

Intersection of nuclear structure and high-energy nuclear collision 2026

13–24 Apr. @YITP, Kyoto

Nuclear Shapes Seen through Nuclear Responses

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大阪大学

THE UNIVERSITY OF OSAKA

Atomic nucleus: *Mysterious object*

Emerging **simplicity** in **complex** systems:

classical and intuitive pictures for **quantum** many-body systems
collective behaviors: **shape**, vibration, and rotation

Emerging **richness** from nucleonic degrees of freedom:

static properties

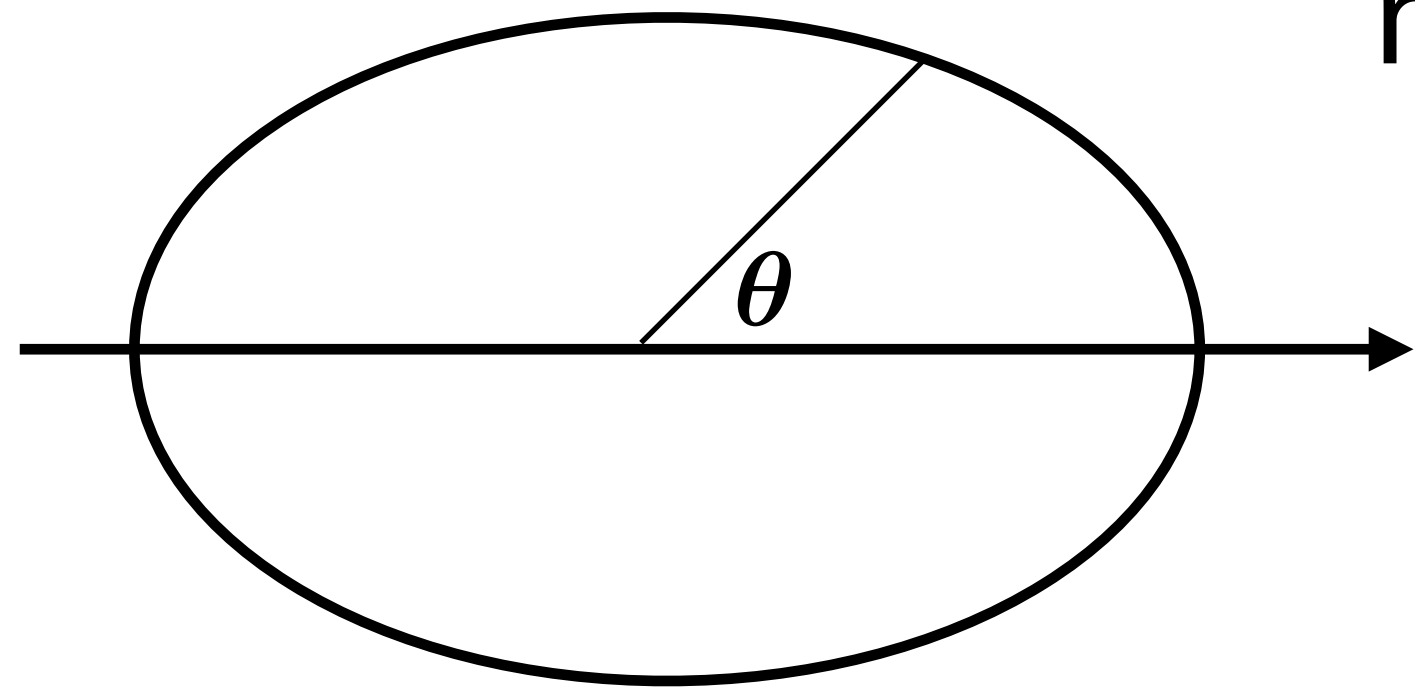
deformation, skin/halo, and superfluidity/superconductivity

dynamic properties

single-particle and collective modes of excitation

explored by nuclear responses

Is a nucleus deformed?



nuclear "shape": classical picture

$$R(\theta, \phi) = \bar{R}_0 \left(1 + \sum_{\lambda\mu} \alpha_{\lambda\mu}^* Y_{\lambda\mu}(\theta\phi) \right) \quad \alpha_{2\mu} \neq 0$$

The **ground state** of all the even-even nuclei is **spherical!**

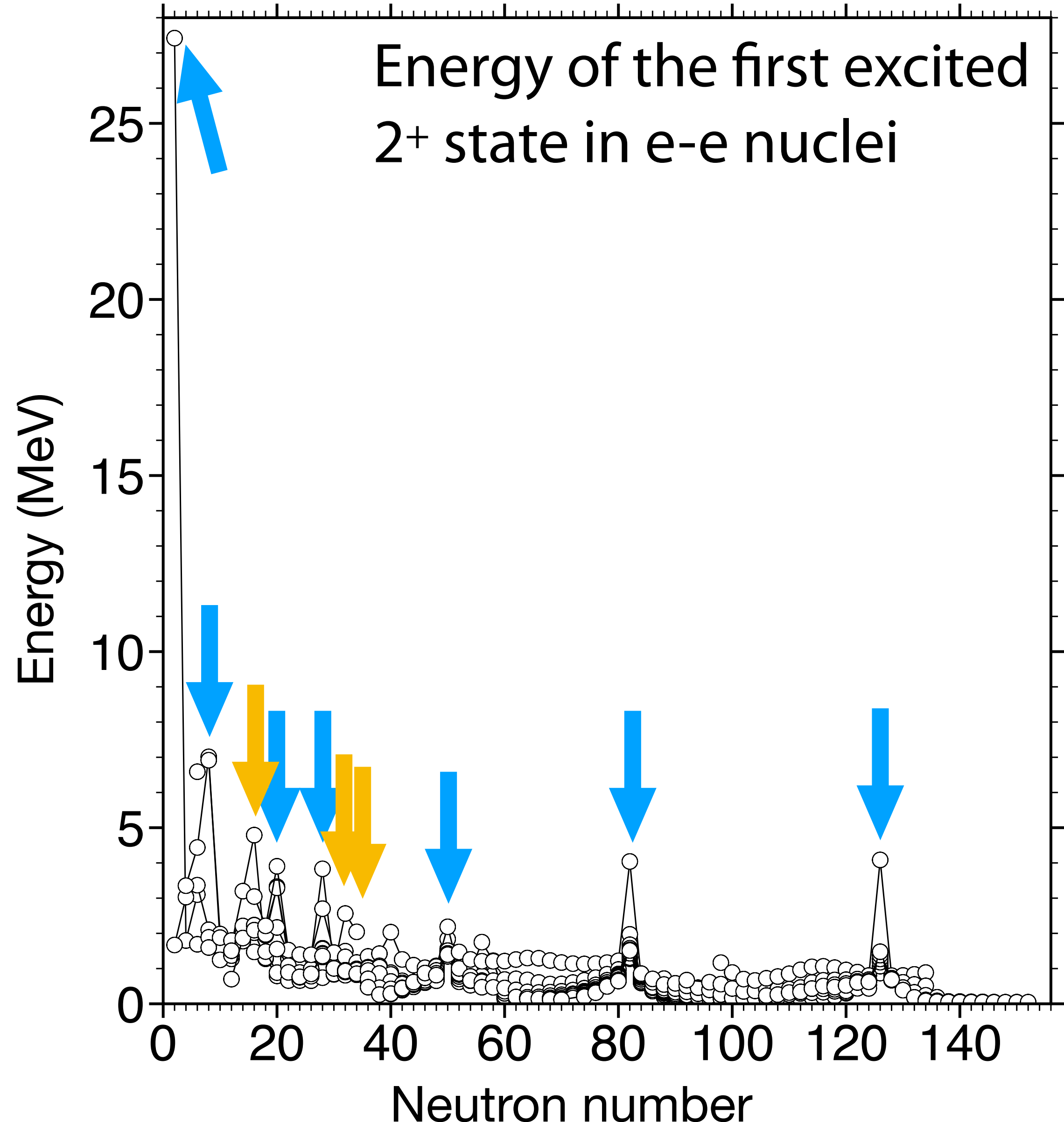
$$\langle J = 0 | Y_{2\mu} | J = 0 \rangle = 0$$

How to define the shape of a quantal object?

$$|J^\pi, k\rangle = P^J P^\pi |\Phi\rangle \quad \text{mean field/intrinsic state}$$

We need to look at the **excited states** to see the intrinsic state.

Quadrupole (2+) state energy: magic number

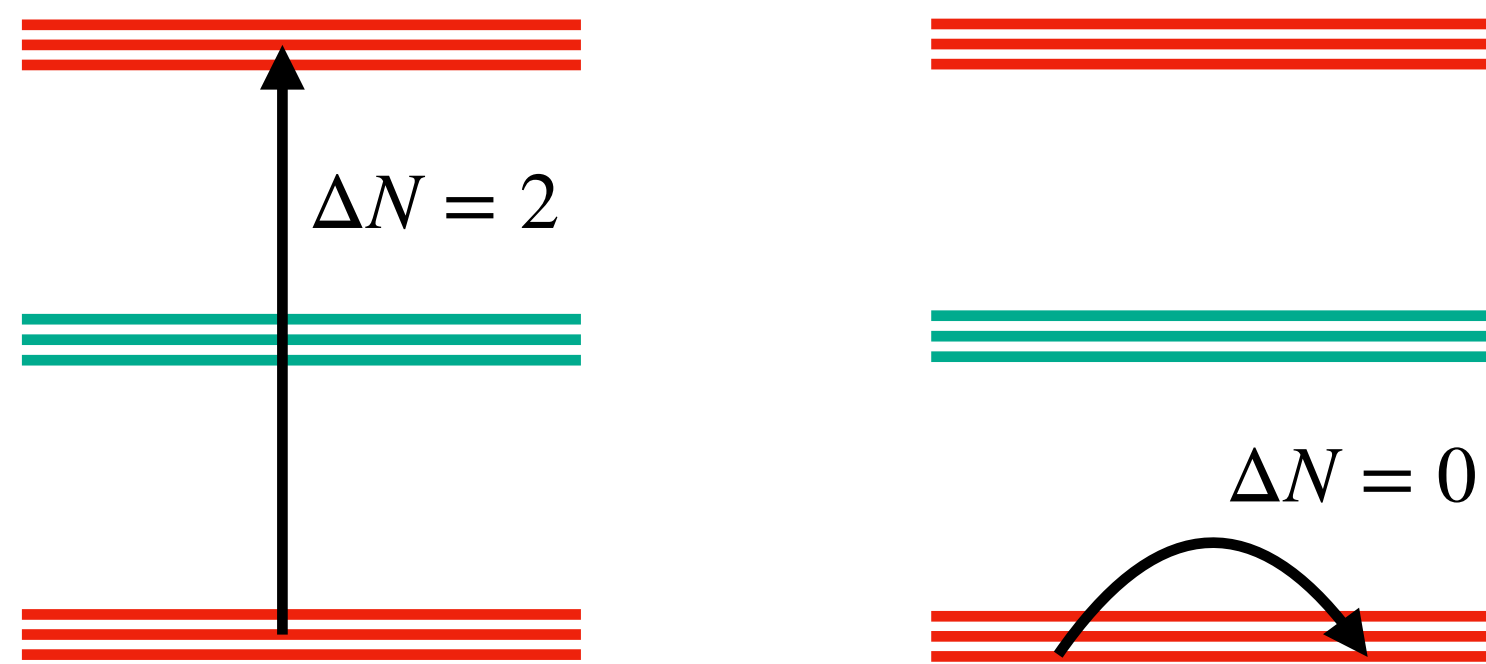


Data from NNDC

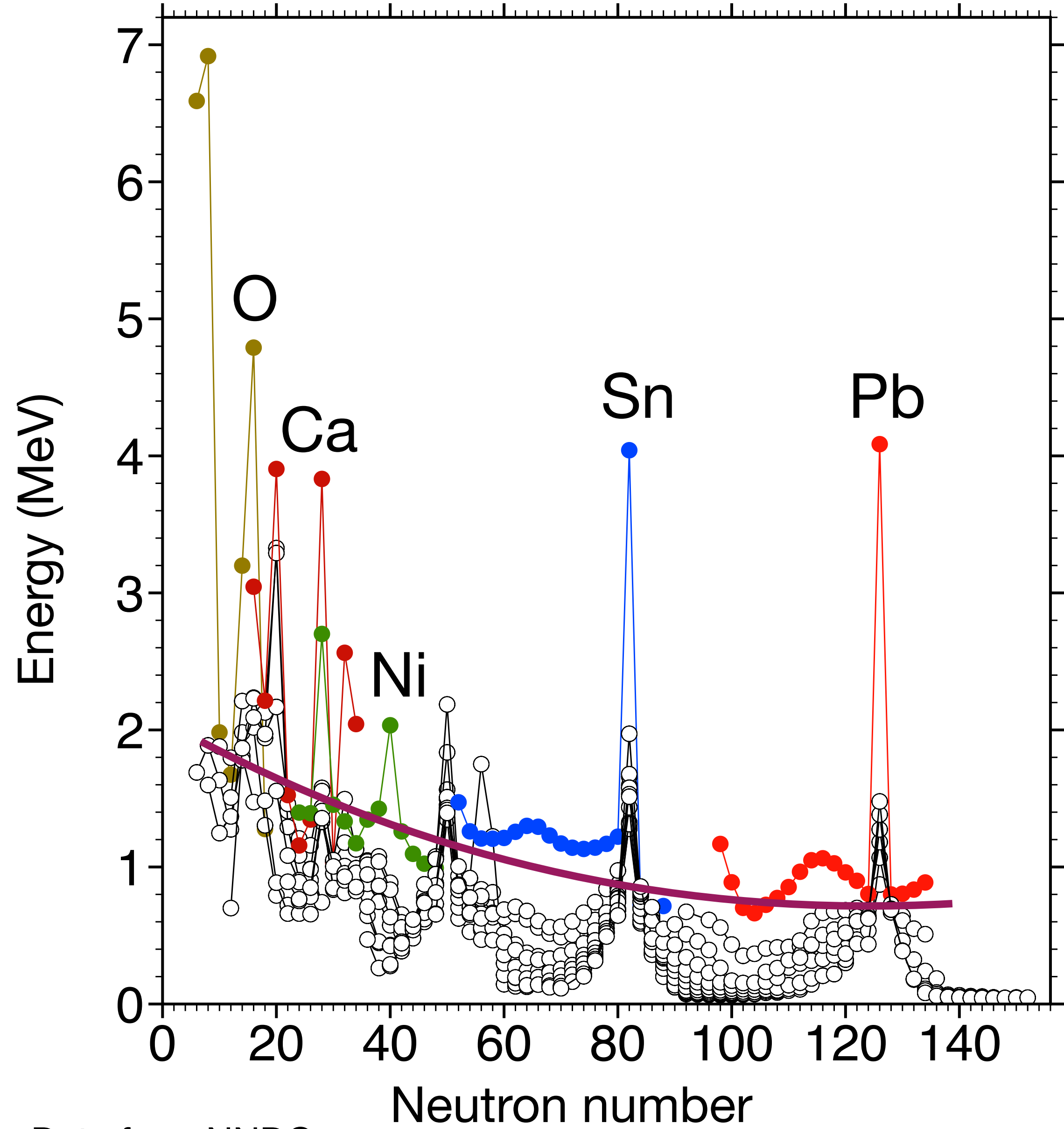
Canonical magic numbers
2, 8, 20, 28, (40), 50, 82, 126

New magic numbers in neutron-rich nuclei
16, 32, 34, ...

magic nuclei are hard to excite



Quadrupole (2+) state energy: pairing

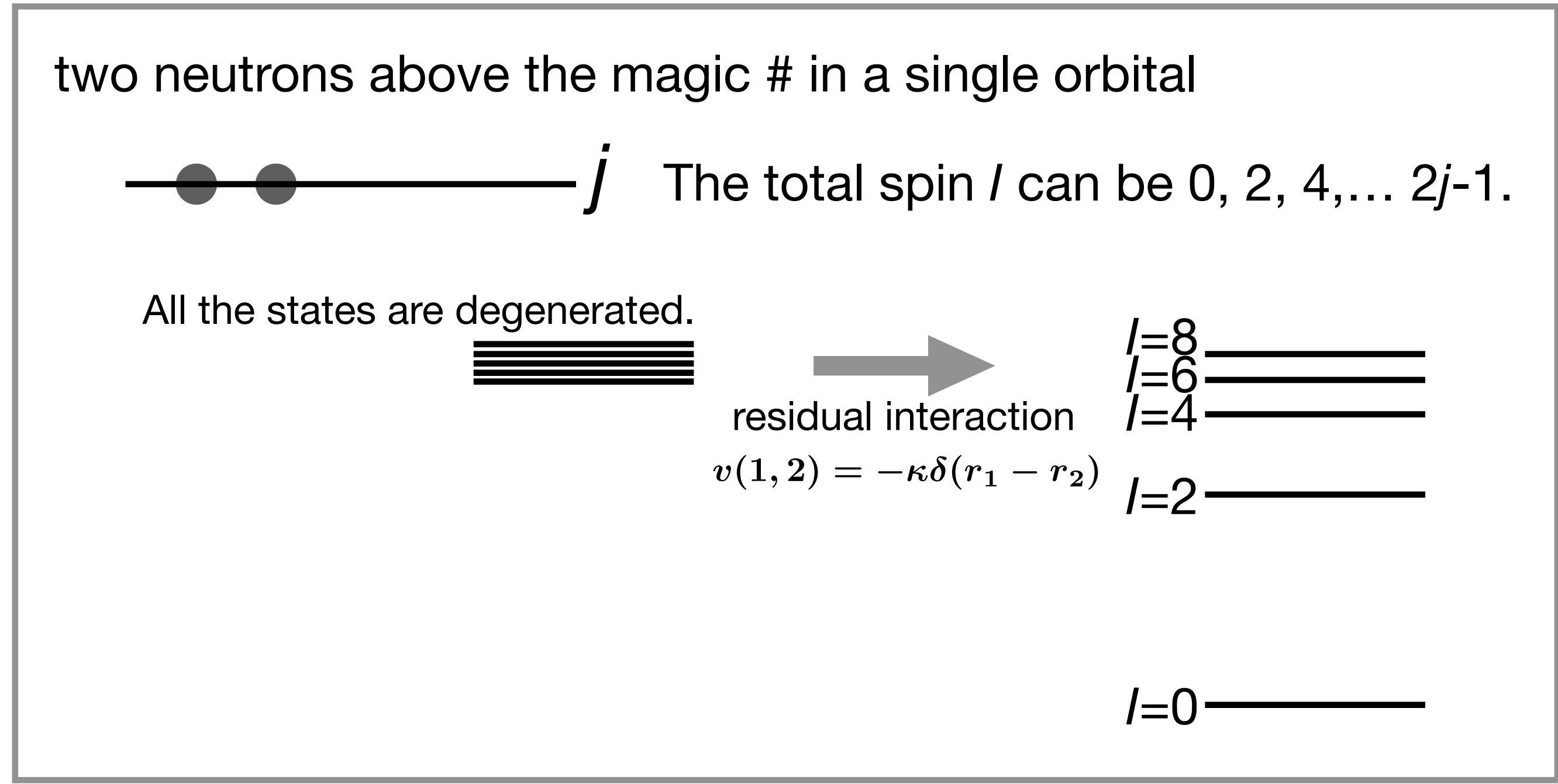


Data from NNDC

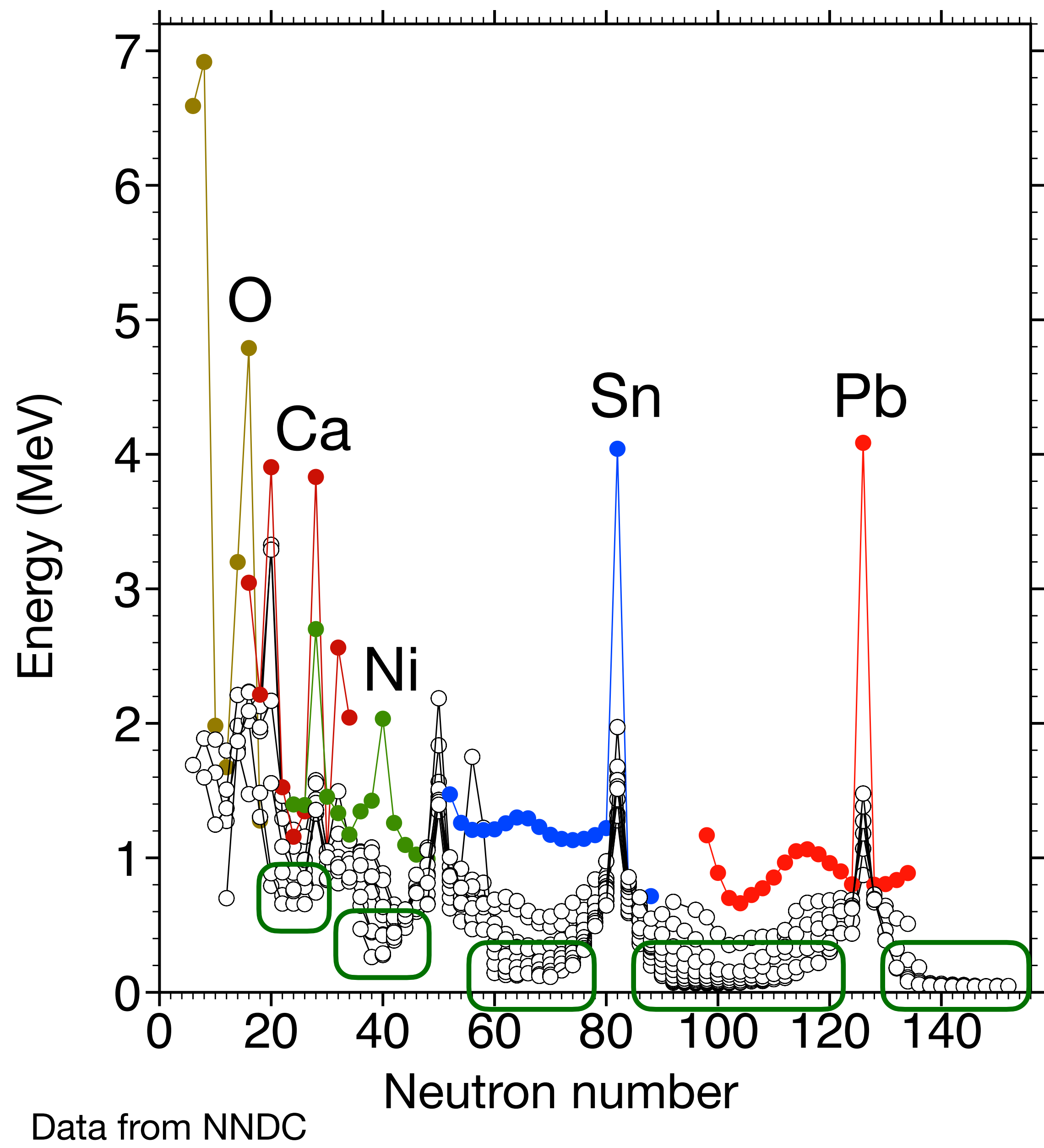
Singly-closed ($Z=\text{magic \#}$) nuclei have

an energy gap

The ground 0^+ state is lowered by the **pairing**.



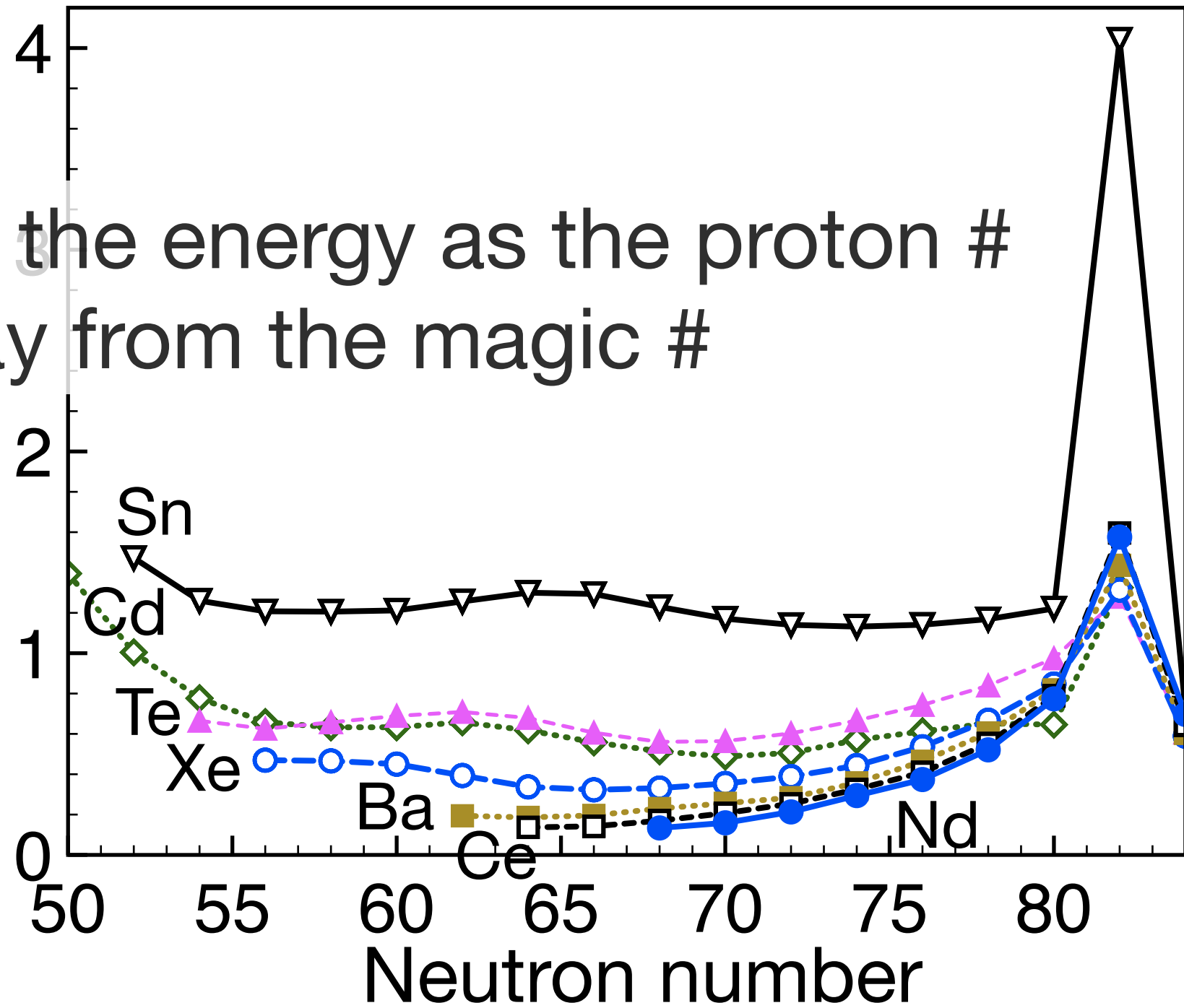
Quadrupole (2^+) state energy: zero mode



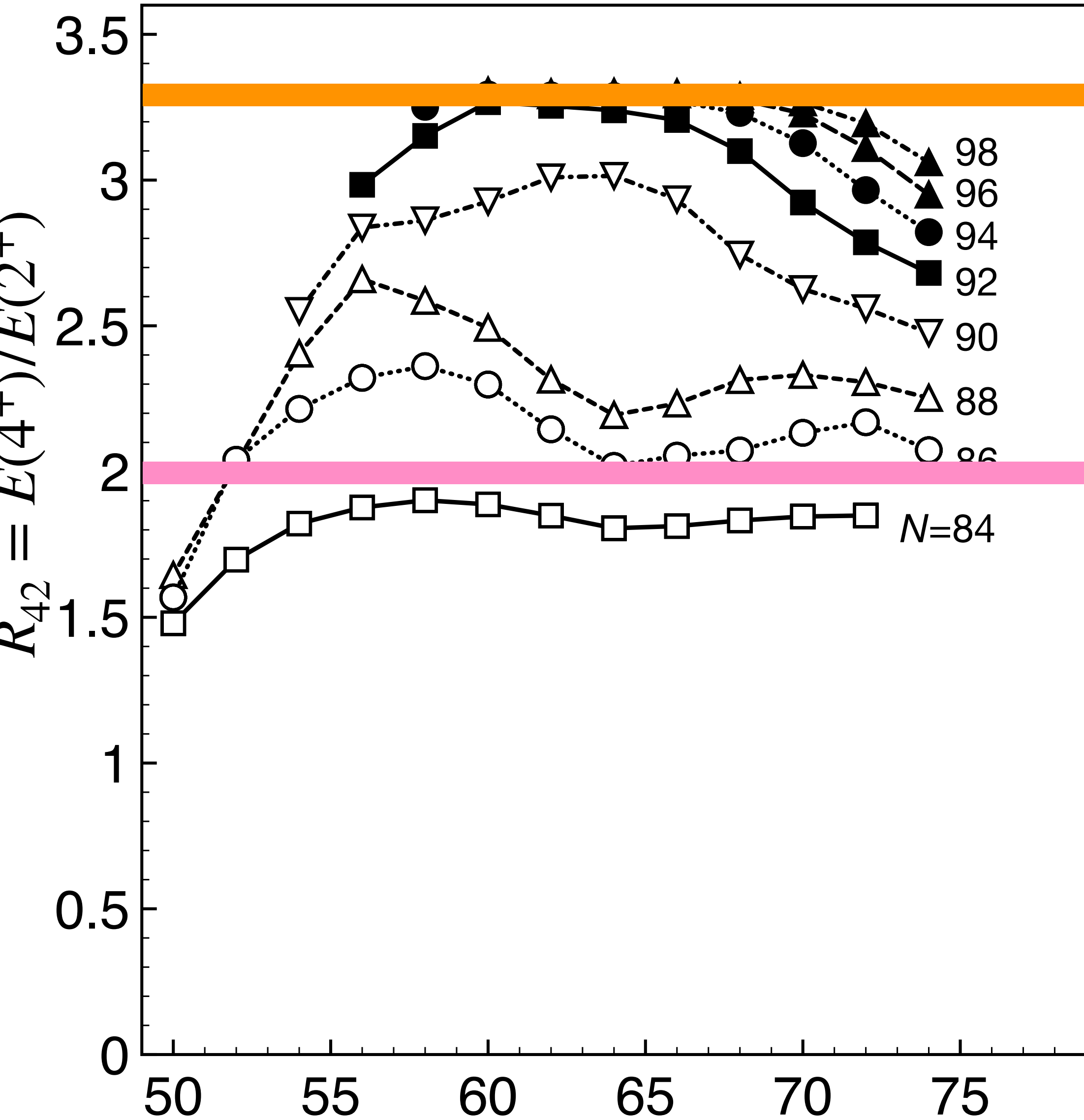
The singly-closed ($Z=\text{magic \#}$) nuclei have
an energy gap
 The ground 0^+ state is lowered by the **pairing**.

“zero-energy” mode in mid-shell nuclei

Lowering of the energy as the proton #
 moving away from the magic #



Excitation energies of 2+ and 4+ states



Data from NNDC Proton number

occurrence of collective excitations

Rotation ← breaking of the rotational symmetry

$$\hat{H} = \frac{\hat{I}^2}{2\mathcal{J}} \quad E(I) = \frac{I(I+1)}{2\mathcal{J}} \quad R_{42} = \frac{4 \times 5}{2 \times 3} = 3.33 \dots$$

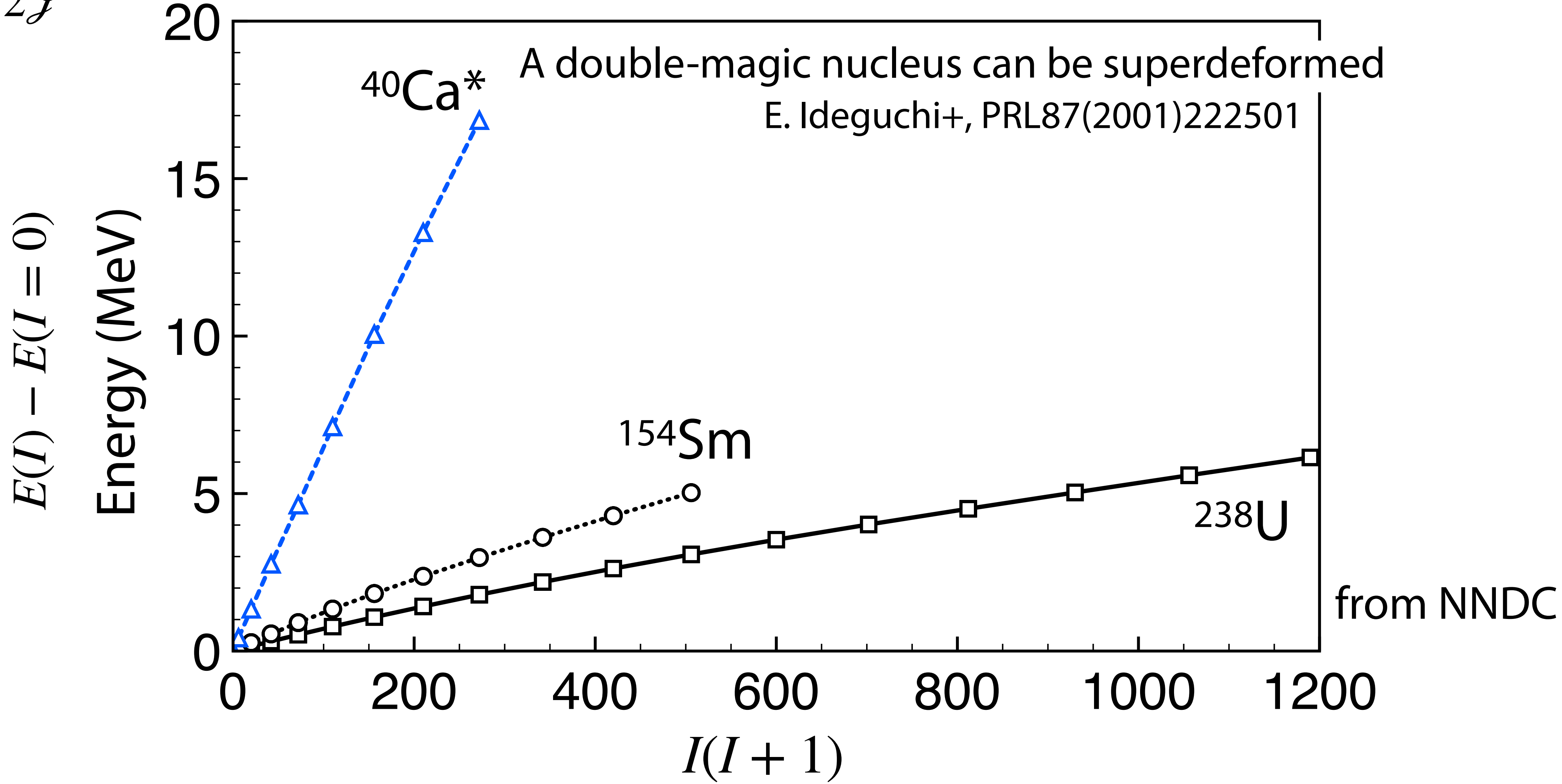
quadrupole deformation

Vibration

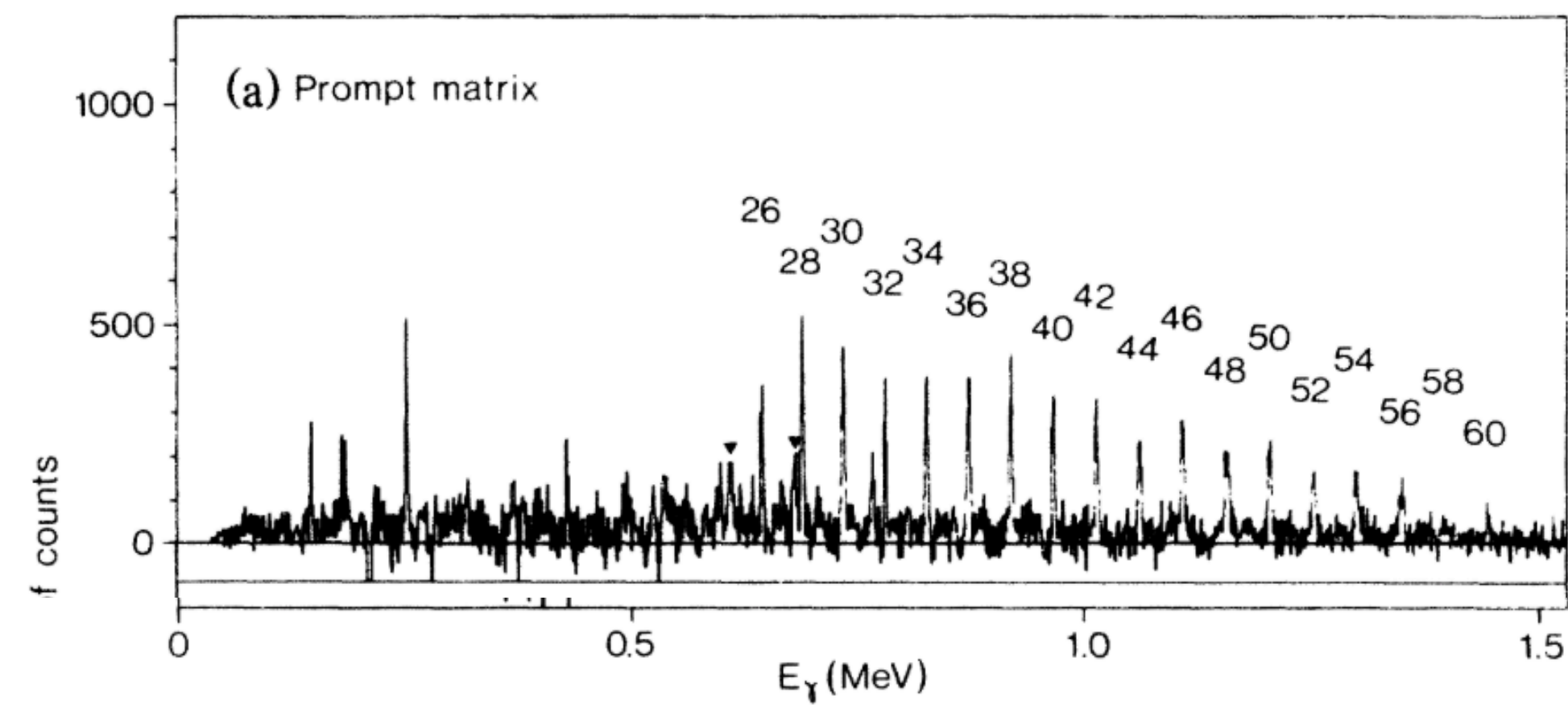
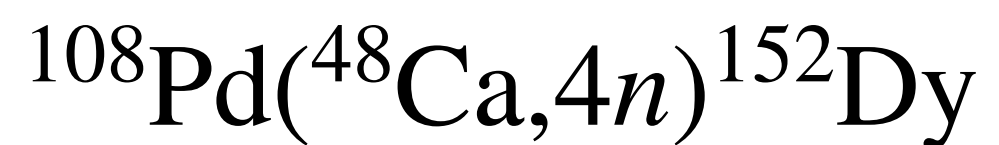
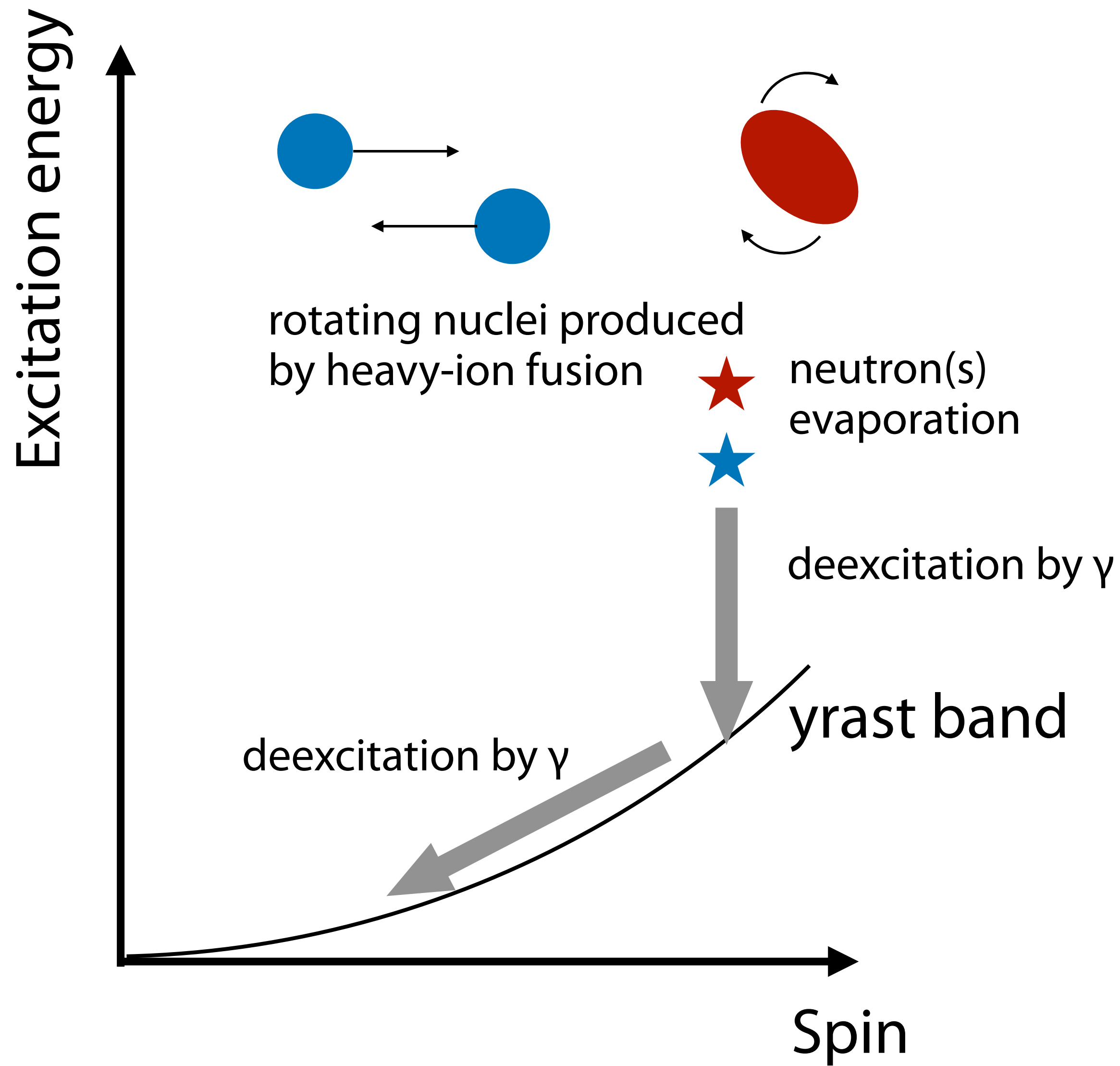
$$\hat{H} = \omega \hat{\Gamma}^\dagger \hat{\Gamma} \quad E(I) = \frac{I}{2} \omega \quad R_{42} = \frac{4}{2} = 2$$

Emergence of the rotational band

$$E = \frac{I(I + 1)}{2\mathcal{J}}$$



Low-energy heavy ion collision for rotating nuclei



Twin+, PRL57(1986)811

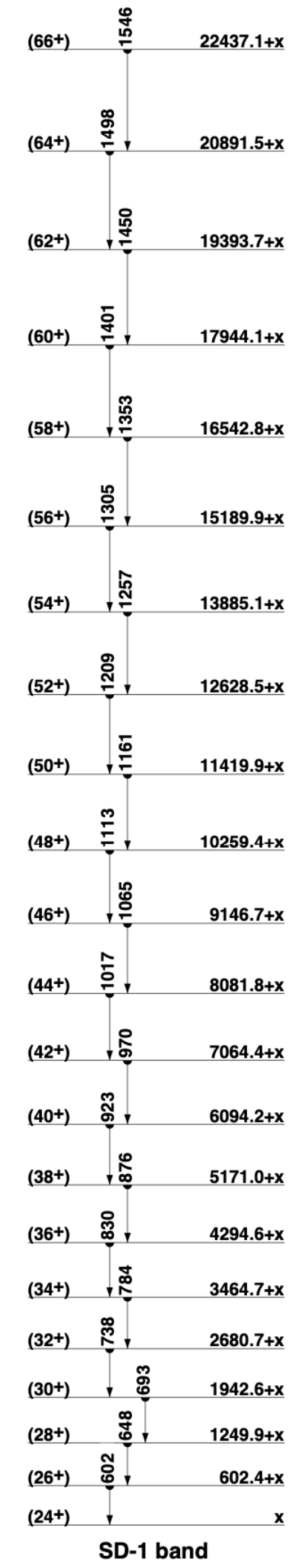
rotational frequency

$$\omega_{\text{rot}}(I) = \frac{\partial E}{\partial I}$$

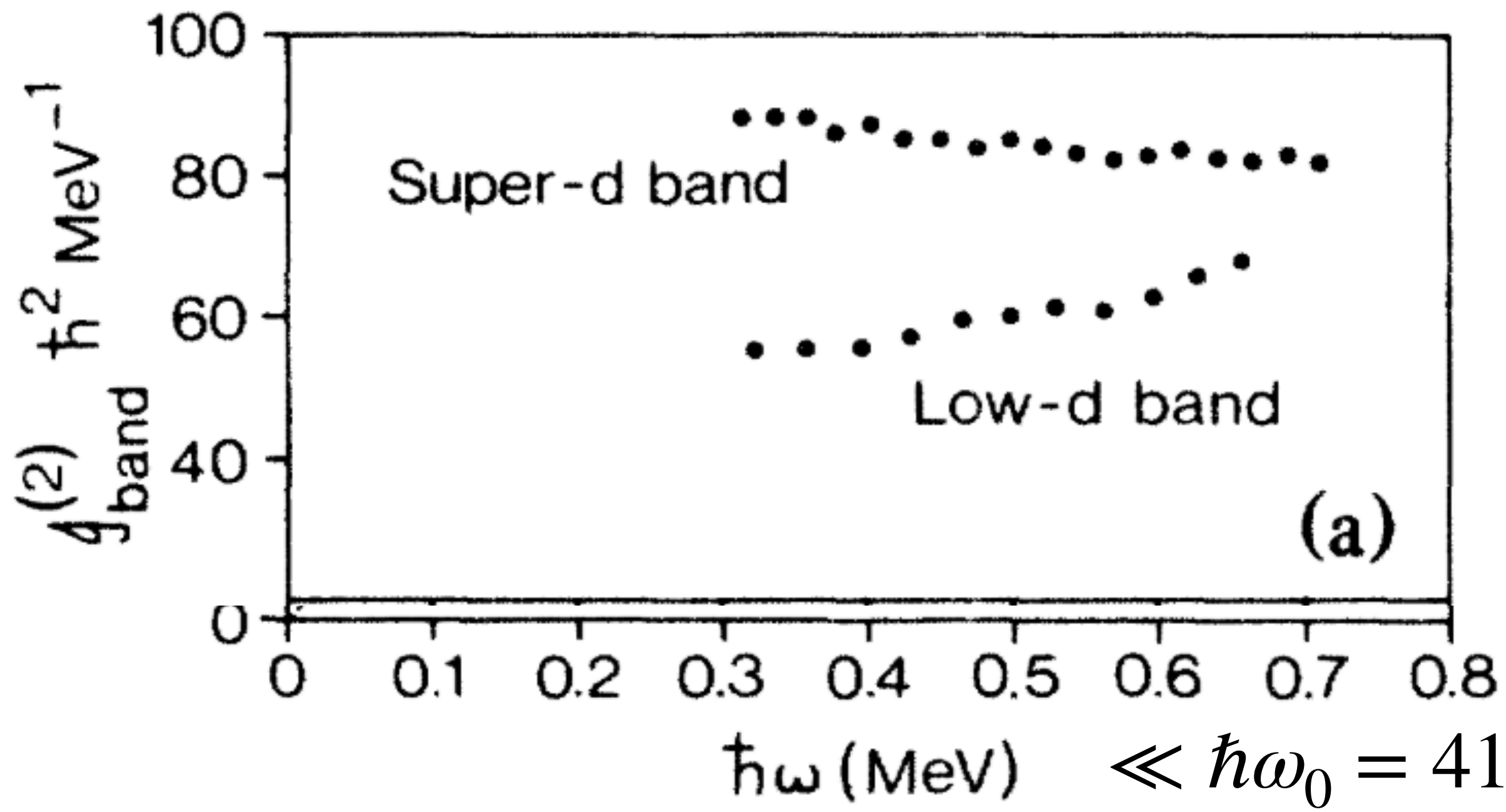
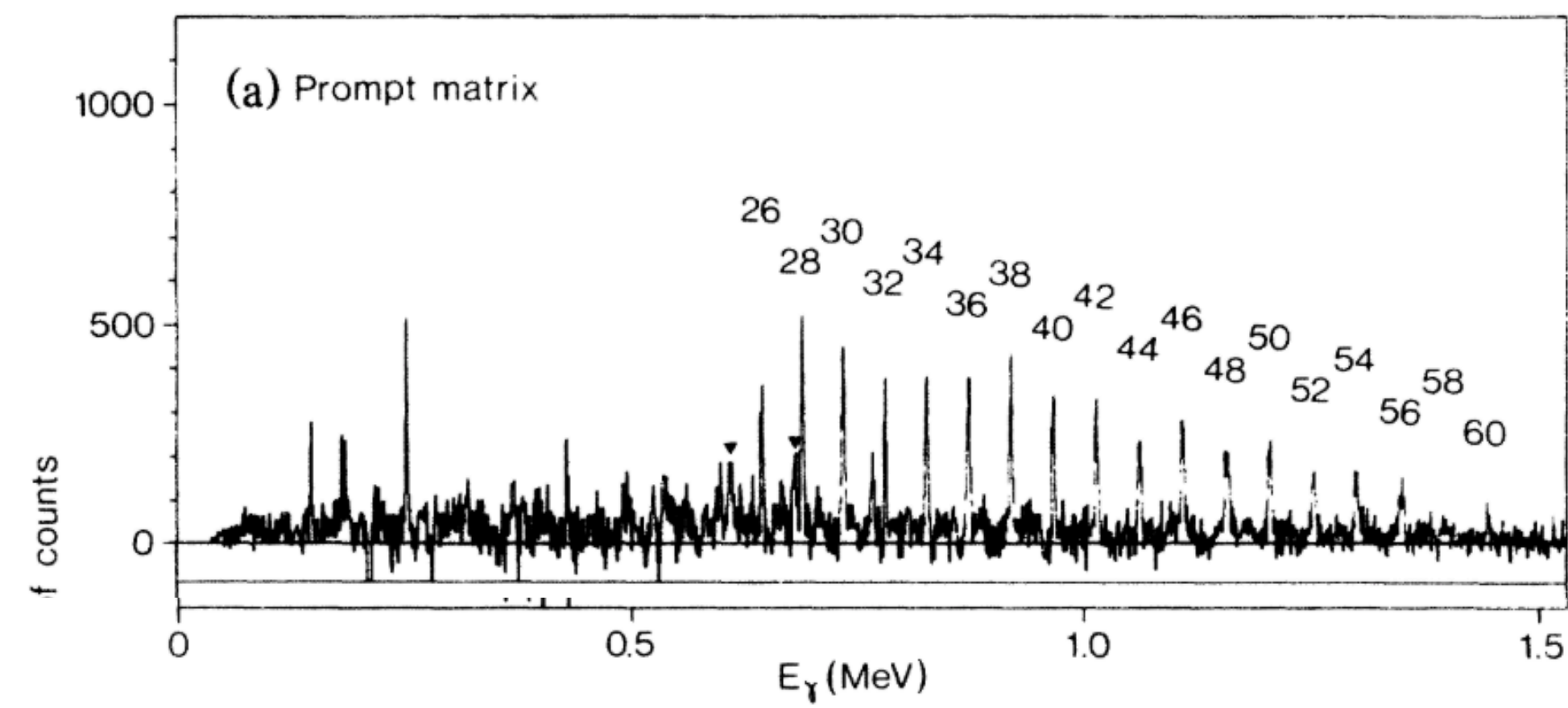
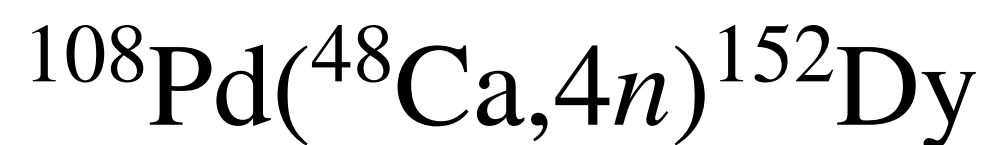
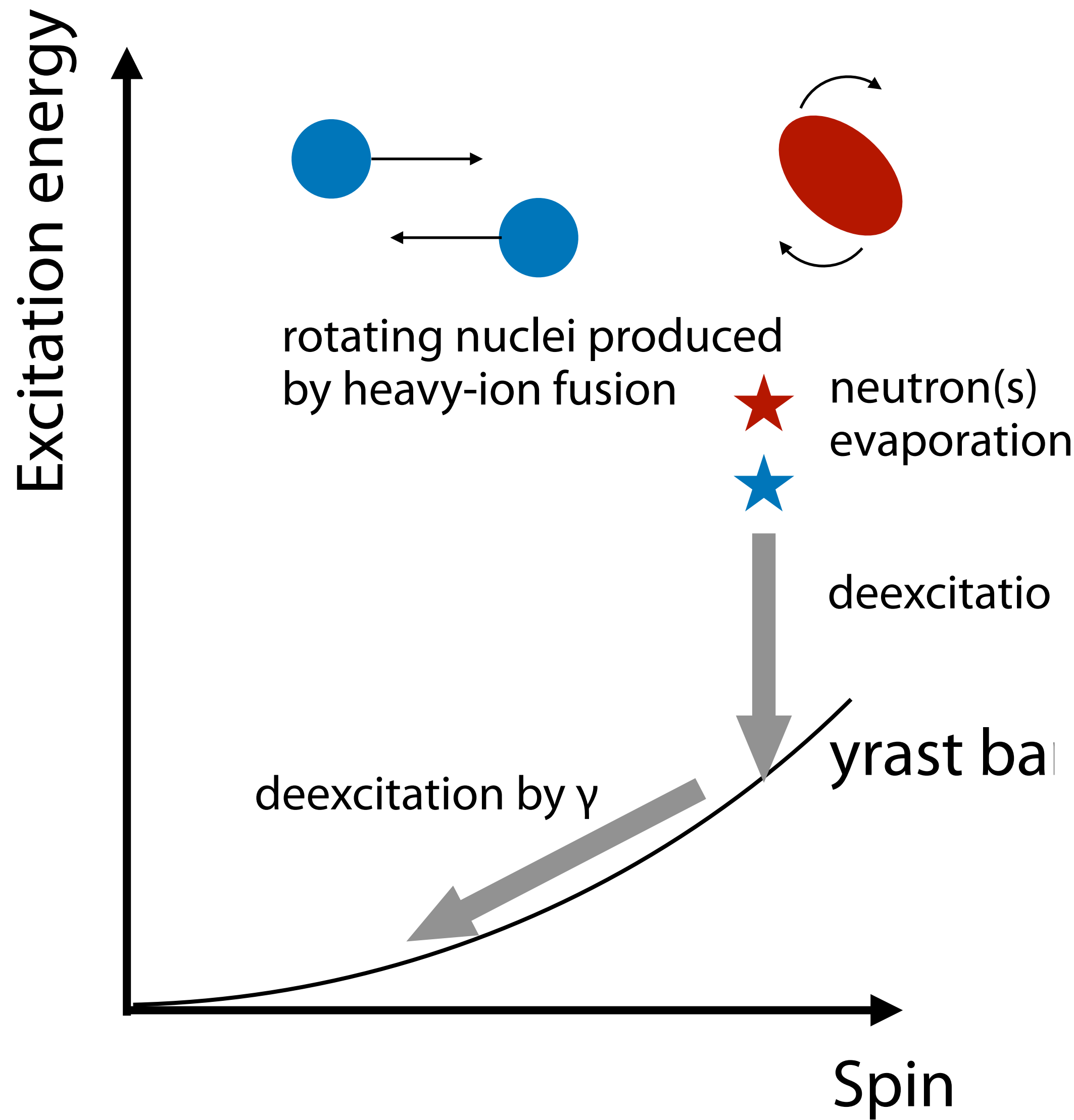
$$\approx \frac{E(I+1) - E(I-1)}{2} = \frac{E_\gamma(I)}{2}$$

moment of inertia

$$\mathcal{J}^{(2)} := \frac{dI}{d\omega_{\text{rot}}} = \left(\frac{d^2E}{dI^2} \right)^{-1} = \frac{4}{\Delta E_\gamma}$$



Low-energy heavy ion collision for rotating nuclei

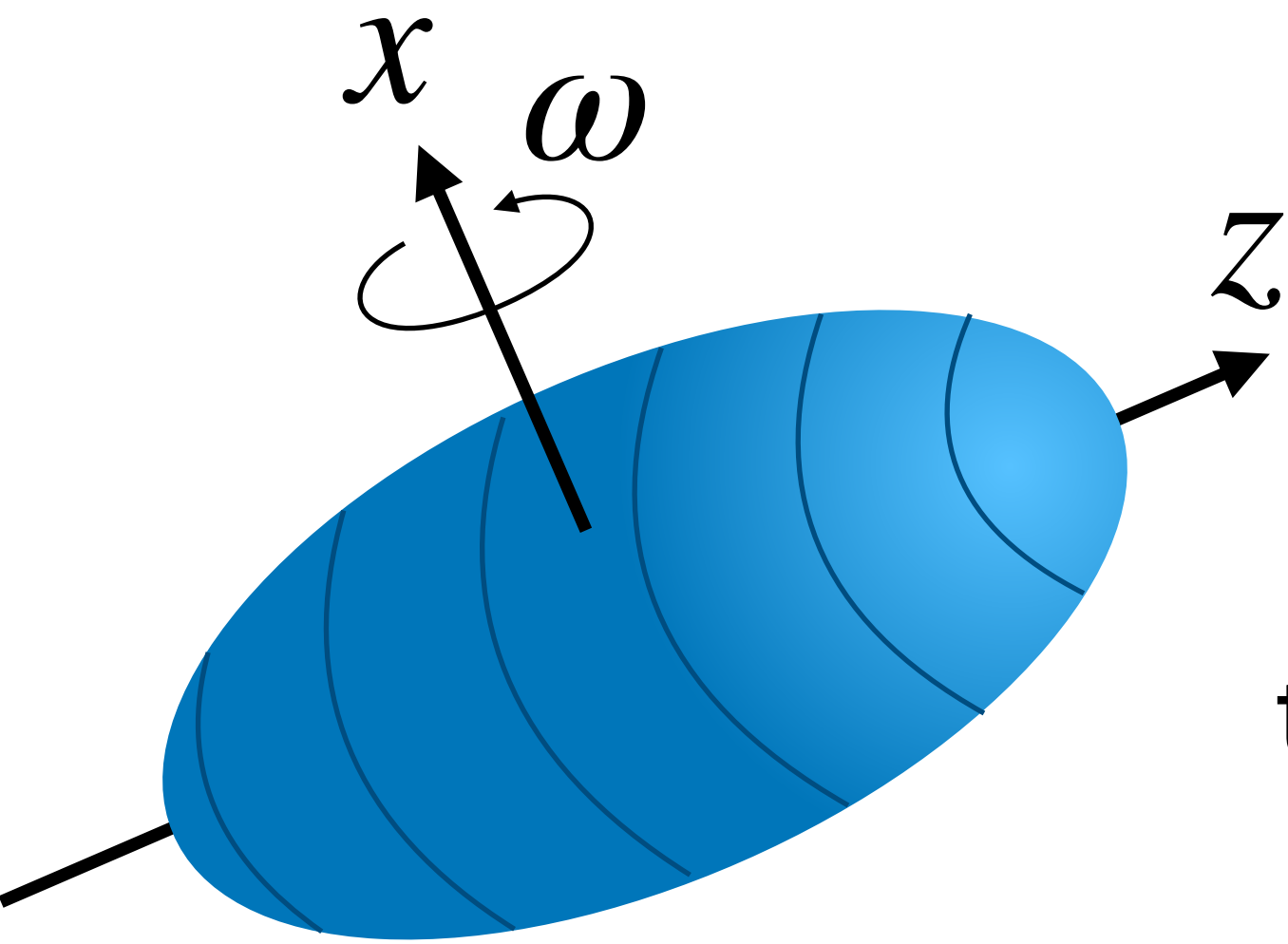


(66+)	1546	22437.1+x
(64+)	1498	20891.5+x
(62+)	1450	19393.7+x
(60+)	1401	17944.1+x
(58+)	1353	16542.8+x
(56+)	1305	15189.9+x
(54+)	1257	13885.1+x
(52+)	1209	12628.5+x
(50+)	1161	11419.9+x
		10259.4+x
		9146.7+x
		8081.8+x
		7064.4+x
		6094.2+x
		5171.0+x
		4294.6+x
		3464.7+x
		2680.7+x
		1942.6+x

The cranking model

simultaneous description of s.p. and rotational motions

the rotation is treated **classically** within the **quantum mechanics**



$$x_1 = x$$

$$x_2 = y \cos \omega t + z \sin \omega t$$

$$x_3 = -y \sin \omega t + z \cos \omega t$$

uniform rotation along
the x-axis

the time-dependent wave functions in the two systems must satisfy

up to the phase

$$\psi^\omega(x_1, x_2, x_3, t) = \psi(x, y, z, t)$$

$$\left(\frac{\partial \psi}{\partial t}\right)_{x,y,z} = \left(\frac{\partial \psi^\omega}{\partial t}\right)_{x_1,x_2,x_3} + \frac{\partial \psi^\omega}{\partial x_2} \frac{\partial x_2}{\partial t} + \frac{\partial \psi^\omega}{\partial x_3} \frac{\partial x_3}{\partial t} \quad \frac{\partial x_2}{\partial t} = \omega x_3, \quad \frac{\partial x_3}{\partial t} = -\omega x_2$$

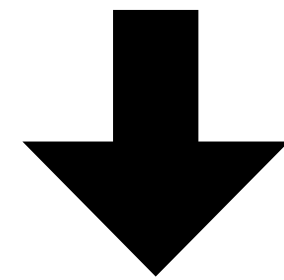
$$\longrightarrow \frac{\partial \psi(x, y, z, t)}{\partial t} = \left(\frac{\partial}{\partial t} - i\omega \ell_1\right) \psi^\omega(x_1, x_2, x_3, t)$$

The cranking model

$$\frac{\partial \psi(x, y, z, t)}{\partial t} = \left(\frac{\partial}{\partial t} - i\omega \ell_1 \right) \psi^\omega(x_1, x_2, x_3, t)$$

➔ time-dependent Sch. eq.

$$i\partial_t \psi(x, y, z, t) = h\psi(x, y, z, t)$$



$$i\partial_t \psi^\omega(x_1, x_2, x_3, t) = \underline{(h - \omega \ell_1)} \psi^\omega(x_1, x_2, x_3, t)$$

the cranking Hamiltonian (Routhian): $\hat{H}' = \hat{H} - \omega \hat{J}_1$

↔ $\delta \langle \psi | \hat{H} | \psi \rangle = 0$ under the constraint $\langle \psi | \hat{J}_1 | \psi \rangle = J_1$

$\delta(E[\rho] - \omega J_1) = 0$ in DFT

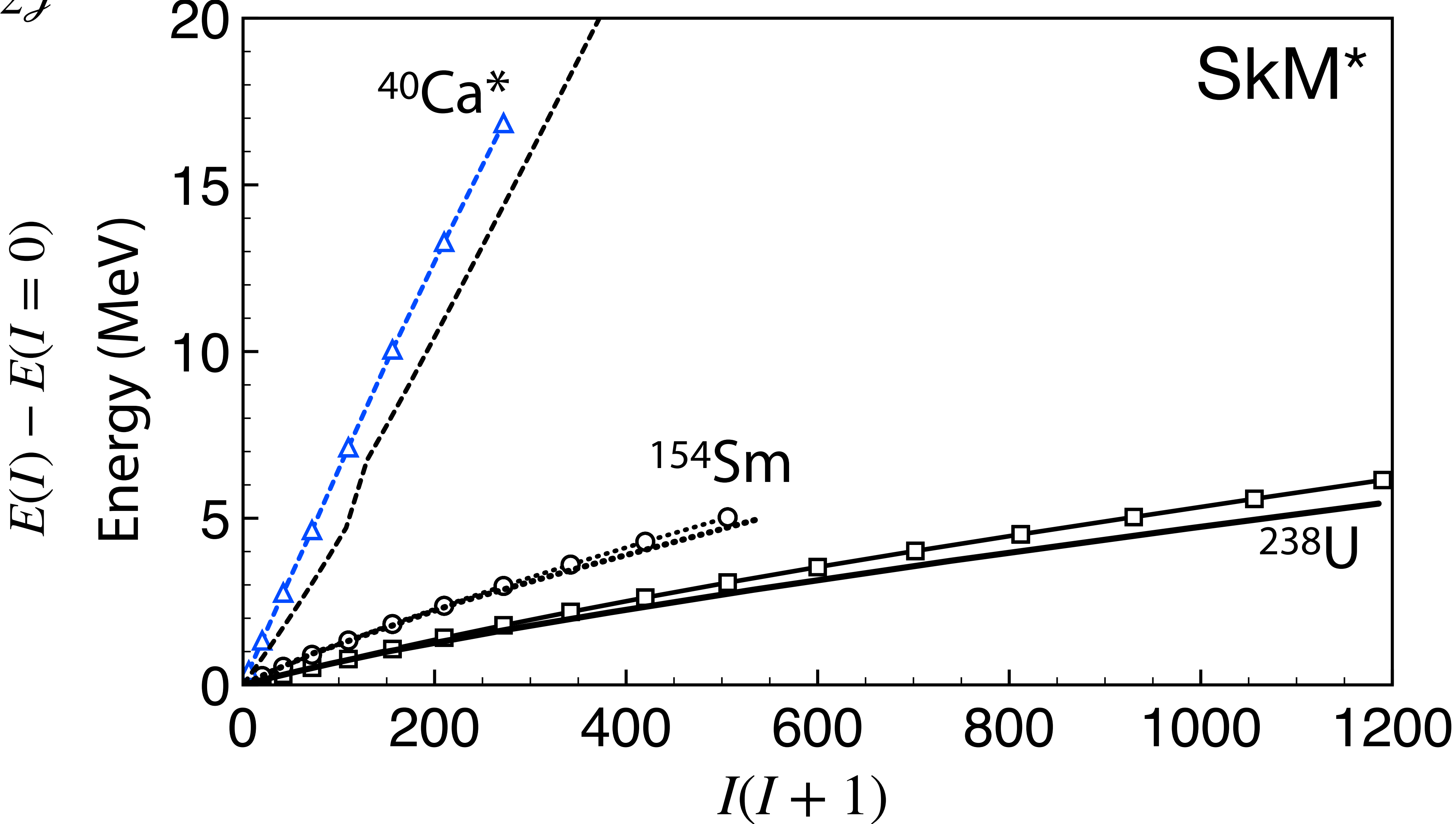
ω : a Lagrange multiplier

note:

$$\begin{aligned} \ell_1 &= -i \left(x_2 \frac{\partial}{\partial x_3} - x_3 \frac{\partial}{\partial x_2} \right) \\ &= -i \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right) = \ell_x \end{aligned}$$

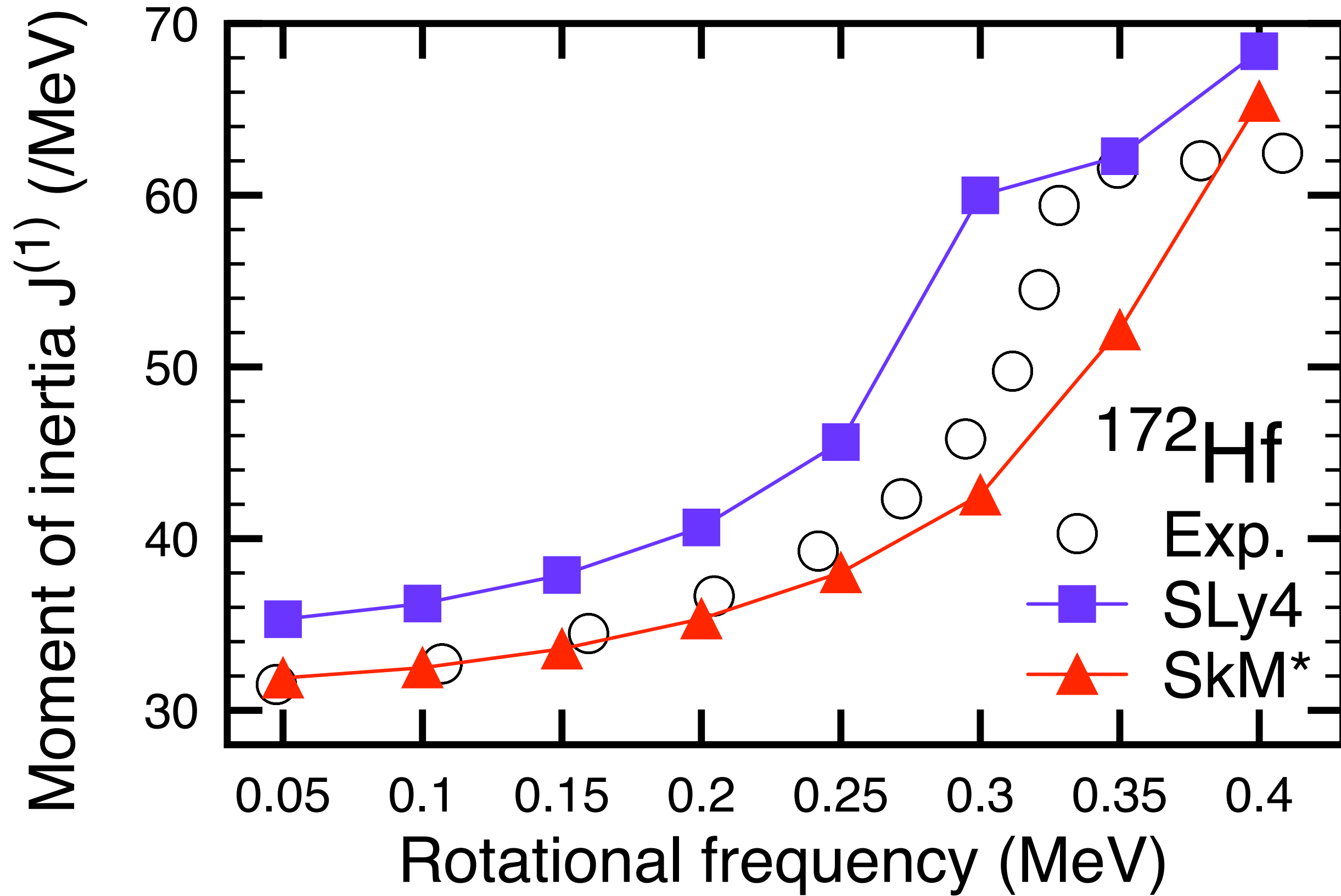
DFT for the rotational band

$$E = \frac{I(I + 1)}{2\mathcal{J}}$$

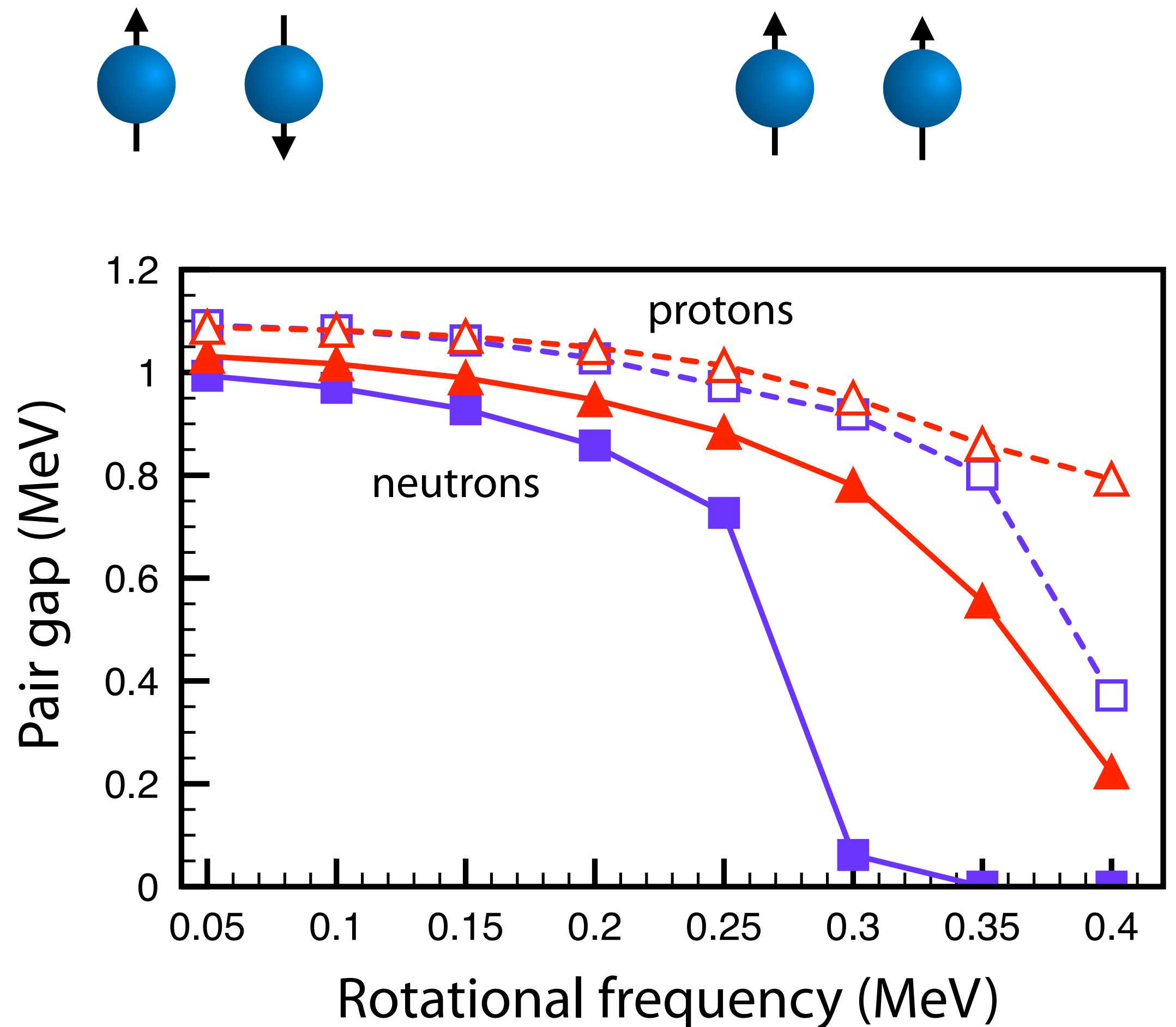


Superfluidity in response to rotation

KY, PRC105(2022)024313

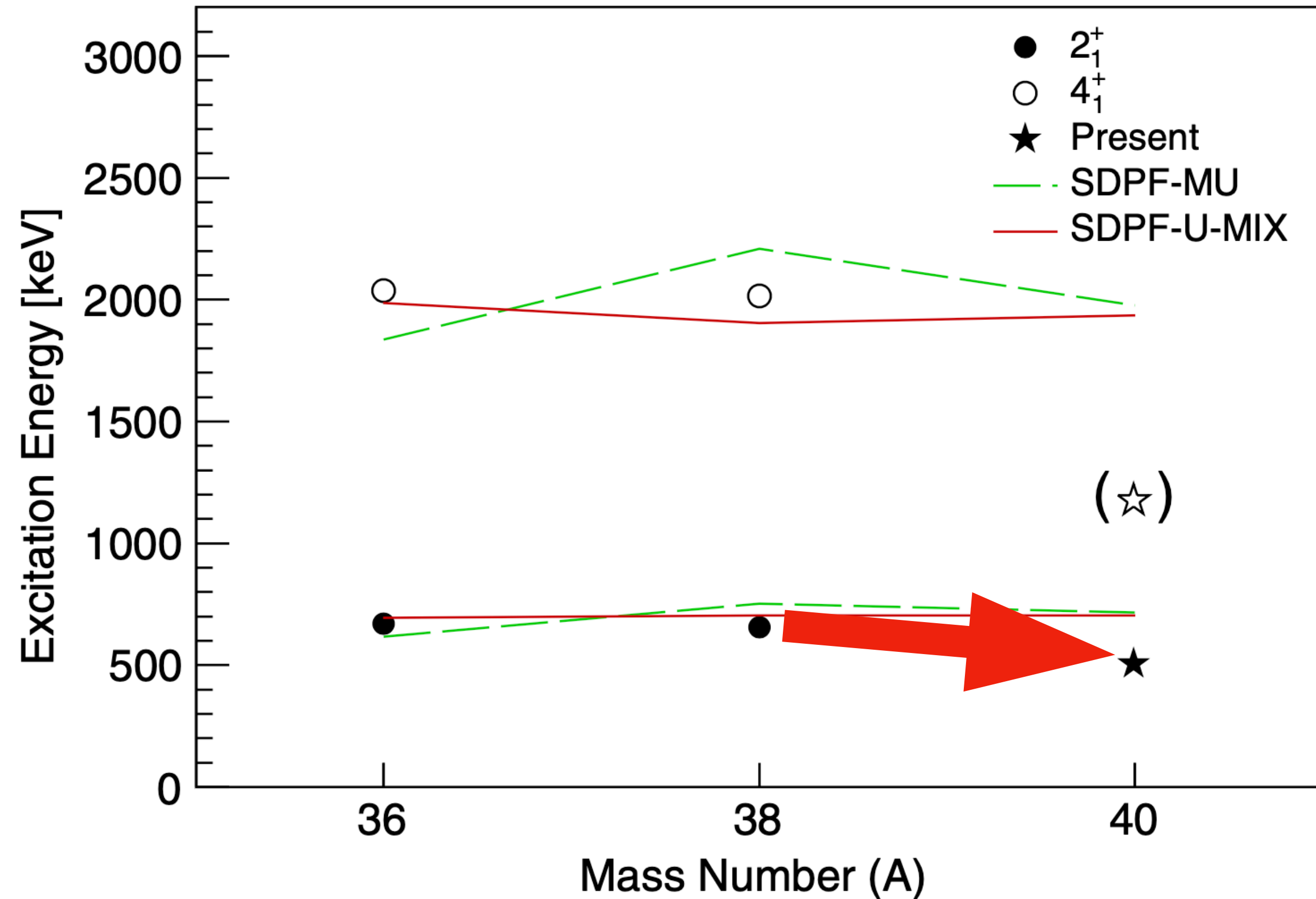


At $\omega_{\text{rot}} \sim 0.3 \text{ MeV}$
breaking of the Cooper pairs



Anomalous 2^+ state in neutron-rich Mg isotopes?

Crawford+, PRL122(2019)052501

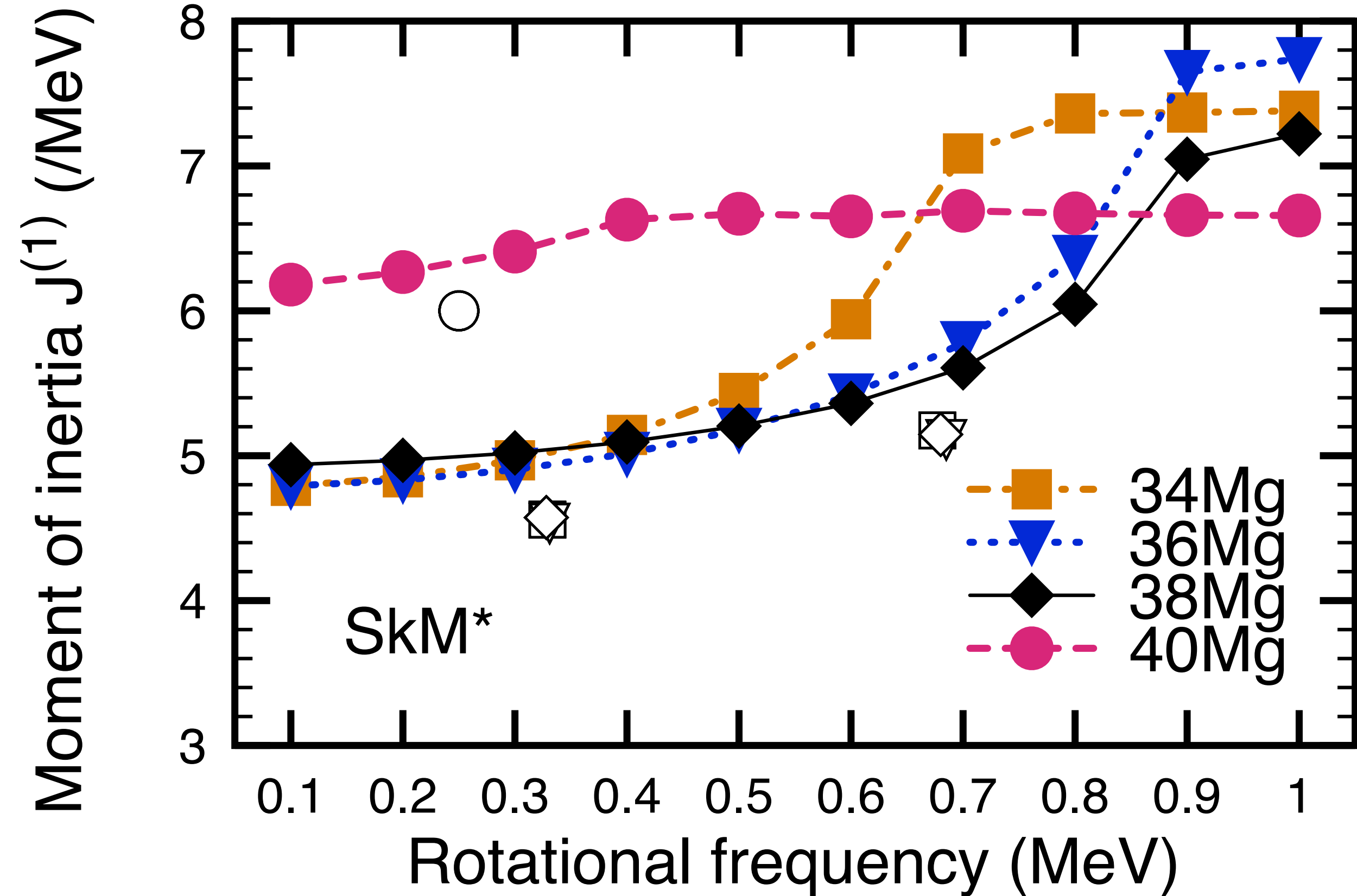


635(6) keV \rightarrow 500(14) keV

~20% decrease in energy

Is it a qualitatively unique feature?

KY, PRC105(2022)024313



weakening of the pairing

deformed shell gap at N=28

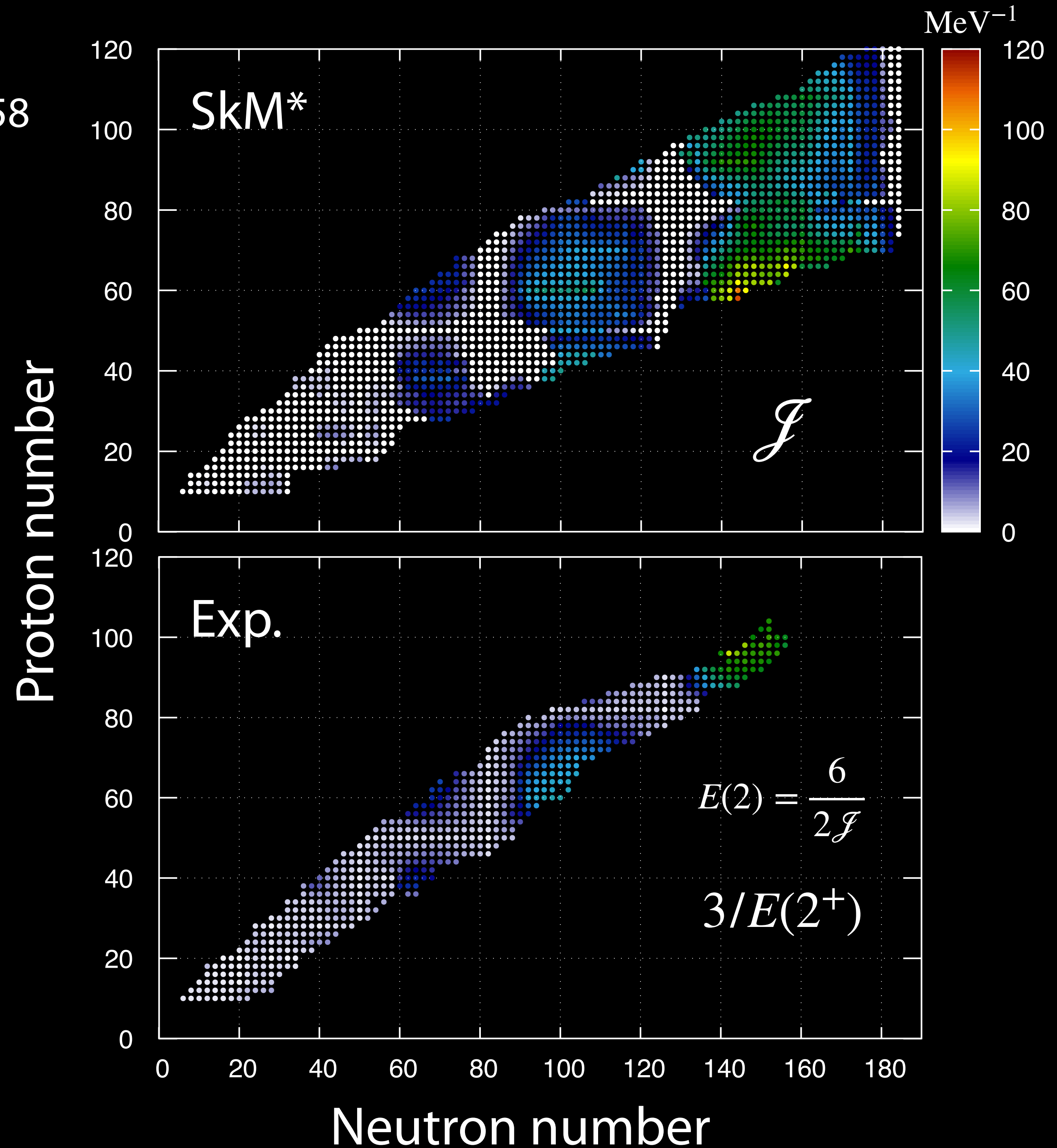
Large-scale DFT calculation

KY, PLB834(2022)137458

low $E(2^+)$ or high \mathcal{J}

an indicator of quadrupole deformation

a pattern of deformation
magic numbers



TDDFT for vibration

vibration around the ground state: $\rho(\mathbf{r}, t) = \underline{\rho_0(\mathbf{r})} + \delta\rho(\mathbf{r}, t) + \text{h.c.}$

Kohn–Sham (–Bogoliubov–de Gennes)

linear response to the vibrating external field: $e^{-i\omega t} \hat{F} = e^{-i\omega t} \int d\mathbf{r} f(\mathbf{r}) \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r})$

$$\delta\rho(\mathbf{r}, t) \sim \delta\rho(\mathbf{r}) e^{-i\omega t} \quad \delta\rho(\mathbf{r}) = \int d\mathbf{r}' \chi_0(\mathbf{r}, \mathbf{r}') \left[\left. \frac{\delta^2 E[\rho]}{\delta^2 \rho} \right|_{\rho=\rho_0} \delta\rho(\mathbf{r}') + f(\mathbf{r}') \right]$$

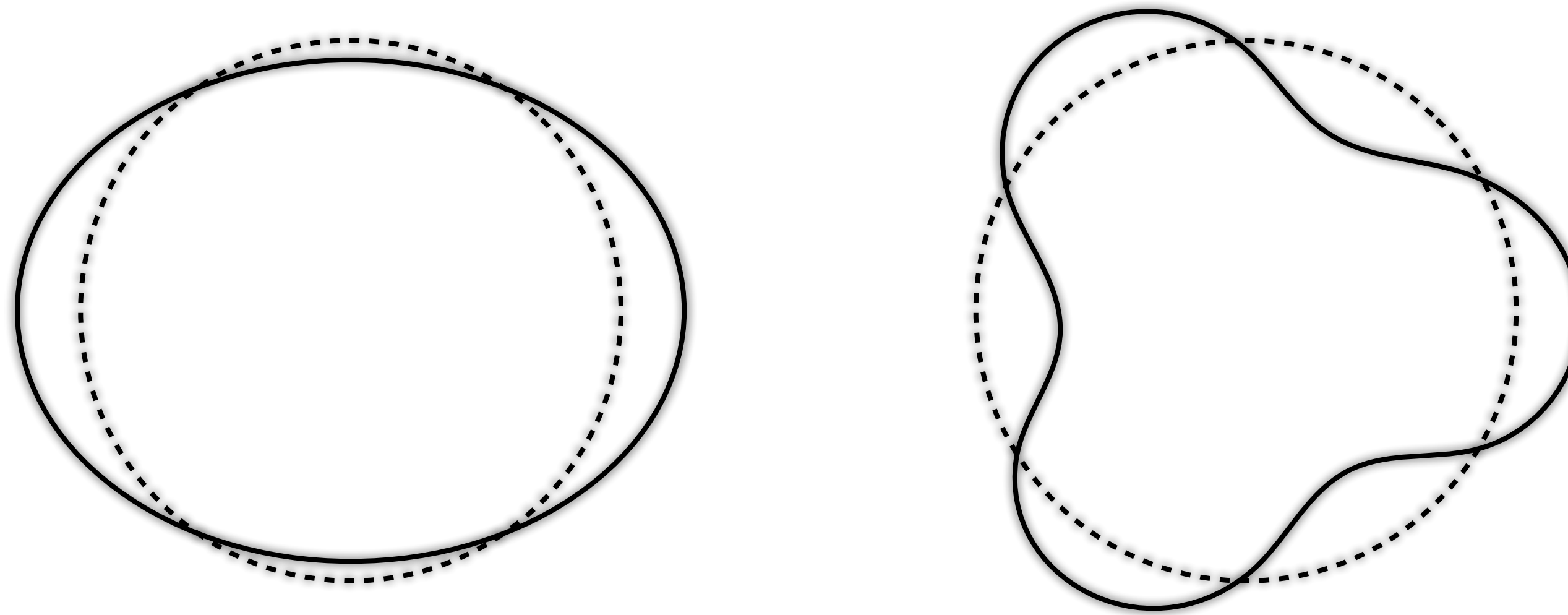
vibration in space/spin-space/isospin-space/gauge-space and coupling among them

$$\hat{F}_L = \int d\mathbf{r} \sum_{\sigma\sigma'} \sum_{\tau\tau'} r^L Y_L(\hat{r}) O(\sigma\tau, \sigma'\tau') \hat{\psi}^\dagger(\mathbf{r}\sigma\tau) \hat{\psi}(\mathbf{r}\sigma'\tau')$$

$$\hat{\psi}^\dagger(\mathbf{r}\sigma\tau) \hat{\psi}^\dagger(\mathbf{r}\tilde{\sigma}'\tilde{\tau}')$$

$$\hat{\psi}(\mathbf{r}\sigma\tau) \hat{\psi}(\mathbf{r}\tilde{\sigma}'\tilde{\tau}')$$

Rich variety of collective vibrations



$$F = \sum_{\sigma, \sigma'} \sum_{\tau, \tau'} \int_{\text{space}} d\mathbf{r} r^L Y_L(\hat{\mathbf{r}}) \psi^\dagger(\mathbf{r} \sigma \tau) \langle \sigma | \begin{Bmatrix} 1 \\ \vec{\sigma} \end{Bmatrix} | \sigma' \rangle \langle \tau | \begin{Bmatrix} 1 \\ \vec{\tau} \end{Bmatrix} | \tau' \rangle \psi(\mathbf{r} \sigma' \tau')$$

space
spin
isospin

observables: $S(\omega) = \sum_n |\langle n | \hat{F} | 0 \rangle|^2 \delta(\omega - \omega_n)$

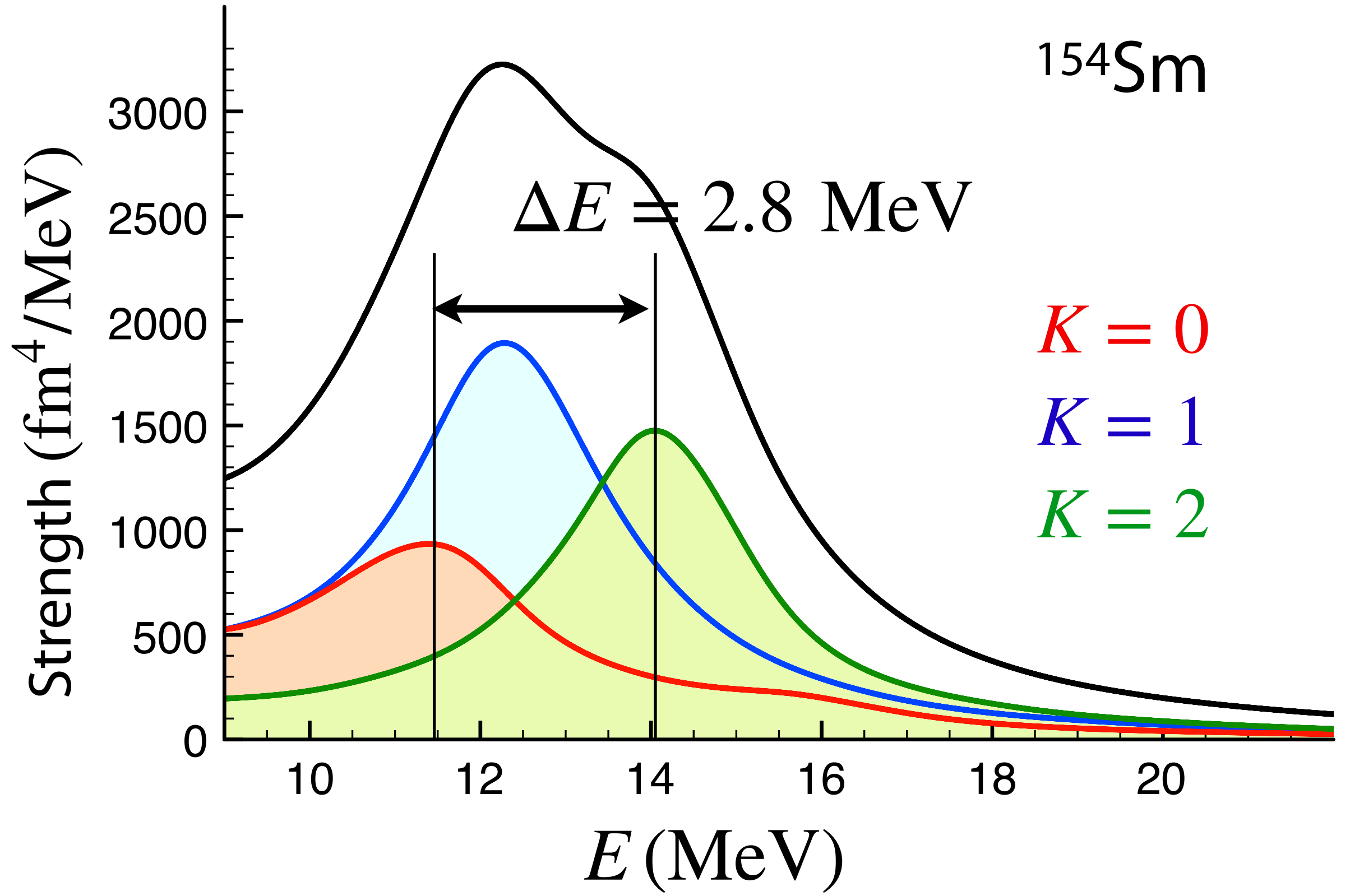
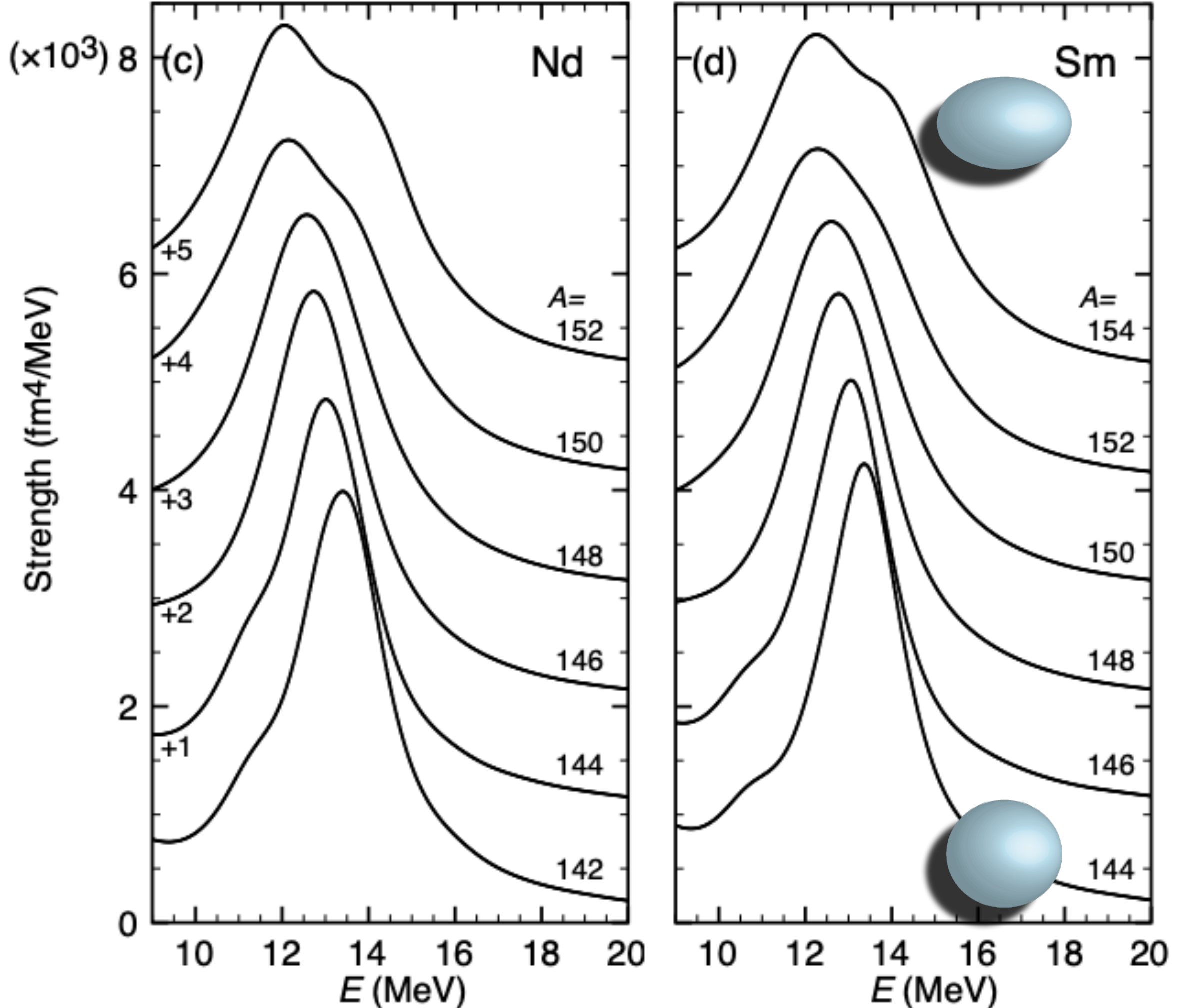
high-frequency modes

$$\omega_{\text{GR}} > \omega_0 = 41 \times A^{-1/3}$$

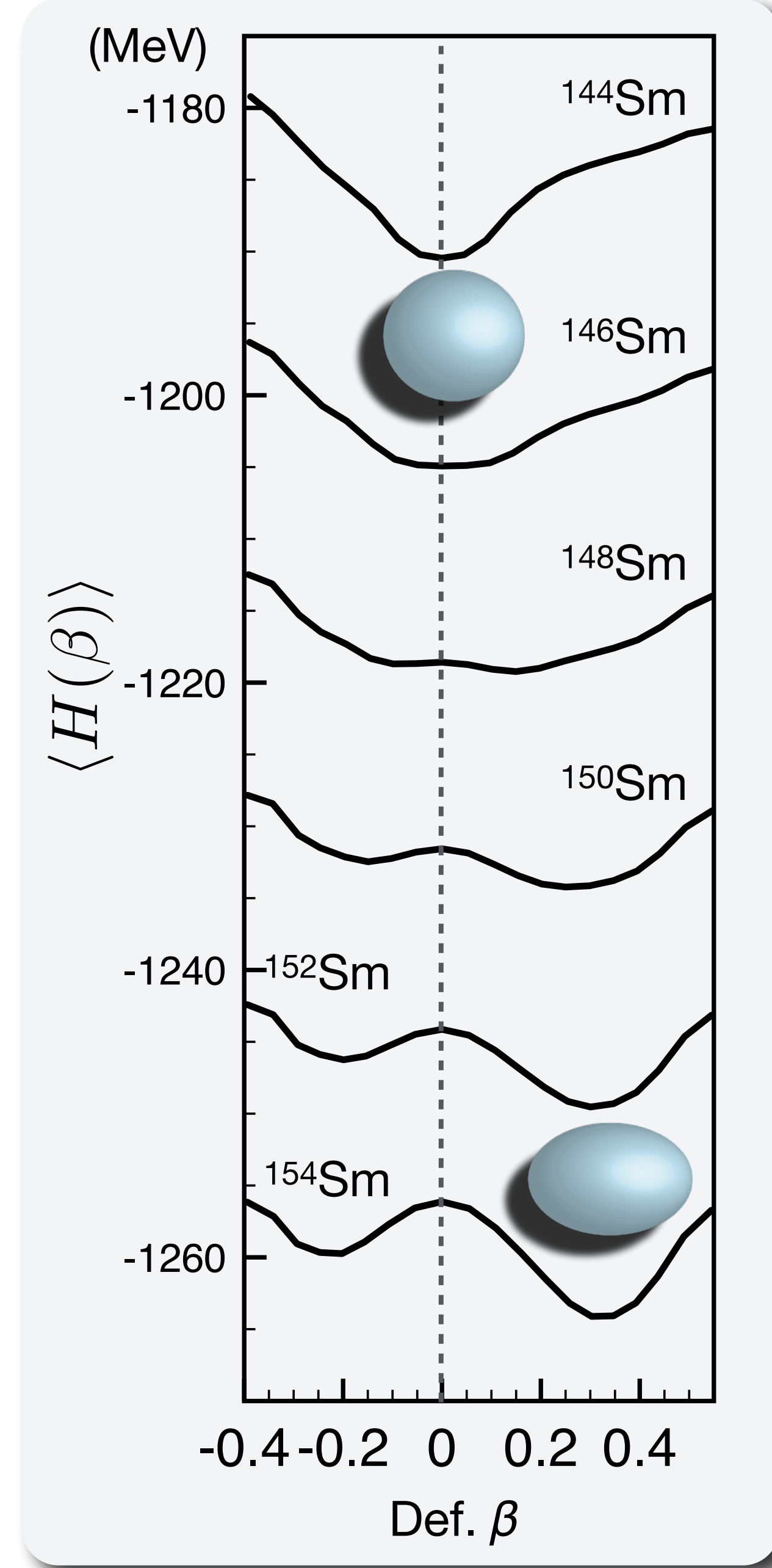
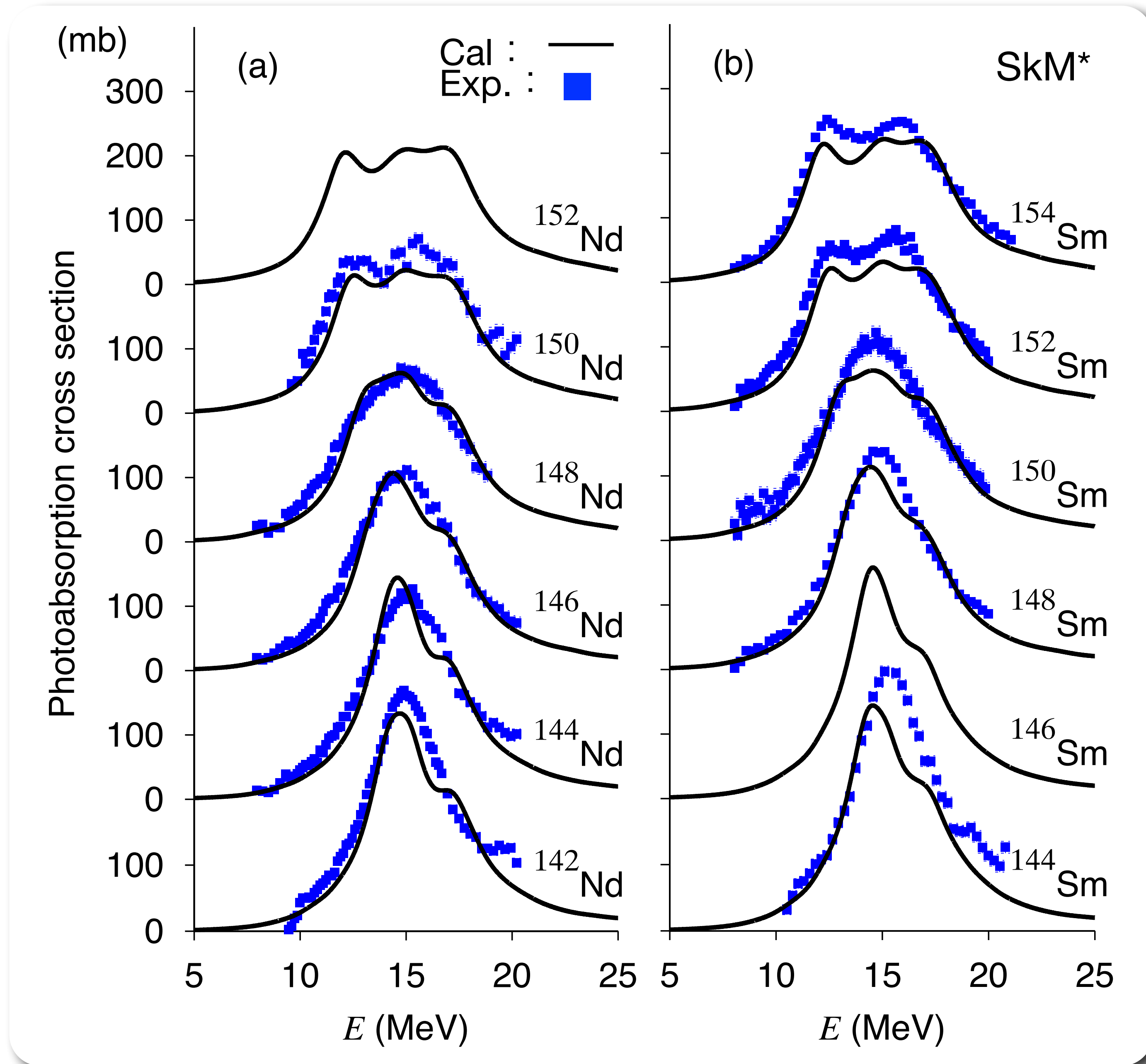
GR is strongly excited by a one-body operator, and **exhausts a sum-rule value**

Splitting/broadening of the resonance due to deformation

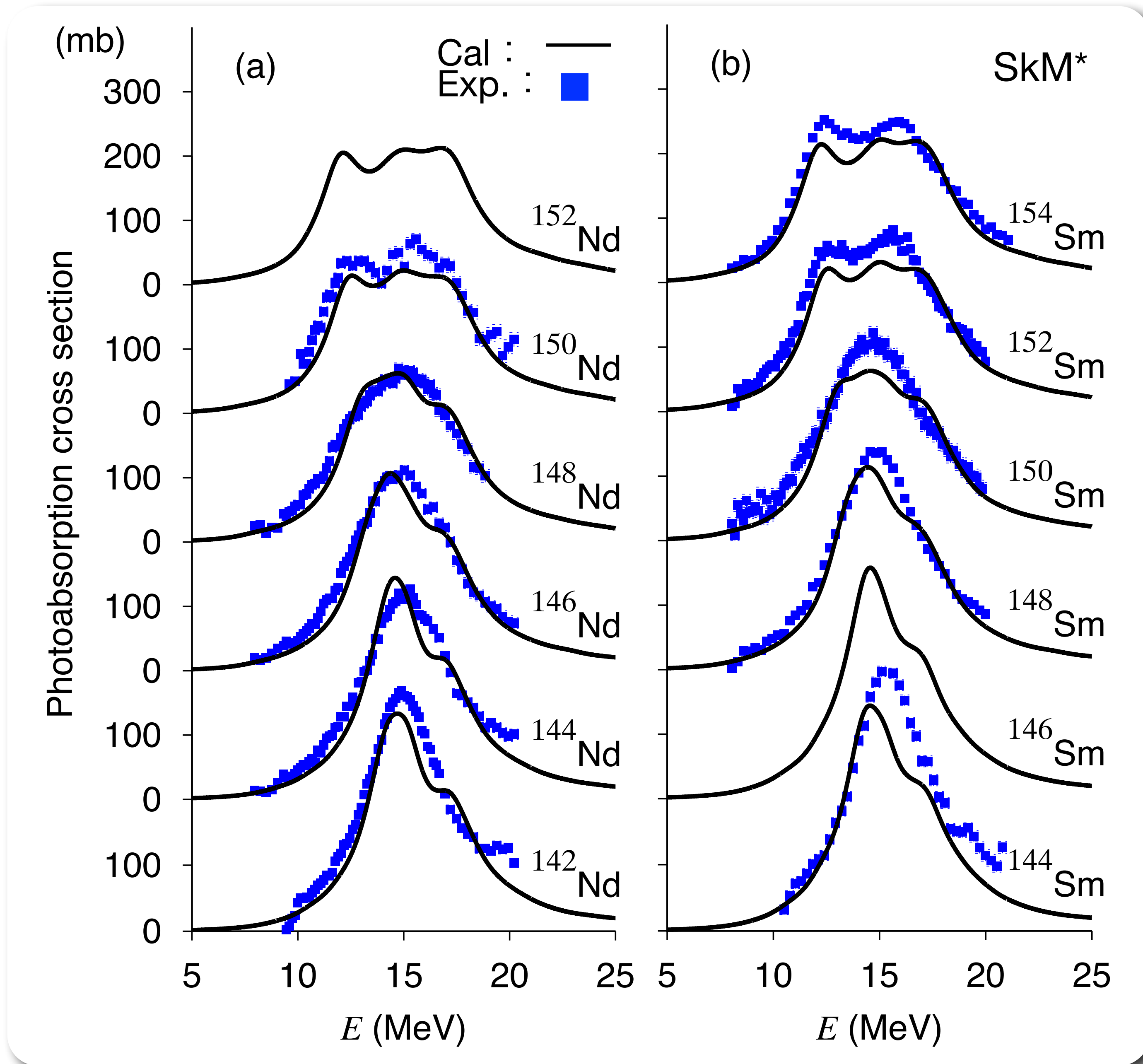
$$S(\omega) = \sum_n |\langle n | \hat{F} | 0 \rangle|^2 \delta(\omega - \omega_n), \quad \hat{F} = \int r^2 Y_{2K}(\hat{r}) \psi^\dagger \psi$$



Shape evolution seen in Giant Dipole Resonance

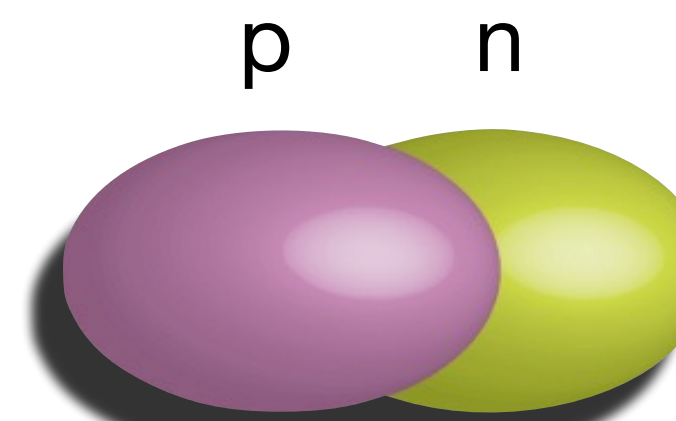


Shape evolution seen in Giant Dipole Resonance



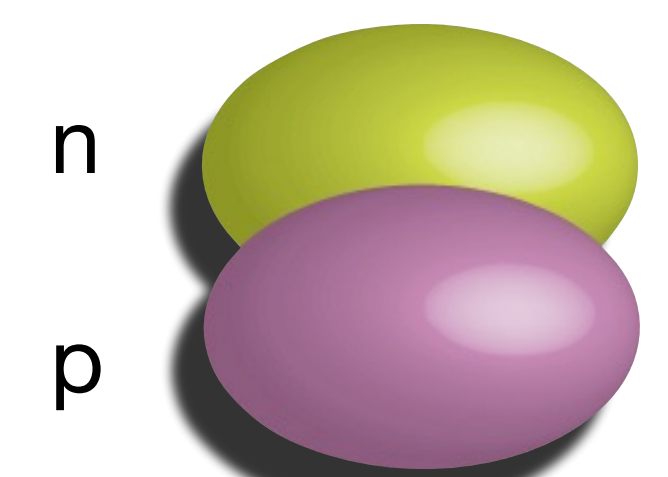
collective oscillation $\omega \propto \frac{1}{R}$

$$\omega_{\parallel} \propto \frac{1}{R_{\parallel}}$$



$K=0$

$$\omega_{\perp} \propto \frac{1}{R_{\perp}}$$



$K=1$

classically understood

Deformation effect in Giant Monopole Resonance?

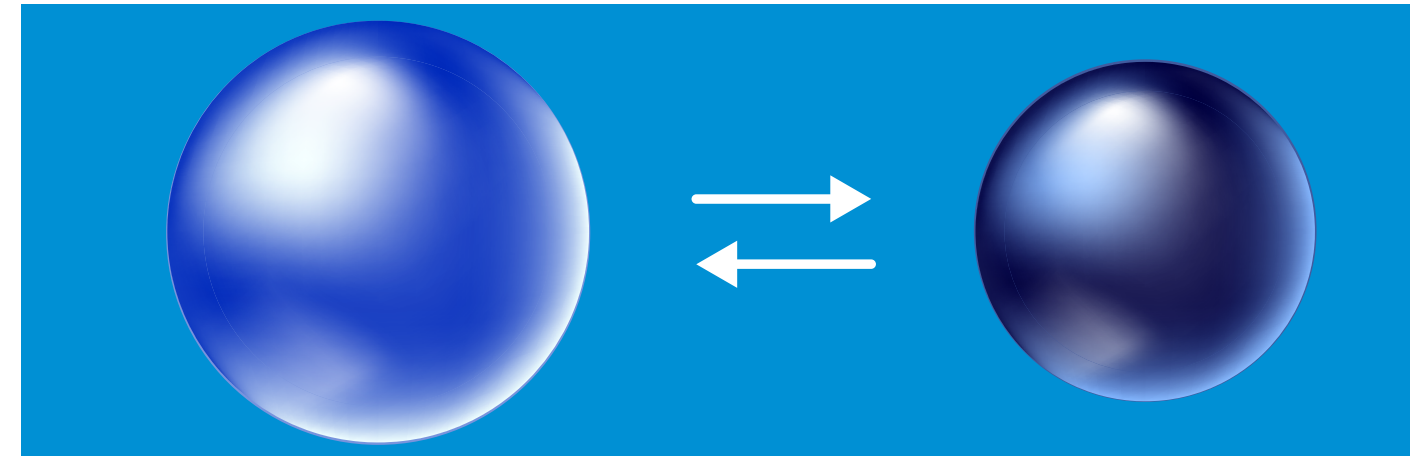
$$F = \int d\vec{r} r^2 \psi^\dagger(\vec{r}) \psi(\vec{r})$$

volume change



incompressibility of nuclear matter

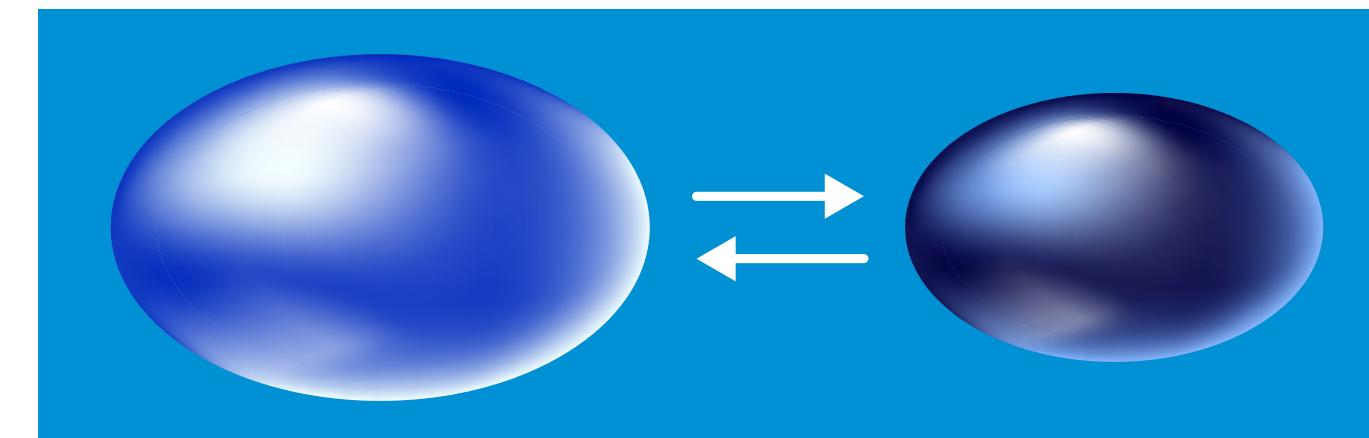
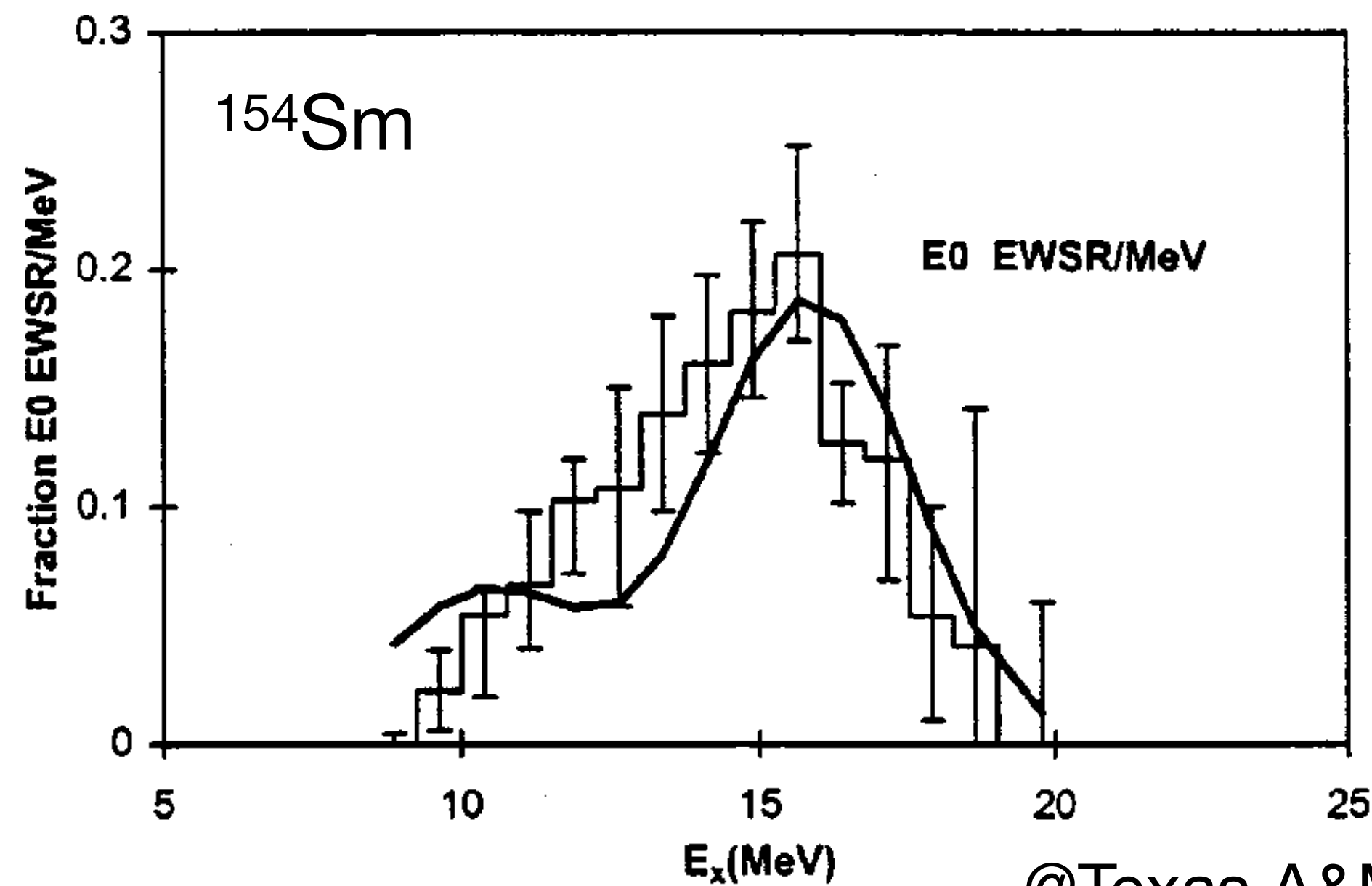
J.-P. Blaizot,
Phys. Rep. 64(1980)171



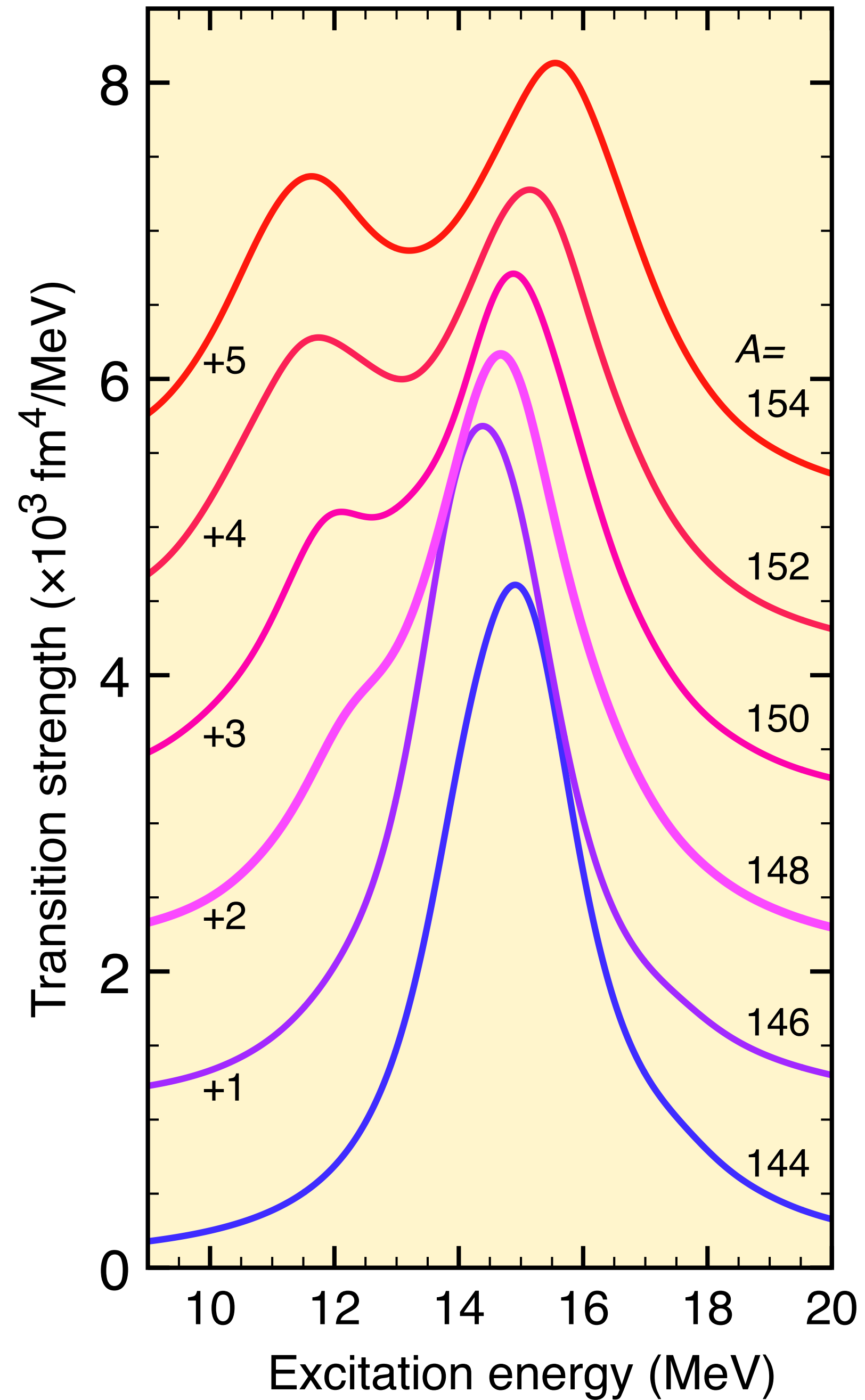
deformation splitting?

U. Garg, *et al.*, PRL45(1980)1670

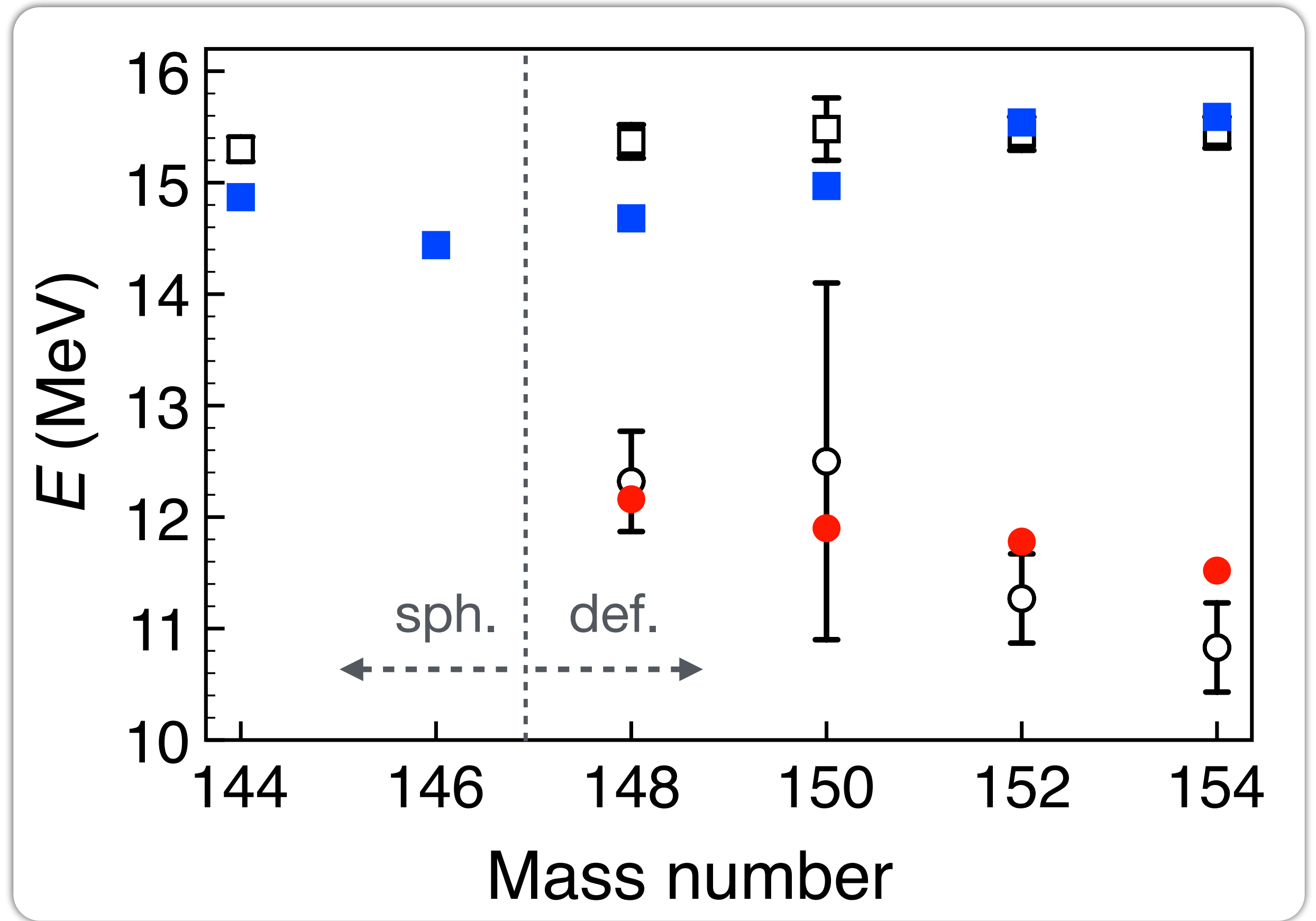
D. H. Youngblood, *et al.*, PRC60(1999)067302



no angle dependence as in GDR



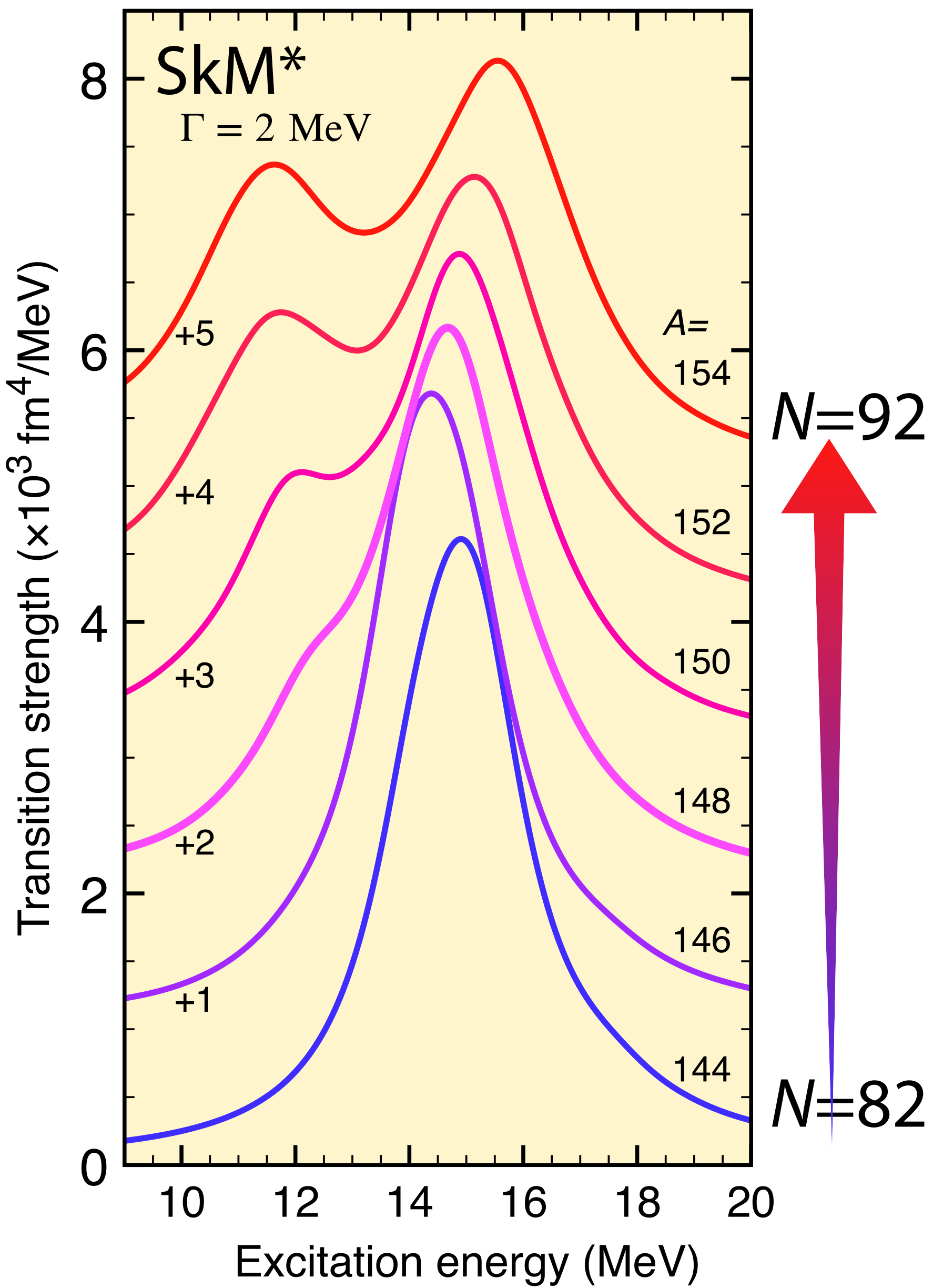
GMR energy in the Sm ($Z=62$) isotopes



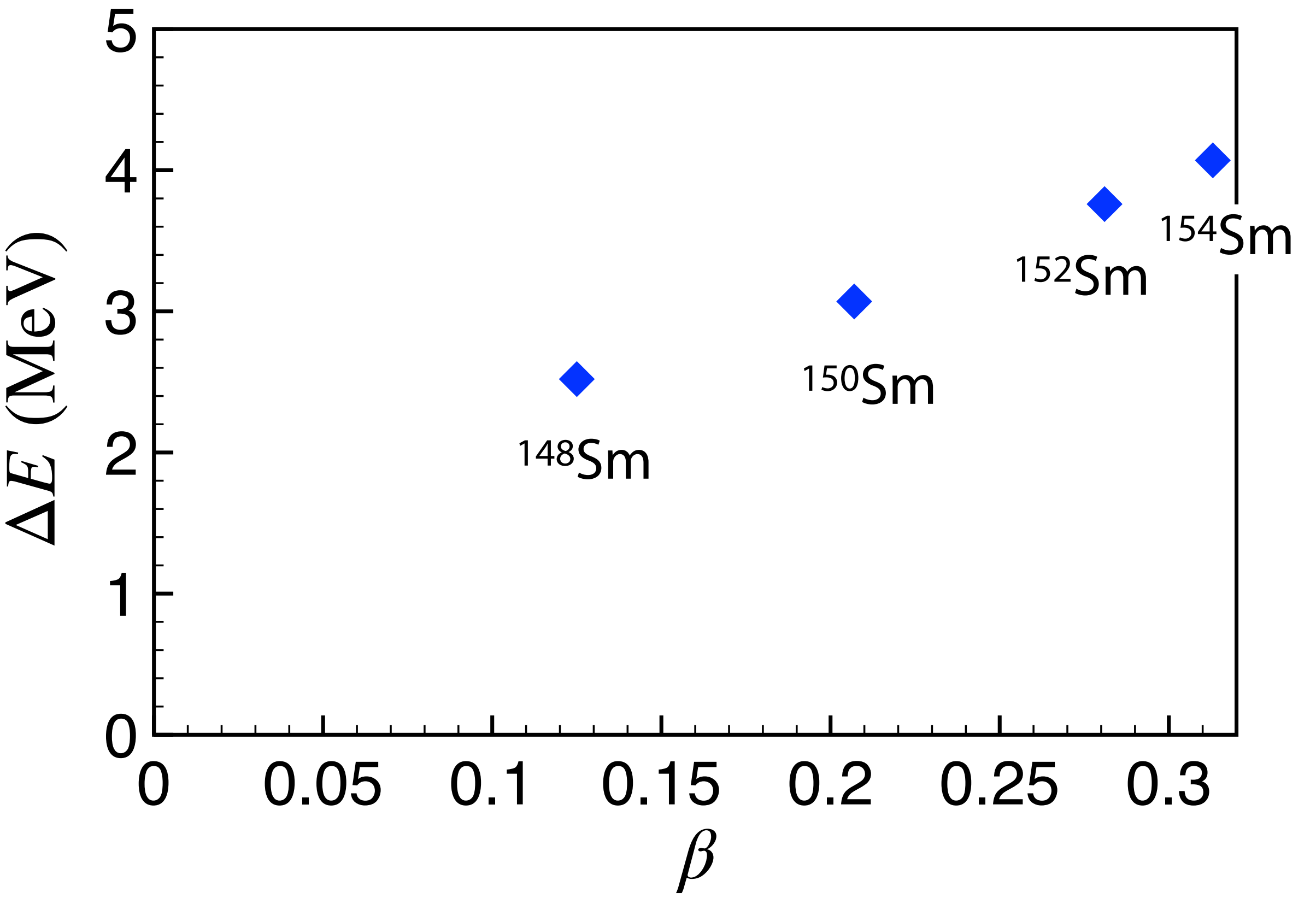
Exp.: M. Itoh, *et al.*, PRC68(2003)064602

GMR in the Sm isotopes

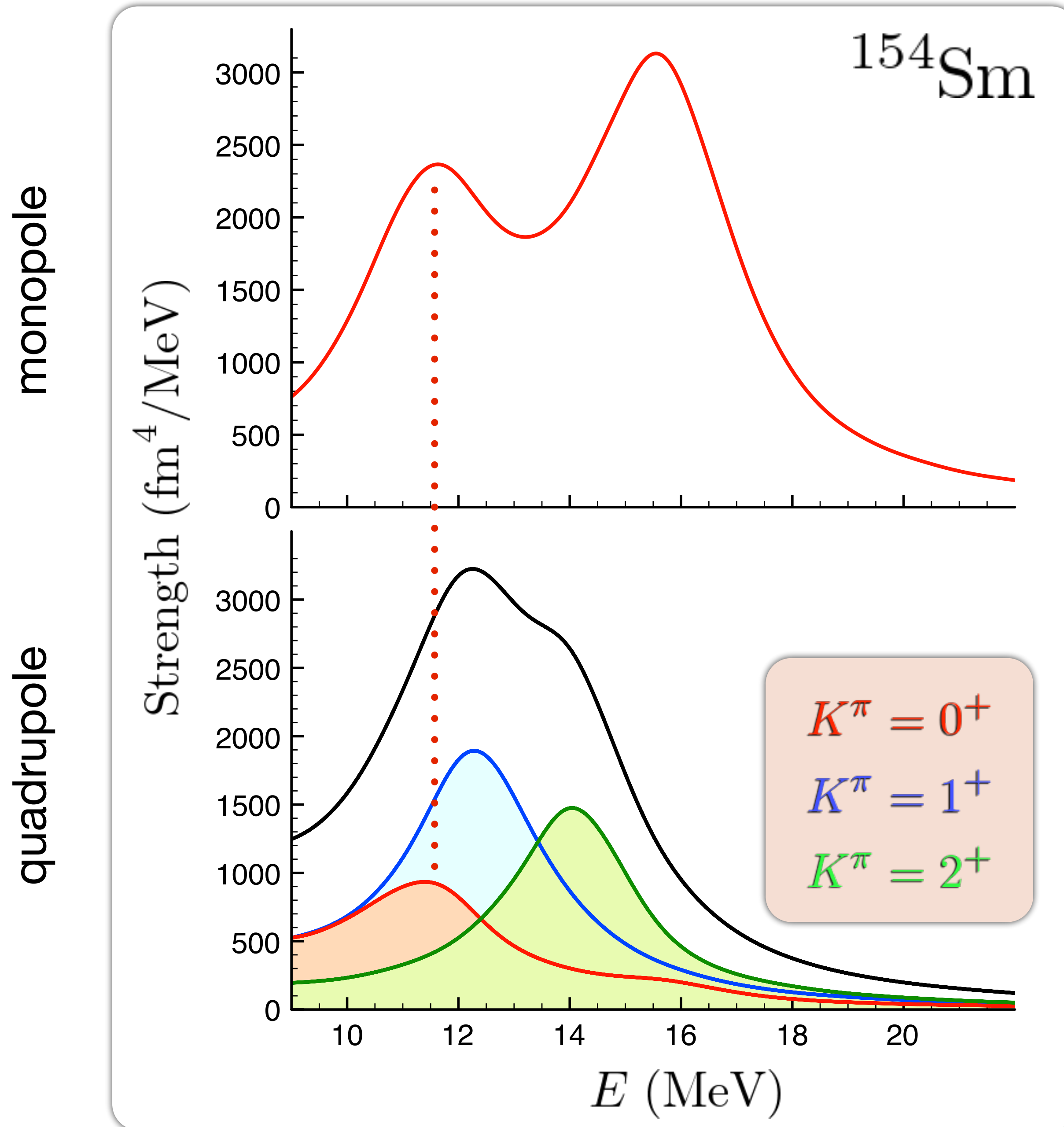
Yoshida–Nakatsukasa ('13)



deformation splitting



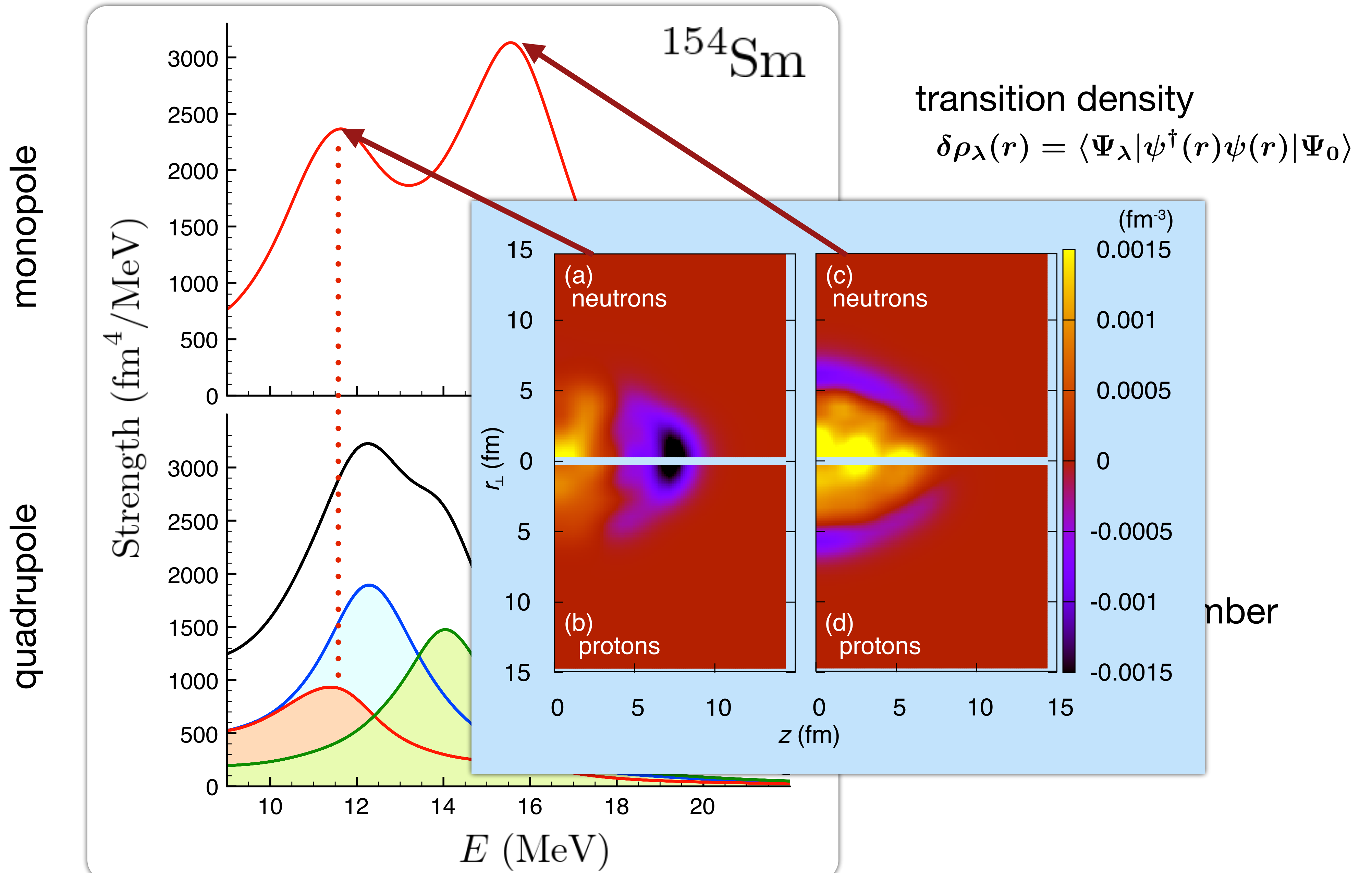
Coupling/mixing of GMR and GQR



$$K=J_z$$

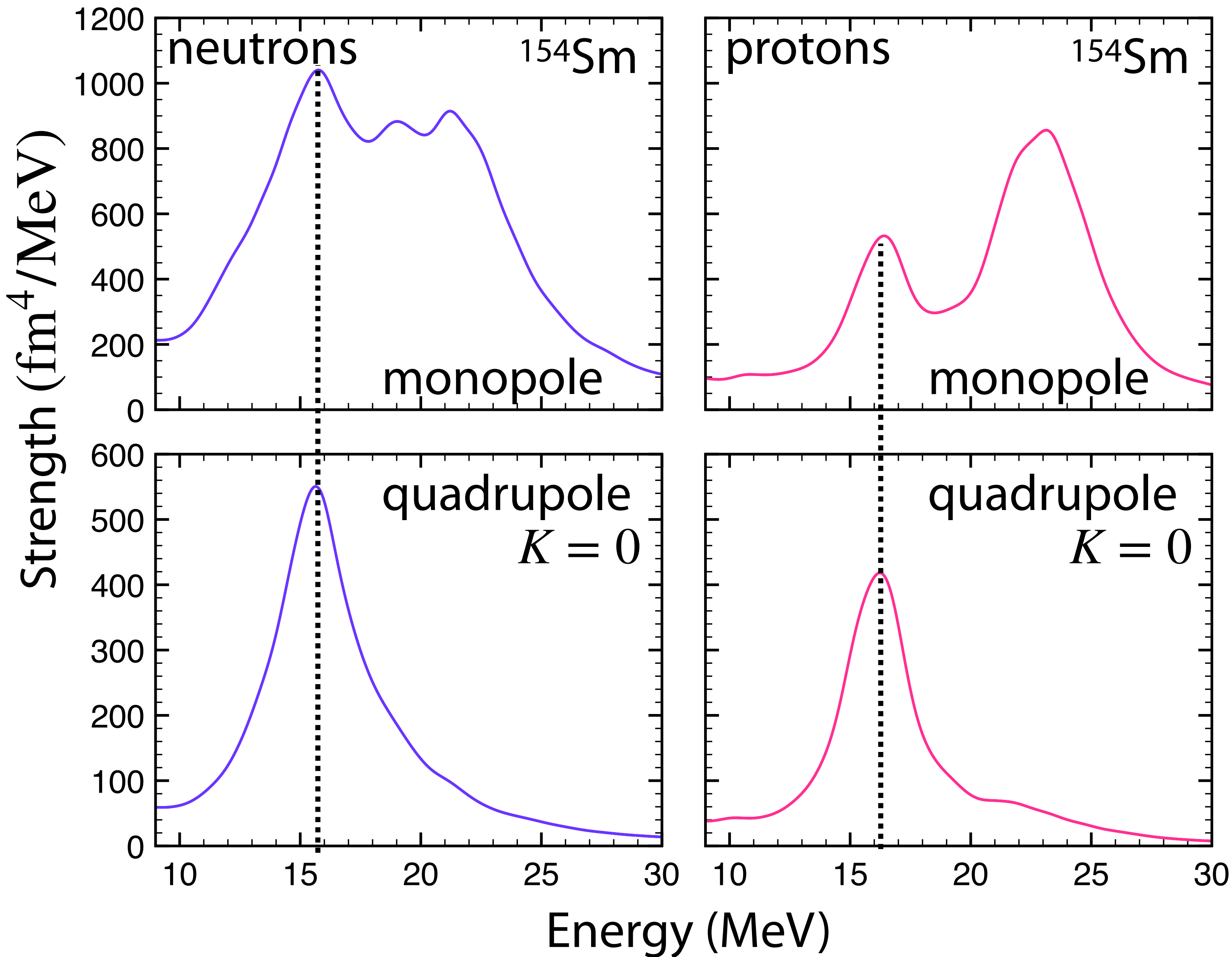
good quantum number

Coupling/mixing of GMR and GQR



Coupling/mixing at the static level

KY, PRC104(2021)044309

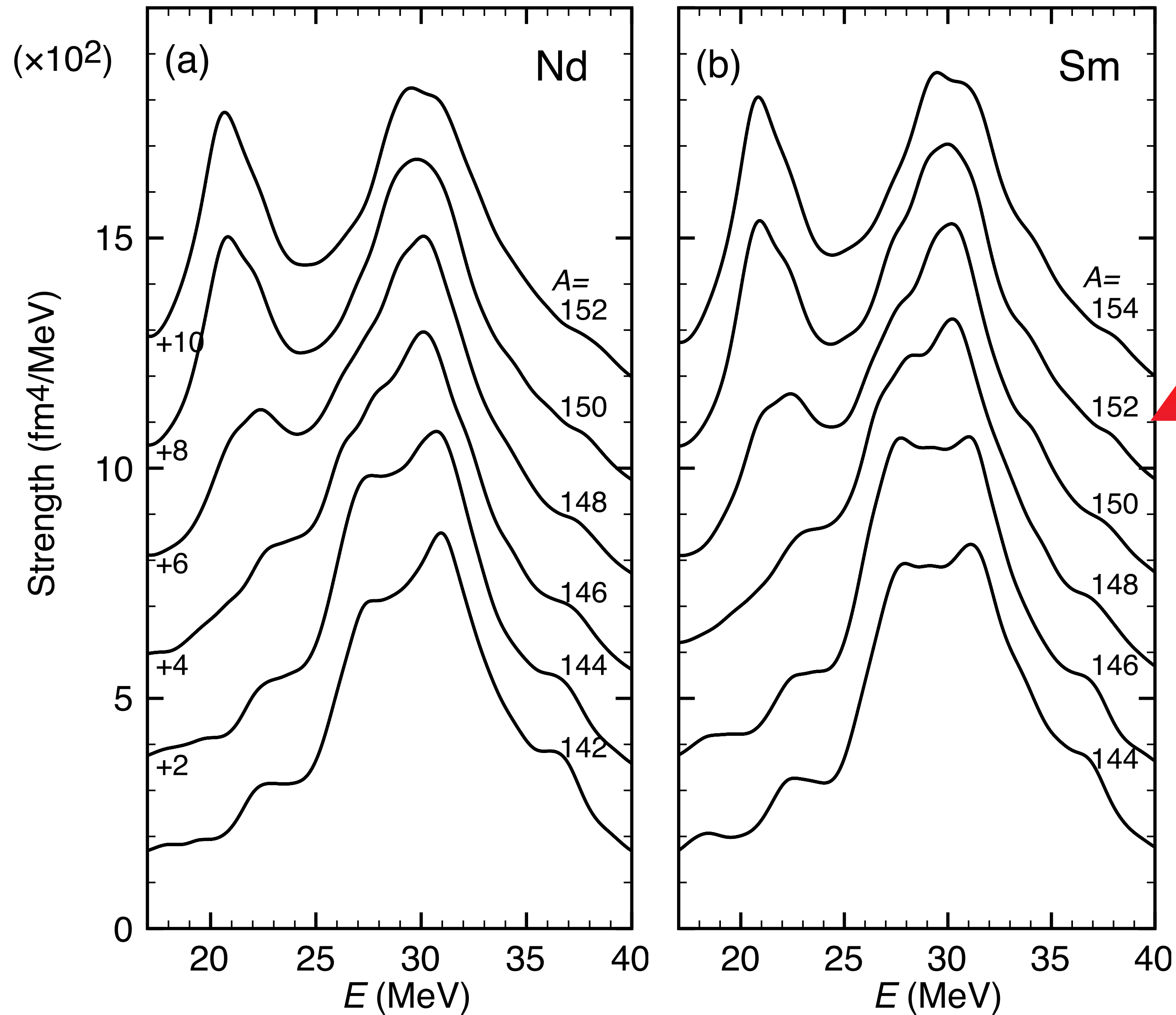


Unperturbed strengths
w/o the RPA (dynamic) correlations
deformation-induced coupling

Peaks
↓
monopole and $K=0$ quadrupole
coincide in energy
residual interactions
↓
Coexistence persists

Isovector (IV)-GMR in deformed nuclei

$$F = \sum_{\tau\tau'} \int d\vec{r} r^2 \psi^\dagger(\vec{r}\tau) \langle \tau | \tau_3 | \tau' \rangle \psi(\vec{r}\tau')$$



emergence of
deformation "splitting"

$$\Delta E \sim 10 \text{ MeV} @ ^{154}\text{Sm}$$

$$\sim 2 \times \Delta E(\text{ISGMR})$$

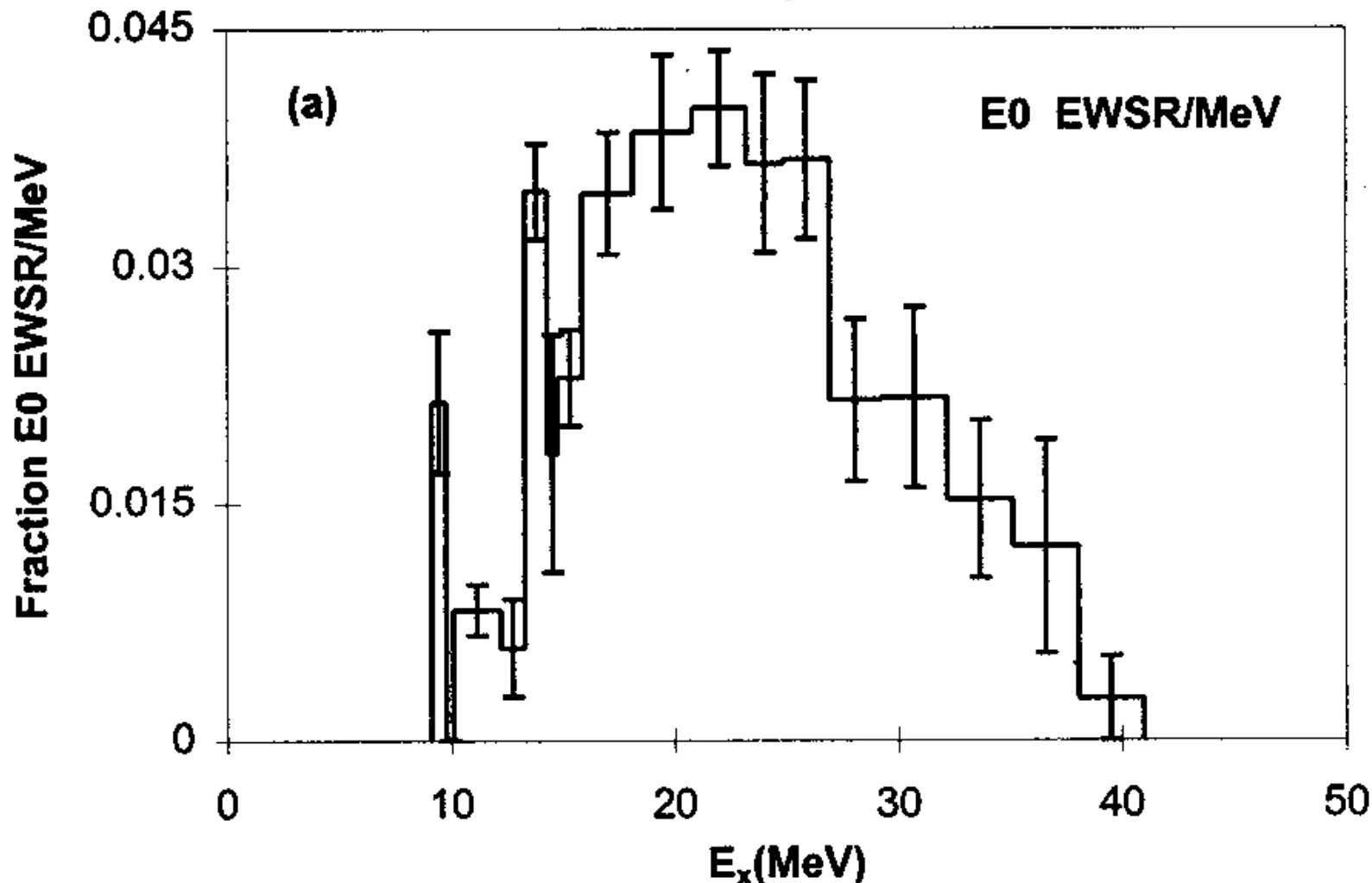
due to the coupling to
the $K = 0$ of IV-GQR

Deformation effect on GMR in light nuclei: universality

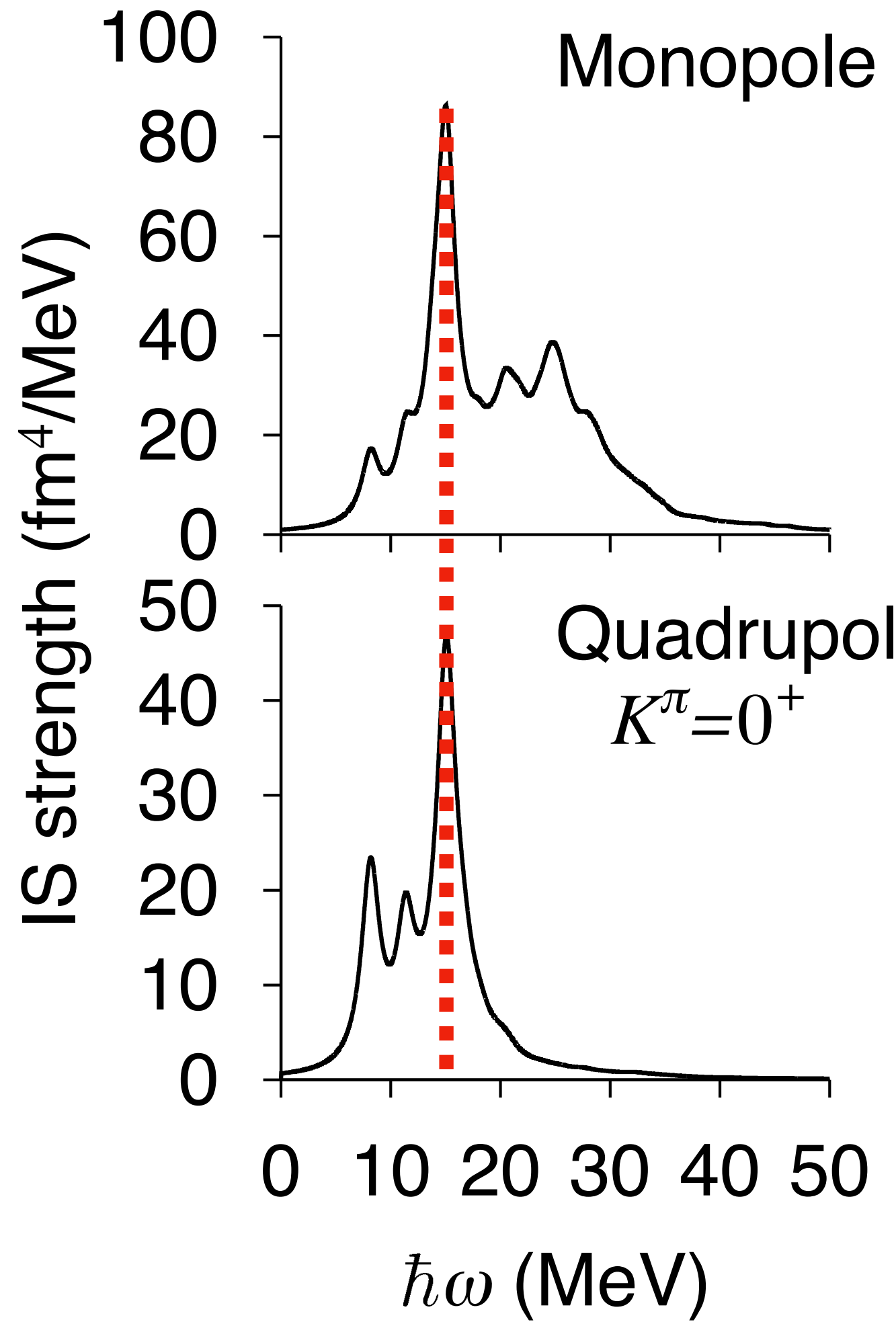
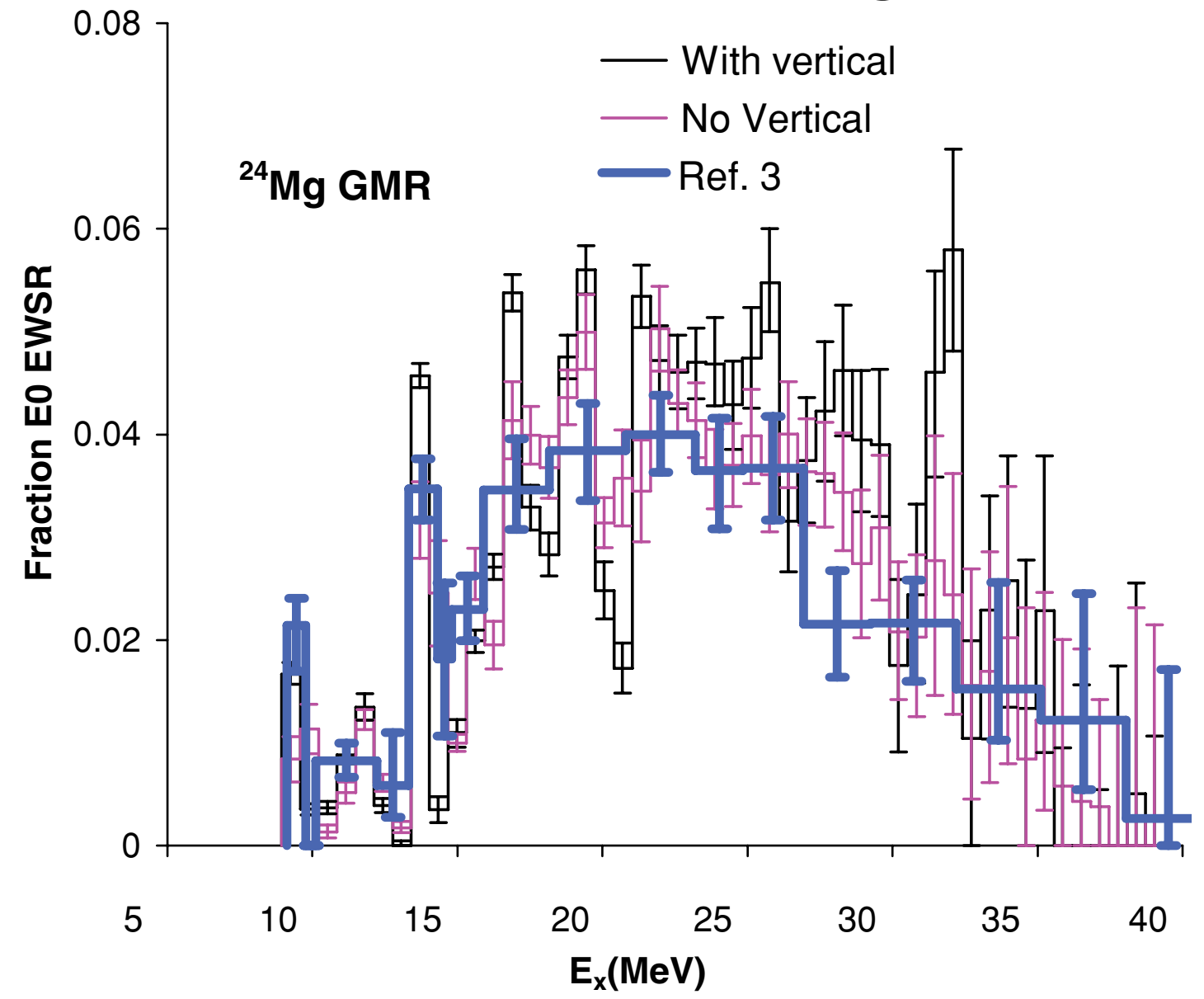
KY, Mod. Phys. Lett. A 25 (2010), 1783

^{24}Mg @ TAMU

Youngblood+('99)



Youngblood+('09)



occurrence of the "lower-energy (~ 15 MeV)" peak due to coupling to the $K=0$ of GQR

Deformation splitting in a light nucleus

Physics Letters B 748 (2015) 343–346



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Splitting of ISGMR strength in the light-mass nucleus ^{24}Mg due to ground-state deformation

Y.K. Gupta^{a,1}, U. Garg^a, J.T. Matta^a, D. Patel^a, T. Peach^a, J. Hoffman^{a,2}, K. Yoshida^{b,c}, M. Itoh^{d,3}, M. Fujiwara^d, K. Hara^d, H. Hashimoto^d, K. Nakanishi^d, M. Yosoi^d, H. Sakaguchi^e, S. Terashima^e, S. Kishi^e, T. Murakami^e, M. Uchida^{e,4}, Y. Yasuda^e, H. Akimune^f, T. Kawabata^{g,5}, M.N. Harakeh^h

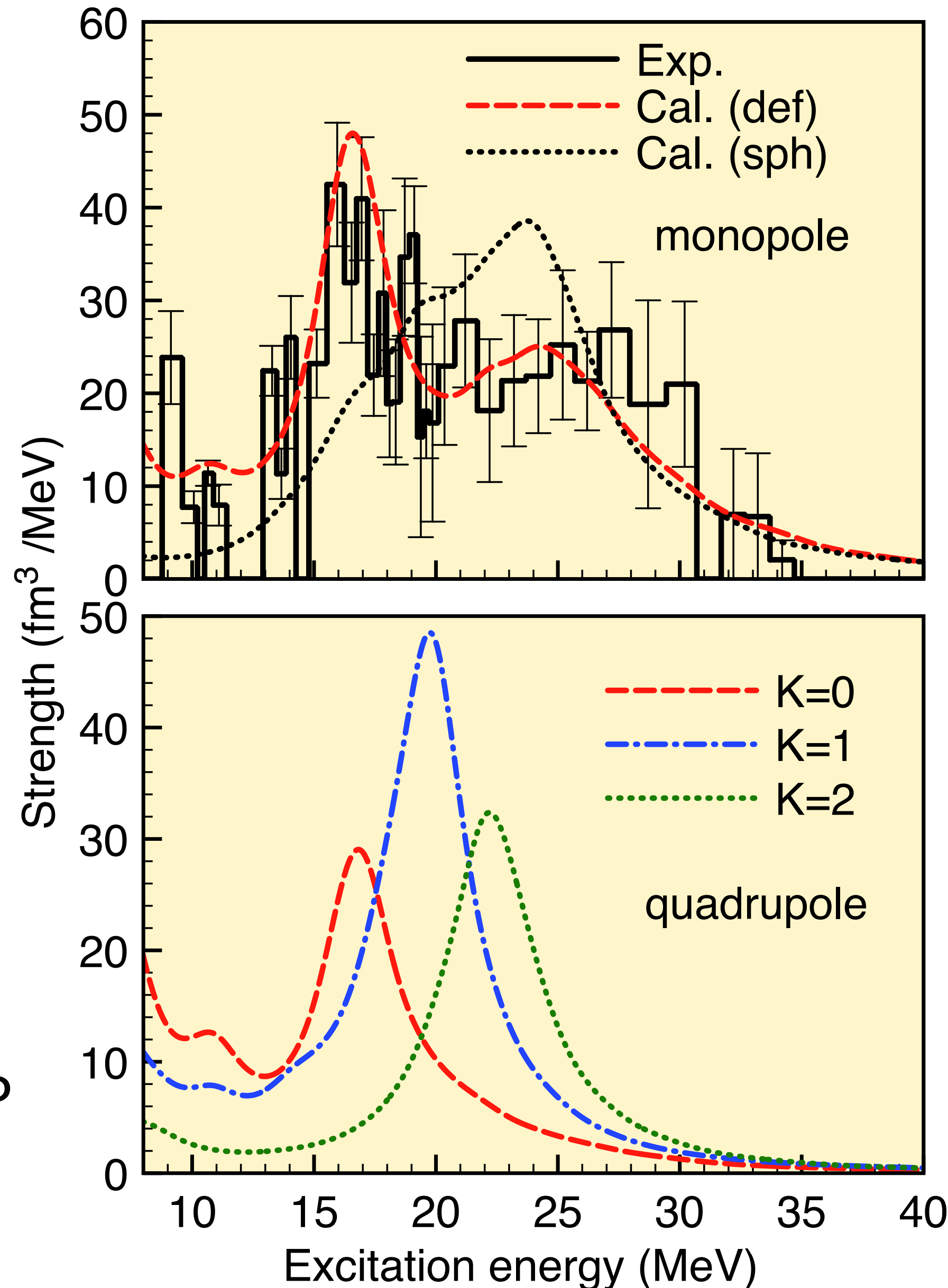
First observation of the splitting of GMR strengths in a light system

universal feature in deformed nuclei



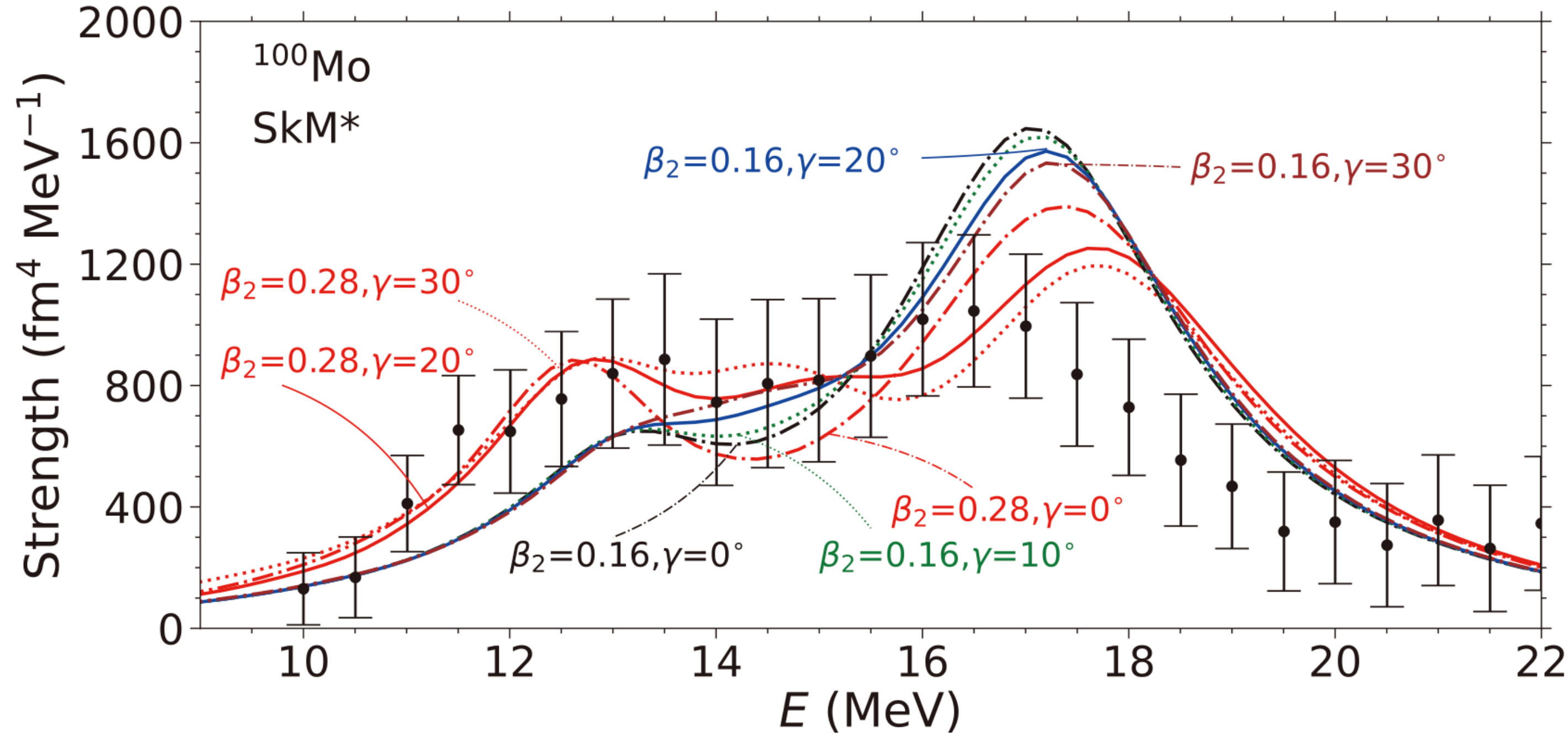
background-free high-resolution experiment @RCNP

parameter-free nuclear DFT calculation



A role of the triaxiality

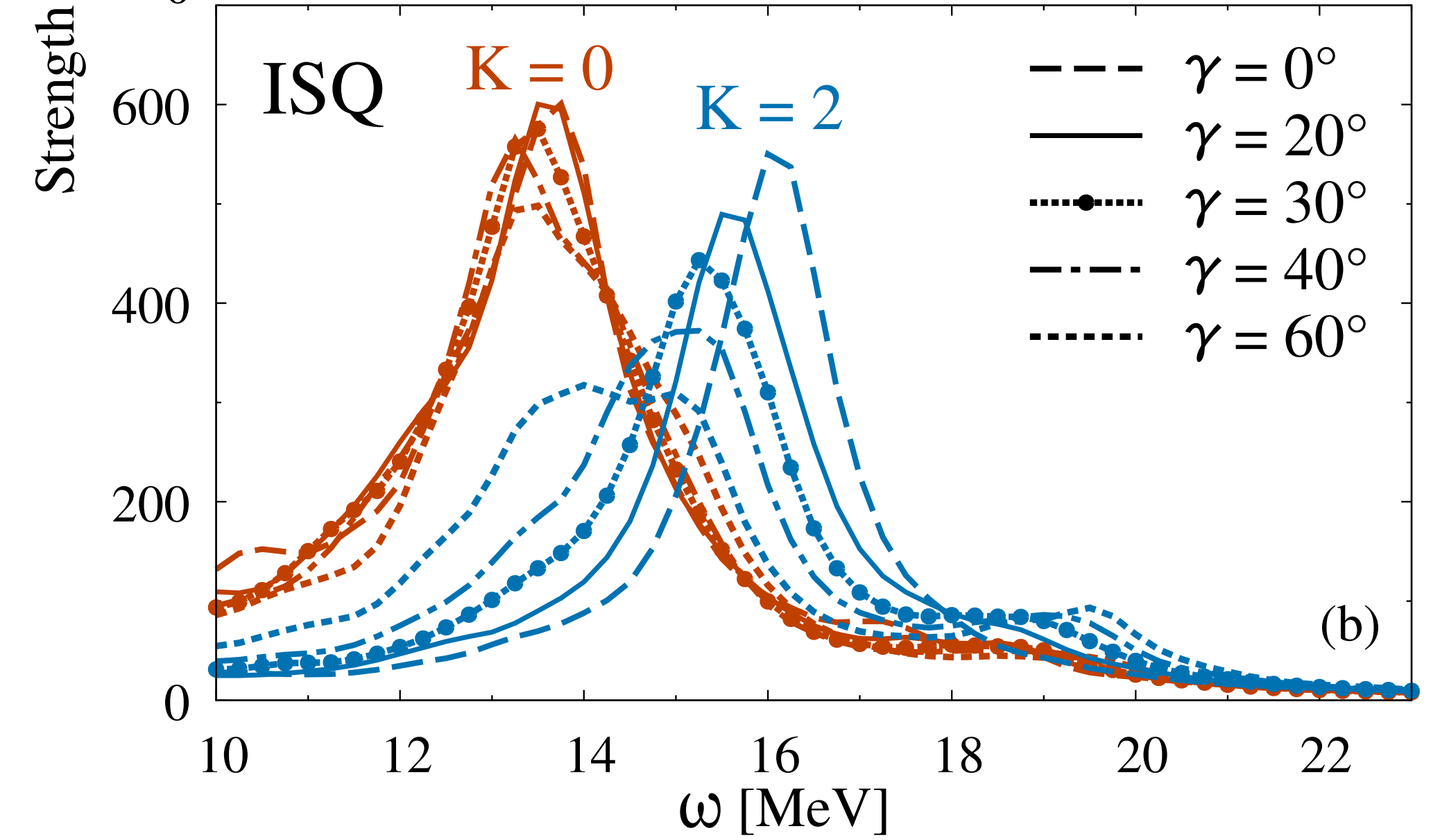
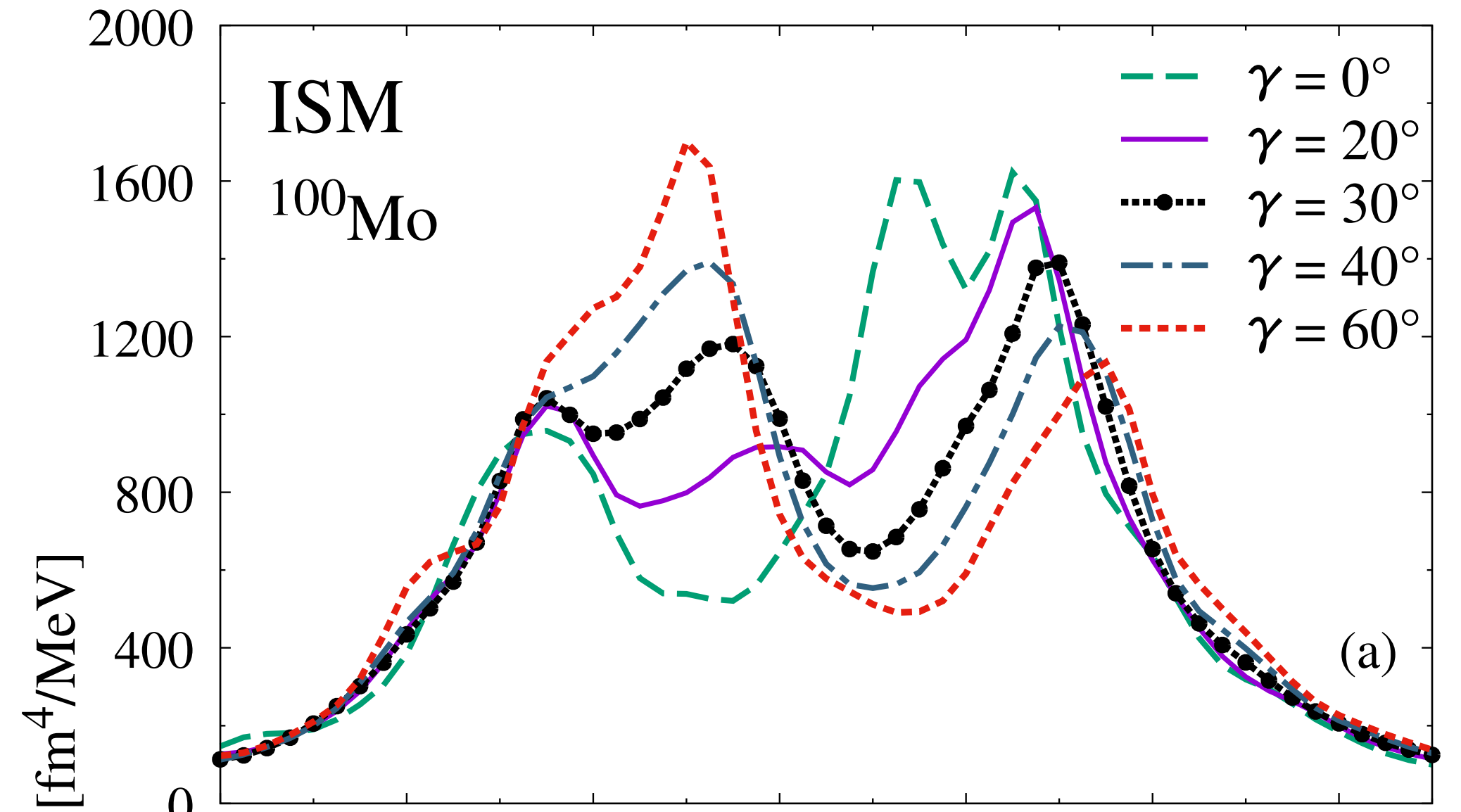
Shi–Stevenson, CPC47(2023)034105



Exp. @RCNP

Howard, Garg, Ito+, PLB807(2020)135608

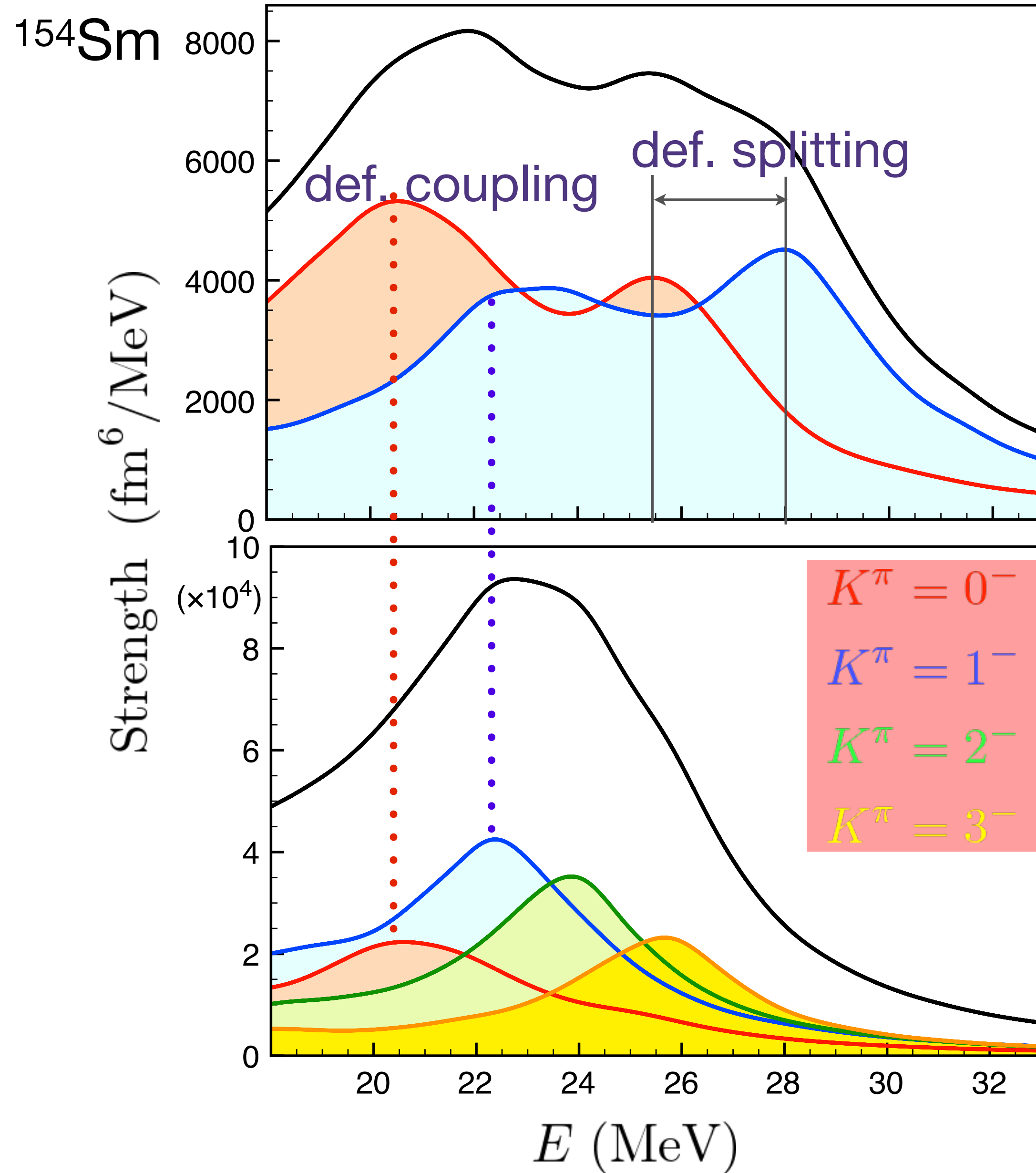
Washiyama–Ebata–Yoshida, PRC109(2024)024317



Isoscalar GDR and high-energy octupole resonance

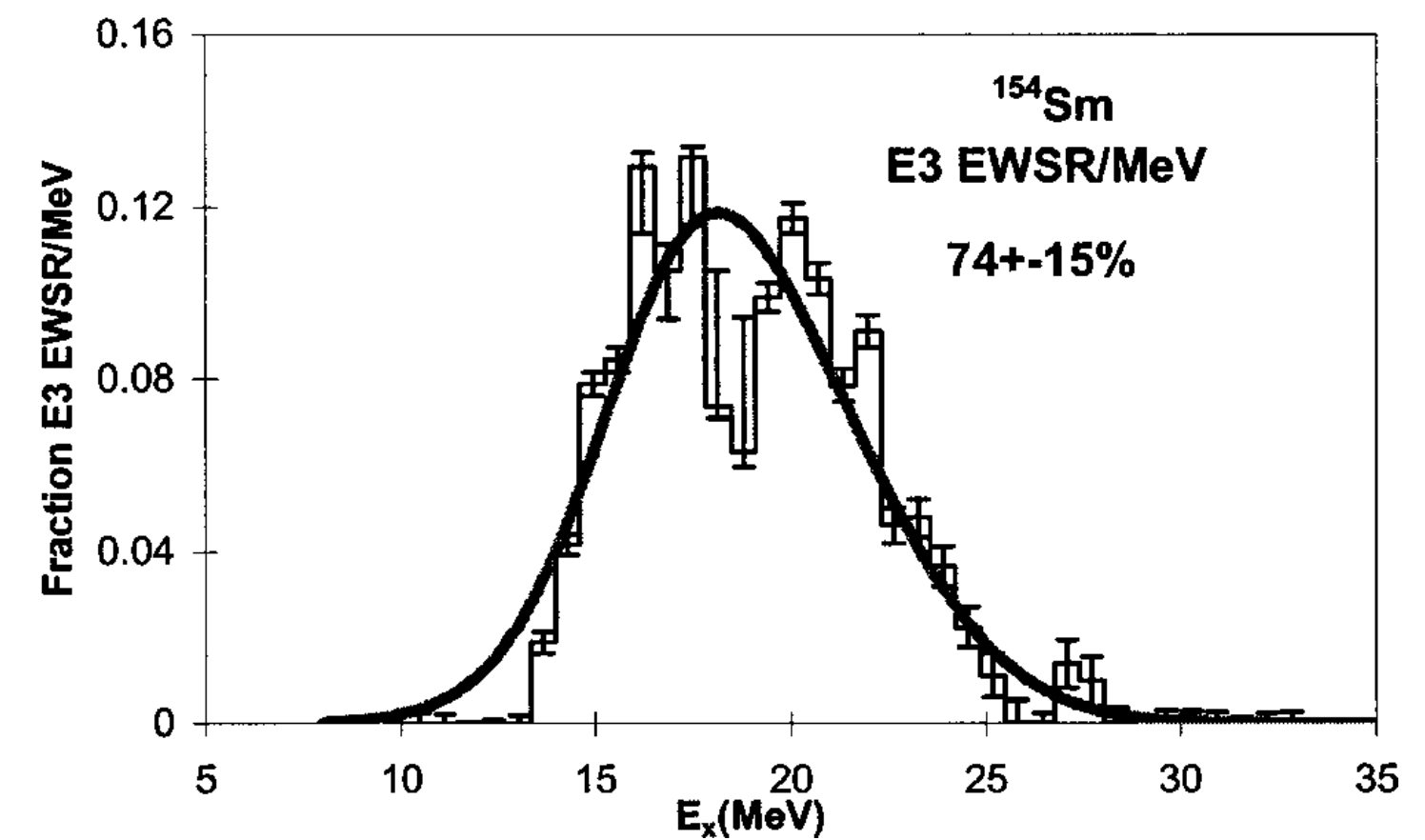
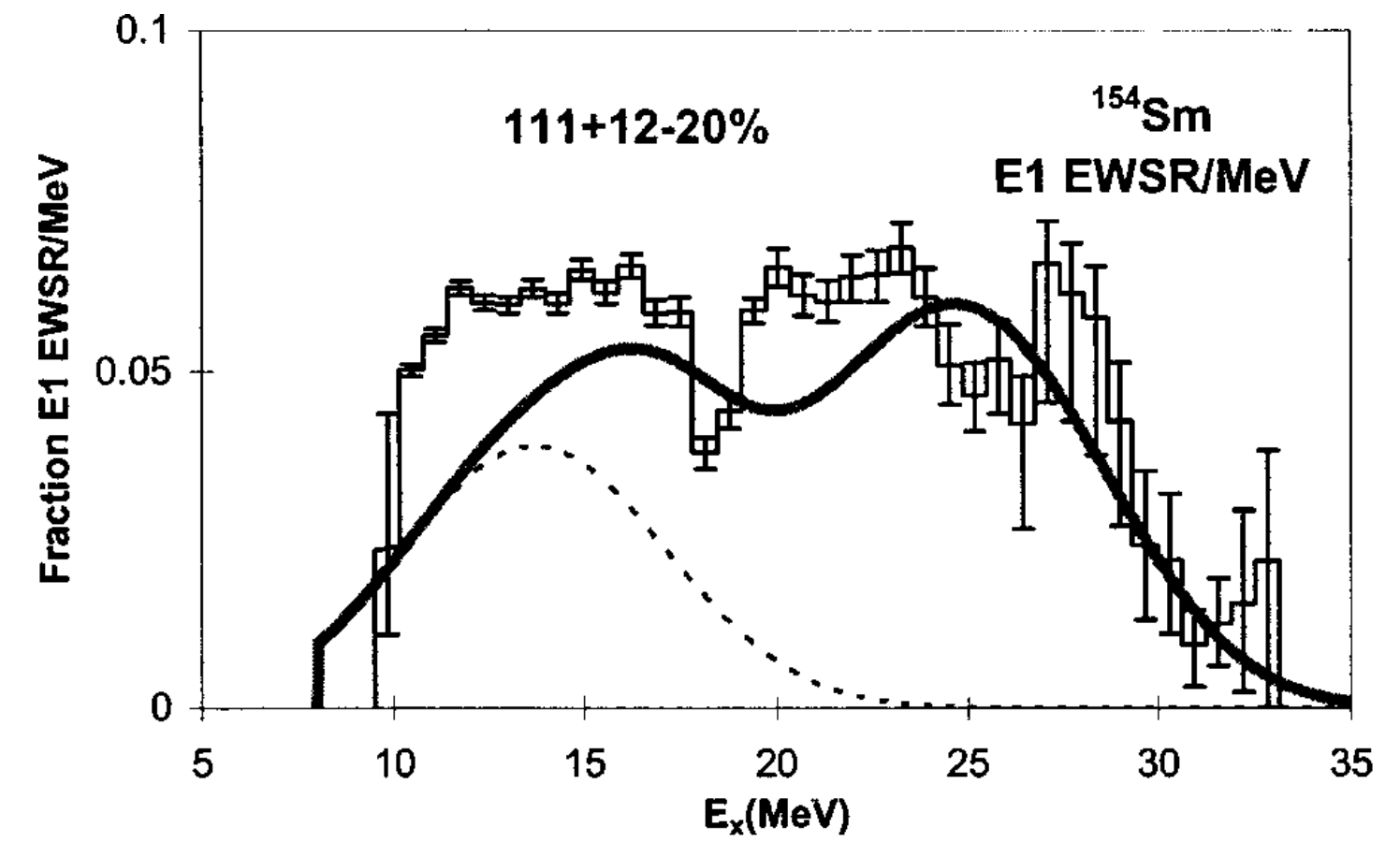
KY, T. Nakatsukasa, PRC88(2013)034309

$$F = \sum_{\tau} \int d\vec{r} r^3 Y_1(\hat{r}) \psi^{\dagger}(\vec{r}\tau) \psi(\vec{r}\tau)$$



large width of the ISGDR

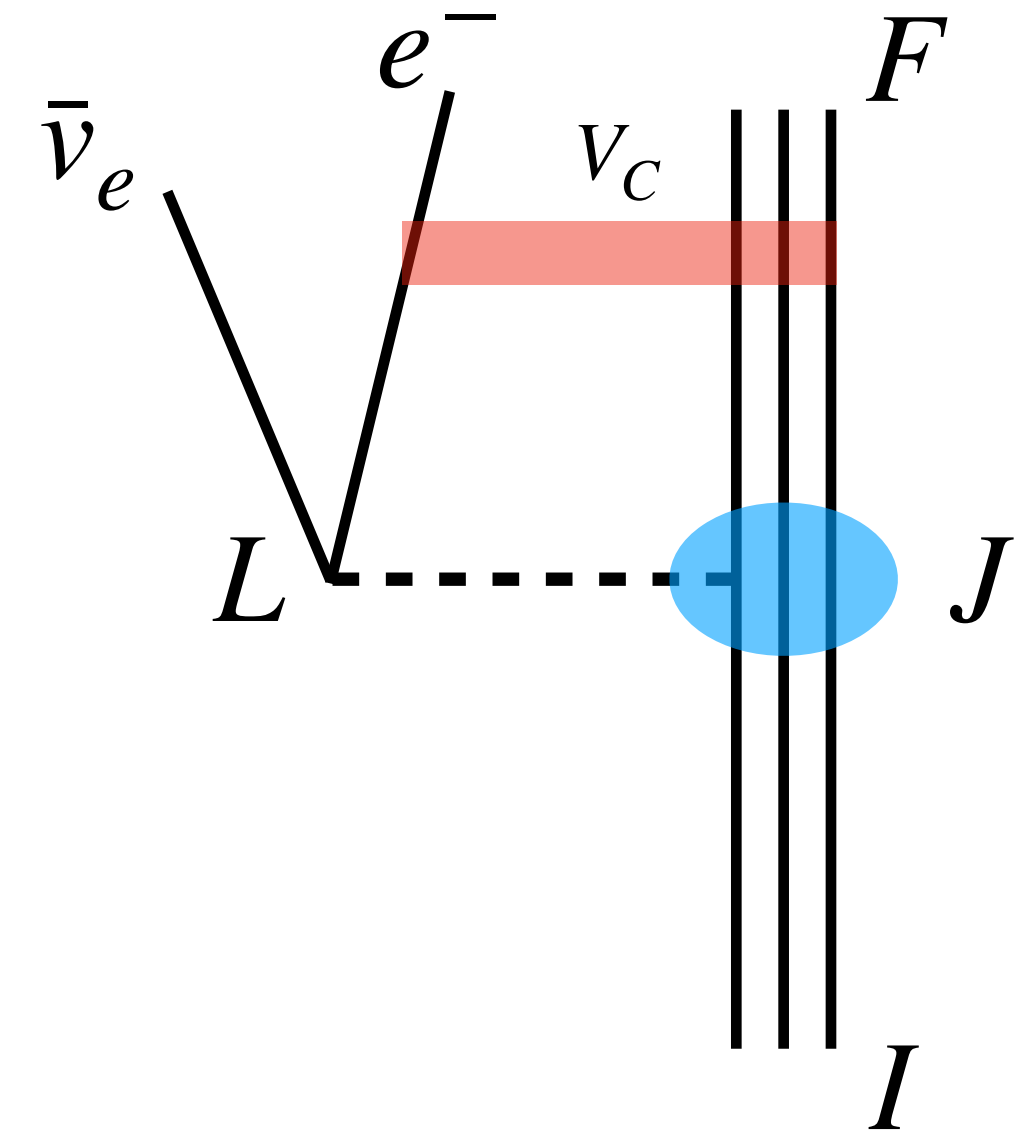
D. H. Youngblood et al.,
PRC69(2004)034315



Nuclear beta decay

transition matrix element:

$$T_{fi} = \frac{G_F V_{ud}}{\sqrt{2}} \int dx \bar{\psi}_{e^-}(\mathbf{x}) \gamma^\mu (1 - \gamma_5) \psi_\nu(\mathbf{x}) \langle F | \underline{J_\mu(\mathbf{x})} | I \rangle$$



Nuclear currents involving not only the nuclear many-body wave functions but the form factors and momentum transfer

$$J^\mu(\mathbf{x}) = \mathcal{V}^\mu(\mathbf{x}) - \mathcal{A}^\mu(\mathbf{x})$$

vector currents $\mathcal{V}^\mu = (V^0, \mathbf{V})$ axial-vector currents $\mathcal{A}^\mu = (A^0, \mathbf{A})$

decay rate:
$$\Gamma = \frac{(G_F V_{ud})^2}{\pi^2} \int_{m_e}^{E_0} dE_e p_e E_e (E_0 - E_e)^2 \sum_{J, \kappa_e, \kappa_\nu} \frac{1}{2J_i + 1} \left| \sum_L \langle f | \underline{\Xi_{JL}(\kappa_e, \kappa_\nu)} | i \rangle \right|^2$$

multipole operator

One-body charged-current operators: Impulse Approx.

Gamow–Teller type
spatial component

$$\mathbf{A}(\mathbf{r}) = \sum_{j=1}^A \delta(\mathbf{r} - \mathbf{r}_j) g_A \boldsymbol{\sigma}_j \tau_j^\pm$$

momentum transfer:
 $\mathbf{q} = \mathbf{p}'_j - \mathbf{p}_j$

$$\langle f || \sum_L \Xi_{JL}(\kappa_e, \kappa_\nu) || i \rangle = \text{sign}(\kappa_e) \int_0^\infty dr r^2 [\rho_{J-1J}^\sigma(r) \phi_a(r) + \rho_{J+1J}^\sigma(r) \phi_b(r)]$$

$$g_A(q^2 = 0) = g_A$$

$$g_V(q^2 = 0) = g_V$$

leptons wfs

nuclear transition density:

$$\rho_{LJ}^\sigma(r) = \langle f || \sum_{j=1}^A \int d\Omega_r \delta(\mathbf{r} - \mathbf{r}_j) \tau_j^\pm [Y_L(\hat{r}) \otimes \boldsymbol{\sigma}_j]_J || i \rangle$$

usually $J = 1, L = 0$ is only considered
"GT"

Fermi type
time component

$$V^0(\mathbf{r}) = \sum_{j=1}^A \delta(\mathbf{r} - \mathbf{r}_j) g_V \tau_j^\pm$$

$$\langle f || \sum_L \Xi_{JL}(\kappa_e, \kappa_\nu) || i \rangle = \text{sign}(\kappa_e) \int_0^\infty dr r^2 \rho_J(r) \phi_A(r)$$

$$\rho_J(r) = \langle f || \sum_{j=1}^A \int d\Omega_r \delta(\mathbf{r} - \mathbf{r}_j) \tau_j^\pm Y_J(\hat{r}) || i \rangle$$

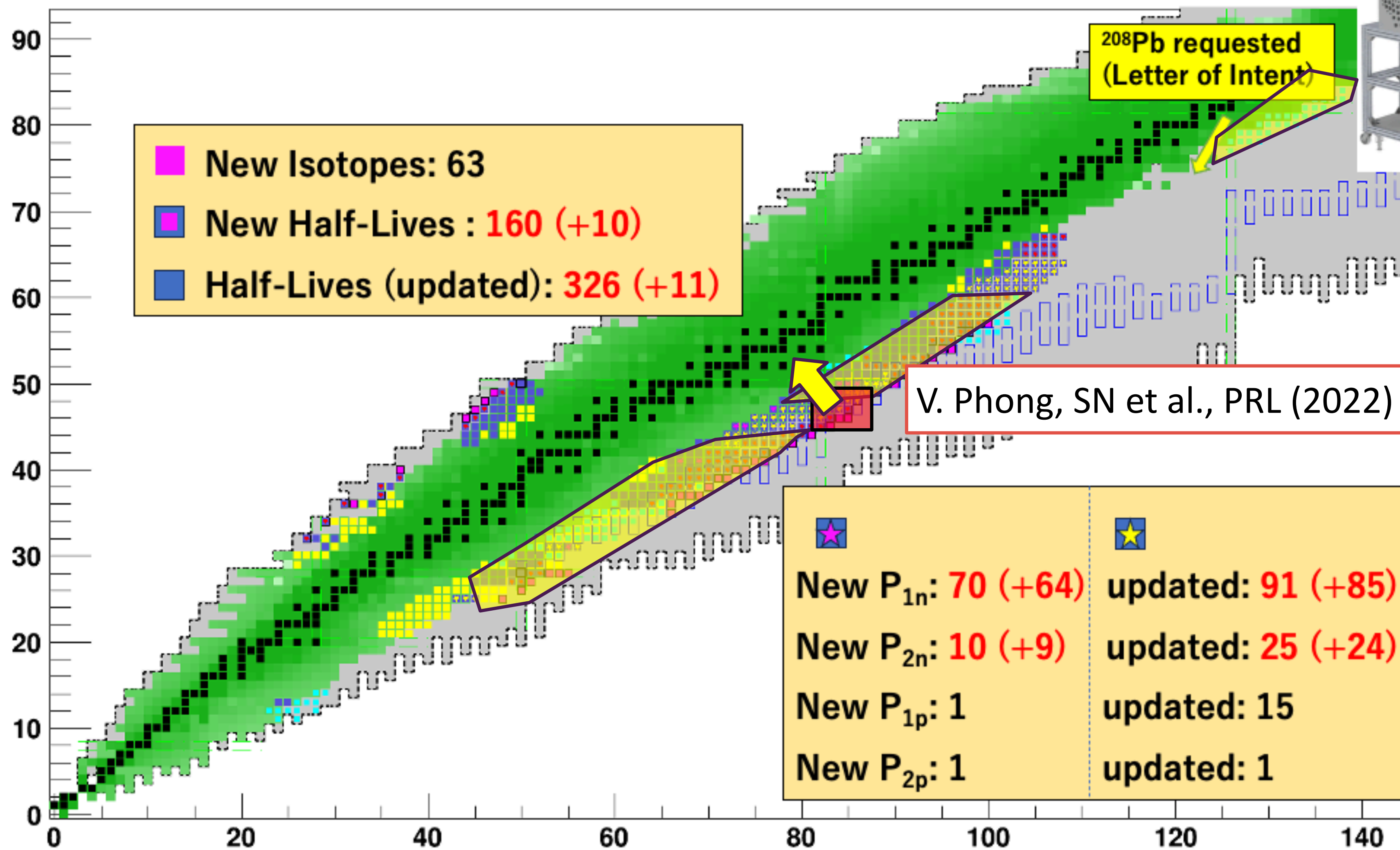
usually $J = L = 0$ is only considered

Decay Properties Surveyed

courtesy of S. Nishimura (RIKEN)

EURICA

+ BRIKEN (2019 ~ 2023)

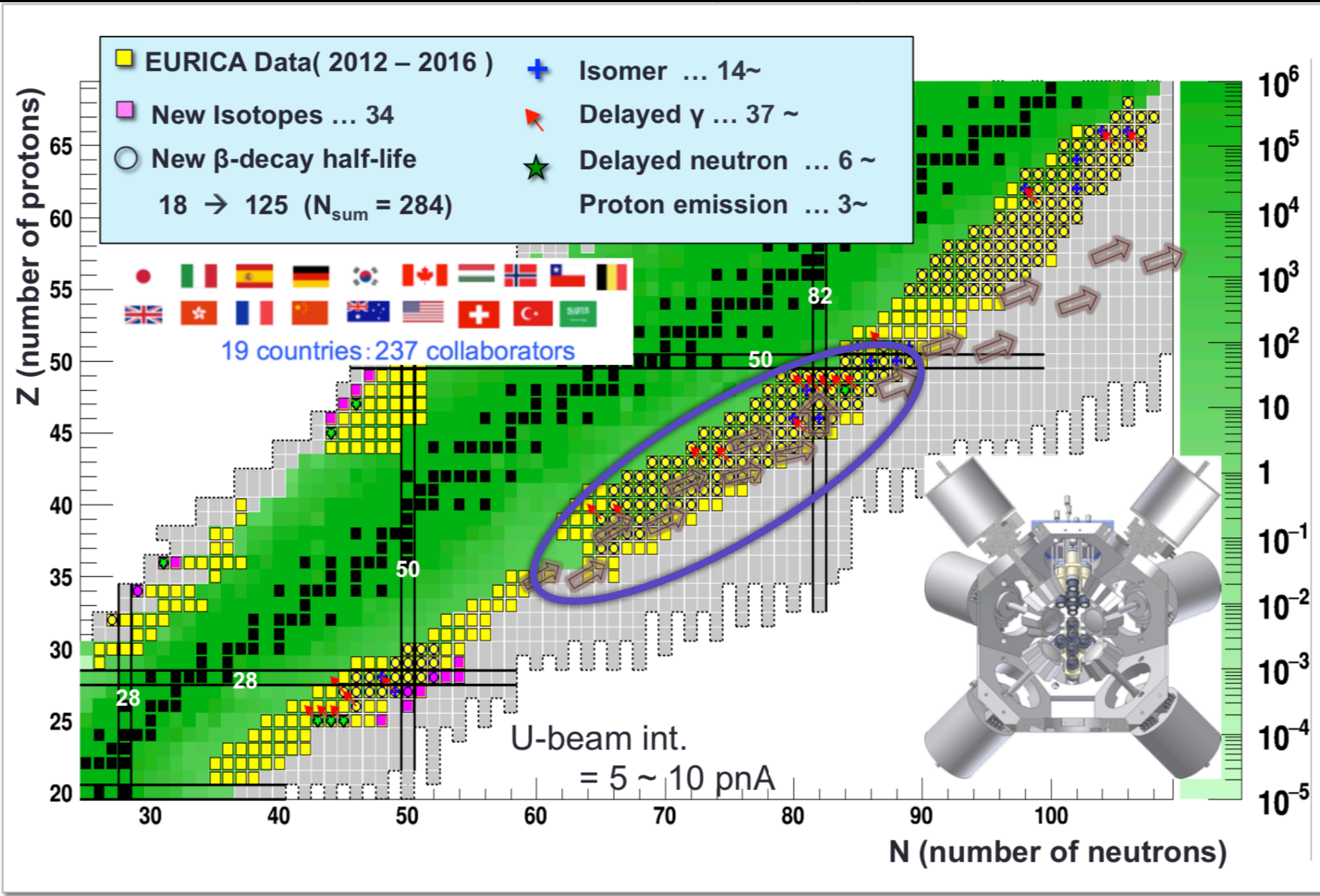


★	New P_{1n} : 70 (+64)	updated: 91 (+85)
★	New P_{2n} : 10 (+9)	updated: 25 (+24)
	New P_{1p} : 1	updated: 15
	New P_{2p} : 1	updated: 1

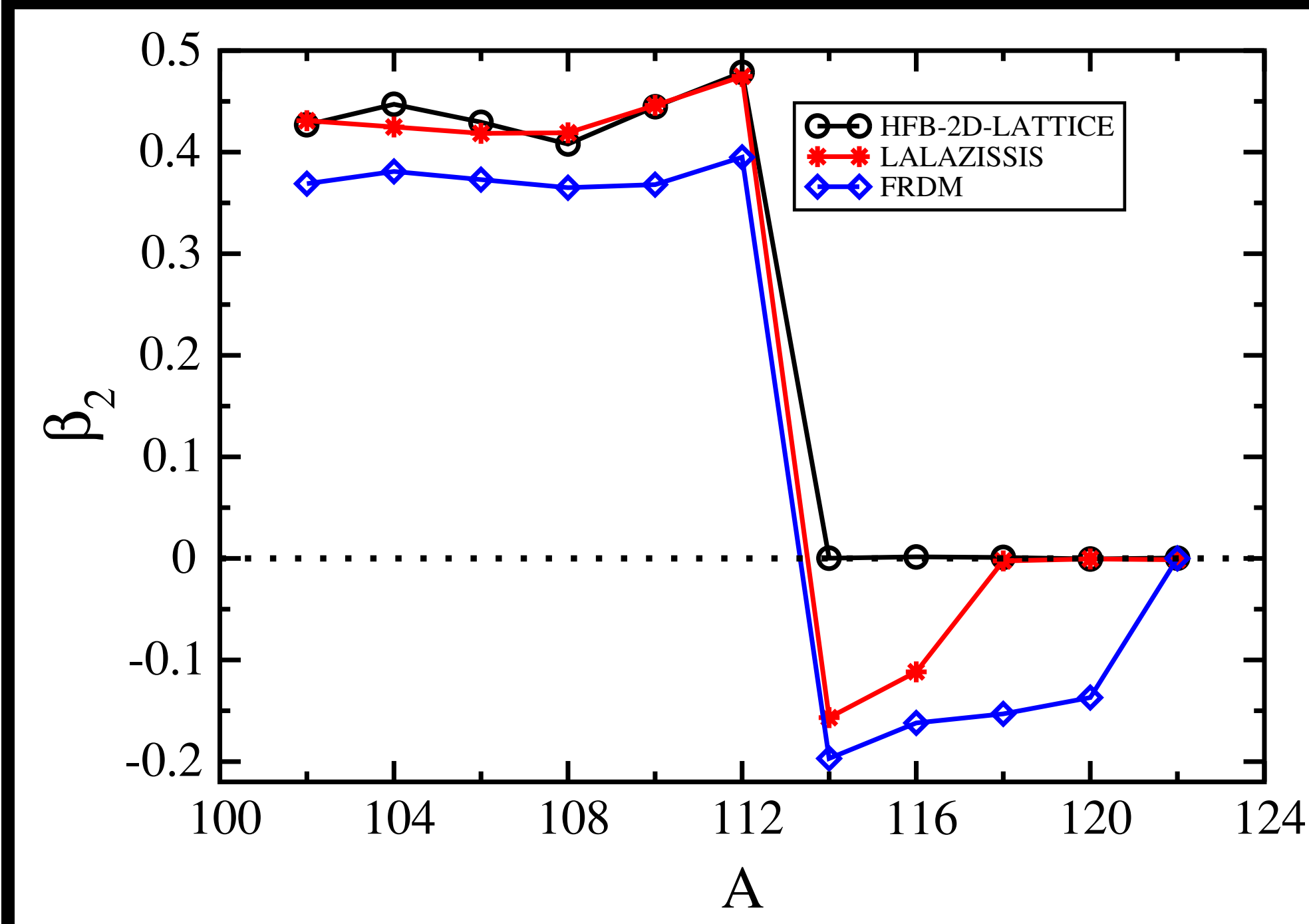
More Decay Data ($T_{1/2}$, P_{xn}) ... + ~ 200 Isotopes expected from BRIKEN

Systematic measurement of β -decay@RIBF

S. Nishimura *et al.* β -decay half-lives of r-process nuclei



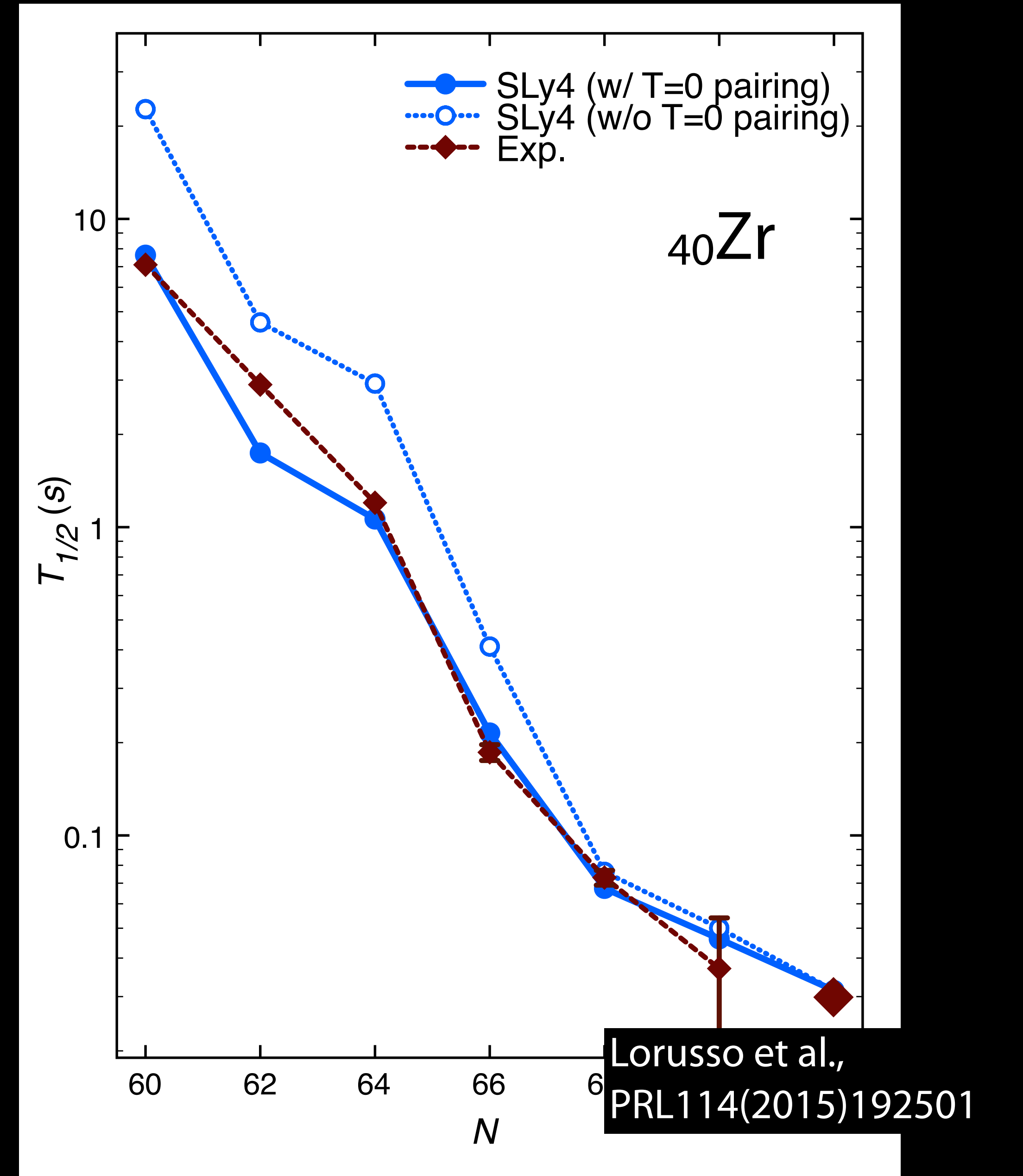
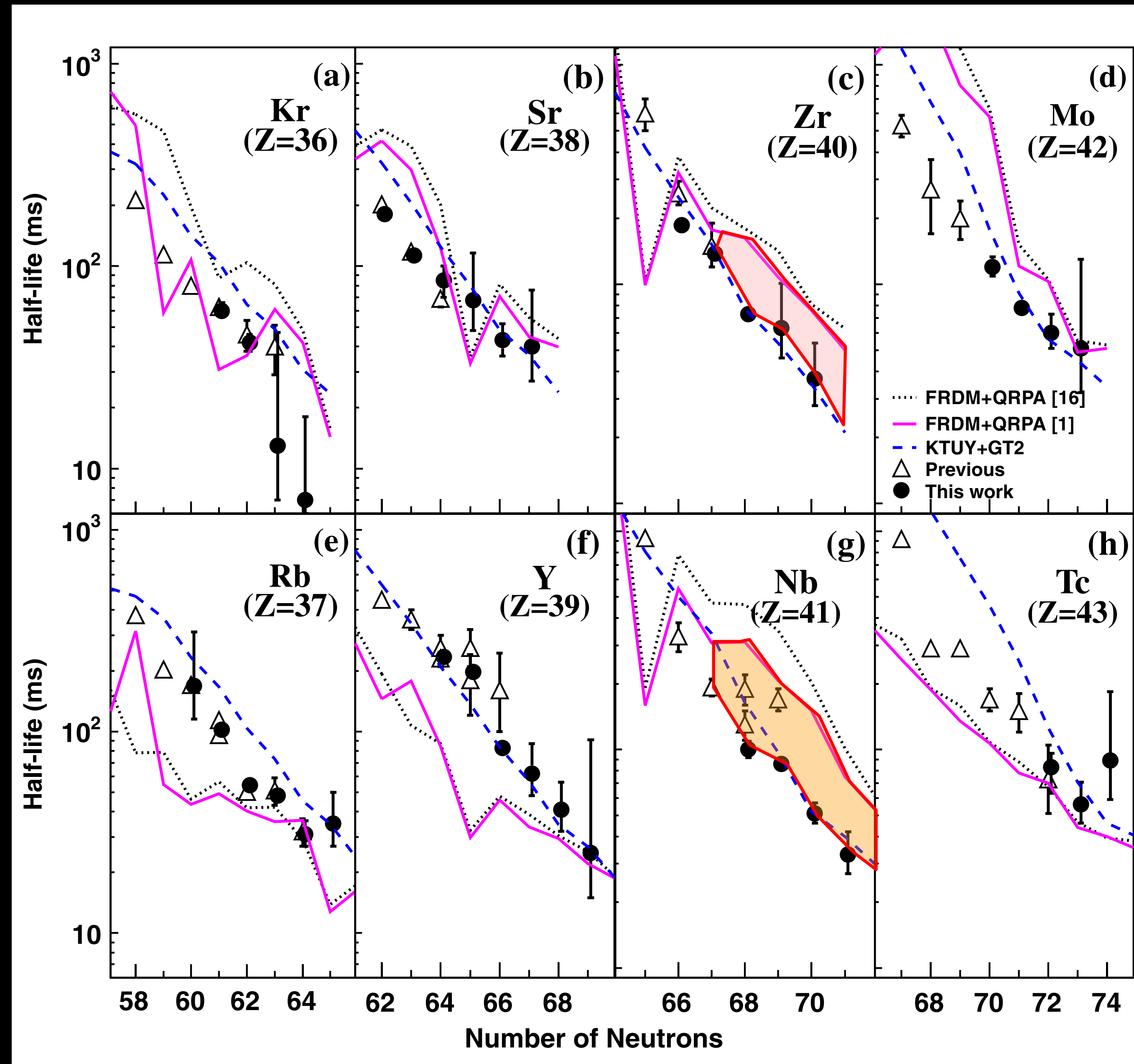
neutron-rich Zr isotopes:
predicted to be well deformed
by DFT cal.



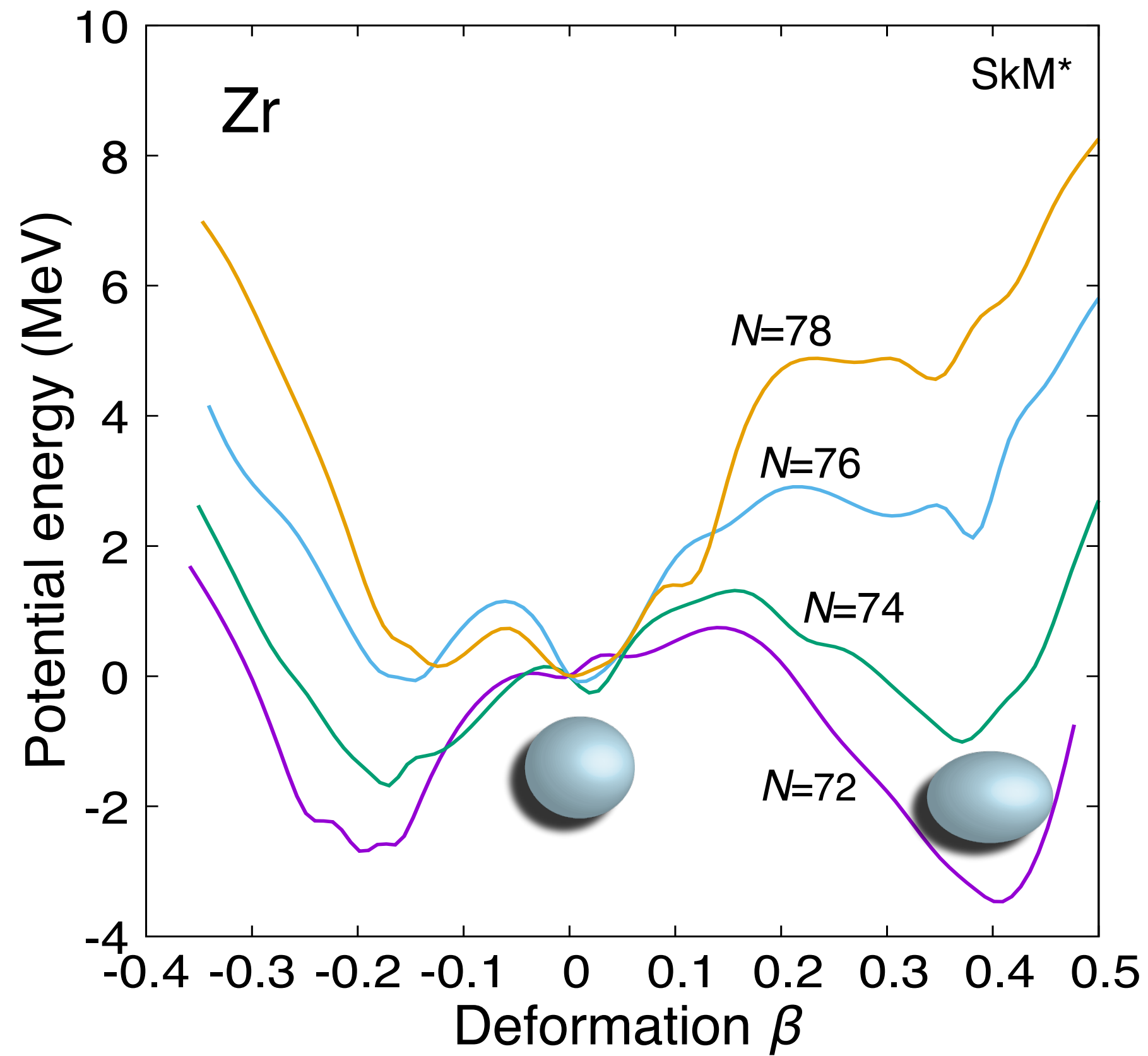
A. Blazkiewicz+, PRC71(2005)054321

Short half-lives in the Zr region

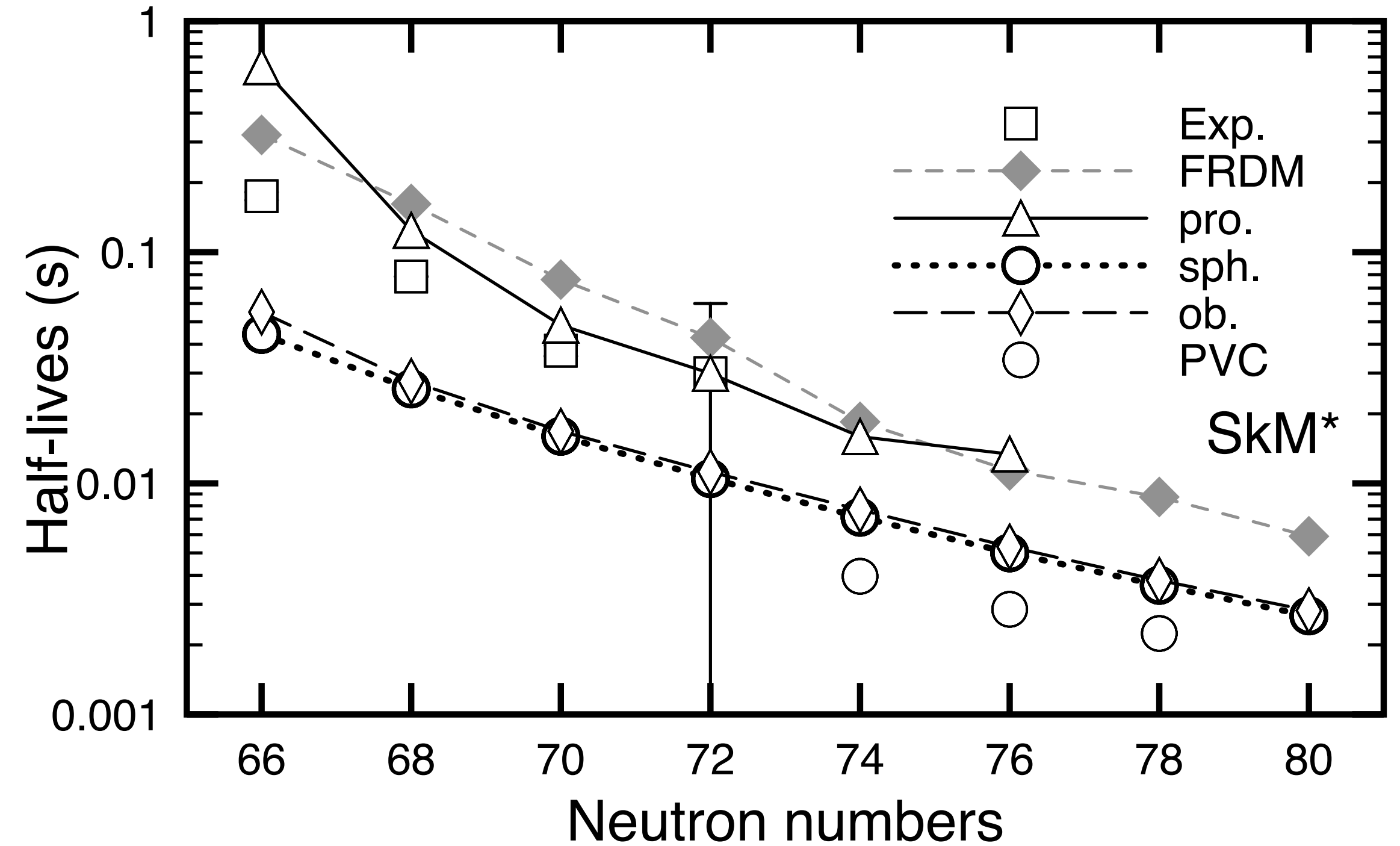
KY, PTEP(2013)113D02



Nuclear shapes in beta-decay



Yoshida–Niu–Minato,
PRC108(2023)034305



nuclear structures besides just the Q_β value

the shape change can be seen!

Summary

Nuclear shapes investigated under the external field

Lower-energy: $\omega < \omega_0$

nuclear deformation \equiv emergence of the rotational band
moments of inertia

Higher-energy: $\omega > \omega_0$

intrinsic shape can be seen in the vibrational modes

deformation splitting

coupling/mixing of the states with different angular momenta

breaking of the rotational sym.

Question to HI experts

What I have learnt: HI collision tells us about the correlations

$$\langle \psi^\dagger \psi^\dagger \psi \psi \rangle \quad \text{and even higher}$$

should contain the information on the superfluidity

$$\langle \psi^\dagger \psi^\dagger \psi \psi \rangle = \langle \psi^\dagger \psi^\dagger \rangle \langle \psi \psi \rangle + \dots$$

possible observables?

$$\langle \psi_p^\dagger \psi_p^\dagger \psi_p \psi_p \rangle = \langle \psi_p^\dagger \psi_p^\dagger \rangle \langle \psi_p \psi_p \rangle + \dots$$

distinguishable?

$$\langle \psi_p^\dagger \psi_n^\dagger \psi_p \psi_n \rangle = \langle \psi_p^\dagger \psi_n^\dagger \rangle \langle \psi_p \psi_n \rangle + \dots$$