A Few Topics in Nuclear Deformation and High-energy Physics

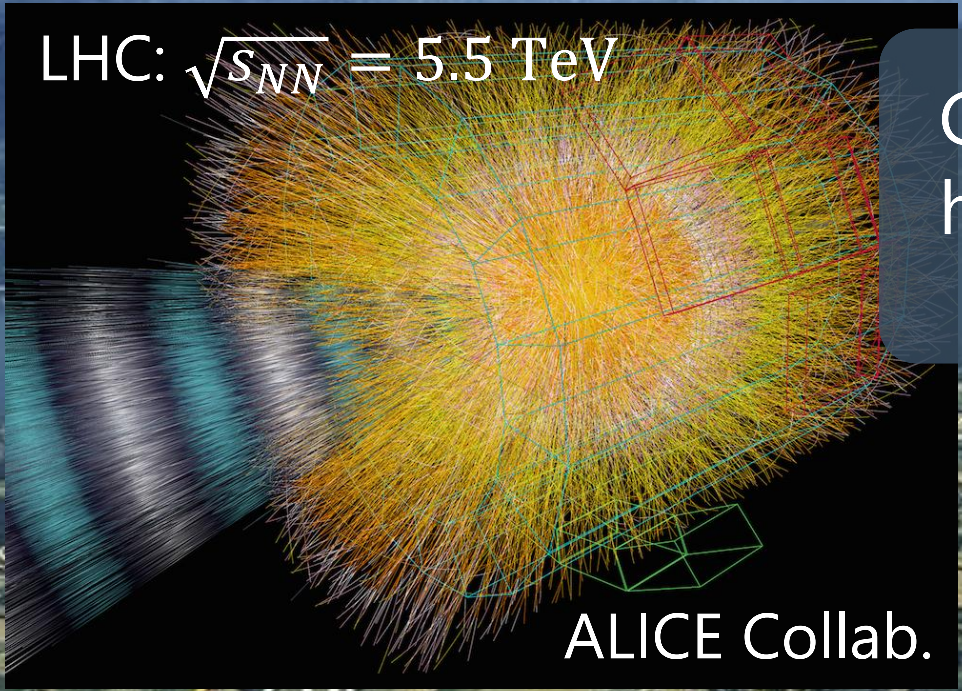
Masakiyo Kitazawa
(YITP, Kyoto)

Hagino, MK, PRC **112** (2025) L041901 [2508.05125];
MK, Esumi, Niida, Nonaka, PTEP (2026) in press, [2510.18383];
Gubler, MK, Minato, Nara, Taya, in progress.

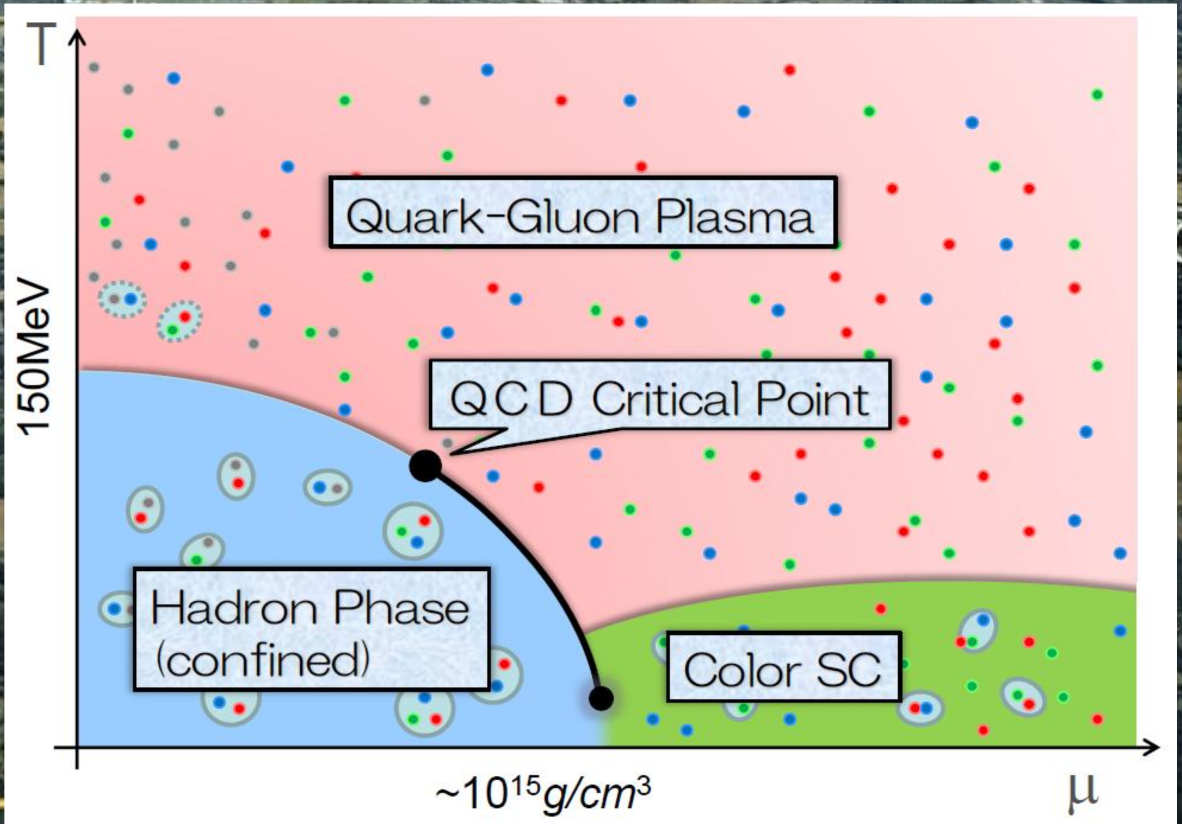
Contents

Contents

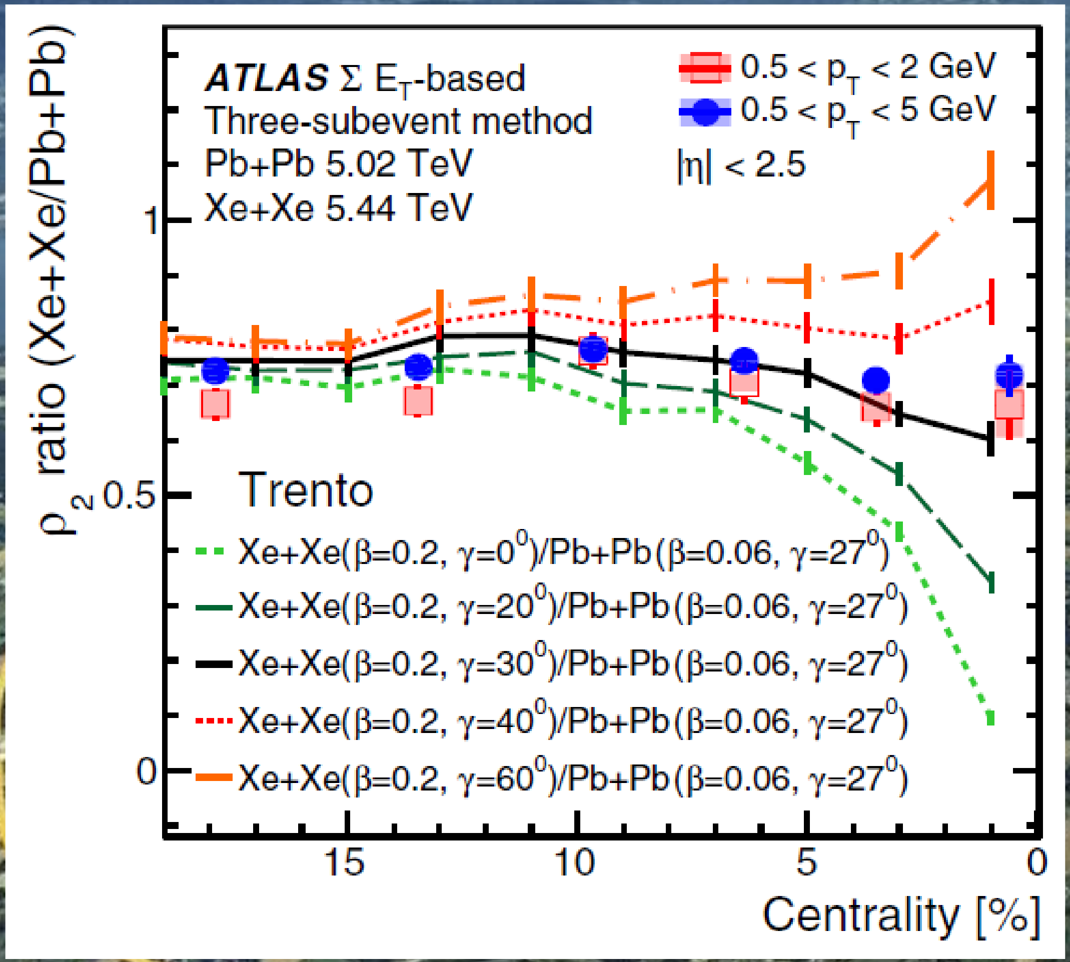
Relativistic Heavy-Ion Collisions



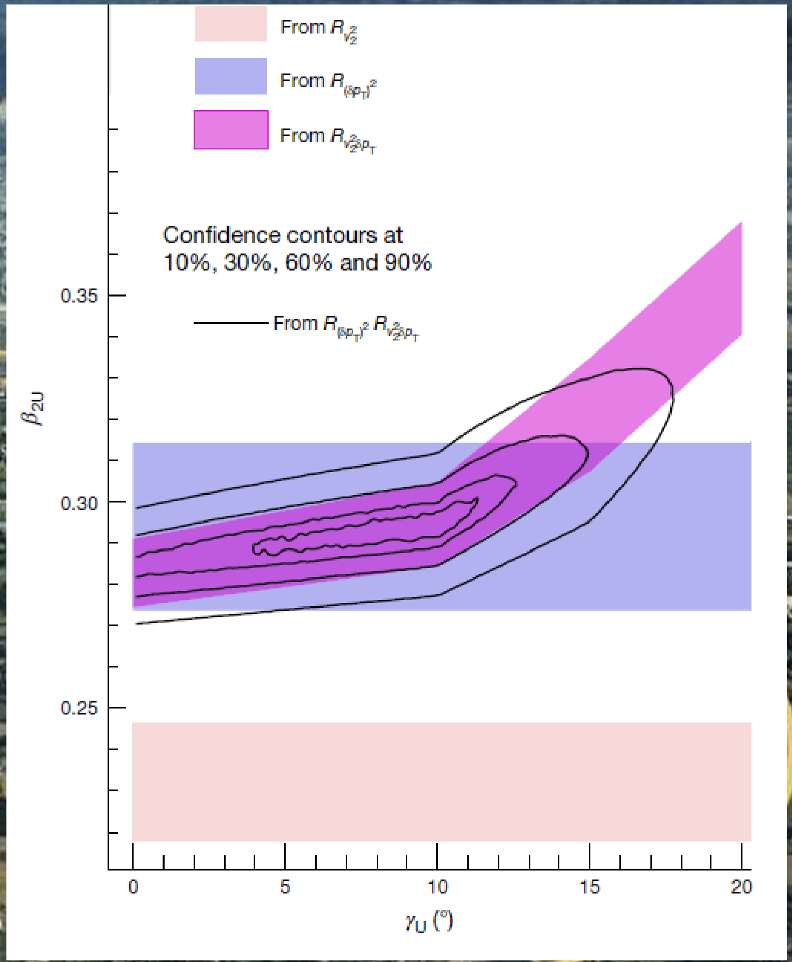
Creation of hot/dense matter ▶ Study of
—quark-gluon plasma
—QCD phase diagram



Probing Nuclear Shapes in RelHIC?!

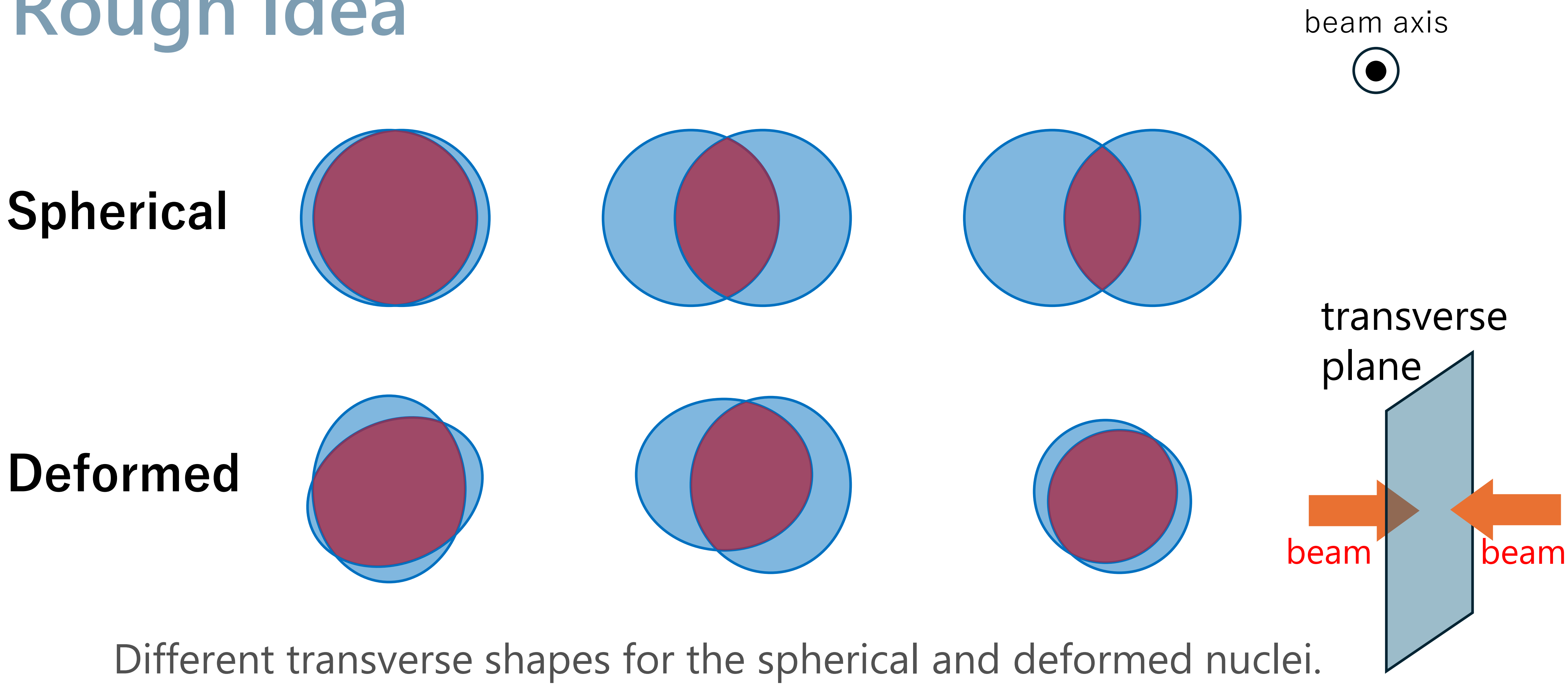


ATLAS, PRC 107, 054910 (2023)



STAR, Nature 635, 67 (2024)

Rough Idea



Different transverse shapes for the spherical and deformed nuclei.
Distribution is reflected into anisotropic flows in the final state.

High-energy collisions → **snapshot** of the overlapping region of **intrinsic states**

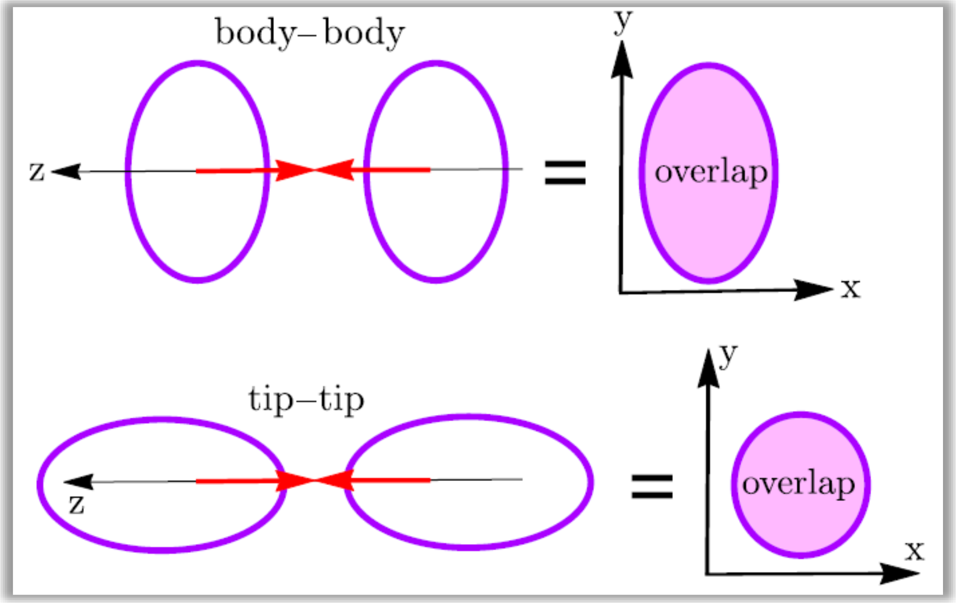
Ultra-Central Collisions (UCC)

UCC ➔ Almost all particles participate in the collisions

Collision of Prolate nuclei

	mean radius	anisotropy
tip-tip	small	small
body-body	large	large

Giacalone, PRL ('20)



Ultra-Central Collisions (UCC)

UCC ➔ Almost all particles participate in the collisions

Collision of Prolate nuclei

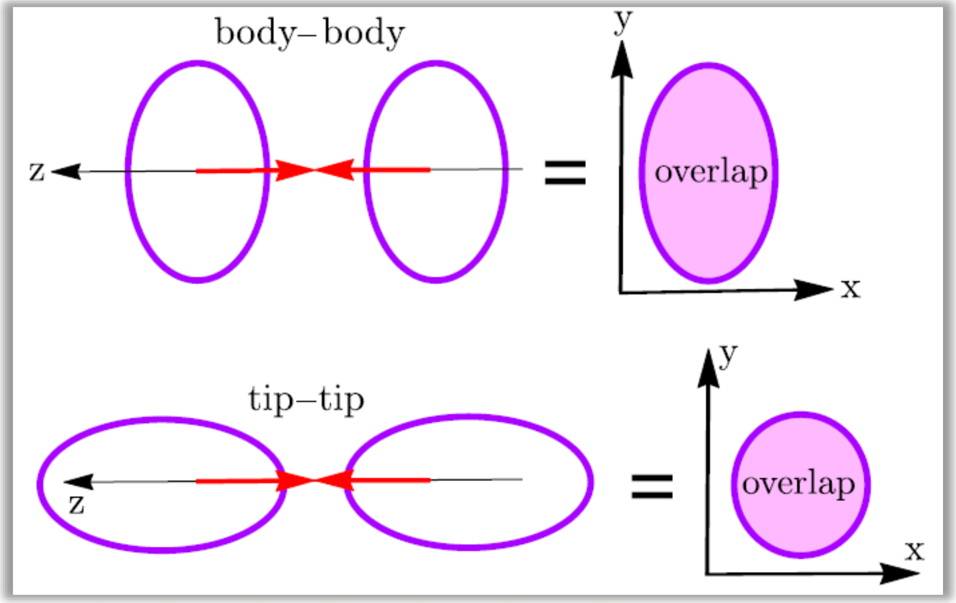
	mean radius	anisotropy
tip-tip	small	small
body-body	large	large

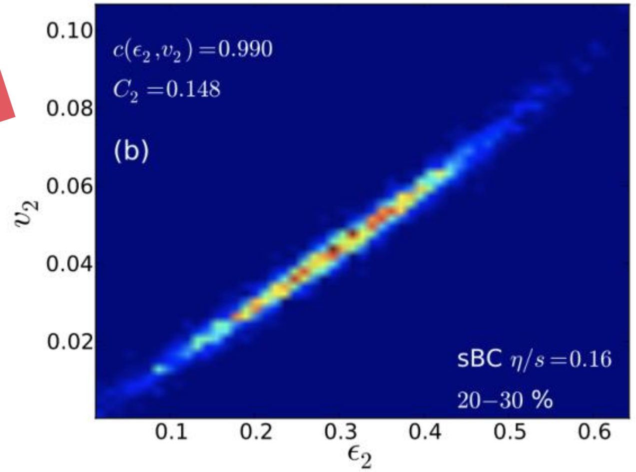
hydro. evolution

tip-tip	large $\overline{p}_T$ / small v_2
body-body	small $\overline{p}_T$ / large v_2

➔Inverse correlation of v_2 & $\overline{p}_T$

Giacalone, PRL ('20)





Experimental Result @STAR

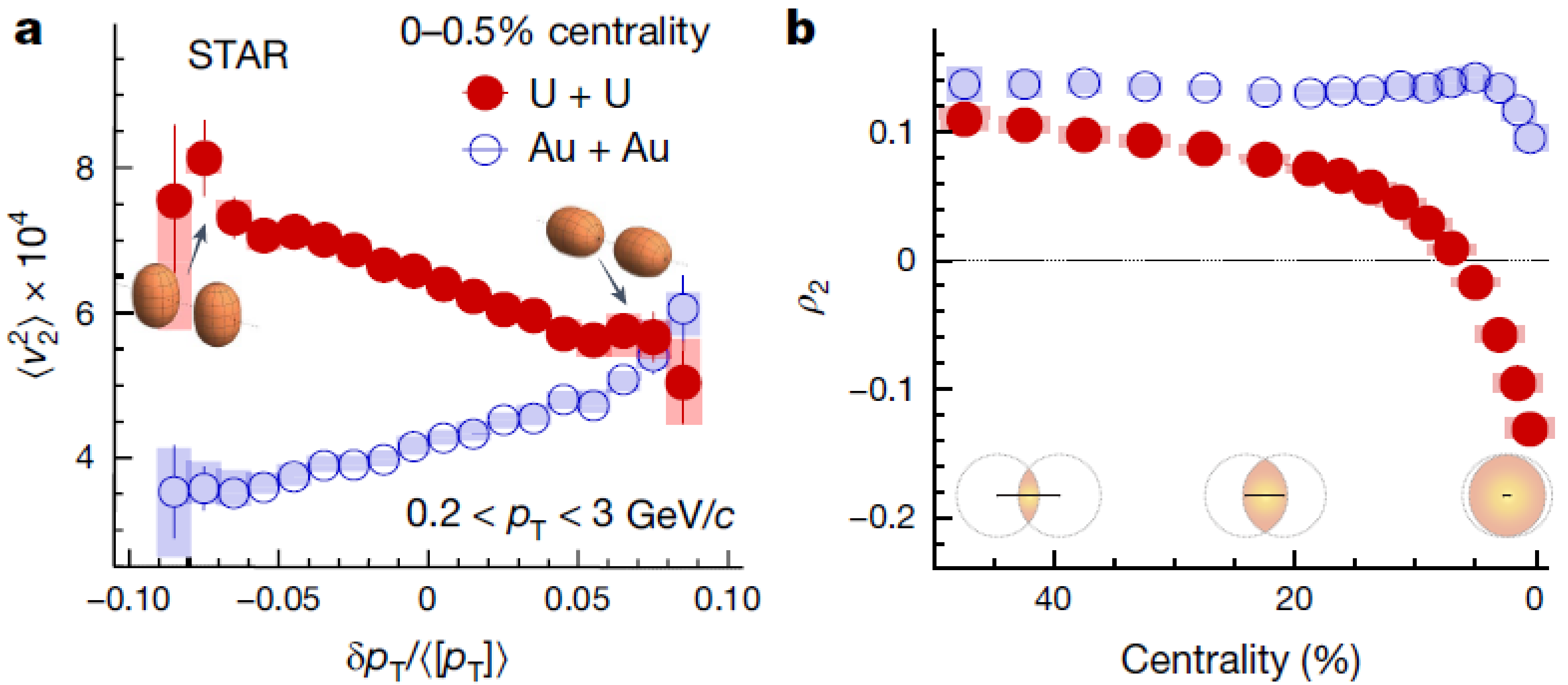
tip-tip	large $\overline{p}_T$ / small v_2
body-body	small $\overline{p}_T$ / large v_2

➔Inverse correlation of v_2 & $\overline{p}_T$

$v_n - p_T$ correlation:

$$\rho(v_n\{2\}^2,[p_\perp]) = \frac{\mathrm{cov}(v_n\{2\}^2,[p_\perp])}{\sqrt{\mathrm{Var}(v_n^2)_{\mathrm{dyn}} C_{p_\perp}}}$$

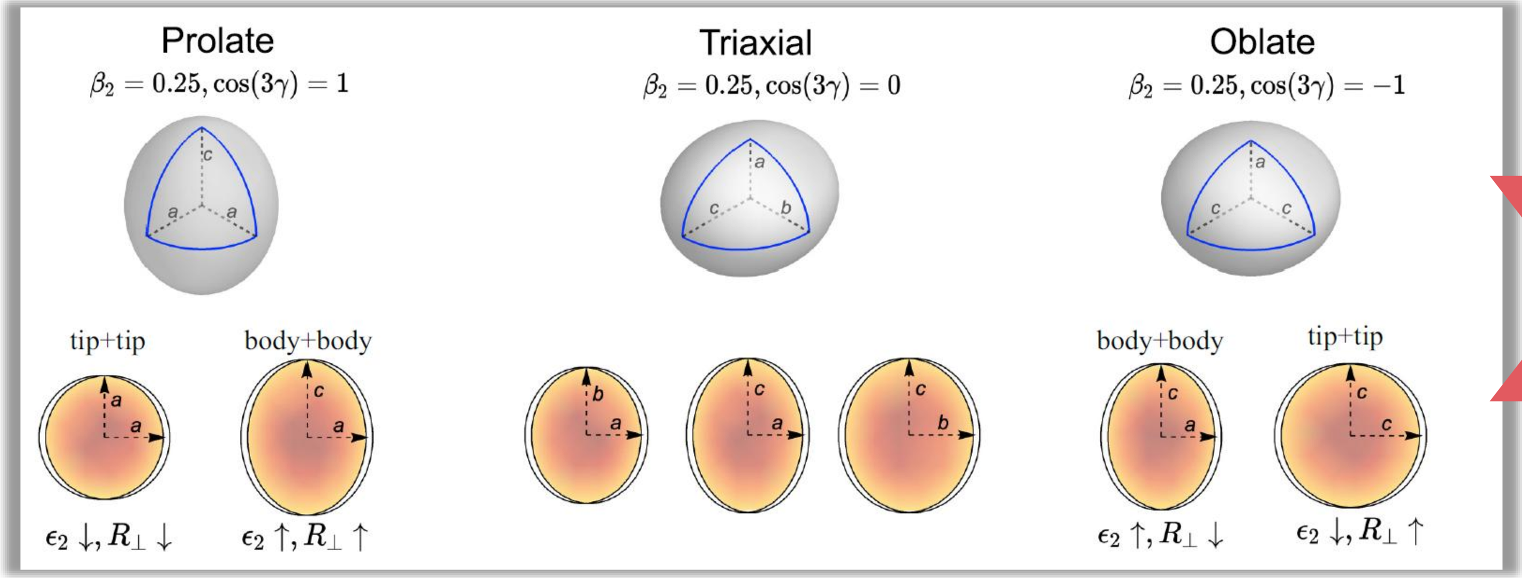
Bozek, PRC 93 ('16)



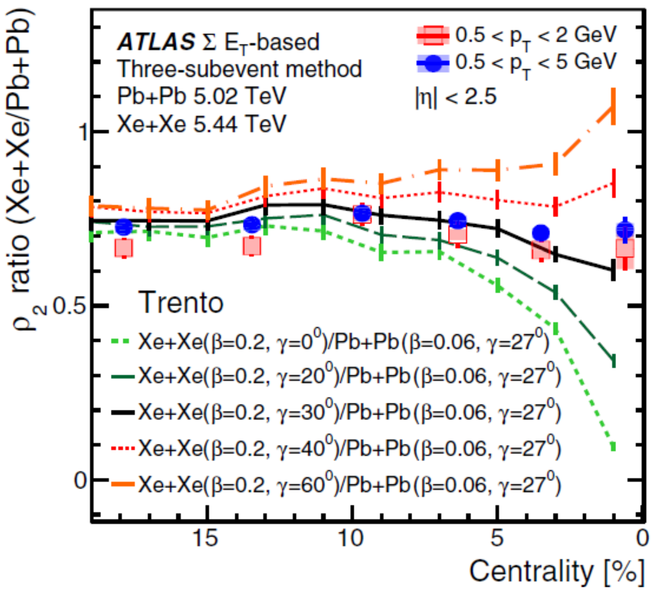
STAR, Nature 635 ('24)

Triaxial Deformation

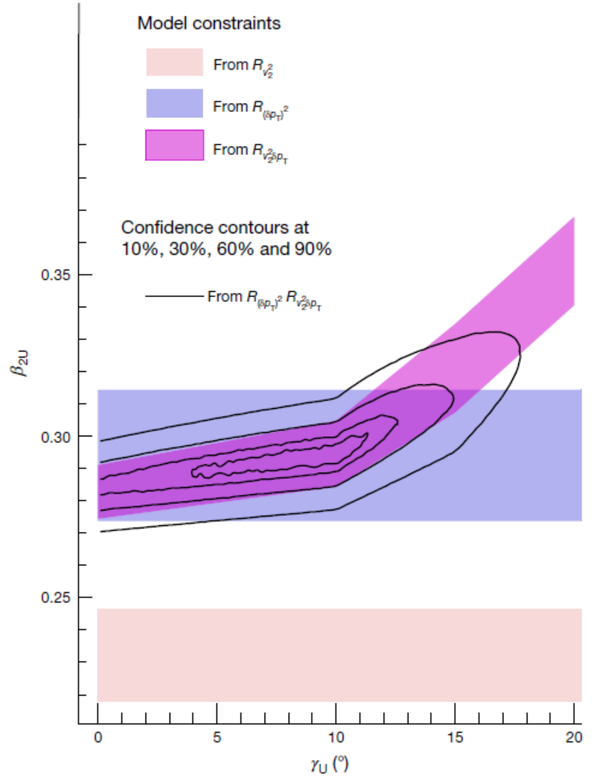
Fig.: Jia, PRC ('22)



ATLAS, PRC 107 (2023)



STAR, Nature 635 (2024)



- v_2-p_T correlation is sensitive to deform. param. γ.
- Other correlations sensitive to β_2 and γ.

$$\left\langle \varepsilon_2^2 \frac{\delta d_\perp}{d_\perp} \right\rangle = -\frac{3\sqrt{5}}{28\pi^{3/2}} \cos(3\gamma)\beta_2^3. \qquad \left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^2 \right\rangle = \frac{1}{16\pi}\beta_2^2$$

Further Extension

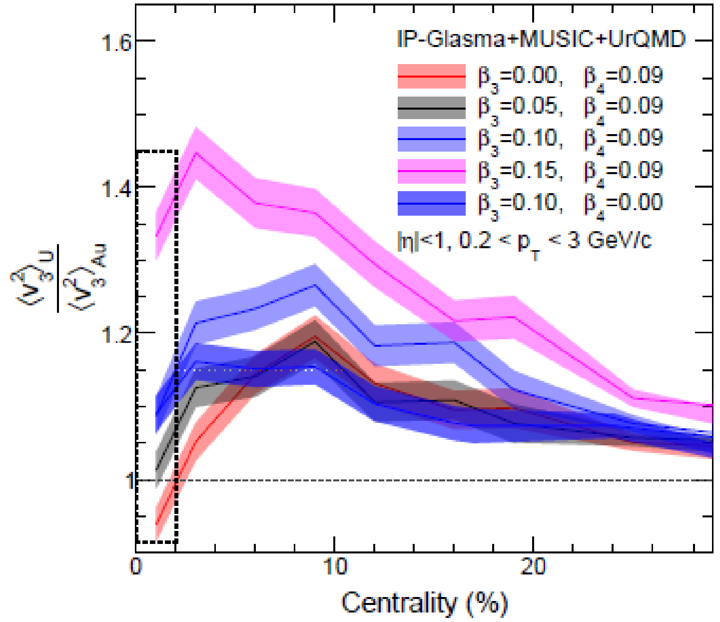
Probing

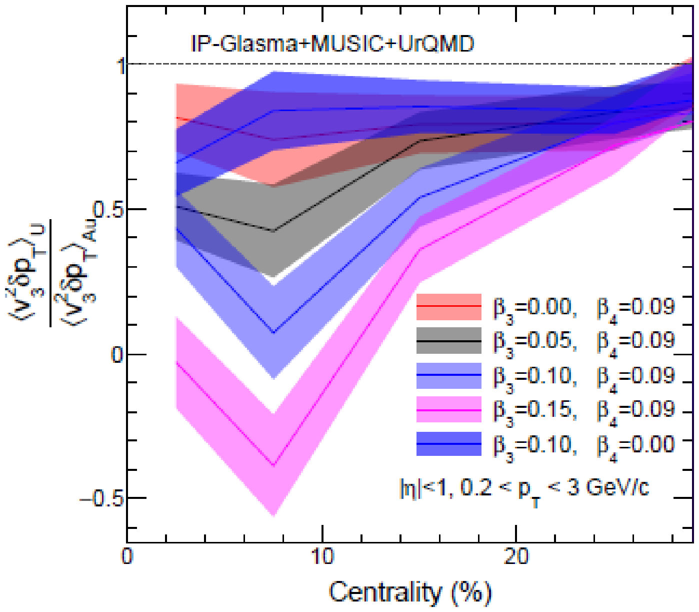
— Shape fluctuations

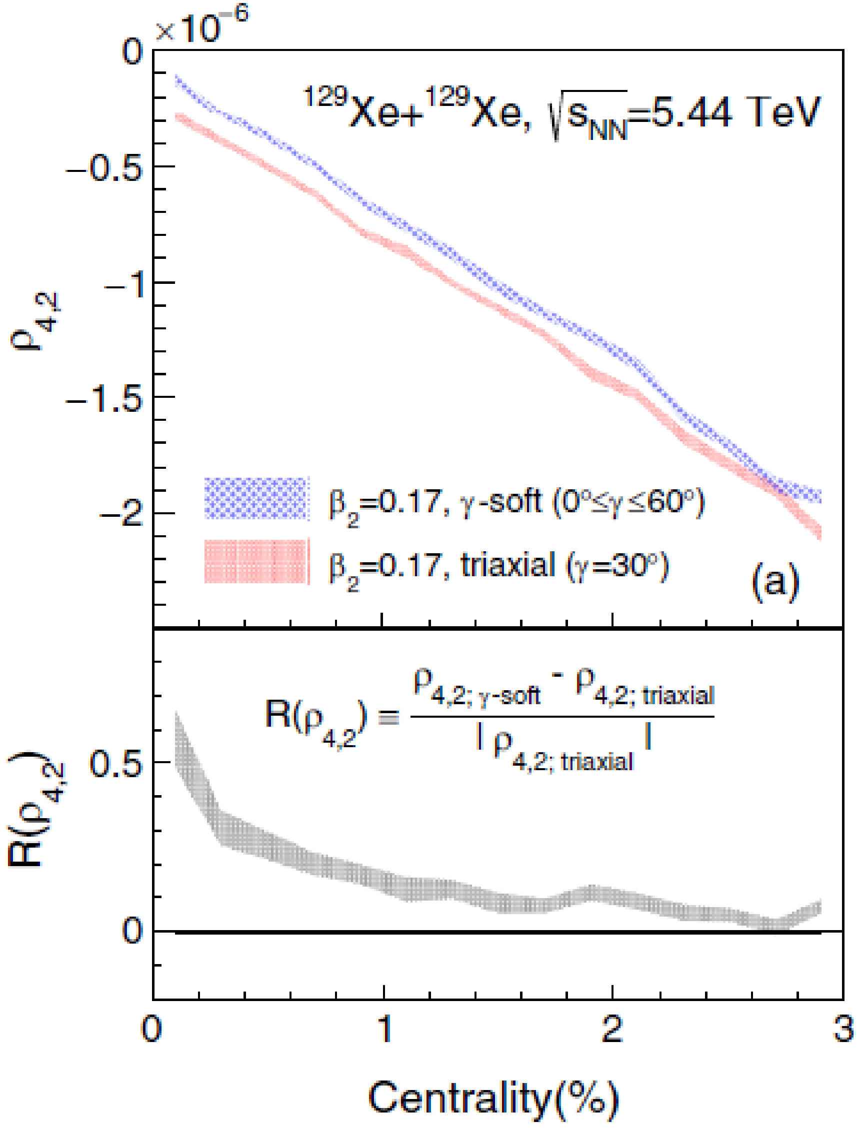
Zhao, Xu, Zhou, Liu, Song, PRL ('24); Hagino, MK, PRC ('25); Xu+, 2504.19644; Liu+, 2509.09376; ...

— Octupole/hexadecapole deformation

Zhang+, 2504.15245; Xu+, 2504.19644; ...

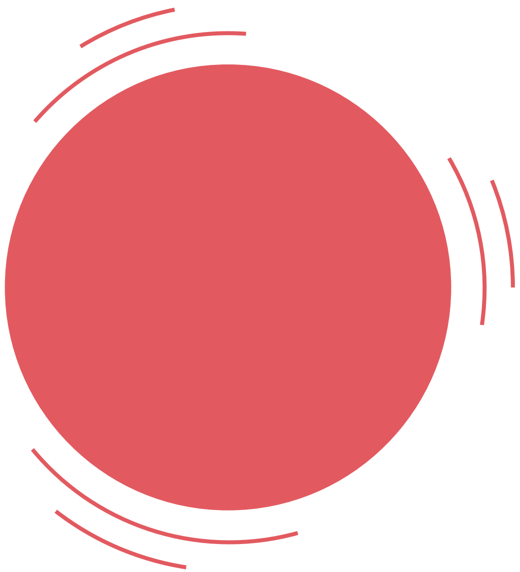






Quantum Surface Vibration

Hagino, MK, PRC ('25)

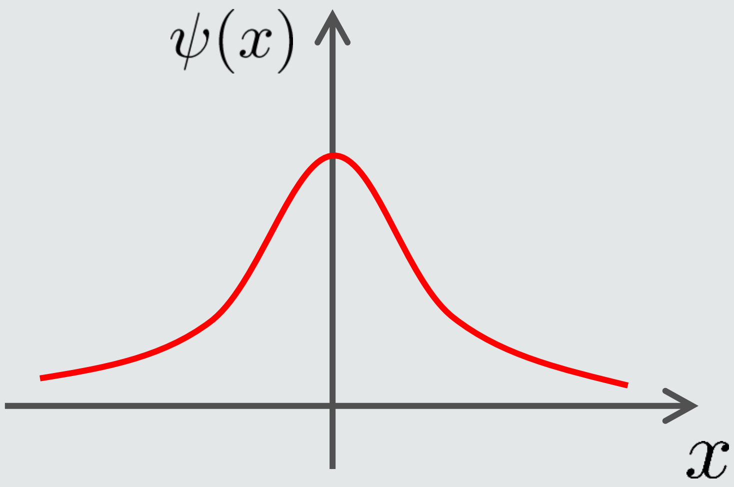


- Shape of a nucleus is **quantumly** vibrating even on the ground state.
- timescale of HIC ≫ surface vibration

HIC takes a snapshot of shape fluctuation.

Harmonic Oscillators

ground state



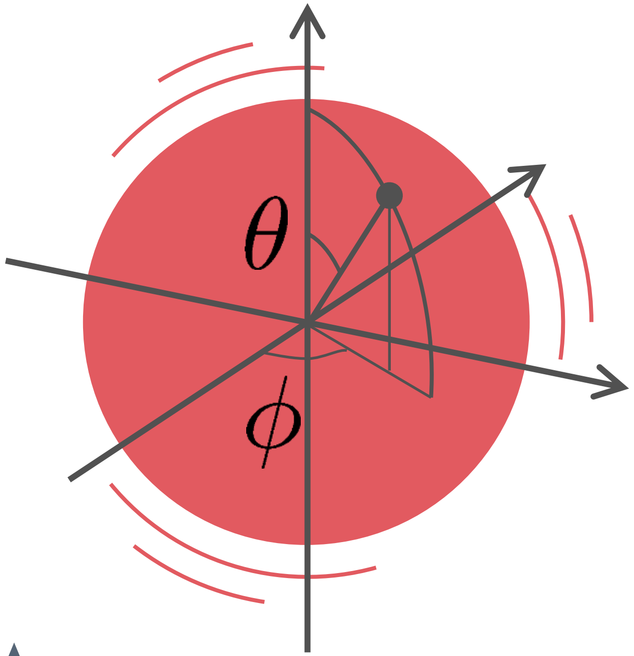
$\langle x \rangle = 0 \quad \langle x^2 \rangle \neq 0$

See also
Zhao, Xu, Zhou, Liu, Song ('24)
Xu, Xu, Zhao, Zhao, Song, Wang ('25)
Liu+, 2509.09376

Spherical Nuclei

Hagino, MK, PRC ('25)

$$\rho(\boldsymbol{r}) = \frac{\rho_0}{1 + e^{(r - R(\theta,\phi))/a}}$$



Space-fixed coordinates

$$R(\theta,\phi) = R_0 \left(1 - \frac{1}{4\pi}\sum_{\lambda,\mu}|\alpha_{\lambda\mu}|^2 + \sum_{\lambda,\mu}\alpha_{\lambda\mu}Y^*_{\lambda\mu}(\hat{\boldsymbol{r}})\right)$$

Harmonic-oscillator model for surface vib.

$$H = \frac{1}{2}\sum_{\lambda,\mu}\left(B_\lambda|\dot{\alpha}_{\lambda\mu}|^2 + C_\lambda|\alpha_{\lambda\mu}|^2\right)$$

$$\left\langle \sum_{\mu}|\alpha_{\lambda\mu}|^2 \right\rangle = (\beta_\lambda)^2$$

More complicated in **body-fixed coordinates**

Constraint from low-E exp. of $B(E\lambda)$

$$\beta_\lambda = \frac{4\pi}{3ZR_0^\lambda}\sqrt{\frac{B(E\lambda)\uparrow}{e^2}}$$

Hagino, Takigawa ('12)
Hagino, Ogata, Moro ('22)

Treatment of surface vibration is apparent in the space-fixed coordinates.
Deformation params. β_λ can be constrained from transition probability.

Transverse Distribution

Hagino, MK, PRC ('25)

Initial Transverse Distr.

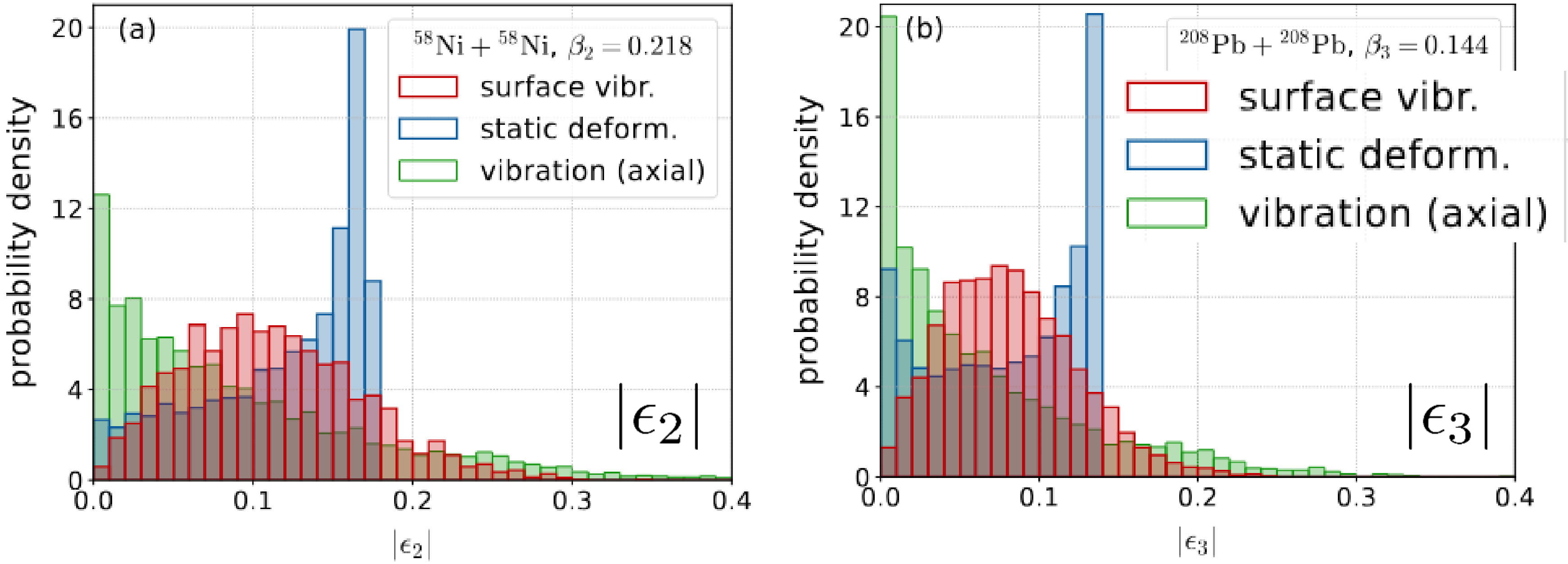
droplet full-overlap model

$$\rho^{(z)}(\boldsymbol{r}_\perp) = \int_{-\infty}^{\infty} dz\, \rho(\boldsymbol{r})$$

Jia ('22)

$$\epsilon_n = -\frac{\langle\langle (x-iy)^n \rangle\rangle}{\langle\langle (x^2+y^2)^{n/2} \rangle\rangle}$$

^{58}Ni, Quadrupole | ^{208}Pb, Octupole



- Distribution differs significantly between the surface vibration and static deformation.
- Axial deformation is insufficient to describe the surface vibration.

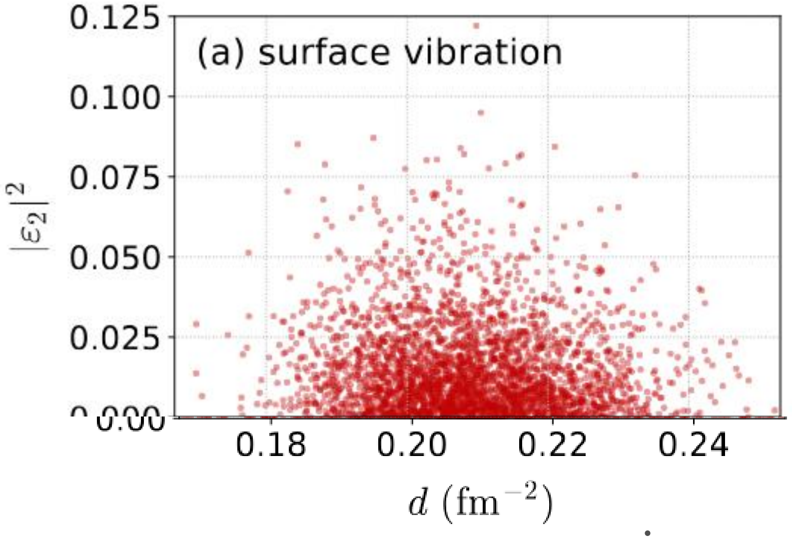
		mean	std. dev.	skewness	kurtosis
^{58}Ni, $\lvert\epsilon_2\rvert$	SV	0.112(1)	0.0554(7)	0.49(3)	-0.02(11)
	SD	0.119(1)	0.0500(5)	-0.79(3)	-0.62(6)
	SV-A	0.090(1)	0.0816(13)	1.22(4)	1.12(20)
^{208}Pb, $\lvert\epsilon_3\rvert$	SV	0.0822(8)	0.0416(5)	0.55(4)	0.15(11)
	SD	0.0821(8)	0.0461(4)	-0.38(3)	-1.29(3)
	SV-A	0.0650(12)	0.0649(11)	1.35(5)	1.49(22)

Transverse Distribution 2

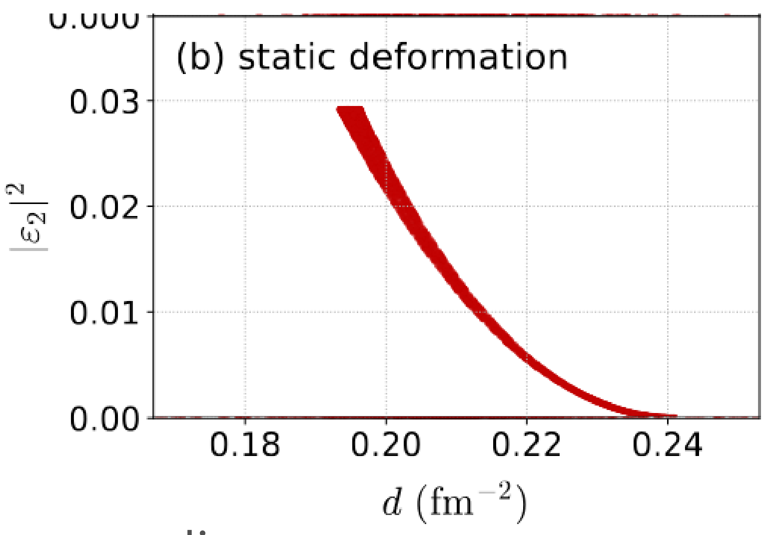
Hagino, MK, PRC ('25)

^{58}Ni, Quadrupole, $\beta_2 = 0.218$

Surface Vibration



Static Deform.



inverse mean radius

$$\epsilon_n = -\frac{\langle\langle (x-iy)^n \rangle\rangle}{\langle\langle (x^2+y^2)^{n/2} \rangle\rangle},$$

$$d = \frac{1}{\sqrt{\langle\langle x^2 \rangle\rangle \langle\langle y^2 \rangle\rangle}}$$

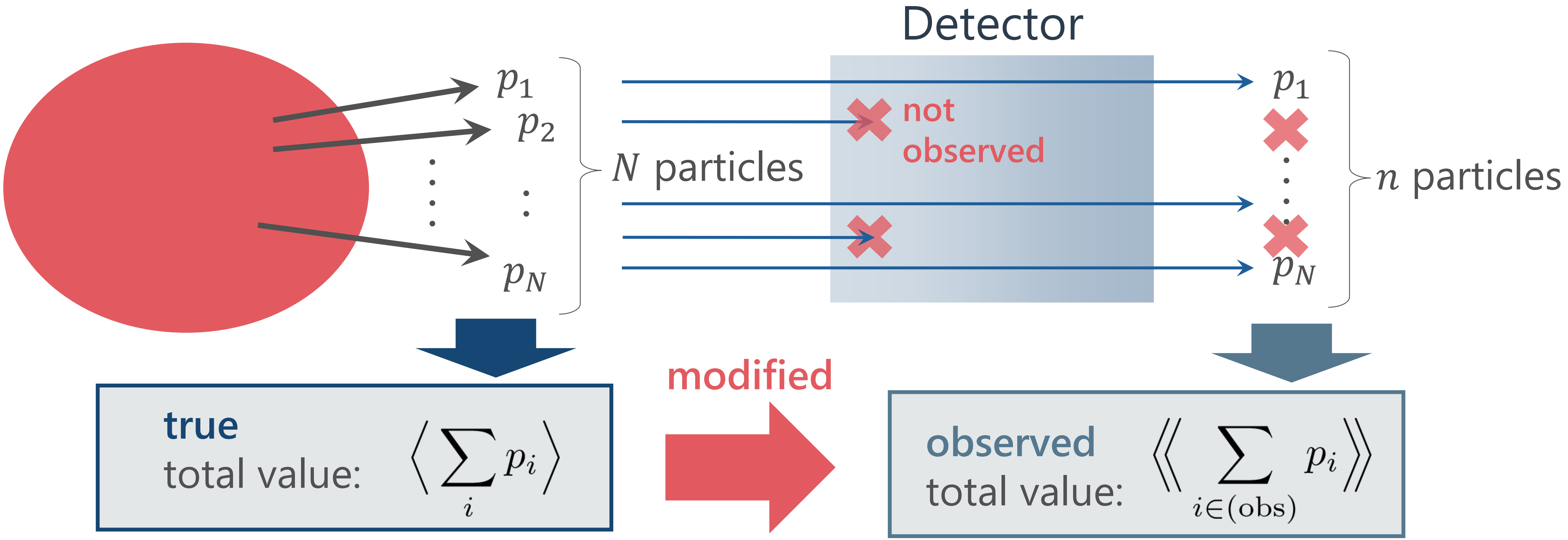
Short Summary

- Surface vibration and static deformation are discriminable through the distributions of ϵ_2, d.
- Space-fixed prescription is more convenient in treating the surface vibration of spherical nuclei.

Contents

1. Probing Surface Vibrations in high-E Heavy-ion Collisions
   Hagino, MK, PRC **112** (2025) L041901 [2508.05125]
2. Efficiency Correction of Flow Correlations
   MK, Esumi, Niida, Nonaka, PTEP (2026) in press, [2510.18383]
3. Exploiting Nuclear Deformation for searching for QCD Phase Structure
   Gubler, Nara, Minato, Taya, MK, in progress

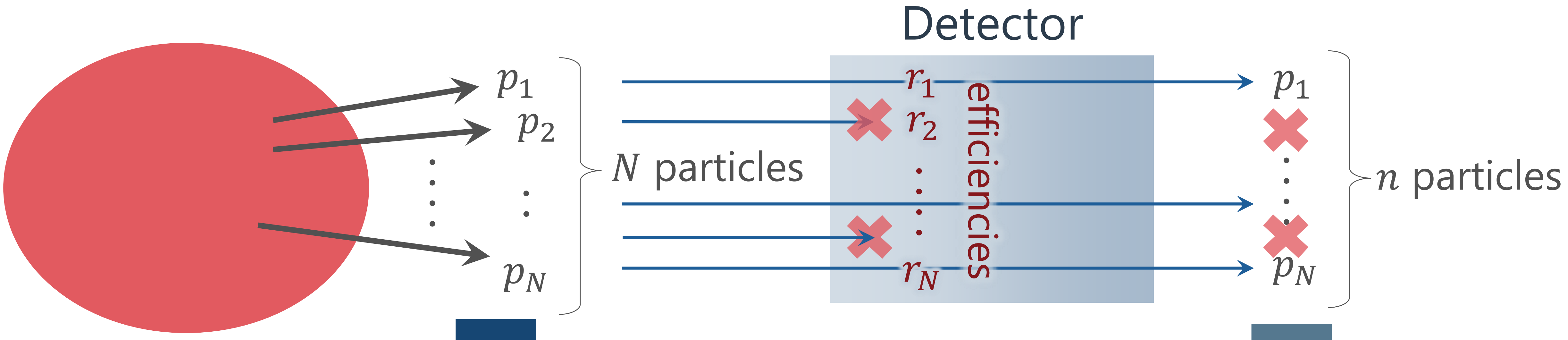
Detector Efficiency



Real detectors lose some particles

Observed results are modified.
Effects must be corrected to obtain the true result.

Efficiency Correction: Total Number



true total value: $\left\langle \sum_i p_i \right\rangle$

modified

observed total value: $\left\langle\!\left\langle \sum_{i\in(\mathrm{obs})} p_i \right\rangle\!\right\rangle$

Correction Formula: $$\left\langle \sum_i p_i \right\rangle = \left\langle\!\left\langle \sum_i \frac{p_i}{r_i} \right\rangle\!\right\rangle$$

Moments (Cumulants) of Total Number

$$\left\langle \left(\sum_i p_i \right)^n \right\rangle$$

Correction Procedure:

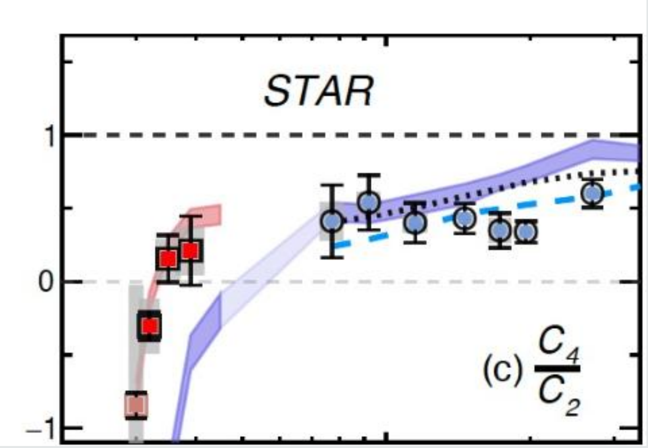
Use factorial moments/cumulants

$$\left\langle \left(\sum_i p_i \right)^n \right\rangle_{\mathrm{f}} = \left\langle\!\!\left\langle \left(\sum_i \frac{p_i}{r_i} \right)^n \right\rangle\!\!\right\rangle_{\mathrm{f}}$$

Assumption: efficiencies of individual particles are independent

Nonaka, MK, Esumi ('17)
Asakwa, MK, PPNP ('16); MK, Luo ('17)

Note
Search for QCD-CP using conserved-charge fluctuations



Long history of efficiency correction:
MK, Asakawa ('12); Bzdak, Koch ('12,'15); Luo ('14); MK ('16); Nonaka+ ('16); Bzdak, Holtzman, Koch ('16); MK, Luo ('17); Nonaka, MK, Esumi ('17); …

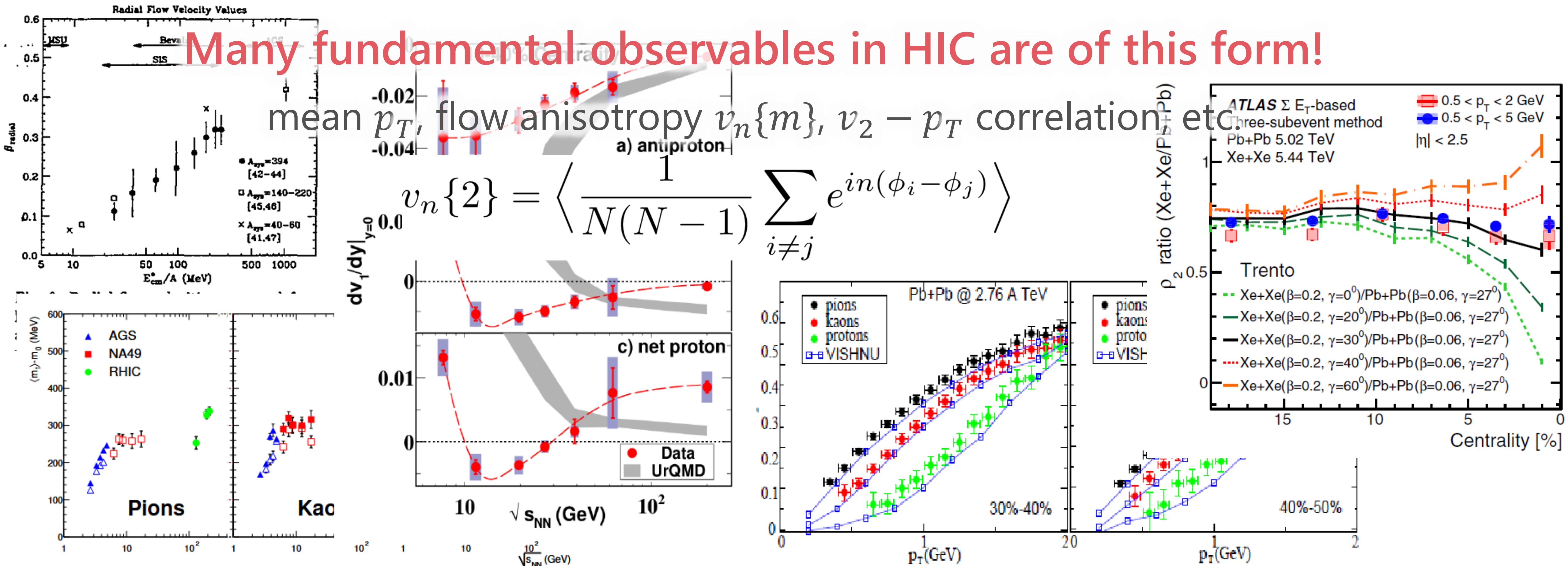
Particle-Averaged Quantities

$$\left\langle \frac{1}{N} \sum_i p_i \right\rangle, \quad \left\langle \left(\frac{1}{N} \sum_i p_i \right)^n \right\rangle, \quad \left\langle \frac{1}{N(N-1)} \sum_{i \neq j} p_i^{(1)} p_j^{(2)} \right\rangle$$

Many fundamental observables in HIC are of this form!

mean p_T, flow anisotropy $v_n\{m\}$, $v_2 - p_T$ correlation, etc.

$$v_n\{2\} = \left\langle \frac{1}{N(N-1)} \sum_{i \neq j} e^{in(\phi_i - \phi_j)} \right\rangle$$



Particle-Averaged Quantities

$$\left\langle \tfrac{1}{N}\textstyle\sum_i p_i \right\rangle, \quad \left\langle \left(\tfrac{1}{N}\textstyle\sum_i p_i \right)^n \right\rangle, \quad \left\langle \frac{1}{N(N-1)} \textstyle\sum_{i\neq j} p_i^{(1)} p_j^{(2)} \right\rangle$$

Many fundamental observables in HIC are of this form!

"Conventional" Correction Formulas

$$\left\langle \frac{1}{N(N-1)} \sum_{i\neq j} p_i^{(1)} p_j^{(2)} \right\rangle = \left\langle\!\!\left\langle \frac{\sum_{i\neq j} p_i^{(1)} p_j^{(2)} / r_i r_j}{\sum_{i\neq j} 1/r_i r_j} \right\rangle\!\!\right\rangle$$

e.g. ATLAS, PRC**107**, 054910 ('23); STAR, Nature **635**, 67 ('24)

Question: Are these formulas correct?

Correct Correction Formulas

MK, Esumi, Niida, Nonaka, arXiv:25010.13838

Mean

$$\left\langle \frac{Q}{N} \right\rangle = \left\langle\!\left\langle \sum_{i=1}^{n} \xi_i k_i \right\rangle\!\right\rangle_{n\neq 0}$$

$$k_i = \frac{1}{r_i} \int_0^1 d\sigma \prod_{j\neq i} \frac{\sigma + r_j \alpha_j}{r_j}$$

2nd Order

$$\left\langle \frac{\{Q_1 Q_2\}}{N(N-1)} \right\rangle = \left\langle\!\left\langle \sum_{i\neq j} q_{1,i} q_{2,j} k_{2;i,j} \right\rangle\!\right\rangle_{n\neq 0,1}$$

$$k_{2;i,j} = \frac{1}{r_i r_j} \int_0^1 d\sigma' \int_0^{\sigma'} d\sigma \prod_{l\neq i, l\neq j} \left(\frac{\sigma}{r_l} + \alpha_l \right)$$

- Correction formulas are written in forms including integral.
- This formula can reproduce the correct result for the previous simple model.

$$\{Q_{w_1} Q_{w_2}\} \equiv \sum_{i\neq j} \xi_i^{(w_1)} \xi_j^{(w_2)}$$

$$\alpha_i = \frac{1 - r_i}{r_i}$$

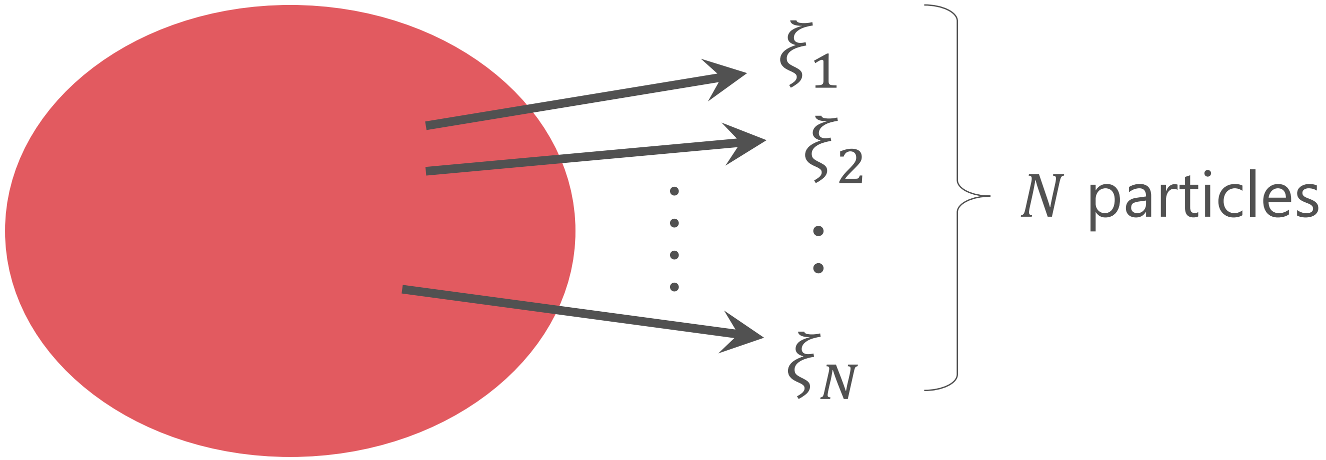
Derivation of Correction Formulas

Assumptions

1. Particle production is described by a classical prob. distr. func. $P(N;\vec{p}_T)$.
2. **Probs. to observe individual particles are independent.**
3. **For each observed particle, the value of efficiency r_i can be specified.**
4. Other detectors' effects are not considered.

True distr. func.

$$P(N;\vec{\xi})$$



Connecting True/Observed Distr. Funcs.

True distr. func.

$$P(N;\vec{\xi})$$

Observed distr. func.

$$\tilde{P}(n;q)$$

- n: observed particle number
- $q = \Sigma_{i\in(\mathrm{obs.})}\xi_i$: observed sum

Probability Distr. of Observed Quantities (uniform r)

$$\tilde{P}(n;q) = \sum_{N=1}^{\infty} \int d\vec{\xi} \sum_{\{b_i\}} \left[\prod_{i=1}^{N} (1-r)^{1-b_i} r^{b_i} \right] \delta_{n,\sum_i b_i} \delta(q - \Sigma_i b_i \xi_i) P(N;\vec{\xi})$$

$$b_i = 0,1$$

Generating Function

MK, Esumi, Niida, Nonaka, PTEP2026

Prob. distr. func: $\tilde{P}(n;q) = \sum_{N=1}^{\infty} \int d\vec{\xi} \sum_{\{b_i\}} \left[\prod_{i=1}^{N} (1-r)^{1-b_i} r^{b_i} \right] \delta_{n,\sum_i b_i} \delta(q - \Sigma_i b_i \xi_i) P(N;\vec{\xi})$

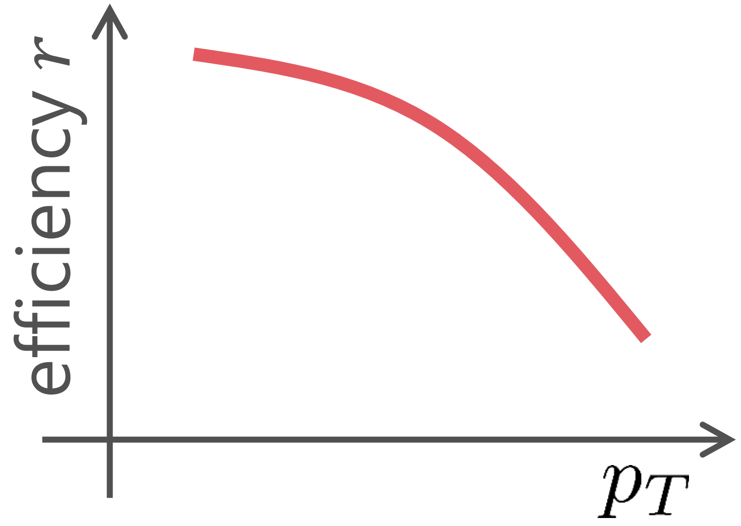
Generating func: $\tilde{G}(s,t) = \sum_n \int dq \tilde{P} s^n t^q = \sum_N \int d\vec{\xi} P \prod_i (1 - r + rst^{\xi_i})$

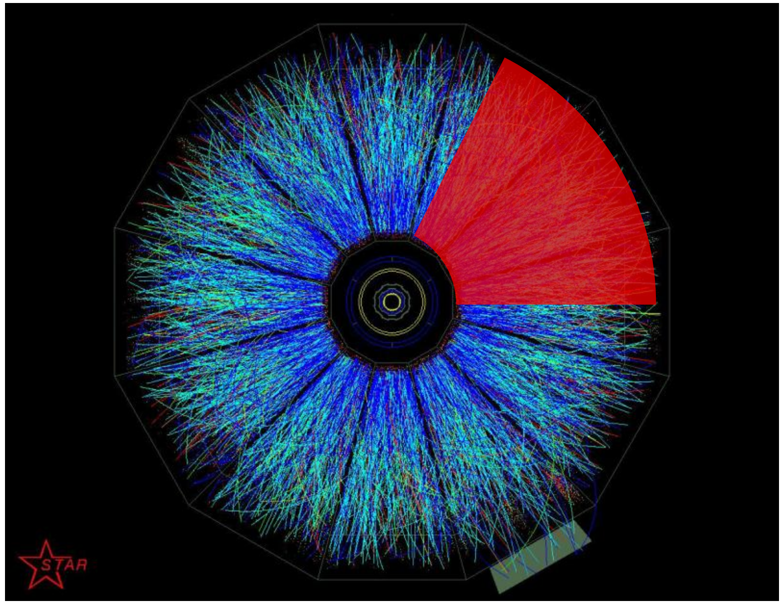
Represent the quantity that you want to express by the derivative of the generating function.
Then, represent it in terms of the observed variables.

$$\left\langle \frac{\sum_i \xi_i}{N} \right\rangle_{\rm true} = \int_\alpha^1 ds \frac{r}{s} \left[\partial_t \tilde{G}(s,t) \right]_{t=1} = \left\langle \frac{\sum_i \xi_i}{n} \left(1 - \alpha^n\right) \right\rangle_{\rm obs} \qquad \alpha = \frac{r-1}{r}$$

Note: $\left\langle \frac{\sum_i \xi_i}{N} \right\rangle_{\rm true} \neq \left\langle \frac{\sum_i \xi_i}{n} \right\rangle_{\rm obs}$ α^n term compensates the $n=0$ contribution.

Correction is Necessary!!





p_T-dependent efficiency

alter mean p_T

Azimuthally nonuniform efficiency

produce unphysical $v_n\{m\}$

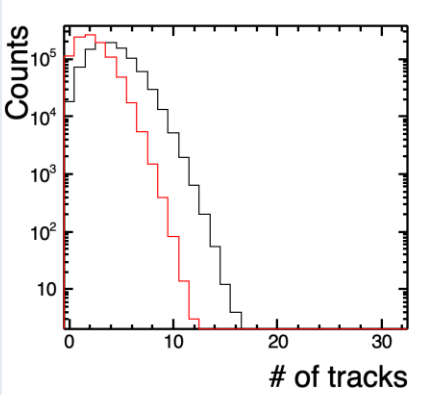
More serious effects on higher-order correlations!

Test in Toy Models

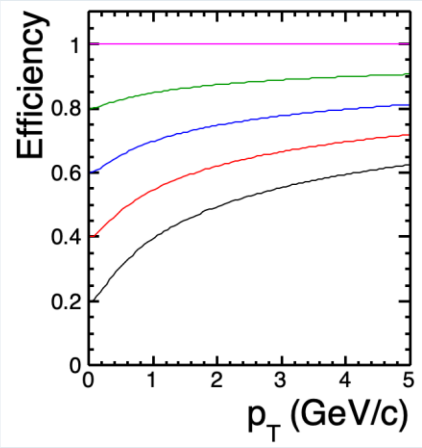
Nonaka+, in prep.

Simulation Procedure

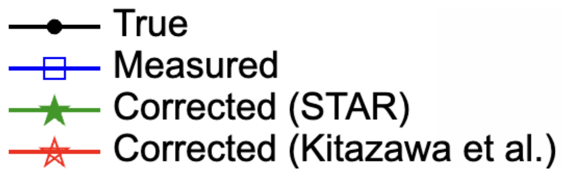
Generate
p_T distr.

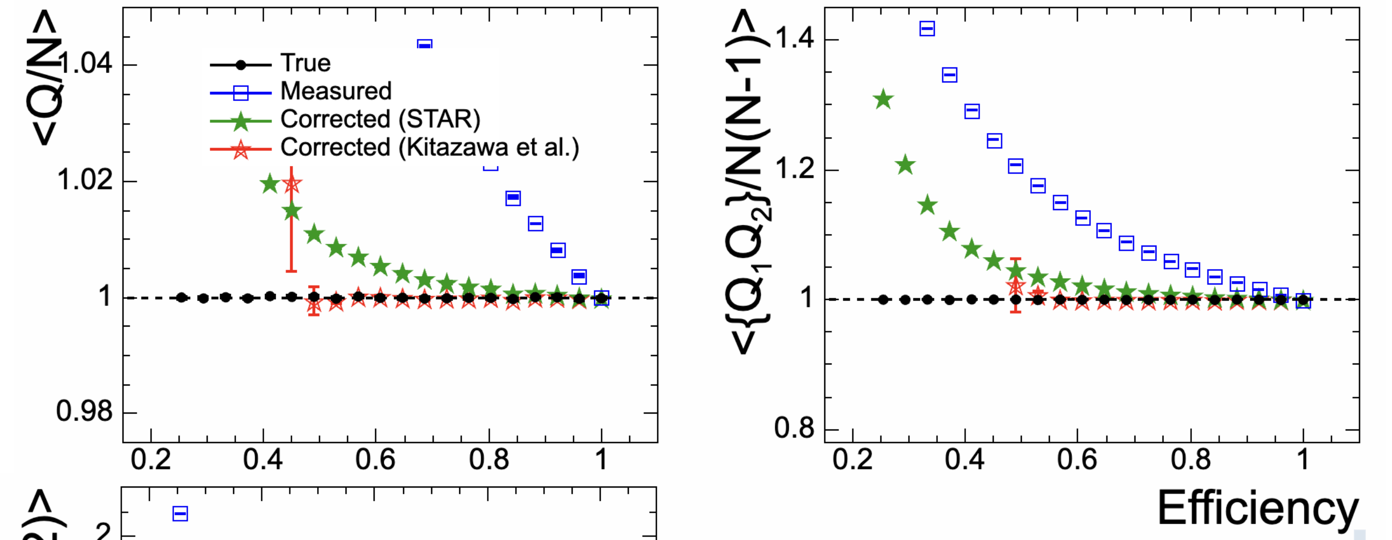


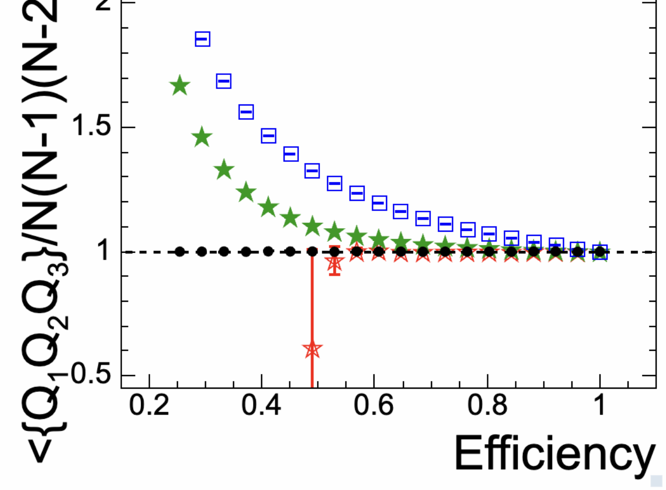
"Observation"
with
p_T dependent
efficiency



Results (mean p_T)







Large effect in higher order correlations

"Conventional" formulas cannot reproduce the correct value

Short Summary

Correct Efficiency Correction Formulas

$$\left\langle \frac{Q}{N} \right\rangle = \left\langle\!\left\langle \sum_{i=1}^{n} \xi_i k_i \right\rangle\!\right\rangle_{n\neq 0} \qquad \left\langle \frac{\{Q_1 Q_2\}}{N(N-1)} \right\rangle = \left\langle\!\left\langle \sum_{i\neq j} q_{1,i} q_{2,j} k_{2;i,j} \right\rangle\!\right\rangle_{n\neq 0,1},$$

$$k_i = \frac{1}{r_i} \int_0^1 d\sigma \prod_{j\neq i} \frac{\sigma + r_j \alpha_j}{r_j}, \qquad k_{2;i,j} = \frac{1}{r_i r_j} \int_0^1 d\sigma' \int_0^{\sigma'} d\sigma \prod_{l\neq i, l\neq j} \left(\frac{\sigma}{r_l} + \alpha_l \right),$$

They will play crucial roles in experimental studies in HIC.
e.g. flow (dv_1/dy, $v_n\{m\}$, v_2-p_T correlation, etc.)

Further Development

General "unbiased estimator" that is applicable for any observables

Murase, MK, in prep.

Contents

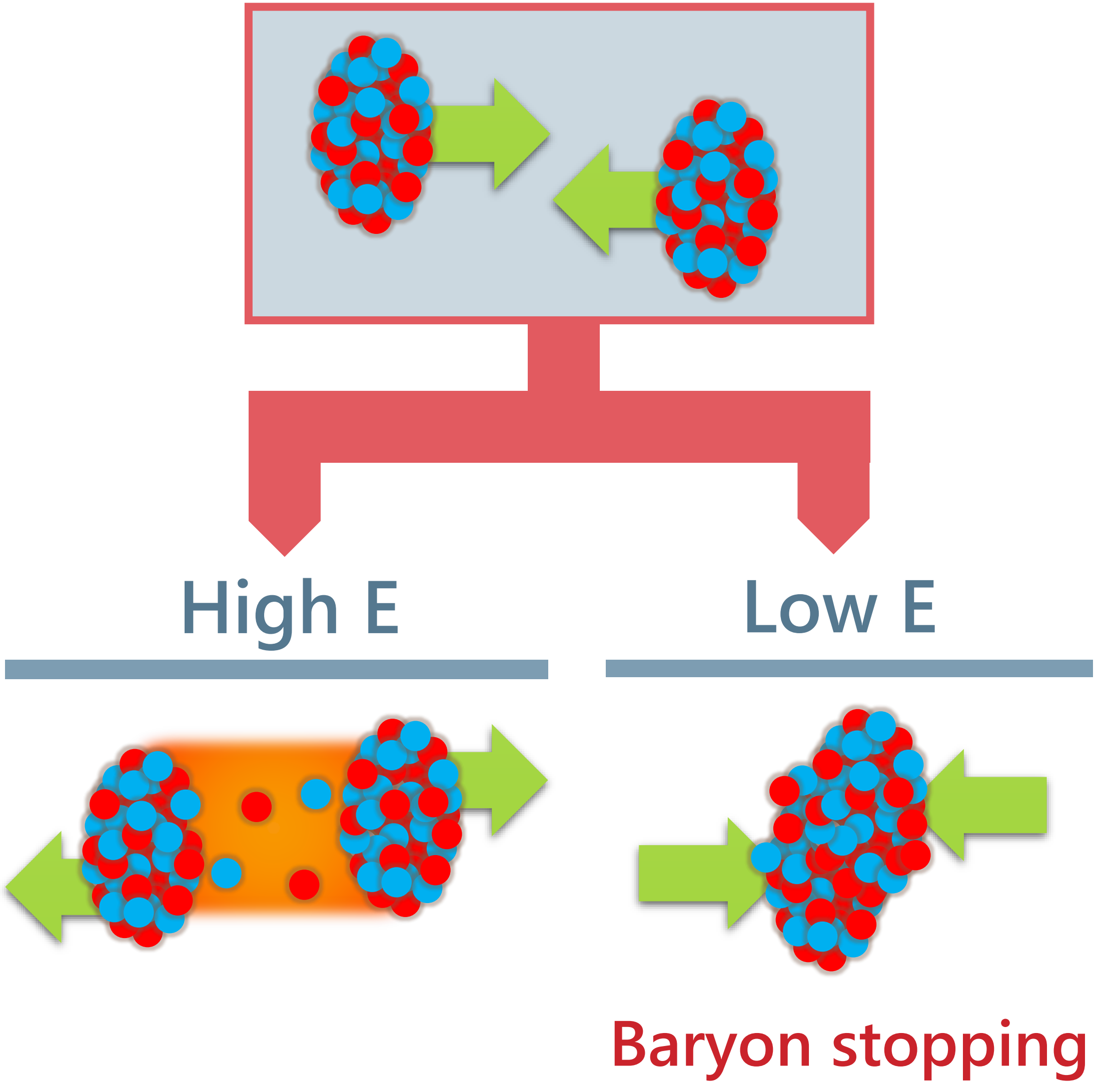
1. Probing Surface Vibrations in high-E Heavy-ion Collisions
   Hagino, MK, PRC **112** (2025) L041901 [2508.05125]

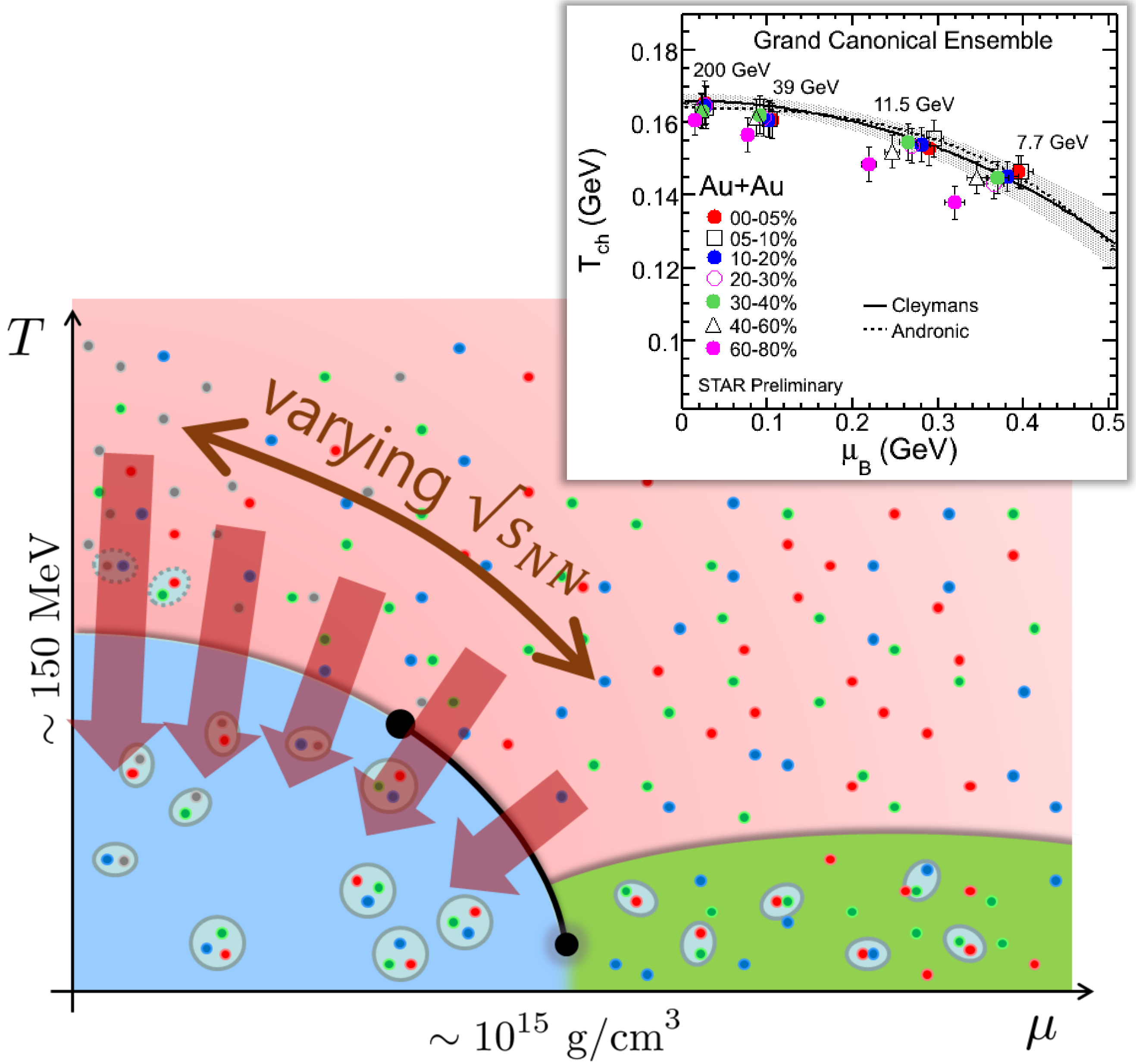
2. Efficiency Correction of Flow Correlations
   MK, Esumi, Niida, Nonaka, PTEP (2026) in press, [2510.18383]

3. Exploiting Nuclear Deformation for searching for QCD Phase Structure
   Gubler, Nara, Minato, Taya, MK, in progress

Beam-Energy Scan

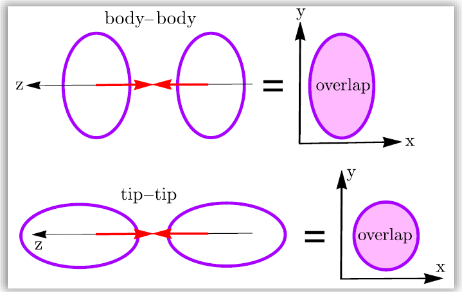
STAR, 2012

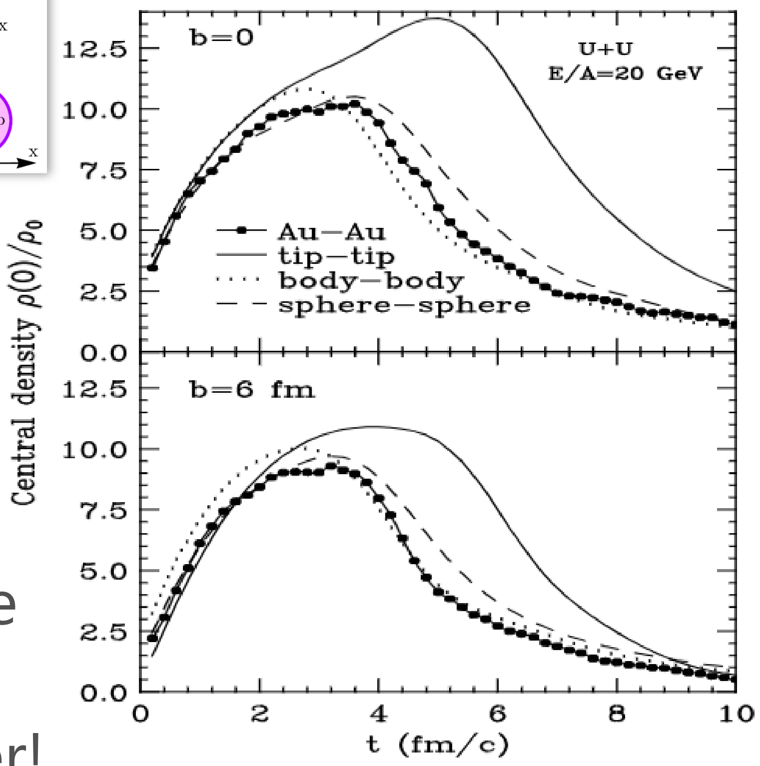




Further Utilization of Deformation
at low collision-energy region

Creating the highest baryon density medium

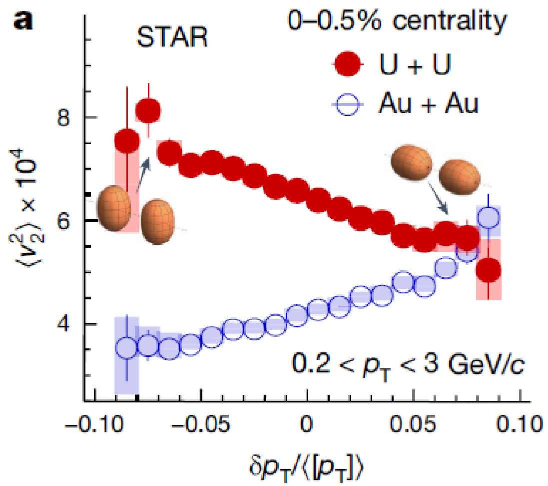


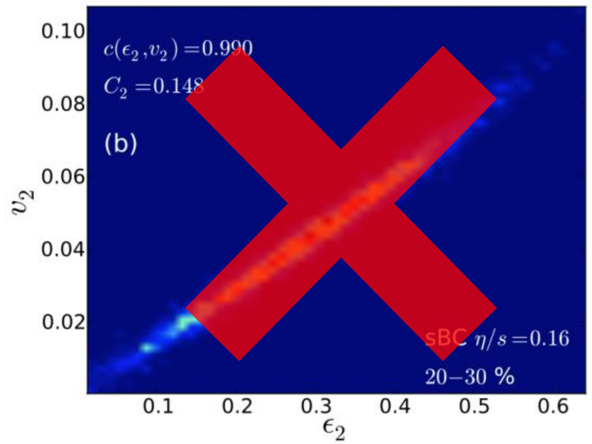


Li, nucl-th/9910030

Tip-tip is obviously more suitable to create denser matter!

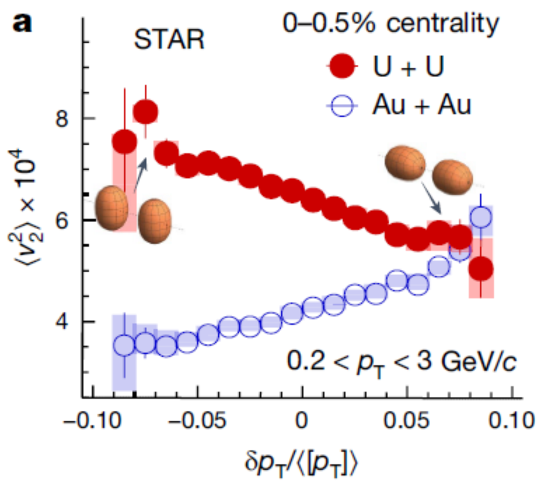
Violation of hydro. picture / EOS Change @ 1st-order tr.



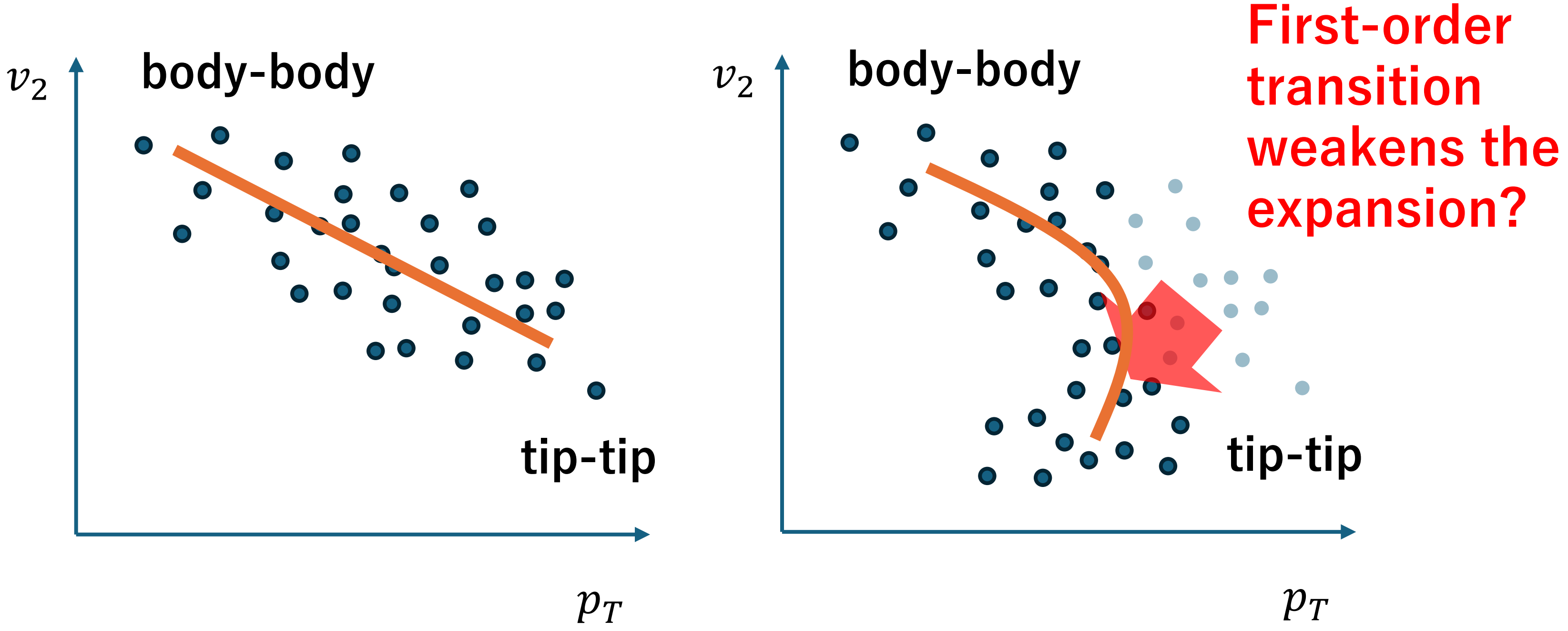


How is this behavior modified?

Signal for First-order Phase Transition?



Event-by-event distribution



Gubler, Nara, Minato, Taya, MK, in progress

Short Summary

- Let's consider how the nuclear deformation can be utilized in the search for the QCD phase structure!

Physics of Heavy-Ion Collisions

Various physical picture / research purposes / theoretical methods
depending on the collision E

Low collision-E ⟷ **High collision-E**

Nuclear reaction

Nuclear liquid-gas transition

Fragmentation

Heavy elements

Nuclear Deformation

EOS

compressed baryonic matter

QGP

Quark deconfinement

TDHF

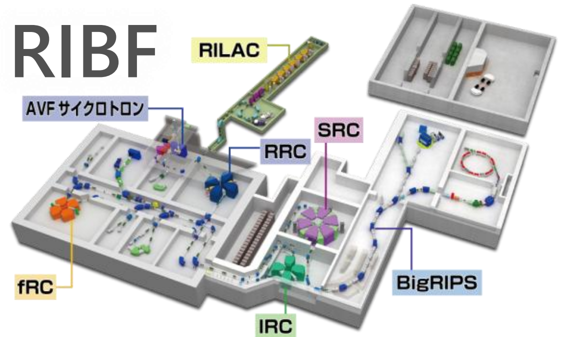
Langevin

AMD

BUU

hadron cascade

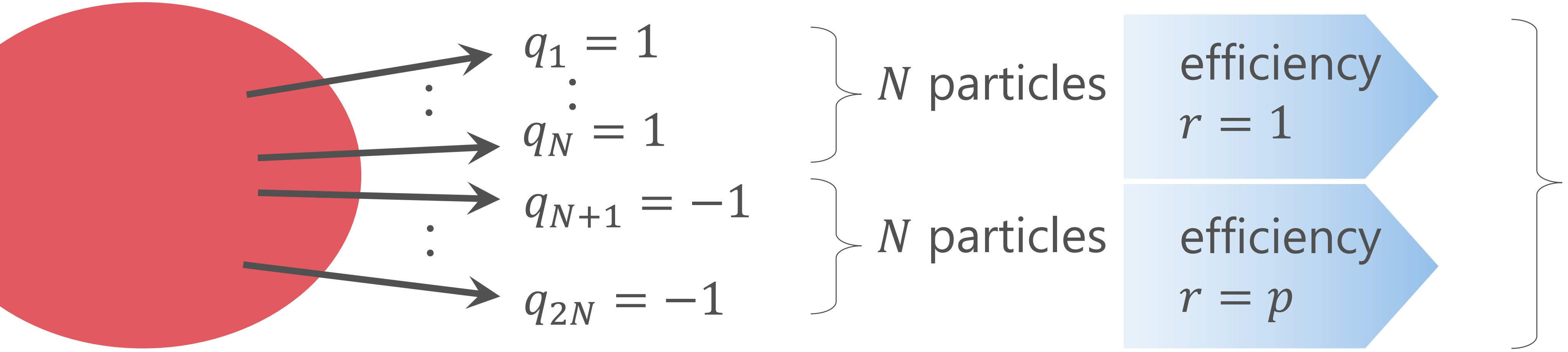
hydro. models



MK, Esumi, Niida, Nonaka,
arXiv:2510.13838

Check in a Simple Model

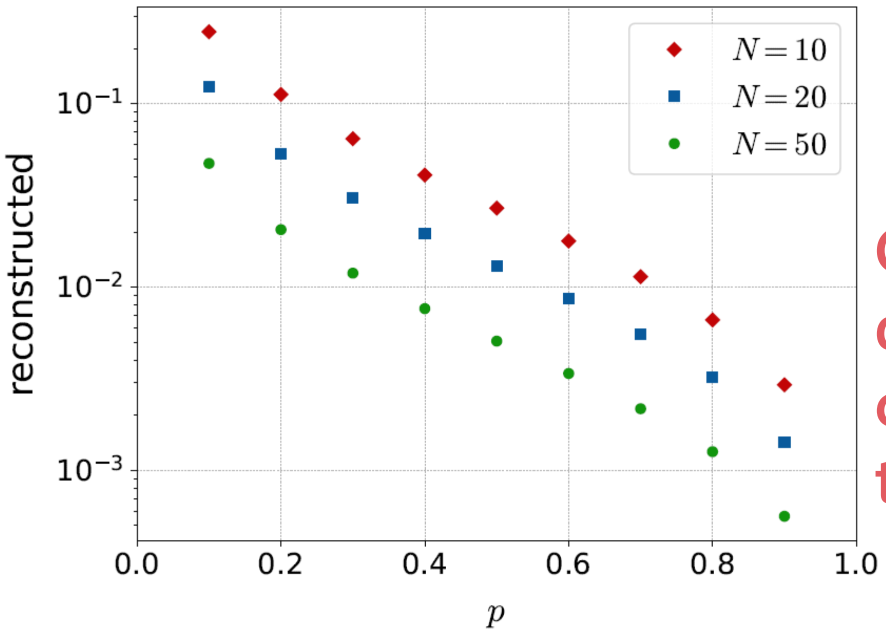
$2N$: fixed for all events



Mean:

True result $\left\langle \frac{Q}{N} \right\rangle = 0$

Reconstructed $\left\langle \frac{\sum_i q_i / r_i}{\sum_i 1/r_i} \right\rangle$



Conventional formula does not reproduce the correct result even for the mean!!