A microscopic approach to $^{12}C+^{12}C$ fusion

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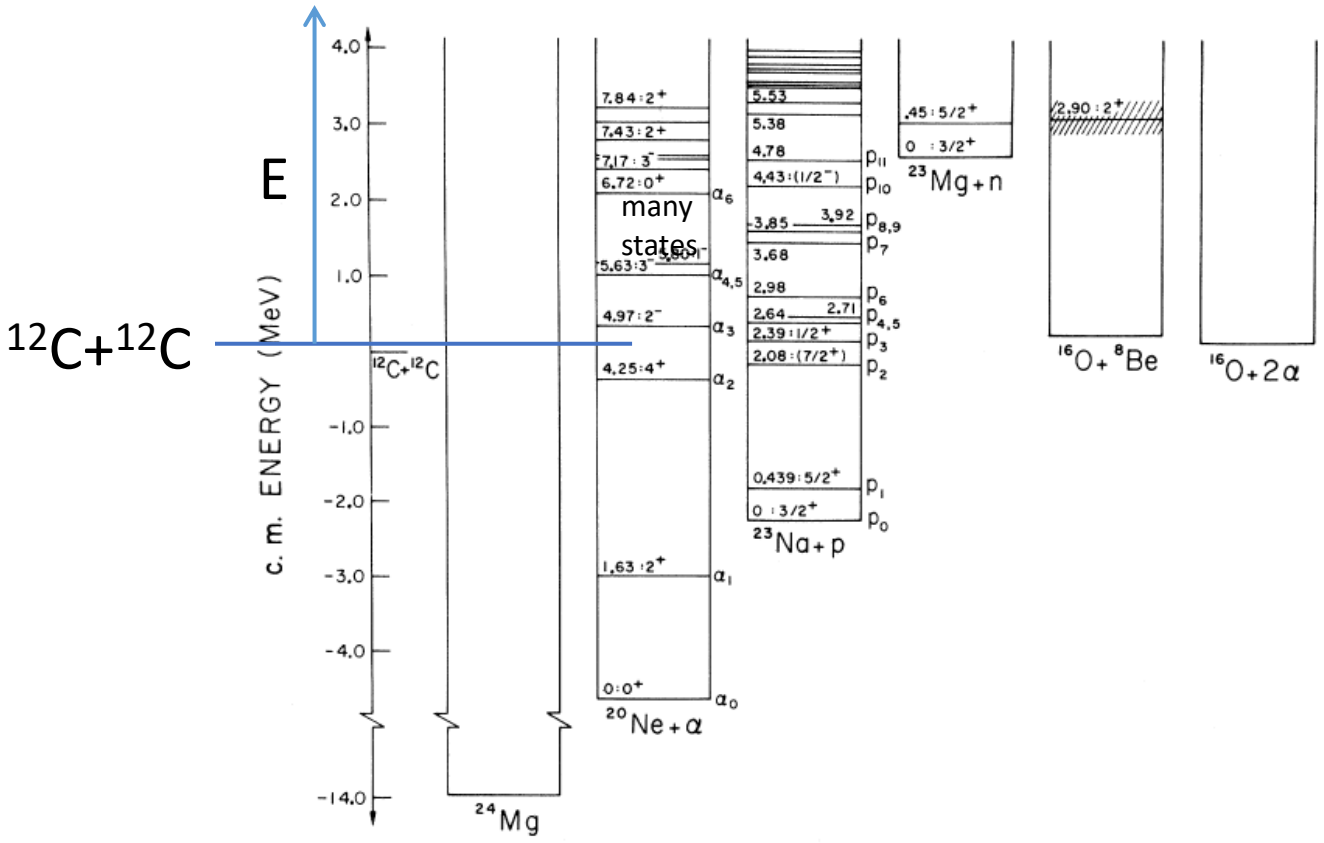
Ref.: P. D., Phys. Rev C. 113, 034613 (2026)

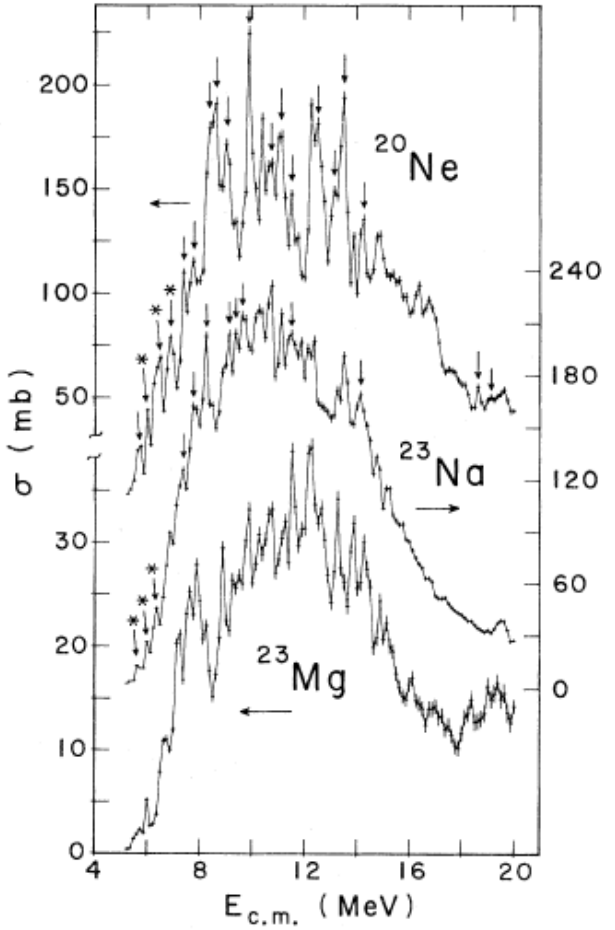
Recent reviews about $^{12}C+^{12}C$:

- M. Wiescher et al., Rev. Mod. Phys. 97, 025003 (2025).
- A. Chieffi et al., Eur. Phys. J. A 61, 280 (2025).

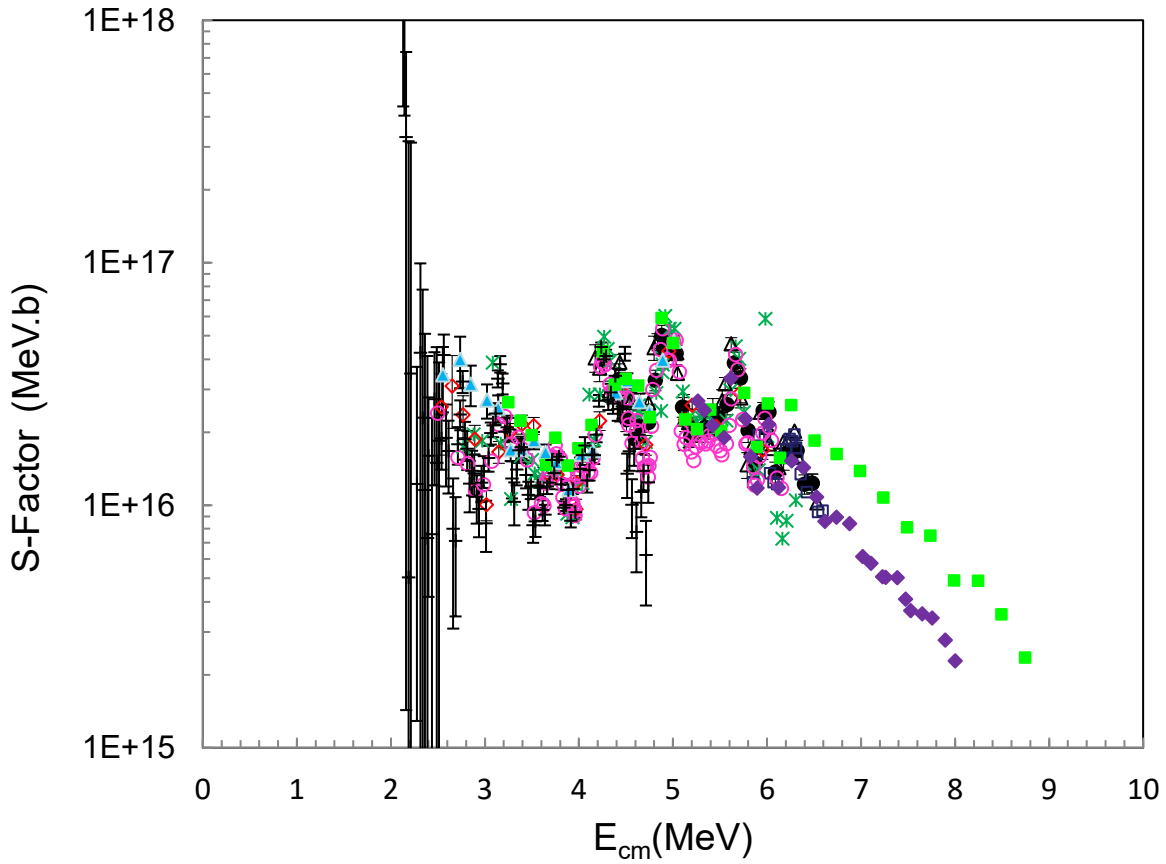
- $^{12}C+^{12}C$ reaction very important in massive stars
- Many measurements and calculations
- Standard problem in nuclear astrophysics : extrapolation is needed at very low energies
  → very low cross sections
- Fusion cross section: includes many channels
  Essentially $^{20}Ne+\alpha$, $^{23}Na+p$, $^{23}Mg+n$
  → absorption simulated by a complex potential



experimental cross section
Satkowiak et al. PRC 26 (1982) 2027



Presence of low energy resonances (« molecular resonances »)



- Modified S factor:
  $\tilde{S}(E) = \sigma(E)E\exp(2\pi\eta + 0.46E)$
- Spillane et al: PRL 98 (2007) 122501
  consistent with a low-energy resonance
- Jiang et al.: PRL 110 (2013) 072701
  « Low-level » density of ^{24}Mg → less absorption (compared to ^{25}Mg and ^{26}Mg)

Problems with resonances in fusion calculations:

- Fusion process needs an imaginary part
  → absorption to fusion channels
  → resonances get broader → disappear

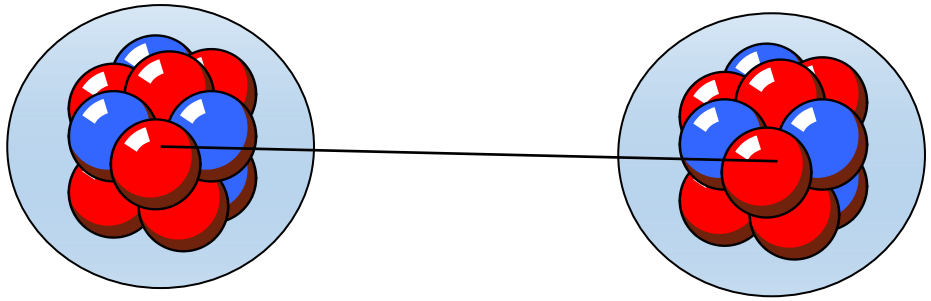
- Main problem for theory
  real potential: resonances YES, fusion NO
  complex potential: fusion YES, resonances NO (or strongly hindered)

- Ideally: coupling to transfer channels ^{20}Ne+α, ^{23}Na+p, ^{23}Mg+n

  → **new multichannel/multiconfiguration GCM calculation**

2. The Generator Coordinate Method (GCM)

The goal is to study $^{12}C+^{12}C$ fusion within a microscopic approach:



Hamiltonian given by

$$H = \sum_i T_i + \sum_{j>i} V_{ij}$$

with
T_i = kinetic energy of nucleon *i*
V_{ij} = two-body nucleon-nucleon interaction

- Contains a nuclear part V_{ij}^N: short range (here V2 interaction + spin-orbit)
- Contains a coulomb part V_{ij}^C: long range $\sim e^2/r$

- No exact solution (too difficult for ab initio models)
- Needs to be used for reactions (more difficult than spectroscopy)
- Not a new method! The difficulty depends on the system (typical simple system: $\alpha+\alpha$)

Cluster approximation RGM/GCM:**Resonating Group Method**

Single channel: $\Psi = \mathcal{A}\phi_1\phi_2 g(\rho)$ (+ angular-momentum projection + symmetrization)
with

- ϕ_1, ϕ_2 = internal wave functions of ^{12}C (**input, shell-model**)
- $g(\rho)$ =relative wave function (**output, contains the phase shifts for scattering states**)
- $\mathcal{A}$ = antisymmetrization operator

Multi channel: $\Psi = \sum_{\{ij\}} \mathcal{A}\phi_1^i \phi_2^j g_{ij}(\rho) + \sum_{\{ij\}} \mathcal{A}\phi_\alpha \phi'^j_2 g'_{ij}(\rho)$

^{12}C+^{12}C
elastic / inelastic

α+^{20}Ne fusion

Here: ϕ_1^i, ϕ_2^j =^{12}C wave functions

$\phi_\alpha = \alpha$ wave function

ϕ'^j_2= ^{20}Ne wave function

GCM: **Generator Coordinate Method**

- expand the relative function $g(\rho)$ in Gaussians: $g(\rho) = \sum_i f_i \exp\left(-\mu \frac{(\boldsymbol{\rho}-\boldsymbol{R_i})^2}{2b^2}\right)$
- R_i=generator coordinates (typically ~10)
- main property: $\mathcal{A}\phi_1\phi_2 \exp\left(-\mu \frac{(\boldsymbol{\rho}-\boldsymbol{R_i})^2}{2b^2}\right) = \Phi(R_i) =$ Slater determinant
- Then $\Psi = \sum_i f_i \, \Phi(R_i)$: coefficients f_i are determined by the eigenvalue problem (for bound states)

$$\sum_j \left[H(R_i, R_j) - E\, N(R_i, R_j)\right] f_j = 0$$

Main problem in the GCM: to compute the kernels

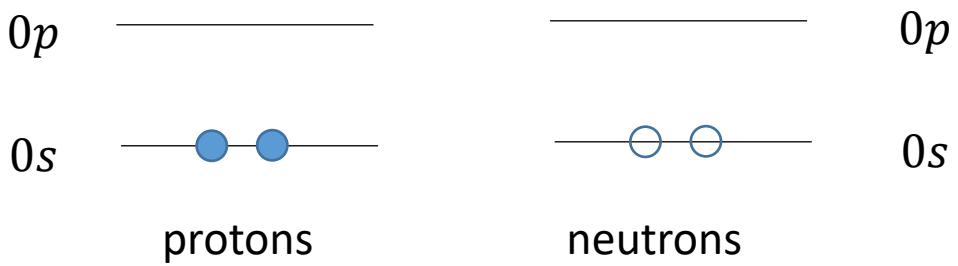
$$N(R_i, R_j) = < \Phi(R_i) | \Phi(R_j) >$$
$$H(R_i, R_j) = < \Phi(R_i) | \mathrm{H} | \Phi(R_j) >$$

With $\Phi(R_i)$, $\Phi(R_j)$=Slater determinants

One-body operators (kinetic energy): 4 double sums over the orbitals (s,p for ^{12}C, s,p,sd for ^{20}Ne)
Two-body operators (NN interaction): 16 quadruple sums : main contribution to the computer time

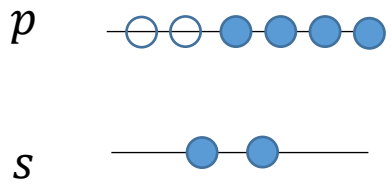
+ Angular-momentum projection: integration over the angle between R_i and R_j

The α particle



1 Slater determinant
→ α+α : 1 SD
Double sums: 1x1x4 (identical)
Quadruple sums: 1x1x16 (identical)
S shell→ ~2^4

The ^{12}C nucleus



$C_4^6 = 15$ possibilities
filled

6 neutrons
6 protons

➔ 225 Slater determinants for ^{12}C: diagonalization of I^2, I_z, L^2, S^2, T^2

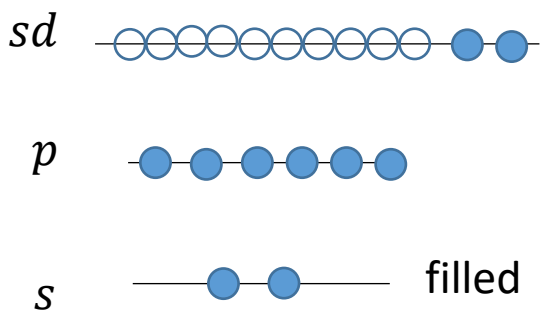
➔ 225 ^{12}C wave functions with good quantum numbers I, M, L, S, T
9 x 0^+, 13 x 1^+, 17 x 2^+, 8 x 3^+, 4 x 4^+ : 51 ^{12}C states

➔ 225^2=50625 SD for ^{12}C+^{12}C : multichannel

Double sums 50625 x 50625 x 4 ≈ 10^{10}
Quadruple sums: 50625 x 50625 x 16 ≈ 4x10^{10}
p shell → ~8^4

First step: determining the minimal number of double and quadruple sums

The ^{20}Ne nucleus

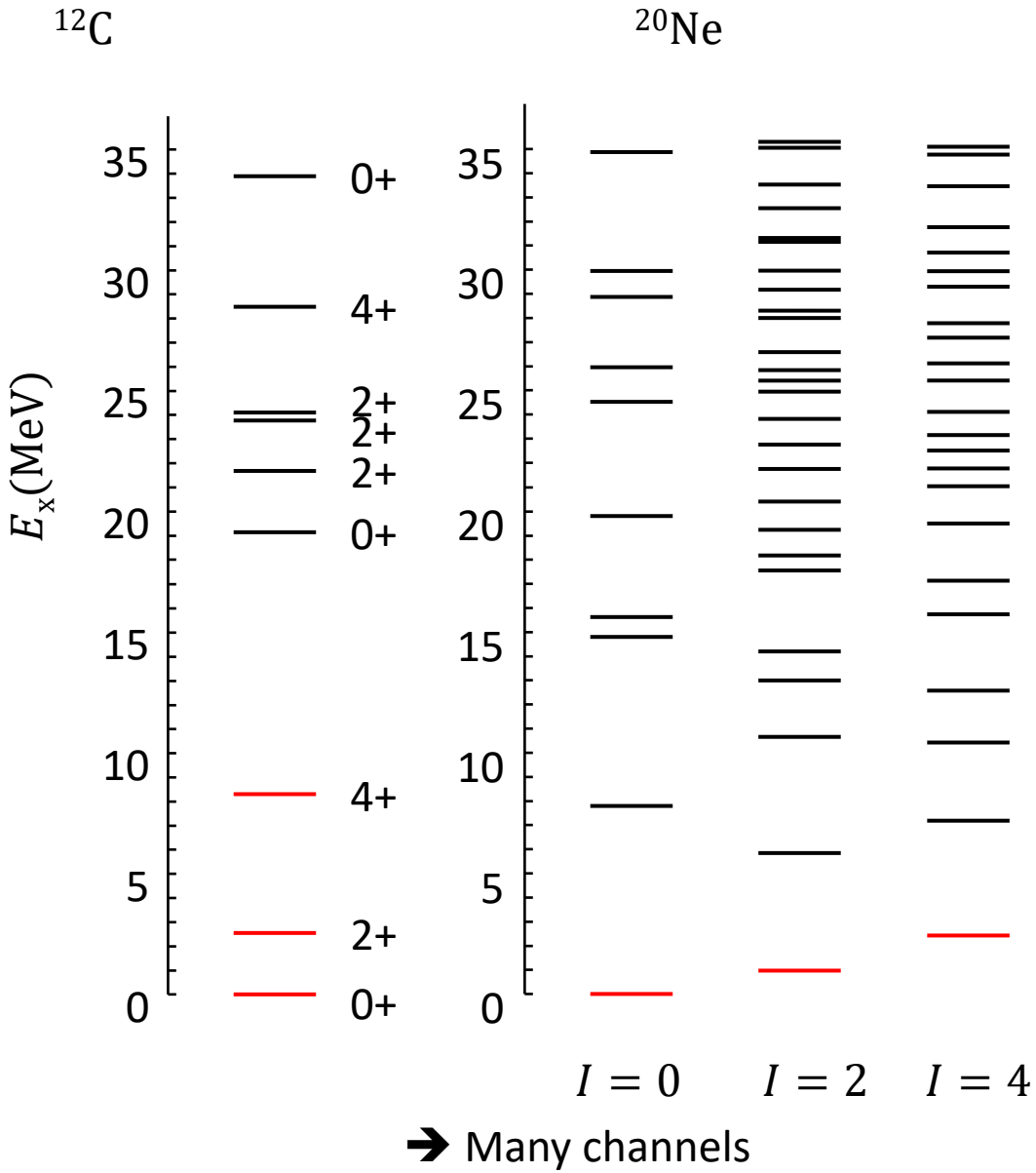


$C_2^{12} = 66$ possibilities

➔ 66^2=4356 Slater determinants for ^{20}Ne

10 neutrons
10 protons

Shell-model states



➔ Many channels

3. The R-matrix method

Microscopic R-matrix: simple generalization of the standard R-matrix theory with GCM basis functions
Based on the existence of 2 regions (radius a):

- Internal: Coulomb+nuclear → GCM wave functions: internal wave function $g_{int}(\rho)$
- external: Coulomb → exact solution: external wave function $g_{ext}(\rho)$

➔ **No approximation for the continuum**

internal region: $\rho \leq a$

$$\Psi_{\text{int}} = \mathcal{A}\phi_1\phi_2 g_{int}(\rho) = \sum_{i=1}^{N} f(R_i)\Phi(R_i)$$

external region: $\rho \geq a$

- Nuclear potential negligible
- Antisymmetrization negligible

$$\Psi_{\text{ext}} = \phi_1\phi_2 g_{ext}(\rho)$$
$$g_{ext}(\rho) = I_\ell(\rho) - U_\ell O_\ell(\rho)$$

- Bound states
- Resonances
- Elastic scattering
- Fusion

Are treated simultaneously
→ Consistency tests are possible

- Schrödinger equation + Matching at $r = a$
  ➔ scattering matrix U_ℓ → elastic, inelastic, transfer (fusion) cross sections
  !! Channel radius a is not a parameter (compromize)
- Test: stability with a and with the number of basis functions N

4. Application to $^{12}C+^{12}C$ fusion

(Limited to the α channel)

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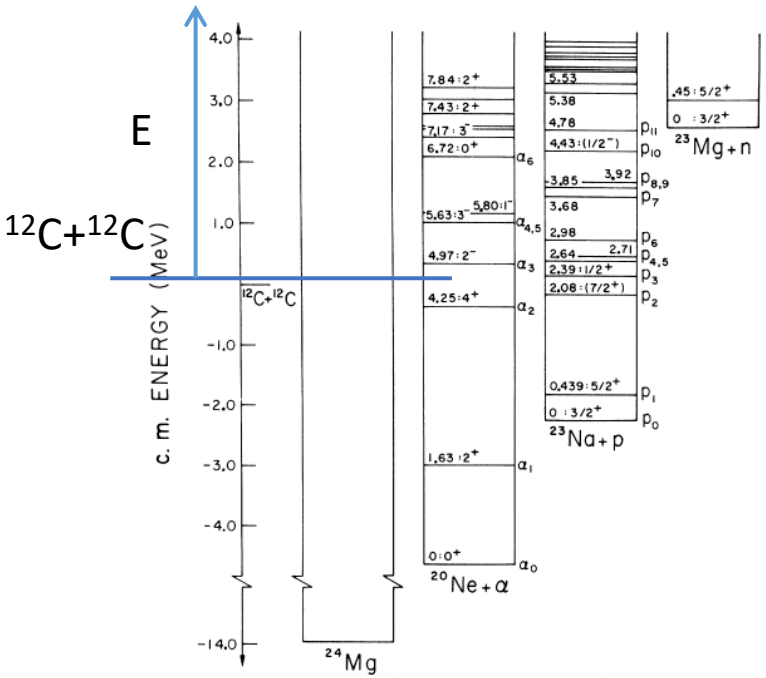
In practice

1. The matrices are 50625x50625 → split in pieces (~50 in parallel)
2. Determination of the minimal number of double and quadruple sums (information stored in files to avoid repetition)
3. Calculation of the matrix elements for all pieces
4. Combination of all pieces/files → matrix elements for 1 set of generator coordinates and 1 angle
5. Must be integrated over the relative angle (angular momentum projection + symmetrization)
6. Must be calculated for ~10 generator coordinates → ~55 pairs (R,R')



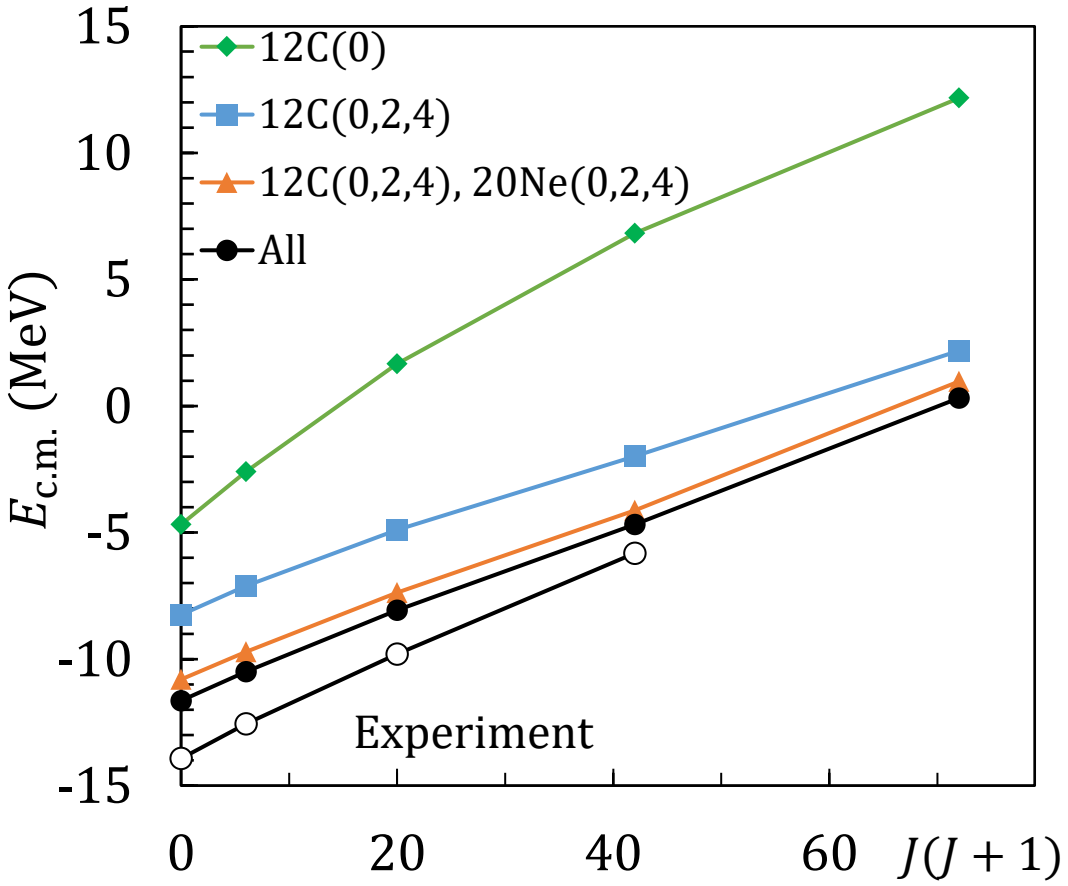
The $\alpha+^{20}Ne$, $p+^{23}Na$, $n+^{23}Mg$ channels should be included

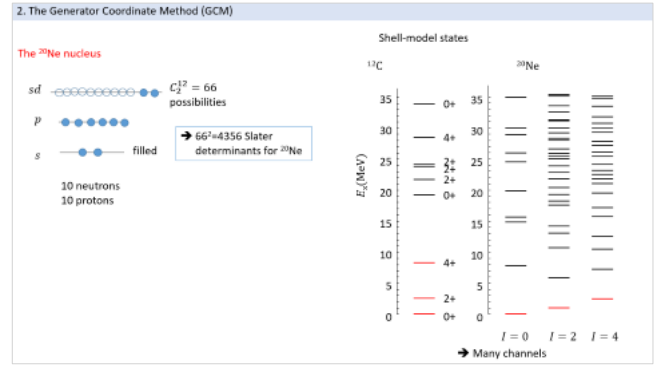
^{20}Ne: 4 particles in the sd shell → 66^2=4356 SD

^{23}Na, ^{23}Mg: 7 particles in the sd shell →108900 SD (x2 for the proton/neutron)

→ not feasible at the moment

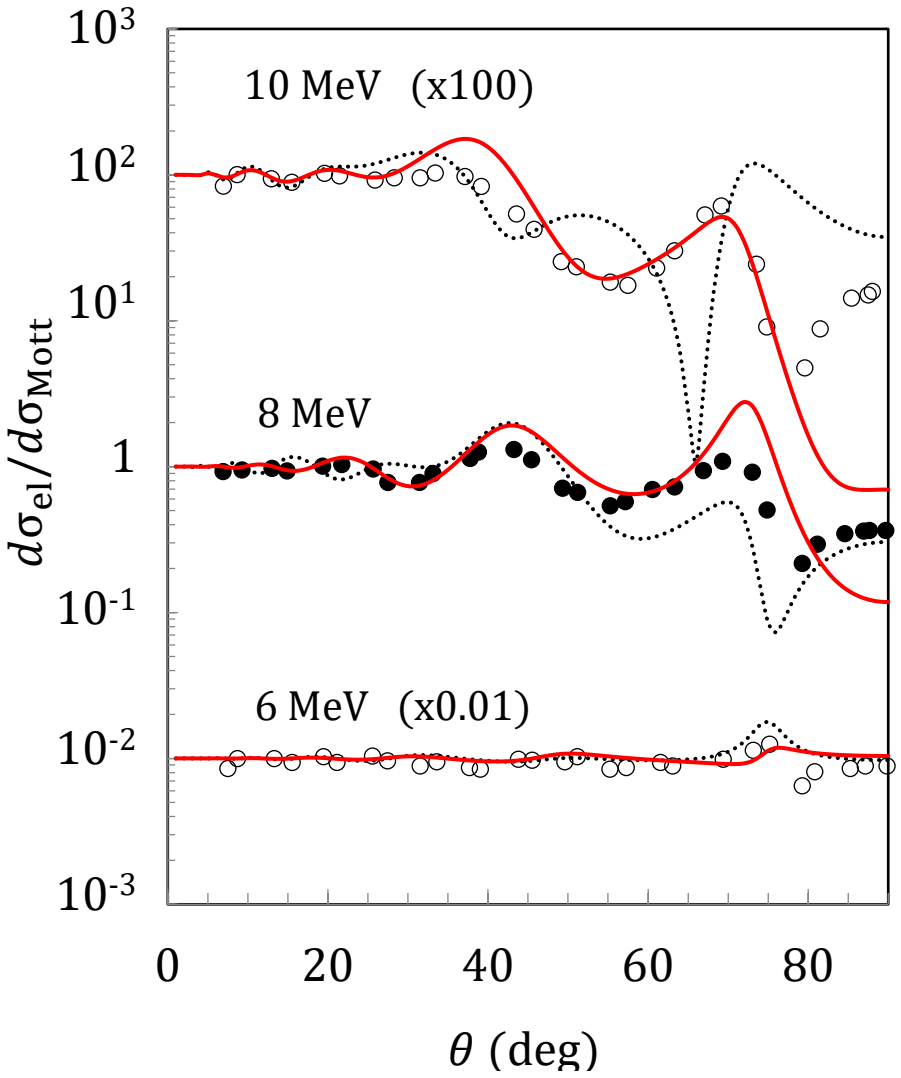
Test on spectroscopy: role of excited configurations on ^{24}Mg ground-state band





➔ Strong influence of excited channels

Elastic cross section (same wave functions) → test

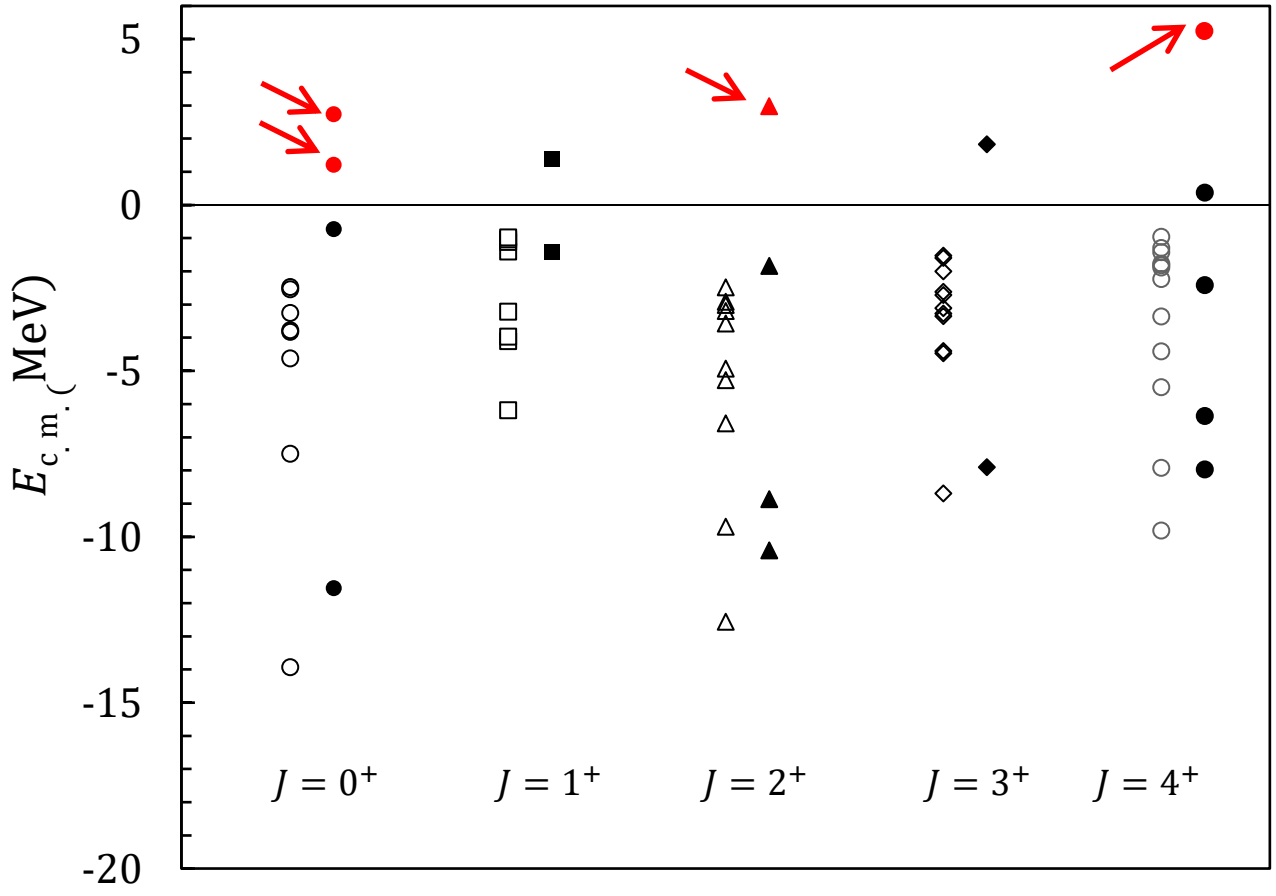


Data from
W. Treu et al., Phys. Rev. C 22, 2462 (1980).

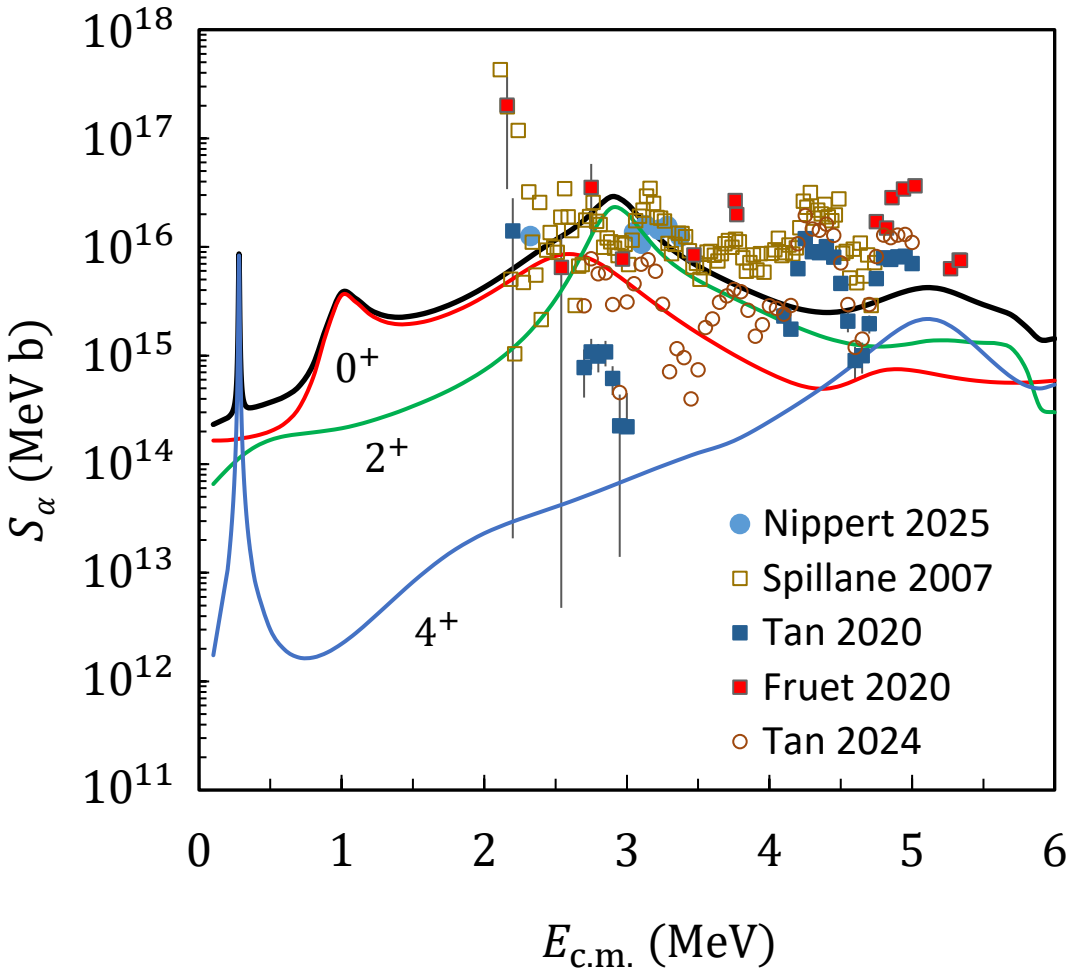
single channel
multichannel

4. Application to $^{12}C+^{12}C$ fusion

^{24}Mg states : multichannel model
Open symbols: experiment
Filled symbols: theory



$^{12}C+^{12}C$ S-factor: α component



Resonance properties

Very small $^{12}C+^{12}C$ widths (below the Coulomb barrier)

Essentially $\alpha+^{20}Ne$ widths

$\mathcal{N}$=component in the wave function (not sensitive to E)

	4^+	**0^+**	**0^+**	**2^+**	**4^+**
Ec.m. (MeV)	0.3	1.14	2.6	2.9	5.1
Γ (MeV)					
$^{12}C+^{12}C$	7.10E-57	1.20E-21	3.90E-09	1.50E-08	1.30E-04
$\alpha+^{20}Ne(0^+)$	8.50E-07	0.026	0.33	0.13	0.25
$\alpha+^{20}Ne(2^+)$	0.0038	0.25	0.58	0.2	0.44
$\mathcal{N}$					
$^{12}C+^{12}C$	0.16	0.04	0.2	0.22	0.16
$\alpha+^{20}Ne(0^+)$	0.05	0.03	0.07	0.04	0.05
$\alpha+^{20}Ne(2^+)$	0.11	0.3	0.19	0.12	0.17

→ No dominant channel

→ Wave functions are spread over several channels

4. Conclusion

4. Conclusion

- No parameter (except the admixture parameter M=0.70 in the V2 interaction, adjusted on the 24Mg ground state)
  Exact treatment of the continuum (microscopic R-matrix)

- Consistent description of
  - ^{24}Mg spectroscopy
  - ^{12}C+^{12}C elastic scattering (influence of excited configurations)
  - ^{12}C+^{12}C fusion

- Resonances are found below the Coulomb barrier (very small ^{12}C+^{12}C widths, wave function spread between several channels)

- Consistent with fusion hindrance at low energies

- p+^{23}Na, n+^{23}Mg: problems with the number of Slater determinants:
  108900 (for ^{23}Mg/^{23}Na) x 2 (for p)=217800
  → 4 times more than for ^{12}C+^{12}C
  → more optimization is needed (memory/computer time)