A microscopic discription of carbon fusion reactions at nuclear astrophysical energy

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Outline

- Introduction
- Models
- Results
- Summary

$^{12}C + {}^{12}C$ fusion reaction : key reaction in nuclear astrophysics

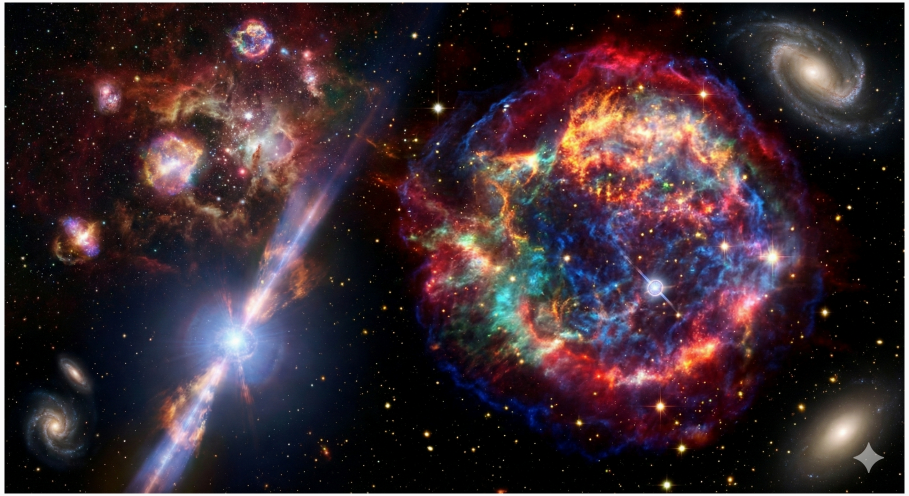


Image by AI (Gemini)

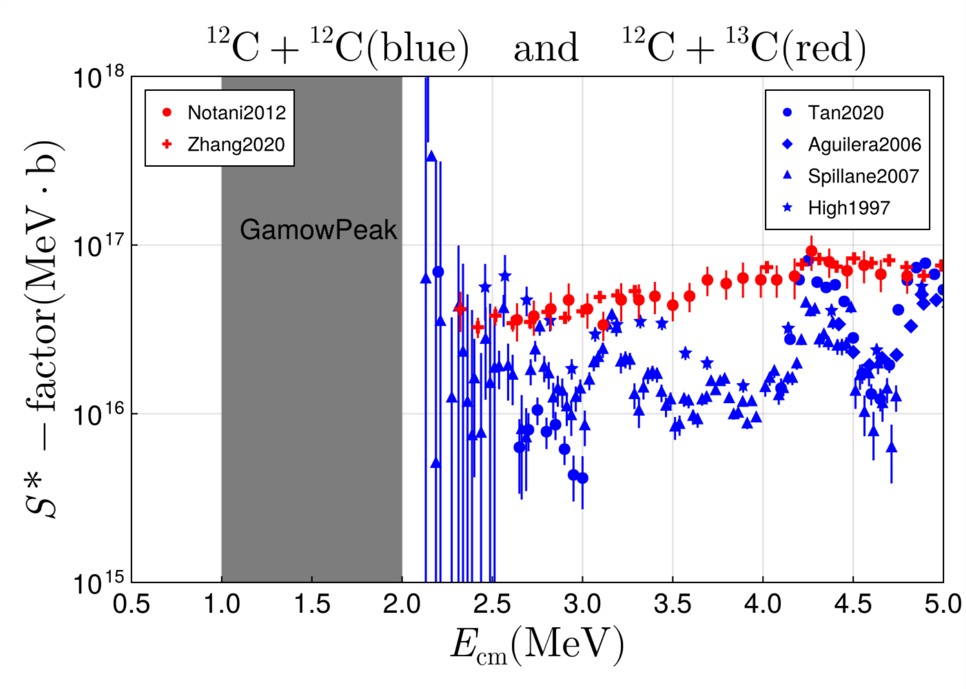
Explosive phenomena

- Evolution in massive stars
- Type-Ia supernovae
- X-ray superbursts

$\Rightarrow$ **Low energy reaction**
  - small cross section

$\Rightarrow$ **Theoretical approach is necessary**

Comparison with neighboring system



cross section for $^{12}C + ^{12}C$, $^{12}C + ^{13}C$

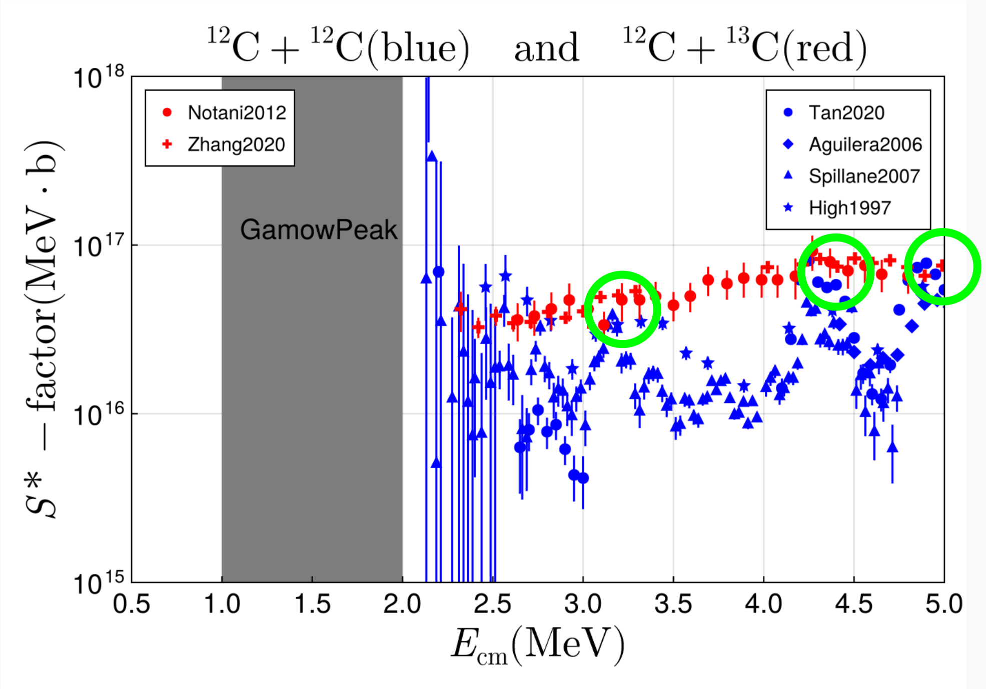
Astrophysical S^* factor

$$S^* = E\sigma(E)\exp(2\pi\eta + 0.46\,\mathrm{MeV}^{-1}E)$$

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar c}\sqrt{(\mu c^2)/(2E)} \;,$$

$\sigma(E)$: fusion cross section

$\Longrightarrow$ Removing the steep decrease in the cross section due to the Coulomb barrier



cross section for $^{12}C + ^{12}C$, $^{12}C + ^{13}C$

- $^{12}C + ^{12}C$
  - **Many pronounced resonances** (Oscillatory behavior)
- $^{12}C + ^{13}C$
  - **Smooth energy dependence**

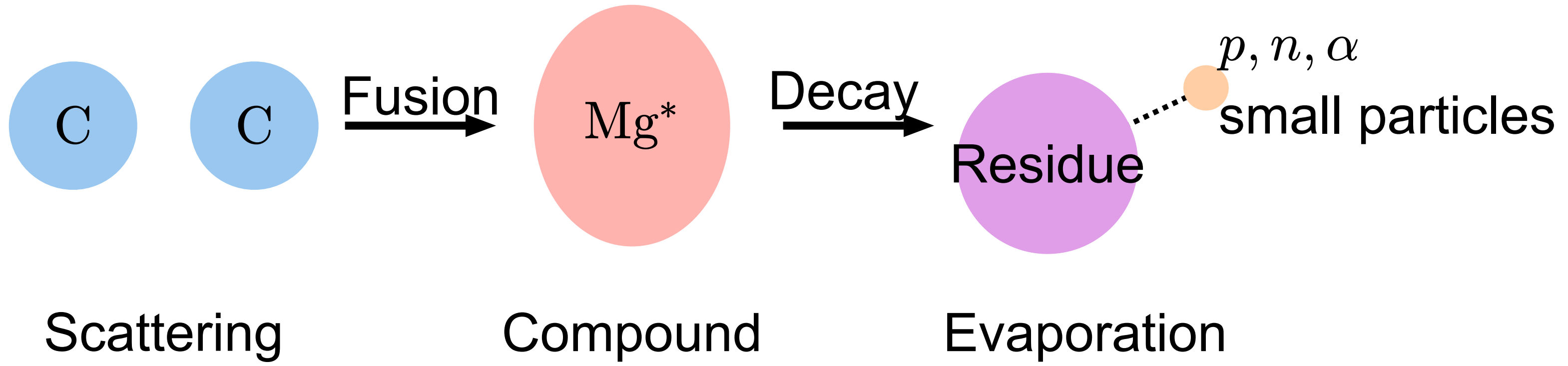
Experimental fact ($E_{c.m.} \gtrsim 2\,\mathrm{MeV}$):

Peaks for $^{12}C + ^{12}C \approx ^{12}C + ^{13}C$

Origin of resonances in $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ fusion cross section

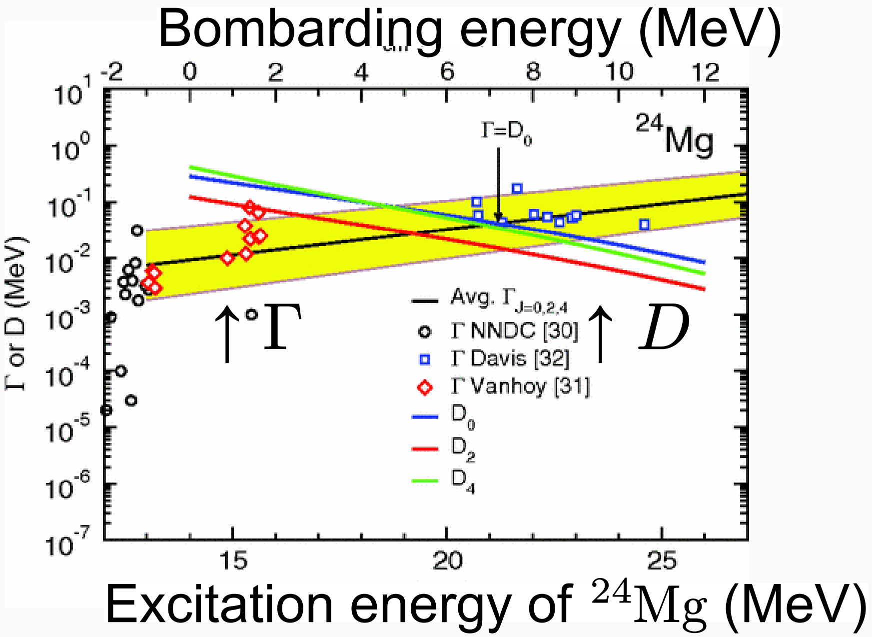
- Properties of the intermediate $^{24}\mathrm{Mg}$ states C.L. Jiang et al., PRL **110**, 72701(2013)

Fusion reaction process



Isolated resonance and overlapping resonance

- Average level spacing D_J and decay width Γ_J



C.L. Jiang et al., PRL **110**, 72701(2013)

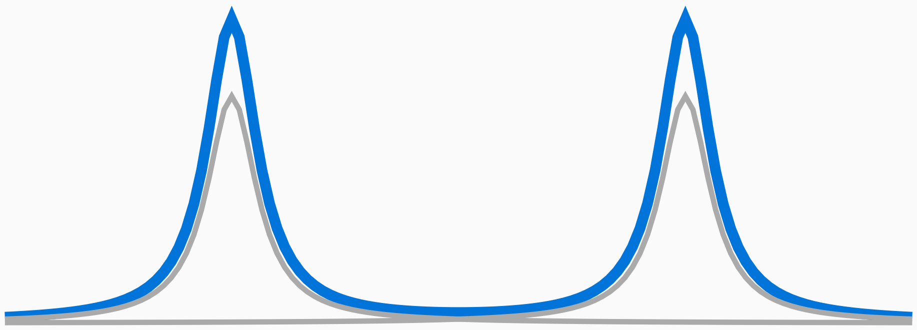
sub-barrier energy

$^{24}\mathrm{Mg}$, $E_{\mathrm{c.m.}} \lesssim V_B \approx 6\,\mathrm{MeV}$:

$$\Gamma < D$$

$\Longrightarrow$ **Isolated resonance** contribution from a single excited state $\Rightarrow \sigma \propto \frac{\Gamma^2}{(E-E_r)^2+\Gamma^2/4}$ (Breit-Wigner)

isolated resonance($\Gamma < D$)



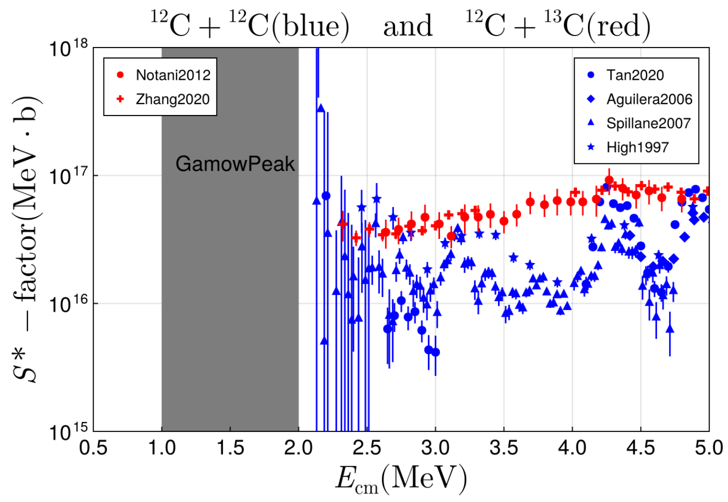
Individual structures appear

overlapping resonance($\Gamma > D$)

Individual structures are not visible

Objectives

- **Calculation of fusion cross sections at low energies**
  (the energy region where direct measurements are difficult).
  - Based on microscopic model to reduce uncertainties



- **Reproduce the difference in fusion CS**
  - Observed in experiments:
    $^{12}C + ^{12}C$ (Resonance)
    $^{12}C + ^{13}C$ (Smooth)
  - **Explicitly includes compound states**

Extension of Model

[1] Extension of **Fanto et al.**
(PRC 98, 014604, 2018)

Previous:

- $\mathrm{n} + {}^{194}\mathrm{Pt}$, Random matrix
- w/o Coulomb, s-wave($l = 0$)

$\Downarrow$ extension

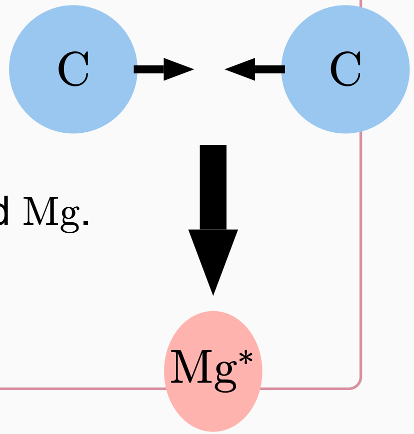
This work:

- $\mathrm{C} + \mathrm{C}$, microscopic compound states
- w Coulomb , all partial waves $l \geq 0$

Hamiltonian in matrix

$$\boldsymbol{H} = \begin{pmatrix} \boldsymbol{H}_{\mathrm{C+C}} & \boldsymbol{V} \\ \boldsymbol{V}^T & \boldsymbol{H}_{\mathrm{CN}} \end{pmatrix}$$

- $\boldsymbol{H}_{\mathrm{C+C}}$:
  relative motion of $\mathrm{C} + \mathrm{C}$
- $\boldsymbol{V}$:
  coupling between $\mathrm{C} + \mathrm{C}$ and Mg.
- $\boldsymbol{H}_{\mathrm{CN}}$:
  internal states of Mg



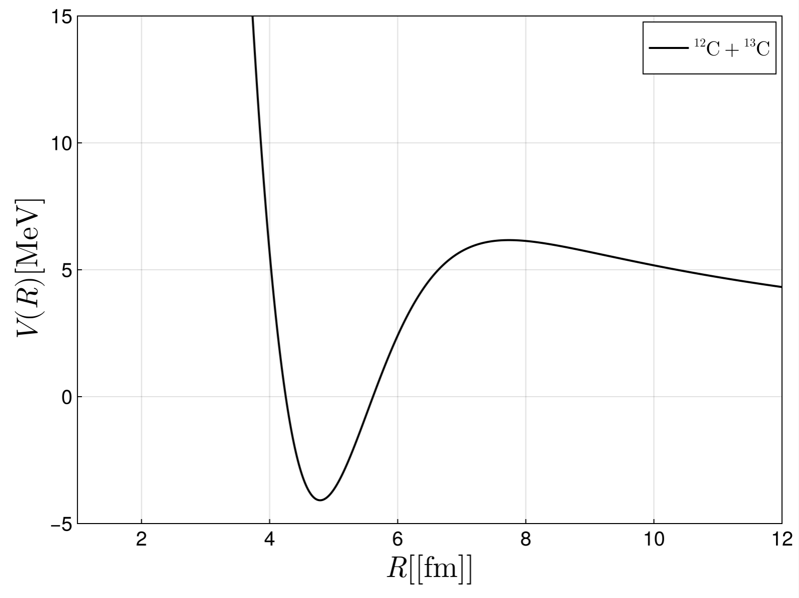
Hamiltonian for the scattering states ($H_{\mathrm{C+C}}$)

- Assumption: $^{12}\mathrm{C}$ and $^{13}\mathrm{C}$ are frozen and treated as spherical shape
- Discribe the relative motion of C isotopes

$\mathrm{C}+\mathrm{C}$ Hamiltonian

$$\boldsymbol{H}^{\mathrm{C+C}} = -\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + V_l(r)$$

- μ : reduced mass
- $V_l(r)$: N-N + centrifugal potential



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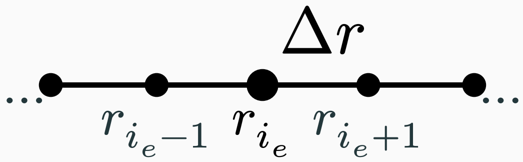

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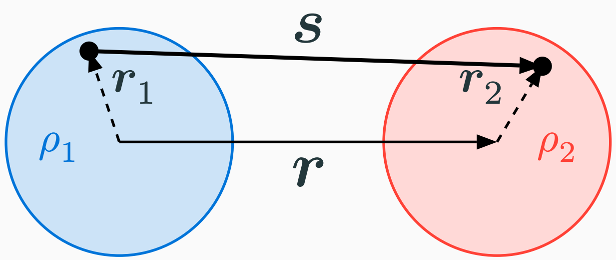
Finate discritized basis

$$\boldsymbol{H}^{\mathrm{C+C}}_{i,j} = [2t + V_l(r_i)]\delta_{ij} - t\delta_{i,i+1} - t\delta_{i,j-1}$$

- $t = \hbar^2/(2\mu\Delta r^2)$
- $i, j = 1, 2, ..., N_{\mathrm{C+C}}$
- Δr : mesh spacing

Double-Folding Potential (M3Y + Repulsion)

[2] H. Esbensen, X. Tang, and C. L. Jiang, PRC **84**, 64613 (2011)



$$U_{\mathrm{M3Y}}(\boldsymbol{r}) = \int \mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2 \rho_1(\boldsymbol{r}_1)\rho_2(\boldsymbol{r}_2) v_{\mathrm{M3Y}}(\boldsymbol{s})$$

where $\boldsymbol{s} = \boldsymbol{r} + \boldsymbol{r}_2 - \boldsymbol{r}_1$

v_{M3Y} : nucleon-nucleon interaction

Repulsive term

Reflecting the nuclear incompressibility :

$$v_r(\boldsymbol{s}) = v_r \delta(\boldsymbol{s})$$

$$U_{\rm M3Y}(\boldsymbol{r}) = \int \mathrm{d}\boldsymbol{r}_1\, \mathrm{d}\boldsymbol{r}_2 \rho_1(\boldsymbol{r}_1)\rho_2(\boldsymbol{r}_2) v_{\rm M3Y}(\boldsymbol{s}), \quad U_r(\boldsymbol{r}) = \int \mathrm{d}\boldsymbol{r}_1\, \mathrm{d}\boldsymbol{r}_2 \rho_{1r}(\boldsymbol{r}_1)\rho_{2r}(\boldsymbol{r}_2) v_r(\boldsymbol{s})$$

Nuclear density distribution ρ

Fermi distribution:

$$\rho_i(r) = \frac{\rho_0}{1 + \exp\left(\frac{r - R_i}{a_i}\right)}$$

- R_i : radius parameter
- a_i : Diffuseness parameter

Determination of parameters

Matter distribution (ρ_i, for M3Y):

- Reproducing the **charge radii**.

Repulsive term (ρ_{ir}):

- same radius parameter ($R_{ir} = R_i$)
- a_{ir} is an adjustable parameter

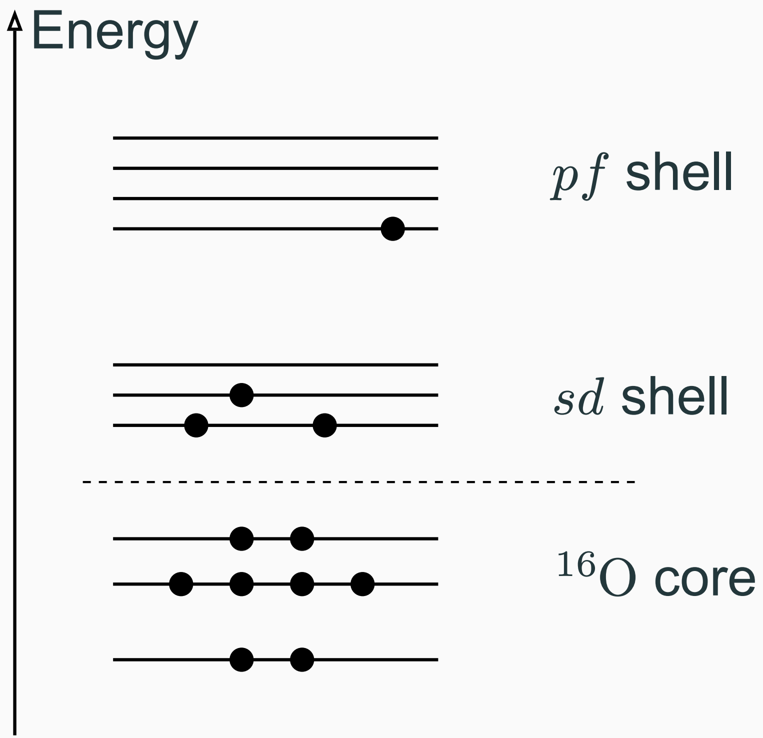
Compound state Hamiltonian (H_{CN})

Description of Compound Nucleus States

Fanto et al.		**This work**
(Random matrix)	$\Longrightarrow$	(shell model)
statistical		**microscopic**

KSHELL

Model Space: $^{16}\mathrm{O}$ core , sd-pf shell

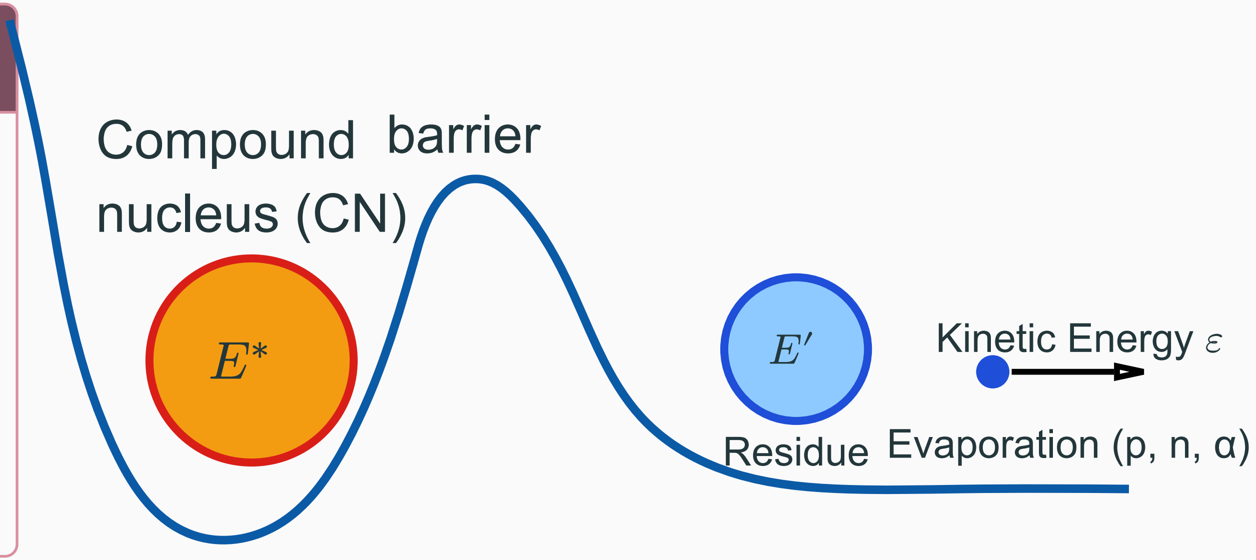


[3] N. Shimizu *et al.*, Comp. Phys. Comm. **244**, 372 (2019)

Statistical Model

$$\Gamma_J(E^*_{\mathrm{CN}}) = \frac{N}{2\pi\rho(E^*_{\mathrm{CN}}, J)}$$

E^*_{CN} : Excitation energy for CN
ρ : Level density of CN(Mg)
N: Number of final states



$$\boldsymbol{H}_{\mathrm{CN},\mu\nu} = \left(\mathcal{E}_\mu - i\frac{\Gamma_\mu}{2}\right)\delta_{\mu\nu}$$

- $\mathcal{E}_\mu$:Excitation energy , Γ_μ : Decay width ($\Gamma(\mathcal{E}_\mu)$) , μ, ν : Index of the state

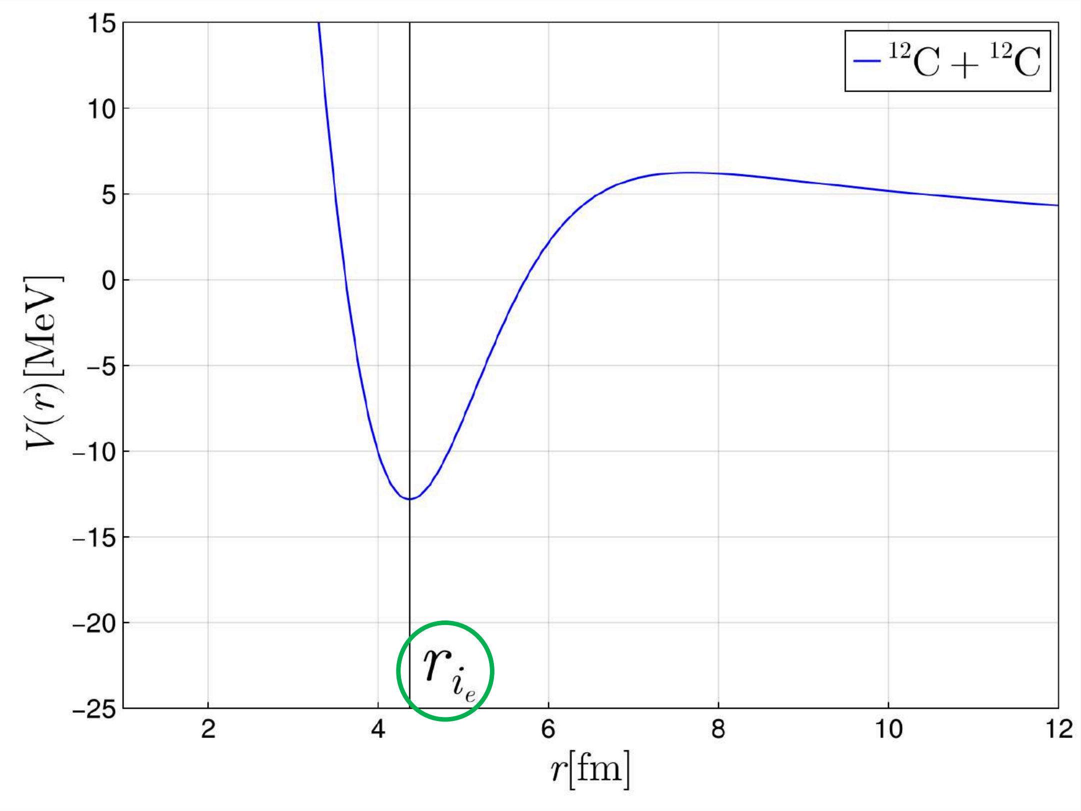
Coupling matrix (V)

Matrix V

$$V_{i,\mu} = v_0 \delta_{i,i_e}$$

- v_0 : coupling strength
- i : index of the relative position
- μ : index of the internal states
- i_e : **Interaction point**

(Fixed at potential minimum)



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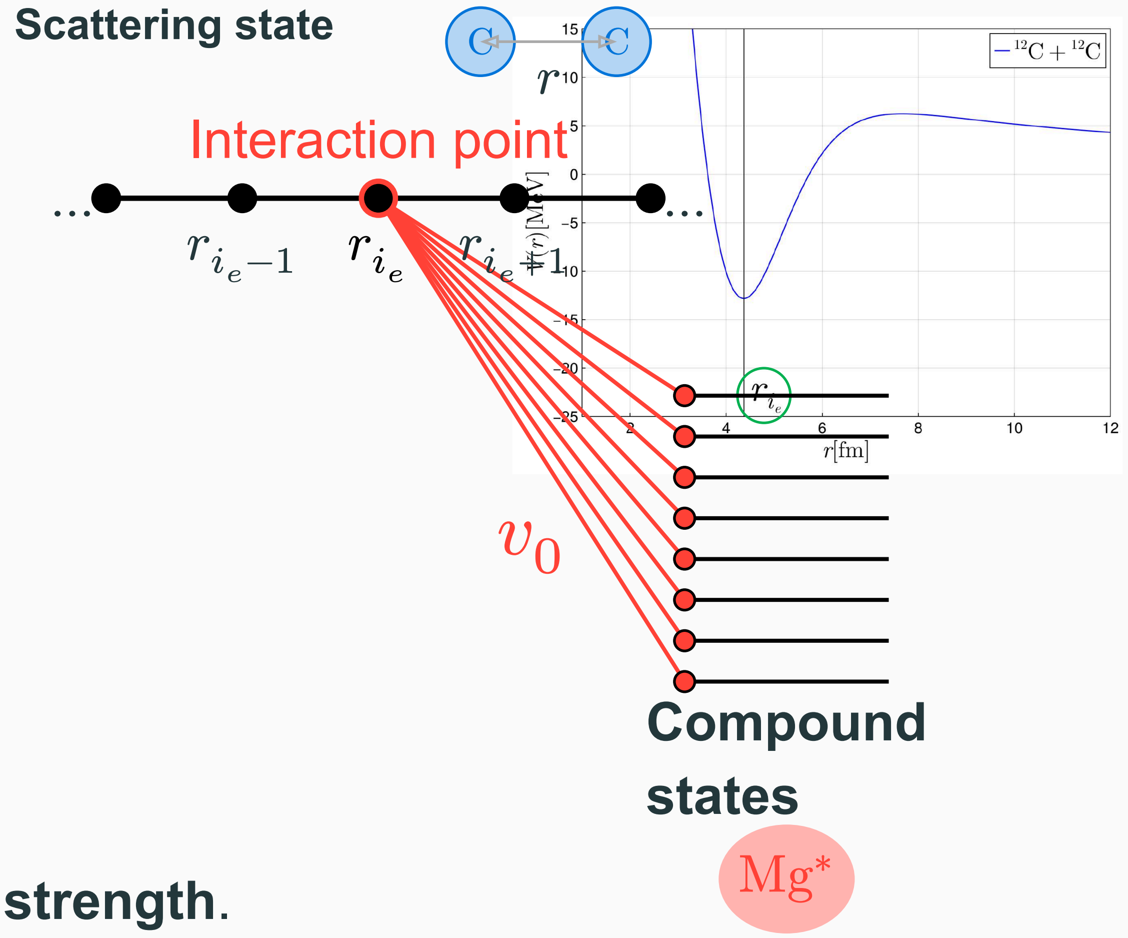


⇒ Couples to all states with the **same strength**.

strength v_0 is a parameter

Eigenvector and Boundary Conditions(BC)

$$\boldsymbol{H}$$

$(N_{\mathrm{C+C}} + N_{\mathrm{CN}}) \times (N_{\mathrm{C+C}} + N_{\mathrm{CN}})$ matrix

$$\downarrow$$

$\vec{u}$: eigenvector

$$\vec{u} = \begin{pmatrix} N_{\mathrm{C+C}} & \text{comp. (C+C)} \\ N_{\mathrm{CN}} & \text{comp.(Mg)} \end{pmatrix}$$

BC (Scattering states)

- **Origin** $(r = 0)$

$$u(0) = 0$$

- **Asymptotic form** $(r \to \infty)$

$$u(r) \to A(k)\left[H_l^-(kr) - S_{\mathrm{CC}}^{l,J} H_l^+(kr)\right]$$

$H_l^\pm$: Coulomb wavefunction
S: S matrix, l: Angular momentum.

$H_l^\pm = G_l \pm F_l$, F_l, G_l : Regular and Irregular Coulomb Function.

Ratio of v_0 between the two systems

The parameters are determined based on the experimental fact that the $^{12}\mathrm{C} + {}^{13}\mathrm{C}$ reaction provides an upper limit for $^{12}\mathrm{C} + {}^{12}\mathrm{C}$.

Projection onto C+C channel:

$$V(r) \to V(r) + V_{\mathrm{eff}}(r)$$

$$V_{\mathrm{eff}}(r) = -\delta\left(r - r_{i_e}\right) \sum_{\mu} \frac{v_0^2}{E - E_\mu + i\frac{\Gamma_\mu}{2}}$$

$\Rightarrow$ Significant difference in the second factor between two systems.

(Level spacing D and Decay width Γ)

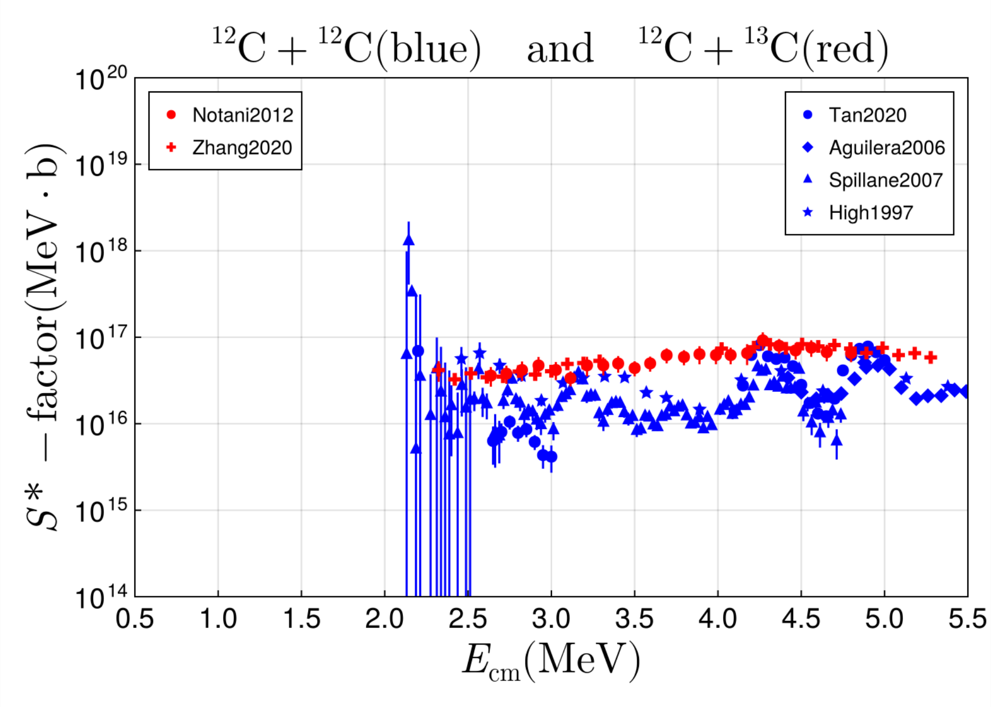
Fusion cross section :

$$\sigma^J_{\mathrm{fus}} \propto (2J+1) \int \mathrm{d}r\, \mathrm{Im}\, V_{\mathrm{eff}} |u(r)|^2$$

$\Rightarrow$ v_0 **is scaled so that** $(2J+1)\,\mathrm{Im}\,V_{\mathrm{eff}}$ **is same for the two systems.**

($2.0\,\mathrm{MeV} < E_{\mathrm{c.m.}} < 5.5\,\mathrm{MeV}$**)**

$\to$ **Resonance peaks of** $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ **coincide with the** $^{12}\mathrm{C} + {}^{13}\mathrm{C}$**.**



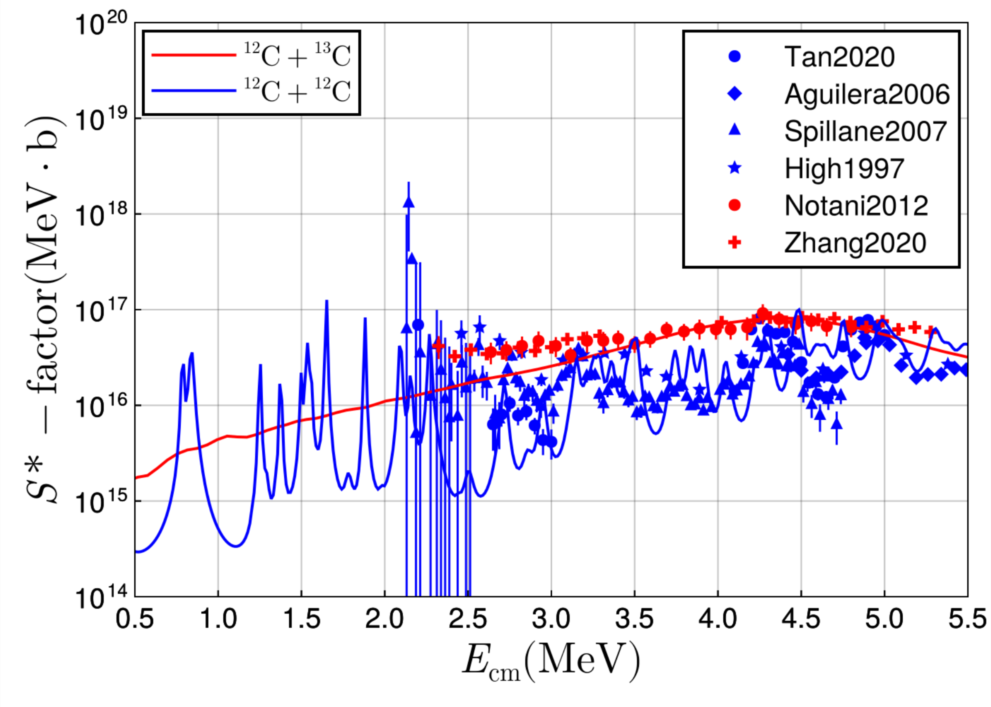
- $^{12}\mathrm{C} + {}^{13}\mathrm{C}$ shows a smooth behavior.

⇒ Fitting the parameters (a_r, v_0) to the $^{12}\mathrm{C} + {}^{13}\mathrm{C}$ data.

⇒ The same a_r and corresponding v_0 are applied to $^{12}\mathrm{C} + {}^{12}\mathrm{C}$.

- **No additional fitting** is repuired for the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ system.

Final Results



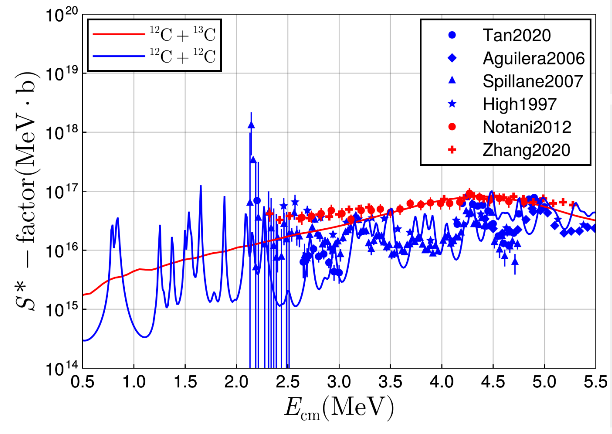
- $2.0\,\mathrm{MeV} < E_{\mathrm{c.m.}} < 5.5\,\mathrm{MeV}$
  - Reproducing the upper-limit behavior.
- $E_{\mathrm{c.m.}} < 2.0\,\mathrm{MeV}$
  - $^{12}\mathrm{C} + {}^{13}\mathrm{C}$ does not provide an upper limit.

Summary

- Calculation of fusion cross sections in the low-energy region
- Reproducing the difference in behavior between the two systems:
  - ${}^{12}\mathrm{C} + {}^{12}\mathrm{C}$: **Resonance structures**
  - ${}^{12}\mathrm{C} + {}^{13}\mathrm{C}$: **Smooth structures**

Outlooks

- Extend the current framework to explicitly incorporate final states ($\alpha + {}^{20}\mathrm{Ne}$)



Bibliography

[1] P. Fanto, G. F. Bertsch, and Y. Alhassid, Phys. Rev. C **98**, 14604 (2018).

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