A New "Skin Thickness" in High-spin Isomers as a Probe for EOS of Spin-polarized Matter

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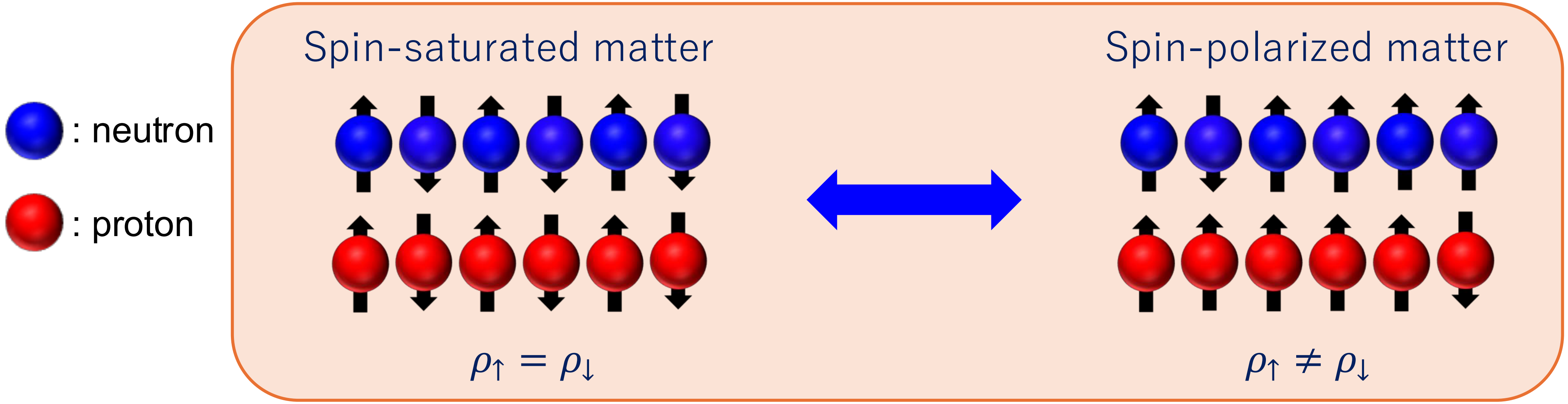
Outline

1. **Motivation – Spin-polarized matter –**
2. The relativistic point-coupling model
3. Spin slope parameter
4. New “skin thickness” in high-spin isomers
5. Summary

Spin-polarized Nuclear Matter



- The ferromagnetic phase in high-density region ▶ Origin of the magnetic field of magnetar

  I. Vidana, I. Bombaci, Phys. Rev. C 66, 045801 (2002)

  T. Maruyama, T. Tatsumi, Nucl. Phys. A 693 (2001) 710-730

- Strong magnetic field in the remnant of neutron star merger ▶ Possible formation of spin-polarized matter

  B. D. Metzger, et al., Astrophys. J. Lett. 856, 101 (2018)

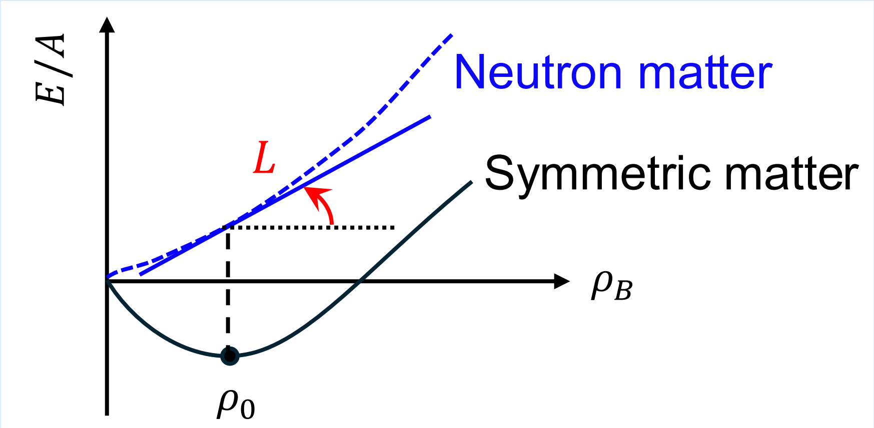
- Changes in particle fraction due to spin polarization ▶ Affecting neutron star cooling.

  N. H. Khoa, N. H. Tan, D. T. Khoa, Phys. Rev. C 105, 065802 (2022)

▶ EOS for spin-polarized matter has been increasingly important.

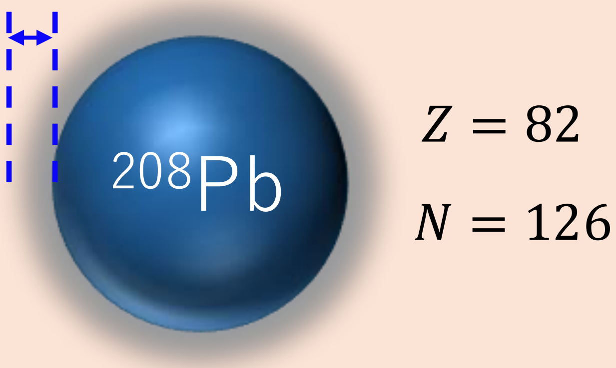
Neutron skin thickness and Slope parameter

Slope parameter L



Linear correlation

Neutron skin thickness Δr_{np}

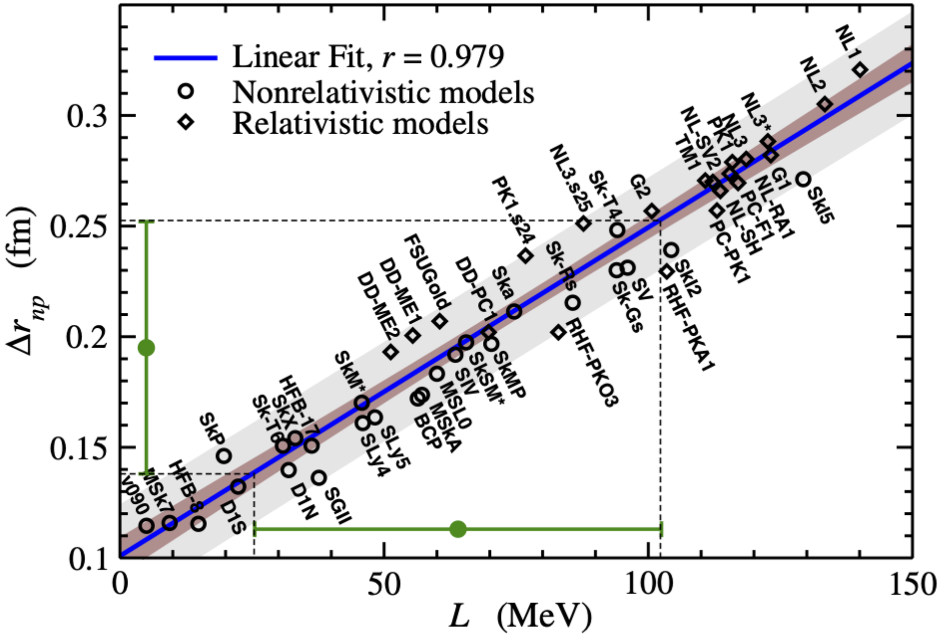


PREX

D. Adhikari, et al., Phys. Rev, Lett. 126, 172502 (2021)

p elastic scattering

J. Zenihiro, et al., Phys. Rev. C 82, 044611 (2010)



X. Roca-Maza, et al., Phys. Rev. Lett. 106, 252501 (2011)

Current State

Neutron skin thickness is an experimental probe to constrain the EOS for asymmetric nuclear matter.

Almost no analogous probes for constraining the EOS for spin-polarized matter has been considered.

Short Summary

Our goals

- Proposing a method to constrain the EOS for spin-polarized matter by terrestrial nuclear experiments.

This Study

- Calculation of the EOS for spin-polarized matter in the relativistic model, which can naturally treat the spin d.o.f. of nucleons
- Correlation between the slope parameter and the spin slope parameter (details to follow)
- Exploring a new skin "thickness" correlated with the spin slope parameter

Outline

1. Motivation – Spin-polarized matter –
2. **The relativistic point-coupling model**
3. Spin slope parameter
4. New “skin thickness” in high-spin isomers
5. Summary

Relativistic Point-coupling Model

Q. Zhao, et. al., Phys. Rev. C 106, 034315 (2022)

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}\alpha_S(\bar{\psi}\psi)^2 - \frac{1}{2}\alpha_{tS}(\bar{\psi}\vec{\tau}\psi)^2 - \frac{1}{2}\alpha_V(\bar{\psi}\gamma^\mu\psi)^2 - \frac{1}{2}\alpha_{tV}(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 - \frac{1}{2}\alpha_{PS}(\bar{\psi}\gamma_5\psi)^2 - \frac{1}{2}\alpha_{tPS}(\bar{\psi}\gamma_5\vec{\tau}\psi)^2$$
$$-\frac{1}{2}\alpha_{PV}(\bar{\psi}\gamma_5\gamma^\mu\psi)^2 - \frac{1}{2}\alpha_{tPV}(\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2 - \frac{1}{2}\alpha_T(\bar{\psi}\sigma^{\mu\nu}\psi)^2 - \frac{1}{2}\alpha_{tT}(\bar{\psi}\sigma^{\mu\nu}\vec{\tau}\psi)^2$$

Mean Field approx.

$$\psi = \sum_{\boldsymbol{p},s,t}\left[u(p,s,t)a_{\boldsymbol{p},s,t}e^{-ip\cdot x} + \cancel{v(p,s,t)b^\dagger_{\boldsymbol{p},s,t}e^{ip\cdot x}}\right]$$

(no sea approximation)

$$\langle[\bar{\psi}(\mathcal{O}\Gamma)_i\psi]^2\rangle = \sum_{\alpha,\beta}\int_{V_{p_F}} dp^3dq^3[\bar{u}(p,\alpha)(\mathcal{O}\Gamma)_i u(p,\alpha) \times \bar{u}(q,\beta)(\mathcal{O}\Gamma)_i u(q,\beta)$$
$$-\underline{\bar{u}(p,\alpha)(\mathcal{O}\Gamma)_i u(q,\beta) \times \bar{u}(q,\beta)(\mathcal{O}\Gamma)_i u(p,\alpha)}]$$

$\mathcal{O} \in \{1, \tau_3\}$

$\Gamma \in \{1, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}\}$

Fierz transf.

$$\blacktriangleright\ \Lambda_{ij}\,\bar{u}(p,\alpha)(\mathcal{O}\Gamma)_j u(p,\alpha) \times \bar{u}(q,\beta)(\mathcal{O}\Gamma)_j u(q,\beta)$$

$$= (\delta_{ij} - \Lambda_{ij})\left[\sum_\alpha\int_{V_{p_F}} dp^3\bar{u}(p,\alpha)(\mathcal{O}\Gamma)_j u(p,\alpha)\right]^2$$

Relativistic Point-coupling Model

Q. Zhao, et. al., Phys. Rev. C 106, 034315 (2022)

Total Energy Density

$$\mathcal{E} = \langle \mathcal{H} \rangle = \mathcal{E}_{\mathrm{kin}} + \frac{1}{2}\alpha_S \rho_S^2 + \frac{1}{2}\alpha_{tS}\rho_{tS}^2 + \frac{1}{2}\alpha_V \rho_V^2 + \frac{1}{2}\alpha_{tV}\rho_{tV}^2 - \frac{1}{2}\alpha_{PV}\rho_{PV}^2 - \frac{1}{2}\alpha_{tPV}\rho_{tPV}^2 + \alpha_T \rho_T^2 + \alpha_{tT}\rho_{tT}^2$$

$$\rho_S = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} \bar{u}(p,s,t)\, u(p,s,t) \ , \qquad \rho_V = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p,s,t)\, u(p,s,t)$$

$$\rho_{PV} = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p,s,t)\, \gamma^0 \gamma^3 \gamma_5\, u(p,s,t) = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p,s,t)\, \Sigma_3\, u(p,s,t)\,,$$

$$\Sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

$$\rho_T = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p,s,t)\, \gamma_0 \sigma^{12}\, u(p,s,t) = \sum_{s,t} \int \frac{d^3p}{(2\pi)^3} u^\dagger(p,s,t)\, \gamma_0 \Sigma_3\, u(p,s,t)\,,$$

$u(p,s,t)$: positive-energy plane wave spinor

Relativistic Point-coupling Model

Q. Zhao, et. al., Phys. Rev. C 106, 034315 (2022)

Total Energy Density

$$\mathcal{E} = \langle \mathcal{H} \rangle = \mathcal{E}_{\mathrm{kin}} + \frac{1}{2}\alpha_S\rho_S^2 + \frac{1}{2}\alpha_{tS}\rho_{tS}^2 + \frac{1}{2}\alpha_V\rho_V^2 + \frac{1}{2}\alpha_{tV}\rho_{tV}^2 - \frac{1}{2}\alpha_{PV}\rho_{PV}^2 - \frac{1}{2}\alpha_{tPV}\rho_{tPV}^2 + \alpha_T\rho_T^2 + \alpha_{tT}\rho_{tT}^2$$

Independent parameters are only $\alpha_S, \alpha_V, \alpha_{tS}, \alpha_{tV}, \alpha_T,$ (because $\mathrm{rank}(1 - \Lambda) = 5$)

and other coupling constants are determined by coefficients of Fierz transf. :

$$\alpha_{tT} = \tfrac{1}{18}(-\alpha_S + 3\alpha_{tS} + 2\alpha_V - 6\alpha_{tV} + 6\alpha_T),$$
$$\alpha_{PV} = \tfrac{1}{3}(2\alpha_S + 3\alpha_{tS} + 2\alpha_V + 3\alpha_{tV} + 6\alpha_T),$$
$$\alpha_{tPV} = \tfrac{1}{9}(2\alpha_S + 3\alpha_{tS} + 5\alpha_V - 6\alpha_{tV} + 6\alpha_T).$$

The four coupling constants $\alpha_S, \alpha_V, \alpha_{tS}$ and α_{tV} are density-dependent :

$$\alpha_i(\rho_B) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \times \alpha_i(\rho_0) \qquad \left(x = \frac{\rho_B}{\rho_0} \right) \qquad i = S, V, tS, tV$$

Relativistic Point-coupling Model

Dirac eq. for nucleons (s: spin index, t: isospin index)

$$[\boldsymbol{p}\cdot\boldsymbol{\alpha} + (M + \alpha_S\rho_S + \tau_3\alpha_{tS}\rho_{tS})\beta - (\alpha_{PV}\rho_{PV} + \tau_3\alpha_{tPV}\rho_{tPV})\Sigma_3 + 2(\alpha_T\rho_T + \tau_3\alpha_{tT}\rho_{tT})\beta\Sigma_3]u(p,s,t)$$
$$= (E - \alpha_V\rho_V - \alpha_{tV}\rho_{tV})u(p,s,t)$$

$$\Sigma_3 = \gamma^0\gamma^3\gamma_5 = \begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix},\quad \rho_S = \langle\bar{\psi}\psi\rangle,\quad \rho_V = \langle\psi^\dagger\psi\rangle(=\rho_B),\quad \rho_{PV} = \langle\psi^\dagger\Sigma_3\psi\rangle,\quad \rho_T = \langle\psi^\dagger\beta\Sigma_3\psi\rangle$$

We have two positive energy solutions ► $u(p,s=+1,t)$ and $u(p,s=-1,t)$

$$\Delta_t = \frac{(\rho_{s=+1,t} - \rho_{s=-1,t})}{\rho_t},\qquad \rho_{s=\pm1,t} = \int\frac{d^3p}{(2\pi)^3}u^\dagger(p,s=\pm1,t)\,u(p,s=\pm1,t)$$

However, $u(p,s,t)$ is not an eigenvector of Σ_3. (when $\boldsymbol{p}=0$, it is an eigenvector of Σ_3)

► $\langle\Sigma_3\rangle$ is true spin polarization rate, and $\Delta \neq \langle\Sigma_3\rangle$

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Numerical Results (EOS)

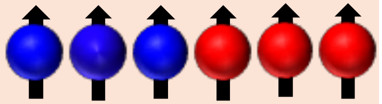
EOS for symmetric nuclear matter (using PCF-PK1)

Q. Zhao, et. al., Phys. Rev. C 106, 034315 (2022)

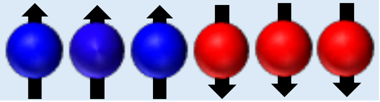
$\delta \equiv (\rho_n - \rho_p)/\rho_B, \ \Delta_t \simeq (\rho_{\uparrow,t} - \rho_{\downarrow,t})/\rho_t$

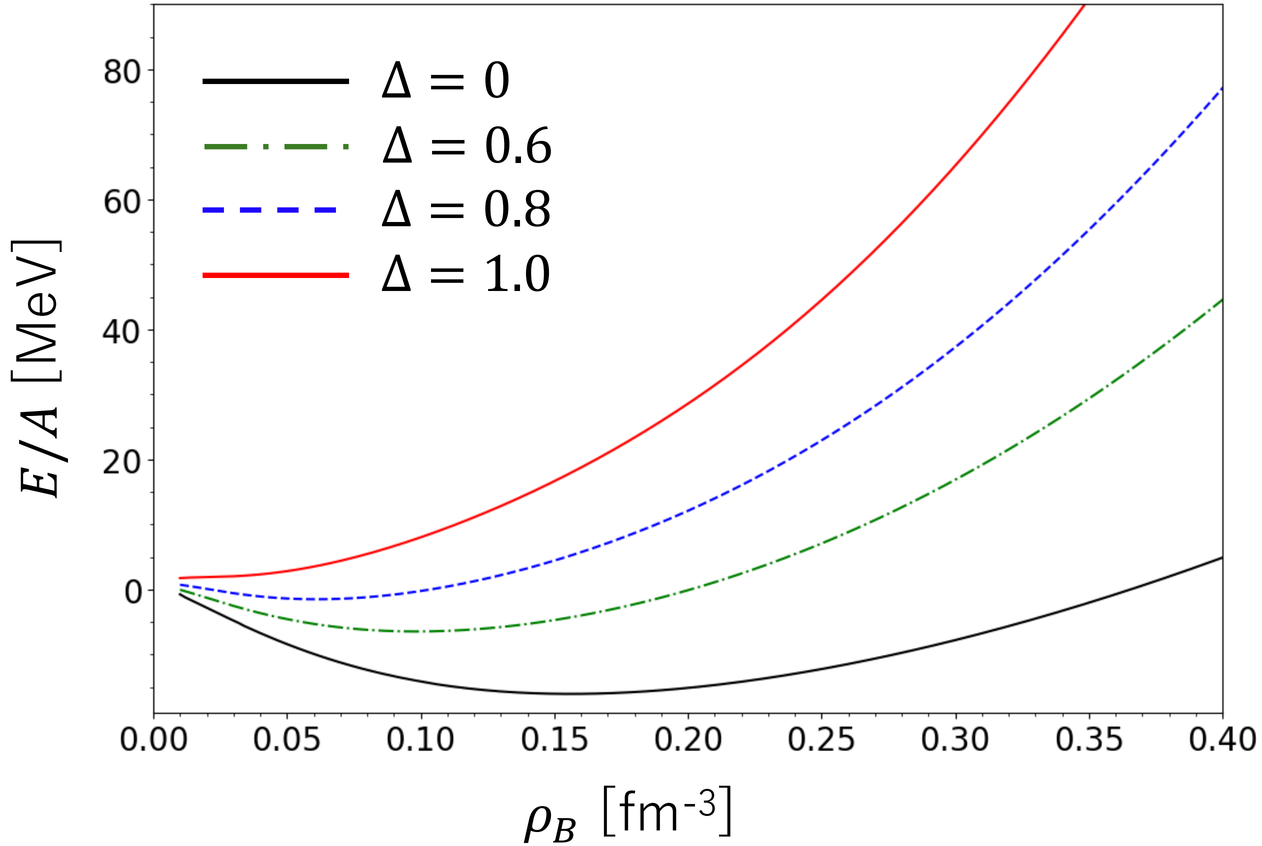
For simplicity, we assume $|\Delta_n| = |\Delta_p|$ ▶ $E(\rho_{\uparrow,n}, \rho_{\downarrow,n}, \rho_{\uparrow,p}, \rho_{\downarrow,p}) = E(\rho_B, \delta, \Delta)$

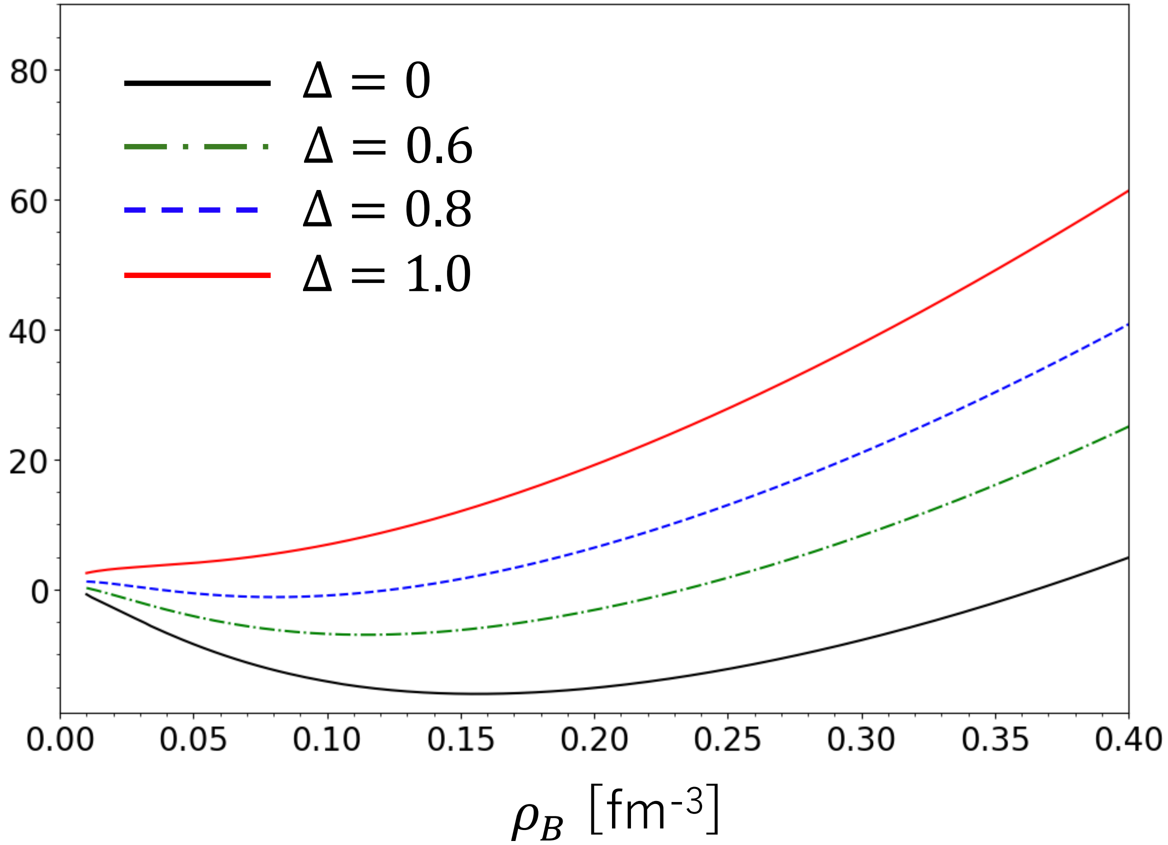
IS(Isoscalar): $\Delta_n = \Delta_p = \Delta$



IV(Isovector): $\Delta_n = -\Delta_p = \Delta$







Spin Slope parameter

Slope parameter

$$\frac{E}{A}(\rho,\delta) \cong \frac{E}{A}(\rho,\delta=0) + \underline{S(\rho)}\delta^2$$

Symmetry energy

$$L = 3\rho_0 \frac{\partial S(\rho)}{\partial \rho}\bigg|_{\rho=\rho_0}$$

Spin slope parameter

$$\frac{E}{A}(\rho,\langle\Sigma_3\rangle,\delta) \cong \frac{E}{A}(\rho,\langle\Sigma_3\rangle=0,\delta) + \underline{W(\rho,\delta)}\langle\Sigma_3\rangle^2$$

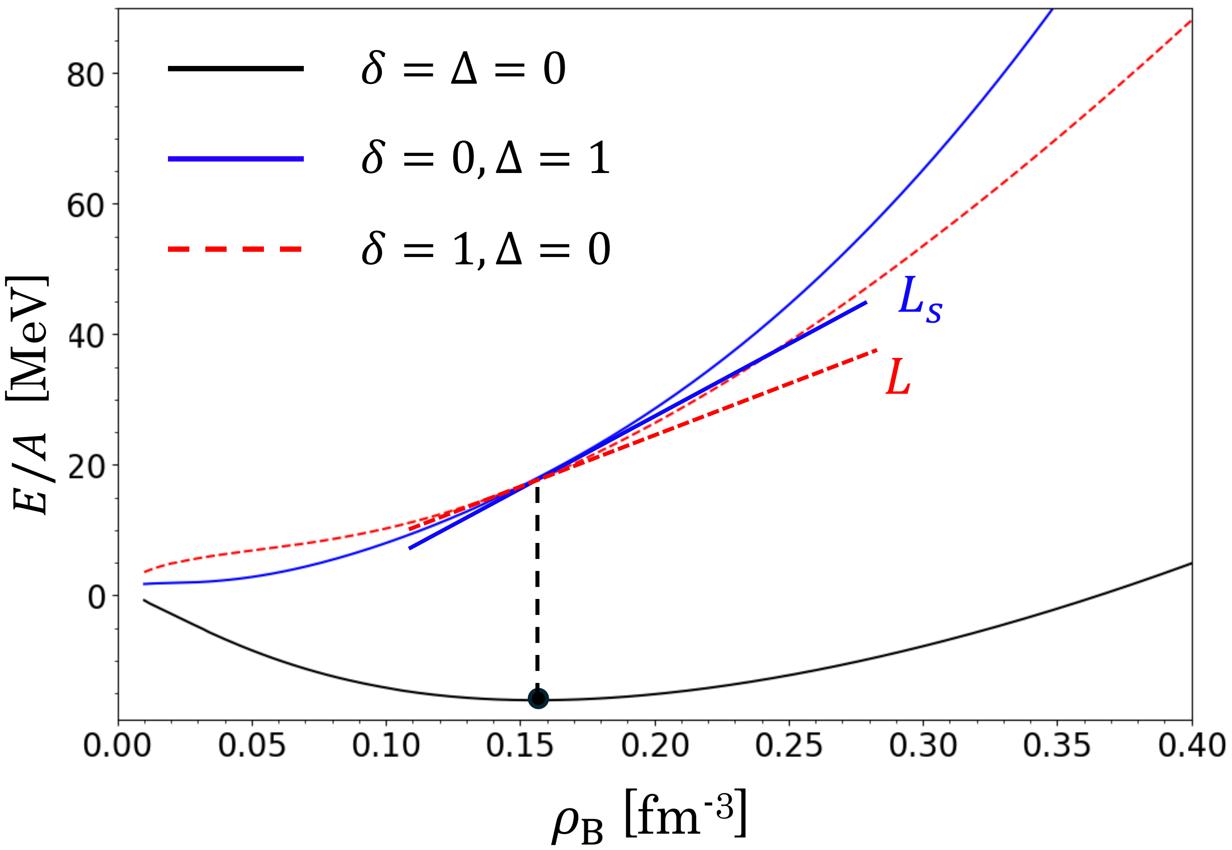
Spin symmetry energy

$$\Sigma_3 = \begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix}$$

$$L_s = 3\rho_0 \frac{\partial W(\rho)}{\partial \rho}\bigg|_{\rho=\rho_0}$$

$$\delta \equiv (\rho_n - \rho_p)/\rho_B, \quad \Delta_\tau \simeq (\rho_{\uparrow,\tau} - \rho_{\downarrow,\tau})/\rho_\tau$$

$$\Delta \equiv |\Delta_n| = |\Delta_p| \ (\neq \langle\Sigma_3\rangle)$$



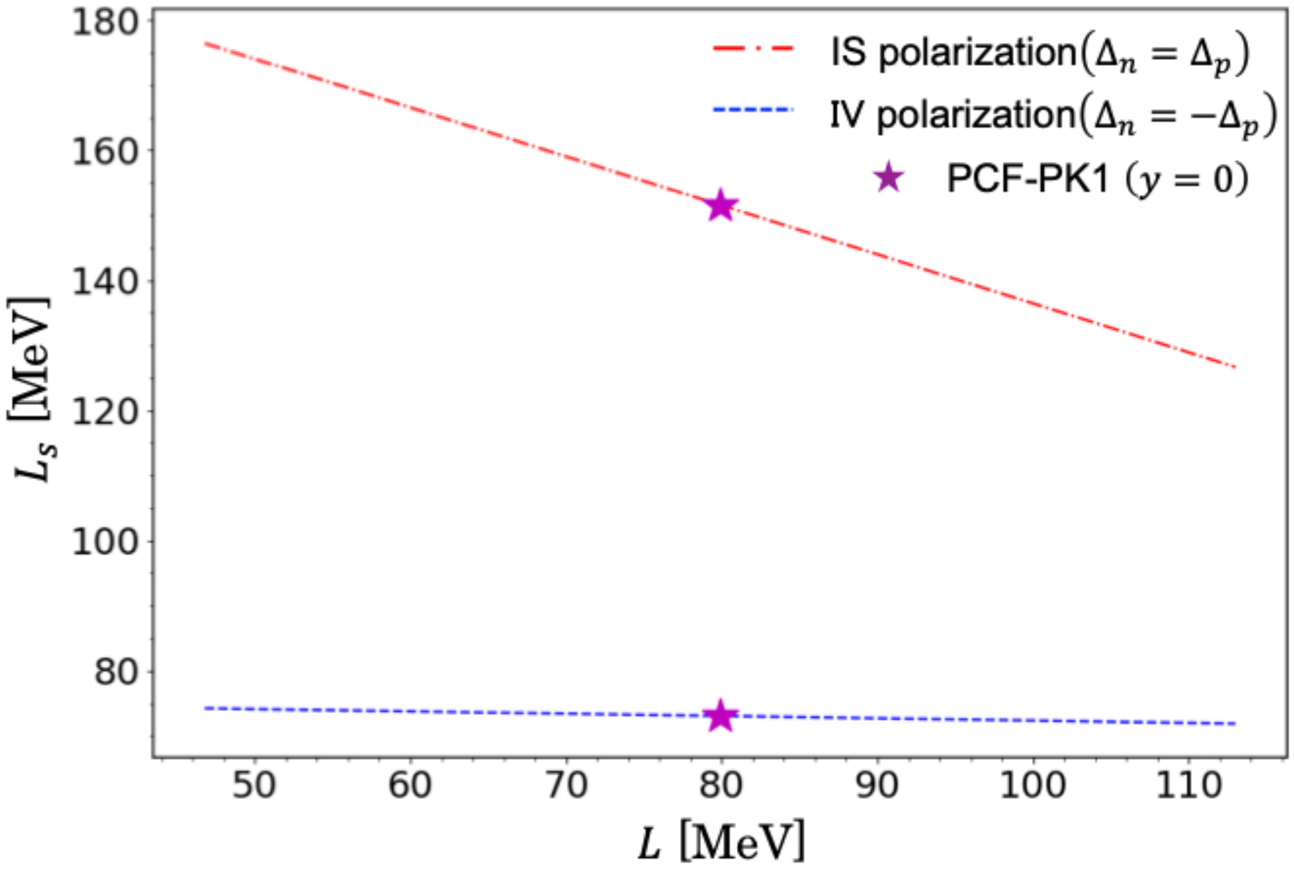
► We investigated the correlation between L and L_s.

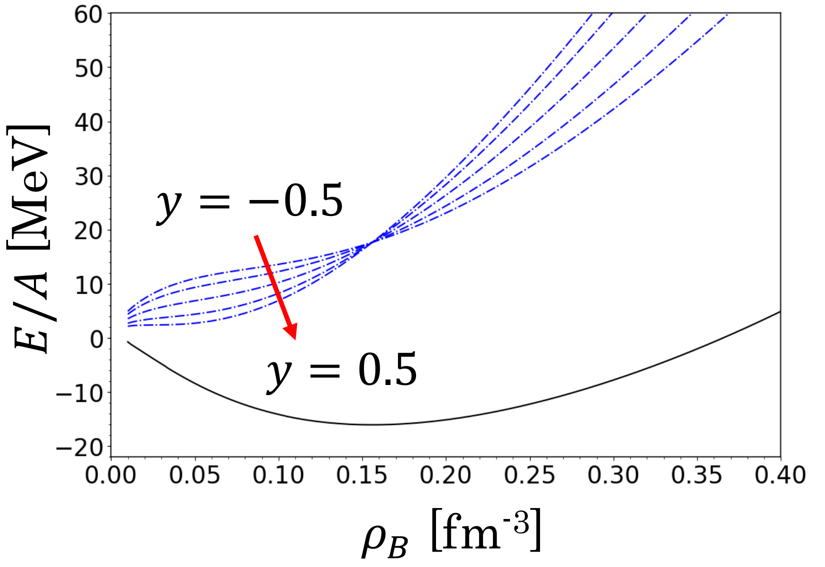
Correlation between L and L_s

Introducing a new variable y:

$$\alpha_{tV}(\rho) \rightarrow (1-y)\alpha_{tV}(\rho) + y\alpha_{tV}(\rho_0)$$

cf. T. Inakura, H. Nakada, Phys. Rev. C 92, 064302 (2015)





- In the case of IS polarization $\Delta_n = \Delta_p = \Delta$, L_s and L are linearly correlated.
- In the case of IV polarization $\Delta_n = -\Delta_p = \Delta$, L_s is nearly independent of L.

Correlation between L and L_s

$$\alpha_{tV}(\rho) \to (1-y)\alpha_{tV}(\rho) + y\alpha_{tV}(\rho_0)$$

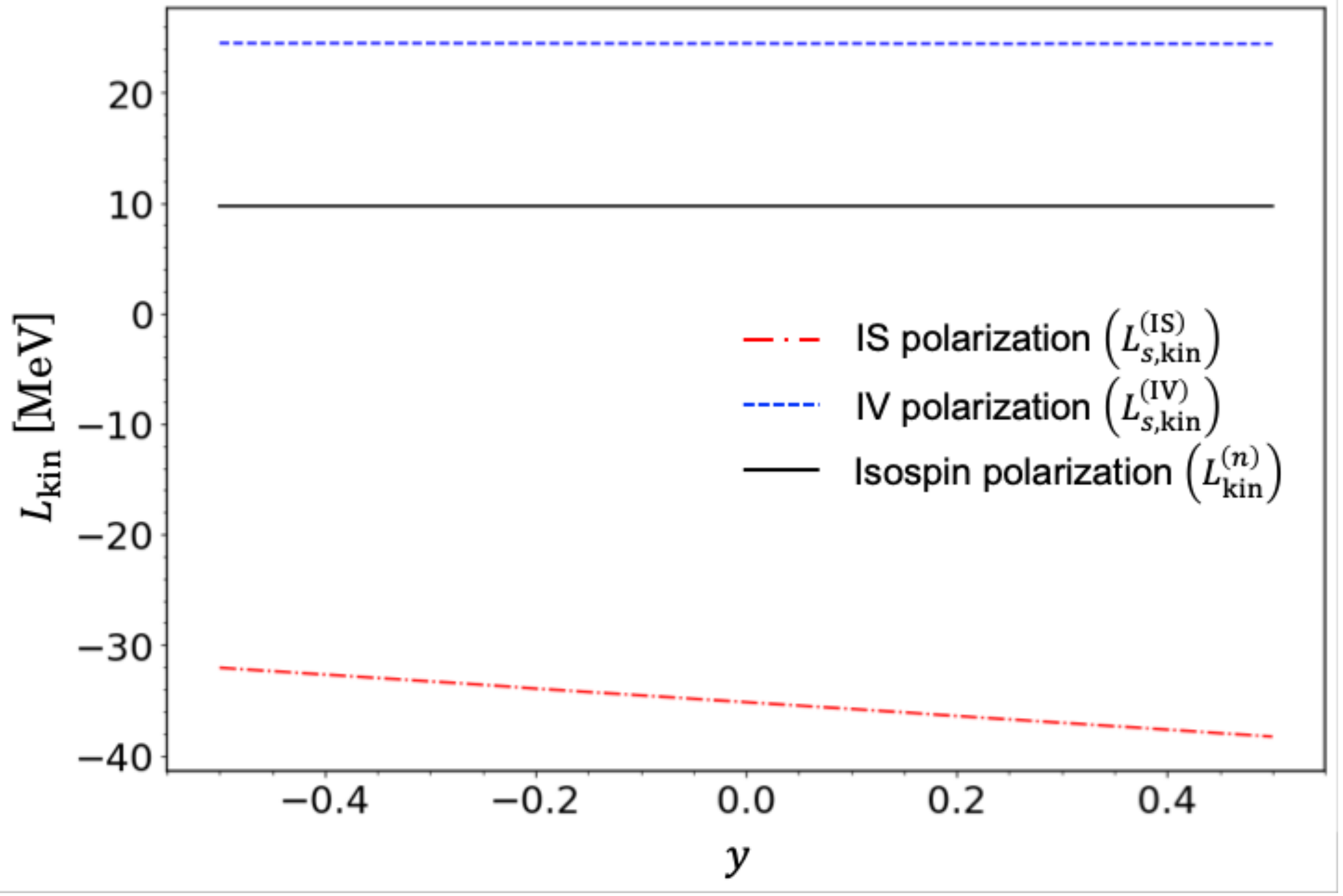
$$\alpha_{tT} = \frac{1}{18}(-\alpha_S + 3\alpha_{tS} + 2\alpha_V - 6\alpha_{tV} + 6\alpha_T),$$
$$\alpha_{PV} = \frac{1}{3}(2\alpha_S + 3\alpha_{tS} + 2\alpha_V + 3\alpha_{tV} + 6\alpha_T),$$
$$\alpha_{tPV} = \frac{1}{9}(2\alpha_S + 3\alpha_{tS} + 5\alpha_V - 6\alpha_{tV} + 6\alpha_T).$$

$L + L_s^{\mathrm{IS}} \simeq const.$ ▶ $L \simeq -L_s^{\mathrm{IS}} + const.$

* $L + L_s^{\mathrm{IS}} \simeq L_{\mathrm{kin,\,n}} + L_{\mathrm{kin}}^{\mathrm{IS}} - (\alpha_S + \alpha_V)\rho_0$

$L_s^{\mathrm{IV}} \simeq const.$

* $L_s^{\mathrm{IV}} = L_{\mathrm{kin}}^{\mathrm{IV}} - \frac{\rho_0}{2}(\alpha_S + \alpha_V)$



Correlations between L and L_s arise from the relativistic structure of the model

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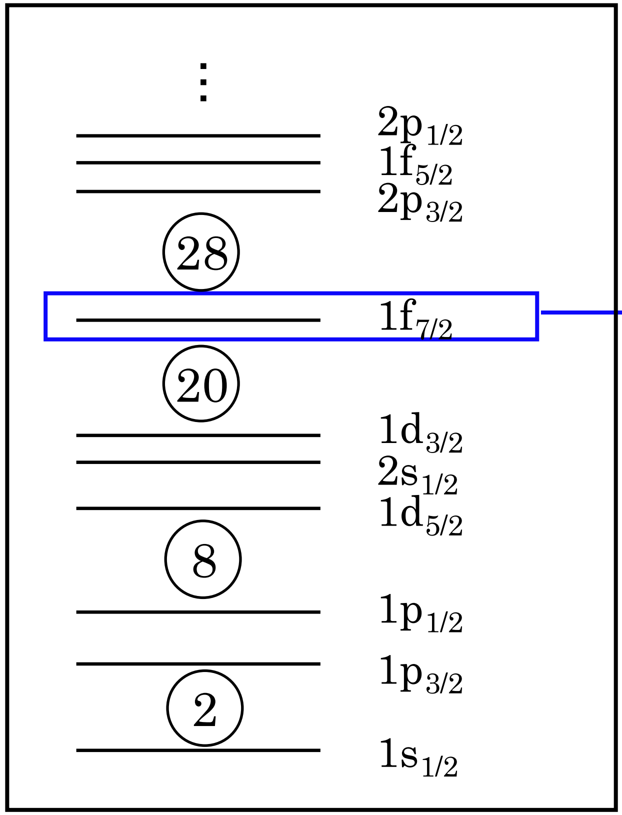
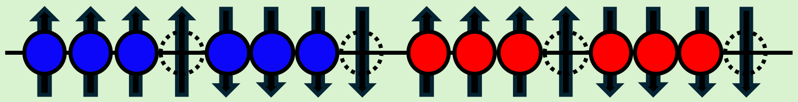
Nuclear Isomer and 12⁺ Excited State of ^{52}Fe ($N = Z = 26$)

<u>Nuclear isomer</u> : excited state with a long lifetime

Lifetime of 12^+ excited state of ^{52}Fe is 45.9s. ▶ Isomer with high spin

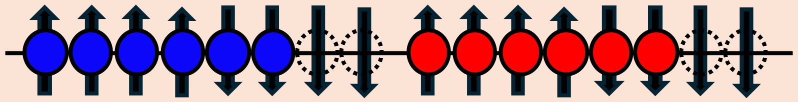


If 6 nucleons occupy f7/2 orbit,

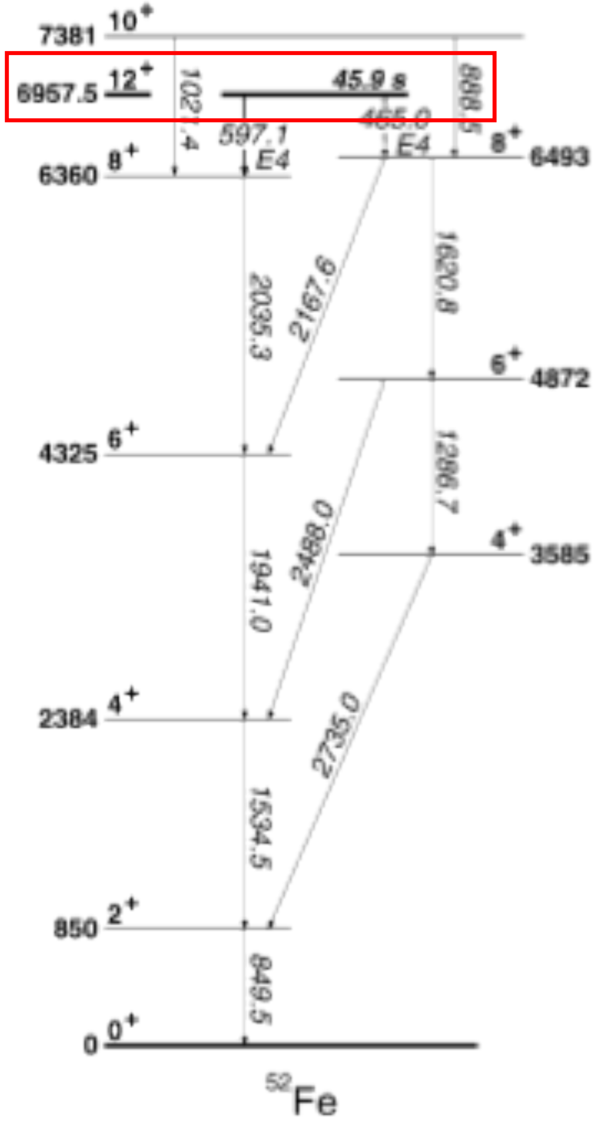


▶ ^{52}Fe(0^+)

In ^{52}Fe(12^+), j_z is fully aligned.



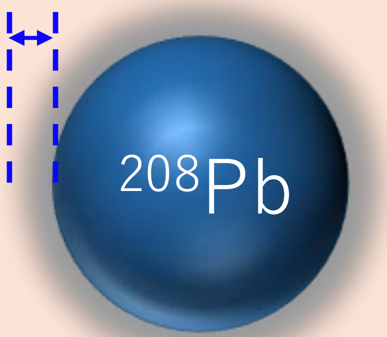
$$\frac{5}{2} + \frac{7}{2} = +6 \qquad \frac{5}{2} + \frac{7}{2} = +6$$

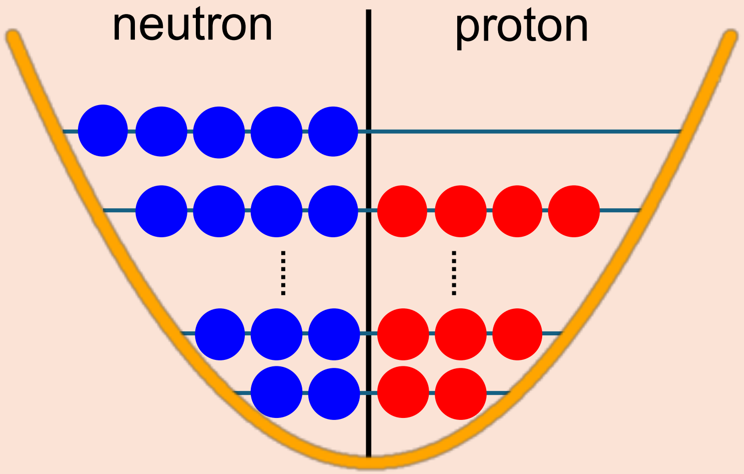


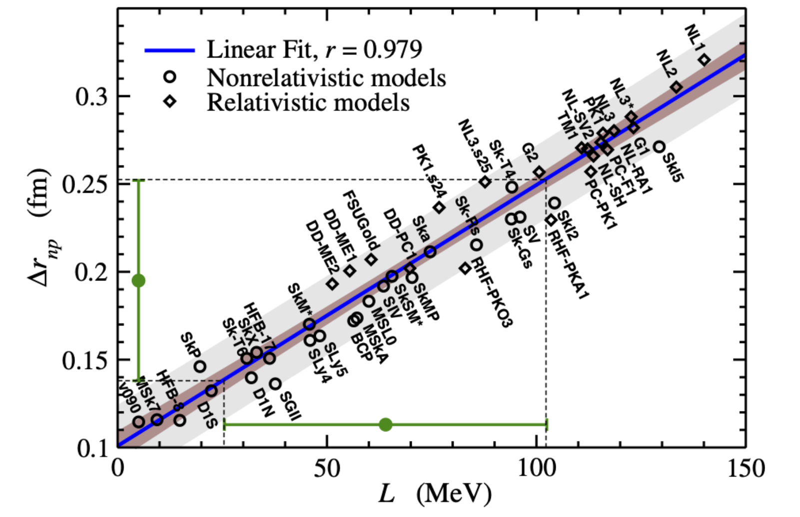
A. Gadea, et al., Phys. Lett. B, 619 (2005) 88

Why High-spin Isomers?

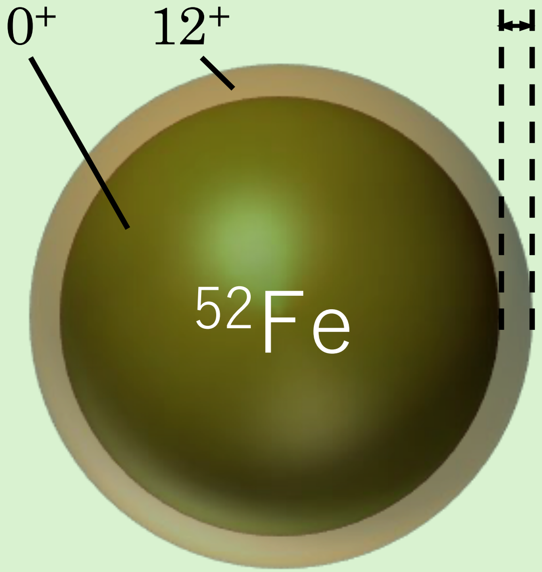
Neutron skin thickness

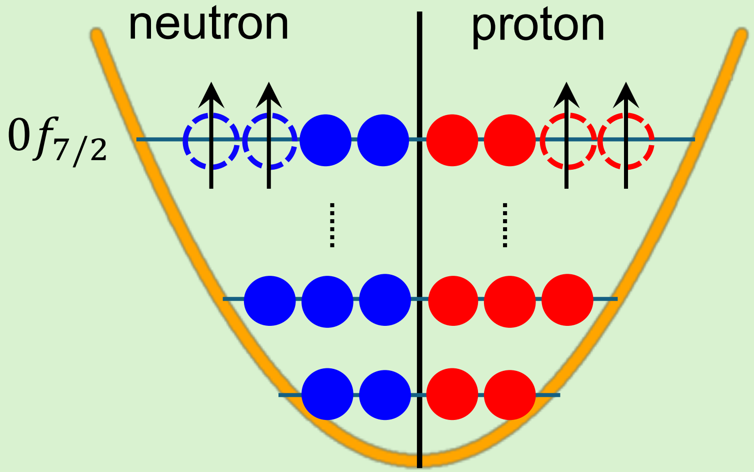




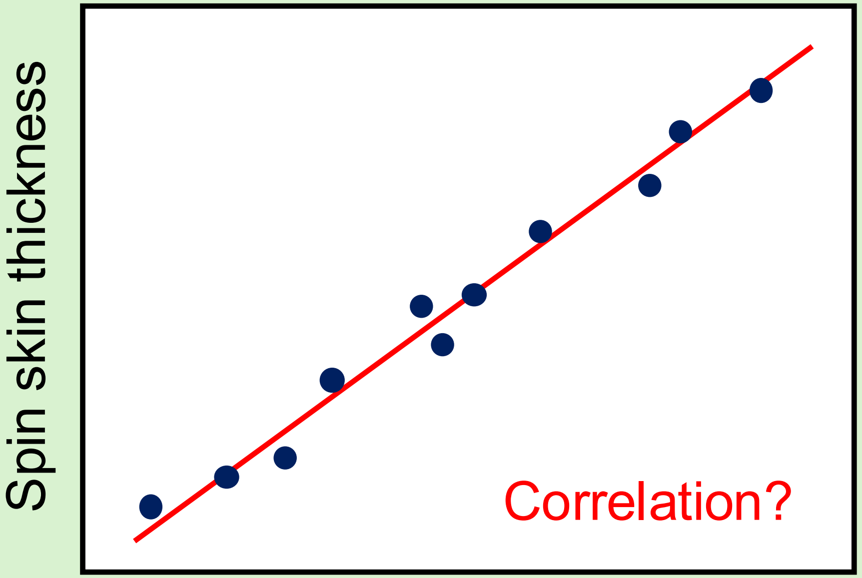


Spin skin thickness?





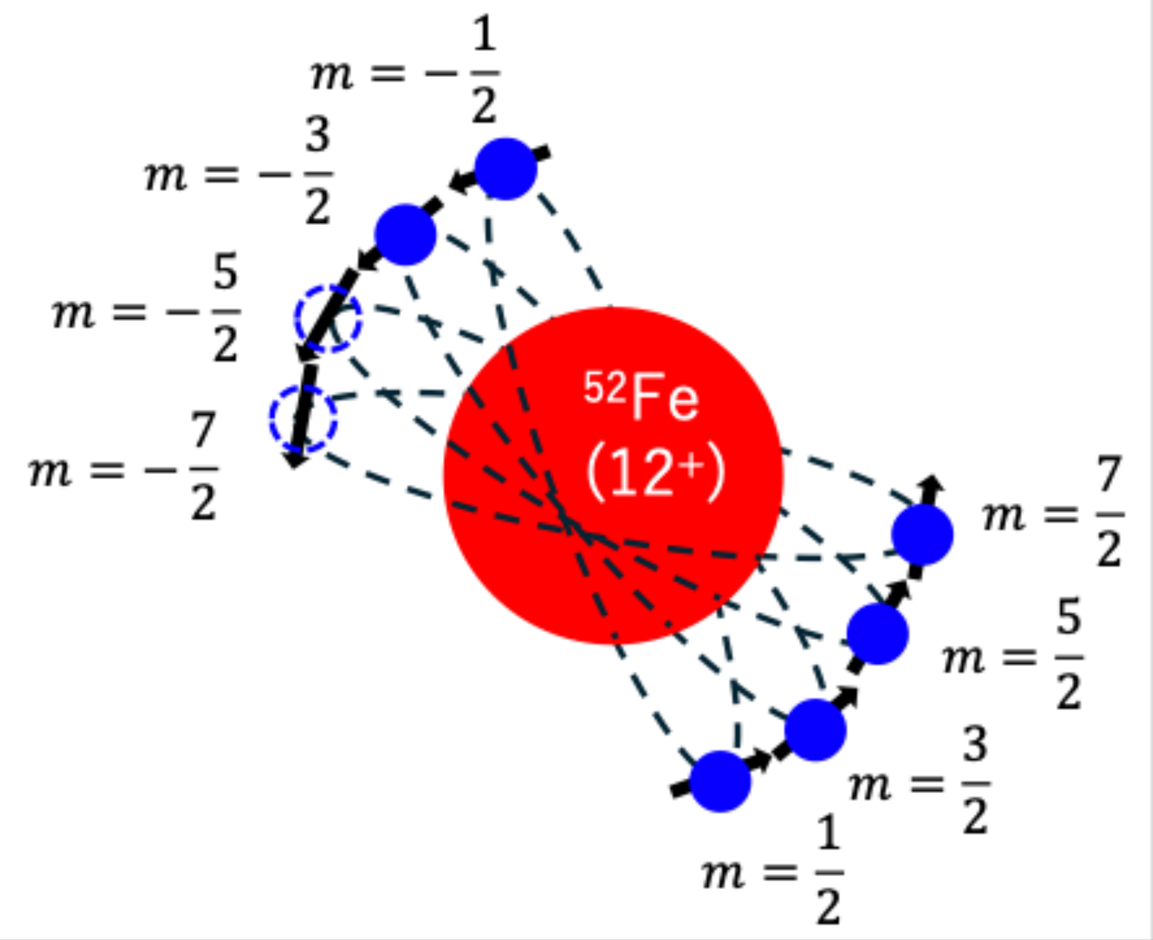
Spin slope parameter



Calculation of 0^+ g.s. and 12^+ isomer of ^{52}Fe

<u>0^+ ground state</u> : spherical HFB calculation with separable paring interaction

<u>12^+ isomer state</u> : HF calculation for ^{52}Fe with the fixed configuration



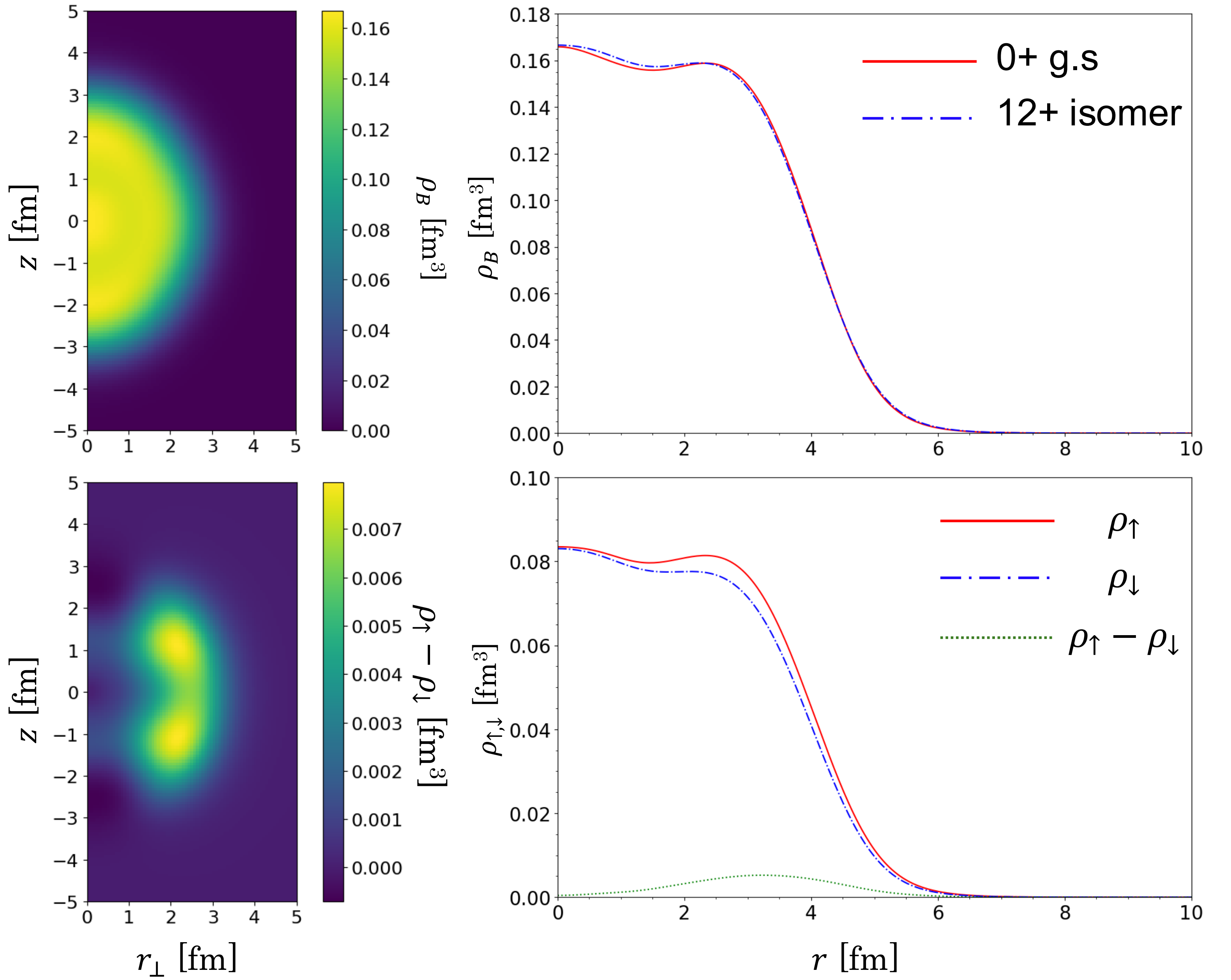
- The configurations in $f7/2$ orbit are fixed by blocking $m = -5/2$ and $m = -7/2$ orbits.

▼

The total angular momentum is 6 for both neutrons and protons.

- Pairing correlations are neglected in calculation of 12^+ isomer state.

Nucleon and Spin Density of $^{52}Fe(12+)$ State



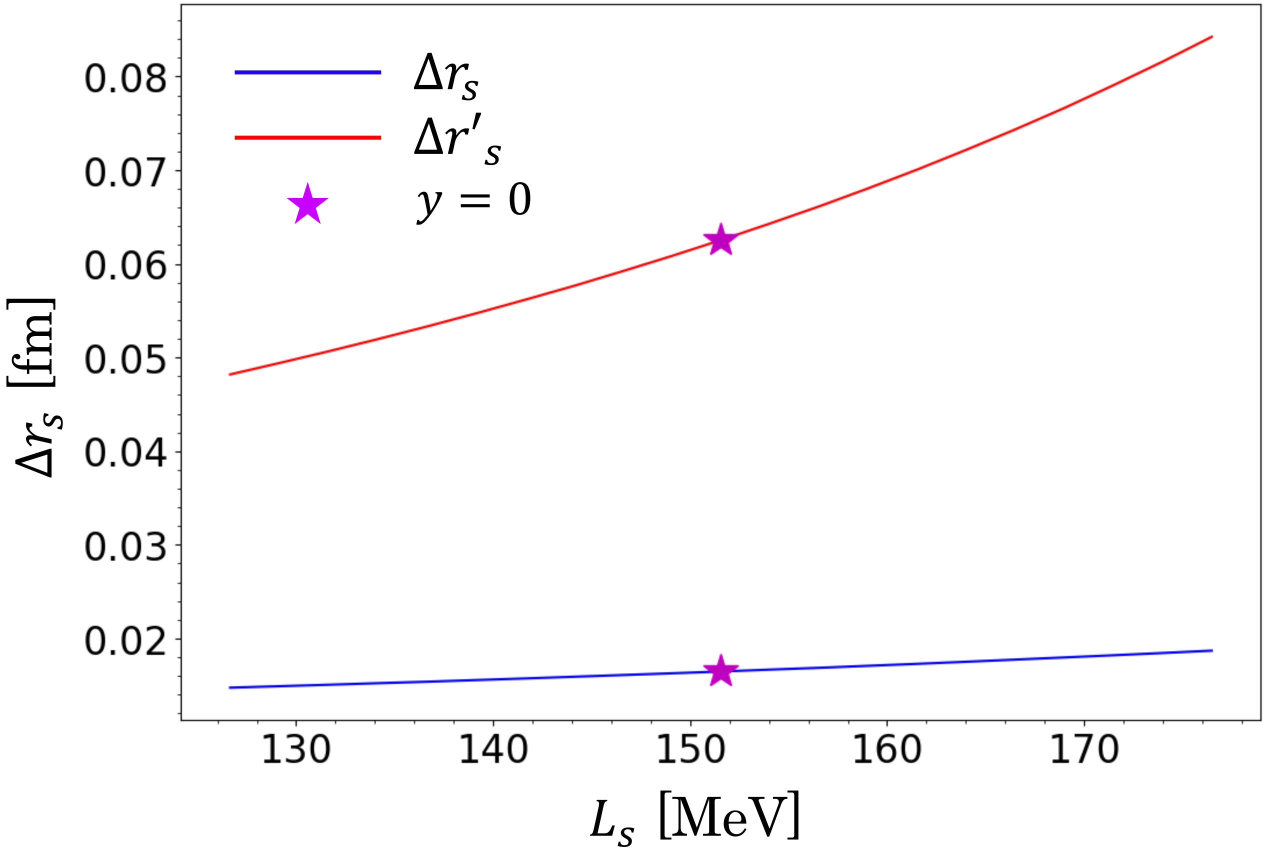
Spin densities are localized near the surface!

Correlation between spin skin thickness and spin slope parameter

Introducing a new variable y:

$$\alpha_{tV}(\rho) \rightarrow (1-y)\alpha_{tV}(\rho) + y\alpha_{tV}(\rho_0)$$

cf. T. Inakura, H. Nakada, Phys. Rev. C 92, 064302 (2015)



Two definitions of the "spin skin thickness"

1. $\Delta r_s = (\text{rad. of } 12^+ \text{ isomer}) - (\text{rad. of } 0^+ \text{ g.s.})$

2. $\Delta r'_s = r_+ - r_-$

   $r_\pm$: rms radius of $\rho_\pm = \langle \psi^\dagger (1 \pm \Sigma_3) \psi \rangle / 2$,
   the spin-up (-down) nucleon density.

$\Delta r'_s$ is determined only by the properties of 12^+ state of ^{52}Fe.

Δr_s : positive but weak correlation

cf. $\Delta R_{\rm exp} \approx 0.003$ fm

A. R. Vernon, et al., PRL 134 252501 (2025)

$\Delta r'_s$: positive and stronger correlation

However, how to observe?

Summary

- Correlation between slope parameter and spin slope parameter
  - a <u>negative correlation</u> in the case of $\Delta_n = \Delta_p = \Delta$
  - L_s is <u>nearly independent</u> of L in the case of $\Delta_n = -\Delta_p = \Delta$
- Correlation between spin skin thickness Δr_s ($\Delta r'_s$) and spin slope parameter L_s
  - Δr_s : the difference in radii between 0^+ ground state and 12^+ isomer state
  - $\Delta r'_s$: the difference in radii between spin-up and spin-down nucleon in 12^+ state
  - a <u>positive but weak correlation</u> between Δr_s and L_s
  - a <u>positive and stronger correlation</u> between $\Delta r'_s$ and L_s
- Future work
  - Calculating high-spin isomer state of other nuclei
  - Including the paring correlation in the calculation of high-spin isomers