Longitudinal flow decorrelations in Xe+Xe collisions and observables

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Including ongoing works with

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Hiroshima University

M. Kitazawa

YITP, Kyoto University

Computational resources

Supercomputer for Quest to Unsolved Interdisciplinary Datascience

A part of calculations is performed within **Project K25A04** on the supercomputer **SQUID** in the D3 Center of the University of Osaka

Workshop “Intersection of nuclear structure and high-energy nuclear collisions 2026,” 2025-04-23, YITP, Kyoto

Outline

1. **Simulation of Xe+Xe/Pb+Pb collisions**
   Current status of ongoing calculations
   – For longitudinal decorrelations
     with A. Sakai, Y. Nara
   – For Bayesian analysis for deformation parameters
     with E. Miyoshi, C. Abdi, A. Sakai, N. Chiho, Y. Nara
2. **Efficiency correction for the general event average**
   with M. Kitazawa
   – General unbiased estimator
   – Interpretation
   – Standard error

Originally requested to talk about this

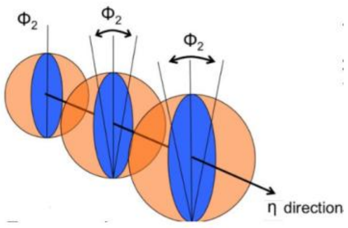
Simulation of Xe+Xe/Pb+Pb

Longitudinal flow decorrelation: r_n and F_n

A flow observable: **Factorization ratio "r_n"**

$$r_n(\eta_p^a, \eta_p^b) = \frac{V_{n\Delta}(-\eta_p^a, \eta_p^b)}{V_{n\Delta}(\eta_p^a, \eta_p^b)}$$

$$\sim \langle \cos n[\Psi_n(-\eta_p^a) - \Psi_n(\eta_p^a)] \rangle$$



J. Jia and P. Huo, PRC **90** 034905 (2014)

Roughly related to flow angles Ψ_n at different rapidity

Ideal case (rapidity independent)

$\Psi_n(-\eta_p^a) = \Psi_n(\eta_p^a) \rightarrow r_n = 1$

Reality (rapidity dependent)

$\Psi_n(-\eta_p^a) \neq \Psi_n(\eta_p^a) \rightarrow r_n < 1$ ***"decorrelation"***

+ flow magnitude fluctuations

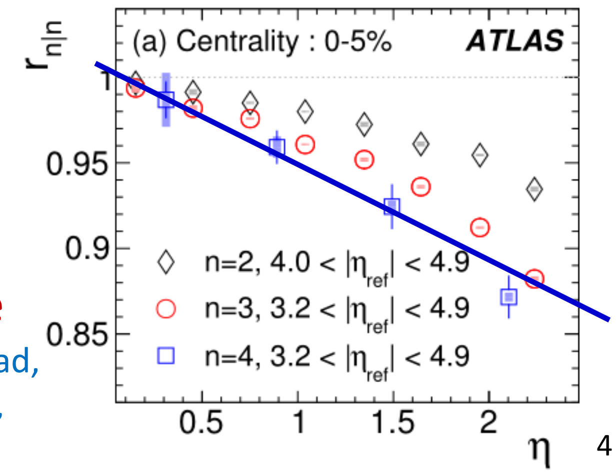
Extracting **slope F_n**

Rapidity dependence wrt η^a is typically *linear* → slope

$$r_n(\eta_p) = 1 - 2F_n\eta_p$$

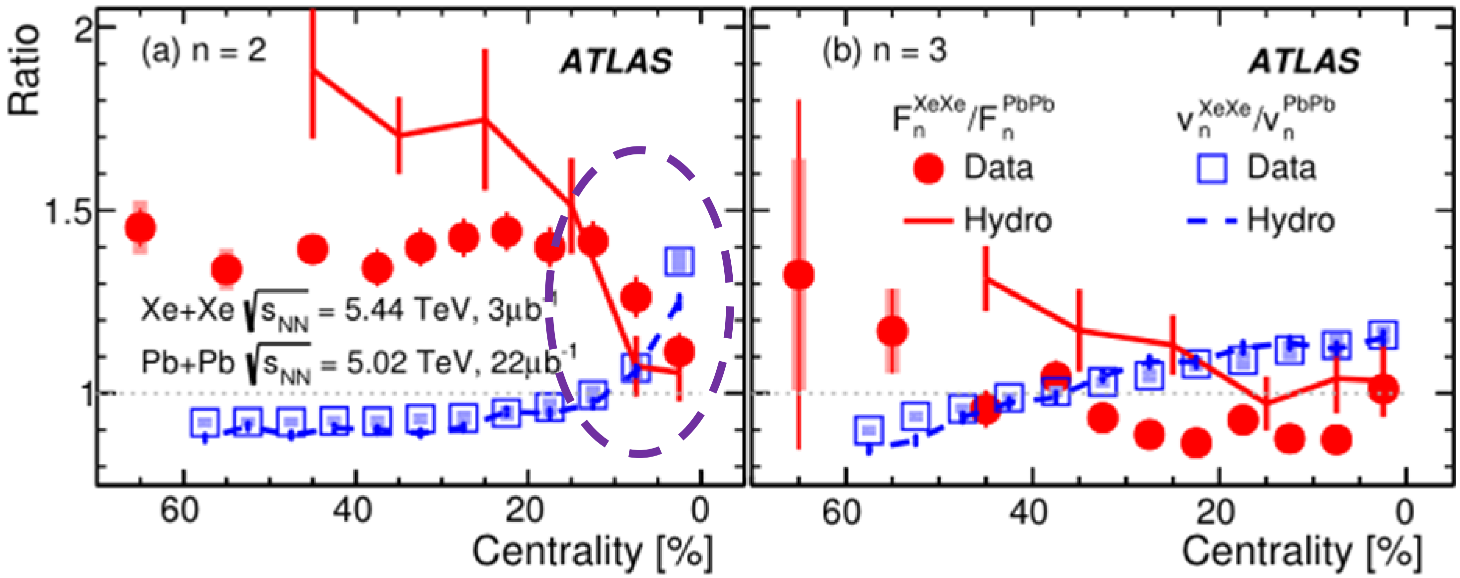
Decorrelation in Xe+Xe

ATLAS Collaboration, Georges Aad, Phys. Rev. Lett. **126** (2021) 12, 122301.



Xe+Xe/Pb+Pb ratio of v_2 and F_2

ATLAS Collaboration, Phys. Rev. Lett. **126** (2021) 12, 122301;
A. Behera, M. Nie, J. Jia, PRR 2, 023362 (2020)



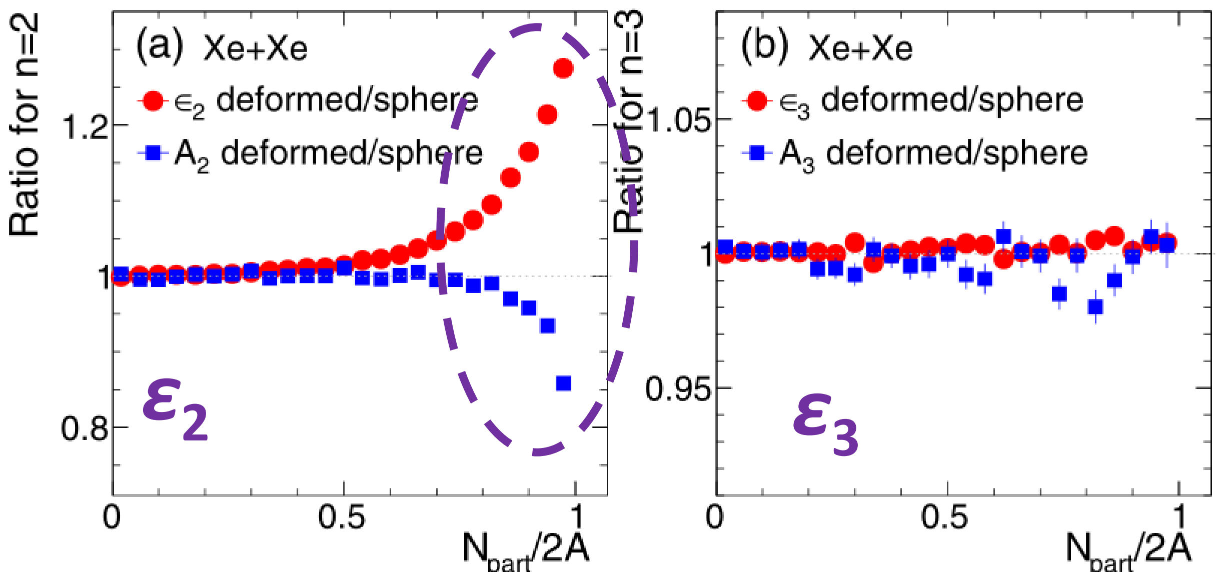
Hydro results from
G. Giacalone, J. Noronha-Hostler, M. Luzum, J.-Y. Ollitrault, PRC 97, 034904 (2018).
L.-G. Pang, H. Petersen, X.-N. Wang, PRC 97, 064918 (2018).
X.-Y. Wu, L.-G. Pang, G.-Y. Qin, and X.-N. Wang, PRC 98, 024913 (2018).

v_n is well described by models
F_n has a deviation between data and a model
Increase/decrease at central (n = 2)

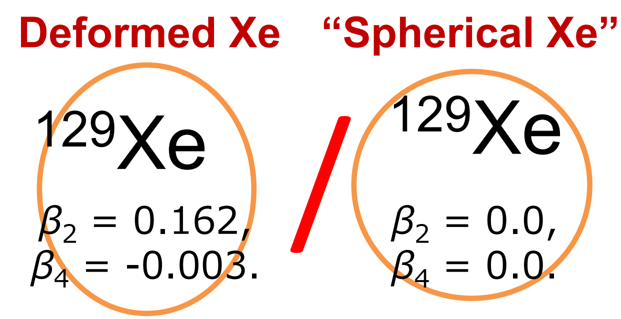
Deformed/spherical ratio for Xe+Xe

A. Behera, M. Nie, J. Jia, PRR 2, 023362 (2020)

based on Monte-Carlo quark Glauber model



← **peripheral** → **central**



ε_n and A_n (initial) correspond to v_n and F_n (final):

$$v_n \sim \kappa_n \varepsilon_n$$

$$F_n \sim \kappa_n A_n$$

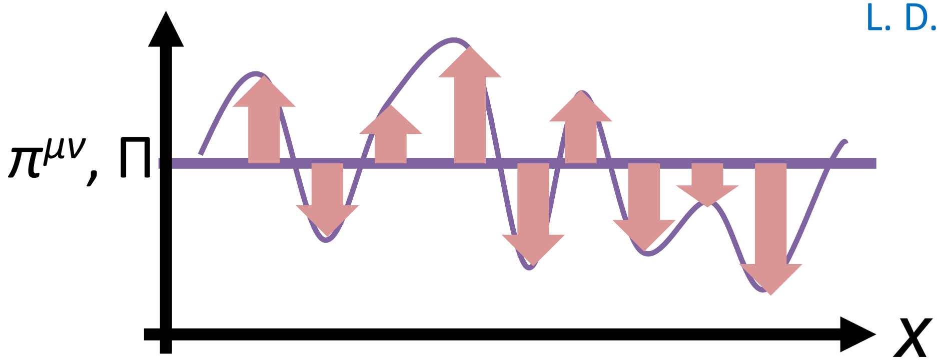
ε_2 increases / A_2 decreases in central collisions

ε_3 and A_3 unaffected by deformation

Longitudinal decorrelation by hydro fluctuations

Hydrodynamic fluctuations (HF) are *thermal fluctuations of fluid fields*

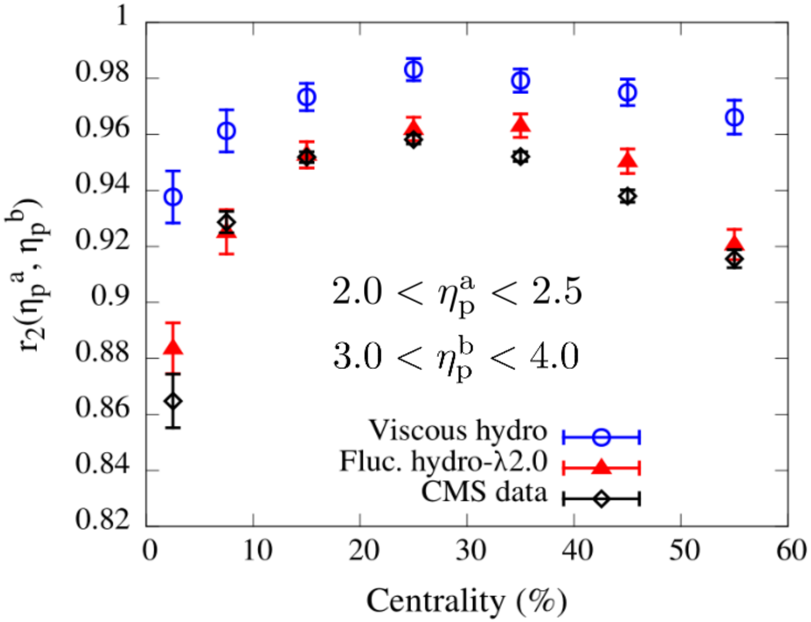
L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (1959)



Spontaneous fluctuations of fields $\pi^{\mu\nu}$, Π, etc. arising at each spacetime point

A. Sakai, KM, T. Hirano, PLB 829 (2022) 137053

Both hydrodynamic fluctuations and initial longitudinal fluctuations are necessary to reproduce the centrality dependence of factorization ratio $r_n(\eta,\eta)$



How HF affects the longitudinal correlation in Xe+Xe (deformed)?

Model

Integrated dynamical model

Ref. Sakai:2020pjw, Sakai:2021rug

1. **Initial condition** (**`mckln`**)
   MC-Glauber × modified BGK
   MC-Glauber × PYTHIA

2. **Hydro** (**`rfh`**)**:** (3+1)-dim stochastic/viscous hydrodynamics,
   EoS: lattice QCD & HRG, $\eta/s = 1/4\pi$

3. **Particlization** at T_{sw} = 155 MeV
   Cooper-Frye formula: $f_0 + \delta f$

4. **Cascades** (**`JAM1` → `JAM2`**)
   (only decays at the moment)

Upgrade to JAM2

Initial condition: Xe+Xe 5.44 TeV

Woods-Saxon with deformation

$$\rho = \frac{\rho_0}{1+\exp(\frac{r-R(\theta)}{a})}.$$

R_0 = 5.42,
a = 0.55,
ρ_0 = 0.166

G. Giacalone, et al, Phys. Rev. C 97, 034904 (2018); P. Mller, et al, Atom. Data Nucl. Data Tabl. 109-110, 1 (2016).

$$R(\theta) = R_0\,[1 + \beta_2\,Y_2{}^0(\theta,0) + \beta_4\,Y_4{}^0(\theta,0)]$$

Compare different deformation

Deformed Xe	**Spherical Xe**
^{129}Xe	^{129}Xe
β_2 = 0.162, β_4 = -0.003.	β_2 = 0.0, β_4 = 0.0.

Also, two other deformation of Xe

- **Xe (larger β_2)**: β_2 = 0.2
- **Xe (triaxial)**: β_2 = 0.2, γ = 27 deg

Relativistic fluctuating hydrodynamics

Causal fluctuating hydrodynamics KM (2019)

Causal viscous hydro

Conservation

$$\bar{\partial}_\mu T^{\mu\nu} = 0,$$ Energy-momentum tensor

$$\bar{\partial}_\mu N_i^\mu = 0.$$ conserved charge current

$\bar{\partial}_\mu$ Covariant derivative

HF here

Constitutive relations (2nd order)

$$\tau_\Pi \bar{\mathrm{D}}\Pi + \sigma_\Pi \theta \Pi + \Pi = -\zeta\theta + \xi_\Pi,$$ bulk pressure

$$\tau_\pi \Delta^{\mu\nu}{}_{\alpha\beta} \bar{\mathrm{D}}\pi^{\alpha\beta} + \sigma_\pi \theta \pi^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \xi_\pi^{\mu\nu},$$ shear stress

$$\tau_{ij} \Delta^\mu{}_\alpha \bar{\mathrm{D}}\nu_j^\alpha + \sigma_{ij}\theta\nu_j^\mu + \nu_i^\mu = \kappa_{ij} T \bar{\nabla}^\mu \frac{\mu_j}{T} + \xi_i^\mu.$$ diffusion

Fluctuation-dissipation relations (in uniform background)

$$\langle \xi_\Pi(x) \xi_\Pi(x') \rangle = 2T\zeta\delta^{(4)}(x-x'),$$

$$\langle \xi_\pi^{\mu\nu}(x) \xi_\pi^{\alpha\beta}(x') \rangle = 4T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x-x'),$$

$$\langle \xi_i^\mu(x) \xi_j^\alpha(x') \rangle = -2T\kappa_{ij}\Delta^{\mu\alpha}\delta^{(4)}(x-x').$$

$$\mathrm{D} = u^\mu \partial_\mu,$$

matter derivative

$$\nabla^\mu = \Delta^{\mu\nu}\partial_\nu.$$

spatial derivative in LRF

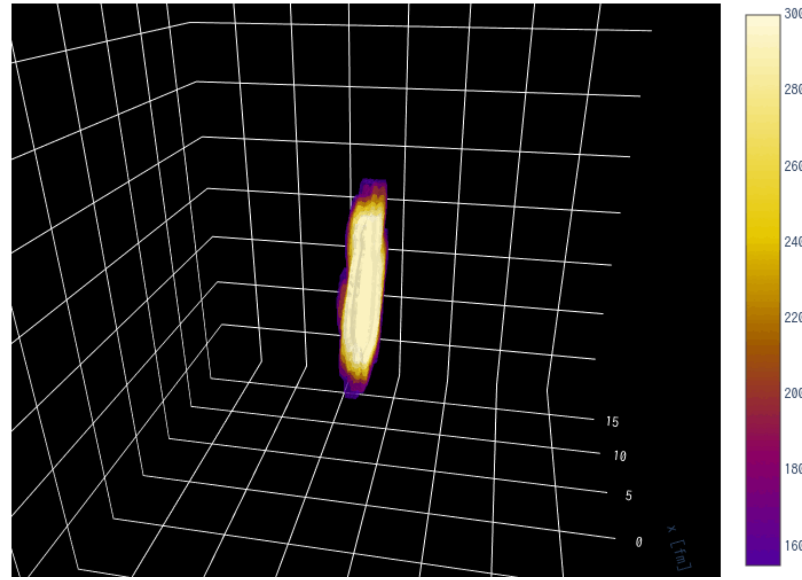
Xe+Xe & Hydro fluctuations: Animation

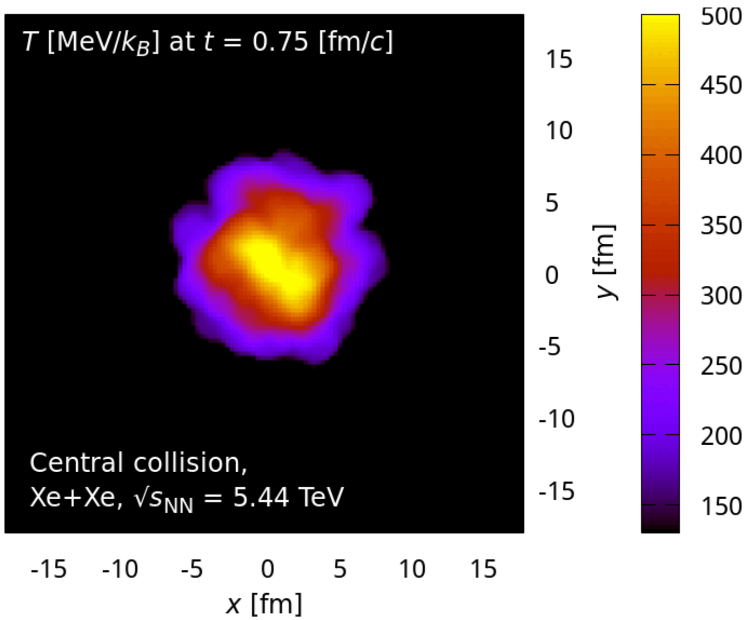
rfh (relativistic fluctuating hydro)

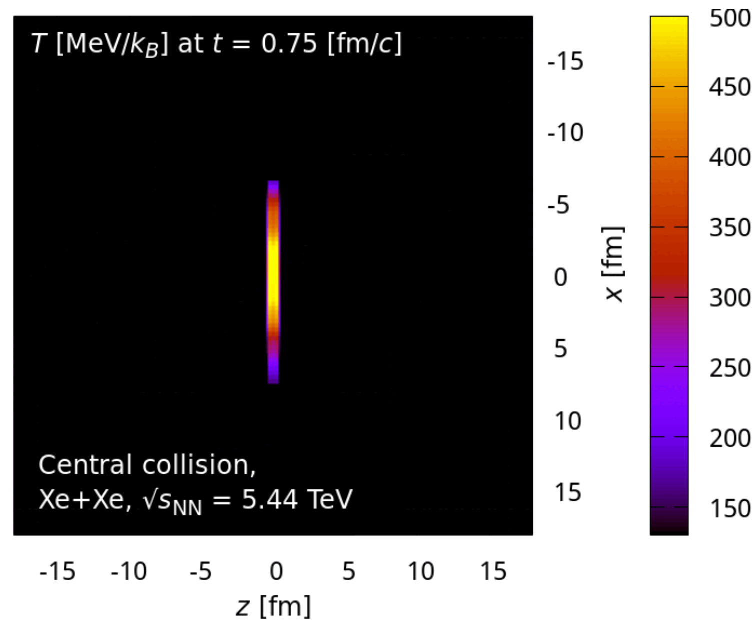
4000 events for each setup (2022)

- deformed/spherical Xe+Xe
- with & without hydro fluctuations

Animations show one of the events with
deformed Xe+Xe with hydro fluctuations

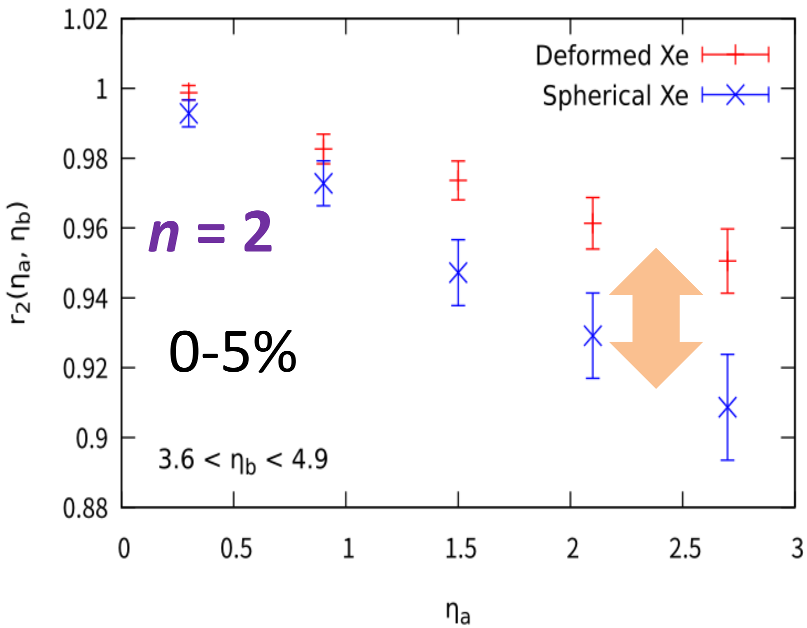


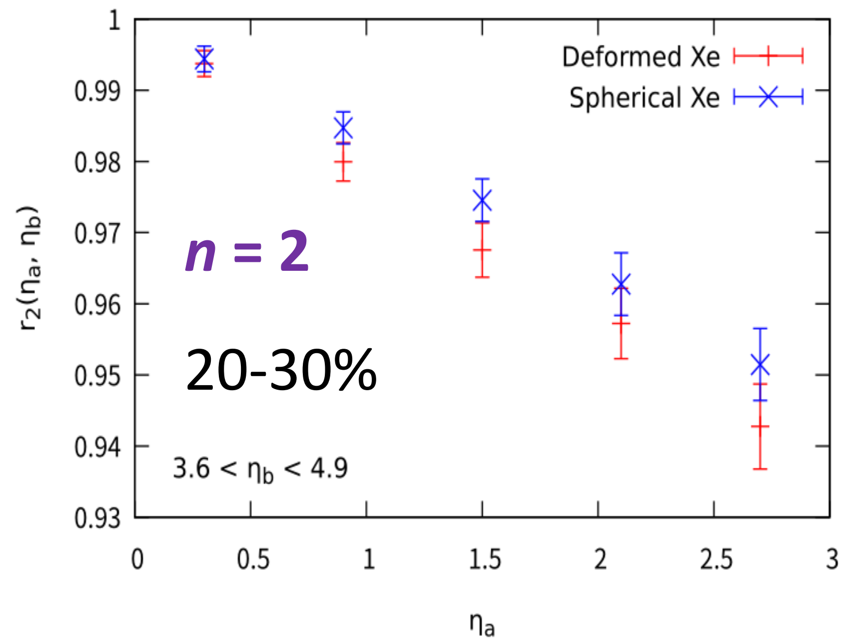




Result: $r_2(\eta^a,\eta^b)$ for deformed vs spherical Xe

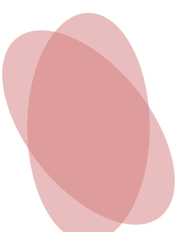
2022 calculations



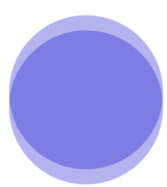


Decorrelation by HF is affected by the deformation in central collisions

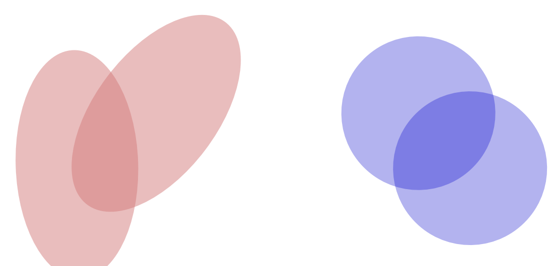
No clear difference in non-central collisions



less decorrelate ← stronger geometry



more decorrelate ← weaker geometry



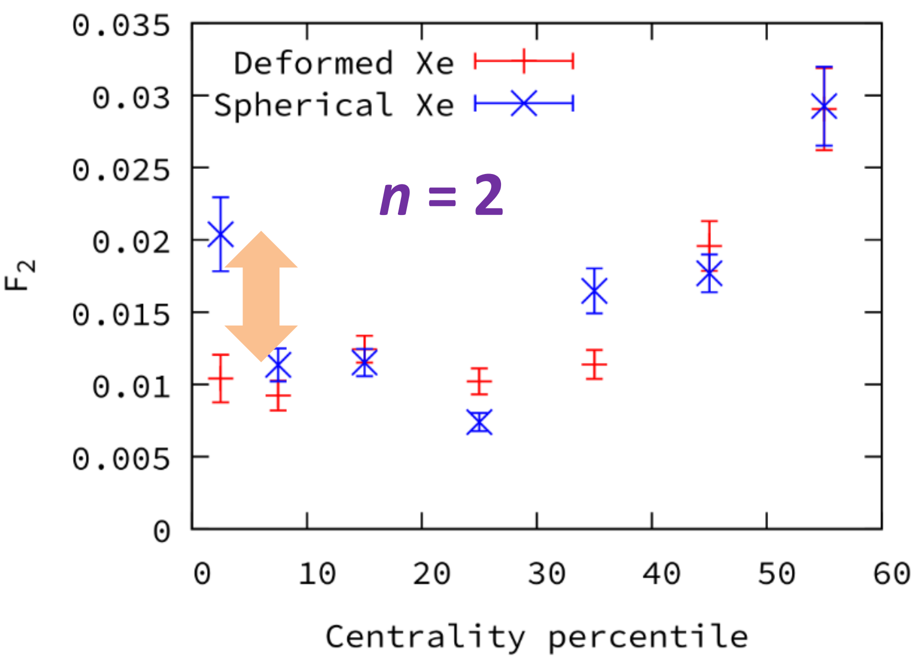
KM, A Sakai, poster at Quark Matter 2022

Result: $r_n(\eta^a,\eta^b)$ for deformed vs spherical Xe

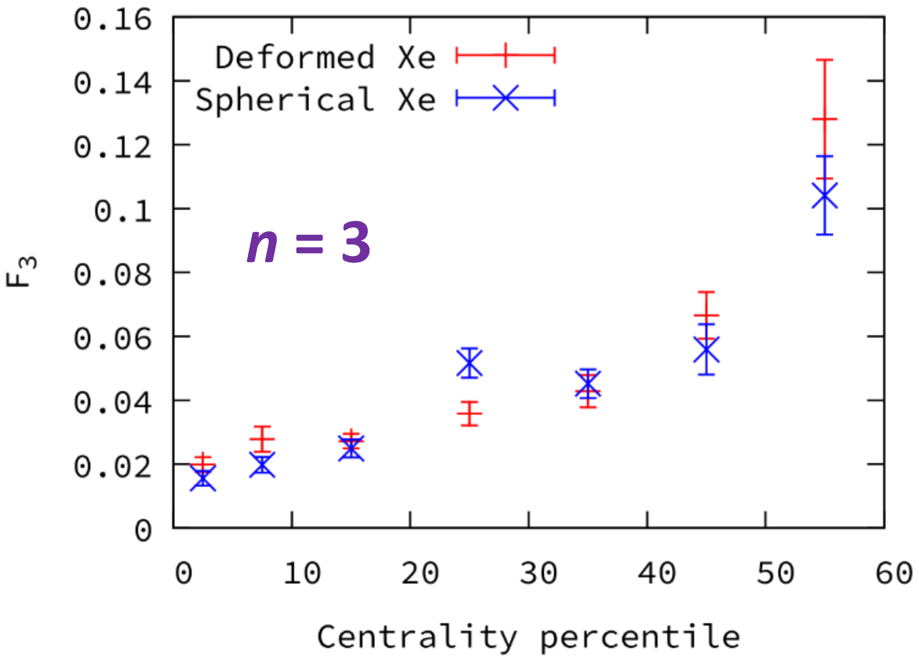
2022 calculations

Centrality dependence of

Slope parameter F_n: $r_n(\eta_a) = 1 - 2 F_n \eta_a$ … magnitude of decorrelation



Elliptic-flow decorrelation
in central collision is suppressed



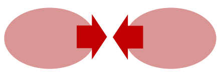
Higher-order (n = 3, 4, 5) do not
receive clear effects

KM, A Sakai, poster at Quark Matter 2022

Result: Event selection by q_2

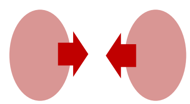
Effect of nuclei orientation?

tip-tip collisions



smaller eccentricity
larger multiplicity

body-body collisions



larger eccentricity
smaller multiplicity

Q. How is the decorrelation changed by the orientation?

2022 calculations

Event-shape engineering

Schukraft, Timmins, Voloshin, Phys. Lett. B719, 394 (2013)

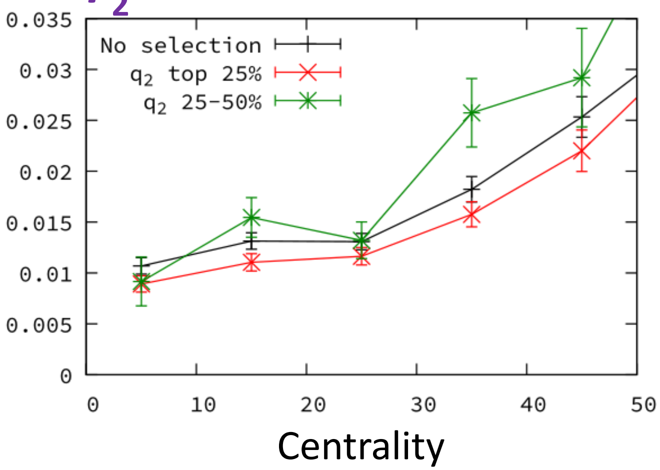
Event selection by "the shape"

$$q_2 = \frac{1}{\sqrt{M}}|Q_2|, \quad Q_n = \sum_{i=1}^{M} e^{in\phi_i}$$

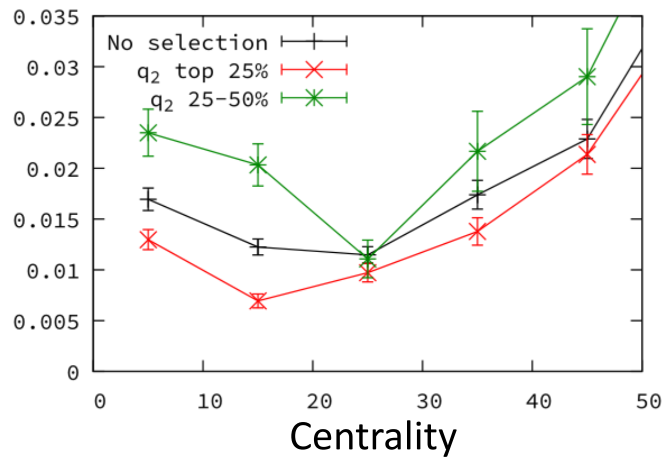
$1.7 < |\eta_p| < 3.7$

→ classify events by the magnitude of q_2

F_2



Deformed Xe+Xe



Spherical Xe+Xe

q_2 dependence becomes weaker by deformation?

Needs selection in ultracentral collisions

We need more statistics

Model update: switch to JAM2

2026 Update

Integrated dynamical model

1. **Initial condition** (**mckln**)
   MC-Glauber × modified BGK
   MC-Glauber × PYTHIA

2. **Hydro** (**rfh**): (3+1)-dim
   stochastic/viscous
   hydrodynamics,

3. **Particlization**
   Cooper-Frye: $f_0 + \delta f$

4. **Cascade** (**JAM1** → **JAM2**)
   (only decays at the moment)

For the hadronic afterburner, we have used the cascade mode of **JAM1** (Fortran) for a long time, but we have now switched to **JAM2** (C++17) [Y. Nara and A. Ohnishi, PRC 105, 014911 (2022)].

JAM2 Updates

- The latest hadron list from PDG data
- Pythia 8 for scattering/decay processes (JAM1 used Pythia 6)
- Various other updates in cross sections, etc.
- Spatial division for efficient collision tests
- Covariant cascade scheme [Y. Nara, A. Jinno, KM, et al, PRC108 (2023)]
- Mean-field update: RQMD2 [Y. Nara, A. Jinno, KM, arXiv:2507.23294], etc. (The mean field is currently not turned on in the afterburner)
- Etc.

Model update: switch to JAM2

2026 Update

Integrated dynamical model

1. **Initial condition** (**mckln**)
   MC-Glauber × modified BGK
   MC-Glauber × PYTHIA

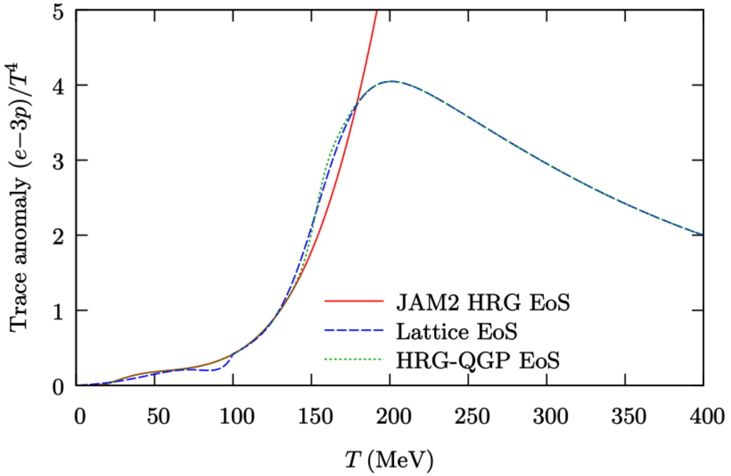
2. **Hydro** (**rfh**): (3+1)-dim
   stochastic/viscous
   hydrodynamics ← EoS

3. **Particlization**
   Cooper-Frye: $f_0 + \delta f$

4. **Cascade** (**JAM1** → **JAM2**)
   (only decays at the moment)

Hydro EoS update for the matching of entropy etc. between hydro and afterburner ← The update of the particle list affects the effective equation of state (EoS) of the cascade stage (hadron resonance gas; HRG).

- JAM1: we used *s95p*-v1.1 [Huovinen:2009yb].
- JAM2: we constructed a new EoS by connecting JAM2 HRG and Lattice EoS [HotQCD:2014kol] using the same way as 4d EoS [Monnai:2024pvy] [connect p(T) with tanh].



New calculations on SQUID

2026 Update

Supercomputer for Quest to Unsolved Interdisciplinary Datascience

SQUID is a supercomputer
installed in the D3 Center of the University of Osaka.

Project K25A04 for Xe+Xe/Pb+Pb simulation
was approved with 100k [Node hour] = 7.6M [CPU hour]

We only used 3.4M [CPU h]

Obtained dataset (Last month)

- Xe+Xe 5.44 TeV ($\beta 2$=0.16, $\beta 4$=-0.003)
- Xe+Xe 5.44 TeV (spherical)
- Xe+Xe 5.44 TeV ($\beta 2$=0.2, γ=27 deg)
- Xe+Xe 5.44 TeV ($\beta 2$=0.2, γ=0)
- Pb+Pb 5.02 TeV (spherical)
- Pb+Pb 2.76 TeV (spherical)

4 settings for each

Initial: MC-Glauber × modified BGK
(2 hydro) x (2M hydro x 20 cascades)
exception: 1M hydro for Pb+Pb 2.76 TeV

Initial: MC-Glauber × PYTHIA
(2 hydro) x (1M hydro x 20 cascades)

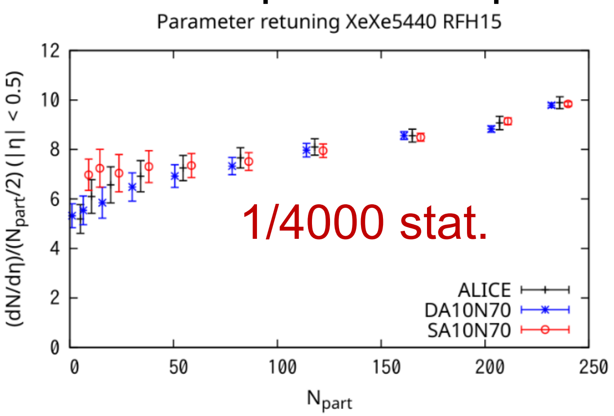
- Xe+Xe 5.44 TeV Parameter scan (for Bayesian analysis):
  $\beta 2$[-0.1,0.3], $\beta 4$[-0.05,0.05], γ[0, 60°], normalization,
  η/s min&slope, relaxation time, (hydro fluctuation cutoff)
  (1000 Sobol points x 40k hydro x 10 cascades) for modified BGK & viscous
  (1000 Sobol points x 1k hydro x 10 cascades) for others

Ongoing Analysis

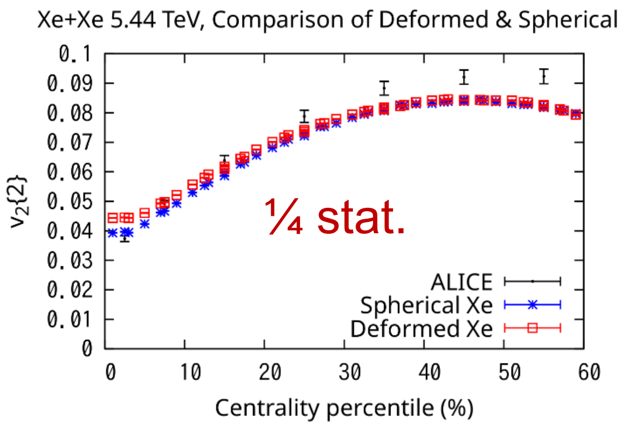
2026 Update

Actually, we haven't yet started the analysis….

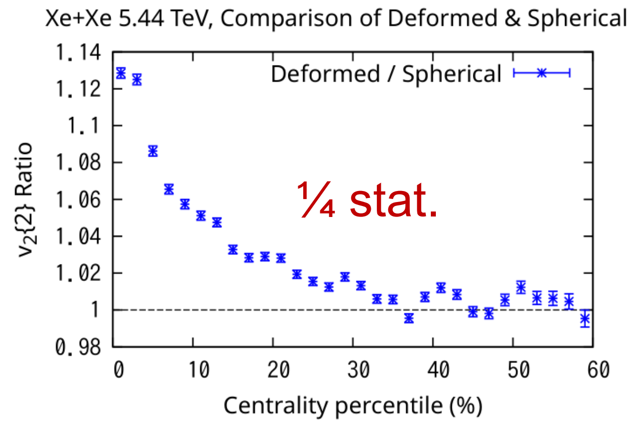
$(dN/d\eta)/(N_{part}/2)$ vs N_{part}



$v_2\{2\}$ vs centrality



$v_2\{2\}$ ratio Deformed/Spherical



Outlook (KM, Sakai)

- Basic observables, mean pt, pt fluct, v2-pt correlation, etc. [Jia:2021qyu]
- Longitudinal observables:
  flow decorrelations $r_n(\eta_a, \eta_b)$, pt decorrelations $r_{pT}(\eta_1, \eta_2)$ [Liu:2025fbu]
- Event selection by q_2 (event shape engineering)
  angle distributions or fraction of tip-tip / body-body collisions
- The effect of hydrodynamic fluctuations

Outlook (Miyoshi, Cendikia, Nonaka)

- Training Gaussian process emulator, Bayesian analysis

Outlook (KM, Nara)

- JAM1 vs JAM2 comparison

Efficiency correction
for the general event average

with M. Kitazawa

Detector efficiency correction

See also slides of the talk by M. Kitazawa on 4/13 (Mon).

- Not all particles emitted in an event are correctly detected in experiments. Some particles may be unobserved, and some may be misidentified.
- ***Physical* observables** … defined in terms of event averages of all-particle quantities. However, only observed particles are accessible in experiment.
- **Efficiency correction** = How do we *estimate* the observables defined in terms of all particles when only a part of particles is available?

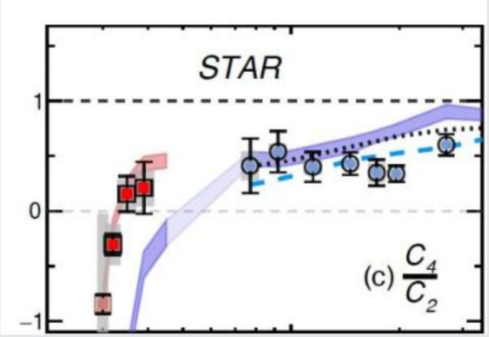
Detector efficiency correction

History of efficiency corrections: Many works starting with a simple observable to more complicated ones. A correction formula has been designed for each form of the observables.

From slides of M. Kitazawa 4/13

Baryon number cumulants

Search for QCD-CP using conserved-charge fluctuations



Long history of efficiency correction:
MK, Asakawa ('12); Bzdak, Koch ('12,'15); Luo ('14); MK ('16); Nonaka+ ('16); Bzdak, Holtzman, Koch ('16); MK, Luo ('17); Nonaka, MK, Esumi ('17); ...

Simple polynomials

Correction Procedure:

Use factorial moments/cumulants

$$\left\langle \left(\sum_i p_i \right)^n \right\rangle_{\mathrm{f}} = \left\langle\!\!\left\langle \left(\sum_i \frac{p_i}{r_i} \right)^n \right\rangle\!\!\right\rangle_{\mathrm{f}}$$

Averages within an event

Mean

$$\left\langle \frac{Q}{N} \right\rangle = \left\langle\!\!\left\langle \sum_{i=1}^{n} \xi_i k_i \right\rangle\!\!\right\rangle_{n\neq 0}$$

$$k_i = \frac{1}{r_i} \int_0^1 d\sigma \prod_{j\neq i} \frac{\sigma + r_j \alpha_j}{r_j}$$

2nd Order

$$\left\langle \frac{\{Q_1 Q_2\}}{N(N-1)} \right\rangle = \left\langle\!\!\left\langle \sum_{i\neq j} q_{1,i} q_{2,j} k_{2;i,j} \right\rangle\!\!\right\rangle_{n\neq 0,1}$$

$$k_{2;i,j} = \frac{1}{r_i r_j} \int_0^1 d\sigma' \int_0^{\sigma'} d\sigma \prod_{l\neq i, l\neq j} \left(\frac{\sigma}{r_l} + \alpha_l \right)$$

MK, Esumi, Niida, Nonaka, PTEP in press [arXiv:2510.13838]

Master formula for the general observable

Detector efficiency correction

History of efficiency corrections: Many works starting with a simple observable to more complicated ones. A correction formula has been designed for each form of the observables.

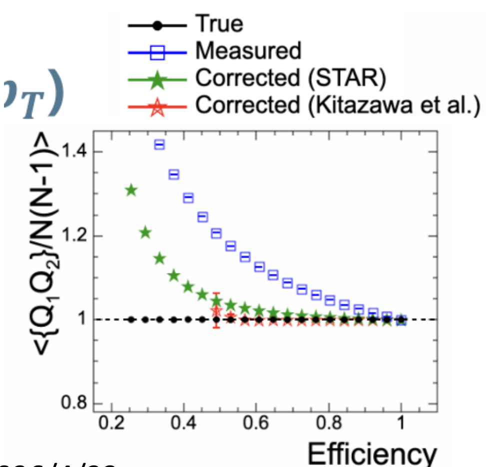
Various non-trivial form of averaged quantities

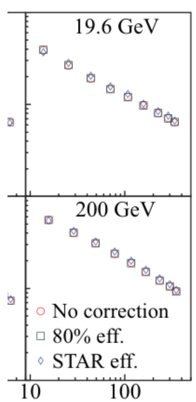
including "mean pT" which is considered important for deformation

From slides of M. Kitazawa 4/13

[Jia:2021qyu]

$$\left\langle \frac{1}{N}\sum_i p_i \right\rangle, \quad \left\langle \left(\frac{1}{N}\sum_i p_i \right)^n \right\rangle, \quad \left\langle \frac{1}{N(N-1)} \sum_{i\neq j} p_i^{(1)} p_j^{(2)} \right\rangle$$





STAR:2019dow

Event-plane resolution

$\sqrt{\langle \cos n(\Psi_A - \Psi_B)\rangle}$

Scalar product

$v_n\{2; \mathrm{SP}\}$

etc

Master formula for the general observable

Simplified mathematical model

For each event random variables

1. N particles $\{\gamma_i\}_{i=1}^N$ are generated with the probability $\Pr(N, \{\gamma_i\}_{i=1}^N)$.
2. The efficiency $r(\gamma_i; \{\gamma_j\}_{j=1}^N)$ (the probability to detect particle i) are determined

   Efficiency $r(\gamma)$ can be different for each event depending on the surrounding tracks

3. $b_i = 1$ (particle i is observed) or 0 (not) is sampled by a binomial trial with the probability $r(\gamma_i)$.

   random variable

To calculate observables, the information of observed particles $\{\gamma_i \mid b_i = 1\}$ and $\{r_i = r(\gamma_i) \mid b_i = 1\}$ are available. Mis-identification and other effects are not considered.

What we want

Defs. The set of indices of observed particles is denoted by B. E.g., $B = \{1,2\}$ *when particles 1 and 2 are observed. The set of observed particles are denoted by* $\Gamma_B = \{\gamma_i \mid i \in B\}$. The set of all indices are denoted by $G = \{1,\ldots,N\}$. The set of all generated particles are denoted by $\Gamma_G = \{\gamma_j\}_{j=1}^{N}$.

We want the event average of an arbitrary function X

$$\langle X(\Gamma_G)\rangle$$

In particular, we want *a good estimator* $X\hat{}(\Gamma_B)$ so that

$$\langle X\hat{}(\Gamma_B)\rangle = \langle X(\Gamma_G)\rangle \quad \text{when } N_{\text{event}} \to \infty$$

$X\hat{}(\Gamma_B)$: An expression independent of the physical process $\Pr(\Gamma_G)$. We do not know the true $\Pr(\Gamma_G)$.

Good estimator?

For estimator X^,

Unbiasedness

$$E[\langle X\hat{}(\Gamma_B)\rangle - \langle X(\Gamma_G)\rangle] = 0, \text{ even with finite } N_{\text{event}}$$

Convergence in probability

$$\langle X\hat{}(\Gamma_B)\rangle \rightarrow \langle X(\Gamma_G)\rangle, \text{ with } N_{\text{event}} \rightarrow \infty$$

Note: there are difference levels of convergence

Faster convergence ~ reasonable upper bound to the standard error

$$V[\langle X\hat{}(\Gamma_B)\rangle] < M$$

For example, if the standard error exponentially diverges wrt N , the error bar is too large to get meaningful values

Unbiased estimator for detector efficiency

Using the unbiasedness condition,
we may derive the general formula purely as a math problem.

Solution

$$\hat{X}(\Gamma_B) = \sum_{I \subseteq B} \left(\prod_{j \in I} \alpha_j \right) \left(\prod_{j \notin I} \bar{\alpha}_j \right) X(\Gamma_I)$$

$$\alpha_j = 1/r_j \text{ and } \bar{\alpha}_j = 1 - 1/r_j$$

Σ is a summation for all subsets of *B* ($2^{|B|}$ terms)

Γ_I is a subset of particles corresponding to index subset *I*

This is a generalization of existing efficiency correction formula:
All known efficiency correction formulae
can be easily derived using the above expression.

Why subsets?

Consider e.g. $\Pr(N) = \delta(N=N_0)$ [always N_0 particles are generated]

- The true average is $X(N_0)$
- If we simply calculate the average 〈$X(n)$〉 using the observed number of particles n ($0 \leqq n \leqq N_0$), the result is contaminated by all different $X(n)$ ($n \neq N_0$). One needs to subtract all contaminations with any combinations of unobserved particles.

The "expectation" value E[] is only given by rare events where $B = G$ (though one cannot select such events experimentally). Other contributions cancel and vanish.

However, other terms contribute to reduce the "variance" V[] (and hence the error bar) when $X(N\text{-}1) \sim X(N)$.

Discussions

$$\hat{X}(\Gamma_B) = \sum_{I \subseteq B} \left(\prod_{j \in I} \alpha_j \right) \left(\prod_{j \notin I} \bar{\alpha}_j \right) X(\Gamma_I)$$

- Excessive cancellation of + terms and – terms → Large errors (cf. “sign problem”).
- Many terms ~ 2^N … The direct use of the formula is not practical.
  - Starting point for an efficient formula.
  - The MC sampling out of 2^N terms
- The standard error is bounded if $GM_i[4(1-r_i)/r_i] < 1$ or $r_i > 80\%$ (sufficient cond.)

geometric mean   on average

Summary for efficiency correction

Master formula (unbiased estimator)

$$\hat{X}(\Gamma_B) = \sum_{I \subseteq B} \left(\prod_{j \in I} \alpha_j \right) \left(\prod_{j \notin I} \bar{\alpha}_j \right) X(\Gamma_I)$$

$$\alpha_j = 1/r_j \text{ and } \bar{\alpha}_j = 1 - 1/r_j$$

Γ_B is a set of observed particles, r_i are efficiency

Σ is a summation for all subsets of *B* ($2^{|B|}$ terms)

Γ_I is a subset of particles corresponding to index subset *I*

- *Literally,* an arbitrary function *X*() … does not need to be continuous. Can include pt or eta cuts.
- All known efficiency correction formulae can be derived using the above expression.
- The standard error: bounded if $GM_i[4(1-r_i)/r_i] < 1$ (sufficient cond.)