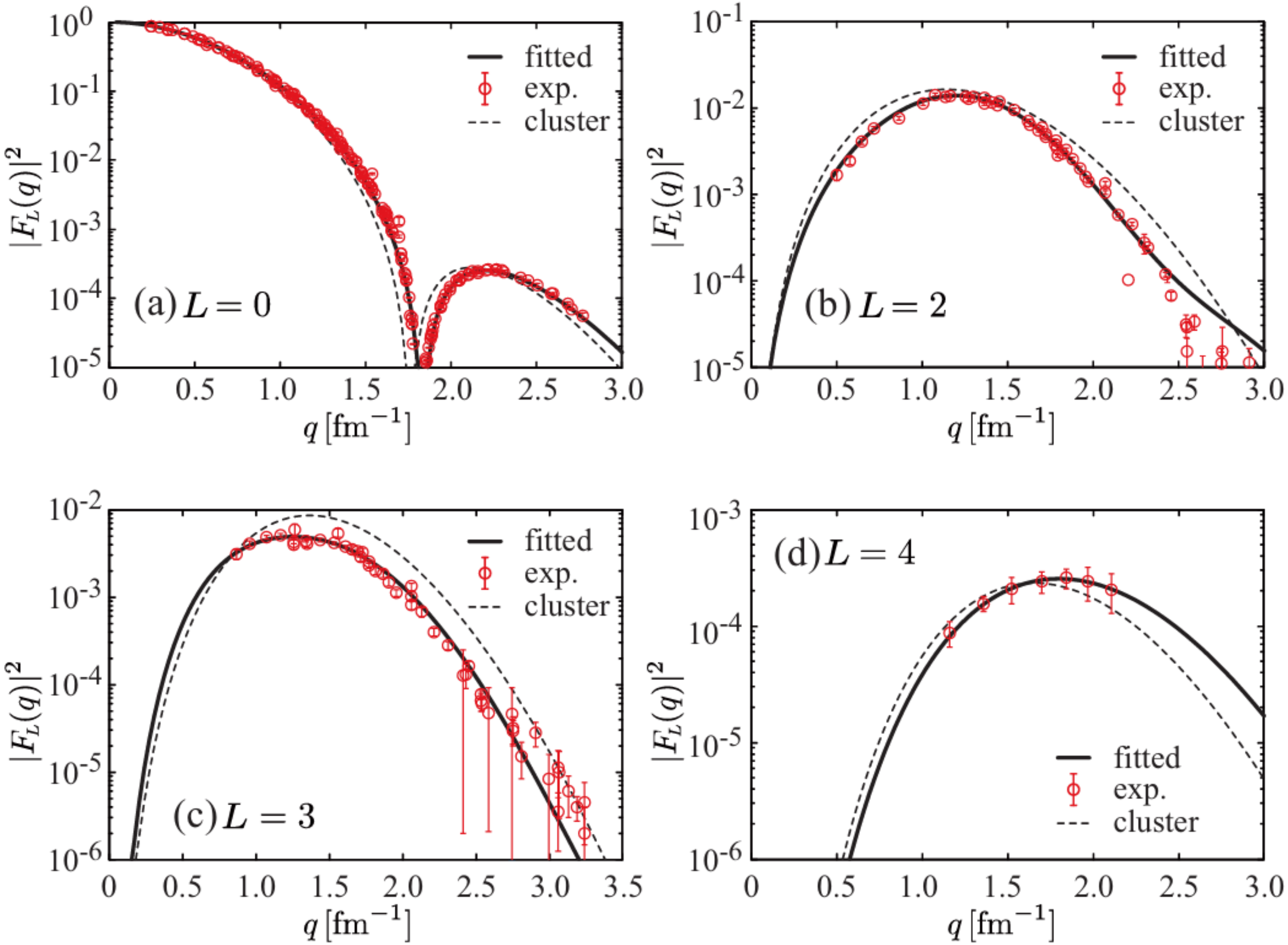
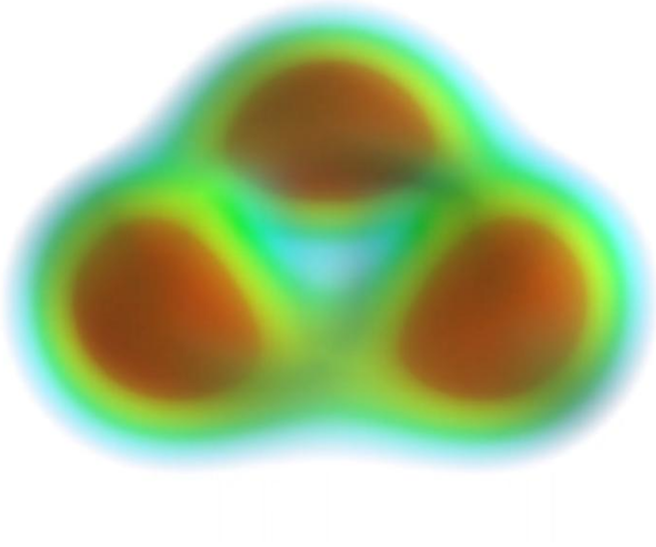
Shape of Carbon-12



M. Kimura (RIKEN)

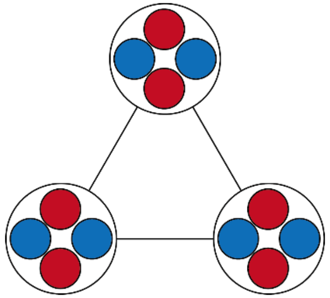


Spinning triangle of three alphas

Spinning triangle of three alphas

Three alpha particles locate
in a triangle configuration (D_{3h} symmetry)

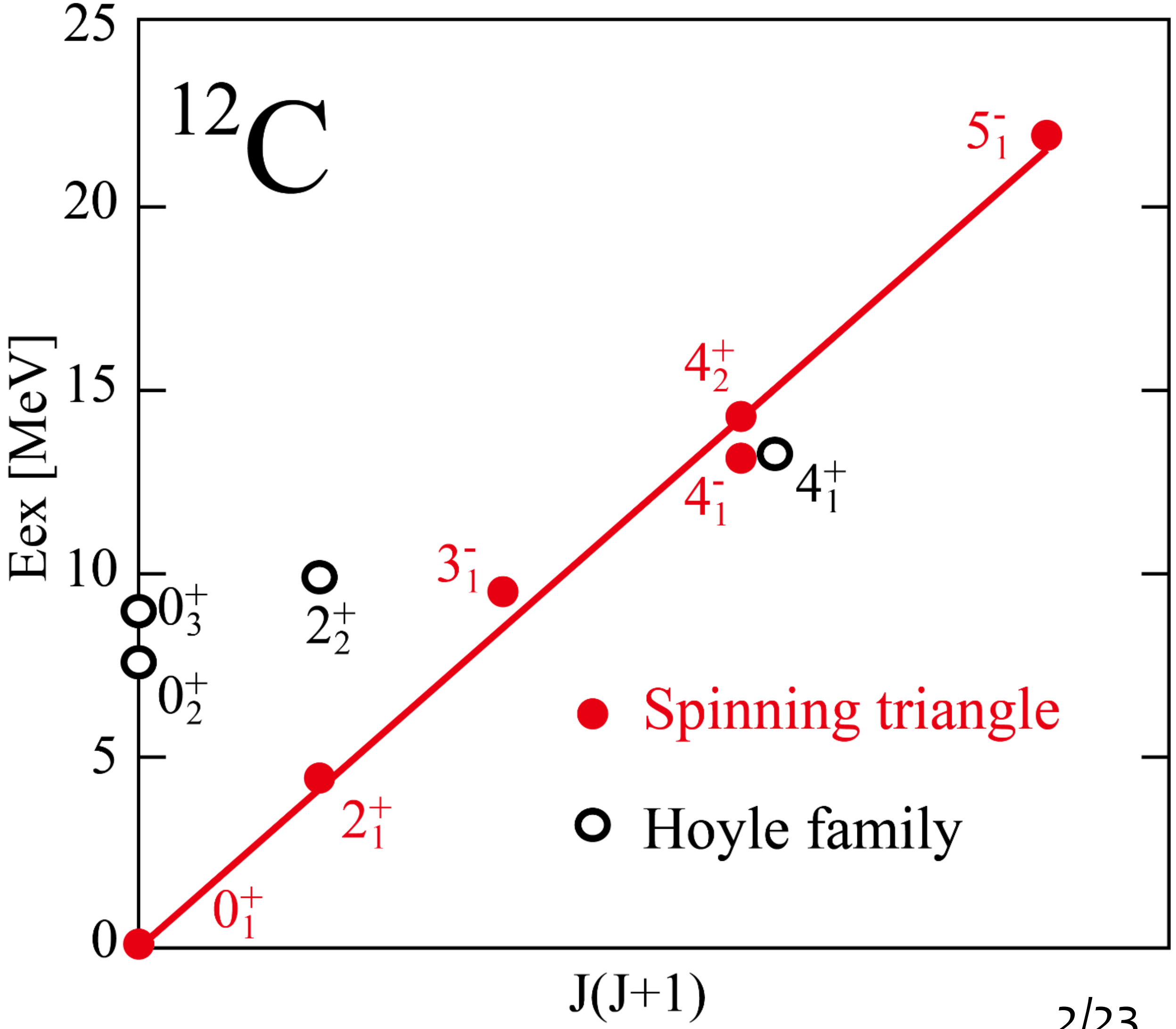
R. Bijker and F. Iachello, PRC61, 067305 (2000).



◎ Due to the symmetry, 0^+, 2^+, 3^-, $4^\pm$, 5^- states constitute "ground band"

◎ 4^+ and 4^- states should degenerate due to "symmetry"

D.J. Marín-Lámbarri, et al. PRL113, 012502 (2014)



Spinning triangle of three alphas

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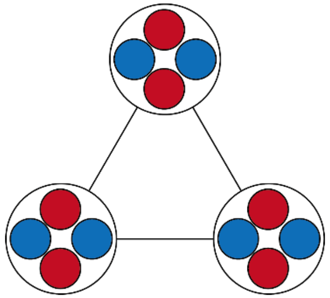
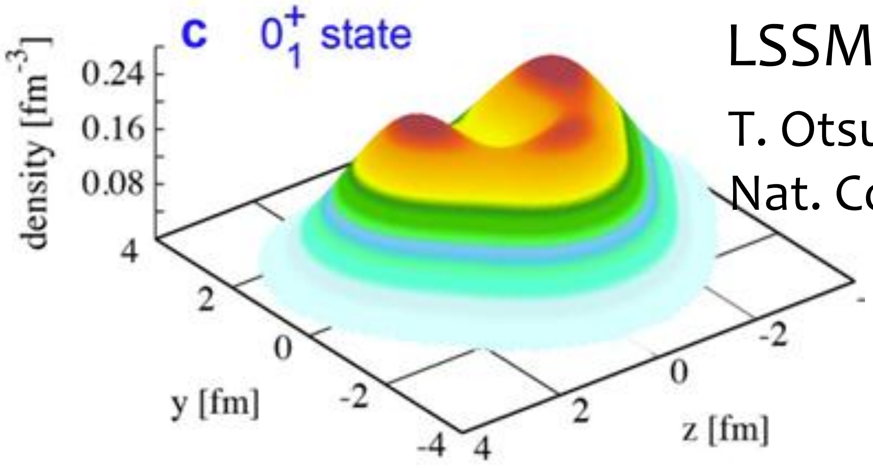


◎ Due to the symmetry, 0^+, 2^+, 3^-, $4^{\pm}$, 5^- states constitute "ground band"

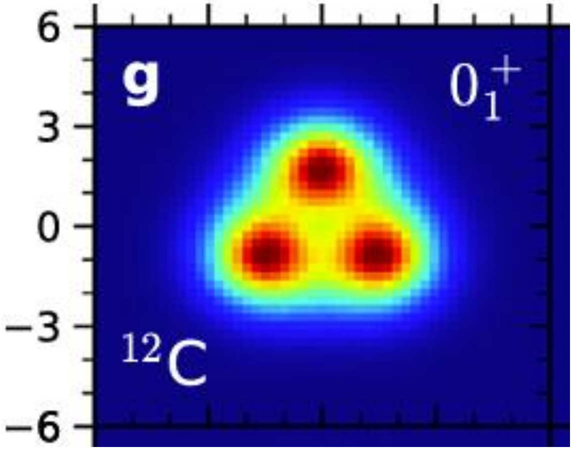
◎ 4^+ and 4^- states should degenerate due to "symmetry"

Shape of ^{12}C from theories



LSSM

T. Otsuka et al., Nat. Comm. 13 (2022)



Lattice EFT

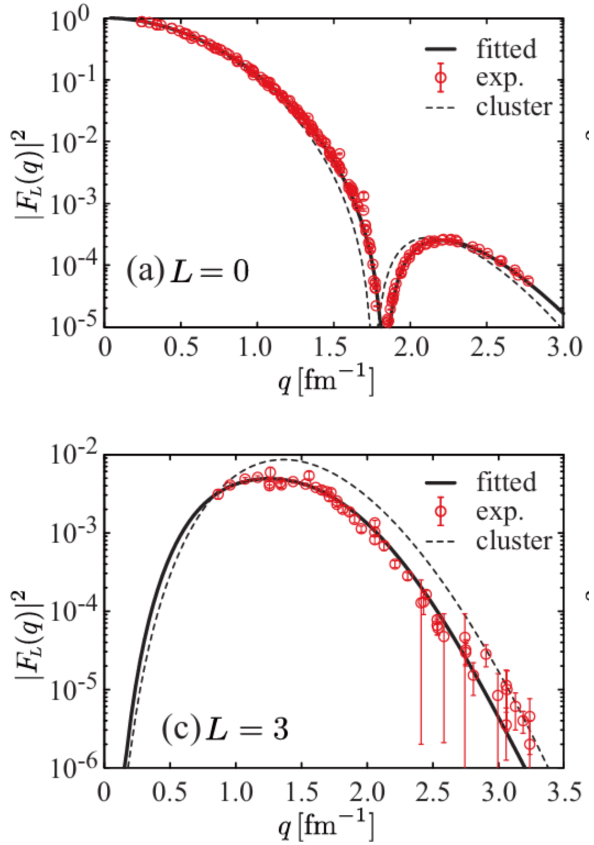
S. Shen et al., Nat. Comm. 17 (2023)

Rebuilding nuclear shape from Exp. data

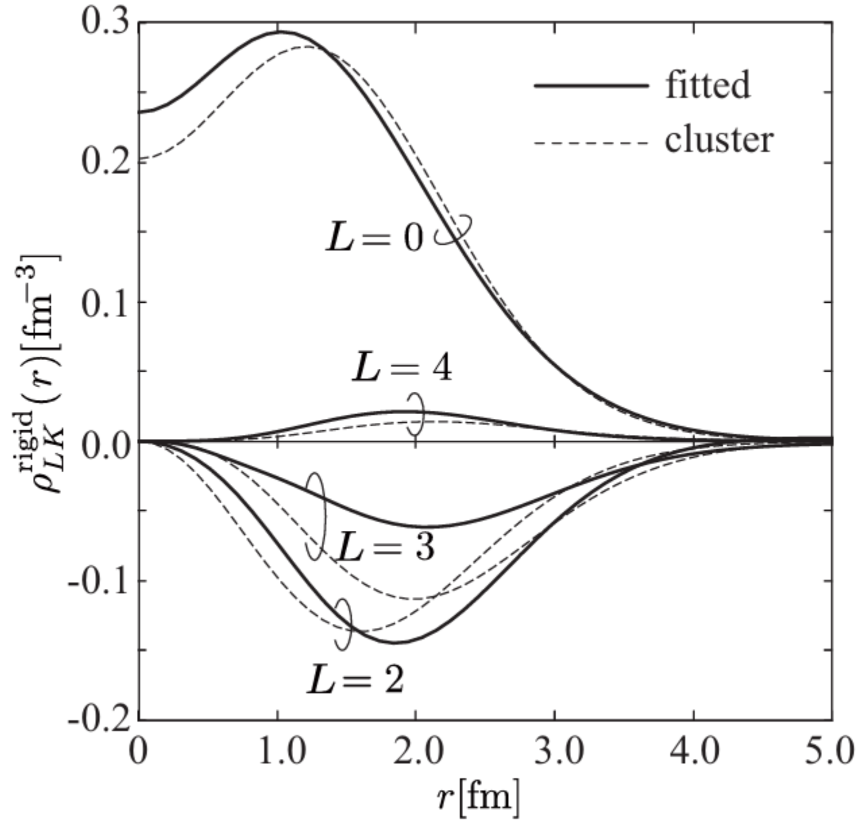
M.K. and Y. Taniguchi, EPJA 60, (2024)

Rebuilding nuclear shape from observables

$^{12}C(e,e')$



Transition Density



Shape

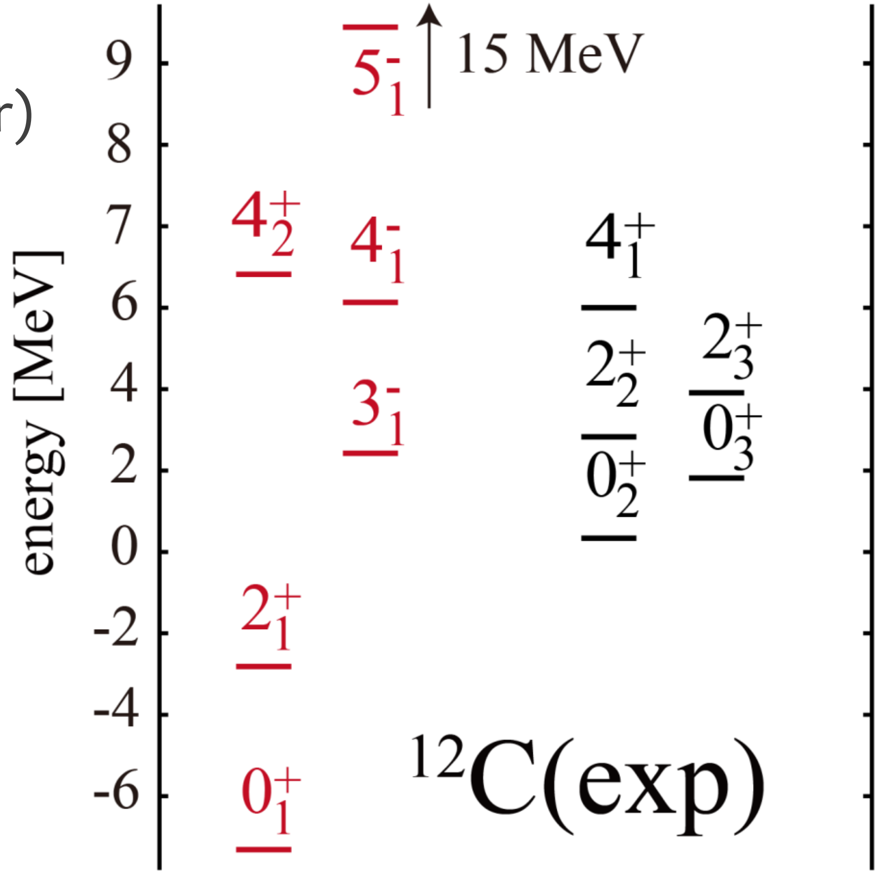
?

Rebuilding nuclear shape from observables

I assume that

"the 0^+, 2^+, 3^- and 4^+ states constitute *the ground band* **sharing the same intrinsic state**"

(*) This is the only assumption I make in this talk.

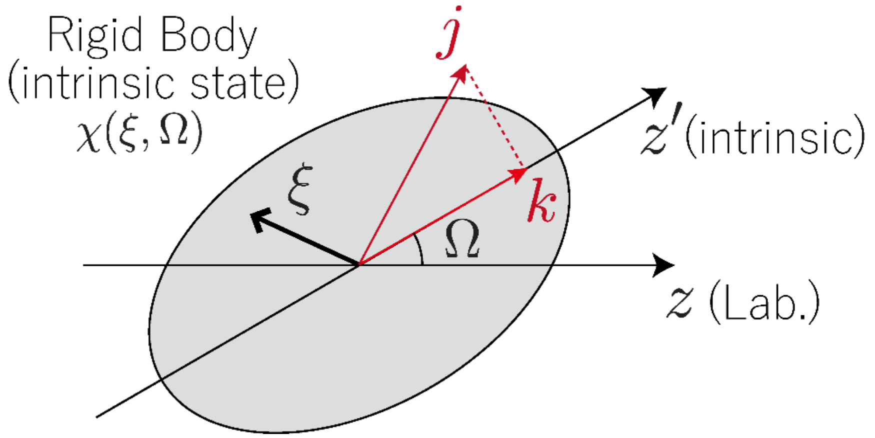


Non-axial and parity-asymmetric rigid rotor (a textbook matter)

$$\Psi^{L^\Pi}_{L_z K} = \sqrt{\frac{2L+1}{16\pi^2(1+\delta_{K0})}}\,\chi(\xi,\Omega)\,\{\, D^L_{L_z K}(\Omega) + \Pi(-)^{L+K} D^L_{L_z -K}(\Omega)\,\}$$

laboratory (observable) — $\Psi^{L^\Pi}_{L_z K}$

intrinsic (unobservable) — $\chi(\xi,\Omega)$



Rebuilding nuclear shape from observables

Electron scattering: Within PWIA, the cross section is proportional to square of From factor

$$\frac{d\sigma_L^{\rm obs}}{d\theta} = |F_L(q)|^2 \frac{d\sigma_L^{\rm Mott}}{d\theta},$$

$$F_L(q) := \frac{\sqrt{4\pi(2L+1)}}{Z}\int_0^\infty r^2 dr\, j_L(qr)\,\rho^{0\to L^\Pi}(\boldsymbol{r}) / Y_{L0}(\hat{r}), \qquad \rho^{0^+\to L^\Pi}(\boldsymbol{r}) := \langle L^\Pi, 0|\rho(\boldsymbol{r})|0_1^+, 0\rangle$$

Form Factor **transition density**

Insert the rigid-rotor wave function into this definition, one gets

$$\rho^{0\to L^\Pi}(\boldsymbol{r}) = \frac{1}{8\pi^2}\left(\frac{2L+1}{2(1+\delta_{K0})}\right)^{1/2}\int d\Omega \left\{D_{L_zK}^{L*}(\Omega) + \Pi(-)^K D_{L_z-K}^{L*}(\Omega)\right\} \langle\chi(\xi,\Omega)|\rho(\boldsymbol{r})|\chi(\xi,\Omega)\rangle.$$

transition density **rigid-body density (shape of 12C)**

$$= \frac{1}{\sqrt{2L+1}}\left(\frac{1+\Pi(-)^L}{1+\delta_{K0}}\right)^{1/2} \rho_{LK}^{\rm rigid}(r) Y_{L0}(\hat{r}),$$

Multipode decomposition of the rigid-body density

(*) Multipole decomposition of the rigid-body density

$$\langle\chi(\xi,\Omega=0)|\rho(\boldsymbol{r})|\chi(\xi,\Omega=0)\rangle = \frac{1}{2}\sum_{lm}\rho_{lm}^{\rm rigid}(r)\left\{Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r})\right\}.$$

Rebuilding nuclear shape from observables

Summary of our assumption and numerical procedure

Assumption: The 0^+, 2^+, 3^- and 4^+ states constitute "the ground band" sharing the same intrinsic state

① Get the multipole decomposition of the rigid-body density from the observed transition densities

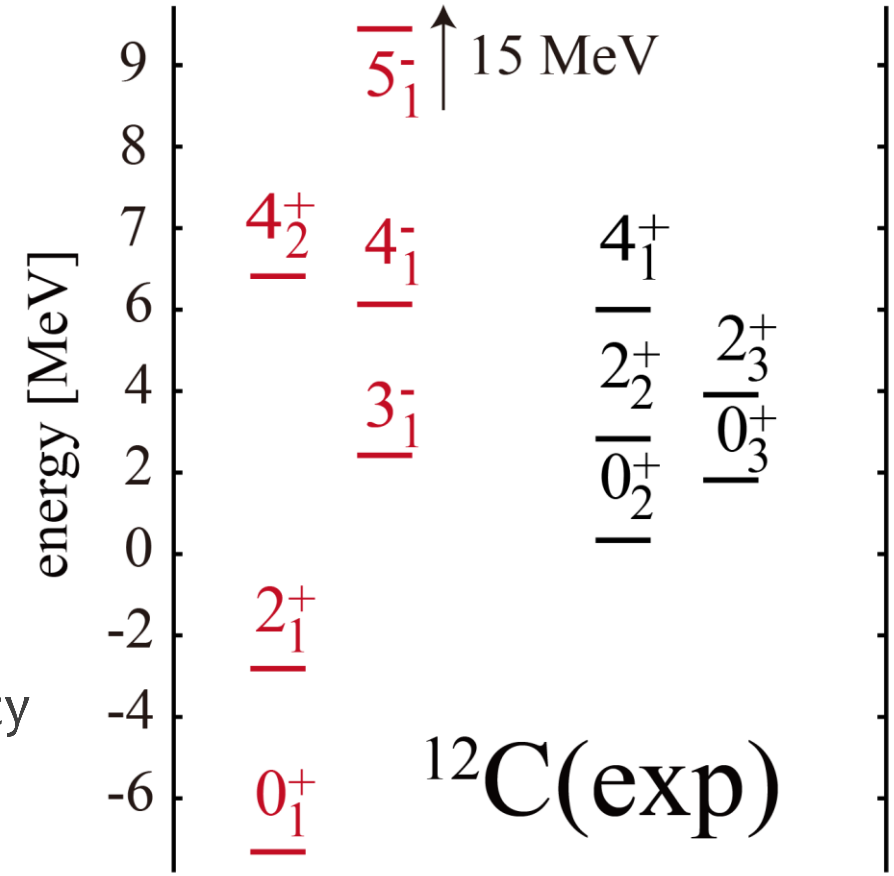
$$\rho^{0\to L^\Pi}(\boldsymbol{r}) = \frac{1}{\sqrt{2L+1}}\left(\frac{1+\Pi(-)^L}{1+\delta_{K0}}\right)^{1/2}\rho^{\rm rigid}_{LK}(r)Y_{L0}(\hat{r}),$$

transition density from 12C(e,e') data

Multipole decomposition of intrinsic density

② Sum up all the multipole to reconstruct the rigid-body density

$$\rho^{\rm rigid}(\boldsymbol{r}) = \frac{1}{2}\sum_{lm}\rho^{\rm rigid}_{lm}(r)\left\{Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r})\right\}$$

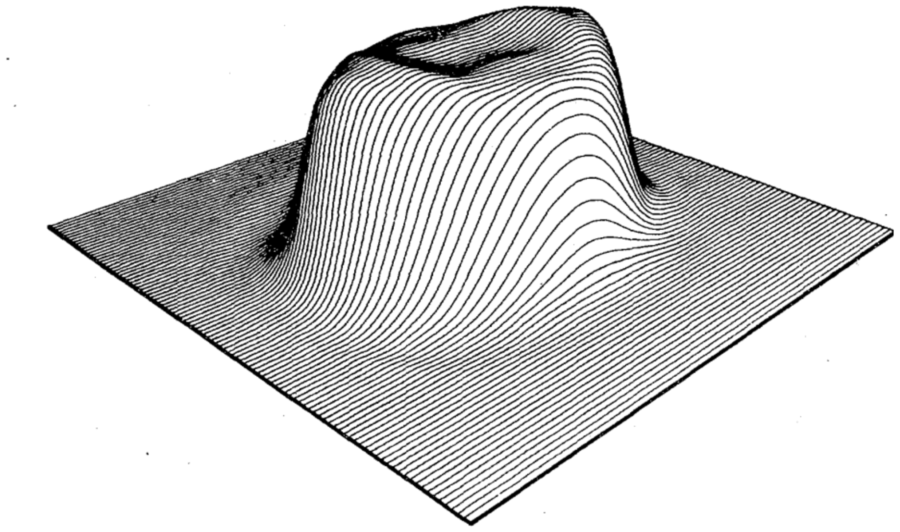


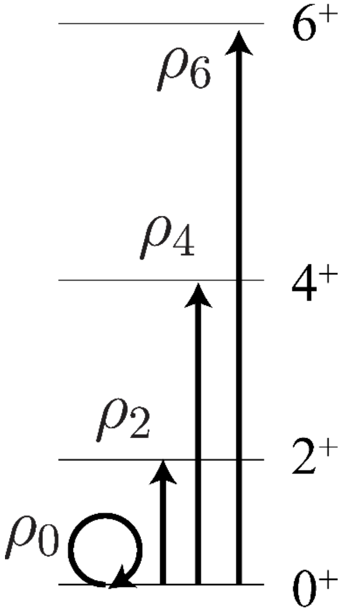
Rebuilding nuclear shape from observables

Actually, this is not a new idea.
In 1970's, it has been already applied to axial and parity-symmetric shapes.

^{152}Sm; rigid-body density

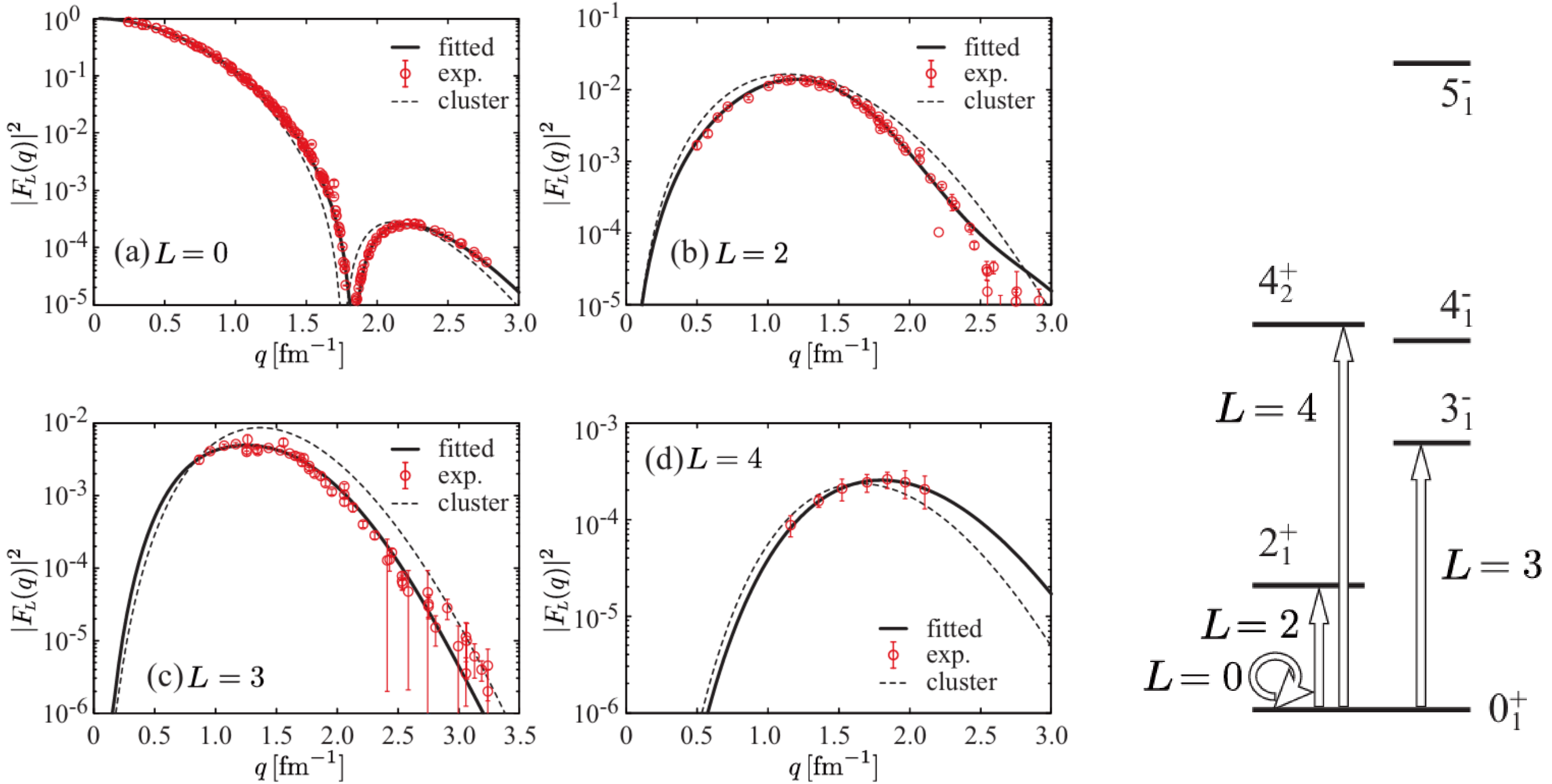
L.S. Carcman et al., PRC78, 1388 (1978).





Rebuilding shape of ^{12}C

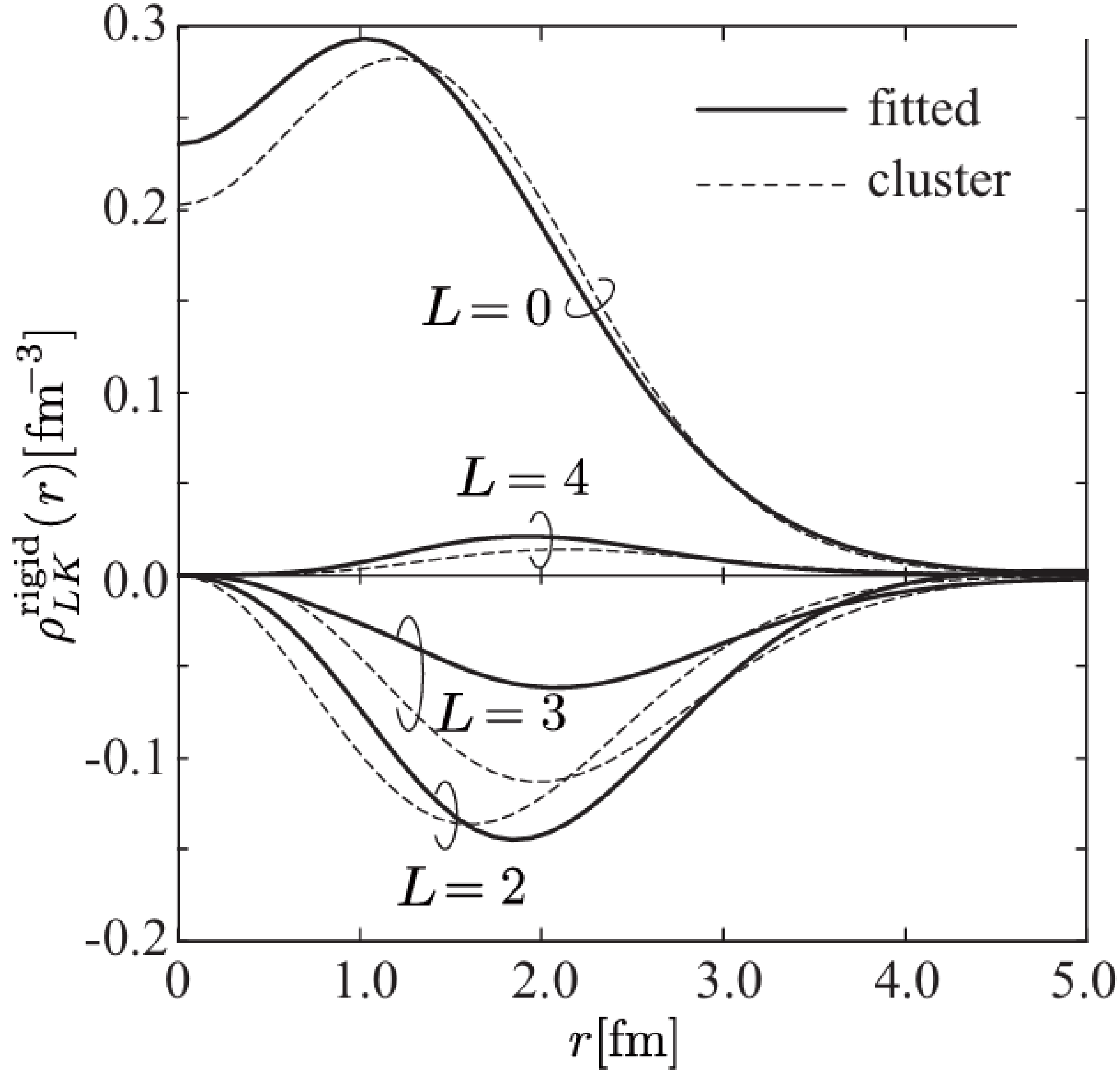
There are plenty of data for ^{12}C(e,e') $$\frac{d\sigma_L^{\text{obs}}}{d\theta} = |F_L(q)|^2 \frac{d\sigma_L^{\text{Mott}}}{d\theta},$$

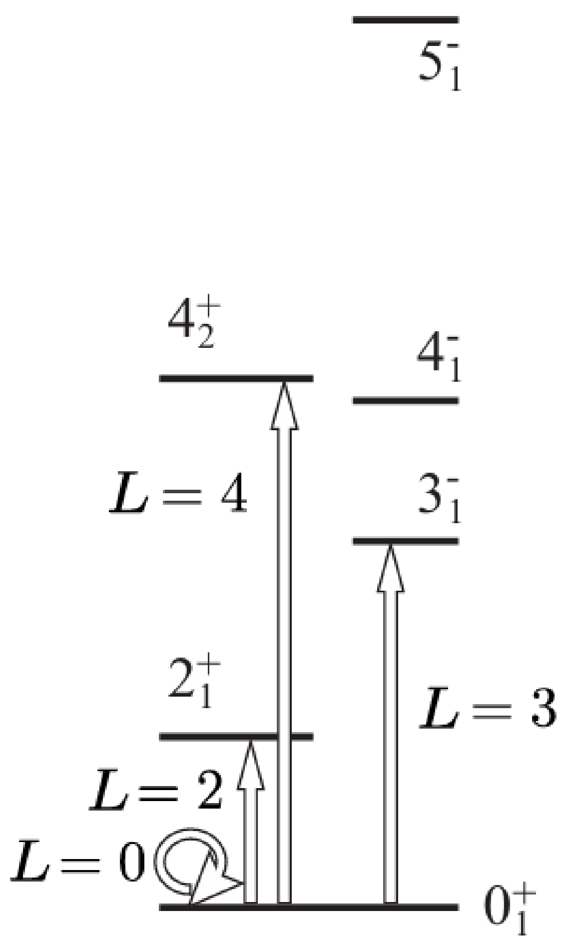


Rebuilding shape of ^{12}C

Inverse Fourier to get the densities.

$$F_L(q) := \frac{\sqrt{4\pi(2L+1)}}{Z}\int_0^\infty r^2 dr\ j_L(qr)\,\rho^{0\to L^\Pi}(\boldsymbol{r})/\,Y_{L0}(\hat{r}),$$





Rebuilding shape of ¹²C

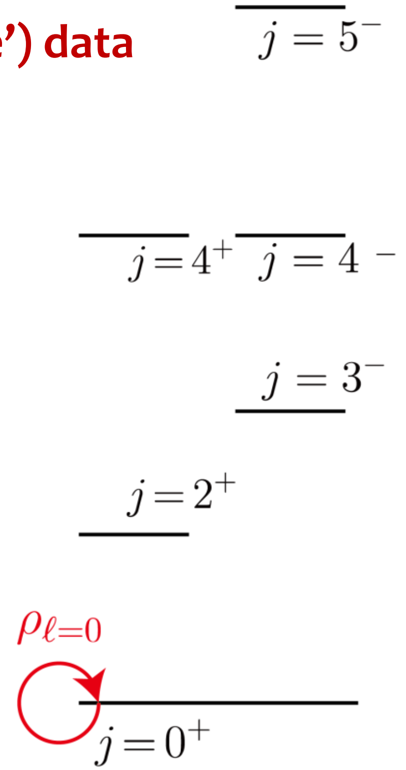
We rebuild the rigid density of ¹²C by superposing the "observed" transition densities

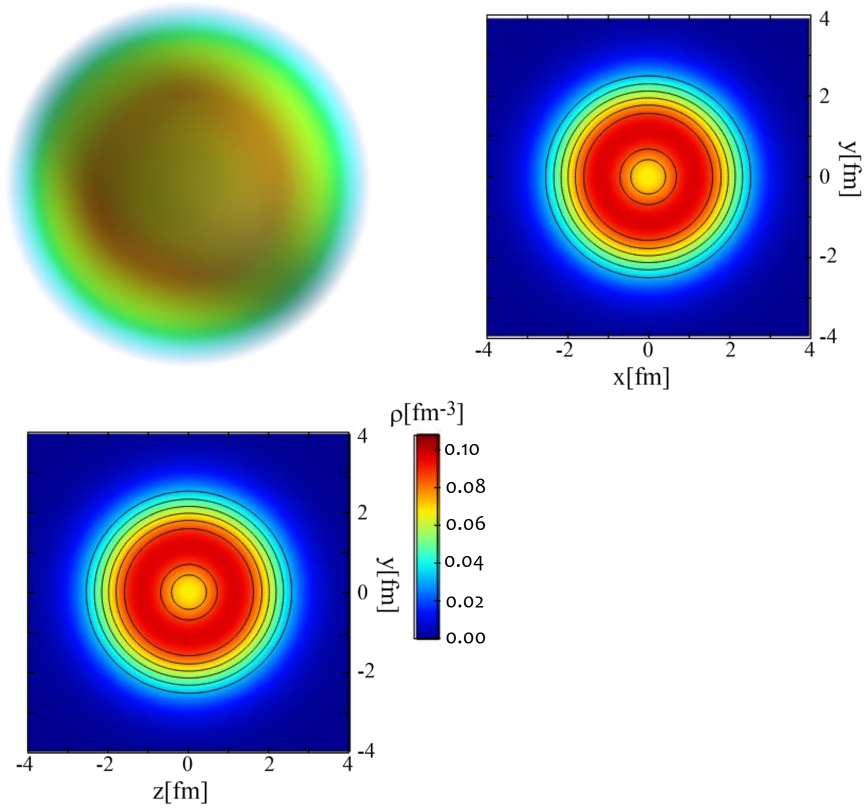
$$\rho^{\text{rigid}}(\boldsymbol{r}) = \frac{1}{2}\sum_{lm} \rho^{\text{rigid}}_{lm}(r)\,\{\,Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r})\,\}$$

Shape of 12C

multipole decomposition of intrinsic density from 12C(e,e') data

$\ell = 0$





- With only the $\ell = 0$ density, it is spherical

Rebuilding shape of ^{12}C

We rebuild the rigid density of ^{12}C by superposing the "observed" transition densities

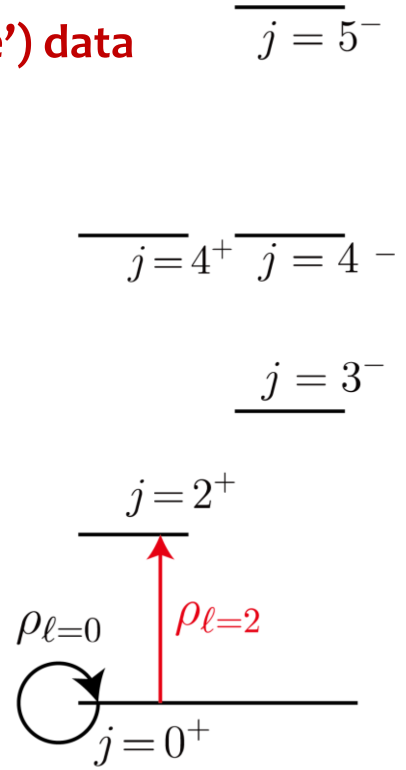
$$\rho^{\mathrm{rigid}}(\boldsymbol{r}) = \frac{1}{2}\sum_{lm} \rho^{\mathrm{rigid}}_{lm}(r)\left\{ Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r}) \right\}$$

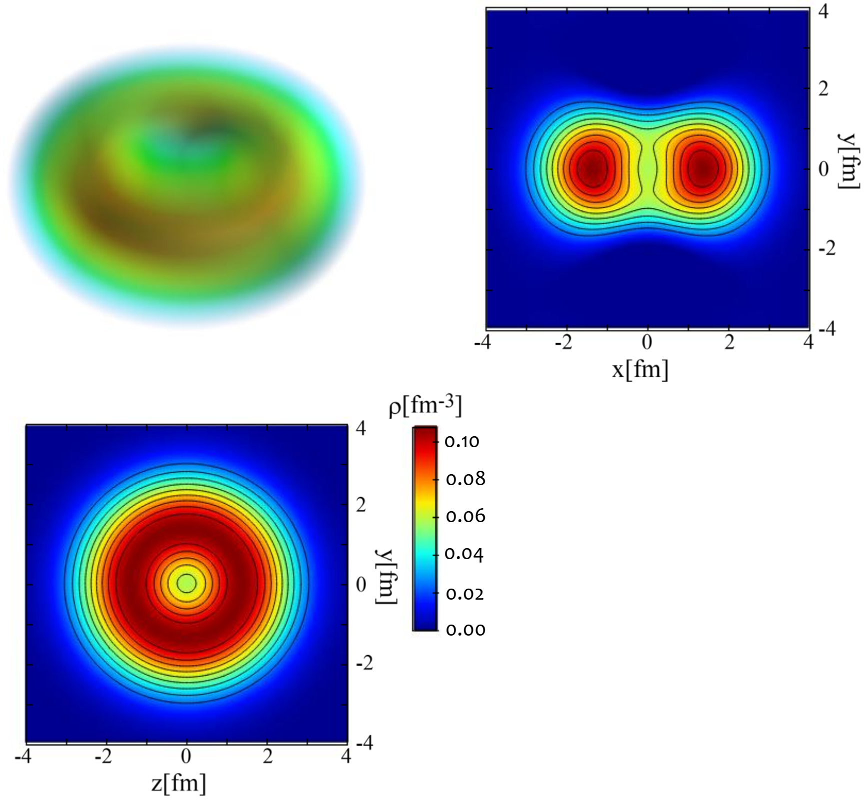
$$\ell = 0 + 2$$

Shape of 12C

multipole decomposition of intrinsic density from 12C(e,e') data

- With only the $\ell = 0$ density, it is spherical
- The $\ell = 2$ density makes it toroidal shape





Rebuilding shape of ^{12}C

We rebuild the rigid density of ^{12}C by superposing the “observed” transition densities

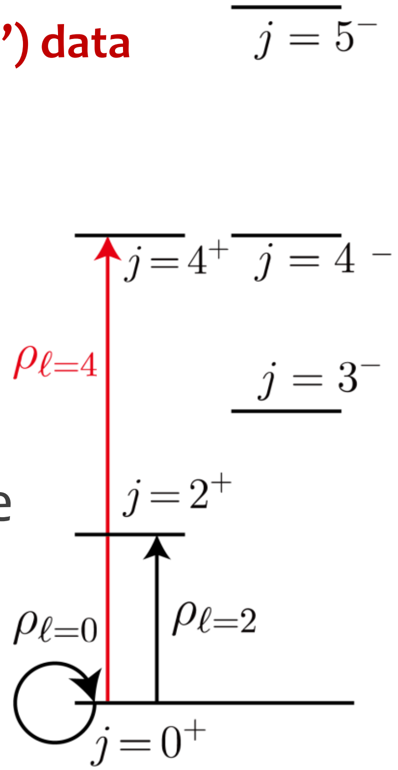
$$\rho^{\mathrm{rigid}}(\boldsymbol{r}) = \frac{1}{2}\sum_{lm} \rho^{\mathrm{rigid}}_{lm}(r)\left\{ Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r}) \right\}$$

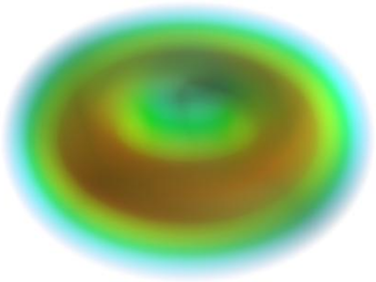
$$\ell = 0^+ + 2^+ + 4^+$$

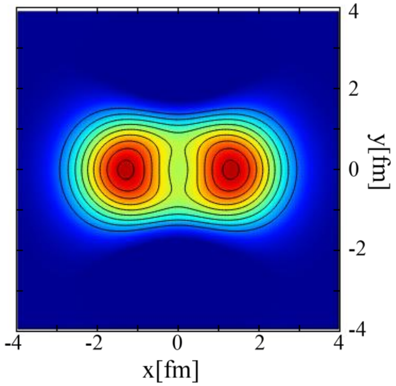
Shape of 12C

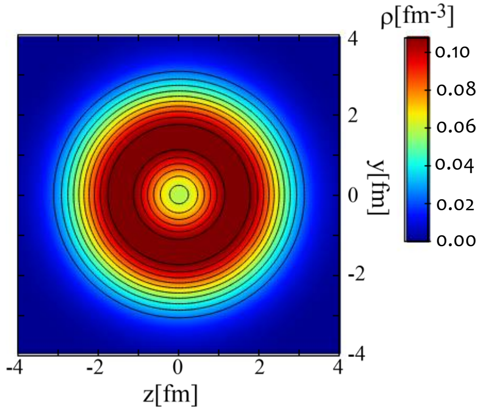
multipole decomposition of intrinsic density from 12C(e,e’) data

- With only the $\ell = 0$ density, it is spherical
- The $\ell = 2$ density makes it toroidal shape
- The $\ell = 4$ density emphasizes toroidal shape









Rebuilding shape of ^{12}C

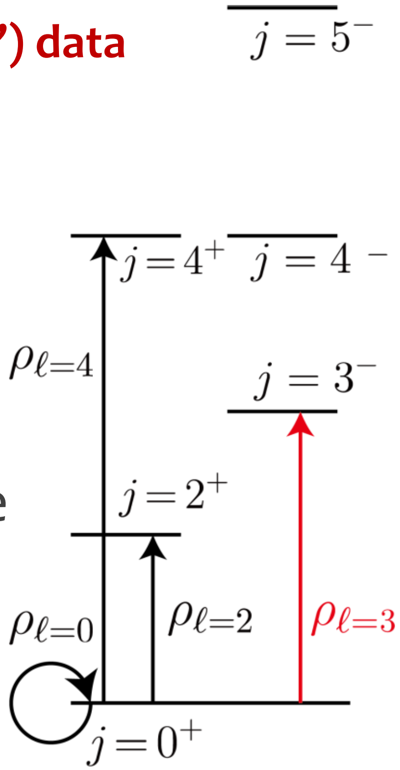
We rebuild the rigid density of ^{12}C by superposing the "observed" transition densities

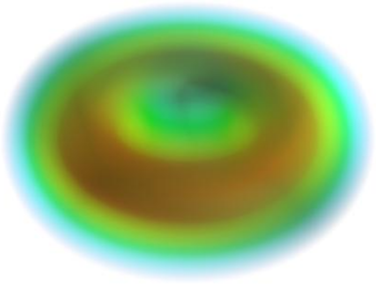
$$\rho^{\text{rigid}}(\boldsymbol{r}) = \frac{1}{2}\sum_{lm} \rho^{\text{rigid}}_{lm}(r)\left\{ Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r}) \right\}$$

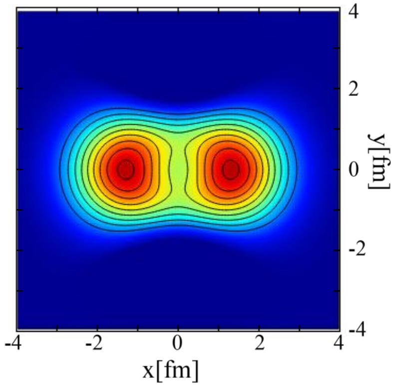
$$\ell = 0^+ + 2^+ + 4^+$$

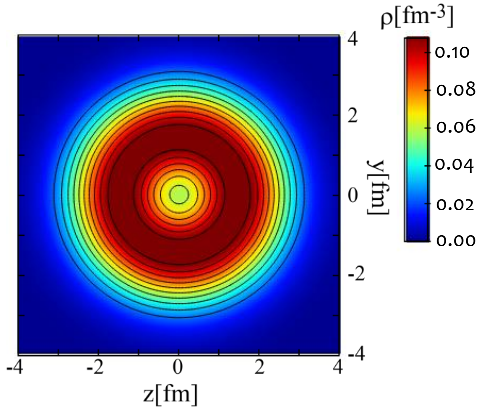
Shape of 12C

multipole decomposition of intrinsic density from 12C(e,e') data









- With only the $\ell = 0$ density, it is spherical
- The $\ell = 2$ density makes it toroidal shape
- The $\ell = 4$ density emphasizes toroidal shape

Rebuilding shape of ^{12}C

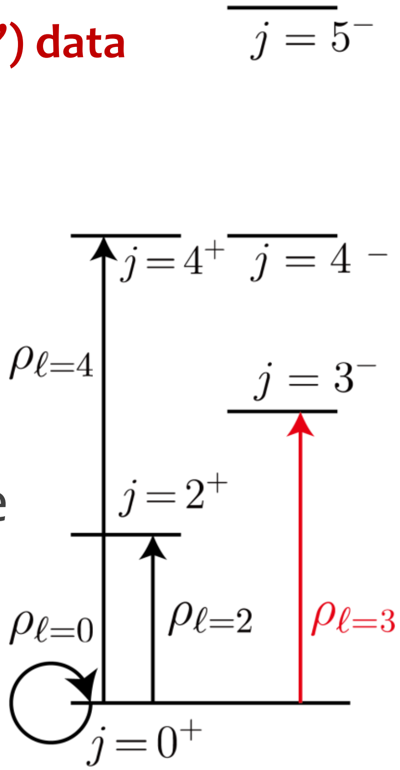
We rebuild the rigid density of ^{12}C by superposing the "observed" transition densities

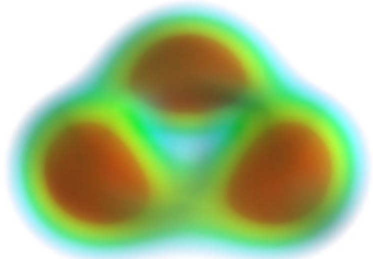
$$\rho^{\text{rigid}}(\boldsymbol{r}) = \frac{1}{2}\sum_{lm}\rho^{\text{rigid}}_{lm}(r)\left\{Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r})\right\}$$

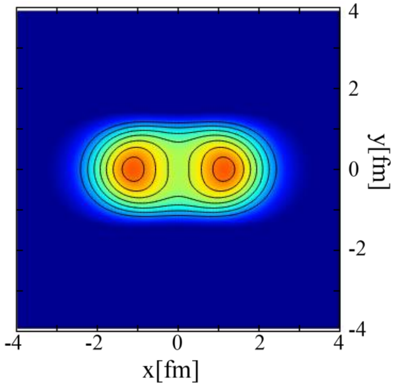
$$\ell = 0^+ + 2^+ + 4^+ + 3^-$$

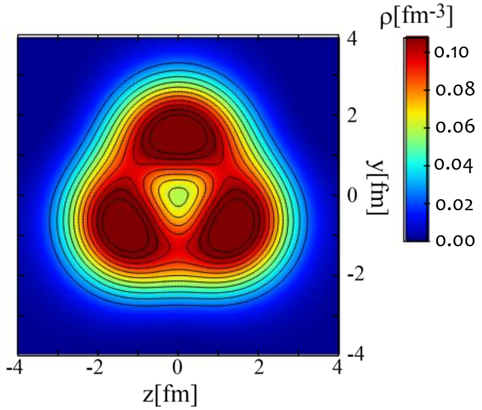
Shape of 12C

multipole decomposition of intrinsic density from 12C(e,e') data







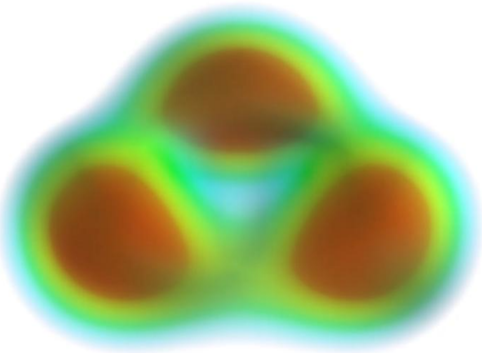


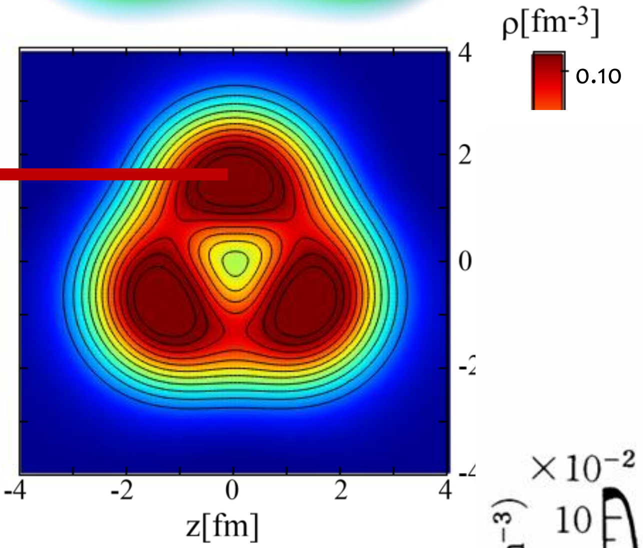
- With only the $\ell = 0$ density, it is spherical
- The $\ell = 2$ density makes it toroidal shape
- The $\ell = 4$ density emphasizes toroidal shape
- The $\ell = 3$ density makes it very exotic.
  - Emphasized triangular shape
  - Three prominent peaks of density

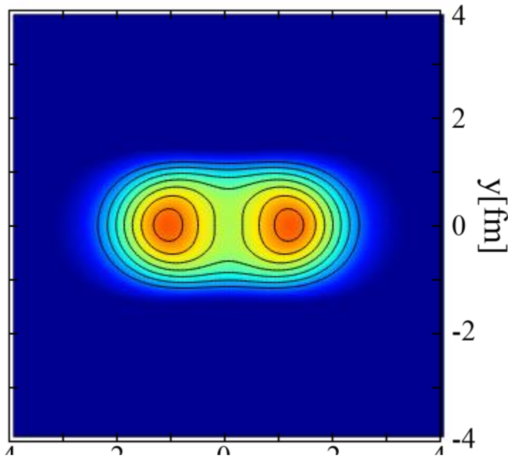
Summary: shape of ^{12}C

Assumption: The 0^+, 2^+, 3^- and 4^+ states constitute "the ground band" sharing the same intrinsic state

An exotic triangular shape of ^{12}C emerges



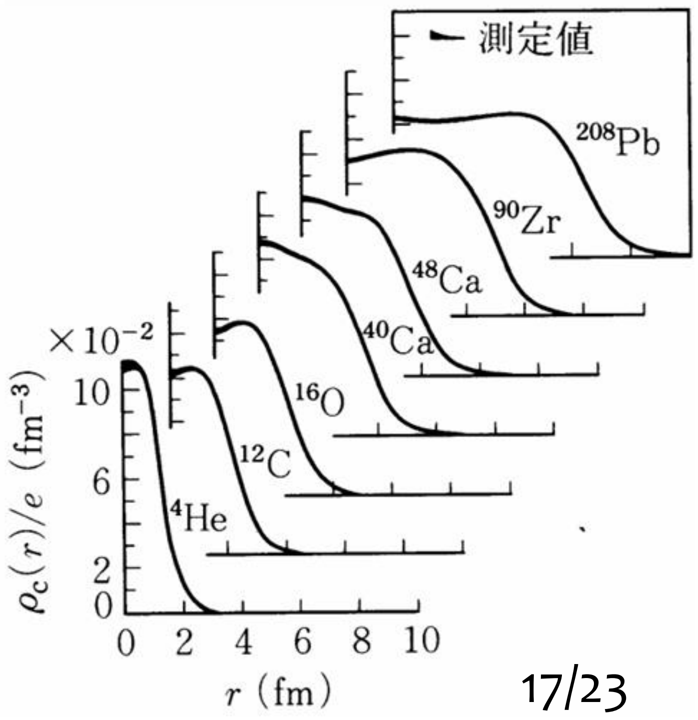




Is this real?

Density at the vertex is as high as 0.11 fm^{-3} which is much larger than typical density (～ 0.08 fm^{-3}).

Among all nuclei, there is only one nucleus which has such high central density.
The central density of alpha particle is 0.11 fm^{-3} !!!



Question: Is this an accidental coincidence?

How can we confirm the shape of 12C ?

A novel experimental probe: α knockout reaction

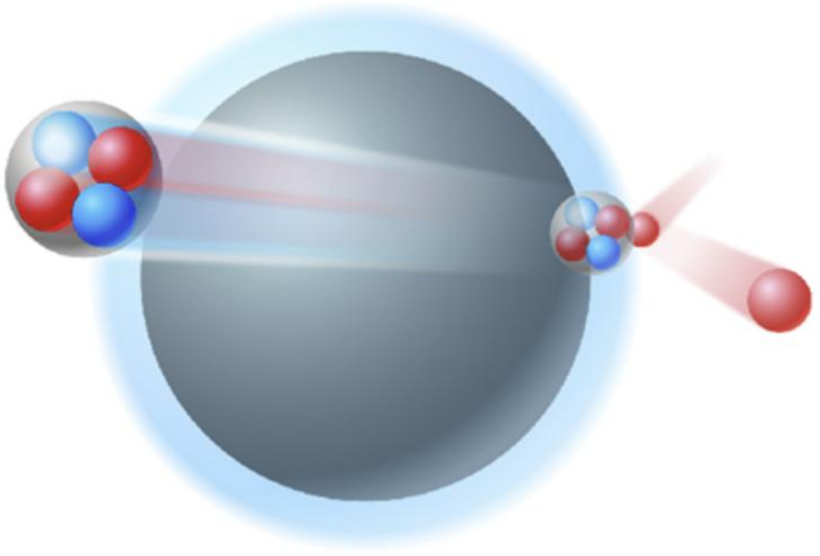
REPORT

J. Tanaka, Z. Yang et al., Science 2021

NUCLEAR PHYSICS

Formation of α clusters in dilute neutron-rich matter

Junki Tanaka[1,2,3]*, Zaihong Yang[3,4]*, Stefan Typel[1,2], Satoshi Adachi[4], Shiwei Bai[5], Patrik van Beek[1],



Theoretical works

^{20}Ne(p,pα) : K. Yoshida et al., PRC 100, 044601 (2019)

^{48}Ti(p,pα): Y. Taniguchi et al., PRC 103, L031305 (2021).

^{A}Be(p,pα), ^{A}B (p,pα), : H.Motoki et al., PTEP2022, 113D01 (2022)
Q. Zhao et al., PRC 106, 054313 (2022)

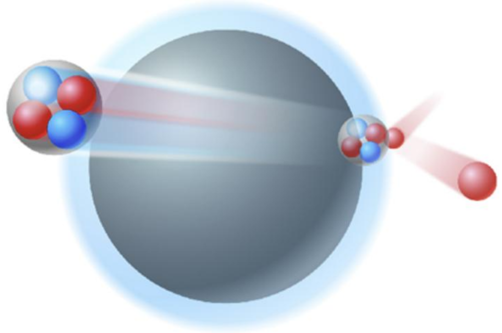
(p,pα) reaction as a probe for α particle preformed in nuclei

High energy (p,pα) reaction is a clean probe for the alpha-particle preformation on the surface of nuclei

DWIA for $(p, p\alpha)$ reaction

K. Yoshida and K. Ogata PRC 94, 044604 (2016)

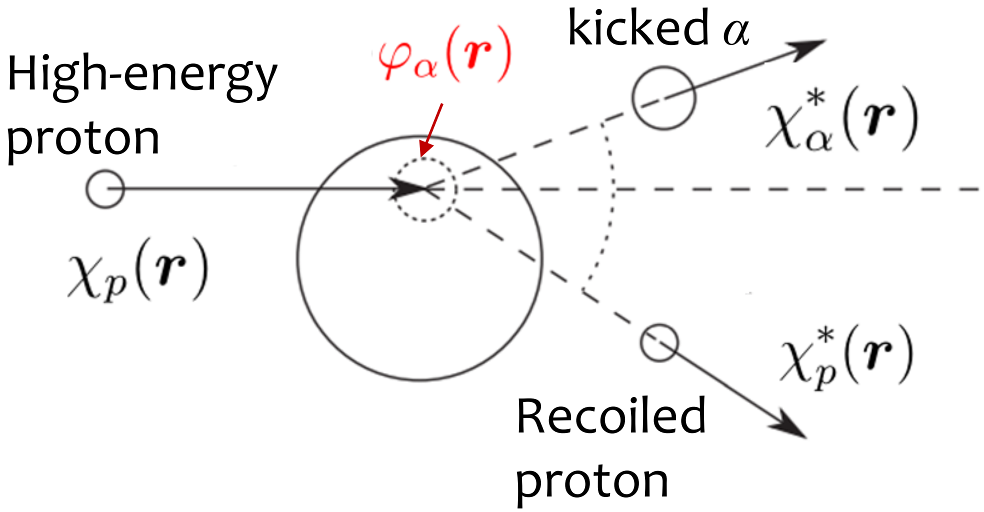
$$\frac{d^3\sigma}{dt_p d\Omega_p d\Omega_\alpha} = F_{\mathrm{kin}} \frac{d\sigma_{p\alpha}}{d\Omega_{p\alpha}} |T|^2$$



Transition matrix

$$T = \int \chi_p^*(\boldsymbol{r})\chi_\alpha^*(\boldsymbol{r})\chi_p(\boldsymbol{r})\underline{\varphi_\alpha(\boldsymbol{r})}e^{-i\boldsymbol{k}_0\cdot\boldsymbol{r}}$$

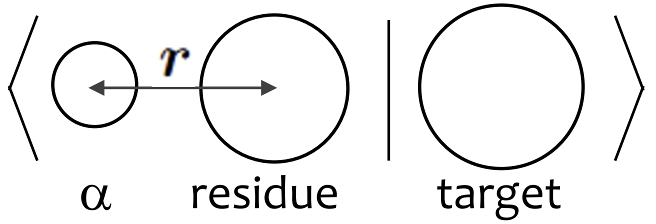
α particle wave function inside nucleus



A benchmark calculation for 20Ne (α+^{16}O)

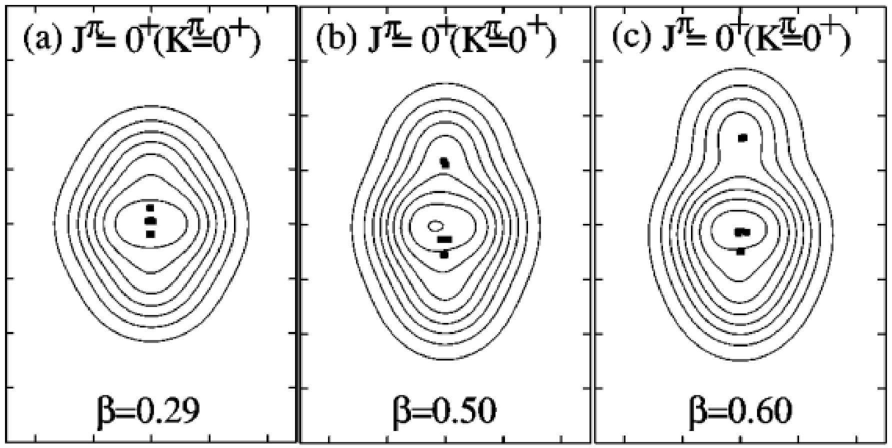
Alpha particle wave function preformed in the target nucleus is defined as the overlap
of target nucleus and decay channel = probability amplitude to observe alpha particle at distance r

$$\varphi(\boldsymbol{r}) = \sqrt{{}_A C_4}\,\langle \delta^3(\boldsymbol{r}' - \boldsymbol{r})\Phi_\alpha \Phi_{residue} | \Phi_{target} \rangle \;=$$



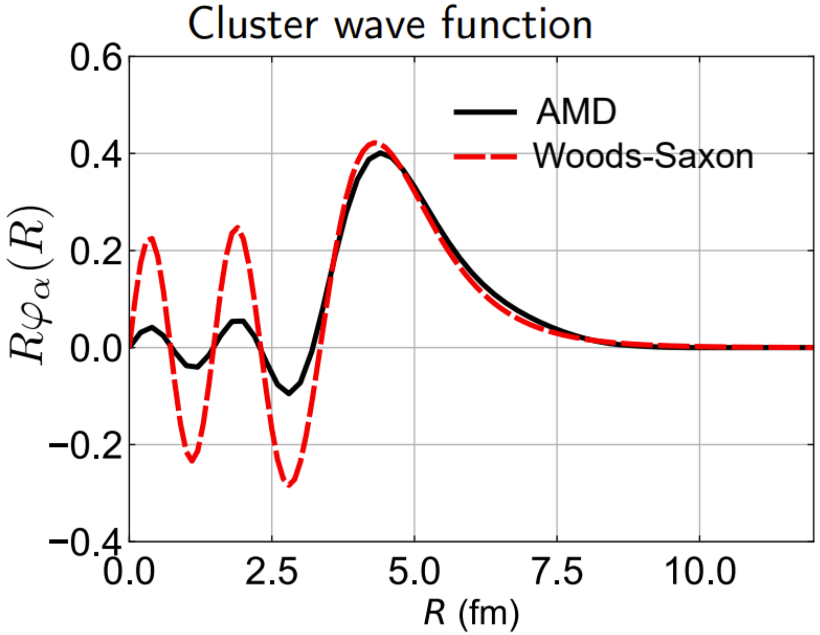
We have a reliable wave function for ^{20}Ne

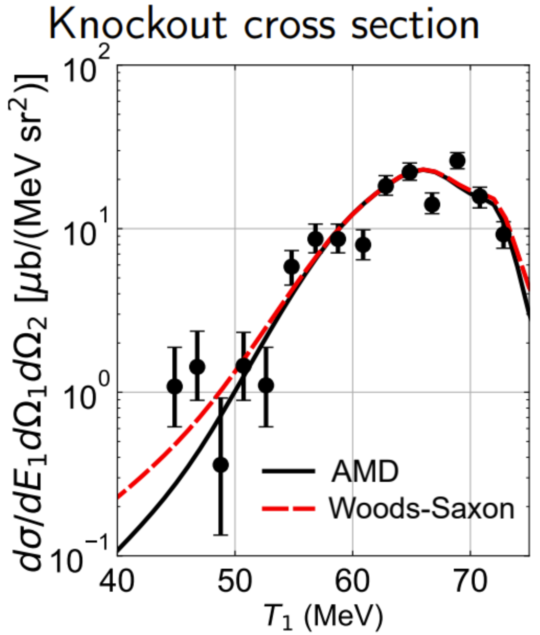
- Spectrum of ^{20}Ne
- α decay width of the excited states
- ^{16}O(^{6}Li,d)^{20}Ne reaction



M. Kimura, PRC69, 044319 (2004)

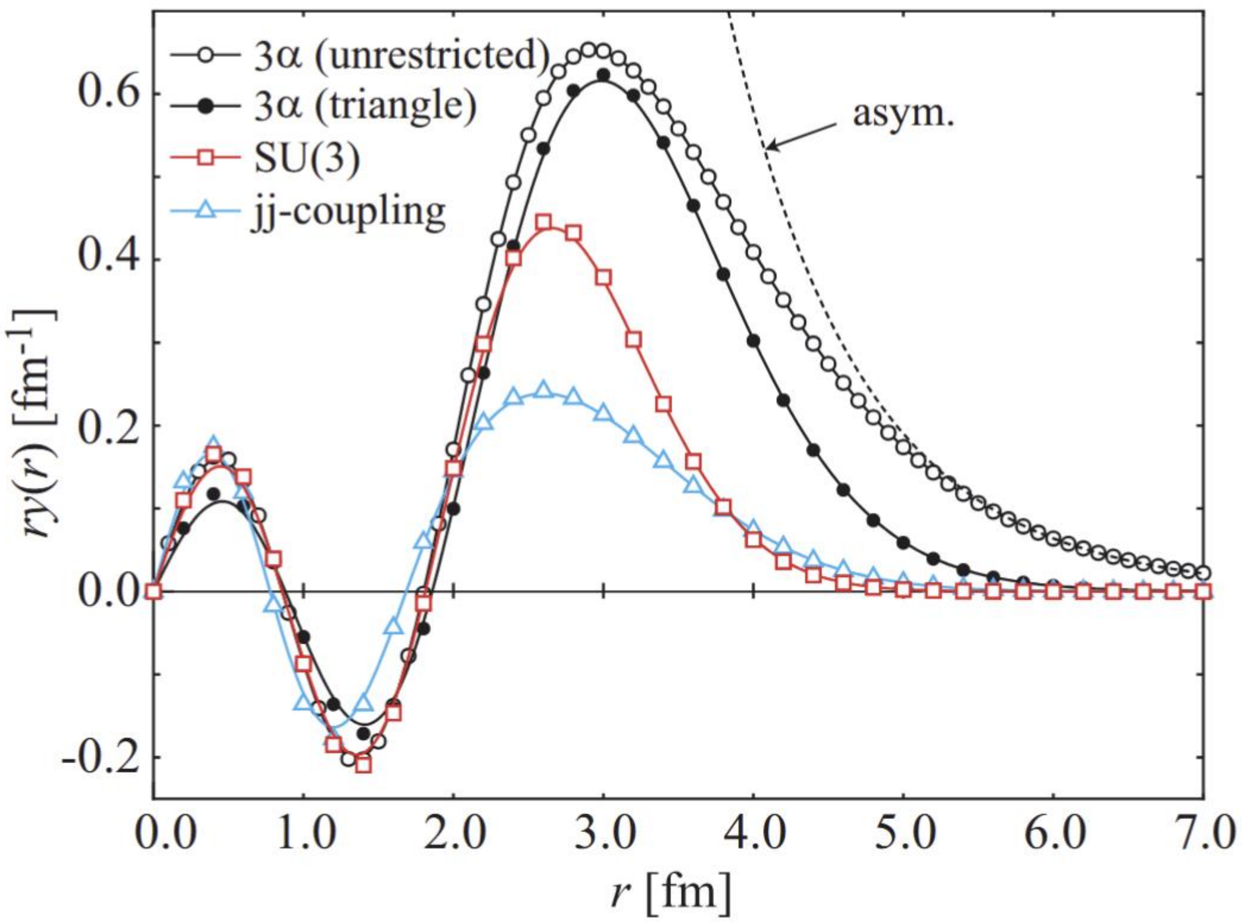
^{20}Ne(p,pα) : K. Yoshida et al., PRC 100, 044601 (2019)

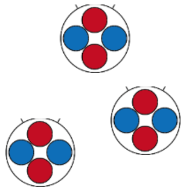




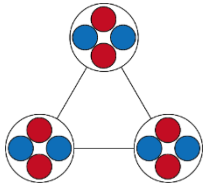
Numerical experiments for ^{12}C

Numerical experiments.

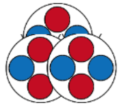




3α cluster model (RGM, GCM, THSR)
3α system solved
without any symmetry assumption



Fixed 3α triangle (≈ Algebraic cluster model)

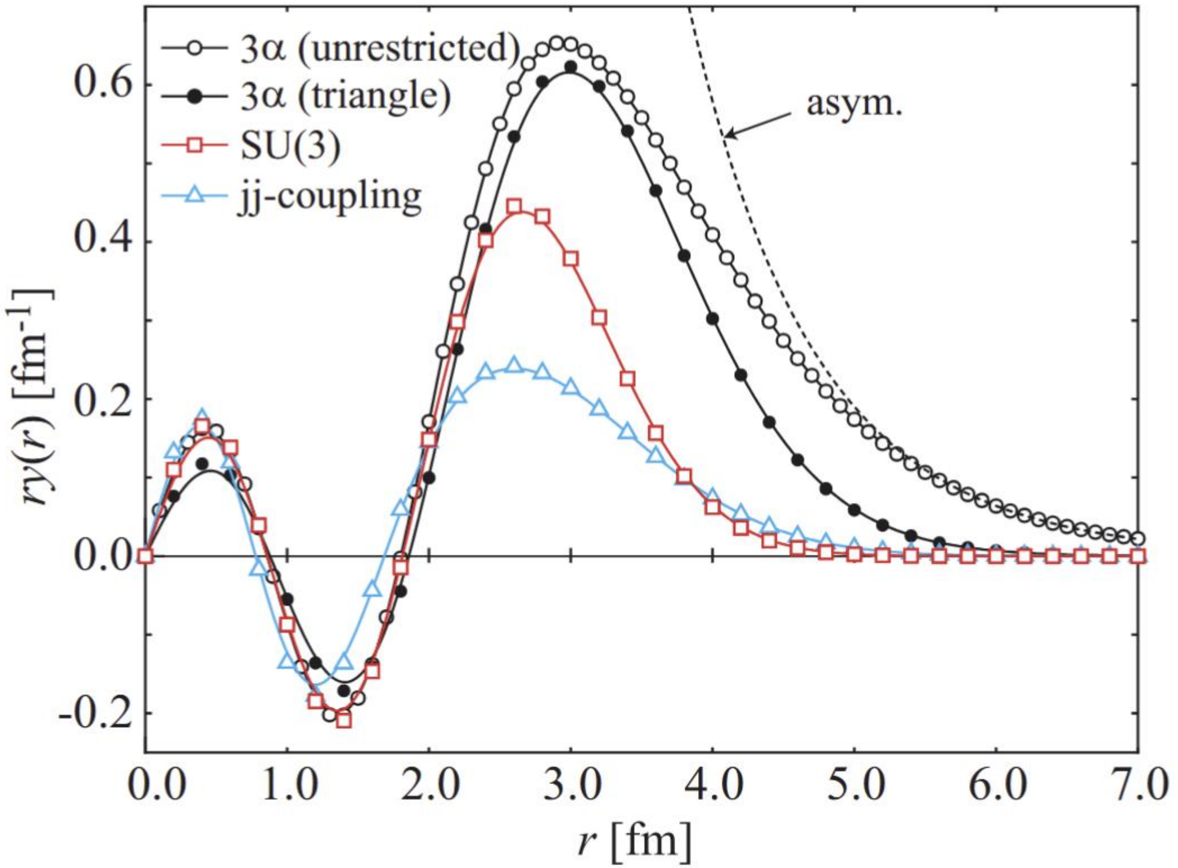


A simple shell model (SU3 limit)
$(0s)^4(0p)^8$ Harmonic oscillator
jj-coupling

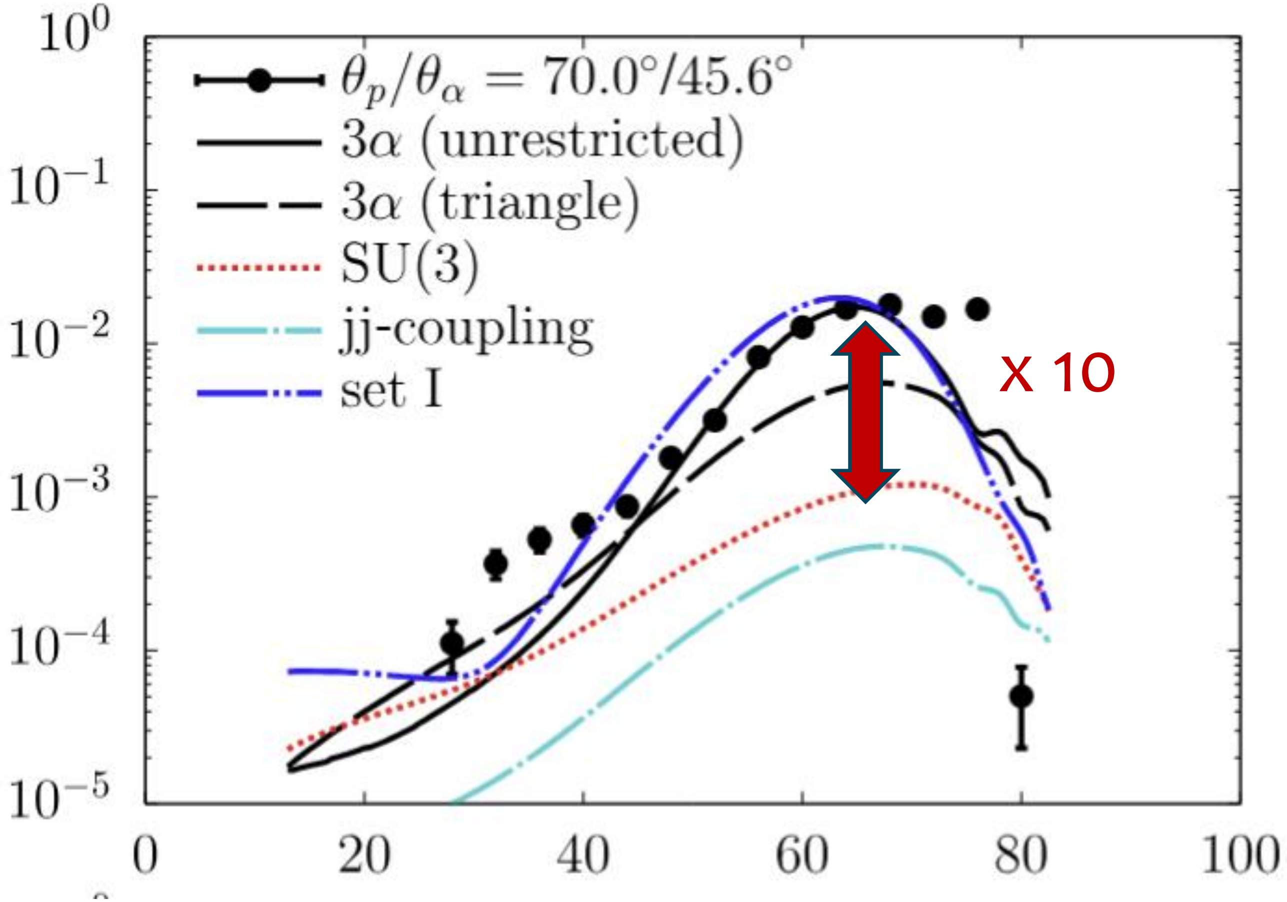
NOTE
All of them reproduce the observed radius of 12C,
but yields different alpha particle preformation

12C seems to be clustered!!

Numerical experiments.



The ordinary cluster model explains the data.
Fixed triangle somewhat undershoot.



Exp: J. Mabiala et al., PRC79, 054612 (2009)

Summary

(1) Shape of ^{12}C constructed from electron scattering data

Assumption: The 0^+, 2^+, 3^- and 4^+ states constitute "the ground band" sharing the same intrinsic state

⇒ An exotic triangular shape of ^{12}C emerges

(2) Alpha knockout from the ground state (ongoing)

Comparison of three different models
3α cluster model, Fixed 3α triangle, A simple shell model

⇒ Only 3α cluster model can explain the observation

⇒ More detailed analysis is ongoing for $^{12}C(p,p\alpha)^{8}Be(2^+)$, $^{16}O(p,p\alpha)^{12}C(0^+)$, etc…