Effect of dynamical flow responses on NeNe and OO flow ratios

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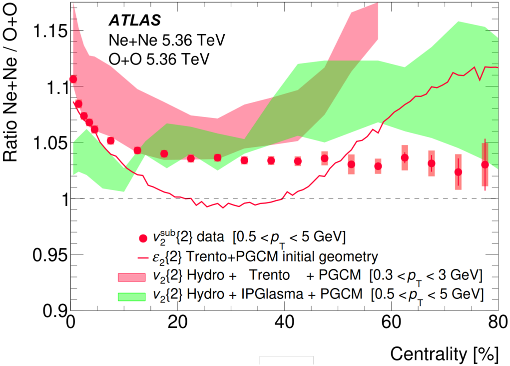
based on WiP with Victor Ambruș,

Giuliano Giacalone and Fabian Zhou



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From nuclear structure to particle spectra

Relevant processes:

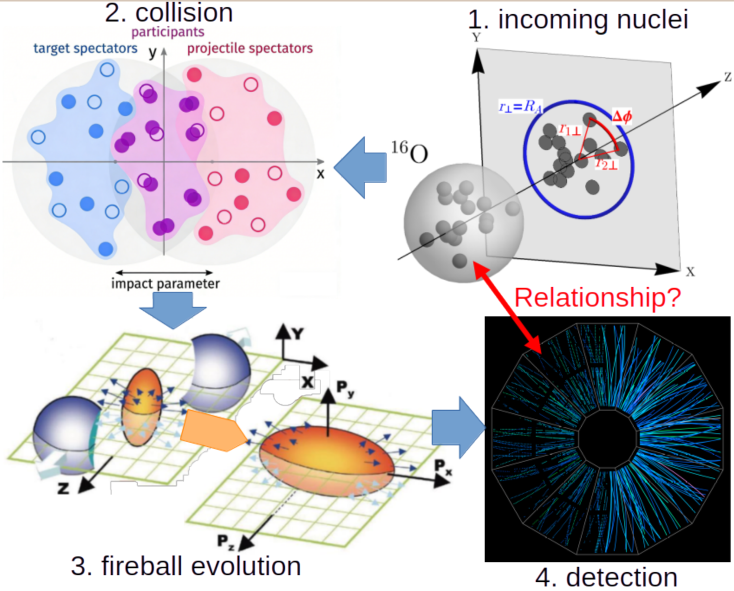
- **energy deposition**

How is nuclear structure converted to initial state $e(\tau_0, \mathbf{x}_\perp)$ of HIC? (e.g. eccentricities ϵ_n) →

- **dynamical evolution**

(This talk's topic)

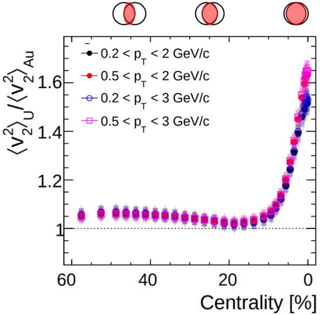
How is the encoded information transported to the detector? (flow response $v_n = \kappa_n \epsilon_n$) →



Extracting initial state information

<u>idea:</u> look at very central collisions (full overlap), cancel dynamical response in ratios $\frac{\langle v_n \rangle_{\rm U}}{\langle v_n \rangle_{\rm Au}} \approx \frac{\langle \epsilon_n \rangle_{\rm U}}{\langle \epsilon_n \rangle_{\rm Au}}$

$\Rightarrow$ assumes constant κ_n, works well in large systems



STAR, Rept.Prog.Phys. 88 (2025) 108601

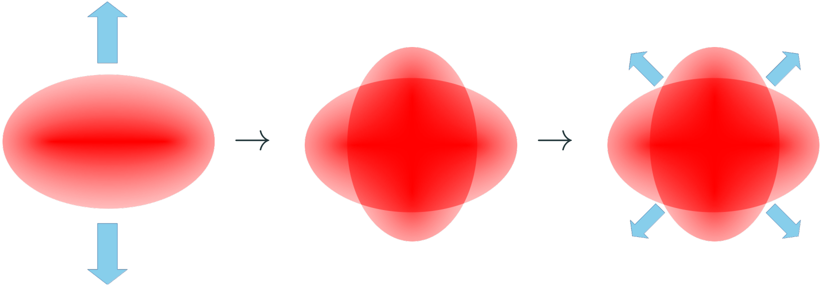
<u>problems described in this talk:</u>

- nonlinear flow responses, e.g. $v_2 \propto \epsilon_2 \epsilon_4$, $v_2 \propto \epsilon_2^3$
- κ depends on "system size" (opacity $\hat{\gamma}$) & on geometry
- ϵ_n can change before being converted to flow
- Modeling evolution via hydrodynamics may not always be accurate. Requirements:
  - small spacetime gradients $\hat{=}$ small Knudsen number ${\rm Kn} \sim \ell_{\rm mfp}/L$
  - closeness to equilibrium $\hat{=}$ small inverse Reynolds number ${\rm Re}^{-1} \sim |\pi_{\mu\nu}|/P$

(can be accounted for in Bayesian inference framework with appropriate modeling)

Why are there nonlinear flow responses?

- must be permitted by rotational symmetry!
  E.g. v_{n+m} and $v_{n-m} \propto \epsilon_n \epsilon_m$ like in $\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$
- possible dynamical mechanism: first response couples again, example of $v_4 \propto \epsilon_2^2$:



Are nonlinear flow responses a problem?

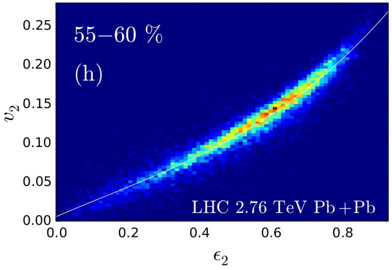
Coupling different eccentricities: event-by-event "$v_2 \propto \epsilon_2 \epsilon_4 \sin[2\Psi_2 - 4\Psi_4]$"

$\Rightarrow$ may cancel on average, depending on correlations between Ψ_n (and ϵ_n)

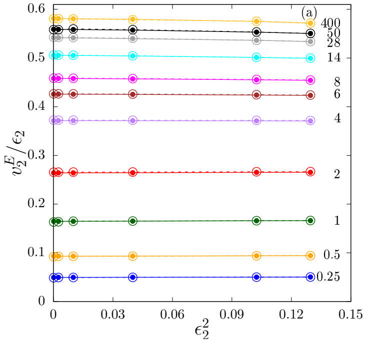
coupling same eccentricity: e.g. v_{2n} always has sizeable ϵ_2^{2n} contribution!

even for v_2 and v_3, there are responses $\propto \epsilon_n^3$:

event-by-event (EKRT)



Niemi, Eskola, Paatelainen PRC 93 (2016) 024907



idealized average

Ambruș, Schlichting, CW PRD 105 (2022) 014031

$\Rightarrow$ some geometric dependence beyond just ϵ_2!

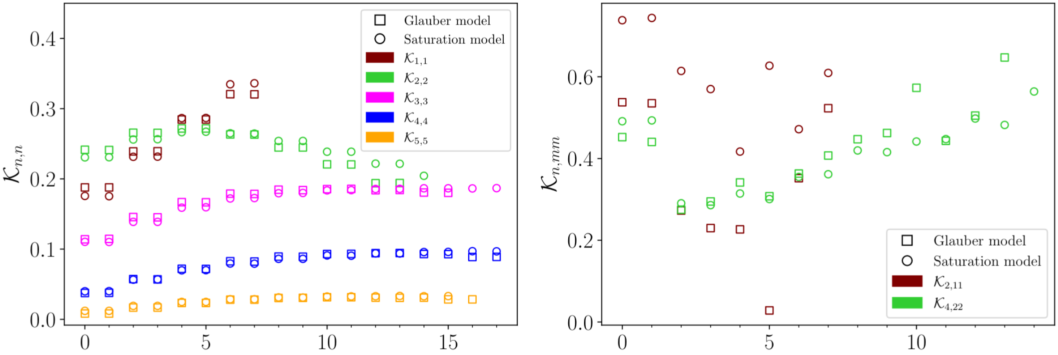
Geometrical differences in flow response

Statistical analysis of event-by-event energy density profiles (Φ_i):

Borghini, Borrell, Feld, Roch, Schlichting, CW PRC 107 (2023) 034905

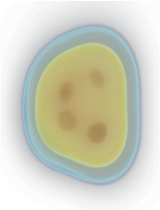
- diagonalization of covariance matrix $\rho = \langle \Phi\Phi^T \rangle - \langle \Phi \rangle \langle \Phi^T \rangle$
  gives statistically independend "modes" (and weights) $\rho_{\text{diag}=} \sum_l \lambda_l \Psi_l \Psi_l^T$
- flow responses from $\langle \Phi \rangle + \Psi_l$ can vary a lot (only difference is geometry):

linear $\mathcal{K}_{n,n} = v_n/\epsilon_n$ $(b = 0)$ quadratic $\mathcal{K}_{n,mm} = v_n/\epsilon_m^2$

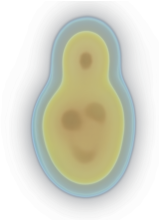


Initial state

From here on, I will connect points to simulations of recent light ion runs at the LHC:



^{16}O



^{20}Ne

- nucleon positions from PGCM
- T$_{\mathrm{R}}$ENTo initial state:

  parameters from bayesian inference for 5.36 TeV (Trajectum)

  Govert Nijs, Wilke van der Schee, PRC 106 (2022) 044903 and PLB 872 (2026) 140061

Dynamical descriptions

kinetic theory in conformal relaxation time approximation (RTA): dynamics of $f = \frac{\mathrm{d}N}{\mathrm{d}^3p\mathrm{d}^3x}$

- some microscopic info "integrated out"
  $\Rightarrow$ no diluteness requirement
- dynamics depends only on opacity $\hat{\gamma}$
  Aleksi Kurkela, Urs A. Wiedemann, Bin Wu, EPJC 79 (2019) 11, 965

$$p^\mu \partial_\mu f = -\frac{f - f_{\mathrm{eq}}}{\tau_R}\,, \quad \tau_R = 5T^{-1}\frac{\eta}{s}$$

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1}\left(\frac{1}{a\pi}R\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4}$$

total interactions $\hat{\gamma} \sim \mathrm{Kn}^{-1}$: depends on shear viscosity, transverse size and energy scale

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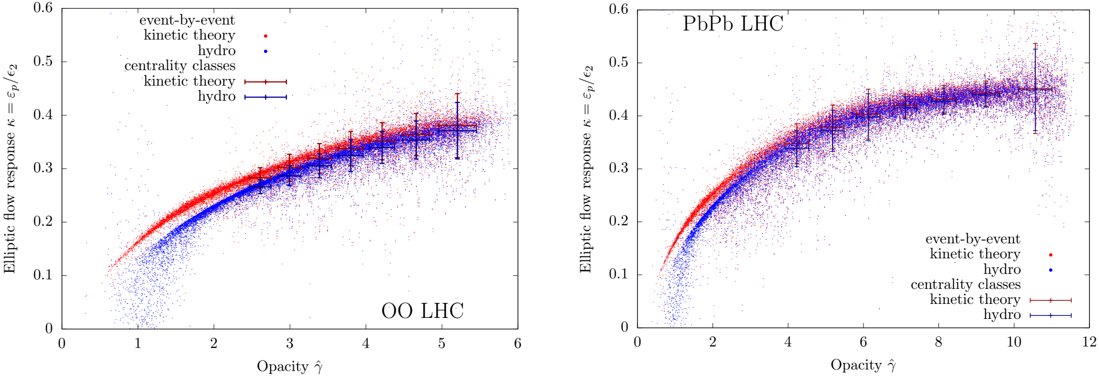
viscous hydrodynamics:

- conformal equation of state $\mathcal{E} = 3P = aT^4$
- transport coefficients from RTA
- elliptic flow of energy ε_p: extracted directly from hydro, no need for particlization

$$\varepsilon_p = \frac{\int_{\mathbf{x}_\perp} T^{xx} - T^{yy} + 2iT^{xy}}{\int_{\mathbf{x}_\perp} T^{xx} + T^{yy}}$$

Event-by-event Flow Responses

$\eta/s = 0.12$



- some event-by-event spread, but mainly following universal curves
- hydro $\rightarrow$ kinetic theory as $\hat{\gamma} \rightarrow \infty$

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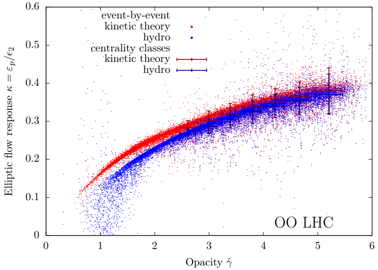
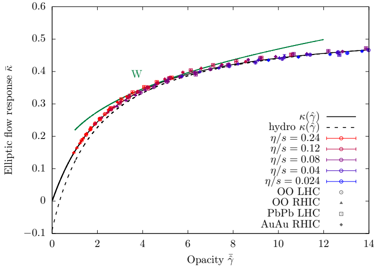
Universal flow response curve $v_2/\epsilon_2 = \kappa(\hat{\gamma})$

Hadronic collision simulations





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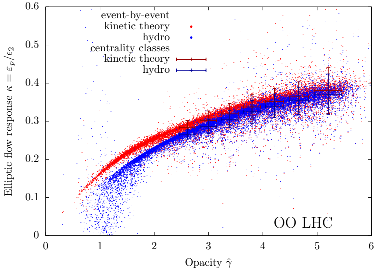
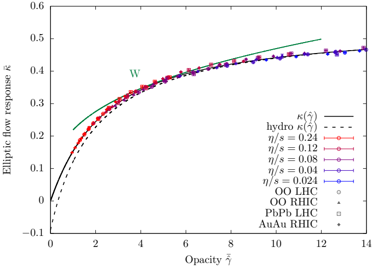
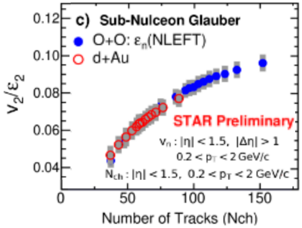


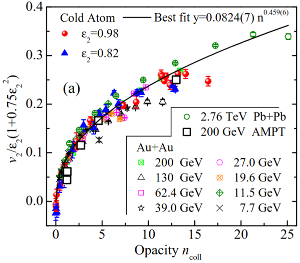


Experiment



Hadronic collisions

Andrew Tamis @ IS25



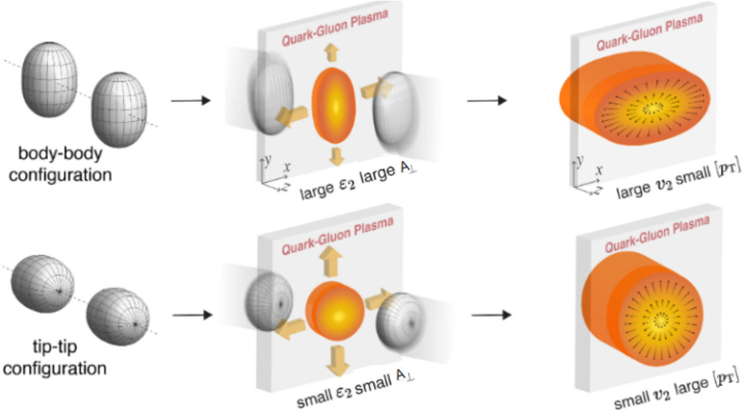
Cold atoms

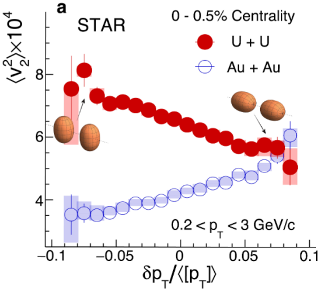
Kei Li, Hong-Fang Song, Hao-Jie Xu, Yu-Liang Sun, Fuqiang Wang, Newton 1 (2025) 100237

$\Rightarrow$ for flow ratio of systems with very different κ, **have** to model dynamical response

Geometry and flow dependence on orientation of nuclei

Orientation of nuclei gives correlated fluctuations in flow and mean transv. momentum





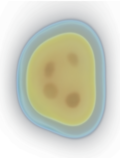
STAR, Nature 635 (2024) 8037, 67-72

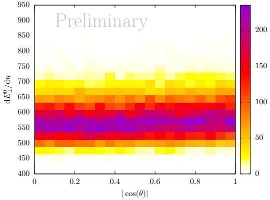
Orientation dependent response coefficient κ could also matter for v_2!

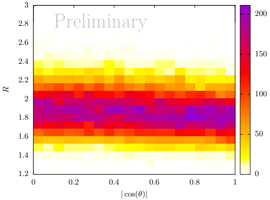
Response dependence on orientation of nuclei

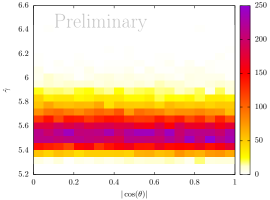
System | Energy $\frac{\mathrm{d}E_\perp}{dy}$ | Radius R | Opacity $\hat{\gamma} \propto \left(R\frac{\mathrm{d}E_\perp}{dy}\right)^{\frac{1}{4}}$

OO 5.36 TeV 0-1%

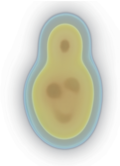


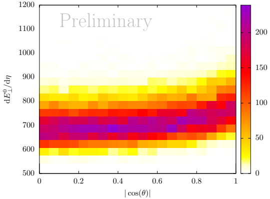


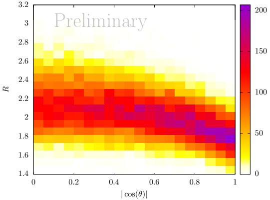


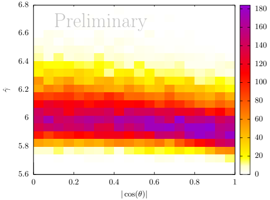


NeNe 5.36 TeV 0-1%









$\Rightarrow$ in OO and NeNe, no orientation dependence of response $\kappa(\hat{\gamma})$ expected!

Regime of applicability of hydrodynamics

timescale of hydrodynamization:

$$\tau_{\text{Hydro}} \approx 0.84\,\hat{\gamma}^{-4/3} R$$

timescale of transv. expansion:

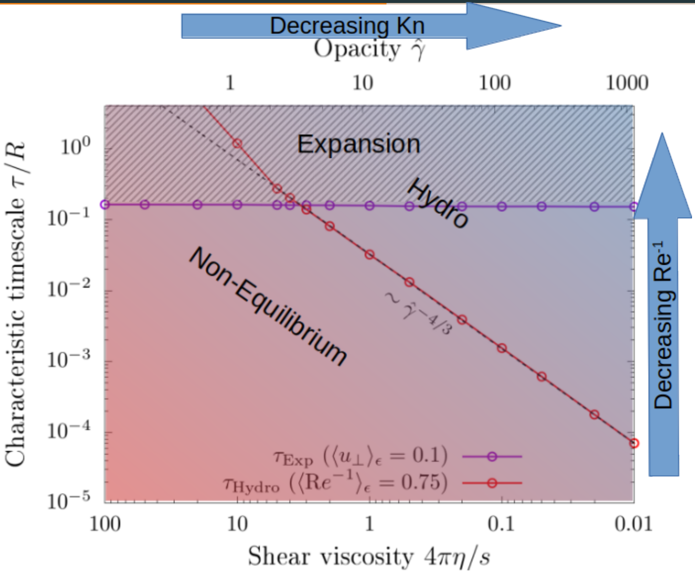
$$\tau_{\text{Exp}} \approx 0.2 R$$

- for $\hat{\gamma} \lesssim 3$, need non-equilibrium description of transverse expansion

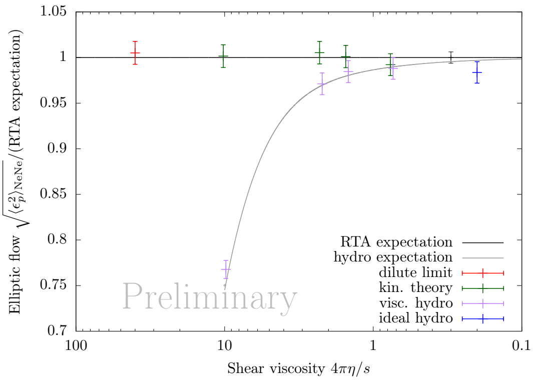
Victor E. Ambruș, Sören Schlichting, CW PRL 130 (2023) 152301 and PRD 107 (2023) 094013

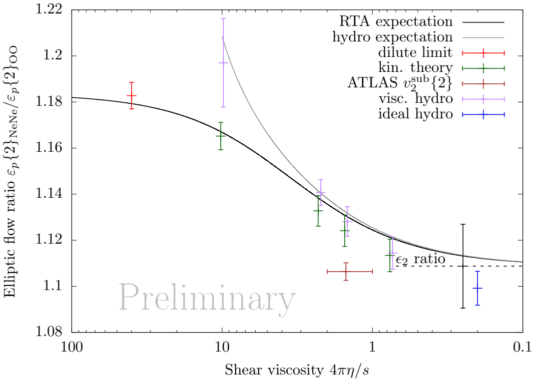
- anisotropic hydro: $\hat{\gamma} \gtrsim 1$

Y. Peng, V. E. Ambruș, CW, S. Schlichting, U. Heinz, H. Song PRC 113 (2026) 024918



NeNe/OO flow ratio





reminder: $\hat{\gamma} \propto (\eta/s)^{-1}$

- flow from simulations matches expectation from response curves $\kappa(\hat{\gamma})$
- ratio modulation with opacity scale, because NeNe is bigger than OO
- taking the ratio reduces hydro error from few % to $< 1\%$.

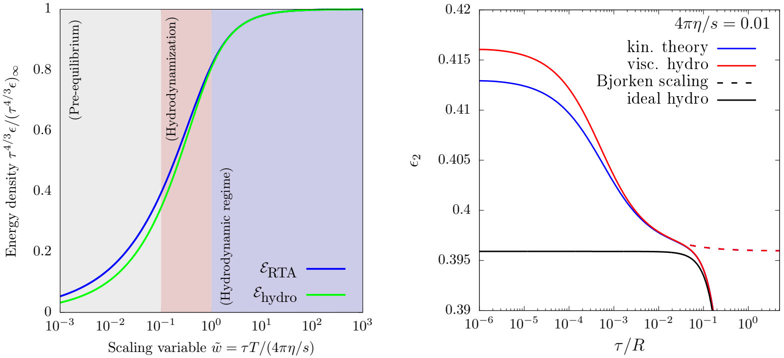
Eccentricity evolution before transverse expansion

$\tau \ll R$: system undergoes local Bjorken flow at each point in transverse plane

$\Rightarrow$ "Attractor behaviour": quick convergence to universal evolution

early times $\epsilon \sim \tau^{-1} \longrightarrow$ equilibration $\tau_{\text{eq}} \propto T^{-1}\eta/s \longrightarrow$ ideal hydro $\epsilon \sim \tau^{-4/3}$

$\Rightarrow$ hotter regions cool more, eccentricities change (amount differs between theories)

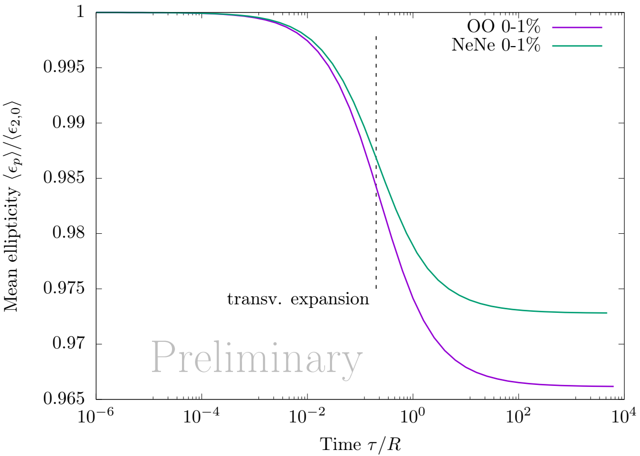


our hydro simulations: scaled initial condition of hydro in anticipation of different pre-equilibrium

Ambruș, Schlichting, CW, PRD 107 (2023) 094013

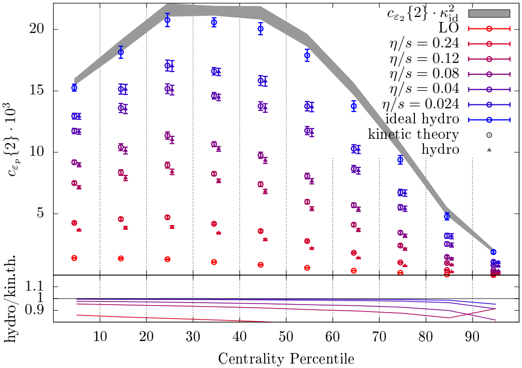
- eccentricity values at onset of transverse expansion determine final state flow
- typically changes eccentricities by $\mathcal{O}(1\%)$, but can be up to 10%

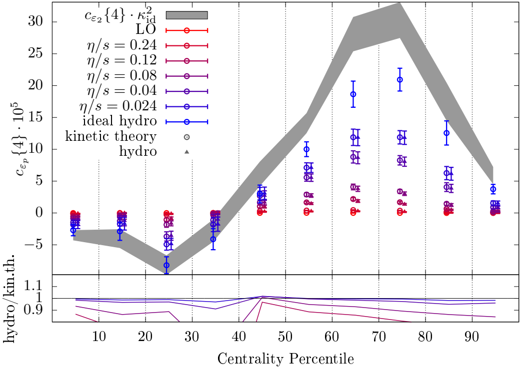
Early time eccentricity evolution in OO and NeNe



- OO: $\sim 3.4\%$, NeNe: $\sim 2.7\%$ total decrease
- when transverse expansion sets in, ϵ_2 has been reduced by a similar fraction $\sim 1.5\%$ in both systems $\Rightarrow$ good cancellation in ratio

Flow Cumulants





$$c_{\mathcal{O}}\{2\} = \langle|\mathcal{O}|^2\rangle \qquad c_{\mathcal{O}}\{4\} = \langle|\mathcal{O}|^4\rangle - 2\langle|\mathcal{O}|^2\rangle^2$$

- $c_{\epsilon_p}\{2k\} \approx \kappa_{\mathrm{id}}^{2k} c_{\epsilon_2}\{2k\}$ in ideal hydro; opacity $\downarrow$: flow response $\downarrow$
- modulation of $c_{\epsilon_p}\{2k\}$ relative to $\propto c_{\epsilon_2}\{2k\}$ from centrality dependence of $\kappa(\hat{\gamma})$
- flow fluct. dominated by avg. response to geometry fluct $\langle(\epsilon_p)^n\rangle = \bar{\kappa}^n \langle(\epsilon_2)^n\rangle + \ldots$
- even "unorthodox" sign $(v_2\{4\} = \sqrt[4]{-c_{v_2}\{4\}})$ follows from initial state!

Summary

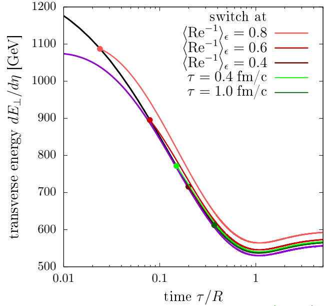
- several dynamical effects modify the correspondence between ϵ_n and v_n
  - nonlinear responses
  - dependencies of κ
  - ϵ_n change in inhomogeneous cooling
  - dynamical model may be inaccurate
- hydrodynamics has limited range of applicability in system size ($\hat{\gamma} \lesssim 3 \,/\, \mathrm{W} \gtrsim 0.5$)
  - however, flow ratios remain accurate in wider range
- flow cumulants follow eccentricity cumulants even when sign is "off" $\Rightarrow$ should be measured

Backup

Comparing hydro to kinetic theory (for now: average profile)

Hydro doesn't work at early times! We can

- scale initial condition of hydro
  - assumes equilibration before transverse expansion
  - works for $\hat{\gamma} \gtrsim 4$



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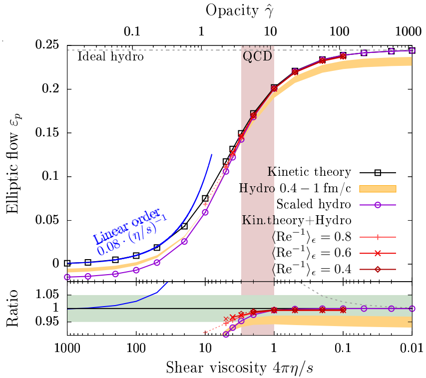
- set up hybrid simulations
  - later switch gives better agreement
  - switching at fixed Re^{-1} gives good handle on accuracy



Hydrodynamization Observable: Definition

measuring hydrodynamization

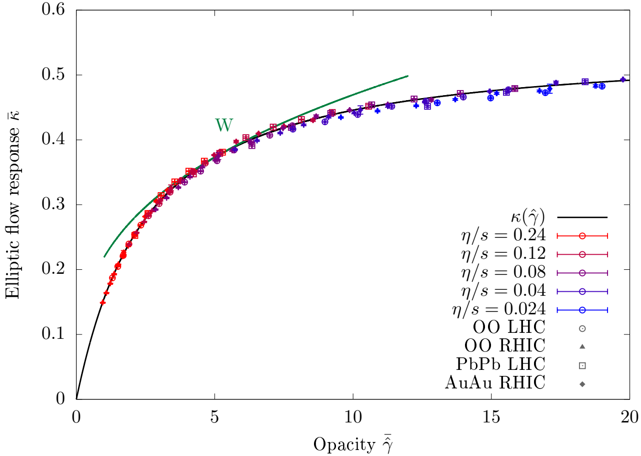
cancel geometry factors between similar systems:

$$\frac{\bar{\kappa}^{2k}_{\mathrm{RHIC}}}{\bar{\kappa}^{2k}_{\mathrm{LHC}}} \approx \frac{c_2^{\mathrm{RHIC}}\{2k\}}{c_2^{\mathrm{LHC}}\{2k\}}$$

$$\frac{\hat{\gamma}_{\mathrm{RHIC}}}{\hat{\gamma}_{\mathrm{LHC}}} \approx \left(\frac{\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}_{\mathrm{RHIC}}}{\frac{\mathrm{d}E_\perp}{\mathrm{d}\eta}_{\mathrm{LHC}}}\right)^{1/4}$$

to combine the two: log turns ratios into differences

$$\mathrm{W} = \frac{2}{k}\frac{\Delta\log(c_2\{2k\})}{\Delta\log(\mathrm{d}E_\perp/\mathrm{d}y)} \approx \frac{\mathrm{d}\log\kappa}{\mathrm{d}\log\hat{\gamma}} \xrightarrow{\hat{\gamma}\to 0} 1 \quad \xrightarrow{\hat{\gamma}\to\infty} 0$$



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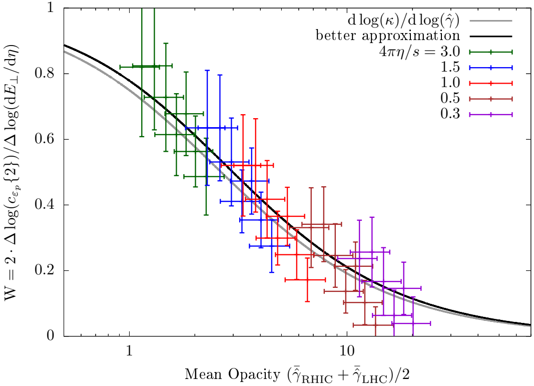
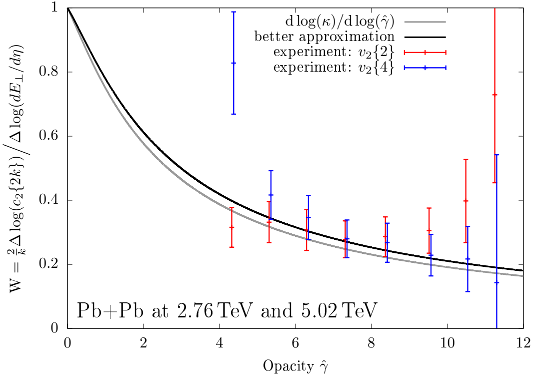
Hydrodynamization Observable: Proof of principle

crosscheck of W-observable:

with simulations



with experimental data



previous hydrodynamization criterion: $\hat{\gamma} \sim 3$ corresponds to $\mathrm{W} \sim 0.5$