^i is the VEV factor of the auxiliary field F^i .

- **SUSY compositeness** produces under NLSUSY transformation

off-shell commutator algebra :

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \quad (28)$$

where $\delta_P(v)$ is a translation with a parameter

$$v^a = 2i(\bar{\zeta}_{1L}^i \gamma^a \zeta_{2L}^i - \bar{\zeta}_{1R}^i \gamma^a \zeta_{2R}^i) \quad (29)$$

- Note that the commutator does not induce the **U(1)** gauge transformation for the off-shell supermultiplet.

- Substituting **SUSY copositeness** into $L_{N=2LSUSYQED}$, we obtain **NL/L SUSY relation for the minimal off-shell supermultiplet:**

$$L_{N=2LSUSYQED} = f(\xi, \xi^i) L_{N=2NLSUSY} + [\text{suface terms}], \quad (30)$$

$$f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1. \quad (31)$$

⇒ **LSUSY multiplet may be regarded as composite eigenstates of global SUSY space-time symmetries.**

- **NL/L SUSY relation bridges naturally the cosmology and the low energy particle physics in NLSGR. (⇒ Sec. 4).**

- **The direct linearization of highly nonlinear SGM action (??) in curved space remains to be carried out.**

→

**In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
from
the cosmological term Λ of SGM and materializes the true vacuum of SGM
(as gravitational composites of NG fermion)**

SM can be a low energy effective theory of SGM/NLSGR.

(II) Systematic Linearization of NLSUSY by the superfield formulation($d = 2$)

- General superfields are given for the $N = 2$ vector supermultiplet by

$$\begin{aligned} \mathcal{V}(x, \theta^i) = & C(x) + \bar{\theta}^i \Lambda^i(x) + \frac{1}{2} \bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2} \bar{\theta}^i \theta^i M^{jj}(x) + \frac{1}{4} \epsilon^{ij} \bar{\theta}^i \gamma_5 \theta^j \underline{\phi}(x) \\ & - \frac{i}{4} \epsilon^{ij} \bar{\theta}^i \gamma_a \theta^j \underline{v}^a(x) - \frac{1}{2} \bar{\theta}^i \theta^i \bar{\theta}^j \underline{\lambda}^j(x) - \frac{1}{8} \bar{\theta}^i \theta^i \bar{\theta}^j \theta^j \underline{D}(x), \end{aligned} \quad (32)$$

and for the $N = 2$ scalar supermultiplet by

$$\begin{aligned} \Phi^i(x, \theta^i) = & \underline{B}^i(x) + \bar{\theta}^i \underline{\chi}(x) - \epsilon^{ij} \bar{\theta}^j \underline{\nu}(x) - \frac{1}{2} \bar{\theta}^j \theta^j \underline{F}^i(x) + \bar{\theta}^i \theta^j \underline{F}^j(x) - i \bar{\theta}^i \not{\partial} B^j(x) \theta^j \\ & + \frac{i}{2} \bar{\theta}^j \theta^j (\bar{\theta}^i \not{\partial} \chi(x) - \epsilon^{ik} \bar{\theta}^k \not{\partial} \nu(x)) + \frac{1}{8} \bar{\theta}^j \theta^j \bar{\theta}^k \theta^k \partial_a \partial^a B^i(x). \end{aligned} \quad (33)$$

e, g.

$$[\underline{\mathbf{1}}(1), \underline{\mathbf{2}}(\frac{1}{2}), \underline{\mathbf{1}}(0)] + [CPT \ conj.] \quad \text{and} \quad [\underline{\mathbf{1}}(\frac{1}{2}), \underline{\mathbf{2}}(0), \underline{\mathbf{1}}(-\frac{1}{2})] + [CPT \ conj.]$$

- Extend the superspace to the following superspace (x'^a, θ') with $-\kappa\psi(x)$,

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i + \kappa\psi^i, \quad (34)$$

and denotes the extended superfields on (x'^a, θ'^i) and their θ -expansions as

$$\mathcal{V}(x'^a, \theta'^i) = \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \Phi(x'^a, \theta'^i) = \tilde{\Phi}(x^a, \theta^i; \psi^i(x)). \quad (35)$$

- **Extended** global SUSY translations on (x'^a, θ'^i) give $\xi_\mu = -i\kappa\bar{\epsilon}\gamma^\mu\psi$:

$$\delta\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \delta\tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\Phi}(x^a, \theta^i; \psi^i(x)), \quad (36)$$

- Therefore, the conditions

$$\tilde{\varphi}_{\mathcal{V}}^I(x) = \xi_{\mathcal{V}}^I(\text{constant}) \quad \tilde{\varphi}_{\Phi}^I(x) = \xi_{\Phi}^I(\text{constant}), \quad (37)$$

make extended superfields invariant under the extended SUSY translation. which provide **SUSY compositeness** by solving them for the original component fields.

- Putting constants in the most general case as follows:

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \xi_\Lambda^i, \quad \tilde{M}^{ij} = \xi_M^{ij}, \quad \tilde{\phi} = \xi_\phi, \quad \tilde{v}^a = \xi_v^a, \quad \tilde{\lambda}^i = \xi_\lambda^i, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad (38)$$

$$\tilde{B}^i = \xi_B^i, \quad \tilde{\chi} = \xi_\chi, \quad \tilde{\nu} = \xi_\nu, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \quad (39)$$

- We obtain straightforwardly **SUSY compositeness** $\varphi_V^I = \varphi_V^I(\psi)$ for the vector supermultiplet

$$C = \xi_c + \kappa \bar{\psi}^i \xi_\Lambda^i + \frac{1}{2} \kappa^2 (\xi_M^{ij} \bar{\psi}^i \psi^j - \xi_M^{ii} \bar{\psi}^j \psi^j) + \frac{1}{4} \xi_\phi \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - \frac{i}{4} \xi_v^a \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \\ - \frac{1}{2} \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \xi_\lambda^j - \frac{1}{8} \xi \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j,$$

$$\Lambda^i = \xi_\Lambda^i + \kappa (\xi_M^{ij} \psi^j - \xi_M^{jj} \psi^i) + \frac{1}{2} \xi_\phi \kappa \epsilon^{ij} \gamma_5 \psi^j - \frac{i}{2} \xi_v^a \kappa \epsilon^{ij} \gamma_a \psi^j \\ - \frac{1}{2} \xi_\lambda^i \kappa^2 \bar{\psi}^j \psi^j + \frac{1}{2} \kappa^2 (\psi^j \bar{\psi}^i \xi_\lambda^j - \gamma_5 \psi^j \bar{\psi}^i \gamma_5 \xi_\lambda^j - \gamma_a \psi^j \bar{\psi}^i \gamma_a \xi_\lambda^j) \\ - \frac{1}{2} \xi \kappa^2 \psi^i \bar{\psi}^j \psi^j - i \kappa \not{\partial} C(\psi) \psi^i,$$

$$\begin{aligned}
M^{ij} &= \xi_M^{ij} + \kappa \bar{\psi}^i \xi_\lambda^j + \frac{1}{2} \xi \kappa \bar{\psi}^i \psi^j + i \kappa \epsilon^{(i|k|} \epsilon^{j)l} \bar{\psi}^k \not{\partial} \Lambda^l(\psi) - \frac{1}{2} \kappa^2 \epsilon^{ik} \epsilon^{jl} \bar{\psi}^k \psi^l \partial^2 C(\psi), \\
\phi &= \xi_\phi - \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \xi_\lambda^j - \frac{1}{2} \xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - i \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \not{\partial} \Lambda^j(\psi) + \frac{1}{2} \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \partial^2 C(\psi), \\
v^a &= \xi_v^a - i \kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \xi_\lambda^j - \frac{i}{2} \xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j - \kappa \epsilon^{ij} \bar{\psi}^i \not{\partial} \gamma^a \Lambda^j(\psi) + \frac{i}{2} \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j \partial^2 C(\psi) \\
&\quad - i \kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^b \psi^j \partial^a \partial_b C(\psi), \\
\lambda^i &= \xi_\lambda^i + \xi \psi^i - i \kappa \not{\partial} M^{ij}(\psi) \psi^j + \frac{i}{2} \kappa \epsilon^{ab} \epsilon^{ij} \gamma_a \psi^j \partial_b \phi(\psi) \\
&\quad - \frac{1}{2} \kappa \epsilon^{ij} \left\{ \psi^j \partial_a v^a(\psi) - \frac{1}{2} \epsilon^{ab} \gamma_5 \psi^j F_{ab}(\psi) \right\} \\
&\quad - \frac{1}{2} \kappa^2 \{ \partial^2 \Lambda^i(\psi) \bar{\psi}^j \psi^j - \partial^2 \Lambda^j(\psi) \bar{\psi}^i \psi^j - \gamma_5 \partial^2 \Lambda^j(\psi) \bar{\psi}^i \gamma_5 \psi^j \\
&\quad - \gamma_a \partial^2 \Lambda^j(\psi) \bar{\psi}^i \gamma^a \psi^j + 2 \not{\partial} \partial_a \Lambda^j(\psi) \bar{\psi}^i \gamma^a \psi^j \} - \frac{i}{2} \kappa^3 \not{\partial} \partial^2 C(\psi) \psi^i \bar{\psi}^j \psi^j, \\
D &= \frac{\xi}{\kappa} - i \kappa \bar{\psi}^i \not{\partial} \lambda^i(\psi)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \kappa^2 \left\{ \bar{\psi}^i \psi^j \partial^2 M^{ij}(\psi) - \frac{1}{2} \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \partial^2 \phi(\psi) \right. \\
& \left. + \frac{i}{2} \epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \partial^2 v^a(\psi) - i \epsilon^{ij} \bar{\psi}^i \gamma_a \psi^j \partial_a \partial_b v^b(\psi) \right\} \\
& - \frac{i}{2} \kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \not{\partial} \partial^2 \Lambda^j(\psi) + \frac{1}{8} \kappa^4 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j \partial^4 C(\psi), \tag{40}
\end{aligned}$$

and **SUSY compositeness for the scalar multiplet** $\varphi_{\Phi}^I = \varphi_{\Phi}^I(\psi)$:

$$\begin{aligned}
B^i &= \xi_B^i + \kappa (\bar{\psi}^i \xi_{\chi} - \epsilon^{ij} \bar{\psi}^j \xi_{\nu}) - \frac{1}{2} \kappa^2 \{ \bar{\psi}^j \psi^j F^i(\psi) - 2 \bar{\psi}^i \psi^j F^j(\psi) + 2i \bar{\psi}^i \not{\partial} B^j(\psi) \psi^j \} \\
&\quad - i \kappa^3 \bar{\psi}^j \psi^j \{ \bar{\psi}^i \not{\partial} \chi(\psi) - \epsilon^{ik} \bar{\psi}^k \not{\partial} \nu(\psi) \} + \frac{3}{8} \kappa^4 \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \partial^2 B^i(\psi), \\
\chi &= \xi_{\chi} + \kappa \{ \psi^i F^i(\psi) - i \not{\partial} B^i(\psi) \psi^i \} \\
&\quad - \frac{i}{2} \kappa^2 [\not{\partial} \chi(\psi) \bar{\psi}^i \psi^i - \epsilon^{ij} \{ \psi^i \bar{\psi}^j \not{\partial} \nu(\psi) - \gamma^a \psi^i \bar{\psi}^j \partial_a \nu(\psi) \}] \\
&\quad + \frac{1}{2} \kappa^3 \psi^i \bar{\psi}^j \psi^j \partial^2 B^i(\psi) + \frac{i}{2} \kappa^3 \not{\partial} F^i(\psi) \psi^i \bar{\psi}^j \psi^j + \frac{1}{8} \kappa^4 \partial^2 \chi(\psi) \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j,
\end{aligned}$$

$$\begin{aligned}
\nu &= \xi_\nu - \kappa \epsilon^{ij} \{ \psi^i F^j(\psi) - i \not{\partial} B^i(\psi) \psi^j \} \\
&\quad - \frac{i}{2} \kappa^2 [\not{\partial} \nu(\psi) \bar{\psi}^i \psi^i + \epsilon^{ij} \{ \psi^i \bar{\psi}^j \not{\partial} \chi(\psi) - \gamma^a \psi^i \bar{\psi}^j \partial_a \chi(\psi) \}] \\
&\quad + \frac{1}{2} \kappa^3 \epsilon^{ij} \psi^i \bar{\psi}^k \psi^k \partial^2 B^j(\psi) + \frac{i}{2} \kappa^3 \epsilon^{ij} \not{\partial} F^i(\psi) \psi^j \bar{\psi}^k \psi^k + \frac{1}{8} \kappa^4 \partial^2 \nu(\psi) \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j, \\
F^i &= \frac{\xi^i}{\kappa} - i \kappa \{ \bar{\psi}^i \not{\partial} \chi(\psi) + \epsilon^{ij} \bar{\psi}^j \not{\partial} \nu(\psi) \} \\
&\quad - \frac{1}{2} \kappa^2 \bar{\psi}^j \psi^j \partial^2 B^i(\psi) + \kappa^2 \bar{\psi}^i \psi^j \partial^2 B^j(\psi) + i \kappa^2 \bar{\psi}^i \not{\partial} F^j(\psi) \psi^j \\
&\quad + \frac{1}{2} \kappa^3 \bar{\psi}^j \psi^j \{ \bar{\psi}^i \partial^2 \chi(\psi) + \epsilon^{ik} \bar{\psi}^k \partial^2 \nu(\psi) \} - \frac{1}{8} \kappa^4 \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \partial^2 F^i(\psi). \tag{41}
\end{aligned}$$

- Choosing the following **Lorentz invariant** and **SUSY invariant constraints** of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \quad (42)$$

gives **previous SUSY compositeness** for the minimal supermultiplet of SUSYQED. and the $N = 2$ NLSUSY action $S_{N=2\text{NLSUSY}}$ is related to $N = 2$ SUSY QED action, NL/L SUSY relation

$$L_{N=2\text{SUSYQED}} \equiv L_{\mathcal{V}\text{free}} + L_{\mathcal{V}f} + L_{\text{gauge}} = f(\xi, \xi^i) S_{N=2\text{NLSUSY}} \quad (43)$$

when $f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1$.

- NL/L SUSY relation bridges **the cosmology** and **the low energy particle physics** in **NLSGR** scenario \implies Sec. 5.

@ SGM scenario predicts the magnitude of the bare gauge coupling constant.

For more general SUSY invariant constraints (vev of 0^+ auxiliary field):

$$\underline{\tilde{C}} = \xi_c, \quad \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \quad (44)$$

NL/L SUSY relation gives

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \quad e = \frac{\ln\left(\frac{\xi^i^2}{\xi^2 - 1}\right)}{4\xi_c}, \quad (45)$$

where e is the bare gauge coupling constant.

- This mechanism is natural and favorable for SGM scenario as a theory of everything.

Broken LSUSY(QED) gauge theory is encoded
in the vacuum of NLSUSY theory
as composites of NG fermion.

4.2 $N = 3$ NL/L SUSY relation and SUSY Yang-Mills theory

- **Physical helicity states of $N = 3$ LSUSY vector supermultiplet:**

$$\left[\underline{1}(+1), \underline{3}\left(+\frac{1}{2}\right), \underline{3}(0), \underline{1}\left(-\frac{1}{2}\right) \right] + [\text{CPT conjugate}], \quad (46)$$

where $\underline{n}(\lambda)$ means the dimension \underline{n} and the helicity λ , are accommodated in $N = 3$ off-shell vector supermultiplet ($d = 2$):

- $N = 3$ superYang-Mills(SUSYYM) minimal off-shell gauge multiplet,

$$\{v^{aI}(x), \lambda^{iI}(x), A^{iI}(x), \chi_{\alpha}^I(x), \phi^I(x), D^{iI}(x)\}, \quad (I = 1, 2, \dots, \dim.G) \quad (47)$$

Each component field belongs to the adjoint representation of the YM gauge group G :

$[T^I, T^J] = if^{IJK}T^K$ and denoted as $\varphi^i = \varphi^{iI}T^I$, etc..

- $N = 3$ (pure) SUSYM action:

$$\begin{aligned}
S_{\text{SYM}} = \int d^2x \operatorname{tr} \left\{ & -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \not{D} \lambda^i + \frac{1}{2}(D_a A^i)^2 + \frac{i}{2}\bar{\chi} \not{D} \chi + \frac{1}{2}(D_a \phi)^2 + \frac{1}{2}(D^i)^2 \right. \\
& -ig\{\epsilon^{ijk} A^i \bar{\lambda}^j \lambda^k - [A^i, \bar{\lambda}^i] \chi + \phi(\bar{\lambda}^i \gamma_5 \lambda^i + \bar{\chi} \gamma_5 \chi)\} \\
& \left. + \frac{1}{4}g^2([A^i, A^j]^2 + 2[A^i, \phi]^2) \right\}, \tag{48}
\end{aligned}$$

where g is the gauge coupling constant, D_a and F_{ab} are the covariant derivative and the YM gauge field strength defined as

$$\begin{aligned}
D_a \varphi &= \partial_a \varphi - ig[v_a, \varphi], \\
F_{ab} &= \partial_a v_b - \partial_b v_a - ig[v_a, v_b]. \tag{49}
\end{aligned}$$

• **SUSY YM action is invariant under $N = 3$ LSUSY transformations:**

$$\begin{aligned}
\delta_\zeta v^a &= i\bar{\zeta}^i \gamma^a \lambda^i, \\
\delta_\zeta \lambda^i &= \epsilon^{ijk} (D^j - i\mathcal{D}A^j) \zeta^k + \frac{1}{2} \epsilon^{ab} F_{ab} \gamma_5 \zeta^i - i\gamma_5 \mathcal{D}\phi \zeta^i \\
&\quad + ig([A^i, A^j] \zeta^j + \epsilon^{ijk} [A^j, \phi] \gamma_5 \zeta^k), \\
\delta_\zeta A^i &= \epsilon^{ijk} \bar{\zeta}^j \lambda^k - \bar{\zeta}^i \chi, \\
\delta_\zeta \chi &= (D^i + i\mathcal{D}A^i) \zeta^i + ig(\epsilon^{ijk} A^i A^j \zeta^k - [A^i, \phi] \gamma_5 \zeta^i), \\
\delta_\zeta \phi &= \bar{\zeta}^i \gamma_5 \lambda^i, \\
\delta_\zeta D^i &= -i\epsilon^{ijk} \bar{\zeta}^j \mathcal{D}\lambda^k - i\bar{\zeta}^i \mathcal{D}\chi + ig(\bar{\zeta}^i [\lambda^j, A^j] + \bar{\zeta}^j [\lambda^i, A^j] - \bar{\zeta}^j [\lambda^j, A^i] \\
&\quad - \epsilon^{ijk} \bar{\zeta}^j [\chi, A^k] + \epsilon^{ijk} \bar{\zeta}^j \gamma_5 [\lambda^k, \phi] + \bar{\zeta}^i \gamma_5 [\chi, \phi]), \tag{50}
\end{aligned}$$

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_P(\Xi^a) + \delta_G(\theta) + \delta_g(\theta), \tag{51}$$

where $\delta_G(\theta)$ means $\delta_G(\theta)\varphi = ig[\theta, \varphi]$ and $\delta_g(\theta)$ is the $U(1)$ gauge transformation only for v^a with $\theta = -2(i\bar{\zeta}_1^i \gamma^a \zeta_2^i v_a - \epsilon^{ijk} \bar{\zeta}_1^i \zeta_2^j A^k - \bar{\zeta}_1^i \gamma_5 \zeta_2^i \phi)$.

- SUSY invariant(composite) relations for $N = 3$ YM off-shell gauge supermultiplet**

$$\begin{aligned}
v^{aI} &= -\frac{i}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma^a\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\psi^l) + \frac{1}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_b(\bar{\psi}^j\gamma_5\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
\lambda^{iI} &= \epsilon^{ijk}\xi^{jI}\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\psi^l) \\
&\quad + \frac{i}{2}\kappa^2\xi^{jI}\partial_a\{\epsilon^{ijk}\gamma^a\psi^k\bar{\psi}^l\psi^l + \epsilon^{ab}\epsilon^{jkl}(\gamma_b\psi^i\bar{\psi}^k\gamma_5\psi^l - \gamma_5\psi^i\bar{\psi}^k\gamma_b\psi^l)\} + \mathcal{O}(\kappa^4), \\
A^{iI} &= \kappa\left(\frac{1}{2}\xi^{iI}\bar{\psi}^j\psi^j - \xi^{jI}\bar{\psi}^i\psi^j\right)(1 - i\kappa^2\bar{\psi}^k\partial\psi^k) - \frac{i}{2}\kappa^3\xi^{iI}\partial_a(\bar{\psi}^i\gamma^a\psi^j\bar{\psi}^k\psi^k) + \mathcal{O}(\kappa^5), \\
\chi^I &= \xi^{iI}\psi^i(1 - i\kappa^2\bar{\psi}^j\partial\psi^j) + \frac{i}{2}\kappa^2\xi^{iI}\partial_a(\gamma^a\psi^i\bar{\psi}^j\psi^j) + \mathcal{O}(\kappa^4), \\
\phi^I &= -\frac{1}{2}\kappa\epsilon^{ijk}\xi^{iI}\bar{\psi}^j\gamma_5\psi^k(1 - i\kappa^2\bar{\psi}^l\partial\psi^l) - \frac{i}{4}\kappa^3\epsilon^{ab}\epsilon^{ijk}\xi^{iI}\partial_a(\bar{\psi}^j\gamma_b\psi^k\bar{\psi}^l\psi^l) + \mathcal{O}(\kappa^5), \\
D^{iI} &= \frac{1}{\kappa}\xi^{iI}|w| - i\kappa\xi^{jI}\partial_a\{\bar{\psi}^i\gamma^a\psi^j(1 - i\kappa^2\bar{\psi}^k\partial\psi^k)\} \\
&\quad - \frac{1}{8}\kappa^3\partial_a\partial^a\{(\xi^{iI}\bar{\psi}^j\psi^j - 4\xi^{jI}\bar{\psi}^i\psi^j)\bar{\psi}^k\psi^k\} + \mathcal{O}(\kappa^5), \tag{52}
\end{aligned}$$

- Arbitrary real constants ξ^{iI} of auxiliary fields D^{iI} bridge $N = 3$ SUSY and the YM gauge group G .

- Substituting (??) into the SYM action (??), we can show the NL/L SUSY relation for $N = 3$ SUSY:

$$L_{\text{SUSY YM}}(\psi) = -(\xi^{iI})^2 L_{\text{NLSUSY}} + [\text{surface terms}]. \quad (53)$$

4.3 LSUSY of higher-spin massless supermultiplet of SGM:

For the massless supermultiplet with the highest-helicity $(\pm 3, \pm \frac{5}{2})$ of SGM, we take **the symmetric tensor(-spinor) gauge field multiplet** $(\varphi_{\mu\nu\rho}, \lambda_{\mu\nu})$.

The equations of motion and the local gauge transformations are given by :
(Ref.: de Wit B. and Freedman D. Z. Phys. Rev. D21(1980), 358-367.)

$$\begin{aligned}
 \Gamma_{\sigma^{\sigma}, \mu_1 \mu_2 \mu_3} &= \partial^{\rho} \partial_{\rho} \varphi_{\mu_1 \mu_2 \mu_3} - \sum_{\mu(1)} \partial_{\mu_1} \partial_{\rho} \varphi^{\rho}_{\mu_2 \mu_3} + \sum_{\mu(2)} \partial_{\mu_1} \partial_{\mu_2} \varphi^{\rho}_{\rho \mu_3} = 0, \\
 \delta_{\xi} \varphi_{\mu_1 \mu_2 \mu_3} &= \sum_{\mu(1)} \partial_{\mu_1} \xi_{\mu_2 \mu_3} \equiv \partial_{\mu_1} \xi_{\mu_2 \mu_3} + \partial_{\mu_2} \xi_{\mu_1 \mu_3} + \partial_{\mu_3} \xi_{\mu_1 \mu_2}, \\
 \gamma^{\rho} \Gamma^{(1)}_{\rho, \mu_1 \mu_2} &= \gamma^{\rho} \partial_{\rho} \lambda_{\mu_1 \mu_2} - \gamma^{\rho} \partial_{\mu_1} \lambda_{\rho \mu_2} - \gamma^{\rho} \partial_{\mu_2} \lambda_{\mu_1 \rho} = 0, \\
 \delta_{\epsilon} \lambda_{\mu_1 \mu_2} &= \partial_{\mu_1} \epsilon_{\mu_2} + \partial_{\mu_2} \epsilon_{\mu_1}, \\
 \xi_{\rho}^{\rho} &= 0, \quad \gamma^{\rho} \epsilon_{\rho} = 0
 \end{aligned} \tag{54}$$

$\Gamma_{\sigma_1 \sigma_2, \mu_1 \mu_2 \mu_3}$ is spin s Christoffel symbol of ordinary gravity.
The arguments are systematic and geometric.

The naive extension of the familiar **global LSUY** for the **on-shell** gauge vector supermultiplet:

$$\begin{aligned}\delta_\zeta \varphi_{abc} &= -i\bar{\zeta}\gamma_{\dot{a}}\lambda_{b\dot{c}}, \\ \delta_\zeta \lambda_{ab} &= \sigma^{cd}\partial_c\varphi_{d\dot{a}b}\zeta,\end{aligned}\tag{55}$$

produce for φ_{abc} the familiar **SUSY** algebra:

$$\begin{aligned}[\delta_2, \delta_1]\varphi_{abc} &= \delta_P + \delta_G, \\ \delta_P &= (i\bar{\zeta}_1\gamma^d\zeta_2)\partial_d\varphi_{abc}, \\ \delta_G &= \partial_{\dot{a}}\{(i\bar{\zeta}_1\gamma^d\zeta_2)\varphi_{d\dot{b}c}\},\end{aligned}\tag{56}$$

While for $\lambda_{\mu\nu}$, the Fierz rearrangement of two supertransformations $\delta_2\delta_1\lambda_{\mu\nu}$

$$\delta_2\delta_1\lambda_{\mu\nu} = (\bar{\epsilon}_2\gamma_a\partial_\rho\lambda_{\mu\nu})\sigma^{\rho\sigma}\epsilon_1 = -\frac{1}{4}\Sigma_I(\bar{\epsilon}_2\Gamma_I\epsilon_1)\sigma^{\rho\sigma}\Gamma_I\gamma_\sigma\partial_{rho}\lambda_{\mu\nu},\tag{57}$$

where $\Gamma_I = (1, \gamma_\mu, 2i\sigma_{\mu\nu}, \gamma_5, \gamma_5\gamma_\mu)$ shows that the non-zero contribution to the commutator $[\delta_2, \delta_1]\lambda_{\mu\nu}$ comes from the vector $\bar{\epsilon}_1\gamma_\mu\epsilon_2$ and the antisymmetric tensor $i\bar{\epsilon}_1\tilde{\sigma}^{\kappa\lambda}\epsilon_2$ terms of the parameters in the rearrangement and becomes as follows with the **pathological** extra term δ_Σ containing $i\bar{\zeta}_1\sigma^{de}\zeta_2$:

$$\begin{aligned}
[\delta_2, \delta_1]\lambda_{ab} &= \delta_P + \delta_G + \delta_{\tilde{G}} + \delta_\Sigma, \\
\delta_P &= -24(i\bar{\zeta}_1\gamma^d\zeta_2)\partial_d\lambda_{ab}, \\
\delta_G &= \partial_{\dot{a}}\{(i\bar{\zeta}_1\gamma^d\zeta_2)(8\lambda_{db} - 4\gamma_b\gamma^c\lambda_{dc} + \dots)\}, \\
\delta_{\tilde{G}} &= \partial_{\dot{a}}\{(i\bar{\zeta}_1\sigma^{cd}\zeta_2)(\gamma_c\lambda_{db} + \dots)\}, \\
\delta_\Sigma &= 2(i\bar{\zeta}_1\sigma^{cd}\zeta_2)(\gamma_{\dot{a}}\partial_c\lambda_{db} + \gamma_c\partial_d\lambda_{\dot{a}b}), \tag{58}
\end{aligned}$$

The generalization of simple SUSY transformation (55) and the **off shell** extension of the multiplet must be considered, e.g., minimal **off-shell** case: $[\#40]_F = [\#40]_B$.

4.4 Linearization of $L_{SGM}(e, \psi)$ in curved space-time

@ Commutator-based linearization:

• Show

NL/L SUSY relation (equivalence):

$$L_{NLSGR}(w) = L_{SGM}(e, \psi) = L_{LSUSY(\text{globalSUGRA})}(e^a{}_\mu, \psi_\nu, v^a, \lambda, \phi, M, N, \dots).$$

under

• NLSUSY(**global**) transformations

$$\delta_\zeta \psi^i = \frac{1}{\kappa} \zeta^i + \xi^\mu \partial_\mu \psi^i, \quad \delta_\zeta e^a{}_\mu = 2i\kappa \bar{\zeta}^i \gamma^\nu \psi^i \partial_{[\mu} e^a{}_{\nu]}, \quad \xi^\mu = i\kappa \bar{\psi}^j \gamma^\mu \zeta^j.$$

and the commutator algebra,

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] = \delta_{GL(4,R)}(\Xi^\mu), \quad \Xi^\mu = 2(i\bar{\zeta}_1^i \gamma^\mu \zeta_2^i - \xi^\nu{}_1 \xi^\kappa{}_2 e^\mu{}_a \partial_{[\mu} e^a{}_{\nu]})$$

• Find **global SUSY, off-shell SUGRA supermultiplet**
and the SUSY composite expression. e. g. $16_B = 16_F$

- In curved space-time, Lorentz- and Riemann-tensor functionals with γ -matrices

$$F_M^{IA} = F_M^{IA}(\psi^i, e_\mu^a, \partial\psi^i, \partial_{[\nu}e^a{}_{\kappa]})$$

satisfies the commutator algebra,
which is favourable to **the linearized SUGRA-like LSUSY** structure.

Now under study

5. Low energy particle physics of NLSGR

@ As we have seen in the toy model that

$N = 2$ SGM is in the lowest order essentially $N=2$ NLSUSY action in tangent (flat) space-time, we focus on $N=2$ NLSUSY action for extracting physical implications of SGM.

- The low energy theorem for NLSUSY gives the following **superon(massless NG fermion)-vacuum coupling**

$$\langle \psi^j_\alpha(x) | J^{k\mu}_\beta | 0 \rangle = i \sqrt{\frac{c^4 \Lambda}{8\pi G}} (\gamma^\mu)_{\alpha\beta} \delta^{jk} + \dots, \quad (59)$$

where $J^{k\mu} = i \sqrt{\frac{c^4 \Lambda}{8\pi G}} \gamma^\mu \psi^k + \dots$ is the conserved supercurrent.

$\sqrt{\frac{c^4 \Lambda}{8\pi G}} = \frac{1}{\sqrt{2\kappa}}$ is the coupling constant ($\equiv g_{sv}$) of superon with the vacuum.

$$\implies m_\nu^4 \sim \rho_{DE},$$

- How to extract the vacuum configuration of SGM.

In Riemann-flat space-time, **NL/L SUSY relation(equivalence)** gives:

$$L_{N=2\text{SGM}} \longrightarrow L_{N=2\text{NLSUSY}} + [\text{surface terms}] = L_{N=2\text{SUSYQED}}. \quad (60)$$

- We study vacuum structures of $N = 2$ **LSUSY QED** action in stead of $N = 2$ SGM.

The vacuum is given by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2\text{LSUSYQED}}$,

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{ \frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e|B^i|^2 \right\} D + \frac{e^2}{2}(A^2 + \phi^2)|B^i|^2. \quad (61)$$

- Substituting the solution of the equation of motion for the auxiliary field D we obtain

$$V(A, \phi, B^i) = \frac{1}{2}f^2 \left\{ A^2 - \phi^2 - \frac{e}{2f}|B^i|^2 - \frac{\xi}{f\kappa} \right\}^2 + \frac{1}{2}e^2(A^2 + \phi^2)|B^i|^2 \geq 0. \quad (62)$$

- Two different types of vacua $V = 0$ exist in (A, ϕ, B^i) -space:

$$(I) \quad A = \phi = 0, \quad |\tilde{B}^i|^2 = -k^2 \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}}B^i, \quad k^2 = \frac{\xi}{f\kappa} \right) \quad (63)$$

and

$$(II) \quad |\tilde{B}^i|^2 = 0, \quad A^2 - \phi^2 = k^2. \quad (64)$$

- Expansions of A, ϕ, \tilde{B}^i around vacuum values give low energy particles $\hat{A}, \hat{\phi}, \hat{B}^i$ in the true vacuum.

- For the type (I) vacuum with $SO(2)$ symmetry for $(\tilde{B}^1, \tilde{B}^2)$, $e\xi < 0$,

$$\begin{aligned}
L_{N=2\text{SUSYQED}} = & \frac{1}{2}\{(\partial_a\rho)^2 - 2(-ef)k^2\rho^2\} \\
& + \frac{1}{2}\{(\partial_a\hat{A})^2 + (\partial_a\hat{\phi})^2 - 2(-ef)k^2(\hat{A}^2 + \hat{\phi}^2)\} \\
& - \frac{1}{4}(F_{ab})^2 + (-ef)k^2v_a^2 \\
& + \frac{i}{2}\bar{\lambda}^i\partial\lambda^i + \frac{i}{2}\bar{\chi}\partial\chi + \frac{i}{2}\bar{\nu}\partial\nu + \sqrt{-2ef}(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu) + \dots,
\end{aligned}
\tag{65}$$

and following mass spectra

$$\begin{aligned} m_{\rho}^2 &= m_{\hat{A}}^2 = m_{\hat{\phi}}^2 = m_{\nu_a}^2 = 2(-ef)k^2 = -\frac{2\xi e}{\kappa}, \\ m_{\lambda^i} &= m_{\chi} = m_{\nu} = 0. \end{aligned} \tag{66}$$

- The vacuum breaks **both SUSY and the local $U(1)(O(2))$ spontaneously** and Higgs-Kibble mechanism works.
- All bosons have the same mass and all fermions remain massless.
- λ^i are NG fermions.

- For the type (II) vacuum with $SO(1, 1)$ symmetry for (A, ϕ) , $f\xi > 0$,

$$\begin{aligned}
L_{N=2\text{SUSYQED}} = & \frac{1}{2}\{(\partial_a \hat{A})^2 - 4f^2 k^2 \hat{A}^2\} \\
& + \frac{1}{2}\{|\partial_a \hat{B}^1|^2 + |\partial_a \hat{B}^2|^2 - e^2 k^2(|\hat{B}^1|^2 + |\hat{B}^2|^2)\} \\
& + \frac{1}{2}(\partial_a \hat{\phi})^2 \\
& - \frac{1}{4}(F_{ab})^2 \\
& + \frac{1}{2}(i\bar{\lambda}^i \not{\partial} \lambda^i - 2fk\bar{\lambda}^i \lambda^i) \\
& + \frac{1}{2}\{i(\bar{\chi} \not{\partial} \chi + \bar{\nu} \not{\partial} \nu) - ek(\bar{\chi} \chi + \bar{\nu} \nu)\} + \dots.
\end{aligned} \tag{67}$$

and following mass spectra:

$$\begin{aligned}
 m_{\hat{A}}^2 &= m_{\lambda^i}^2 = 4f^2k^2 = \frac{4\xi f}{\kappa}, \\
 m_{\hat{B}^1}^2 &= m_{\hat{B}^2}^2 = m_{\chi}^2 = m_{\nu}^2 = e^2k^2 = \frac{\xi e^2}{\kappa f}, \\
 m_{\nu_a} &= m_{\hat{\phi}} = 0,
 \end{aligned}
 \tag{68}$$

which produces mass hierarchy by the factor $\frac{e}{f}$ independent of κ . ($\kappa^{-2} = \frac{c^4 \Lambda}{16\pi G}$)

• The vacuum breaks both SUSY and $SO(1,1)$ for (A, ϕ)
 and restores(maintains) $SO(2)(U(1))$ for $(\tilde{B}^1, \tilde{B}^2)$, spontaneously,

which produces NG-Boson $\hat{\phi}$ and massless photon ν_a
 and gives soft masses $\langle A \rangle$ to λ^i .

- We have shown explicitly that

$N=2$ LSUSY QED, i.e. the matter sector(Λ term) of $N = 2$ SGM (in flat-space), possesses a true vacuum type (II).

- The resulting model describes qualitatively

lepton-Higgs-U(1) sector analogue of SM:

one massive charged Dirac fermion ($\psi_D^c \sim \chi + i\nu$),

one massive neutral Dirac fermion ($\lambda_D^0 \sim \lambda^1 - i\lambda^2$),

one massless vector (a photon) (v_a),

one charged scalar ($\hat{B}^1 + i\hat{B}^2$),

one neutral complex scalar ($\hat{A} + i\hat{\phi}$),

which are **composites of superons**.

**In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
from
the NLSUSY cosmological term of SGM
as composites of NG fermion in the true vacuum.**

6. Cosmological implications of SGM scenario

The variation of SGM action $L_{\text{SGM}}(e, \psi)$ with respect to $e^a{}_\mu$ yields **Einstein equation equipping with matter and cosmological term:**

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = \frac{8\pi G}{c^4} \left\{ \tilde{T}_{\mu\nu}(e, \psi) - g_{\mu\nu} \frac{c^4 \Lambda}{8\pi G} \right\}. \quad (69)$$

where $\tilde{T}_{\mu\nu}(e, \psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

- Note that the cosmological term $-\frac{c^4 \Lambda}{8\pi G}$ can be interpreted as **the negative energy density of space-time,** i.e. **the dark energy density ρ_D .**

- **Big collapse(BC)** induces **3 dimensional (space-like region) rapid expansion of space-time by Pauli principle:**

$$ds^2 = s_{\mu\nu}(x)dx^\mu dx^\nu = \{g_{\mu\nu} + \underline{\Phi_{\mu\nu}(e, \psi)}\}dx^\mu dx^\nu.$$

$$\begin{aligned} \{\psi(x)_a, \bar{\psi}(y)_b\}_{x_0=y_0} = 0 &\Rightarrow \{\psi(x)_a, \bar{\psi}(y)_b\}_{x_0=y_0} = \delta^{(3)}(\mathbf{x} - \mathbf{y})\delta_{ab} \\ (\{\psi(x)_a, \bar{\psi}(y)_b\} = 0 &\Rightarrow \{\psi(x)_a, \bar{\psi}(y)_b\} = iS(x - y)\delta_{ab}) \end{aligned}$$

- **BC produces gravitational composite (massless) eigenstates of SO(N) sP algebra, which is the ignition of the Big Bang(BB) SM scenario.**

- **As shown in the toy model, the vacuum of SGM may explain naturally observed mysterious (numerical) relations:**

$$\text{dark energy density } \rho_D \sim O(\kappa^{-2}) \sim m_\nu^4 \sim (10^{-12} \text{GeV})^4 \sim g_{sv}^2,$$

provided λ_D^0 is identified with neutrino and $f\xi \sim O(1)$.

7. Summary

NLSGR(SGM) for unity of nature:

• **Ultimate entity; Unstable $d = 4$ space-time:** $[x^a, \psi_\alpha^N; x^\mu]$ described by $[L_{\text{NLSGR}}(w^a_\mu)]$: **Global SUSY NLSGR on New space-time with $\Lambda > 0$**

⇒ **Big Collapse (BC)**

• The creation of ordinary Riemann space-time [graviton e^a_μ] and massless NG fermionic matter [superon ψ_α^N], **rapid expansion by Pauli principle** $[L_{\text{SGM}} = L_{\text{EH}}(e) - \Lambda + T(\psi.e)]$: **Einstein GR with $\Lambda > 0$ and N superon**

• The universal attractive force graviton dictates the evolution of $[L_{\text{SGM}}$ by forming gravitational composite **global LSUSY** supermultiplet corresponding to (massless) eigenstates of space-time symmetry **SO(10) sP**.

⇒ **Ignition of Big Bang of the universe toward Λ CDM scenario**

• In flat space-time, **broken N -LSUSY theory** emerges from **the N -NLSUSY cosmological term of $L_{\text{SGM}}(e, \psi)$ [NL/L SUSY relation]**. \longleftrightarrow **BCS vs GL**

The cosmological term is the origin of everything!

Predictions and Speculations

@ SO(10) sP algebra with $\underline{10} = \underline{5}_{\text{SU}(5)} + \underline{5}^*_{\text{SU}(5)}$:
superon-quintet model(SQM) of matter

- **Three 1^C particles** beyond SM :

One neutral spin 1/2 fermion $N: \epsilon^{abc} Q_a Q_b Q_c \epsilon^{mn} Q_m Q_n$

One double-charge spin 1/2 fermion $E^{2\pm}: \epsilon^{abc} Q_a Q_b Q_c \epsilon^{mn} Q^*_m Q^*_n$

One neutral vector boson $S: \delta^{ab} Q_a Q^*_b$

- Proton decay diagrams of SU(5) GUT **in SQM view**
are forbidden by **superon slection rule**. \Rightarrow **stable proton, FCNC suppression,**

@Field theory via Linearization:

- NLSGR(SGM) scenario **predicts** 4 dimensional space-time.
- The bare gauge coupling constants are constrained (unified).
- $N = 10$ **L/NL SUSY relation**(equivalence) bridges cosmology and the particle physics. \iff **superon-quintet model of SM/GUT**

cosmological term \leftrightarrow dark energy density \leftrightarrow SUSY Br. $\rightarrow m_\nu$

@ Many Open Problems! e.g.,

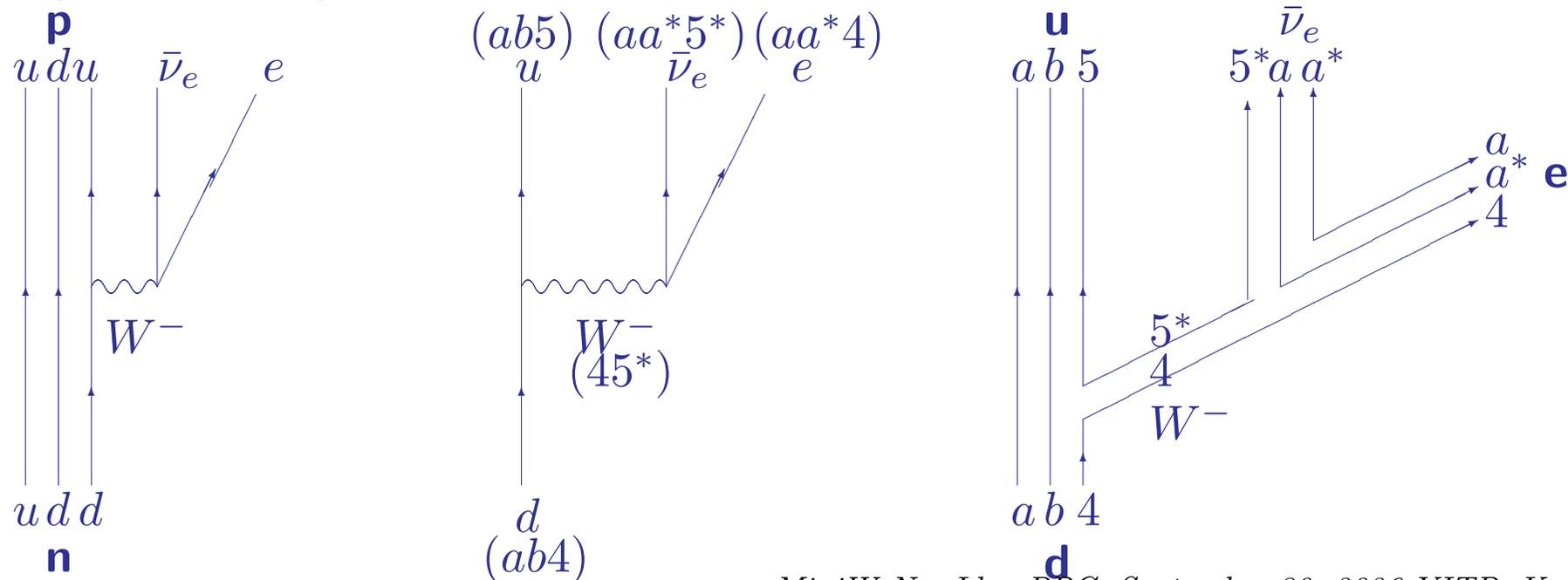
- LSUSY of **higher-spin off shell (massless) supermultiplet of SGM:**

- Direct linearization of **SGM action** in curved space, i.e. **NL/L SUSY relation** in curved space-time. .

- Revisit SM and GUT from $N = 10$ **SQM composite diagram** viewpoints:

$(e, \nu_e): \delta^{ab} Q_a Q_b^* Q_m, (\mu, \nu_\mu): \delta^{ab} Q_a Q_b^* \epsilon^{lm} Q_l Q_m Q_n^*, (\tau, \nu_\tau): \epsilon^{abc} Q_b Q_c \epsilon^{ade} Q_d^* Q_e^* Q_m$
 $(u, d): \epsilon^{abc} Q_b Q_c Q_m, (c, s): \epsilon^{lm} Q_l Q_m \epsilon^{abc} Q_b Q_c Q_n^*, (t, b): \epsilon^{abc} Q_a Q_b Q_c Q_d^* Q_m,$
HiggsBoson: $\epsilon^{abc} Q_a Q_b Q_c Q_m, \text{GaugeBoson: } Q_a Q_b^*, \dots$

e.g. SQM diagram for β -decay.



- The dangerous Feynman diagrams of SM and GUT (e.g., proton decay, FCNC, ...) are forbidden naturally in SQM diagram interpretation.
- The **thermodynamics of the universe** in NLSGR/SGM is open.
- Is Universe **Big Cycle**?

NLSGR → (BC) → SGM → (BB) → SMs → ... → BH → *NLSGR* → ...

[Ref.]

1. K. Shima, Invited talk at the Symposium *100 Years Werner Heisenberg:-Works and Impact-*, September 26-30, 2001, Bamberg, Germany, Proceeding: Fortschr. Phys, 50 (2002) 5-7. ed. D. Leust and W. Schleich.
2. K. Shima, Invited talk at *Conference on Cosmology, Gravitational Waves and Particles*, 6-10, January, 2017, NTU, Singapore(Uploaded at YouTube by IAS). Proceeding of *CCGWP*, ed. Harald Fritzsch, (World Scientific, 2017), 301.
3. All results have been published separately. For references, see Temporal pleriminary report. K. Shima, arXiv: 2012.01646[hep-th].