Utilising Bayesian inference in probing nuclear structures at the LHC

Maxim Virta[1]

1. Niels Bohr Institute, Denmark

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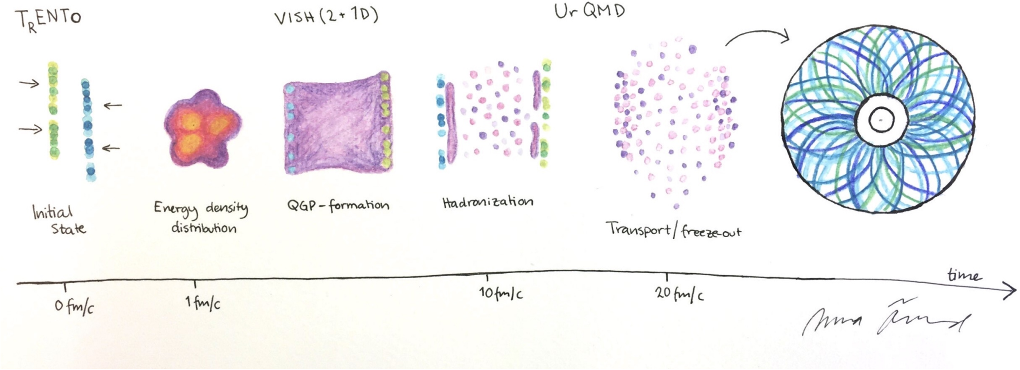
Intersection of nuclear structure and high-energy nuclear collisions
Kyoto, Japan

European Research Council
Established by the European Commission

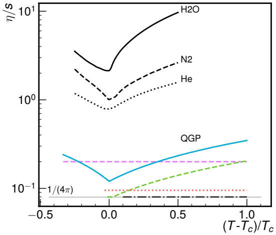


Credits to Anna Önnerstad

$$T^{\mu\nu} = eu^{\mu}u^{\nu} - (P + \Pi)\Delta_{\mu\nu} + \pi^{\mu\nu}, \qquad \delta_{\mu}T^{\mu\nu} = 0$$

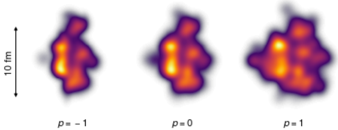
based on https://github.com/Duke-QCD

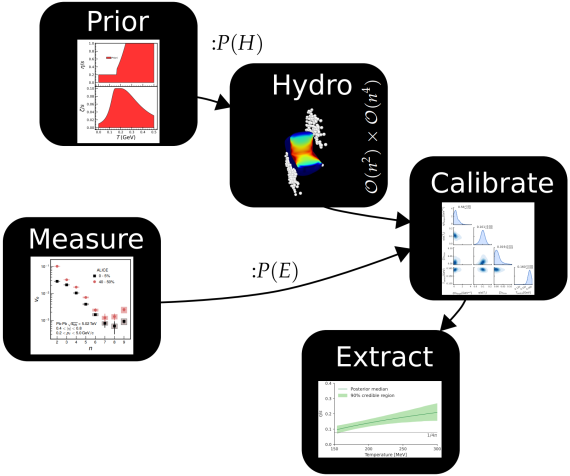
Parameter	Description	
T_c	Temperature of const. $\eta/s(T)$, $T < T_c$	Hydrodynamics
$\eta/s(T_c)$	Minimum $\eta/s(T)$	
$(\eta/s)_{\text{slope}}$	Slope of $\eta/s(T)$ above T_c	
$(\eta/s)_{\text{curve}}$	Curvature of $\eta/s(T)$ above T_c	
$(\zeta/s)_{\text{peak}}$	Temperature of $\zeta/s(T)$ maximum	
$(\zeta/s)_{\text{max}}$	Maximum $\zeta/s(T)$	
$(\zeta/s)_{\text{width}}$	Width of $\zeta/s(T)$ peak	
T_{switch}	Switching / particlization temperature	
N(5.02 TeV)	Overall normalization (Pb–Pb 5.02 TeV)	Initial conditions
N(5.44 TeV)	Overall normalization (Xe–Xe 5.44 TeV)	
p	Entropy deposition parameter	
w	Nucleon width	
σ_k	Std. dev. of nucleon multiplicity fluctuations	
d^3_{min}	Minimum volume per nucleon	
τ_{fs}	Free-streaming time	



shear viscosity

Trento p-value, http://qcd.phy.duke.edu/trento/





Bayes' theorem:

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

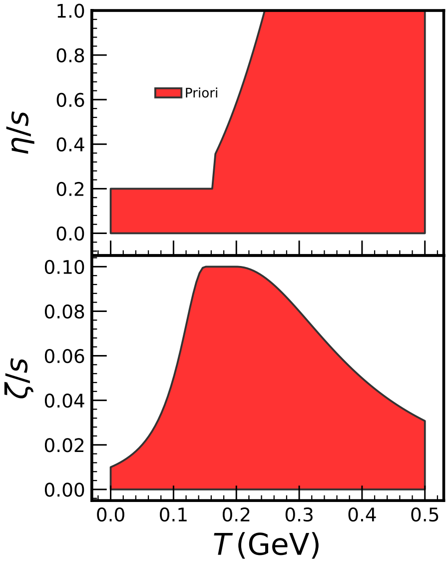
$$P(E) = \sum_{i=1}^{n} P(E|H_i)P(H_i)$$

- Find an optimal set of model parameters that best reproduces the experimental data.
- Utilise constraints, such as flow observables, to help narrow down the $\eta/s(T)$ and such.

The prior knowledge is encoded in the model

- Directly via physics assumptions or formulation
- Indirectly via the ranges of parameters
- These cause inevitable bias

$$\partial_\mu T^{\mu\nu}(x) = 0$$
$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$



The prior knowledge is encoded in the model

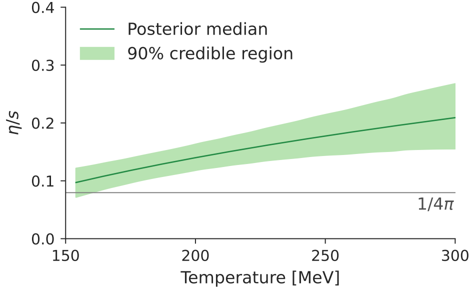
- Directly via physics assumptions or formulation

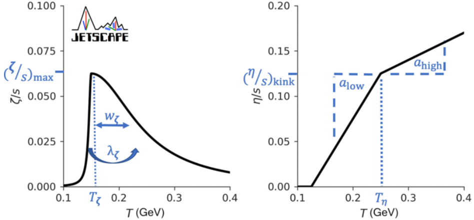
Jyväskylä/Duke

$$\frac{\eta}{s}(T) = \left(\frac{\eta}{s}\right)_{\text{min}} + \left(\frac{\eta}{s}\right)_{\text{slope}} (T - T_c)\left(\frac{T}{T_c}\right)^{\left(\frac{\eta}{s}\right)_{\text{crv}}}$$

JETSCAPE

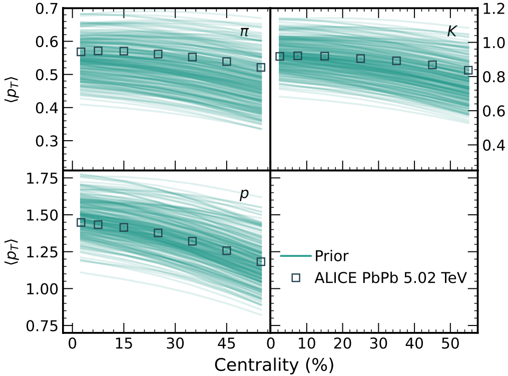
$$\frac{\eta}{s}(T) = a_{\text{low}}(T - T_\eta)\Theta(T_\eta - T) + \left(\frac{\eta}{s}\right)_{\text{kink}} + a_{\text{high}}(T_\eta - T)\Theta(T - T_\eta)$$

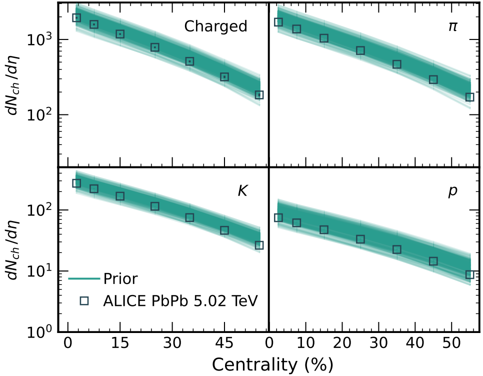




JETSCAPE, *Phys. Rev. C* **103**, 054904

- Parameters on their own are not comparable with the experiment
- Evaluate different observables from the hydrodynamic simulations

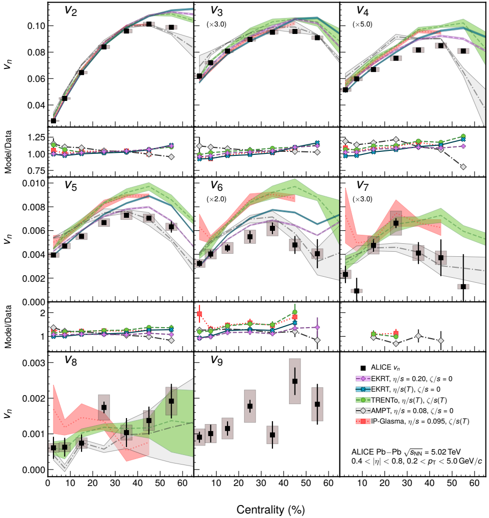




- Experimental measurements define the "truth"
- More observables of different kinds needed for a full picture

	$dN_{ch}/d\eta$	$dN_{p^\pm}/d\eta$	$\langle p_{T,\pi^\pm}\rangle$	$\langle p_{T,K^\pm}\rangle$	$\langle p_{T,p^\pm}\rangle$	v_2	v_3	v_4	v_5	NSC(3, 2)	NSC(4, 2)	NSC(4, 3)
τ_{fs}	0.05	0.87	0.98	0.92	0.78	0.62	1.30	2.09	3.37	0.25	4.58	6.75
$(\eta/s)_{slope}$	0.40	0.53	0.28	0.28	0.25	0.67	1.20	1.72	2.80	0.38	4.97	3.61
T_c	1.18	1.75	0.84	0.74	0.58	0.48	0.66	1.04	1.13	0.41	5.24	2.85
$\eta/s(T_c)$	0.42	0.33	0.31	0.31	0.26	1.35	2.48	4.20	4.93	1.05	20.73	11.94
$(\eta/s)_{crv}$	0.02	0.02	0.02	0.02	0.01	0.04	0.07	0.09	0.16	0.02	0.13	0.14
$(\zeta/s)_{peak}$	0.92	2.44	1.83	1.66	1.31	0.62	1.14	1.89	3.64	0.48	6.22	3.70
$(\zeta/s)_{max}$	0.67	0.89	0.87	0.78	0.59	0.03	0.02	0.05	0.44	0.21	2.20	3.92
$(\zeta/s)_{width}$	0.29	1.00	0.70	0.67	0.57	0.18	0.29	0.50	2.22	0.14	2.33	3.82
T_{switch}	2.23	1.93	0.07	0.07	0.19	1.17	2.02	2.70	1.84	0.37	12.63	4.05

$S[x_j]$ (colour scale: 2, 4, 6, 8)



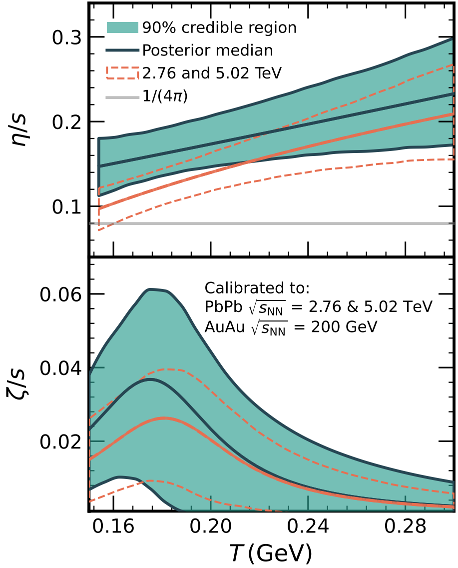
ALICE, *JHEP05* (2020) 085

Updating prior information with measured data

- Output is a probability distribution
- Dependence on **model** chosen
- Dependence on **observables** used

Interpretation

Probability of a parameter lying in a certain interval **given the measured data for the model used**



M. Virta, J. Parkkila, D.J. Kim, *Phys. Rev.* C **111**, 044903

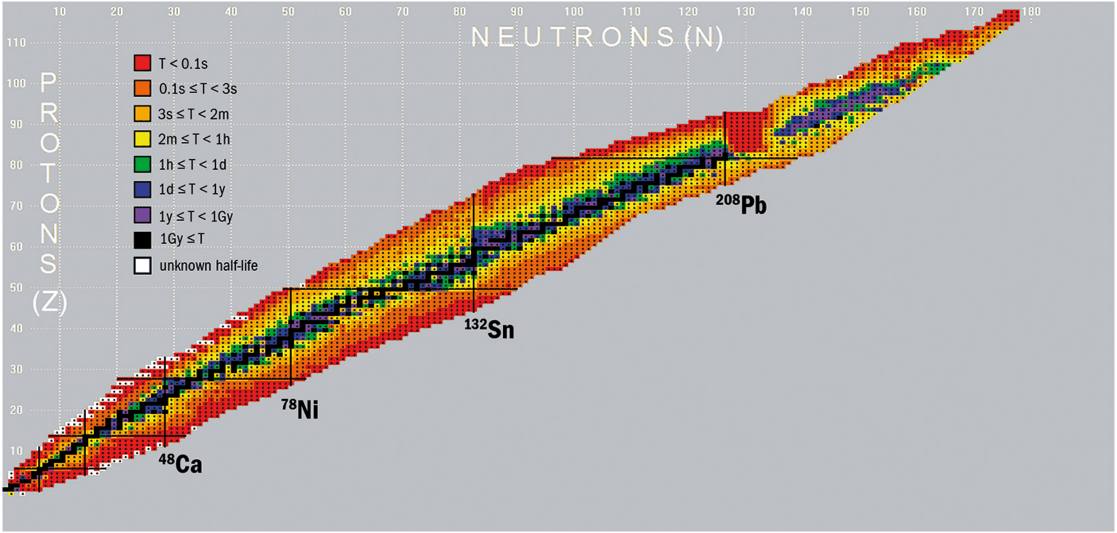
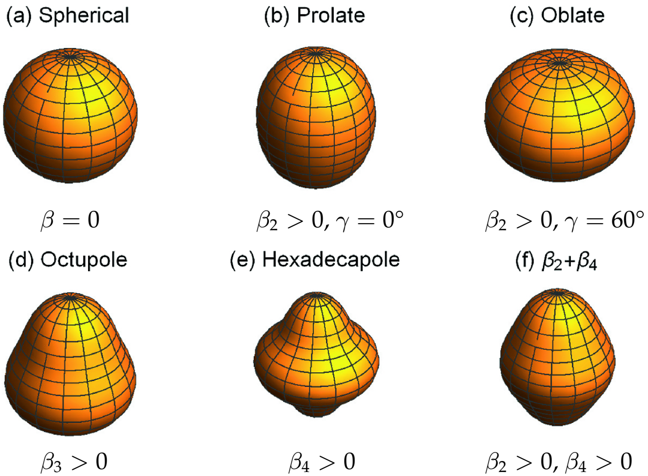
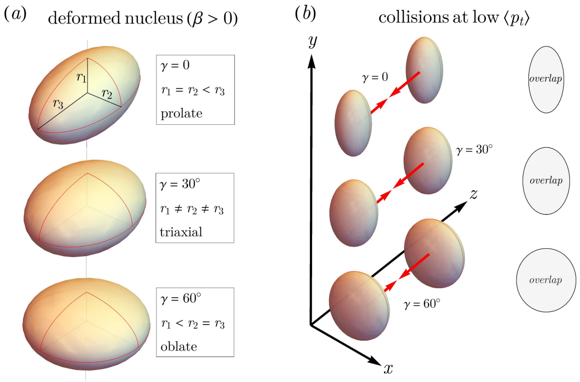


Figure from CERN Courier: https://cerncourier.com/a/exploring-nuclei-at-the-limits/



$$R(\theta, \phi) = R_0\left(1 + \sum_{l,m} \beta_{l,m} Y_{l,m}\right)$$

Figure from: Kota, V.K.B. (2020). Introduction. In: SU(3) Symmetry in Atomic Nuclei. Springer

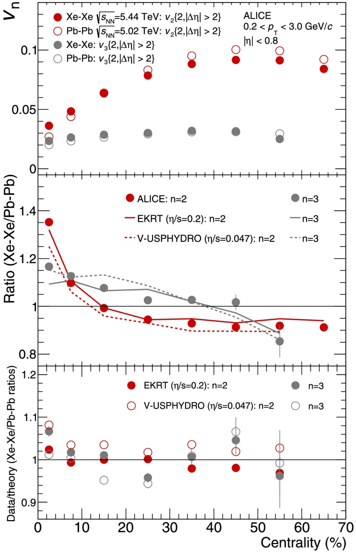


B. Bally *et al.*, *Phys. Rev. Lett.* 128 (2022), 082301

$$R(\theta, \phi) = R_0 \left[1 + \beta_2 \left(\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}\right)\right]$$

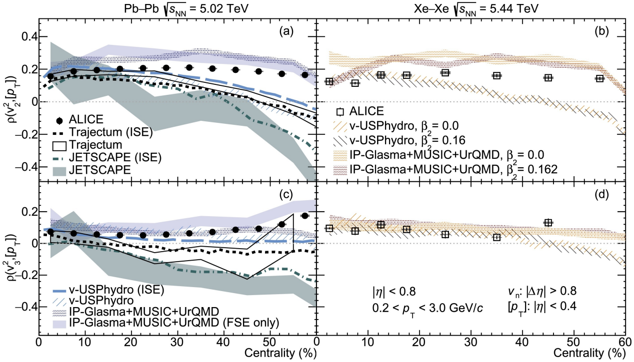
- Xenon nucleus has mostly quadrupole deformation
- Probe the nuclear structure, quadrupole deformation magnitude β_2 and triaxiality angle γ

$$\beta_2 = \sqrt{\beta_{2,0}^2 + 2\beta_{2,2}^2},\ \gamma = \mathrm{atan}(\sqrt{2}\beta_{2,2}/\beta_{2,0})$$



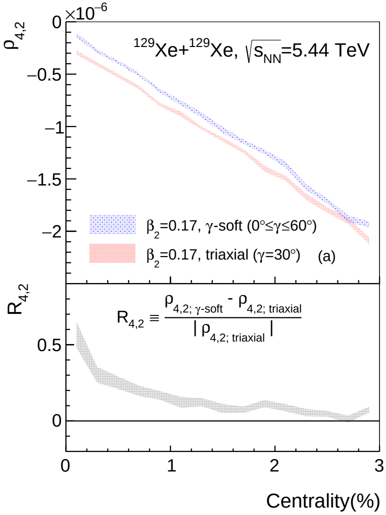
ALICE, *Phys. Lett. B* 784 (2018), 82-95

ALICE, *Phys. Lett. B* 834 (2022), 137393

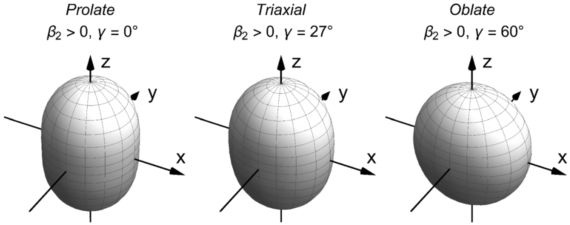


- Deformation of the nucleus affects the collision geometry
- β_2 requires two-particle correlation, while γ needs three-particle correlation
- $\rho(v_2^2, \delta p_T)$ is defined as the Pearson correlation between v_2^2 and δp_T

S. Zhao *et al.*, *Phys. Rev. Lett.* 133 (2024), 192301

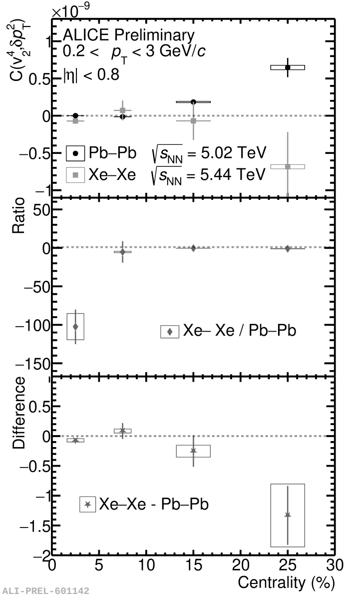


E. Nielsen *et al.*, *Eur.Phys.J.A* 60 (2024) 2, 38

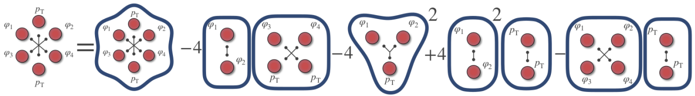


- γ requires a three-particle correlation
  $\rightarrow \gamma$ fluctuations need six-particle correlation
- New 6-particle correlation between flow harmonics and mean-p_{T}, $C(v_2^4, \delta p_{\mathrm{T}}^2)$
- $\rho_{4,2}$ is the corresponding initial-state observable

OBSERVABLES

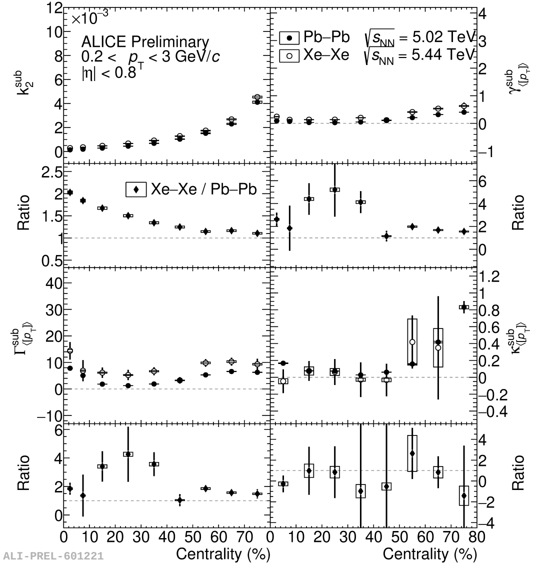


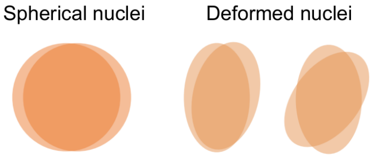
$$C(v_2^4, \delta p_T^2) = \langle\langle v_2^4 \delta p_T^2 \rangle\rangle - 4\langle\langle v_2^2 \rangle\rangle \langle\langle v_2^2 \delta p_T^2 \rangle\rangle - 4\langle\langle v_2^2 \delta p_T \rangle\rangle^2 + 4\langle\langle v_2^2 \rangle\rangle^2 \langle\langle \delta p_T^2 \rangle\rangle - \langle\langle v_2^4 \rangle\rangle \langle\langle \delta p_T^2 \rangle\rangle$$



- Measured for the first time in Pb–Pb and Xe–Xe collisions
- The difference between the systems is used due to fluctuations around zero

Moments of δp_{T}^2



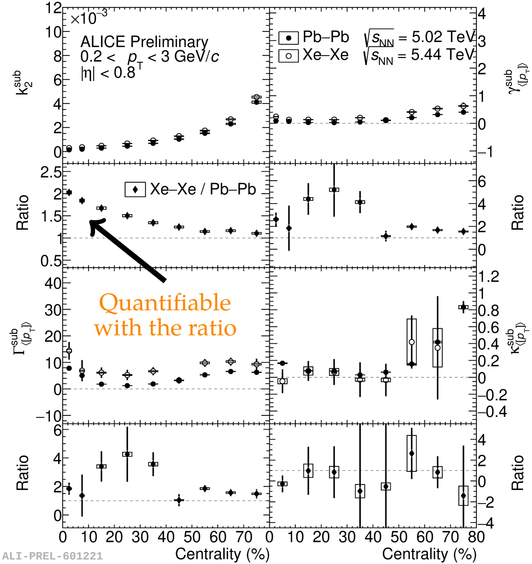


$$k_2^{\mathrm{sub}} = \frac{c_2^{\mathrm{sub}}}{[p_{\mathrm{T}}]_A [p_{\mathrm{T}}]_B} \qquad \gamma^{\mathrm{sub}}_{[p_{\mathrm{T}}]} = \frac{c_3^{\mathrm{sub}}}{(c_2^{\mathrm{sub}})^{\frac{3}{2}}}$$

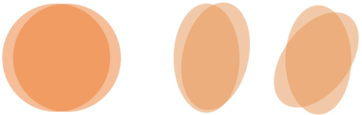
$$\Gamma^{\mathrm{sub}}_{[p_{\mathrm{T}}]} = \frac{[p_{\mathrm{T}}] c_3^{\mathrm{sub}}}{(c_2^{\mathrm{sub}})^2} \qquad \kappa^{\mathrm{sub}}_{[p_{\mathrm{T}}]} = \frac{c_4^{\mathrm{sub}}}{(c_2^{\mathrm{sub}})^2}$$

- Event-by-event meant-p_{T} fluctuations are measured
- Larger fluctuations for deformed nuclei
- Additional constraints for the initial conditions

Moments of δp_{T}^2



Spherical nuclei Deformed nuclei



$$k_2^{\mathrm{sub}} = \frac{c_2^{\mathrm{sub}}}{[p_{\mathrm{T}}]_A [p_{\mathrm{T}}]_B} \qquad \gamma^{\mathrm{sub}}_{[p_{\mathrm{T}}]} = \frac{c_3^{\mathrm{sub}}}{(c_2^{\mathrm{sub}})^{\frac{3}{2}}}$$

$$\Gamma^{\mathrm{sub}}_{[p_{\mathrm{T}}]} = \frac{[p_{\mathrm{T}}] c_3^{\mathrm{sub}}}{(c_2^{\mathrm{sub}})^2} \qquad \kappa^{\mathrm{sub}}_{[p_{\mathrm{T}}]} = \frac{c_4^{\mathrm{sub}}}{(c_2^{\mathrm{sub}})^2}$$

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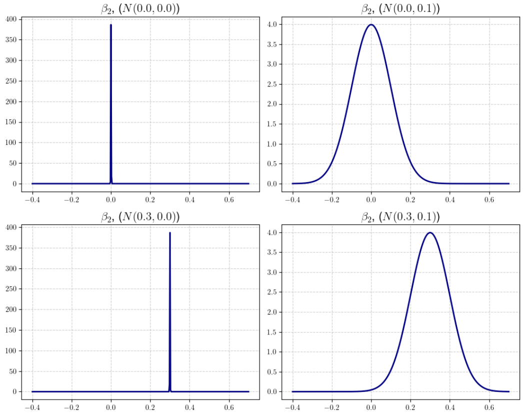
MODEL CALCULATIONS



Hydrodynamic model

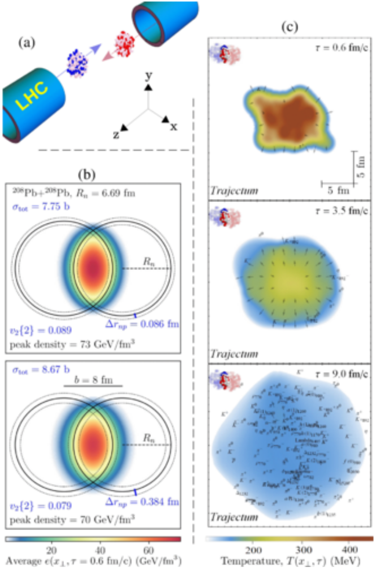
TRENTo + VISH(2+1)+UrQMD

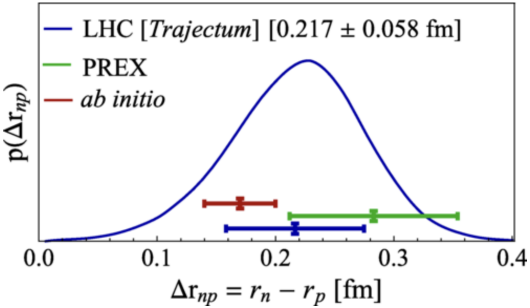
Variable	Prior range
$\overline{\beta_2}$	[0.0, 0.3]
σ_{β_2}	[0.0, 0.1]
$\overline{\gamma}$	[0.0, $\pi/3$]
σ_{γ}	[0.0, $\pi/9$]

- β_2 and γ are drawn from Gaussian distributions
- Four new parameters describing the magnitude and fluctuations of the nuclear structure parameters

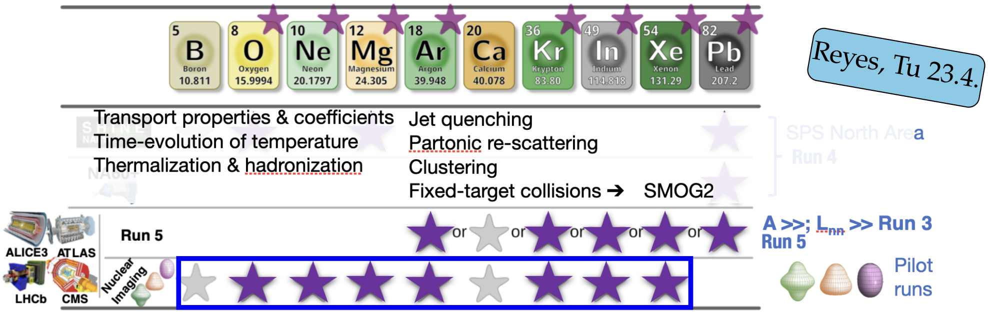
Model setup is similar to those in J. Bernhard *et al.*, *Nat. Phys.* 15, 1113–1117 (2019), J. Parkkila *et al.*, *Phys. Lett. B* 835 (2022) 137485 and M. Virta *et al.*, *Phys. Rev. C* 111, 044903

Recent and future possibilities





- Trajectum study on extracting the neutron utilising Bayesian inference
- Various flow observables included
- Yields similar results with other extractions and calculations

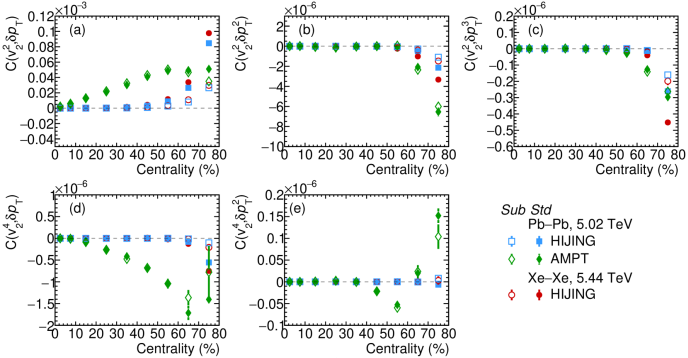


- New collision system proposals for the future at the LHC
- Looks promising for the nuclear imaging
- Lots of data remains to be analysed

THANK YOU!

BACKUP

$$\langle v_2^m \delta p_{\mathrm{T}}^k \rangle = \frac{\sum\limits_{i_1 \neq \ldots \neq i_m \neq i_{m+1} \neq \ldots \neq i_k} \omega_{i_1} \ldots \omega_{i_m} \omega_{i_{m+1}} \ldots \omega_{i_k} e^{in(\Delta\varphi_{i_1} + \cdots + \Delta\varphi_{i_{m/2}})} \delta p_{\mathrm{T},\mathrm{i}_{\mathrm{m}+1}} \ldots \delta p_{\mathrm{T},\mathrm{i}_\mathrm{k}}}{\sum\limits_{i_1 \neq \ldots \neq i_m \neq i_{m+1} \neq \ldots \neq i_k} \omega_{i_1} \ldots \omega_{i_m} \omega_{i_{m+1}} \ldots \omega_{i_k}}$$

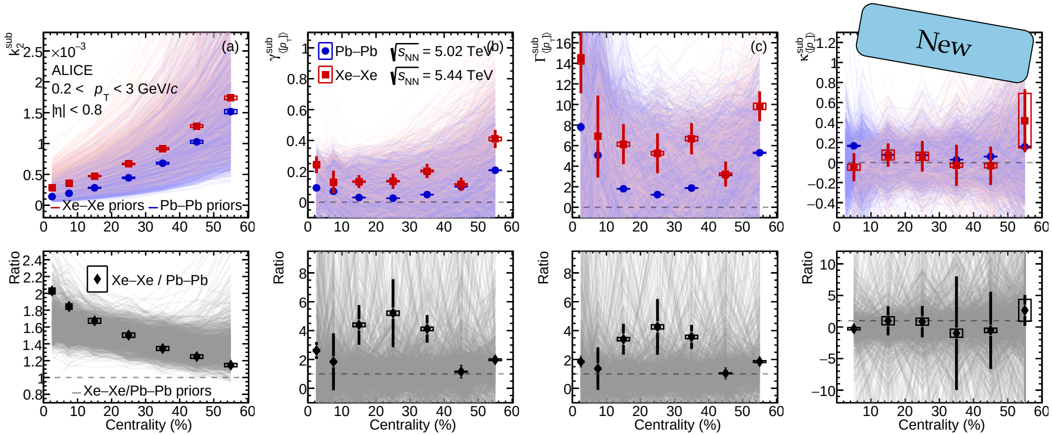


E. Nielsen *et al.*, *arXiv*:2504.03044

- Cumulants are extracted in terms of the moments
- Calculated using the Unified Algorithm

  E. Nielsen *et al.*, *arXiv*:2504.03044
- Evaluated with the two-sub event method

- Correlations between v_2^2 (v_2^4) and different orders of δp_{T}



- 1 500 design points used for each collision system
- Priors cover the measurements nicely
- Ratio or difference of Xe–Xe and Pb–Pb are used to be more sensitive to Xe nuclear structure