In-medium generator coordinate method for nuclear collective excitations and double-beta decays

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Intersection of nuclear structure and high-energy nuclear collisions 2026
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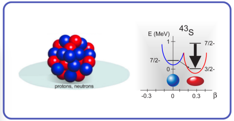
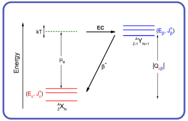
Contents

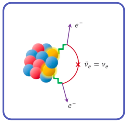
Nuclear physics in a high-precision era



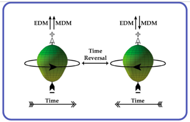
Exotic nuclear structure



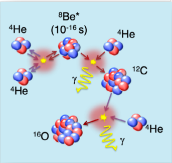
Nuclear processes in astrophysics



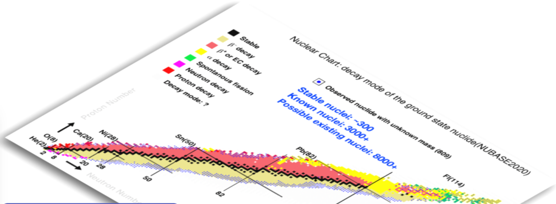
Double beta decay

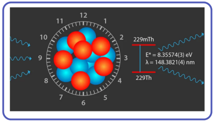


Nuclear Schiff moment

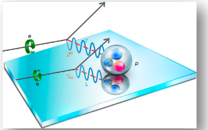


alpha-alpha fusion

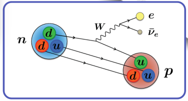




Nuclear clock



Parity-Violating Electron Scattering

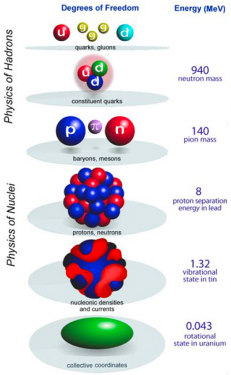


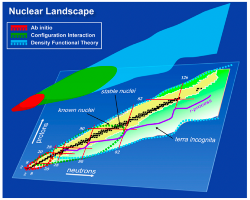
Superallowed Fermi transition

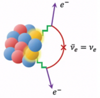
Atomic nuclei: quantum many-body systems, natural laboratories for testing fundamental symmetries and **understanding the origin of elements**.

Advances in high-precision measurements (atomic, nuclear, particle physics) and searches for new physics call for nuclear theory with controlled uncertainties.

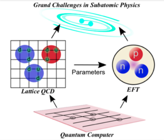
Development of nuclear models in the past century





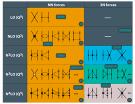


(Lattice) QCD for NP



2010s

ab initio methods



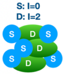
2000s

High-precision NN interactions Entem, Machleidt (2003)

1990s

Chiral EFT for NN interaction Weinberg (1991)

1970s

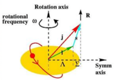


Self-consistent mean-field/EDF approaches

Interacting Boson Models Arima, Iachello (1974) IBM

Hill, Griffin, Wheeler (1953,1957) GCM

1950s

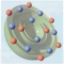


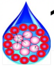
1930s

Collective models Bohr and Mottelson (1953) PRM, BH

Shell models Bohm, Pines (1951-1953) RPA

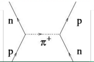
Goeppert-Mayer (1949);
Haxel, Jensen, and Suess (1949) ISM





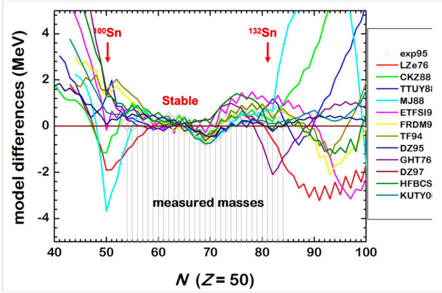
Liquid drop model
von Weizsäcker (1935)

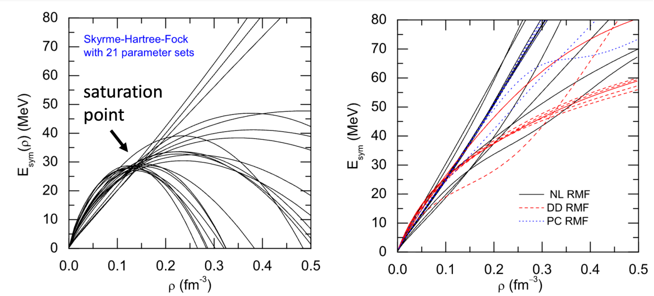
Meson-exchange theory Yukawa (1935)



Phenomenological models for nuclear structure: challenges

- Excellent description of nuclear structure with data (free parameters)
- Divergent description of nuclear structure without data (large model depepdence)
- Difficult to quantify the systematic uncertainty (error uncontrollable effective forces)

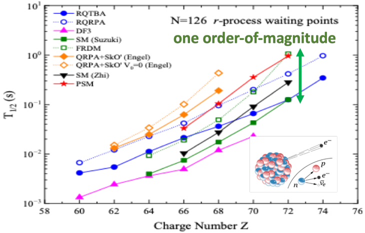




Bao-An Li, Lie-Wen Chen, Che Ming Ko, Physics Reports 464, 113 (2008)

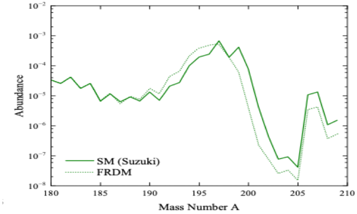
Phenomenological models for weak decays: challenges

Single-beta decay

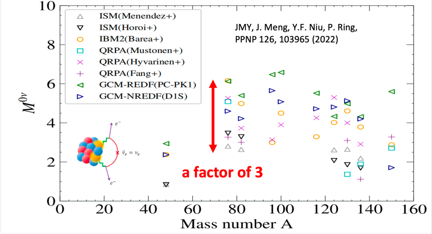


N=126 *r*-process waiting points

one order-of-magnitude

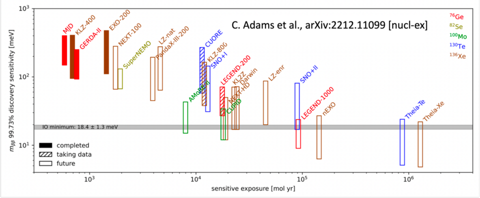


Neutrinoless double-beta decay



JMY, J. Meng, Y.F. Niu, P. Ring, PPNP 126, 103965 (2022)

a factor of 3

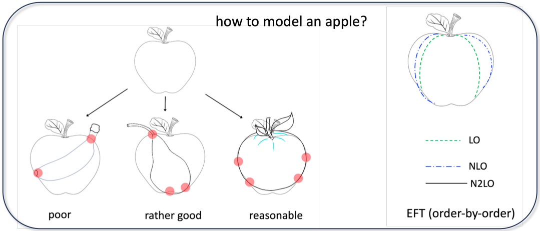


C. Adams et al., arXiv:2212.11099 [nucl-ex]

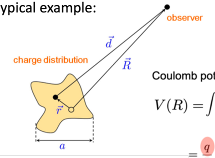
Q: how to reduce the 'LARGE' model dependence (uncertainty)? A nontrivial question!

From phenomenological to systematically improvable models

- Challenging to reduce the discrepancy among different models (Different from the beginning!)
- Why choosing phenomenological models? (Non-perturbative nature of nuclear force and quantum many-body problem, starting from the simplest.)
- Can we choose a model with errors systematically improvable? (perturbative, comparable, convergence pattern)



A typical example:



Coulomb potential:

$$V(R) = \int d^3r \, \frac{\rho(\vec{r}\,)}{|\vec{R} - \vec{r}\,|}$$

$$= \underbrace{\frac{q}{R}}_{\text{LO}} + \underbrace{\frac{1}{R^3}\sum_i R_i P_i}_{\text{NLO}} + \underbrace{\frac{1}{6R^5}\sum_{ij}(3R_iR_j - \delta_{ij}R^2)Q_{ij}}_{\text{N}^2\text{LO}} + \dots$$

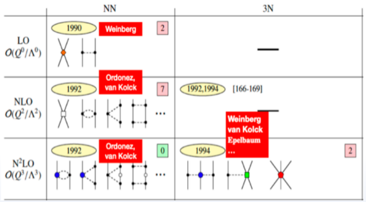
The above expansion converges provided that the quantity $(r/R) << 1$.

Nuclear ab initio methods based on operators from chiral EFT

❑ Basic idea and advantages

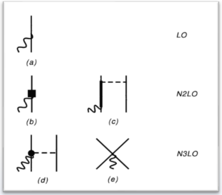
- Nuclear forces & transition operators: consistently constructed within **chiral EFT** (N, π)
- The theoretical uncertainty: controlled by the power-counting $(Q/\Lambda)^n$ with $Q \approx 140$ MeV, $\Lambda \approx 700$ MeV)
- Nuclear wave functions: solved using **exact or systematically improvable many-body methods**

Nuclear forces



Phys. Rep. 503, 1 (2011); 890,1 (2020)

Axial currents



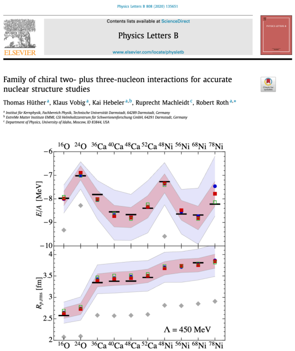
Phys. Rev. C 95, 059902 (2017)
Few Body Syst. 62, 114 (2021)

● Nuclear ab initio methods

- ✓ 无芯壳模型 (NCSM)
- ✓ 介质相似重整化群方法 (IMSRG)
- ✓ 格林函数蒙特卡罗方法 (GFMC)
- ✓ 耦合团簇理论 (CC)
- ✓ 多体微扰论方法 (MBPT)
- ✓ Brueckner Hartree-Fock (BHF)
- ✓ ...

Frontiers in Physics 8, 379 (2020)

Remarkable progress in ab initio studies of atomic nuclei



Physics Letters B 808 (2020) 135651

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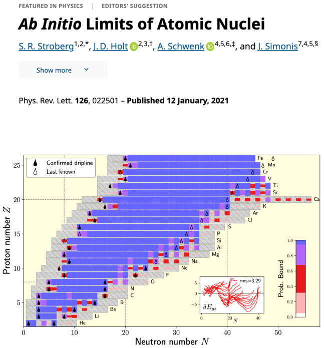
www.elsevier.com/locate/physletb

Family of chiral two- plus three-nucleon interactions for accurate nuclear structure studies

Thomas Hüther [a], Klaus Vobig [a], Kai Hebeler [a,b], Ruprecht Machleidt [c], Robert Roth [a,*]

[a] Institut für Kernphysik, Fachbereich Physik, Technische Universität Darmstadt, 64289 Darmstadt, Germany
[b] ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany
[c] Department of Physics, University of Idaho, Moscow, ID 83844, USA

A systematic uncertainty analysis based on the order-by-oder behavior of nuclear observables, which reveals a very robust convergence pattern.



FEATURED IN PHYSICS | EDITORS' SUGGESTION

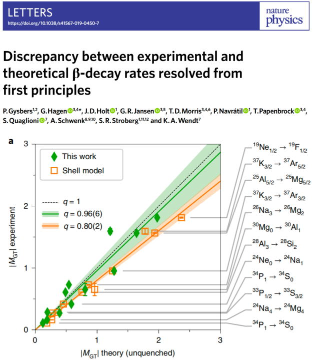
Ab Initio Limits of Atomic Nuclei

S. R. Stroberg[1,2,*], J. D. Holt[2,3,†], A. Schwenk[4,5,6,‡], and J. Simonis[7,4,5,§]

Show more

Phys. Rev. Lett. **126**, 022501 – **Published 12 January, 2021**

First-principles calculations predict the properties of nearly 700 isotopes between helium and iron, with **the rms deviation of 3.3 MeV**.



LETTERS
https://doi.org/10.1038/s41567-019-0450-7

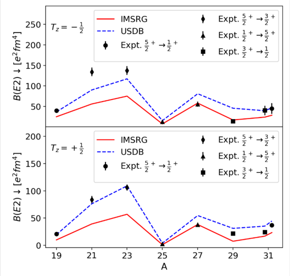
nature physics

Discrepancy between experimental and theoretical β-decay rates resolved from first principles

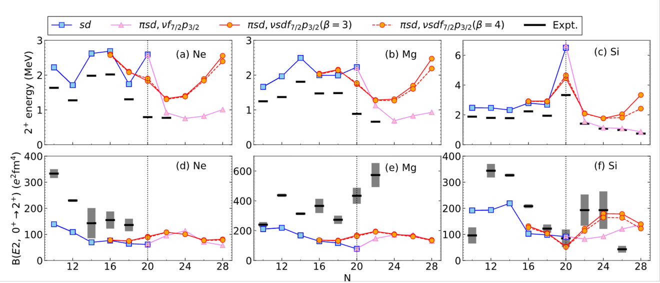
P. Gysbers[1,2], G. Hagen[3,4*], J. D. Holt[1], G. R. Jansen[3,5], T. D. Morris[3,4,6], P. Navrátil[1], T. Papenbrock[3,4], S. Quaglioni[7], A. Schwenk[8,9,10], S. R. Stroberg[1,11,12] and K. A. Wendt[7]

The gA quenching issue is resolved by considering the **two-body current effect** and **many-body correlation**.

Open-shell deformed nuclei: requires a large model space



S.R. Stroberg et al., PRC105, 034333 (2022)

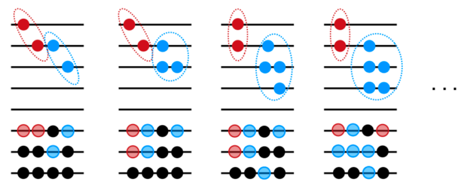


T. Miyagi et al., PRC 102, 034320 (2020)

- **New challenge**: nuclear collective correlations, especially those associated with deformation, remain a major challenge for many ab initio methods due to the model-space truncations.
- **Significance**: deformation effect is crucial for understanding atomic nuclei and the NMEs relevant in the search of new physics, including $0\nu\beta\beta$ decay and atomic EDMs.

Many-body collective correlations in open-shell deformed nuclei

many-particle many-hole excitations



important for deformed nuclei, but challenge for most ab initio methods!

An example: IMSRG(3)

- Computational scaling $O(N^9)$
- memory storage N^6

  computational challenge!

An example: IMSRG(A)

- From a single reference state $|\Phi\rangle$ to exact ground state $|\Psi\rangle$

$$|\Psi\rangle = e^{\hat{\Omega}}|\Phi\rangle,$$

where many-body correlations are

built into the correlation operator $\hat{\Omega}$,

$$\hat{\Omega} = \hat{\Omega}^{(1b)} + \hat{\Omega}^{(2b)} + \hat{\Omega}^{(3b)} + \cdots + \hat{\Omega}^{(Ab)}$$

determined from the IMSRG.

How to overcome this challenge?

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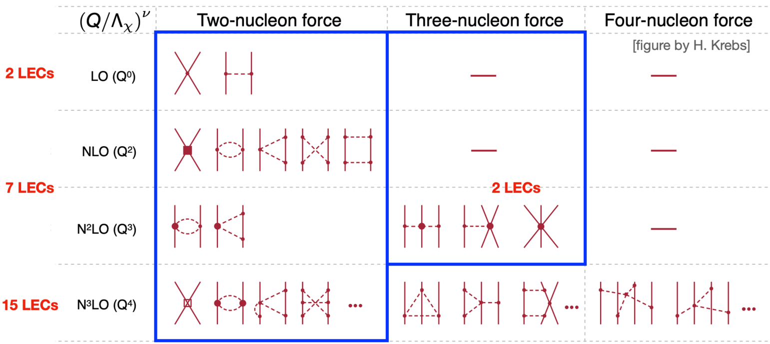
Nuclear Hamiltonian based on chiral interactions

$$\hat{H}_0 = (1 - \frac{1}{A})T^{[1]} + \frac{1}{A}T^{[2]} + \sum_{i<j} V_{ij}^{[2]} + \sum_{i<j<k} W_{ijk}^{[3]},$$

where the kinetic energy terms $T^{[1]} = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_N}, T^{[2]} = -\sum_{i<j} \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{m_N},$



[figure by H. Krebs]

NN+3N (EM): R. Machleidt, D. R. Entem, Phys. Rep. 503, 1 (2011)

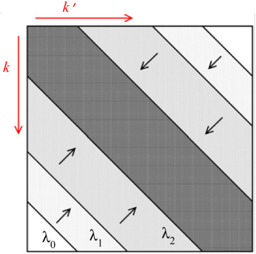
Changing the momentum resolution of nuclear forces with SRG

- Unitary transformations to decouple the off-diagonal elements

$$H_s = U_s H U_s^\dagger \equiv T_{\rm rel} + V_s$$

from which one finds the flow equation

$$\frac{dH_s}{ds} = [\eta_s, H_s], \quad \eta_s = [T_{\rm rel}, H_s]$$



The flow parameter s is usually replaced with $\lambda = s^{-1/4}$ in units of fm^{-1} (a measure of the spread of off-diagonal strength).

Evolution of the potential

$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$

S. Bogner, R. Furnstahl, and A. Schwenk, PPNP65, 94 (2010)

Solving many-body problem (decoupling H) with IMSRG

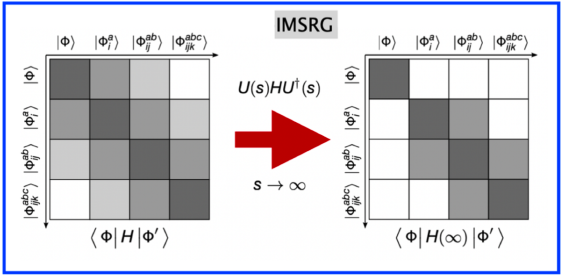
- Unitary transformation

$$\hat{H}(s) = \hat{U}(s)\hat{H}_0\hat{U}^\dagger(s),$$

leads to the flow equation

$$\frac{d\hat{H}(s)}{ds} = [\hat{\eta}(s), \hat{H}(s)]$$

- Generator $\eta(s)$ is chosen in such a way that a given reference state $|\Phi\rangle$ is decoupled from its excitations.



The time complexity increases polynomially (instead of exponentially) with nuclear size.

K. Tsukiyama et al., PRL106, 222502 (2011); H. Hergert et al., Phys. Rep. 621, 165 (2016)

If choosing a HF state as the reference state, one has

$$\hat{H}_0\left|\Psi^{(0)}_{\rm Exact}\right\rangle = E_0\left|\Psi^{(0)}_{\rm Exact}\right\rangle \longrightarrow \hat{H}(\infty)\left|\Phi_{\rm HF}\right\rangle = E_0\left|\Phi_{\rm HF}\right\rangle, \quad \hat{U}(s) = e^{\hat{\Omega}(s)}.$$

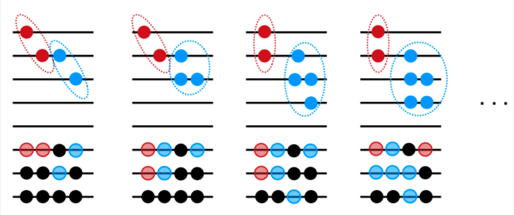
A multi-reference framework for deformed nuclei

Basic idea of MR-IMSRG

Build correlations into the reference state

$$
\begin{aligned}
|\Psi_{\rm Exact}\rangle &= e^{\hat{\Omega}^{(1b)}+\hat{\Omega}^{(2b)}+\cdots+\hat{\Omega}^{(Ab)}}|\Phi_{\rm HF}\rangle \\
&\simeq e^{\hat{\Omega}^{(1b)}+\hat{\Omega}^{(2b)}+\cdots}|\Phi_{\rm Cor}\rangle
\end{aligned}
$$

many-particle many-hole configurations

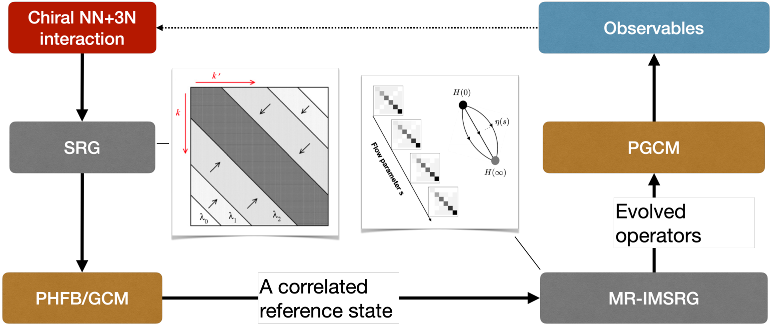


- The correlated reference state $|\Phi_{\rm Cor}\rangle$ is chosen as a state with many-particle many-hole excitations relevant for nuclear collective excitations.
- Based on this idea, we have developed a novel IM-GCM for open-shell deformed nuclei, which opens the door to ab initio calculations of $0\nu\beta\beta$ decay.

  JMY et al., PRL124, 232501 (2020)
- Other recently-developed multi-reference methods: IM-NCSM, PGCM-PT, MR-CC, etc

The in-medium generator coordinate method (IM-GCM)

The Framework of IM-GCM



Chiral NN+3N interaction

SRG

PHFB/GCM

A correlated reference state

MR-IMSRG

Evolved operators

PGCM

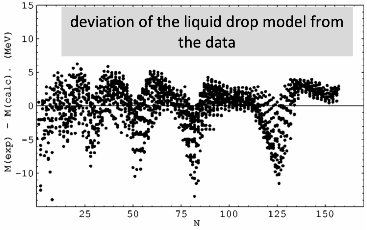
Observables

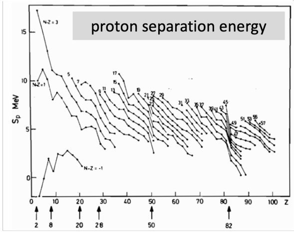
- **In-medium similarity renormalization group (IMSRG)**: capture dynamic correlations associated with high-energy few-particle, few- hole excitations
- **Projected generator coordinate method (PGCM)**: include the collective (static) correlations associated with pairing and deformation.

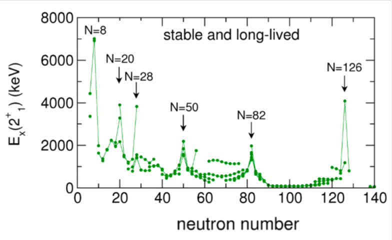
JMY et al., PRC98, 054311 (2018); PRL124, 232501(2020); arXiv:2603.07363 [nucl-th]

Contents

Nuclear shell structure and the ”magic numbers”



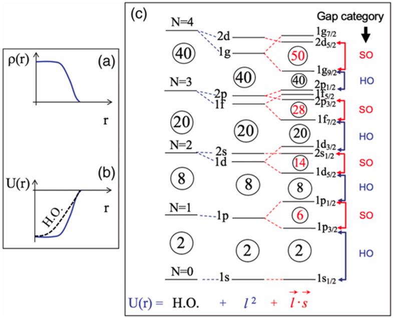


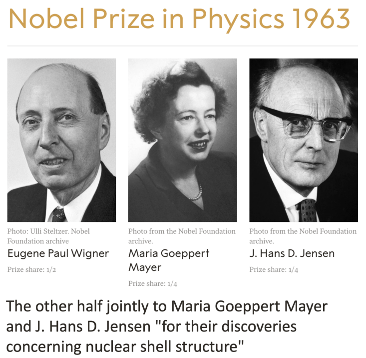


- Nuclei with specific neutron or proton numbers—such as $2, 8, 20, 28, 50, 82$, and 126—exhibit pronounced discontinuities in binding energies and elevated excitation energies compared to their neighboring nuclei.
- The concept of shell structure was introduced to explain these irregular trends and the ”magic numbers”.

Nuclear shell structure and the ”magic numbers”

Otsuka, RMP **92**, 015002 (2020)





Nobel Prize in Physics 1963

Photo: Ulli Steltzer. Nobel Foundation archive	Photo from the Nobel Foundation archive.	Photo from the Nobel Foundation archive.
Eugene Paul Wigner	Maria Goeppert Mayer	J. Hans D. Jensen
Prize share: 1/2	Prize share: 1/4	Prize share: 1/4

The other half jointly to Maria Goeppert Mayer and J. Hans D. Jensen "for their discoveries concerning nuclear shell structure"

- 1949, Goeppert Maye and Jensen independently proposed to add a large spin-orbit potential ”by hand” to explain the ”magic” numbers.

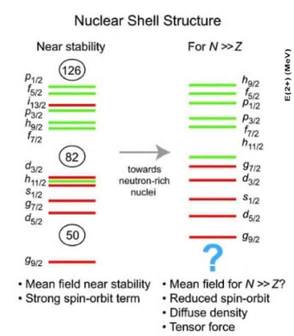
  M. Goeppert Mayer, Phys. Rev. 75, 1969 (1949); O. Haxel.; J.H.D. Jensen, H. E.. Suess, Phys. Rev. 75, 1766 (1949).

- Since then, shell structure has become a fundamental concept for understanding the structure and stability of atomic nuclei.

Nuclear shell structure and the "magic numbers"

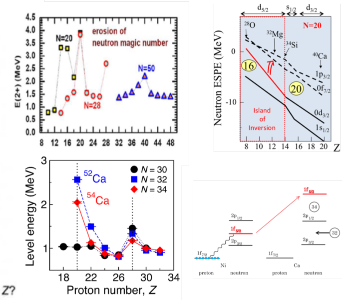
- The study of nuclear shell structure towards dripline has become a key frontier of nuclear physics. persistence or erosion?
- Close relation between shell structure and nuclear low-lying states (e.g., excitation energies, transition strengths).

Evolution of nuclear structure and energy spectrum



- Mean field near stability
- Strong spin-orbit term

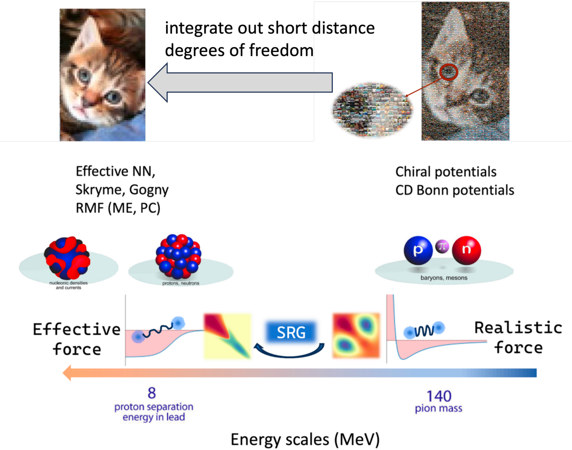
- Mean field for $N >> Z$?
- Reduced spin-orbit
- Diffuse density
- Tensor force



T. Otsuka et al., Rev. Mod. Phys. 92, 015002 (2020)

Qs: Why there is a "large" spin-orbit coupling in atomic nuclei? What is the microscopic origin? How does it change towards dripline nuclei?

Understand shell structure starting from realistic nuclear forces



integrate out short distance degrees of freedom

Effective NN,
Skryme, Gogny
RMF (ME, PC)

Chiral potentials
CD Bonn potentials

Effective force

Realistic force

8
proton separation energy in lead

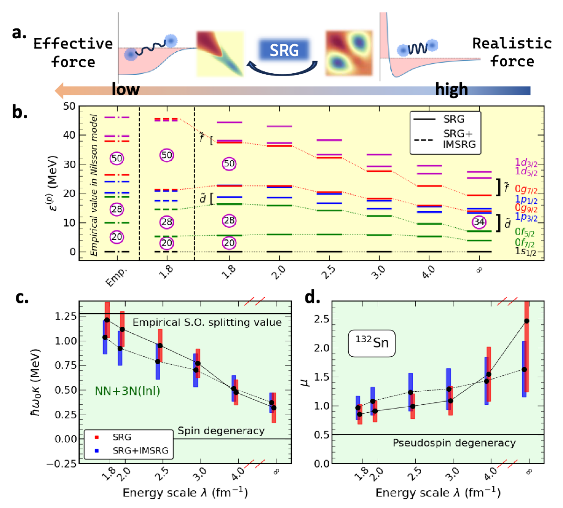
140
pion mass

Energy scales (MeV)

- Build a connect between realistic nuclear forces to effective nuclear potentials.
- Investigate the evolution of shell structure with the momentum resolution of nuclear forces.
- Explore the role of three-body nuclear forces (3NF).

See also: T. Duguet and G. Hagen, PRC85, 034330 (2012); T. Duguet, H. Hergert, J. D. Holt, and V. Soma, PRC92, 034313 (2015).

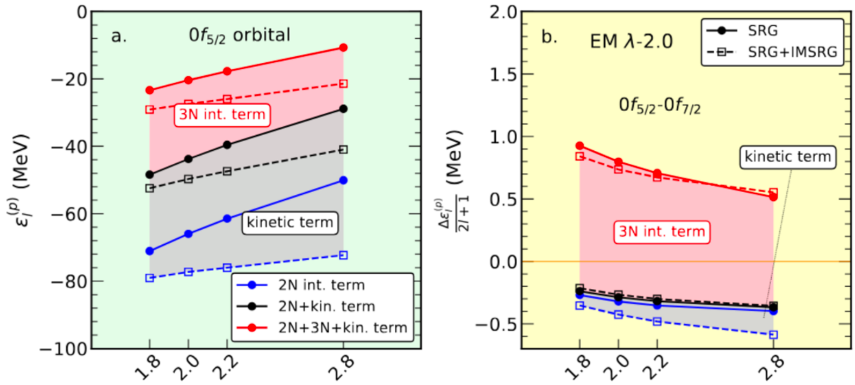
SRG: blurring the resolution of the nuclear force



- The picture of transition from spin symmetry to pseudo spin symmetry is exhibited in the s.p. energy spectrum as the resolution scale decreases.
- Magic numbers beyond 20 emerge naturally: $Z = 28$ ($Z = 34$) shell gap develops (disappears).

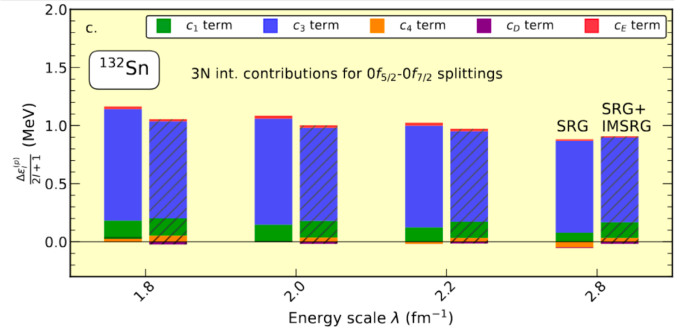
In other words, spin (pseudospin) symmetry dominates at high (low) resolution, and the competition between them produces the observed shell structure.

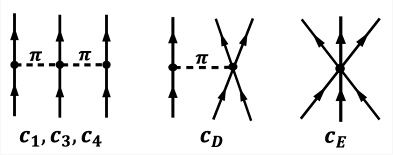
Contributions of nuclear forces to shell structure



- The 3NF predominantly contributes to the energy splitting of these SO doublets.
- The negative contribution of the 2NF to the SO splitting is a result of cancellation among different terms.

Contributions of nuclear forces to shell structure



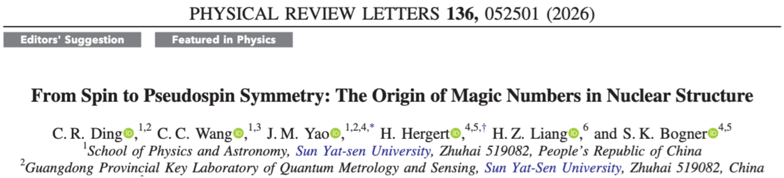


- The 2π exchange potential is the dominant contributor to the energy splitting of SO doublets, in which, the component associated with the LEC c_3

$$W_{c_3}^{(3N)} = \frac{c_3 g_A^2}{4f_\pi^4} \sum_{i\neq j\neq k} \vec{\tau}_i \cdot \vec{\tau}_j \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(q_i^2 + m_\pi^2)(q_j^2 + m_\pi^2)} \vec{q}_i \cdot \vec{q}_j$$

gives the major contribution.

Microscopic origin of the magic numbers



PHYSICAL REVIEW LETTERS **136,** 052501 (2026)

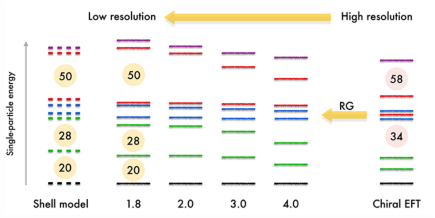
Editors' Suggestion **Featured in Physics**

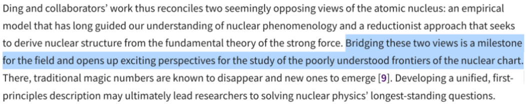
From Spin to Pseudospin Symmetry: The Origin of Magic Numbers in Nuclear Structure

C. R. Ding,[1,2] C. C. Wang,[1,3] J. M. Yao,[1,2,4,*] H. Hergert,[4,5,†] H. Z. Liang,[6] and S. K. Bogner[4,5]

[1]*School of Physics and Astronomy, Sun Yat-sen University, Zhuhai 519082, People's Republic of China*

[2]*Guangdong Provincial Key Laboratory of Quantum Metrology and Sensing, Sun Yat-Sen University, Zhuhai 519082, China*





Ding and collaborators' work thus reconciles two seemingly opposing views of the atomic nucleus: an empirical model that has long guided our understanding of nuclear phenomenology and a reductionist approach that seeks to derive nuclear structure from the fundamental theory of the strong force. Bridging these two views is a milestone for the field and opens up exciting perspectives for the study of the poorly understood frontiers of the nuclear chart. There, traditional magic numbers are known to disappear and new ones to emerge [9]. Developing a unified, first-principles description may ultimately lead researchers to solving nuclear physics' longest-standing questions.

Exposing Nuclear Magic

Vittorio Somà

CEA Paris-Saclay, Gif-sur-Yvette, France

February 2, 2026 • *Physics* 19, 11

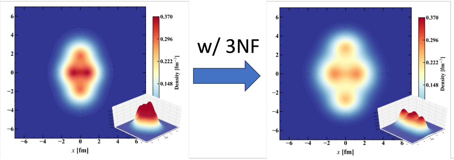
Calculations show how the mysterious "magic numbers" that stabilize nuclear structures emerge naturally from nuclear forces—once these are described with appropriate spatial resolution.

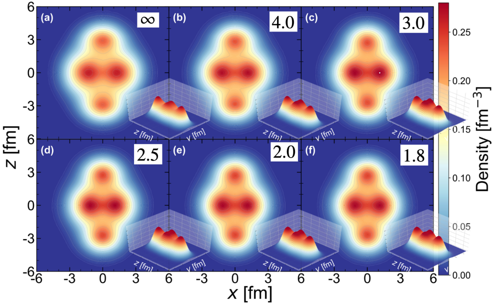
Figure 1: Nuclear "magic numbers" (2, 8, 20, 28, 50, 82…) correspond to particularly stable configurations of atomic nuclei. New work connects their traditional, phenomenological shell-model interpretation to a microscopic explanation rooted in the underlying interactions between nucleons.

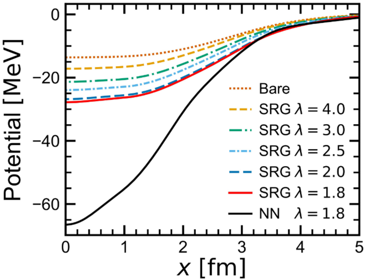
Vittorio Somà (CEA)

Mechanism responsible for nuclear cluster in ^{20}Ne

Clustering structure at mean-field level







- The 3NF increases significantly the clustering structure.
- The effect of changing momentum resolution is relatively moderate.

See the talks by M. Kimura and B.N. Lu for more discussions.

The change of the magic numbers in neutron-rich nuclei?

The erosion of traditional magic number N=20?

PHYSICAL REVIEW C VOLUME 41, NUMBER 3 MARCH 1990

Mass systematics for $A=29$–44 nuclei: The deformed $A\sim32$ region

E. K. Warburton
Brookhaven National Laboratory, Upton, New York 11973
and Universität Heidelberg and the Max-Planck-Institut für Kernphysik, Heidelberg, West Germany

J. A. Becker
Lawrence Livermore National Laboratory, Livermore, California 94550

B. A. Brown
Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan

(Received 11 October 1989)

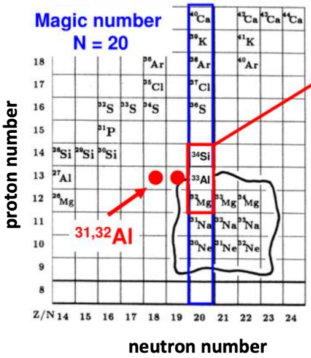
Further evidence for the presence of an anomaly in binding energies for the "island of inversion" centered at $Z=11$, $N=21$ is obtained by comparison of shell-model calculations to experiment. The calculations were done with a shell-model interaction that is applicable to nuclei with active valence nucleons in both the $(1s,0d)$ and $(0f,1p)$ major shells. This interaction is described in detail as are its predictions for binding energies and energy spectra of $Z=8$–20, $N=18$–25 nuclei. These calculations provide the background for the exploration of the "island of inversion." The extent of the "island" and the magnitude of the anomaly is explored by calculating the binding energies of $2\hbar\omega$ excitations of neutrons from the $(1s,0d)$ shell to the $(0f,1p)$ shell relative to the $0\hbar\omega$ ground state. The reason why mixed $(0+2)\hbar\omega$ calculations are not considered reliable is addressed. Truncation schemes and a weak-coupling approximation are used to extend the range of the calculations. It is found that for $Z=10$–12, $N=20$–22 (and possibly $N>22$) nuclei the lowest $2\hbar\omega$ state is more bound than the $0\hbar\omega$ ground state. The role of odd n $n\hbar\omega$ excitations is considered and it is found that the $1\hbar\omega$ ground state always lies below that of $3\hbar\omega$, and for $N=19$, 21, and 23, the lowest $1\hbar\omega$ state is in close competition with $2\hbar\omega$ for the lowest binding energy. Collectivity is considered via $E2$ observables and energy spectra for the $2\hbar\omega$ ground-state bands. The reason for the existence of the "island" is discussed.

This article appears in the following collection:

PHYSICAL REVIEW C
50 YEARS

Physical Review C 50th Anniversary Milestones

E. K. Warburton, J. A. Becker, B. A. Brown, *PRC41, 1147 (1990)*



N=20 isotones
^{34}Si - spherical
^{33}Al - ???
^{32}Mg - deformed

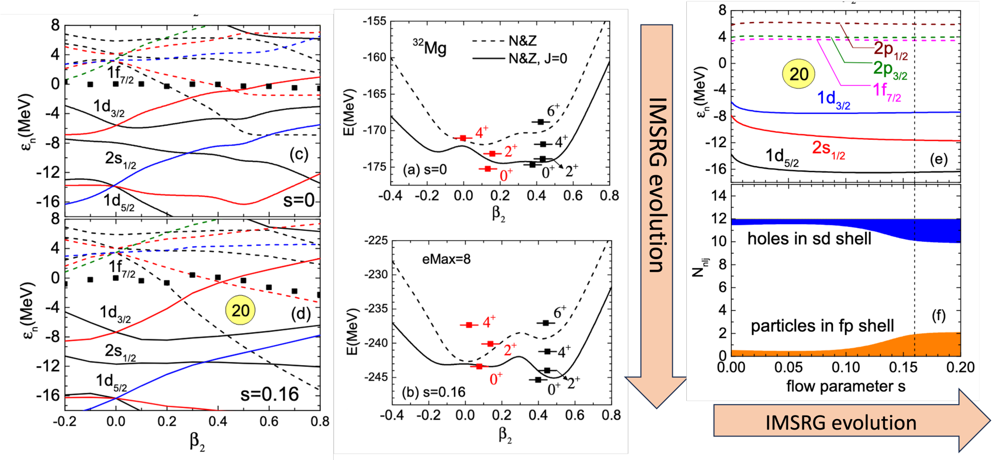
Residual interaction(s)
⇒
breaking of magic number N = 20

Island of Inversion
Ground state of nuclei consists of configurations with two neutrons excited to intruder *fp*-states
⇒
Deformation

... shores of the island are NOT known yet!!!

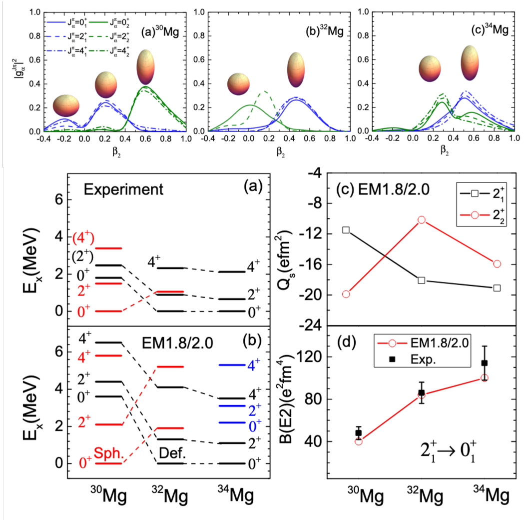
The nuclei at the island of inversion are challenging for most ab initio methods.

Understanding the structure of ^{32}Mg



With the many-body correlations absorbed into the effective *H* gradually by the IMSRG, a transition from the spherical to deformed configuration occurs in the ground state of ^{32}Mg.

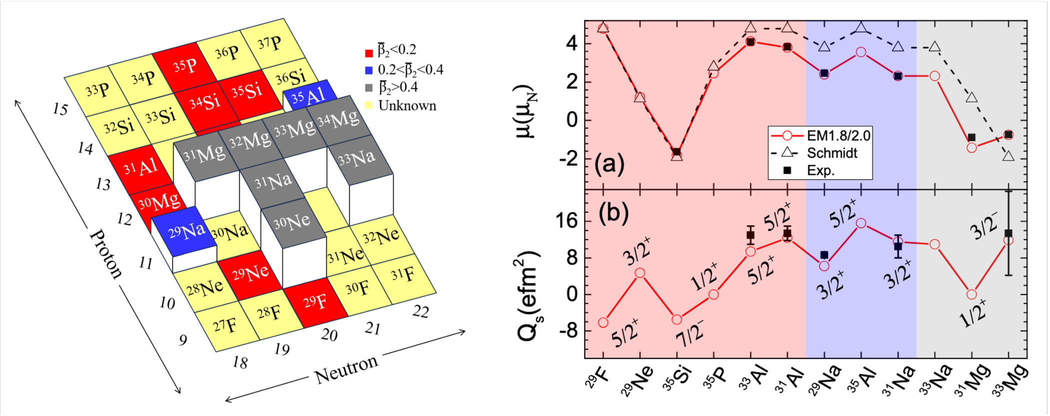
Mg isotopes around ^{32}Mg



E. F. Zhou, C. R. Ding, JMY et al., Phys.Lett.B 865, 139464 (2025).

- Onset of strong collectivity from ^{30}Mg to ^{34}Mg: increase in $B(E2: 2^+_1 \to 0^+_1)$ and decrease in $E_x(2^+_1)$.
- The weakly prolate and oblate deformed ground state of ^{30}Mg, coexisting with strongly prolate deformed second 0^+ state.
- The strongly prolate deformed ground states of ^{32,34}Mg, coexisting with weakly deformed second 0^+ state.

The boundary of the IOI around ^{32}Mg



- Nuclei inside the island of inversion (IOI): ^{30}Ne, ^{29,31,33}Na, ^{31,32,33,34}Mg, and ^{35}Al
- Nuclei outside: ^{29}F, ^{29}Ne, ^{30}Mg, ^{31,33}Al, ^{34,35}Si, and ^{35}P
- More comprehensive conclusion: extenstion to odd-odd nuclei

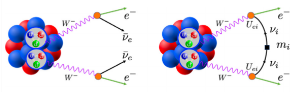
E. F. Zhou, C. R. Ding, Q. Y. Luo, JMY, H. Hergert, arXiv:2603.07363 [nucl-th] (2026).

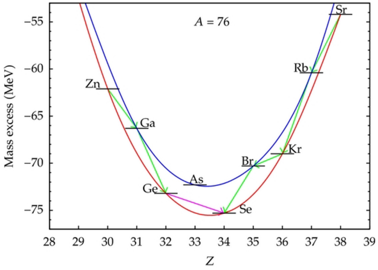
Contents

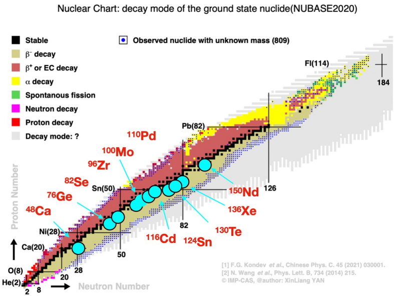
An introduction to $0\nu\beta\beta$ decay

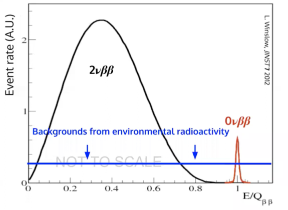
- The two modes of $\beta^-\beta^-$ decay:

$$(A, Z) \rightarrow (A, Z+2) + 2e^- + (2\bar{\nu}_e)$$









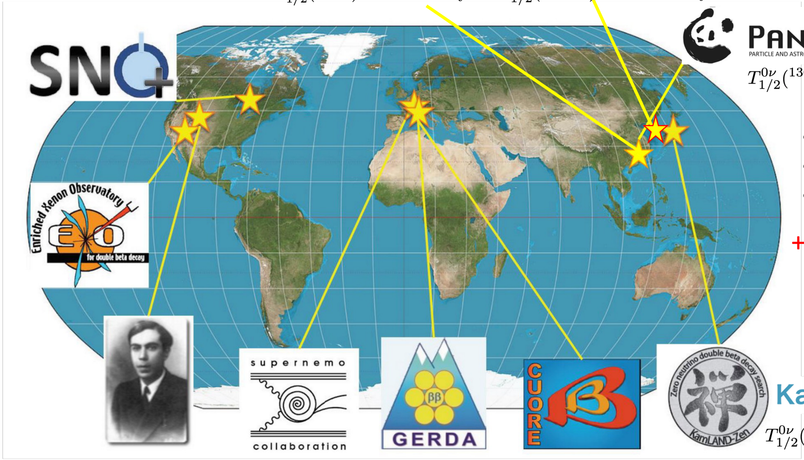
Wordwide experiments searching for $0\nu\beta\beta$ decay

CDEX-1T
China dark-matter experiment

$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 1.0 \times 10^{23}\mathrm{yr}$

AMoRE experiment
Advanced Mo-based Rare process Experiment

$T^{0\nu}_{1/2}(^{100}\mathrm{Mo}) > 2.9 \times 10^{24}\mathrm{yr}$



PANDA X
PARTICLE AND ASTROPHYSICAL XENON TPC

$T^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 2.1 \times 10^{24}\mathrm{yr}$

- NvDEX
- CUPID-China
- JUNO

+several others

KamLAND-Zen

$T^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 2.3 \times 10^{26}\mathrm{yr}$

$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 1.8 \times 10^{26}\mathrm{yr}$

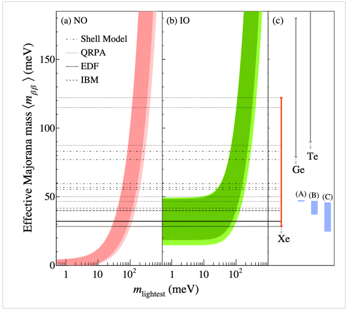
Most sensitive measurements and limits on neutrino mass

Isotope	$G_{0\nu}$ $[10^{-14}\ \mathrm{yr}^{-1}]$	$M^{0\nu}$ [min, max]	$T^{0\nu}_{1/2}$ [yr]	$\langle m_{\beta\beta}\rangle$ [meV]	Experiments with best sensitivity References
^{48}Ca	2.48	[0.85, 2.94]	$> 5.8 \cdot 10^{22}$	[2841, 9828]	CANDLES: PRC78, 058501 (2008)
^{76}Ge	0.24	[2.38, 6.64]	$> 1.9 \cdot 10^{26}$	[75, 200]	LEGEND-200: PRL136, 022701 (2026)
^{82}Se	1.01	[2.72, 5.30]	$> 4.6 \cdot 10^{24}$	[277, 540]	CUPID-0: PRL129, 111801 (2023)
^{96}Zr	2.06	[2.86, 6.47]	$> 9.2 \cdot 10^{21}$	[3557, 8047]	NPA847, 168 (2010)
^{100}Mo	1.59	[3.84, 6.59]	$> 2.9 \cdot 10^{24}$	[210, 610]	AMoRE-I: PRL134, 082501 (2025)
^{116}Cd	0.48	[3.29, 5.52]	$> 2.2 \cdot 10^{23}$	[1766, 2963]	PRD 98, 092007 (2018)
^{130}Te	1.42	[1.37, 6.41]	$> 3.5 \cdot 10^{25}$	[70, 250]	CUORE: Science (2025)
^{136}Xe	1.46	[1.11, 4.77]	$> 3.8 \cdot 10^{26}$	[28, 122]	KamLAND-Zen: PRL135, 262501 (2025)
^{150}Nd	6.30	[1.71, 5.60]	$> 2.0 \cdot 10^{22}$	[1593, 5219]	NEMO-3: PRD 94, 072003 (2016)

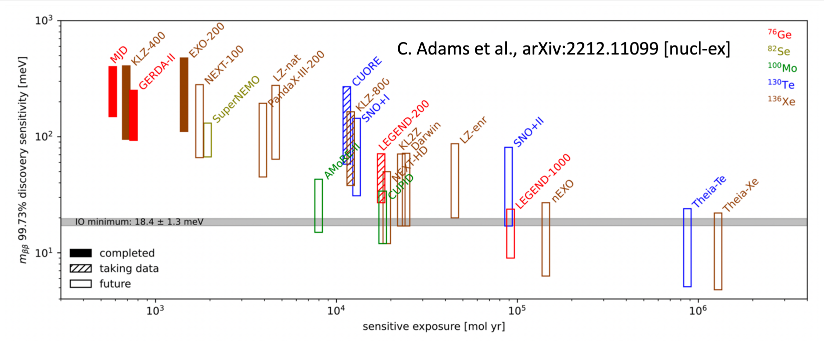
If $0\nu\beta\beta$ decay is driven by exchanging light Majorana neutrinos,

$$\langle m_{\beta\beta}\rangle \equiv \left|\sum_{j=1}^{3} U_{ej}^2 m_j\right| = \left[\frac{m_e^2}{g_A^4\, G_{0\nu}\, T^{0\nu}_{1/2}\, |M^{0\nu}|^2}\right]^{1/2}$$

Accurate values of the NMEs $M^{0\nu} = \langle 0_F^+|\hat{O}^{0\nu}|0_I^+\rangle$ are crucial for designing and interpreting those experiments, as they link the observed decay rate to the effective neutrino mass $\langle m_{\beta\beta}\rangle$.



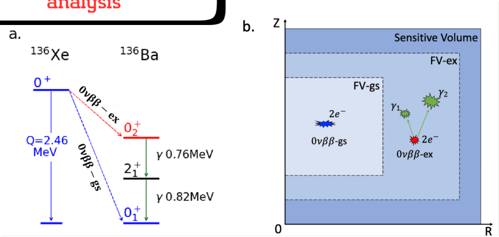
Current and future experimental measurements

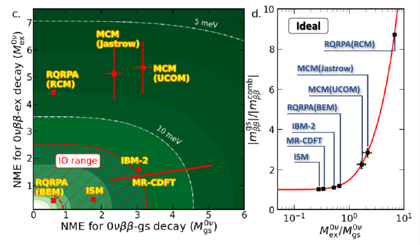


- Lifetime sensitivity of the future ton-scale experiments: $T^{0\nu}_{1/2} > 10^{28}$yr.
- Whether or not the ton-scale experiments are able to cover the entire parameter space for the IO case depends strongly on the employed NME, with a large systematic uncertainty.

A strategy to enhance the sensitity of $0\nu\beta\beta$ decay

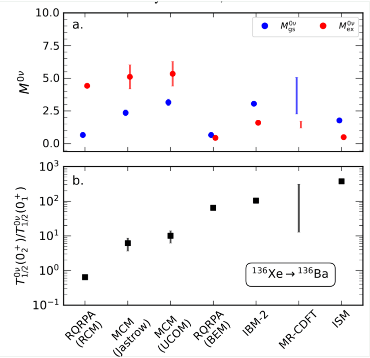
Combined multi-transition analysis





10-year data acquisition on XLZD and the PandaX-xT

Comparison between the NMEs for both decay modes

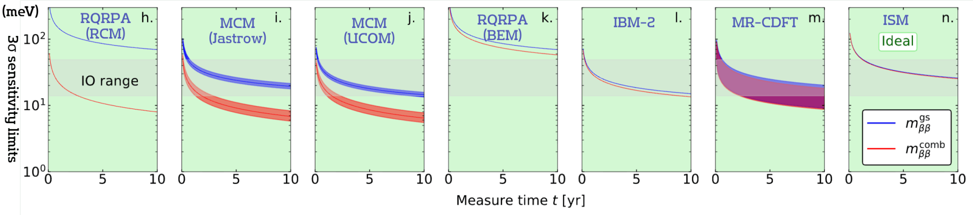


Some models predicted a larger NME for the decay to the excited 0+ state

Ding, Han, Wang, JMY, arXiv2508.17413 [nucl-th]

A strategy to enhance the sensitity of $0\nu\beta\beta$ decay

Combined multi-transition analysis




丁晨蓉

- The combined analysis: can enhance the sensitivity of next-generation experiments, without requiring a larger detector.
- The effectiveness strongly depends on the NMEs, which currently exhibit significant discrepancies across different nuclear models.
- It underscores the importance of achieving theoretically accurate and systematically quantified NME calculations for the NLDBD.

Ding, Han, Wang, JMY, arXiv2508.17413 [nucl-th]

Determine NMEs with relativistic heavy-ion collisions

- angular distribution of heavy-ion collision

$$\frac{dN_{\rm ch}}{d^2\mathbf{p}} \propto \frac{dN_{\rm ch}}{p_T dp_T}\left(1+\sum_{n>1} 2\, v_n \cos n(\phi-\phi_n)\right)$$

where v_2 is strongly correlated with the initial-state ellipticity parameter, ε_2

- observables of interest

$$\Delta(v_2\{2\}) \equiv \frac{1}{2}\Delta\left(\left\langle v_2^2\right\rangle\right),$$

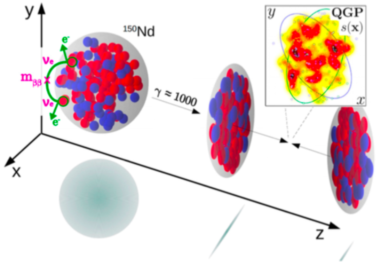
$$\Delta(\delta[p_T]) \equiv \frac{1}{2}\Delta\left(\left\langle(\delta[p_T])^2\right\rangle\right),$$

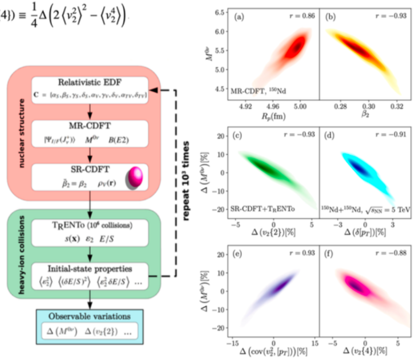
$$\Delta\left(\mathrm{cov}(v_2^2,[p_T])\right) \equiv \frac{1}{3}\Delta\left(\left\langle v_2^2\delta[p_T]\right\rangle\right),$$

$$\Delta(v_2\{4\}) \equiv \frac{1}{4}\Delta\left(2\left\langle v_2^2\right\rangle^2-\left\langle v_2^4\right\rangle\right)$$

- Relative variation of an observable

$$\Delta(O) = \frac{O-\langle O\rangle_{\rm C}}{|\langle O\rangle_{\rm C}|}$$



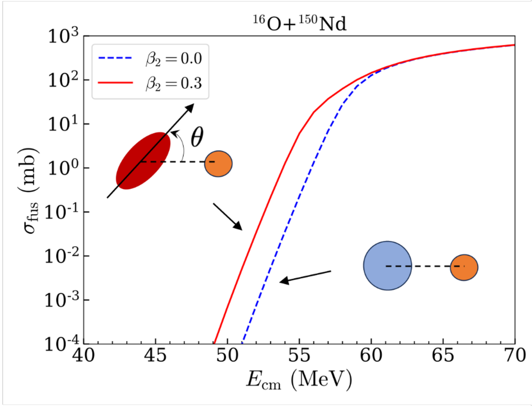


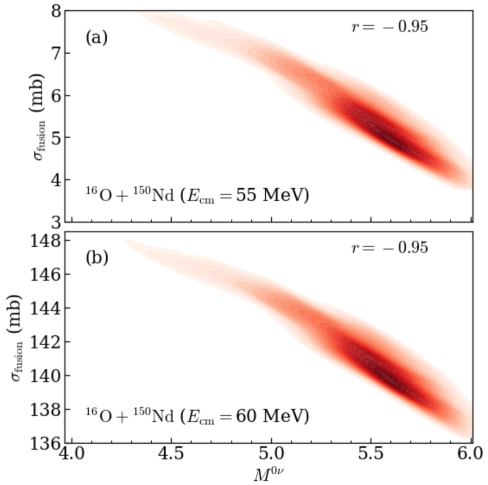
Y. Li, X. Zhang, G. Giacalone, JMY, PRL135, 022301 (2025)

See the talk by Xin Zhang for more details.

Determine NMEs with sub-barrier heavy-ion fusions

Some preliminary results by the CCFULL and MR-CDFT:



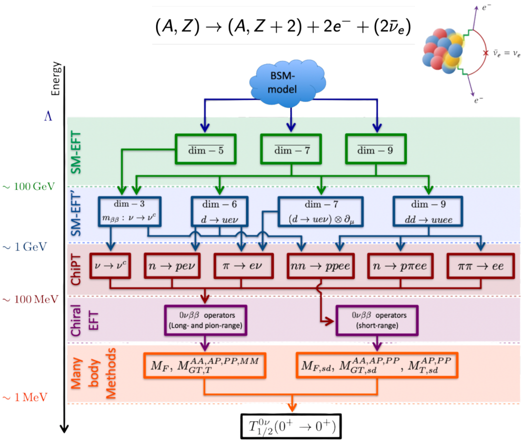


JMY, X. Zhang, K. Hagino, in preparation (2026). See the talk by Kouichi Hagino for more details.

ab initio studies: transition operators

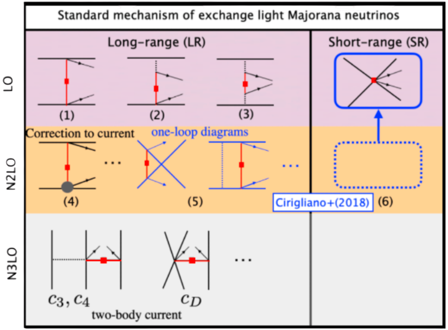
Lepton-number violating operators for neutrinoless double-beta decay

$$(A, Z) \to (A, Z+2) + 2e^- + (2\bar{\nu}_e)$$


e^-
$\bar{\nu}_e = \nu_e$
e^-
Energy
Λ
$\sim 100\,\mathrm{GeV}$
$\sim 1\,\mathrm{GeV}$
$\sim 100\,\mathrm{MeV}$
$\sim 1\,\mathrm{MeV}$
BSM-model
SM-EFT
$\mathrm{dim}-5$ $\mathrm{dim}-7$ $\mathrm{dim}-9$
SM-EFT'
$\mathrm{dim}-3$ $m_{\beta\beta}: \nu \to \nu^c$
$\mathrm{dim}-6$ $d \to ue\nu$
$\mathrm{dim}-7$ $(d \to ue\nu) \otimes \partial_\mu$
$\mathrm{dim}-9$ $dd \to uuee$
ChiPT
$\nu \to \nu^c$ $n \to pe\nu$ $\pi \to e\nu$ $nn \to ppee$ $n \to p\pi ee$ $\pi\pi \to ee$
Chiral EFT
$0\nu\beta\beta$ operators (Long- and pion-range)
$0\nu\beta\beta$ operators (short-range)
Many body Methods
$M_F,\ M_{GT,T}^{AA,AP,PP,MM}$
$M_{F,sd},\ M_{GT,sd}^{AA,AP,PP},\ M_{T,sd}^{AP,PP}$
$T_{1/2}^{0\nu}(0^+ \to 0^+)$

The 0νββ decay operators in the chiral EFT

Power Counting: $\nu = 2A + 2L - 2 + \sum_i \left(\frac{n_f}{2} + d - 2 + n_e\right)_i$


Standard mechanism of exchange light Majorana neutrinos
Long-range (LR)
Short-range (SR)
LO
(1) (2) (3)
N2LO
Correction to current
one-loop diagrams
(4) … (5) …
Cirigliano+(2018) (6)
N3LO
c_3, c_4
c_D
two-body current
…

G. Prézeau, M. Ramsey-Musolf, P. Vogel, PRD68, 034016 (2003); V. Cirigliano et al., JHEP 12, 097(2018)

First-wave ab initio studies of $0\nu\beta\beta$ decay: ^{48}Ca

- Multi-reference in-medium generator coordinate method (IM-GCM)

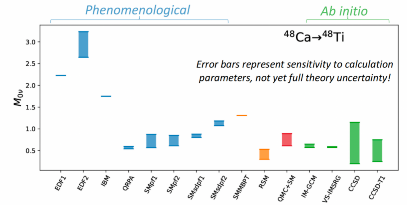
  JMY et al., PRL124, 232501 (2020)

- Valence-space shell model+IMSRG (VS-IMSRG)

  A. Belley et al., PRL126, 042502 (2021)

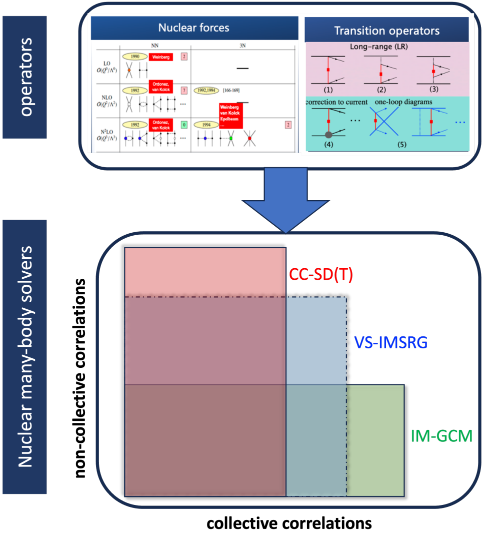
- Coupled-cluster with singlets, doublets, and partial triplets (CCSDT1) .

  S. Novario et al., PRL126, 182502 (2021)



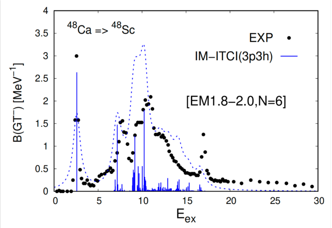
Error bars represent sensitivity to calculation parameters, not yet full theory uncertainty!

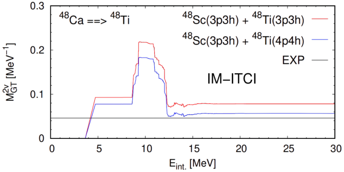
arXiv:2212.11099, adapted from J. Phys. G: Nucl. Part. Phys. 49, 120502 (2022)



A recent study of $0\nu\beta\beta$ decay in ^{48}Ca

IMSRG + Configuration interaction (IMCI-Q) method





More many-body correlations reduce the NME for both 2nu and 0nu DBD

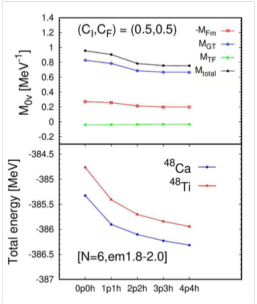


Table: $0v\beta\beta$ matrix elements for ^{48}Ca. ($\hbar\omega = 12$)

Force	Reference state	Cut off	M_F^{0v}	M_{GT}^{0v}	M_T^{0v}	M^{0v}
EM1.8/2.0	^{48}Ca (HF)	N=6,IM-ITCI(4p4h)	-0.376	1.035	-0.088	1.180
EM1.8/2.0	^{48}Ti (HFB)	N=6,IM-ITCI(4p4h)	-0.291	0.740	-0.088	0.833

A consistent NME is obtained between the two methods

In comparison with the IM-GCM calculation

		NME		
Interaction	$\hbar\omega$	$e_{\max} = 6$	$e_{\max} = 8$	$e_{\max} = 10$
EM1.8/2.0	12	0.85	0.70	0.64
EM1.8/2.0	16	1.03	0.78	0.66

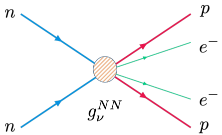
JMY et al., PRL124, 232501 (2020)

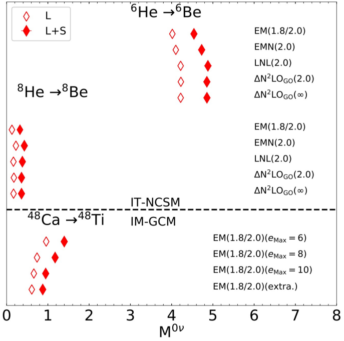
X. Lian, C. R. Ding, C.L. Bai, JMY, in preparation (2025)

Promoting the contact transition operator up to LO

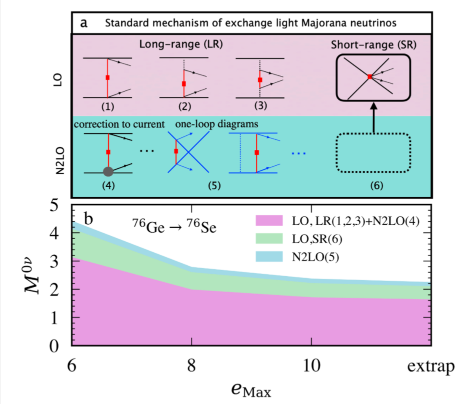
- A contact transition operator which could either enhance or quench the $0\nu\beta\beta$ decay, is needed to be promoted to LO to ensure renormalizibility. V. Cirigliano et al., PRL120, 202001 (2018)
- We determine the unknown LEC g_ν^{NN} of the contact operator, consistent with the employed chiral interaction (EM1.8/2.0), based on the synthetic data for the process $2n \rightarrow 2p + 2e^-$. V. Cirigliano et al., PRL126, 172002 (2021)
- The contact term turns out to enhance the NME for ^{48}Ca by 43(7)%, thus reducing the half-life $T_{1/2}^{0\nu}$ significantly.

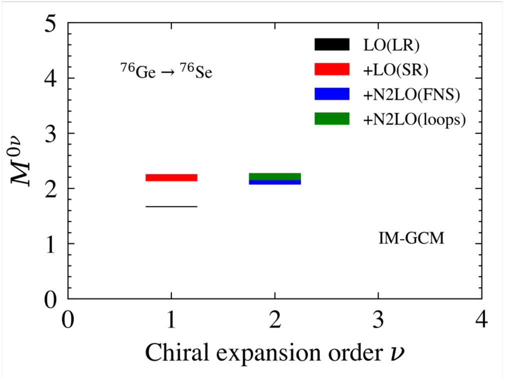
R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)





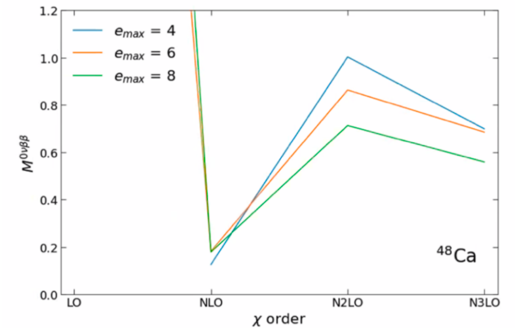
Convergence of the NMEs with the chiral expansion



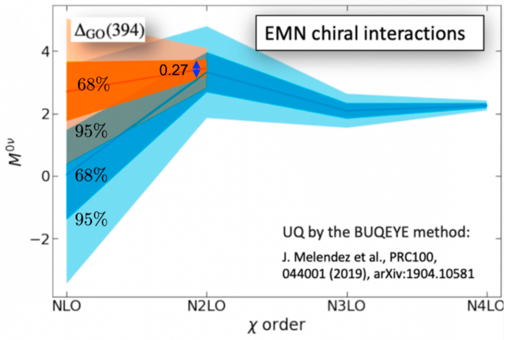


- The contact term enhances the NME by about 30%
- The two parts of the contributions at N2LO (about 5%) almost cancel out.

The impact of the truncation of nuclear forces on the NMEs



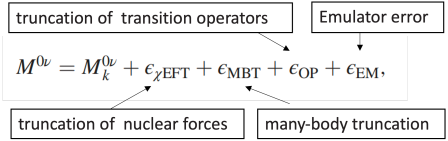
$^{48}\mathrm{Ca} \rightarrow ^{48}\mathrm{Ti} + 2e^-$



$^{76}\mathrm{Ge} \rightarrow ^{76}\mathrm{Se} + 2e^-$

- The NME converges with the chiral expansion order ν of nuclear forces.
- The EFT truncation error is shrinking with the increase of ν.

Quantification of the uncertainty in the NME of ^{76}Ge



truncation of transition operators

Emulator error

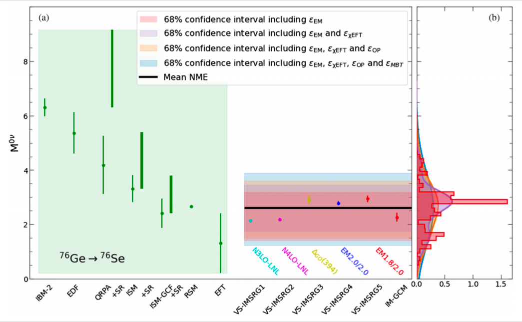
$$M^{0\nu} = M^{0\nu}_k + \epsilon_{\chi\mathrm{EFT}} + \epsilon_{\mathrm{MBT}} + \epsilon_{\mathrm{OP}} + \epsilon_{\mathrm{EM}},$$

truncation of nuclear forces

many-body truncation

TABLE I. The recommended value for the total NME of $0\nu\beta\beta$ decay in ^{76}Ge, together with the uncertainties from different sources.

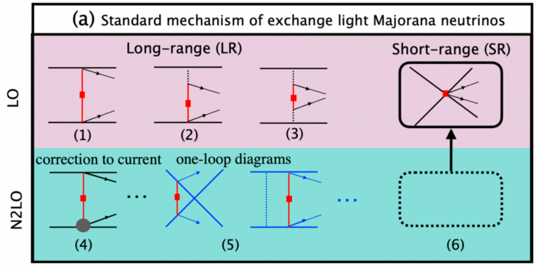
$M^{0\nu}$	ϵ_{LEC}	$\epsilon_{\chi\mathrm{EFT}}$	ϵ_{MBT}	ϵ_{OP}	ϵ_{EM}
$2.60^{+1.28}_{-1.36}$	0.75	0.3	0.88	0.47	< 0.06



- Our recommended value $M^{0\nu} = 2.60^{+1.28}_{-1.36}$.
- Together with the best half-life limit: $> 1.8 \times 10^{26}$ yr, it sets the upper limit $\langle m_{\beta\beta} \rangle = 187^{+205}_{-62}$ meV, and the sensitivity of the next-generation experiment $\langle m_{\beta\beta} \rangle = 22^{+24}_{-7}$ meV, covering almost the entire range of IO hierarchy.

A. Belley, JMY et al., PRL132, 182502 (2024)

Comparison of IM-GCM and VS-IMSRG for ^{136}Xe



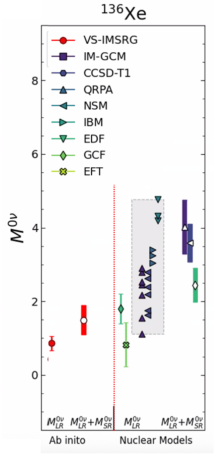
Preliminary results on the NME of 136Xe by the IM-GCM

	LO		N2LO		N3LO	
	long-range	contact	FNS	Residual	TBC	Total
EM1.8–2.0	1.07	0.42(8)	-0.07	0.17	-0.32(3)	1.27(11)
N2LO$_{\rm GO}$(394)	0.88	0.39(7)	-0.07	0.17	-0.09(1)	1.28(8)

- The NMEs are around 1.3 by two different nuclear chiral forces
- The contact term enhances the NME by more than 30%
- The net N2LO enhances the NME about 10%
- The two-body current (TBC) is obtained under the NO2B approximation

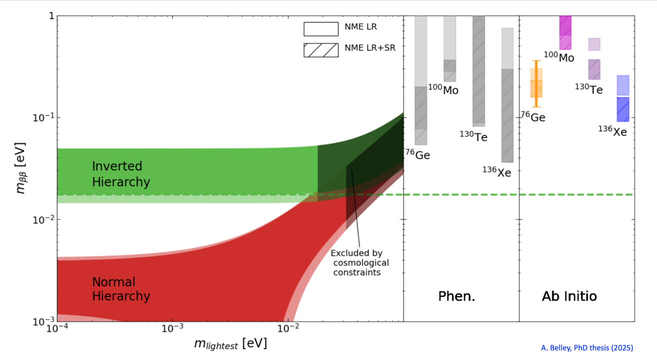
C. R. Ding, JMY, et al. in preparation (2026)

The NME by the VS-IMSRG: [1.08-1.90]
a factor of about two uncertainty



A. Belley et al., arXiv2307.15156 [nucl-th]

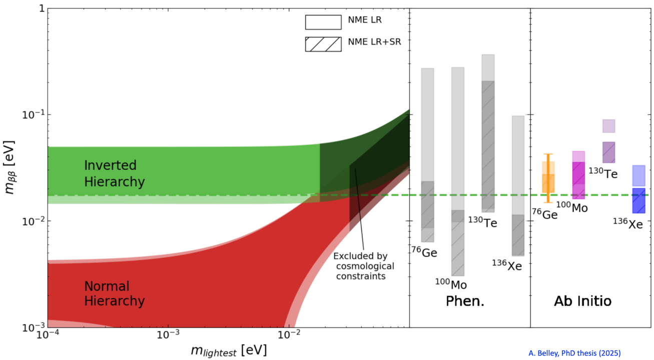
Effective neutrino mass from current $0\nu\beta\beta$-decay experiments



A. Belley, PhD thesis (2025)

- Using the NMEs from the phenomenological models, current ^{136}Xe exp. touches the parameter region for the IH case.
- Using the NMEs from ab initio nuclear models, none of the four candidate nuclei touches the region.

Sensitivity of ton-scale $0\nu\beta\beta$-decay experiments



A. Belley, PhD thesis (2025)

- Using the NMEs from the phenomenological models with the contribution of short-range transition operator, future ^{100}Mo and ^{136}Xe can cover the IH case.
- Using the NMEs from ab initio nuclear models, none of the four candidate nuclei could fully cover the IH case on the ton-scale exp.

Contents

Summary

- The significant model dependence of phenomenological models has become a key bottleneck for understanding and interpreting experimental observations.
- The ab initio methods based on operators from chiral EFT provides a promising uncertainty controllable framework. However, there are significant challenges in describing open-shell nuclei with the strong collective correlations associated with deformation.
- We have developed a novel ab initio method (IM-GCM) for medium-mass deformed open-shell nuclei, which provids a crucial tool for understanding nuclear structure, and in particular for the searching of new physics:
  - explanation of the origin of magic number, clustering structure, and deformation
  - ab initio study of $M^{0\nu}$ for candidate neutrinoless double-beta decay with UQ.

These developments pave the way toward a nuclear many-body theory with controlled uncertainties, enabling high-precision calculations of nuclear structure and key nuclear matrix elements for new-physics researches.

Acknowledgements

Collaborators

- SYSU: C.R. Ding, C.F. Jiao, Y. Li, Q. Y. Luo, B.R. Liu, Y.X. Zhang, E.F. Zhou, etc
- CAEP: Chen-can Wang
- SCU: Xin Lian, Chunlin Bai
- SWU: Longjun Wang
- SJU: Ke Han, Shaobo Wang
- Univ. Tokyo: H. Z. Liang
- Kyoto Univ.: X. Zhang, K. Hagino
- CERN: Giuliano Giacalone
- MSU: Heiko Hergert, Roland Wirth
- UNC: Jonathan Engel
- TRIUMF: Antonie Belly, Jason Holt
- Tsukuba Univ.: Takayuki Miyagi
- Notre-Dame U: Ragnar Stroberg
- UAM: Benjamin Bally, Tomas Rodriguez

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Thank you for your attention!

Evidence of pseudospin symmetry in nuclear shell structure

1969, Arima, Hecht et al., introduced the concept of pseudospin symmetry (PSS) to explain the quasi-degeneracy between two single-particle states. Arima et al., PLB30, 517(1969); Hecht et al., NPA137, 129 (1969)

- Near degeneracy of some doublets

$$(n+1, l, j = l + 1/2)$$
$$(n, l+2, j = l + 3/2)$$

- Pseudo-orbit angular momentum

$$\tilde{l} = l + 1$$

- Pseudo-spin partners

$$(\tilde{l}, j = \tilde{l} \pm 1/2)$$

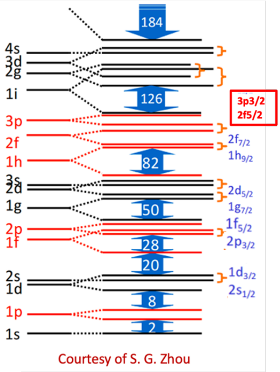
- The single-particle Hamiltonian transforms as

Bahri et al. PRL68, 2133 (1992)

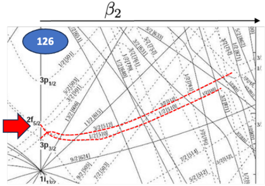
$$h = \hbar\omega_0\left(-\frac{1}{2}\Delta + \frac{1}{2}r^2\right) - \hbar\omega_0\kappa\left(2\boldsymbol{l}\cdot\boldsymbol{s} + \mu\boldsymbol{l}^2\right)$$

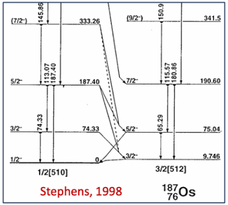
$$= H_0 + v_{ls}l\cdot s + v_{ll}l^2 \underset{\text{pseudo}}{\overset{\text{normal}}{\rightleftarrows}} \tilde{H}_0 + (4v_{ll} - v_{ls})\tilde{l}\cdot\tilde{s} + v_{ll}\tilde{l}^2 + (\hbar\omega + 2v_{ll} - v_{ls}) .$$

Pseudo-spin degeneracy: $4v_{ll} - v_{ls} = 0 \xrightarrow{\text{Nilsson model}} \mu = 1/2$



Courtesy of S. G. Zhou





Stephens, 1998 $^{187}_{76}$Os

Pseudospin partner bands

Explanation of PSS in a relativistic framework

Dirac spinor: $$\psi_\alpha(\mathbf{r}) = \frac{1}{r}\begin{pmatrix} iG_a(r)\mathscr{Y}^{l_a}_{j_a m_a}(\hat{\mathbf{r}}) \\ -F_a(r)\mathscr{Y}^{\tilde{l}_a}_{j_a m_a}(\hat{\mathbf{r}}) \end{pmatrix}$$

$$\tilde{p}_{1/2,3/2} \nearrow 2s_{1/2}: \begin{pmatrix} n=2, l=0, j=l+1/2 \\ \tilde{n}=2, \tilde{l}=1, j=\tilde{l}-1/2 \end{pmatrix}$$

$$\tilde{p}_{1/2,3/2} \searrow 1d_{3/2}: \begin{pmatrix} n=1, l=2, j=l-1/2 \\ \tilde{n}=2, \tilde{l}=1, j=\tilde{l}+1/2 \end{pmatrix}$$

Quantum numbers $$\left\{ \begin{array}{l} j = l \pm 1/2 \\ \kappa = (-1)^{j+l+1/2}(j+1/2) \\ \tilde{l} = l - \mathrm{sign}(\kappa) \end{array} \right.$$

- Dirac equation

$$\begin{pmatrix} \Sigma(r)+M & -\frac{d}{dr}+\frac{\kappa}{r} \\ \frac{d}{dr}+\frac{\kappa}{r} & -\Delta(r)-M \end{pmatrix}\begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = E\begin{pmatrix} G(r) \\ F(r) \end{pmatrix},$$

where $\Sigma(r) = S(r)+V(r), \quad \Delta(r) = S(r)-V(r),$

$\kappa = \mp(j+1/2)$ for $j_G = l_G \pm 1/2.$

- Schrödinger-like equation for the lower component

$$\left\{-\frac{1}{M_-}\frac{d^2}{dr^2} + \frac{1}{M_-^2}\frac{dM_-}{dr}\frac{d}{dr} + \left[(-M-\Delta) + \frac{1}{M_-}\frac{\kappa(\kappa-1)}{r^2} \boxed{- \frac{1}{M_-^2}\frac{dM_-}{dr}\frac{\kappa}{r}}\right]\right\} F = EF,$$

with $M_- = E - M - \Sigma$

Courtesy of H.Z. Liang

Meng, Sugawara-Tanabe, Yamaji, Ring, Arima, *PRC* **58**, R628 (1998)

- The pseudo-orbital angular momentum $\tilde{\ell}$ is the orbital angular momentum of the lower component of the Dirac spinor. Ginocchio, PRL 78, 436 (1997)
- Condition for the PSS: $d(S+V)/dr = 0$ (shallow MF potential). Meng et al., (1998)
- Test of PSS in deformed nuclei and other cases. Liang, Meng, Zhou, Phys. Rep. (2015)

Q: How does the PSS emerge in (low-energy) nuclear shell structure?