On the Quantum Nature of Nuclear Shapes in Heavy-ion Collisions

Intersection of nuclear structure and high-energy nuclear collisions 2026
Yukawa Institute for Theoretical Physics, Apr/20/2026, (online)

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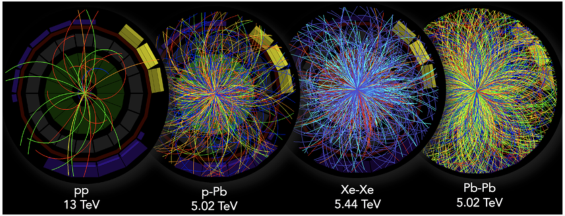
Based on WK 2509.09549 + updates (Y. Bao, WK)

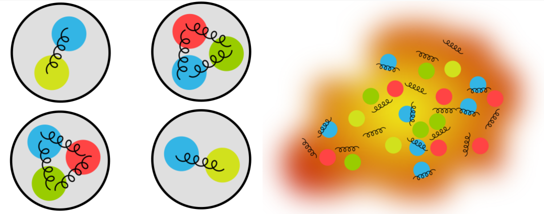
Outline

I. A short introduction of nuclear structures in HIC

Relativistic heavy-ion collisions



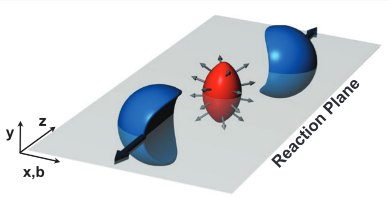
[ALICE event displays from p-p to Pb-Pb]

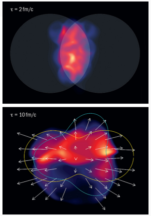


[From hadrons to deconfined quarks and gluons]

- Huge inelasticity: head-on Pb-Pb collision at $\sqrt{s} = 5.02$ TeV yields $>$ 30,000 final particles. Quarks and gluons d.o.f. excited during the collisions.
- A quark-gluon plasma (QGP) is formed: a strongly-coupled, expanding system. Typical numbers for Pb-Pb at LHC: $T_0 \sim 0.5$ GeV, $\tau \sim L_T \sim 10\ \mathrm{fm}/c$.

Collective response bridges initial and final states





$$(e+P)\frac{D\vec{u}}{dt} = -\nabla P + \eta\nabla^2\vec{u}, \quad P = P(e)$$

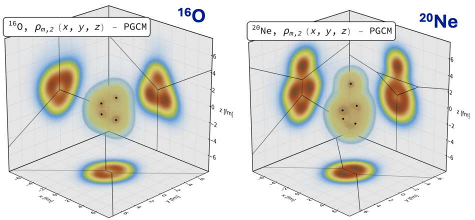
$$\epsilon_n = \frac{\langle x_\perp^n e^{in\phi_x}\rangle}{\langle x_\perp^n\rangle}.$$

$$v_n = \langle e^{in(\phi_p - \Psi_n)}\rangle.$$

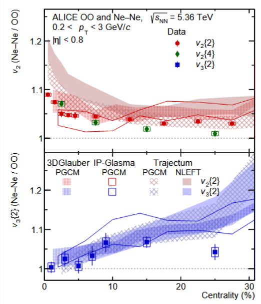
- Final-state collectivity: frequent interactions drive the system towards local equilibrium, dynamics well described by the evolution of local energy density and flow velocity, etc $\Rightarrow$ hydrodynamics
- The produced QGP is eccentric in space ($\epsilon_n \neq 0$). Collective expansion transform that into anisotropy in momentum space ($v_n \neq 0$).
- The response $v_n \approx k_n\epsilon_n + c^n_{m\ell}\epsilon_m\epsilon_\ell$ bridges initial and final-state information.

Impact of nuclear structure in heavy-ion collisions

- In ultra-central collisions of spherical nucleus, ϵ_n come from fluctuations of nucleons positions & subnucleonic d.o.f. $\epsilon_n \propto 1/\sqrt{A}$
- Deviations from this expectation encodes nuclear shape information (as compared to a spherical, independent-nucleon model).
- Collective response of QGP enable the study of nuclear structure impacts in HICs.

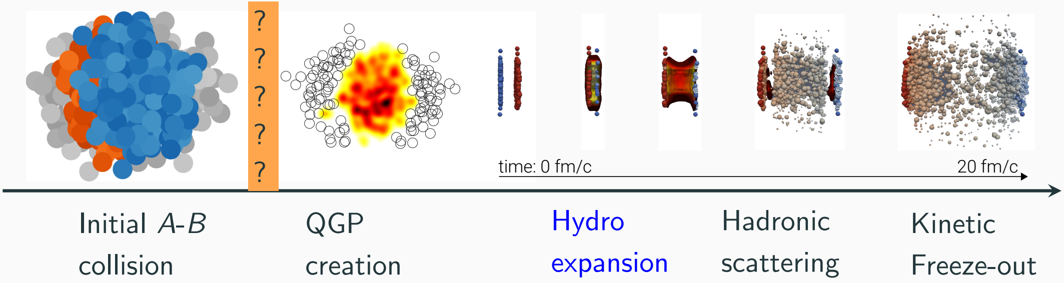


Nucleon densities of ^{16}O and ^{20}Ne showing deformed geometries with well-separated structures. In ^{16}O they form an irregular tetrahedron with two short and two long edges. For ^{20}Ne, one sees a characteristic bowling-pin-like ^{16}O+α.
Ref: Giacalone et al., PRL, 135, 012302 (2025)



[ALICE 2509.06428]

However, there are also fundamental concerns...



- From $|A\rangle \otimes |B\rangle$ to QGP fireball heavily relies on the Monte-Carlo Glauber model.
- Is that enough to capture the quantum nature of the initial state?

However, there are also fundamental concerns...

Regard the quantum-many-body nature of nuclei and the MC modeling in HICs, several questions are raised [Dobaczewski, Gade, Godbey, Janssens, Nazarewicz, Phys.Rev.Res.7(2025)043159.]

Relevant questions for this talk (my personal rephrasing)

- The nuclear ground state (a stationary state) respects rotational symmetry, what is the notion of shape in a relativistic collision?
- Intrinsic shape is a model construct, can it be used in HIC modeling?
- What is driven the fluctuation of QGP event-by-event geometry?

II. The “intrinsic shape” of a nuclei

Build a deformed nuclei: step 1, an independent-particle picture

- A common starting point: find a single-particle basis under mean field, solve the single-particle problem

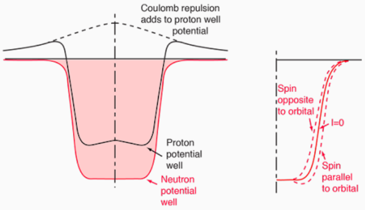
$$\hat{H}_1|\psi_n\rangle = e_n|\psi_n\rangle$$

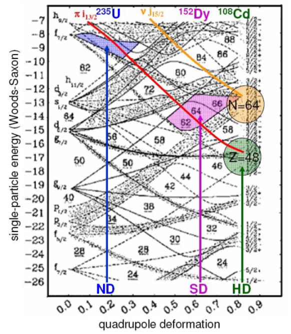
- Nucleons are fermions, building blocks are Slater determinants $\det\{\psi_i(r_j)\}$:

$$\langle r_1, \cdots r_N|\Psi_N\rangle = \begin{pmatrix} \psi_1(r_1) & \psi_1(r_2) & \cdots & \psi_1(r_N) \\ \psi_2(r_1) & \psi_2(r_2) & \cdots & \psi_2(r_N) \\ \cdots & \cdots & \cdots & \cdots \\ \psi_N(r_1) & \psi_N(r_2) & \cdots & \psi_N(r_N) \end{pmatrix}$$

- Sometimes, to get an "energy" efficient basis, single-particle mean-field is chosen to be deformed.

[Fig. from H.-J. Wollersheim's Lecture, https://web-docs.gsi.de/~wolle]





Build a deformed nuclei: step 2, introduce many-body correlations

- A deformed mean field can be parametrized by some labels (collective coordinates): orientation Ω, radius R_0, deformations β_2, γ, β_3, etc,

$$V(r,\theta,\phi) = \frac{1}{1+\exp\left(\frac{r-R(\theta,\phi)}{a}\right)}\,,\;\; R(\theta,\phi) = R_0\left[1+\beta_2 Y_{20}(\theta,\phi)+\cdots\right]$$

- To go beyond independent particle approximations, one consider an efficient way to include many-body correlation: superposing collective coordinates.
- The ground state can be approximated by linear combinations of

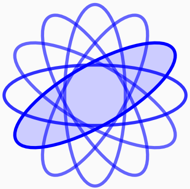
$$|A\rangle = \sum_{\Omega,\beta_2,...} C_{\Omega,\beta_2,...}|\Psi_N;\Omega,\beta_2,...\rangle$$

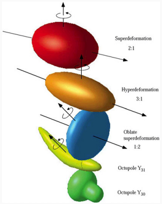
Coefficients $C_{\Omega,\beta_2,...}$ are varied to minimize energy $\langle A|\hat{H}|A\rangle$, subject to constraints: 1) $\langle\psi|\psi\rangle = 1$, and 2) **preserve necessary symmetry**!

Constraints from rotational symmetry of the Hamiltonian

- It is hard to make general statement on the β_n superposition, but superposition in Ω is completely dictated by rotational symmetry, because $[\hat{H}, \hat{\mathbf{J}}] = 0$.

$$|A\rangle \sim \sum_{\Omega} \sum_{\beta_2, \beta_3, \ldots} C_{\Omega, \beta_2, \beta_3, \ldots} |\Omega, \beta_2, \beta_3\rangle,$$





- The known procedure to build $|JM\rangle$ state from deformed configurations (Axial) at orientation Ω [P. Ring and P. Schuck, *The Nuclear Many-Body Problem*] :

$$|A, JM\rangle = \hat{P}^J_{MK} |\Omega\rangle = \frac{2J+1}{8\pi^2} \int_{\alpha,\beta,\gamma} (D^J_{MK})^* (\alpha, \beta, \gamma) \hat{R}(\alpha, \beta, \gamma) |\Omega, \beta\rangle \,,$$

$$\text{For spin-0 state: } |00\rangle \propto \int d\Omega |\Omega, \beta\rangle .$$

Intrinsic shape is a label of the building block $|\Omega, \beta\rangle$ of the full wave function $|A\rangle$.

III. “Shape” in nuclear reactions

Deformation in transitional process $|n\rangle \to |n'\rangle$

- Suppose the ground state is exited to some other states by a weak external EM field, the transitional matrix element is $\langle n|\hat{Q}|00\rangle$. Sum over all possibility

$$P_{\text{excitation}} = \sum_{n\neq 0} P_n = \sum_{n\neq 0} \langle 00|\hat{Q}^\dagger|n\rangle\langle n|\hat{Q}|00\rangle = \langle 00|\hat{Q}^\dagger \hat{Q}|00\rangle$$

$$\text{note that } \langle 00|\hat{Q}|00\rangle \text{ vanishes for spin-1 operator}$$

- EM fields couple to each nucleon individually $\hat{Q} = \sum_i \hat{q}_i$, so $\hat{Q}^\dagger \hat{Q}$ is a two-body operator. One needs correlation to "see" the impact of deformation!

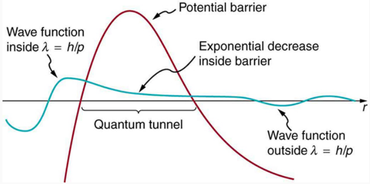
$$|\mathcal{N}|^2 \int d\Omega \int d\Omega' \langle \Omega', \beta|\hat{Q}^\dagger \hat{Q}|\Omega, \beta\rangle$$

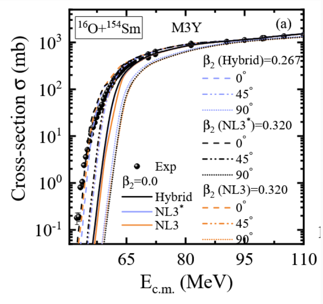
[Thanks to Jiangming Yao's explanation.]

- UPC events are also probing transitional type of matrix elements.

Impact of deformation in sub-threshold fusion

- Incident energy below the Coulomb barrier.
- Coulomb interaction: long range; Strong interaction: short range





$$e^{-iH_{\mathrm{eff}}t}\sum_{\Omega}|\Omega(t_0)\rangle \approx \sum_{\Omega} e^{-B(\Omega)}|\Omega(t)\rangle$$

[From my naïve understanding...]

[Rana, Bhuyan, Kumar, Carlson PRC110(2024)024601]

- Components of tunneling amplitude is sensitive to deformation and orientation.
- Before short-range force take effect, long-range force selects preferred components.

Go to higher energy

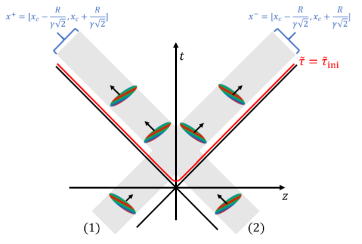


Fig. from H. Matsuda, X.-G. Huang, PRD108(2023)114008

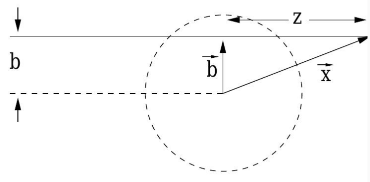
- At higher energy, all components pass the Coulomb barrier. Dynamical phases may causes complicated interference between different orientations. $e^{-iH_{\mathrm{eff}}t}\sum_\Omega |\Omega(t_0)\rangle \approx \sum_\Omega e^{-iB(\Omega)}|\Omega(t)\rangle$
- At ultra-relativistic energy, things gets simplified again.
  - $v \to c$, all interactions take place at the same time.
  - Coulomb field cannot know the shape of the other nucleus before the contact.
  - At $\tau = 0$ of collision, $|00\rangle = \mathcal{N}\int d\Omega|\Omega, \beta\rangle$

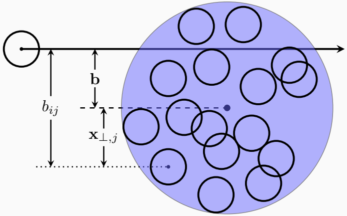
How to model nuclear deformation in HIC (Monte-Carlo Glauber model)

- One should use ground-state nuclear density matrix in the **Lab** frame.
- Collisions with a fast local entropy production (τ_0) $\Rightarrow$ a simultaneous measurement of the transverse position ($x_\perp$) of participant nucleons at τ_0. Remark: not a projective measurement on the collective coordinates (Ω, β).
- In ultra-central collisions, the detectable shape of QGP is due to the many-body correlations + fluctuations.

Outline

The eikonal limit of high-energy interaction



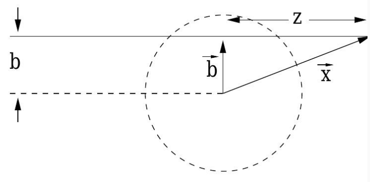


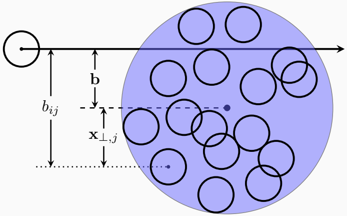
At high energy, S matrix of elastic processes only depends on impact parameter **b**

$$f_{AB}(\mathbf{q}) = \frac{ik}{2\pi} \int d^2\mathbf{b} e^{-i\mathbf{q}\cdot\mathbf{b}} (1 - \underbrace{e^{i\chi_{AB}(\mathbf{b})}}_{S_{AB}(\mathbf{b})}).$$

Inelasticity is reflected in the imaginary part of elastic phase $\chi_{AB} = \mathfrak{Re}\chi_{AB} + i\mathfrak{Im}\chi_{AB}$

Nuclear collisions in the high-E limit





Scattering theory relates $\sigma_{AB}^{\mathrm{el}}$, $\sigma_{AB}^{\mathrm{inel}}$, $\sigma_{AB}^{\mathrm{tot}}$ to χ_{AB}.

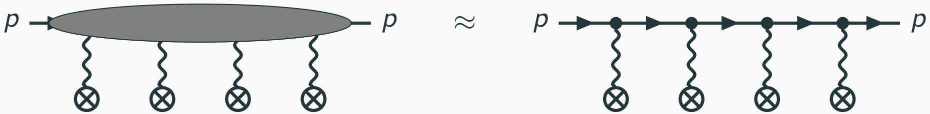
$$f_{AB}(\mathbf{q}) = \frac{ik}{2\pi} \int d^2\mathbf{b} e^{-i\mathbf{q}\cdot\mathbf{b}} (1 - \underbrace{e^{i\chi_{AB}(\mathbf{b})}}_{S_{AB}(\mathbf{b})}).$$

Elastic cross section	$\sigma_{AB}^{\mathrm{el}} = \int \frac{d^2\mathbf{q}}{k^2} \lvert f_{AB}(\mathbf{q}) \rvert^2$
Total cross section	$\sigma_{AB}^{\mathrm{tot}} = \frac{4\pi}{k} \mathfrak{Im} f_{AB}(\mathbf{q} = 0)$
Inelastic cross section	$\sigma_{AB}^{\mathrm{inel}} = \int d^2\mathbf{b} (1 - \lvert e^{i\chi_{AB}(\mathbf{b})} \rvert^2)$

The Glauber's theory for $A+B$ nuclear interactions

The elastic channel of S matrix of AB collision is factorized in binary NN elastic collisions [*Lectures in Theoretical Physics Vol. 1: R.J. Glauber, High-Energy Collision Theory*]

For example, for the phase accumulated by in p-A collisions

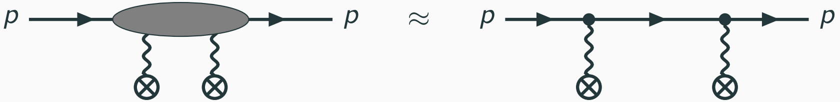


For A-B interaction, generalize to

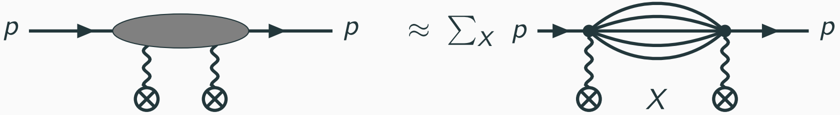
$$e^{i\chi_{AB}(\mathbf{b})} = \langle A|\langle B| \underbrace{\prod_{a=1}^{A}\prod_{b=1}^{B}\left[1-\Gamma_{NN}(\hat{x}_{a,\perp}-\hat{x}_{b,\perp}-\mathbf{b})\right]}_{\hat{S}(\mathbf{b})} |A\rangle|B\rangle$$

The Gribov correction to the Glauber's theory (to do)

- In the Glauber model, the elastic amplitude from multiple scattering, e.g., $p + N_1 + N_2 \to p + N_1 + N_2$, is factorized into products of elastic amplitude:

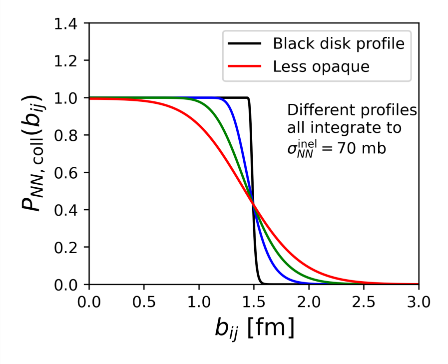


- The Gribov correction: proton is allowed to fluctuates to other intermediate states. Can be important for small systems. But often neglected when describing fireball geometry in $A{+}B$ with significant overlap.



- Can be included in the following study.

The total inelastic cross section in Glauber's theory



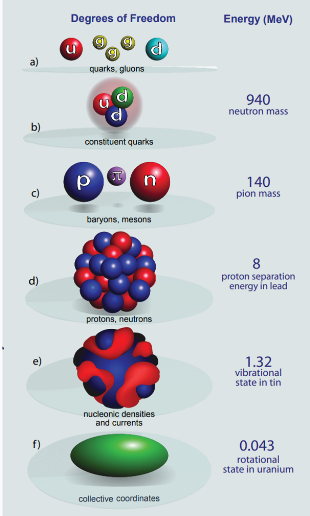
- From scattering theory, the cross-section to any inelastic scattering is

$$\sigma_{AB}^{\text{all inel}} = \int d^2\mathbf{b}\left(1 - \left|\langle A|\langle B|\hat{S}(\mathbf{b})|A\rangle|B\rangle\right|^2\right).$$

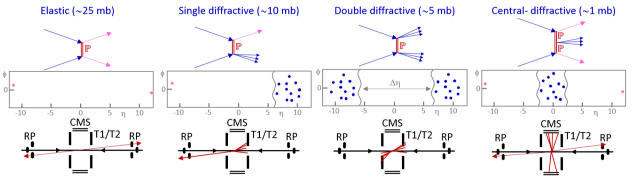
- A-B scattering is reduced to N-N scattering $S(b) = \prod_{a,b}\left[1 - \Gamma_{NN}(\Delta\hat{x}_{ab} - \mathbf{b})\right]$.
- The N-N interaction profile function $\Gamma_{NN}(b)$ is often parametrized as

$$\Gamma_{NN}(b) = \frac{1-it}{2}P_{NN}^{\text{coll}}(b), \quad \int d^2\mathbf{b}P_{NN}^{\text{coll}}(b) = \sigma_{pp}^{\text{tot}}(\sqrt{s}).$$

Select events that breaks the nucleon or the nucleus



- $Q > 1$ GeV: produce quarks and gluons.
- 140 MeV: produce mesons and resonances.
- 8 MeV: nucleon separation energy.
- From 10 keV to 1 MeV: collective excitation.



E.g., elastic & diffractive processes [M. Csanad, 1312.3803]

Minimum-bias events in *pp*: almost all non-diffractive events.

Minimum-bias events in *AB*: non-single diffractive events.

Only focus on inelastic collisions that destroys nucleons (to create QGP!)

- We focus on events where nucleon d.o.f. are destroyed (experimental trigger).

$$\sigma_{AB}^{\mathrm{inel},*} = \int d^2\mathbf{b}\Big\{1 - \underbrace{\Big|\langle A|\langle B|\hat{S}(\mathbf{b})|A\rangle|B\rangle\Big|^2}_{\text{Remain in ground state}} - \underbrace{\sum_{A^*,B^*}\Big|\langle A^*|\langle B^*|\hat{S}(\mathbf{b})|A\rangle|B\rangle\Big|^2}_{\text{Transition to } \geq 1 \text{ excited nuclei/unbound nucleons}}\Big\}$$

$$= \int d^2\mathbf{b}\Big\{1 - \langle A|\langle B|\hat{S}^\dagger(\mathbf{b})\sum_{X_A}|X_A\rangle\langle X_A| \otimes \sum_{Y_B}|Y_B\rangle\langle Y_B|\hat{S}(\mathbf{b})|A\rangle|B\rangle\Big\}$$

X_A, Y_B contains all final states of nuclei & nucleons. They form completeness relation in the Hilbert space of "nuclear structure physics"

- The cross section only involves one power of nuclear density matrices in the Lab frame. Good for a probabilistic (Monte-Carlo) interpretation

$$\sigma_{AB}^{\mathrm{inel},*} = \int d^2\mathbf{b}\left(1 - \langle A|\langle B|\hat{S}^\dagger(\mathbf{b})\hat{S}(\mathbf{b})|A\rangle|B\rangle\right) = \int d^2\mathbf{b}\left(1 - \mathrm{Tr}\{\hat{\rho}_A\hat{\rho}_B|\hat{S}(\mathbf{b})|^2\}\right).$$

Expand the non-diffractive cross-section in many-body operators

$$
\begin{aligned}
\sigma_{AB}^{\mathrm{inel},*} &= \int d^2\mathbf{b}\left(1-\mathrm{Tr}\{\hat{\rho}_A\hat{\rho}_B|\hat{S}(\mathbf{b})|^2\}\right)\\
&= \int d^2\mathbf{b}\left(1-\mathrm{Tr}\{\hat{\rho}_A\hat{\rho}_B\prod_{i\in A, j\in B}(1-\Gamma(\hat{x}_i-\hat{x}_j-\mathbf{b}))\}\right)\\
&= \underbrace{\int d^2\mathbf{b}\sum_{(ij)}\mathrm{Tr}\{\hat{\rho}_A\hat{\rho}_B\Gamma(\hat{x}_i-\hat{x}_j-\mathbf{b})\}}_{\text{“at least a pair of } NN \text{ collide”}} - \int d^2\mathbf{b}\sum_{(ij)\neq(k\ell)}\mathrm{Tr}\{\hat{\rho}_A\hat{\rho}_B\Gamma(\hat{x}_i-\hat{x}_j-\mathbf{b})\}\Gamma(\hat{x}_k-\hat{x}_\ell-\mathbf{b})\} + \cdots
\end{aligned}
$$

- For each nucleus: a sum of one-body, two-body, until A-body operators.
- In calculations using the full wave-functions, e.g., the Complete Glauber calculation [S. Hatakeyama, W. Horiuchi, NPA985(2019)20-37], the cross section depends on such many-body correlations.
- But to study shape of QGP, one needs more information than cross-sections.

Outline

I. A short introduction of nuclear structures in HIC

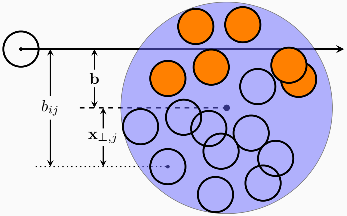
II. The "intrinsic shape" of a nuclei

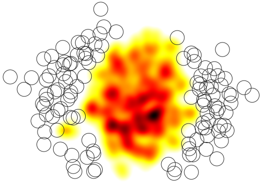
III. "Shape" in nuclear reactions

- Glauber Model and Nuclear Density Matrix
- Entropy production and decoherence of nucleon position
- From intrinsic shape to the detectable shape

IV. Phenomenological Impact of Quantum Superposition

EbE Fireball geometry relies on a probability interpretation of $\sigma_{AB}^{\mathrm{inel}}$



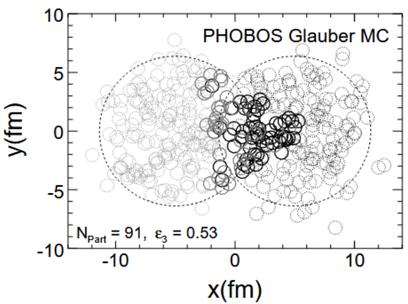


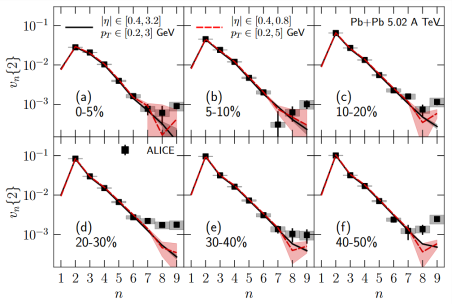
- Glauber's original paper considers $p{+}A$ with an independent nucleon approximation $\rho_A \approx \prod_{i=1}^{A} \rho_i(x_i)$.
- A probabilistic picture of p-A in terms of N-N collisions:

$$P_{pA}^{\mathrm{coll}}(\mathbf{b}) = 1 - \mathrm{Tr}\{\hat{\rho}_A |\hat{S}(\mathbf{b})|^2\} \approx 1 - \prod_{n=1}^{A} \left(1 - P_{\mathrm{NN}}^{\mathrm{coll}}(\mathbf{x}_n - \mathbf{b})\right) .$$

- For event-by-event A-B collisions, sample nucleons from $\rho_A(x_1, \cdots, x_A)$, $\rho_B(x_1, \cdots, x_B) \to$ granularity.
- But notice that this EbE interpretation is beyond the Glauber theory, which only calculates cross section!

Strong experimental evidence that this interpretation is physical





- Nucleon position EbE fluctuations are required to explain higher-order flow [B. Alver, G. Roland, PRC81(2010)054905] .
- Flucuating initial condition + hydro response provide a good agreement up to $n = 7$ order flow [O. Horecny, C. Shen 2506.02930]

Inelasticity & decoherence of participant nucleons

- No later than the hydrodynamization time τ_0, information of bulk dynamics are specified by $T^{\mu\nu}(\tau_0, \mathbf{x}) = \langle\psi(\tau_0)|\hat{T}^{\mu\nu}(\mathbf{x})|\psi(\tau_0)\rangle$.
- In eikonal limit, collision only depends on transverse location of incoming nucleons. Destroy nucleon a, b, create other d.o.f.

$$|\psi(\tau_0)\rangle = \sum_X |X\rangle\langle X| e^{-i\sum_k (\hat{\mathcal{C}}_k(\hat{x}_a,\hat{x}_b)\hat{\mathcal{O}}_k + h.c.)\tau_0} |P_a\rangle|P_b\rangle|0\rangle_{\text{other dof}}$$
$$= \sum_X |X\rangle \int d\mathbf{x}_a d\mathbf{x}_b S_X(\mathbf{x}_a, \mathbf{x}_b)\phi_a(\mathbf{x}_a)\phi_b(\mathbf{x}_b)$$

- Evaluate the expectation value

$$T^{\mu\nu}(\tau_0, \mathbf{x}) = \int d\mathbf{x}'_a d\mathbf{x}_a d\mathbf{x}'_b d\mathbf{x}_b \phi^*_a(\mathbf{x}'_a)\phi_a(\mathbf{x}_a)\phi^*_b(\mathbf{x}'_b)\phi_b(\mathbf{x}_b)$$
$$\sum_{X,Y} S^*_Y(\mathbf{x}'_a, \mathbf{x}'_b) S_X(\mathbf{x}_a, \mathbf{x}_b)\langle Y|\hat{T}^{\mu\nu}(\mathbf{x})|X\rangle$$

Inelasticity & decoherence of participant nucleons

- Assume weakly-coupled particles $\hat{T}^{\mu\nu} = \sum_i \frac{p^\mu p^\nu}{p^0}$, then $\langle Y|\hat{T}^{\mu\nu}(\mathbf{x})|X\rangle \propto \delta_{X,Y}$
- If many d.o.f. are excited, assume decoherence,

$$\sum_{X,Y} S_X^*(\mathbf{x}'_a, \mathbf{x}'_b) S_X(\mathbf{x}_a, \mathbf{x}_b)\delta_{X,Y} \to \delta(\mathbf{x}'_a - \mathbf{x}_a)\delta(\mathbf{x}'_b - \mathbf{x}_b)\sum_X$$

- Specify the expectation value $T^{\mu\nu}$ only requires the probability distribution of individual nucleons $T^{\mu\nu} = |\phi_a(\mathbf{x}_a)|^2|\phi_b(\mathbf{x}_b)|^2 \sum_X \langle X|\hat{T}^{\mu\nu}(\mathbf{x})|X\rangle$.

Not a proof, but to see what approximations are needed. Hopefully, to give an operator definition of what properties of nuclear wave function is probed.

High-E collisions are not measuring the eigenvalue of "intrinsic shape"

- The old/classical-intuitive picture

$$\sum_{\Omega,\beta} C_{\Omega,\beta}|A;\Omega,\beta\rangle \xrightarrow[\text{generalized coordinates}]{\text{Collapse on}} |A;\Omega^*,\beta^*\rangle \xrightarrow[\text{nucleon positions}]{\text{Collapse on}} |\langle\{x_i\}|A;\Omega^*,\beta^*\rangle|^2$$

- The physical picture is simpler

$$\sum_{\Omega,\beta} C_{\Omega,\beta}|A;\Omega,\beta\rangle \xrightarrow[\text{nucleon positions}]{\text{Collapse on}} \sum_{\Omega',\beta'}\sum_{\Omega,\beta} C^*_{\Omega',\beta'} C_{\Omega,\beta}\langle A;\Omega',\beta'|\{x_i\}\rangle\langle\{x_i\}|A;\Omega,\beta\rangle$$

1. High-E collisions are not measuring the eigenvalues of generalized coordinates. Its information is retained when many nucleon wavefunction collapses.
2. Geometry of QGP must contain off-diagonal information of generalized coordinates!

Outline

Let's consider a minimum set-up for deformed nuclei

- Approximated deformed basis by a Slater determinant $\langle \{r_i\} | A, \hat{\mathbf{z}}, \beta_n \rangle = \det \{\phi_i(r_j)\}$.
- To form a spin-0 state, the superposition is isotropic $|A\rangle = \frac{1}{N_A} \int d\Omega |A, \Omega, \beta_n\rangle$.
- Examine the collision probability

$$
\begin{aligned}
P_{AB}^{\text{coll}}(\mathbf{b}) &= \langle A|\langle B|1 - |\hat{S}(\mathbf{b})|^2|A\rangle|B\rangle \\
&= \int d\Omega_A d\Omega'_A d\Omega_B d\Omega'_B \langle \Omega'_A|\langle \Omega'_B| \left[1 - |\hat{S}(\mathbf{b})|^2\right] |\Omega_A\rangle|\Omega_B\rangle
\end{aligned}
$$

and see if we can identify a probabilistic interpretation of the form $1 - \prod_{ij}(1 - p_{ij})$.

Re-interpret the cross section in terms of probabilistic N-N interactions

$$P_{AB}^{\rm coll}(\mathbf{b}) = \int d\Omega_A d\Omega'_A d\Omega_B d\Omega'_B \langle\Omega'_A|\langle\Omega'_B|\left[1-|\hat{S}(\mathbf{b})|^2\right]|\Omega_A\rangle|\Omega_B\rangle$$

$$= \int d\Omega_A d\Omega'_A d\Omega_B d\Omega'_B \langle\Omega'_A|\Omega_A\rangle\langle\Omega'_B|\Omega_B\rangle \frac{\langle\Omega'_A|\langle\Omega'_B|\left[1-|\hat{S}(\mathbf{b})|^2\right]|\Omega_A\rangle|\Omega_B\rangle}{\langle\Omega'_A|\Omega_A\rangle\langle\Omega'_B|\Omega_B\rangle}$$

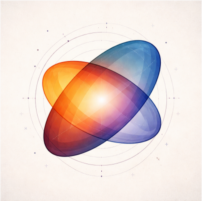
$$= \underbrace{\int d\Omega'_A d\Omega_A d\Omega'_B d\Omega_B}_{\text{Symmetry, off-diagonal}} \times \underbrace{D(\Omega_A,\Omega'_A)D(\Omega_B,\Omega'_B)}_{\text{Decorrelation of off-diagonal ele.}} \times \underbrace{P_{AB}^{\rm inel}(\mathbf{b},\Omega_A,\Omega'_A,\Omega_B,\Omega'_B)}_{\text{Coll. prob. for given configurations}}$$

- Why guessing this specific reorganization? Because after factoring out the overlap function, the first term in $P_{AB}^{\rm inel}(\mathbf{b},\Omega_A,\Omega'_A,\Omega_B,\Omega'_B)$ starts with 1.
- For Slater determinant of even-even nucleus with axial deformation, $D(\Omega,\Omega')$ is real. Positivity?

How important is the off-diagonal effect? A simple estimate:

1. Gaussian approximation for the **decorrelation function**:

$$D(\theta) = \frac{1}{|N_A|^2}\langle A,\theta|A,0\rangle = \frac{1}{|N_A|^2}\det\left\{\langle\phi_i|e^{-i\theta\hat{j}_y}|\phi_j\rangle\right\}$$
$$\approx \frac{1}{|N_A|^2}e^{\mathrm{Tr}_A \ln\langle\phi_i|1-i\theta\hat{j}_y-\frac{1}{2}\theta^2\hat{j}_y^2|\phi_j\rangle}$$
$$\approx \frac{1}{|N_A|^2}e^{-\frac{\theta^2}{2}\mathrm{Tr}\left(\hat{j}_y^2\hat{P}_A-\hat{P}_A\hat{j}_y\hat{P}_A\hat{j}_y\right)}$$

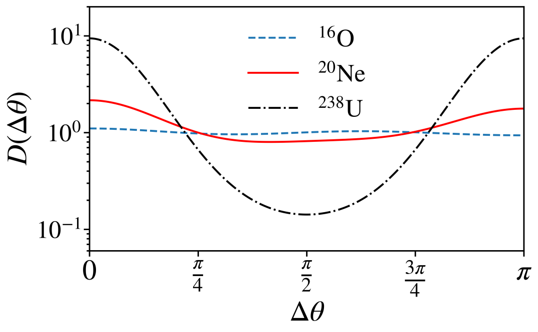


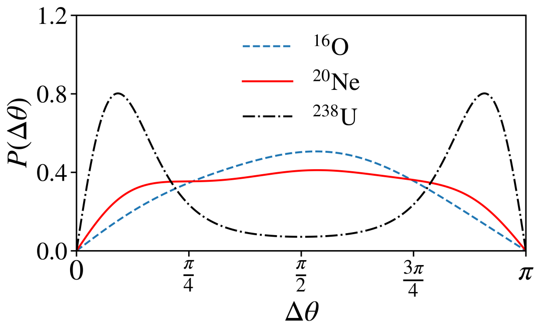
2. Further approximate with just the **one-body density operator in intrinsic frame**

$$D(\theta) \approx \frac{1}{|N_A|^2}\left(\mathrm{Tr}\left\{e^{-i\theta\hat{j}_y}\sqrt{\hat{\rho}_1}e^{i\theta\hat{j}_y}\sqrt{\hat{\rho}_1}\right\}\right)^{A/2}, \text{ stronger decorrelation for large nucleus.}$$

3. Approximate $\hat{\rho}_1$ by mean-field value $\rho_1 = \langle\hat{\rho}_1\rangle \to$ **one-body density distribution in the intrinsic frame**.

How important is the off-diagonal effect? A simple estimate:





- Left: the de-correlation function for O, Ne, U.
- Right: the sampling probability includes the Jacobian $P(\theta) \propto D(\theta)\sin\theta$.
- It will be beneficial to compare this simplified $D(\theta)$ with first-principle calculation.

The orientation-dependent collision probability

For each nuclei, we sample two sets of collective coordinates Ω and Ω', then there is still an approximate probabilistic description (proof for pA ✓, working on the AB case)

$$P_{AB}^{\mathrm{inel}}(\mathbf{b}, \Omega_A, \Omega'_A, \Omega_B, \Omega'_B) \approx 1 - \prod_{a,b} \left[1 - P^{ab}_{\Omega_A \Omega'_A; \Omega_B \Omega'_B}(\mathbf{b}) \right].$$

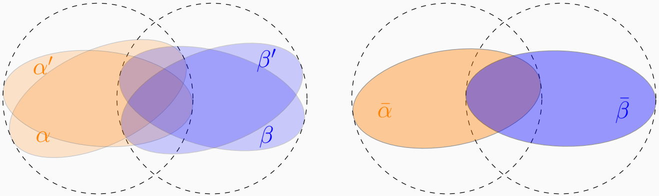
The modified nucleon binary collision probability is

$$P^{ab}_{\Omega_A \Omega'_A; \Omega_B \Omega'_B}(\mathbf{b}) = \int d^2\mathbf{x}_a \int d^2\mathbf{x}_b P_{\mathrm{NN}}^{\mathrm{inel}}(\mathbf{x}_a - \mathbf{x}_b - \mathbf{b}) T_A(\mathbf{x}_a; \Omega_A, \Omega'_A) T_B(\mathbf{x}_b; \Omega_B, \Omega'_B)$$

with the modified one-nucleon thickness function

$$T_A(\mathbf{x}; \Omega_A, \Omega'_A) = \frac{1}{T_{\mathrm{norm}}} \int_{-\infty}^{\infty} dz \sqrt{\rho_A}(\mathbf{r}; \Omega_A) \sqrt{\rho_A}(\mathbf{r}; \Omega'_A).$$

Reduction to classical picture for large nuclei



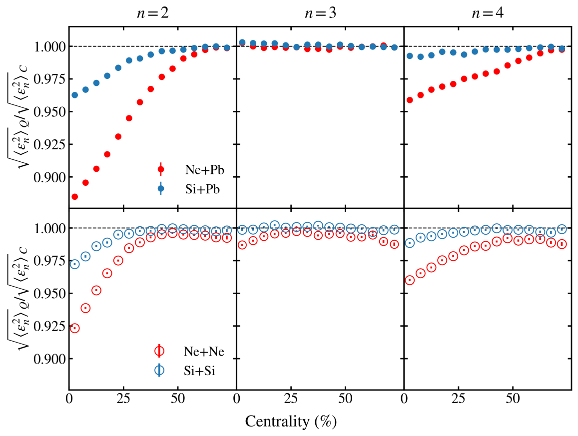
I. Re-interpretation of quantum formula: thickness function contains non-diagonal information in collective coordinates.

II. Semi-classical $A, B \gg 1$: Approximate the thickness functions with only the diagonal terms in the collective coordinates.

- For A, B sufficiently large, the decorrelation approaches $D(\Omega, \Omega') \to \frac{\delta(\Omega - \Omega')}{4\pi}$.
- The modified MC Glauber reduce to traditional MC Glauber for deformed nuclei.
- In general, the superposition effect smears out the intrinsic shape.

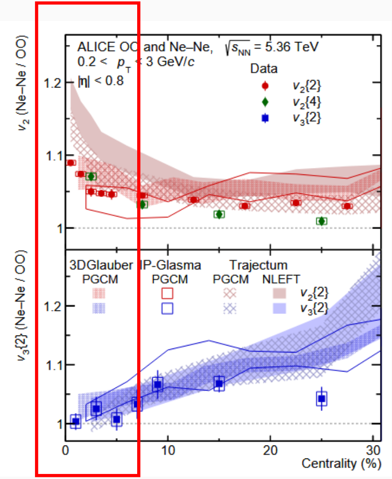
IV. Phenomenological Impact of Quantum Superposition

Phenomenological impact: eccentricity



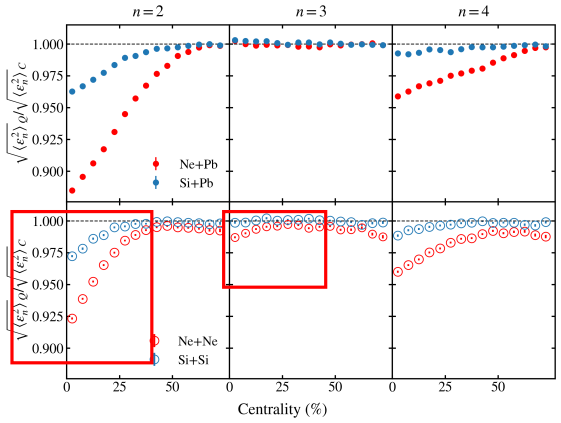
- Central Ne-Pb @ 70 GeV: 12% reduction of $\langle \epsilon_2^2 \rangle^{1/2}$ relative to classical case.
- Central Ne-Ne @ 5.36 TeV: 7.5% reduction of $\langle \epsilon_2^2 \rangle^{1/2}$.
- Effect less important for ϵ_3.
- This estimation uses TRENTo's energy deposition ansatz with $p = 0$.

Central Ne-Ne and O-O



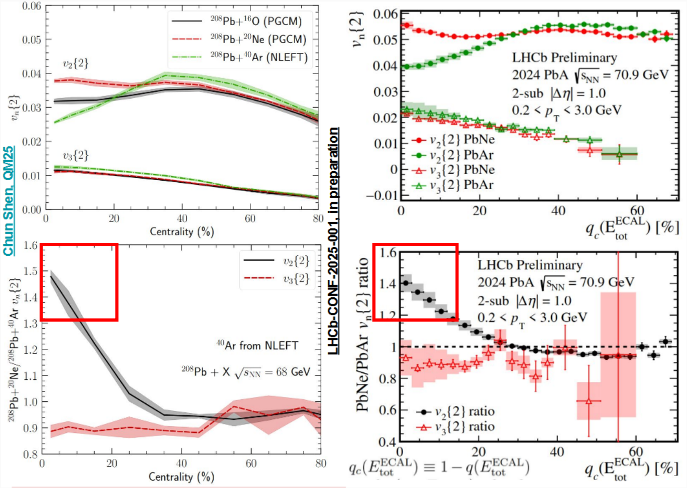
[△ ALICE, 2509.06428]

Superposition suppresses classical treatment of ϵ_2 ratio in central collisions by 7.5%. For ϵ_3, only 2%.

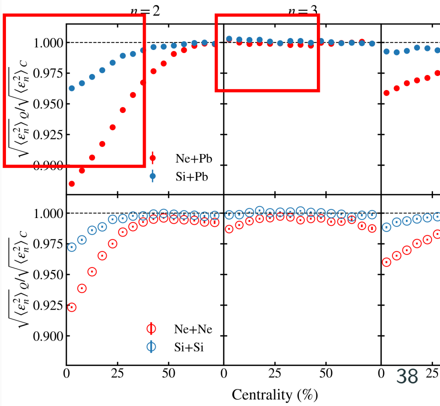


[See also ATLAS, 2509.05171, CMS, CMS-PAS-HIN-25-009]

Fixed-target Ne-Pb and Ar-Pb

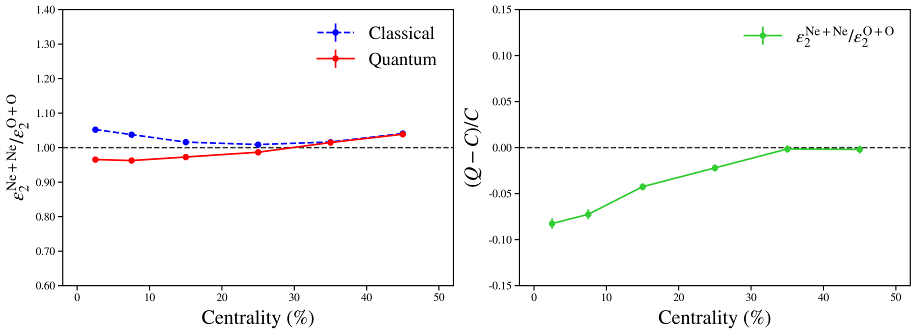


Superposition effect can suppress ϵ_2 ratio in central Ne-Pb up to 12%. For ϵ_3, only affects $< 1\%$.



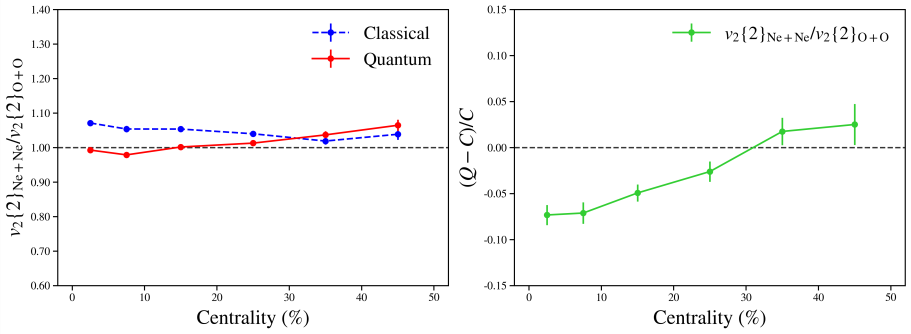
[From talk by S. Mariani for the LHCb Collaboration at IS2025.]

Including hydrodynamic + UrQMD in the final state: small-on-small



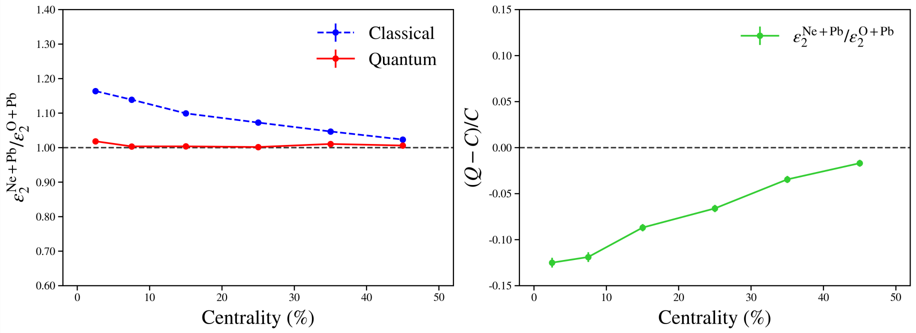
- Left: eccentricity ratio of Ne+Ne over O+O, estimated using classical mixture v.s. quantum superposition.
- Right: change in percentage in the predicted ratio.

Including hydrodynamic + UrQMD in the final state: small-on-small



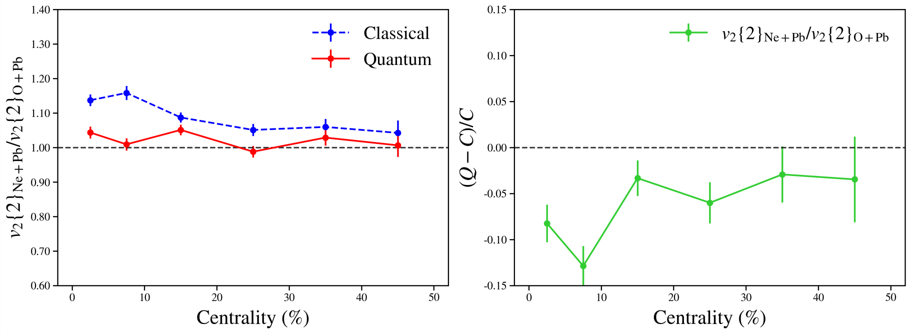
- Left: v_2 ratio of Ne+Ne over O+O, estimated using classical mixture v.s. quantum superposition.
- Right: change in percentage in the predicted ratio.

Including hydrodynamic + UrQMD in the final state: small-on-large



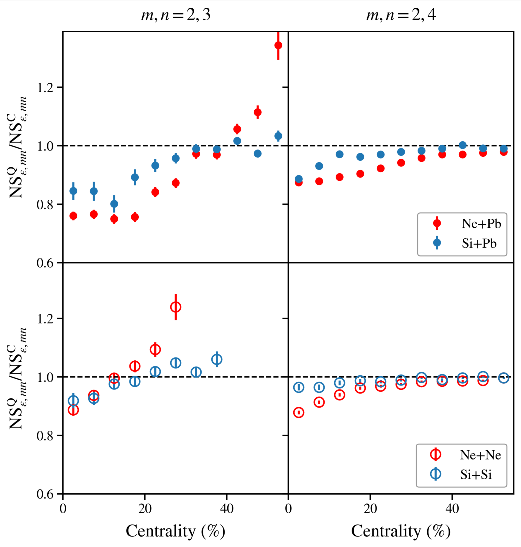
- Left: eccentricity ratio of Ne+Pb over O+Pb, estimated using classical mixture v.s. quantum superposition.
- Right: change in percentage in the predicted ratio.

Including hydrodynamic + UrQMD in the final state: small-on-large



- Left: v_2 ratio of Ne+Pb over O+Pb, estimated using classical mixture v.s. quantum superposition.
- Right: change in percentage in the predicted ratio.

What can be further tested: eccentricity correlation



- Event larger impacts for more eccentricity correlations.

$$\mathrm{NS}\epsilon, mn = \frac{\langle |\epsilon_m|^2 |\epsilon_n|^2 \rangle}{\langle |\epsilon_m|^2 \rangle \langle |\epsilon_n|^2 \rangle} - 1.$$

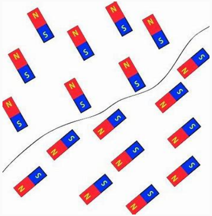
- Something can be checked independently in the future.
- It would be interesting to construct an observable that is only sensitive to such quantum fluctuation!

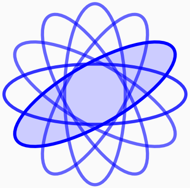
Summary

- Putting nuclear structure information into MC Glauber model is non-trivial:
    1. Symmetry restoration via superposition of deformed configuration in the intrinsic frame $\Rightarrow$ highly-entangled nucleon wave functions.
    2. Inelastic collisions with large local entropy production is a simultaneous measurement of the transverse position of participant nucleons, not generalized coordinates.
- From #1 and #2, the fireball geometry must contain some off-diagonal information in terms of the generalized coordinates $\rho_{\alpha,\alpha'}$.
- We proposed an extension to MC Glauber Model based on 1) intrinsic frame one-body density and 2) superposition under Gaussian overlap approximation.
- Quantum superposition make a difference for light nuclei collisions.

Questions?

Classical mixing versus quantum superposition





- **In classical world**, system undergoes spontaneous symmetry breaking eventually chooses a preferred orientations. (Decoherence at macroscopic level, environmental selection, etc)
- **But quantum mechanically**, the ground state of <u>isolated</u> nucleus should preserve rotational symmetry by superposition.
- Individual configurations $|\Omega, \beta\rangle$
  - neither energy eigen-states nor states with correct symmetry;
  - not orthogonal $\langle \Omega', \beta' | \Omega, \beta \rangle \neq 0$ (like coherent states);
  - not a unique set of choice (reparameterization invariance).