Tip–Tip Collisions of Deformed Nuclei as a Probe of the High-Density Equation of State

Department of physics, Hiroshima University
International Institute for Sustainability with Knotted Chiral Meta Matter／SKCM².
Hiroshima University
Kobayashi Maskawa Institute, Nagoya University

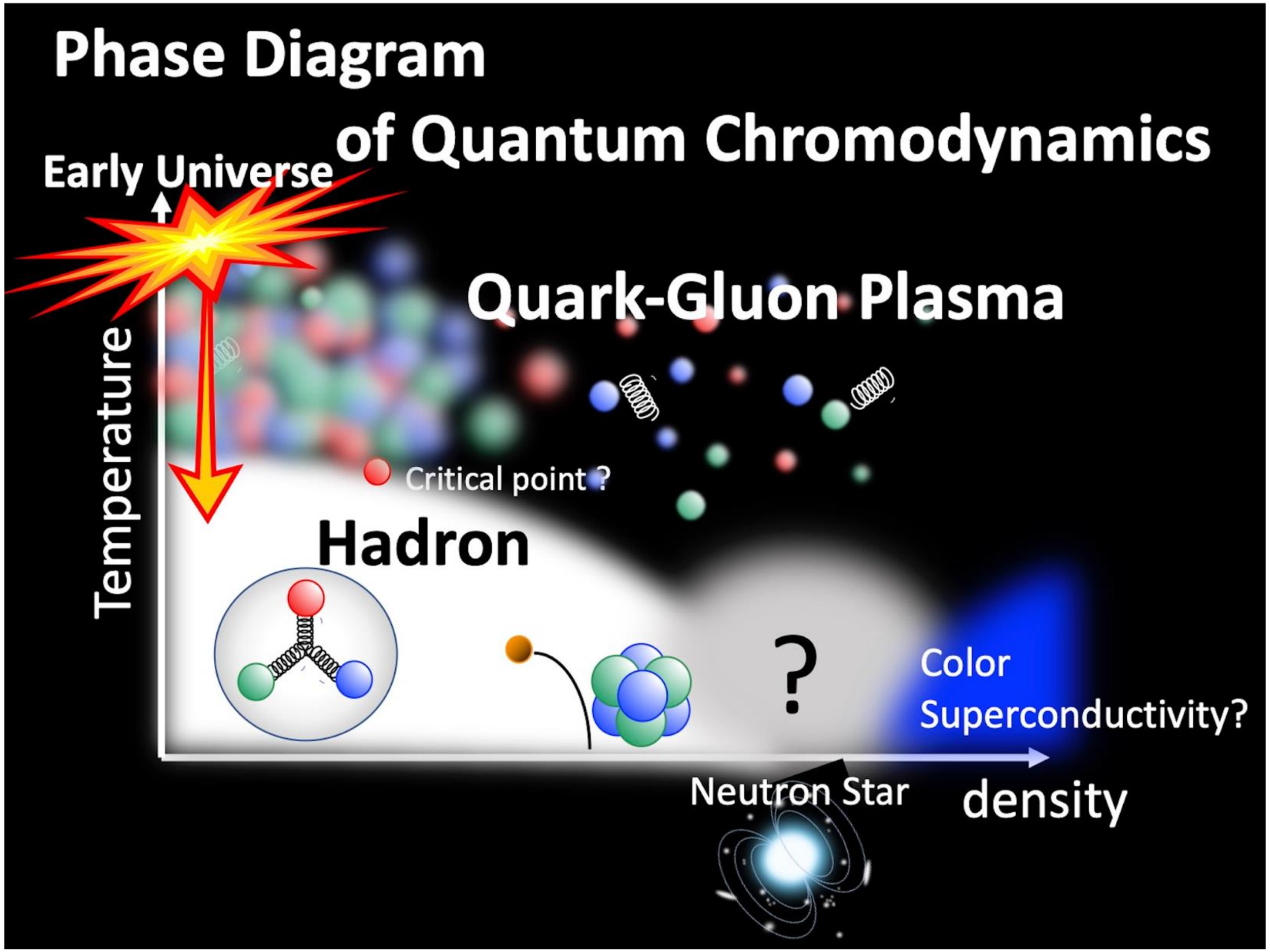
Chiho Nonaka

In collaboration with Eri Miyoshi, Yasushi Nara, Shunji Nishimura

Theoretical Particle and Hadron Physics Group

April 14, 2026 @ Intersection of nuclear structure and high-energy nuclear collisions 2026, YITP

The QCD Phase diagram and Equation of State



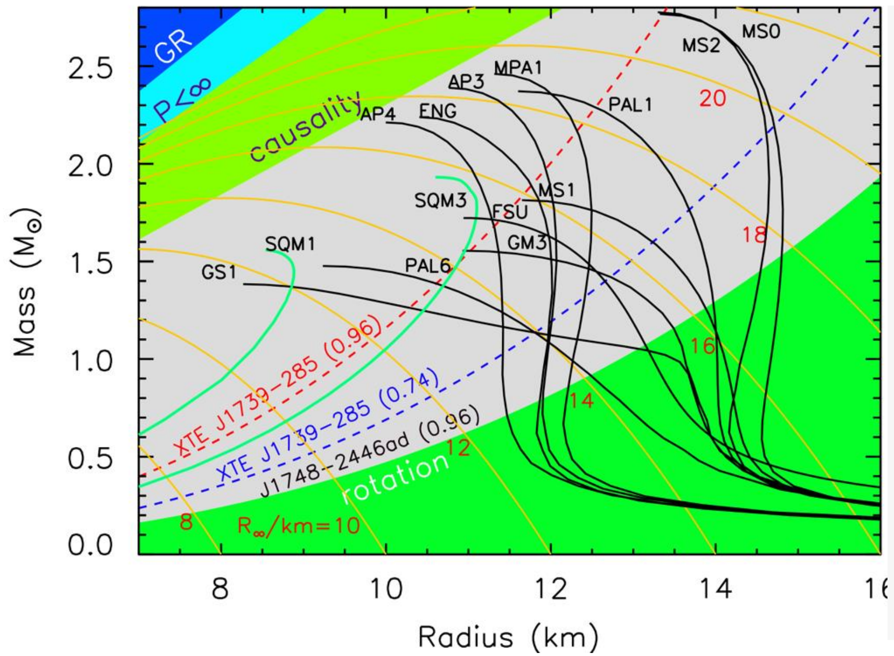
High temperature, low density

- High-energy heavy-ion collisions RHIC, LHC
- Flow, anisotropic flow
- Lattice equation of state

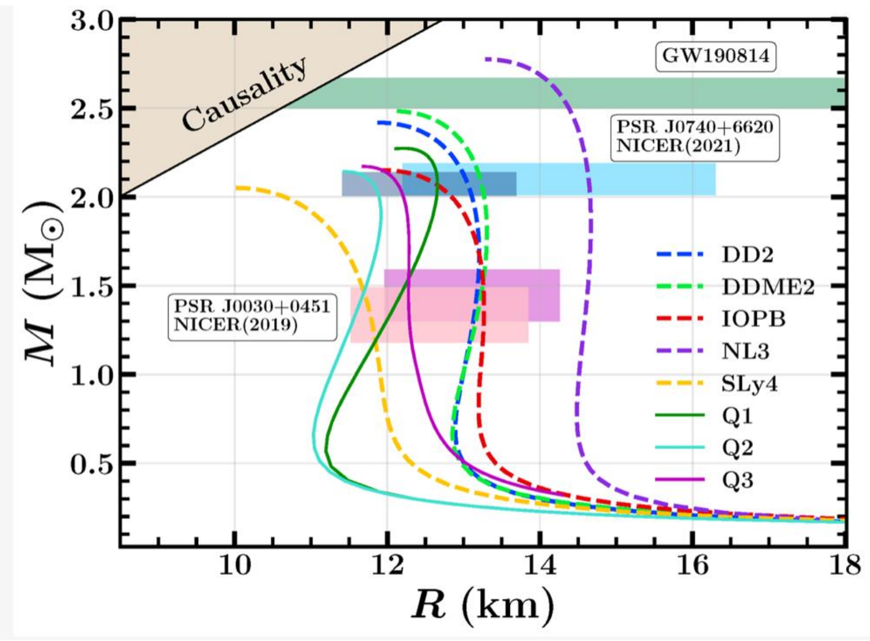
Low temperature, high density

- Neutron star, neutron star merger
- *M-R* relation
- EoS?

M-R Relation and Equation of State



Lattimer, Prakash, PR 442, 2007

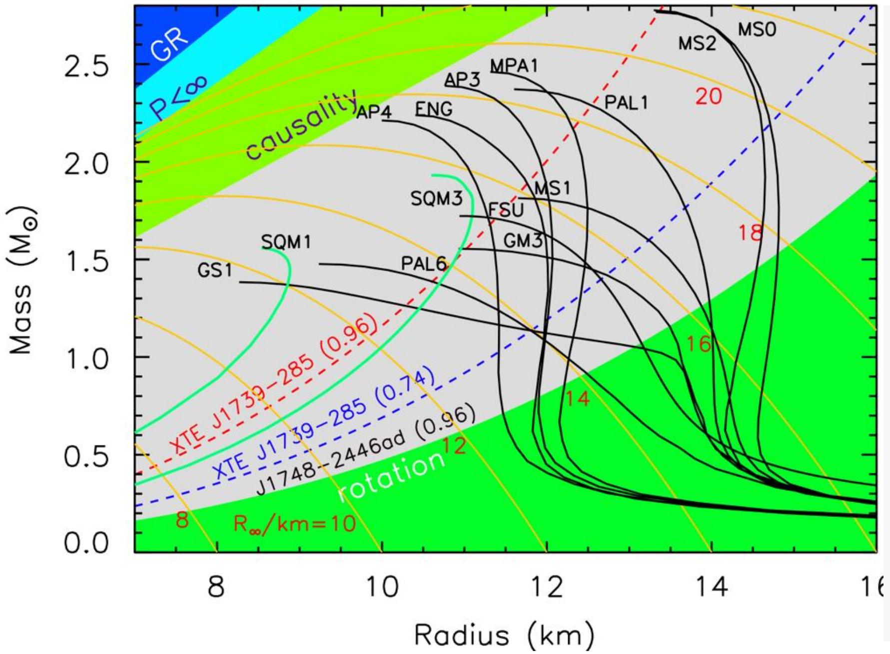


*Sen et al, Galaxies **2023**, 11(2), 60;*

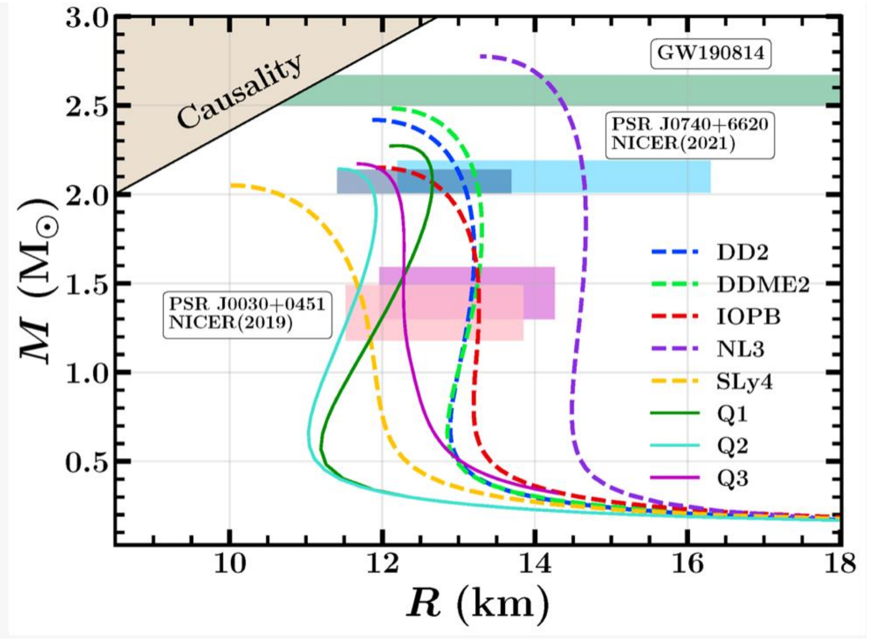
- Stiff EoS → larger radii and higher maximum mass
- Soft EoS → smaller radii and lower maximum mass
- Observations (~2 M⊙) rule out overly soft EoS
- NICER and GW constrain the radius (~11–14 km for 1.4 M⊙)

C. NONAKA

M-R Relation and Equation of State



Lattimer, Prakash, PR 442, 2007



*Sen et al, Galaxies **2023**, 11(2), 60*

- Uncertainty in dense matter physics above nuclear saturation density
- Possible phase transitions (hyperons, quark matter) not well constrained
- Need for consistent treatment of crust and core EoS
- Rotation, magnetic fields, and temperature effects often neglected

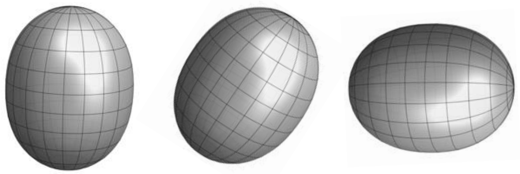
Can we explore the EoS inside of neutron star form high-energy collisions?

Impact of Nuclear Deformation

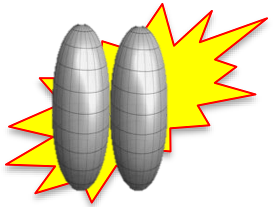
STAR, Nature635, 67 (2024)

- High-Energy Heavy-Ion Collisions: Snapshot of Nuclear Shape
  - Nuclear Deformation

ex. Uranium



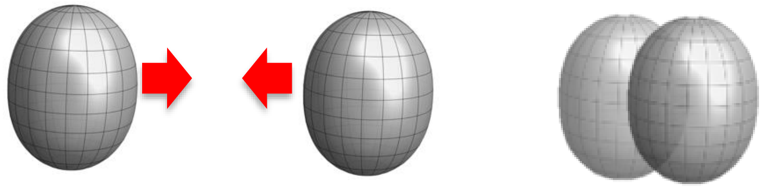
Quantum fluctuations in orientation~ 10^3-10^4 fm



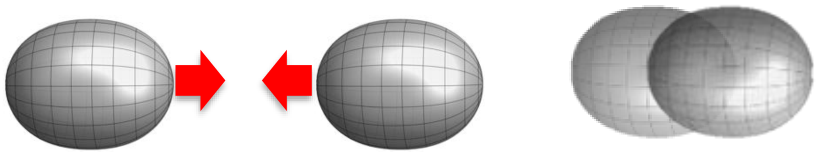
High-Energy Heavy-Ion collision: ~ 0.1 fm

body-body collisions

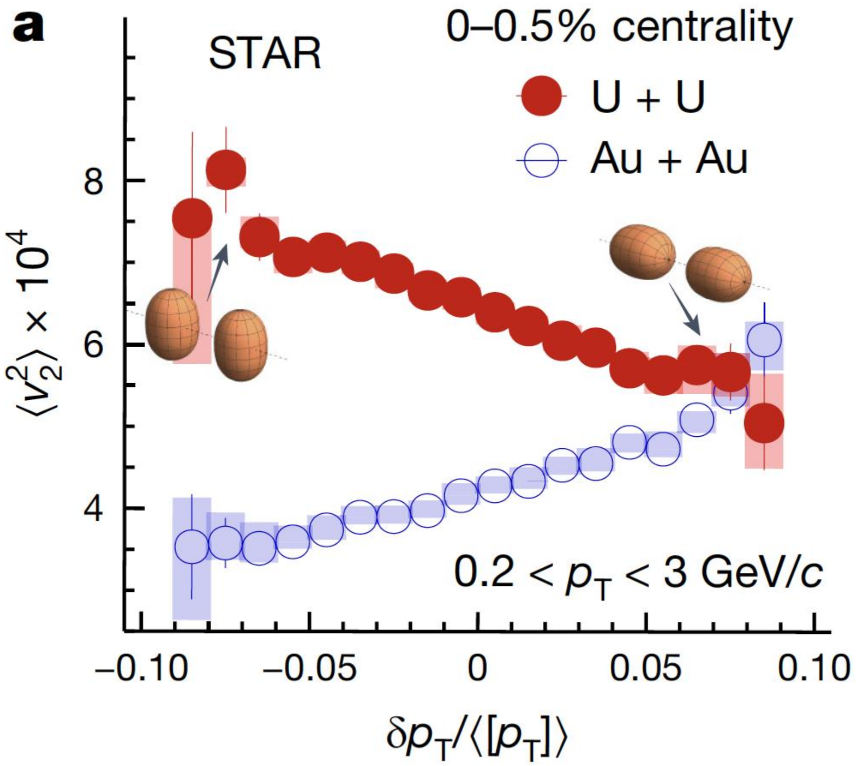
Overlap region



tip-tip collisions



expansion
collective flow



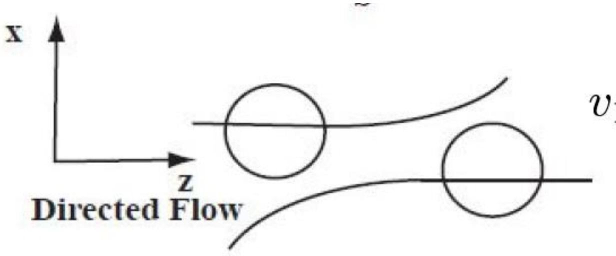
RHIC, LHC
Xe+Xe
O+O, Ne+Ne…

New Way to Extract EoS from Heavy-Ion Collisions

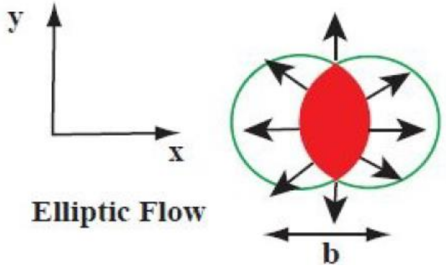
1. Directed and elliptic flow in ultra-central tip-tip collisions of deformed nuclei

- Directed (v_1) and elliptic (v_2): collision geometry, EoS

Danielewicz, Lacey, Lynch, Science 298 (2002)1592



$$v_1 = \langle \cos\phi \rangle = \left\langle \frac{p_x}{p_T} \right\rangle$$

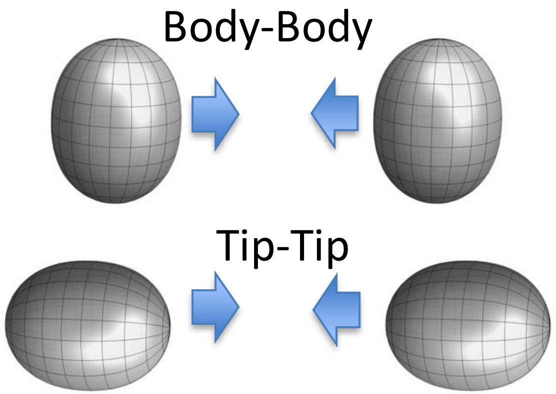


$$v_2 = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

Tilted shape just after collisions, expansion

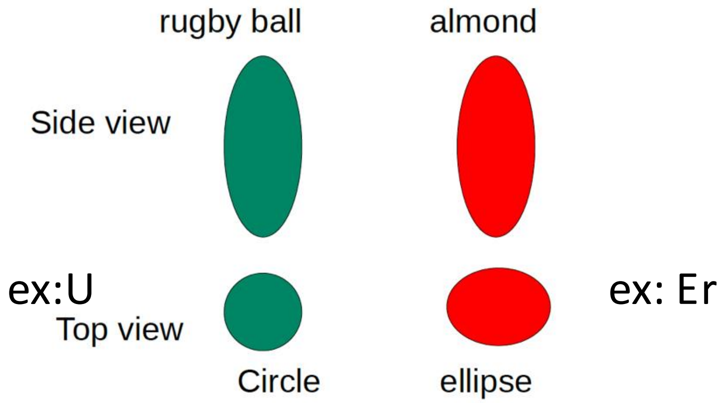
Shadowing (spectator) effect

2. Ultra central collisions of deformed nuclei



Finite anisotropic flow
No shadowing effect

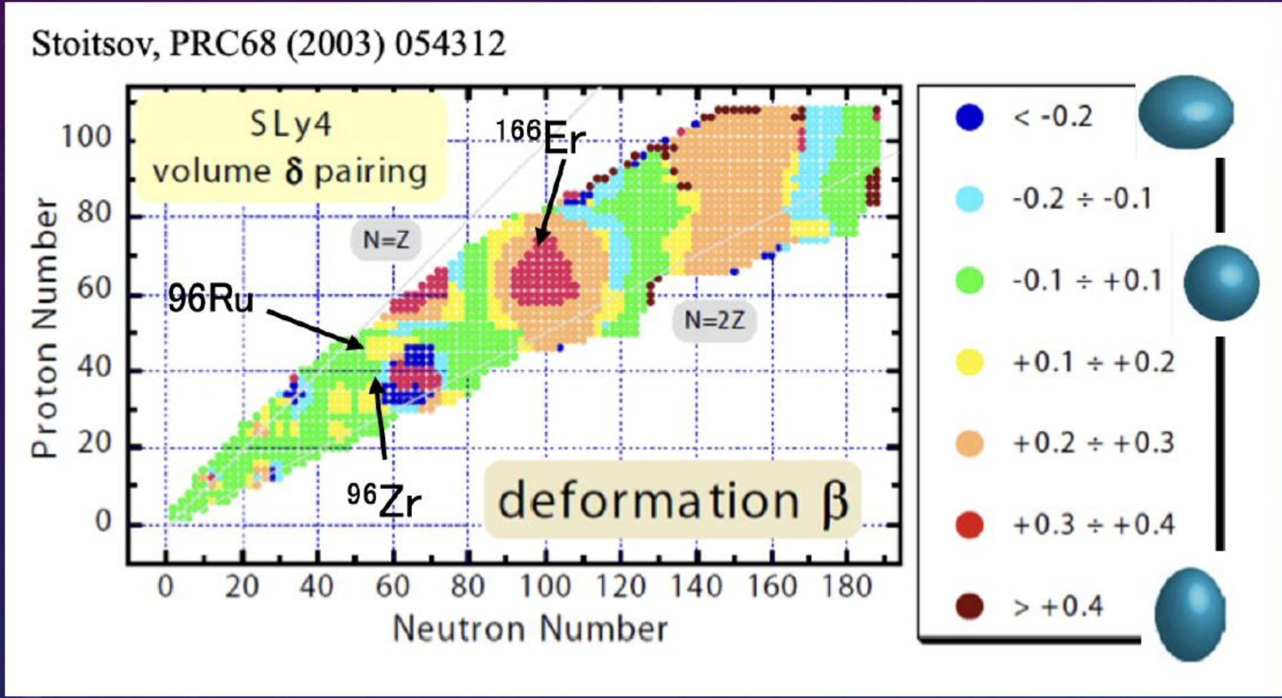
Finite trixiality (γ)



Idea of Deformed Nucleus + Nucleus Collisions

$^{166}Er + ^{166}Er$ Collisions (2.1 GeV/n)

A. Rosenhauer, J.A. Maruhn, H. Stocker, and W. Greiner
Phys. Lett. 159B, (1985)



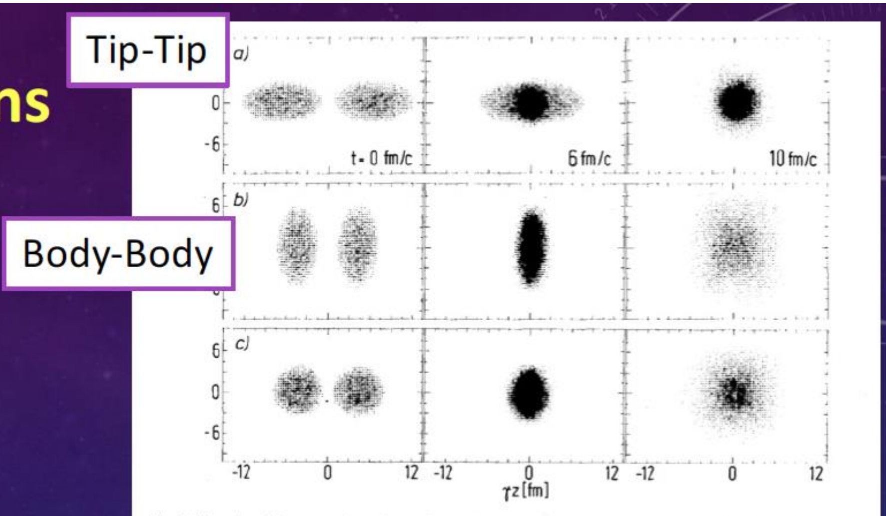


Fig. 1. Illustration of the compression and expansion stage in a central collision. The density at different reaction times as indicated is plotted for the three constellations. The "head on head" collision corresponds to (a), the "belly on belly" orientation to (b) and the reaction of two spherical nuclei ($^{46}Ti + ^{46}Ti$ at 2.1 GeV/n) is illustrated in (c).

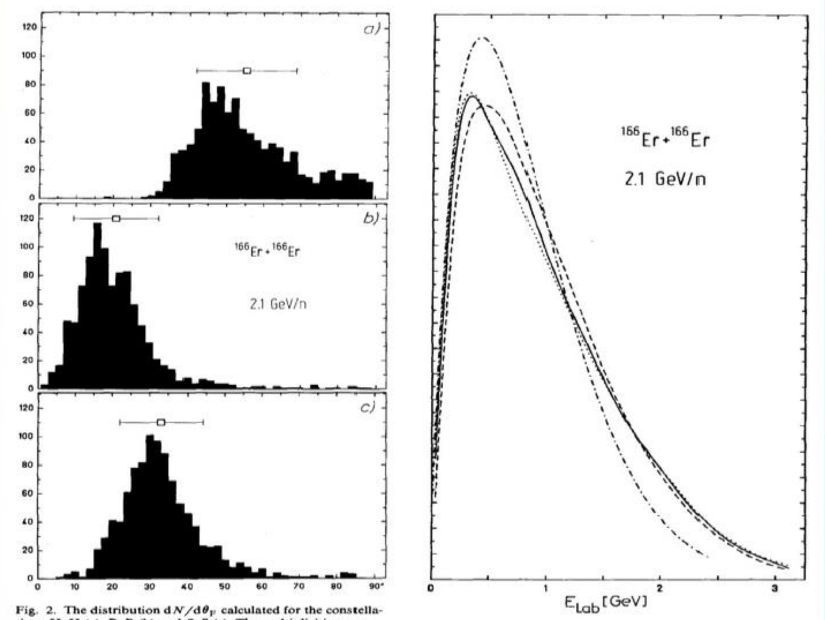


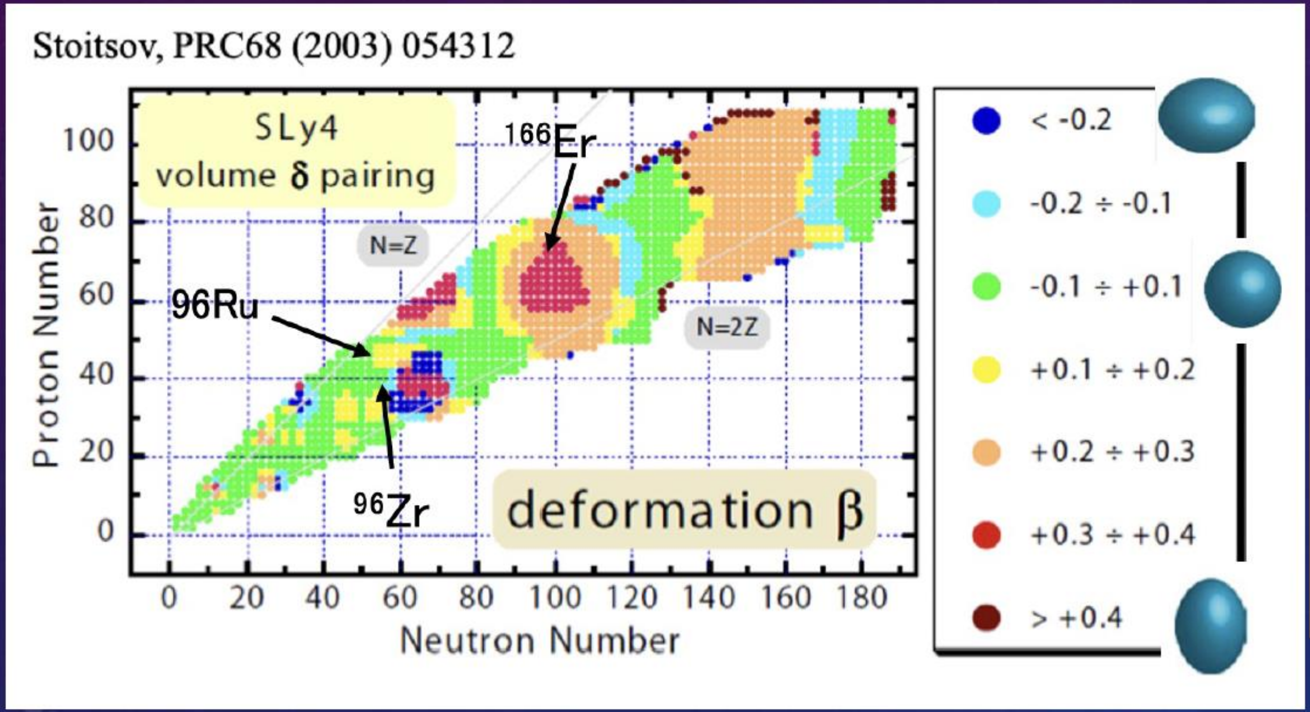
Fig. 2. The distribution $dN/d\theta_F$ calculated for the constellations HoH (a), BoB (b) and SoS (c). The multiplicities as a function of θ_F and the resulting mean value (square) with the corresponding standard deviation (error bars) are plotted on the basis of 1000 evaluated reactions of the system $^{166}Er + ^{166}Er$ at 2.1 GeV/n.

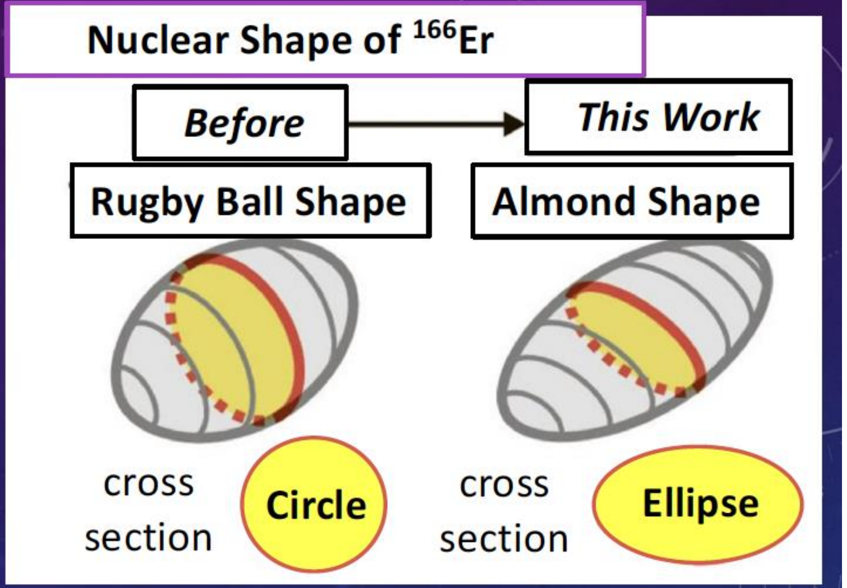
Fig. 3. Energy distribution dN/dE of all participant particles detected by the plastic ball. The spectrum obtained in the lab-system has been plotted for the cases HoH (dashed line), BoB (dotted line) and SoS (full line). The dash–dotted line shows the energy spectrum of a fireball.

By courtesy of S. Nishimura

Deformed Nucleus-Nucleus Collisions : $^{166}Er + ^{166}Er$

A. Rosenhauer, J.A. Maruhn, H. Stocker, and W. Greiner
Phys. Lett. 159B, (1985)





T. Otsuka, Y. Tsunoda et al.,
Eur. Phys. J. A (2025) 61:126
"Prevailing triaxial shapes in atomic nuclei and
a quantum theory of rotation of composite objects"

By courtesy of S. Nishimura

Probe of High-Density EoS

1. High-energy Heavy Ion Collisions

- EoS is well established by lattice QCD calculation
- LHC collisions energy, Relativistic viscous hydrodynamic model
- Fix deformation parameters in initial conditions: ex: β_2 (Prolate, Oblate), γ (trixiality)

$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+\exp\left[\frac{r-R(\theta,\phi)}{a}\right]}$$

$$R(\theta,\phi) = R_0\left[1+\beta_2\left(\cos\gamma Y_{20}(\theta)+\frac{\sin\gamma}{\sqrt{2}}(Y_{22}(\theta,\phi)+Y_{2,-2}(\theta,\phi))\right)+\cdots\right]$$

$\beta_2 > 0 \rightarrow$ prolate (rugby ball)
$\beta_2 < 0 \rightarrow$ oblate
$0^\circ < \gamma < 60^\circ \rightarrow$ triaxial (almond)

2. Explore the EoS in finite density region

- JAM2
- Use deformation parameters fixed in high-energy collisions
- EoS dependence of v_1 and v_2 -> Find the suitable EoS in finite density

Miyoshi, Nara, Nishimura, CN, in preparation

Probe of High-Density EoS

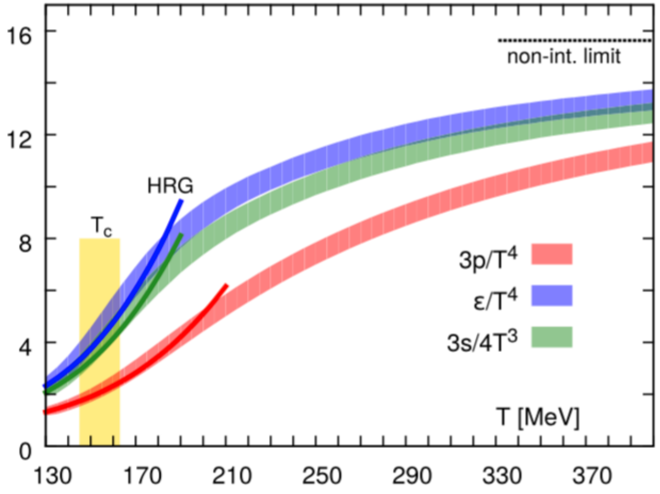
1. High-energy Heavy Ion Collisions

- EoS is well established by lattice QCD calculation
- LHC collisions energy, Relativistic viscous hydrodynamic model
- Fix deformation parameters in initial conditions: ex: β_2 (Prolate, Oblate), γ (trixiality)

$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+\exp\left[\frac{r-R(\theta,\phi)}{a}\right]}$$

$$\beta_2 > 0 \rightarrow \text{prolate (rugby ball)}$$
$$\beta_2 < 0 \rightarrow \text{oblate}$$
$$0^\circ < \gamma < 60^\circ \rightarrow \text{triaxial (almond)}$$

$$R(\theta,\phi) = R_0\left[1+\beta_2\left(\cos\gamma Y_{20}(\theta) + \frac{\sin\gamma}{\sqrt{2}}(Y_{22}(\theta,\phi)+Y_{2,-2}(\theta,\phi))\right)+\cdots\right]$$

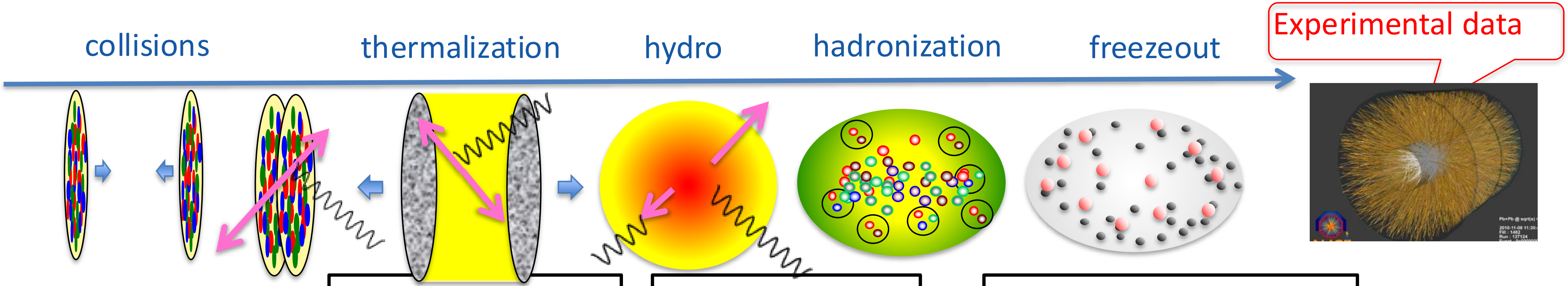


HotQCD,PRD90,094503(2014)

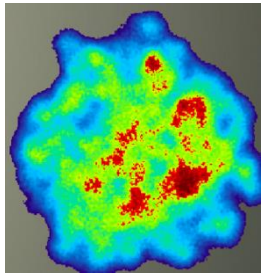
(2+1) flavor, Highly improved staggered quark action
Nt=6,8,10,12,Ns=4Nt → continuum limit
Parametrization of EoS

C. NONAKA

High Energy: Relativistic Hydrodynamic Model



Initial conditions



Energy (entropy) density distributions

Hydrodynamics

QCD phase diagram

EoS: lattice QCD

Shear and bulk viscosities

Final state interactions

Hadron based event generator

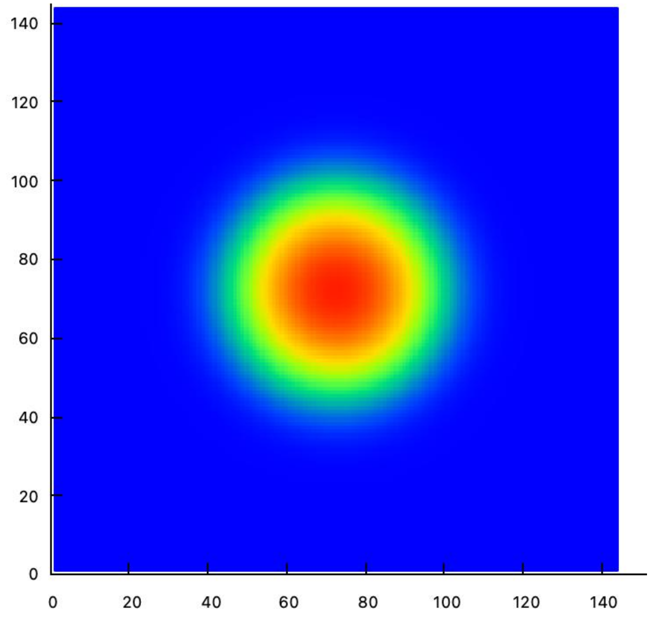
Er collisions

Akamatsu et al, JCP256,34(2014)
Okamoto, Akamatsu, Nonaka, EPJC76,579(2016)
Okamoto and Nonaka, EPJC77,383(2017)

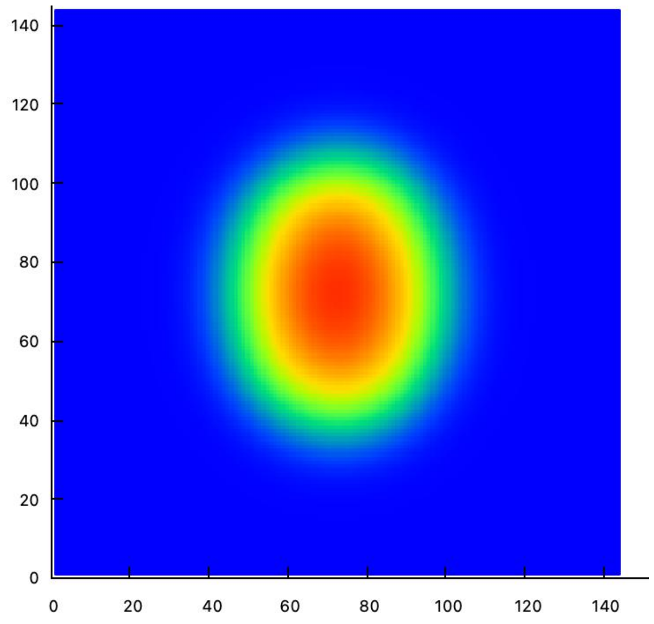
Results from Hydrodynamic Model

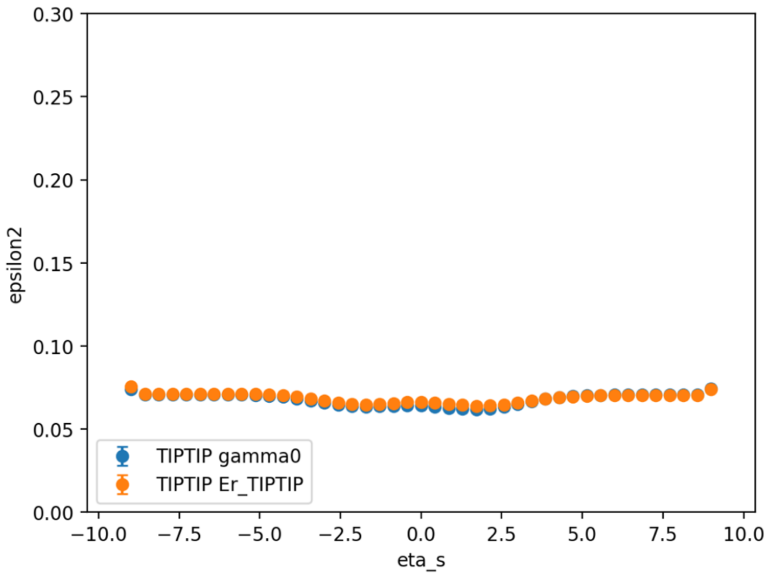
- Initial conditions: Trento
  - Er+Er collisions: b=0 fm: $\sqrt{s_{\rm NN}} = 5.02 \quad \beta_2 = 0.319, \gamma = 0.419$
  - Smoothed distribution

TIP-TIP



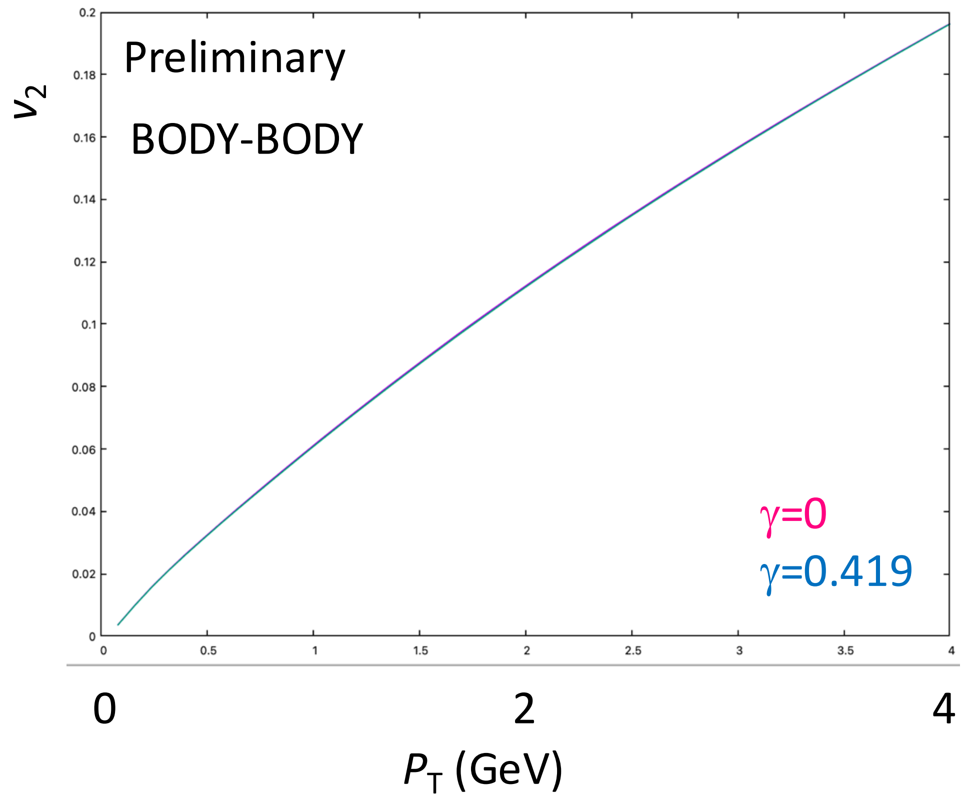
BODY-BODY

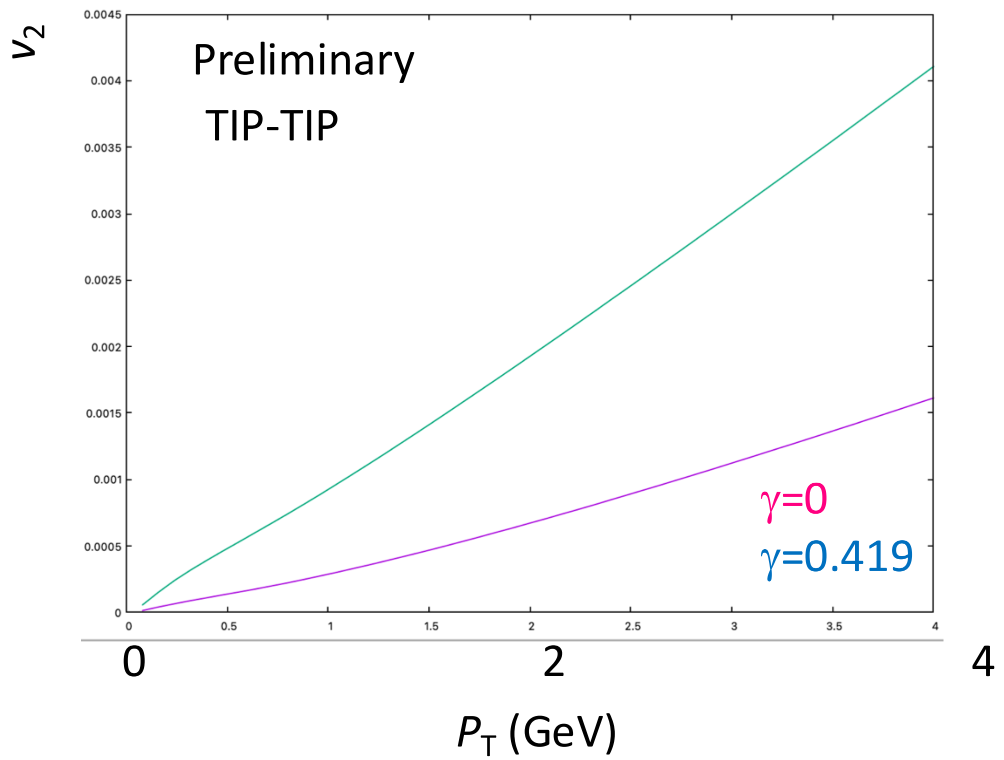




Results from Hydrodynamic Model

- Need to find the suitable normalization
  - The lifetime of TIP-TIP collisions is 20 % larger than that of BODY-BODY collisions
  - v_2: larger g dependence Is observed in TIP-TIP collisions





C. NONAKA

Probe of High-Density EoS

1. High-energy Heavy Ion Collisions

- EoS is well established by lattice QCD calculation
- LHC collisions energy, Relativistic viscous hydrodynamic model
- Fix deformation parameters in initial conditions: ex: β_2 (Prolate, Oblate), γ (trixiality)

$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+\exp\left[\frac{r-R(\theta,\phi)}{a}\right]}$$

$$R(\theta,\phi) = R_0\left[1+\beta_2\left(\cos\gamma Y_{20}(\theta) + \frac{\sin\gamma}{\sqrt{2}}(Y_{22}(\theta,\phi)+Y_{2,-2}(\theta,\phi))\right)+\cdots\right]$$

$\beta_2 > 0 \rightarrow$ prolate (rugby ball)
$\beta_2 < 0 \rightarrow$ oblate
$0^\circ < \gamma < 60^\circ \rightarrow$ triaxial (almond)

2. Explore the EoS in finite density region

- JAM2
- Use deformation parameters fixed in high-energy collisions
- EoS dependence of v_1 and v_2 -> Find the suitable EoS in finite density

Miyoshi, Nara, Nishimura, CN, in preparation

Low Energy: RQMD2(JAM2)

Nara, Ohnishi, Jinno, Murase…

Extension of JAM (hadron + string cascade)

- JAM (hadron–string cascade transport mode)
- Includes mean-field dynamics (RQMD framework)

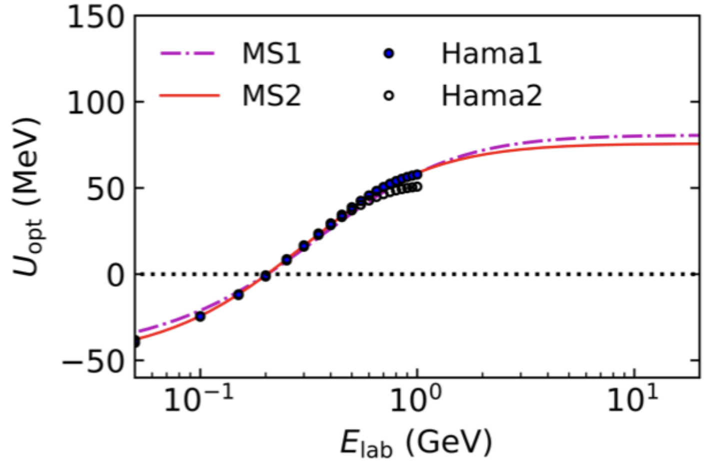
  skyrme type Lorentz vector potential

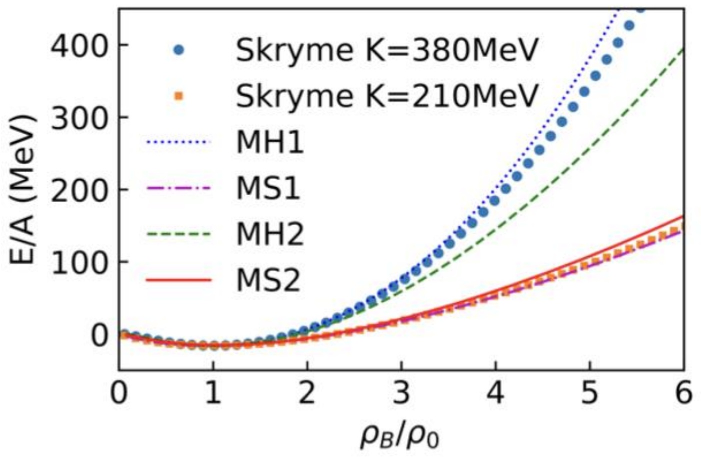
$$\mathrm{p}^{*\mu} = p^\mu - U^\mu_{\mathrm{sk}}(\rho) - U^\mu_m(p).$$

$$U^\mu_{sk} = U_{sk}(n_B)u^\mu, \quad u^\mu = \frac{J^\mu_B}{n_B} \quad n_B = \sqrt{J_B^2}$$

$$U_{\mathrm{sk}}(\rho) = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^\gamma$$

$$U^\mu_m(p) = \frac{C}{\rho_0}\int d^3p' \frac{p^{*\prime\mu}}{e'^*}\frac{f(x,p')}{1+[(\boldsymbol{p}-\boldsymbol{p}')/\mu_k]^2}$$



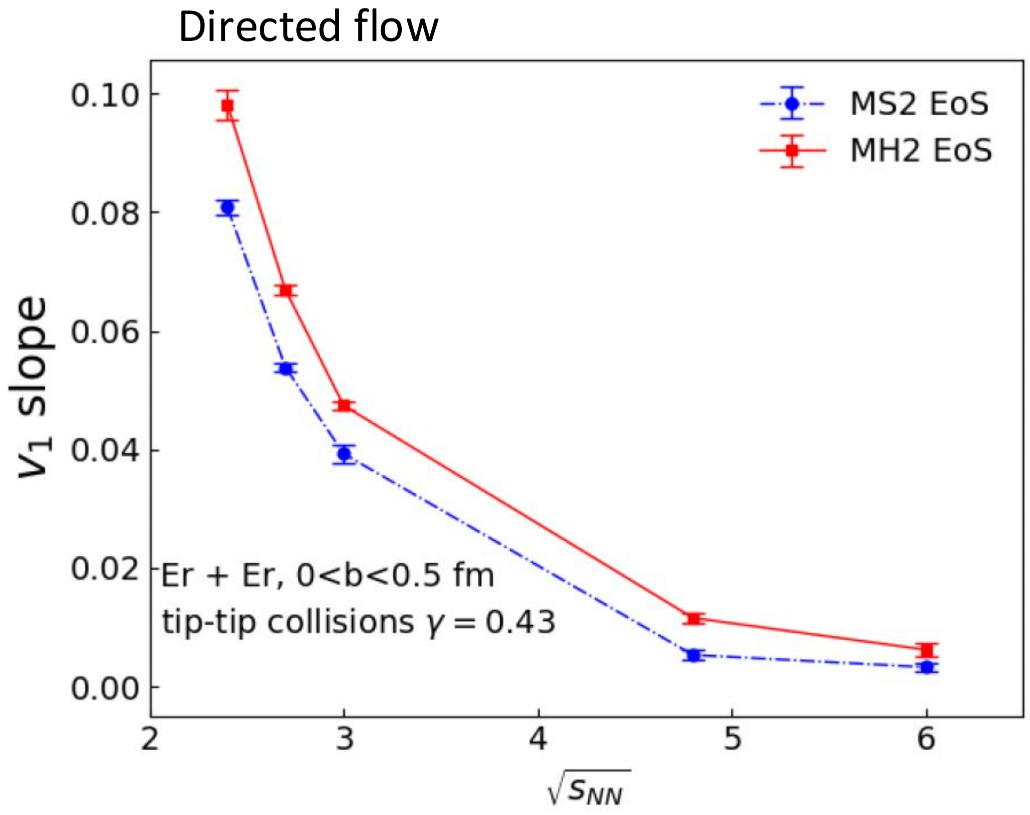


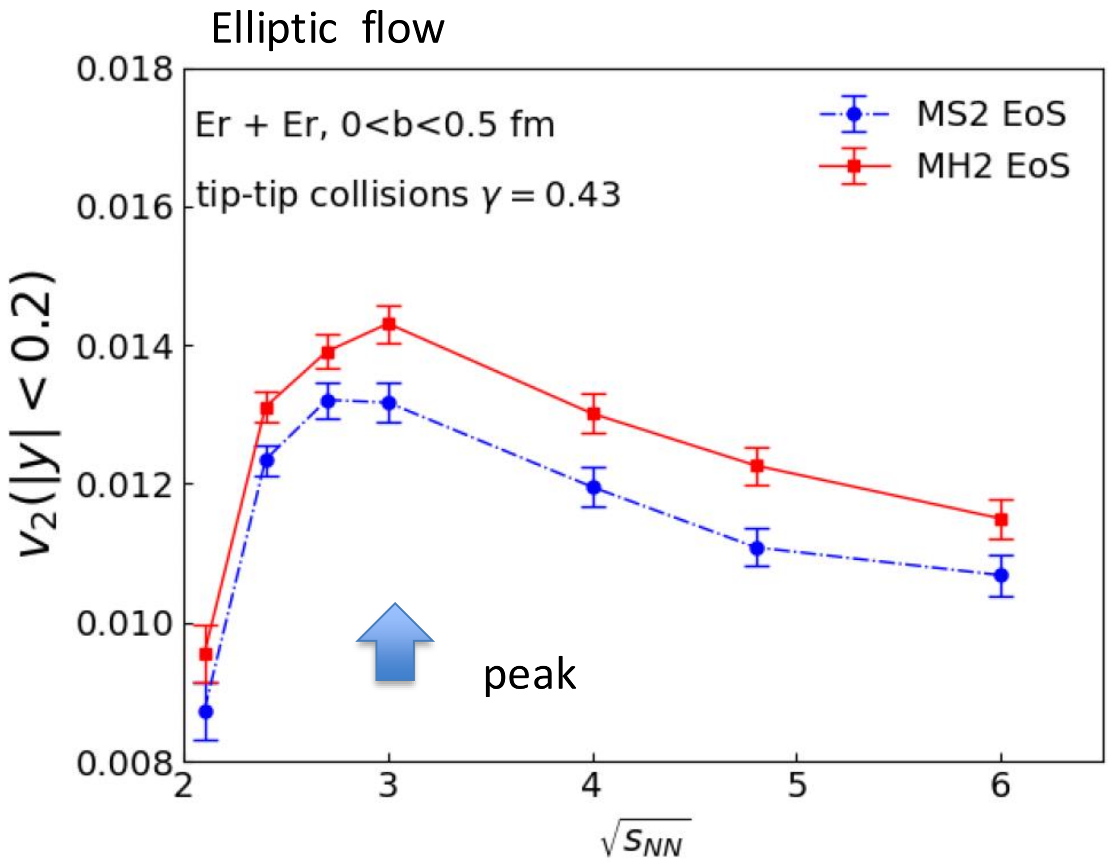
Nara, Ohnisi, PRC, 2022

Finite density region
EoS dependence

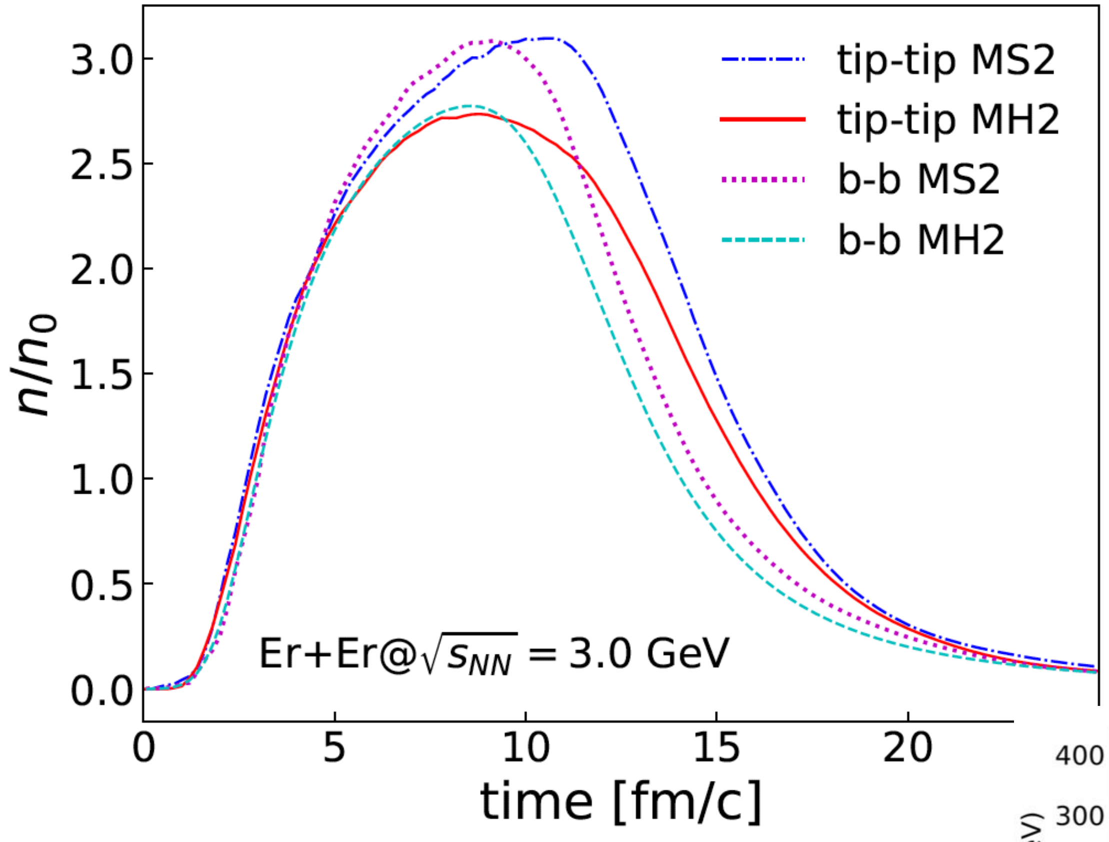
Beam Energy Dependence

- Er+Er central collisions $\beta_2 = 0.319$





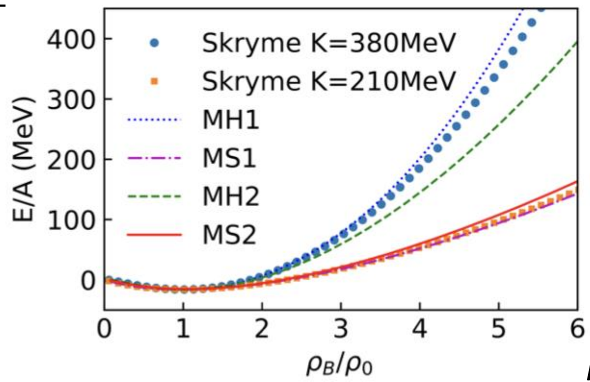
Time Evolution of Baryon Density



We can explore the EoS for neutron star.

- 2-3 times the normal density is achieved at 3 GeV.
- Clear EoS dependence
  - Baryon density is larger in MS2 compared to MH2

- Life time of high density is longer in tip-tip collisions compared to body-body

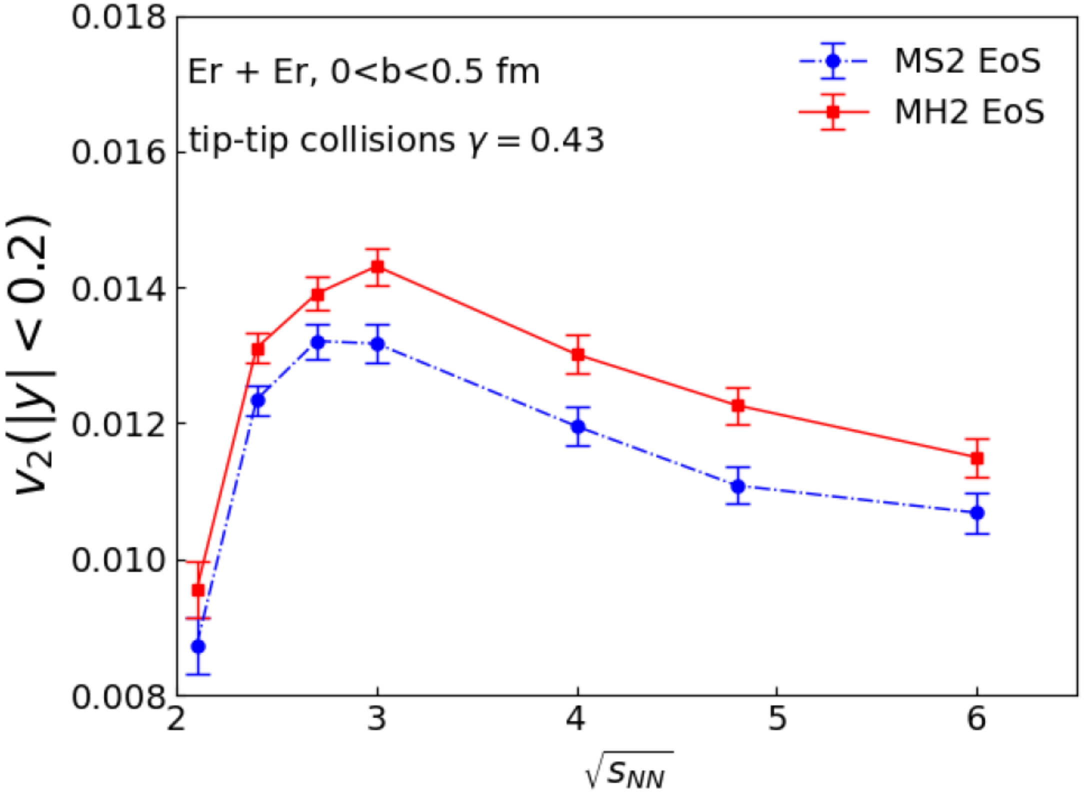


Nara, Ohnisi, PRC, 2022

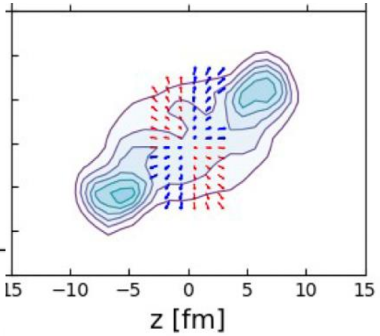
Beam Energy Dependence

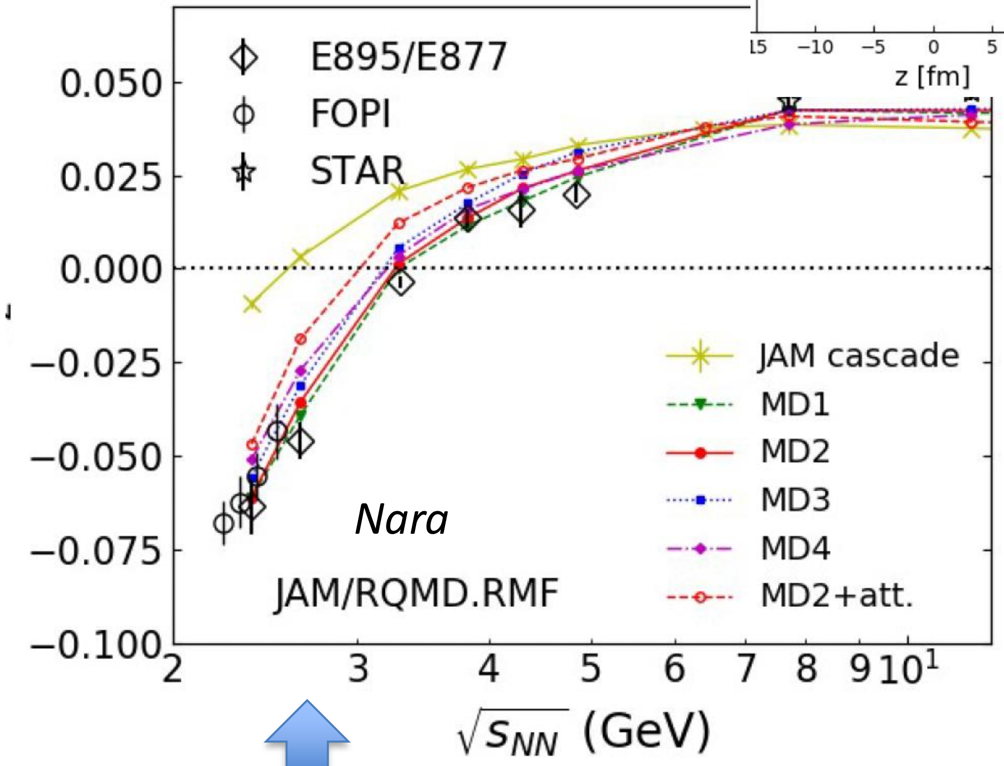
- Er+Er central collisions

Elliptic flow



Au+Au collision, finite impact parameter

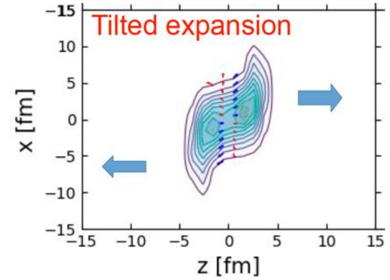




- v_2 remains positive at all collision energies.
- No shadowing effect
- Eos dependence: v_2 of MH2 is larger than that of MS2.

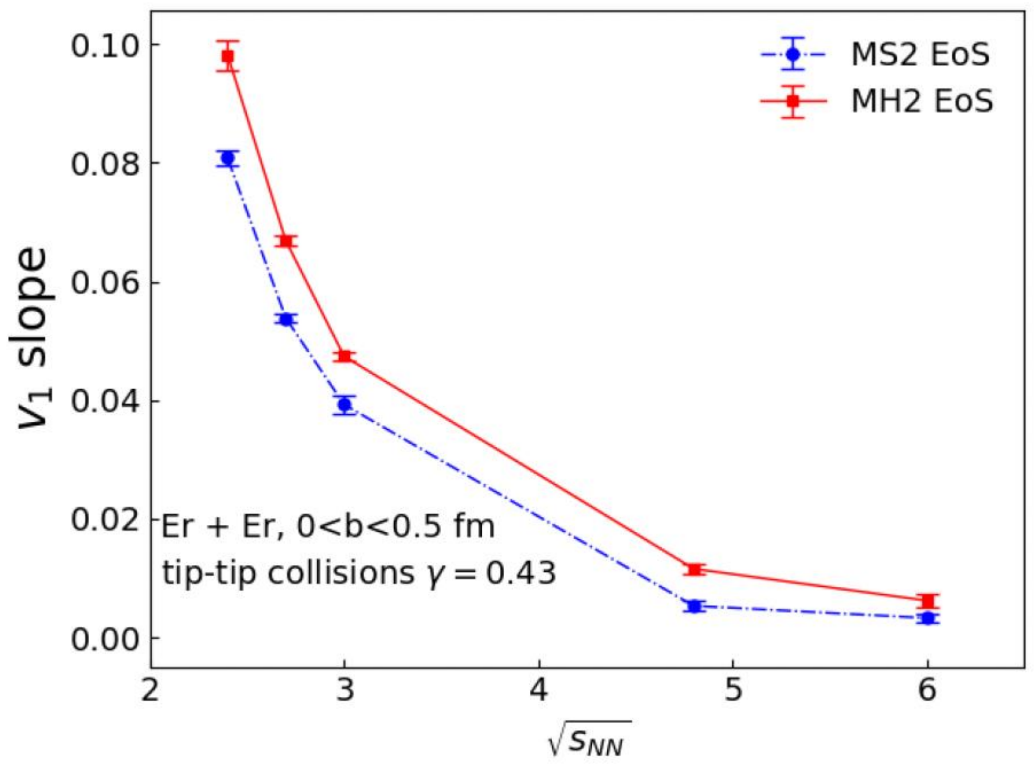
- v_2 becomes negative due to shadowing by spectators.

Beam Energy Dependence

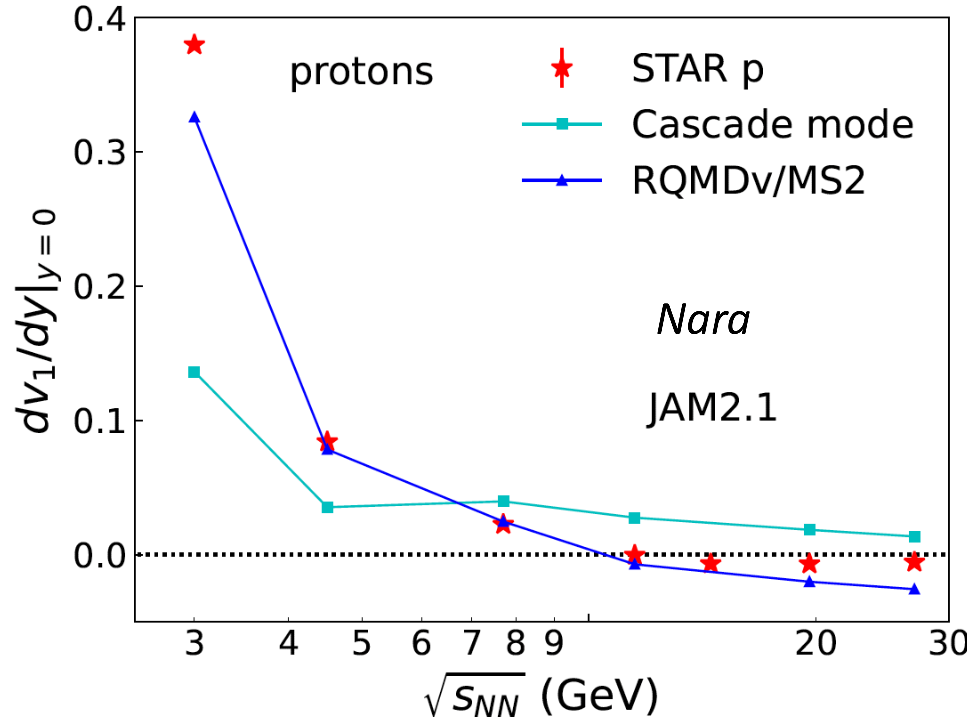


- Er+Er central collisions

Directed flow



Au+Au collision, finite impact parameter



- v_1 remains positive at all collision energies.
- No shadowing effect
- EoS dependence: v_1 of MH2 is larger than that of MS2.

- Positive v_1 at compression, while negative v_1 at expansion

Summary

New way to extract the information about EoS at high densities by using ultra-central tip-tip collisions of deformed nuclei

1. High-energy Heavy Ion Collisions (Hydrodynamic Model), on-going
   - Fix deformation parameters in initial conditions: ex: β_2 (Prolate, Oblate), γ (trixiality)
2. Explore the EoS in finite density region (JAM2)
   - EoS dependence of v_1 and v_2 -> Find the suitable EoS in finite density

Results

- Directed flow and elliptic flow are key observables sensitive to the EoS.
- In Er central collisions, v_1 and v_2 remain positive at all collision energies, with no shadowing effects.
- The results show clear EoS dependence, providing a new way to probe high-density matter relevant to neutron stars.

Miyoshi, Nara, Nishimura, CN, in preparation