Active
Macroscopic Fluctuation Theory

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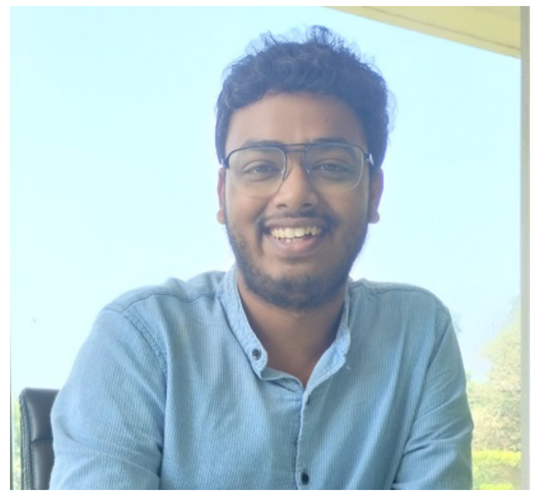
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Sabyasachi


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Juliane Klamser

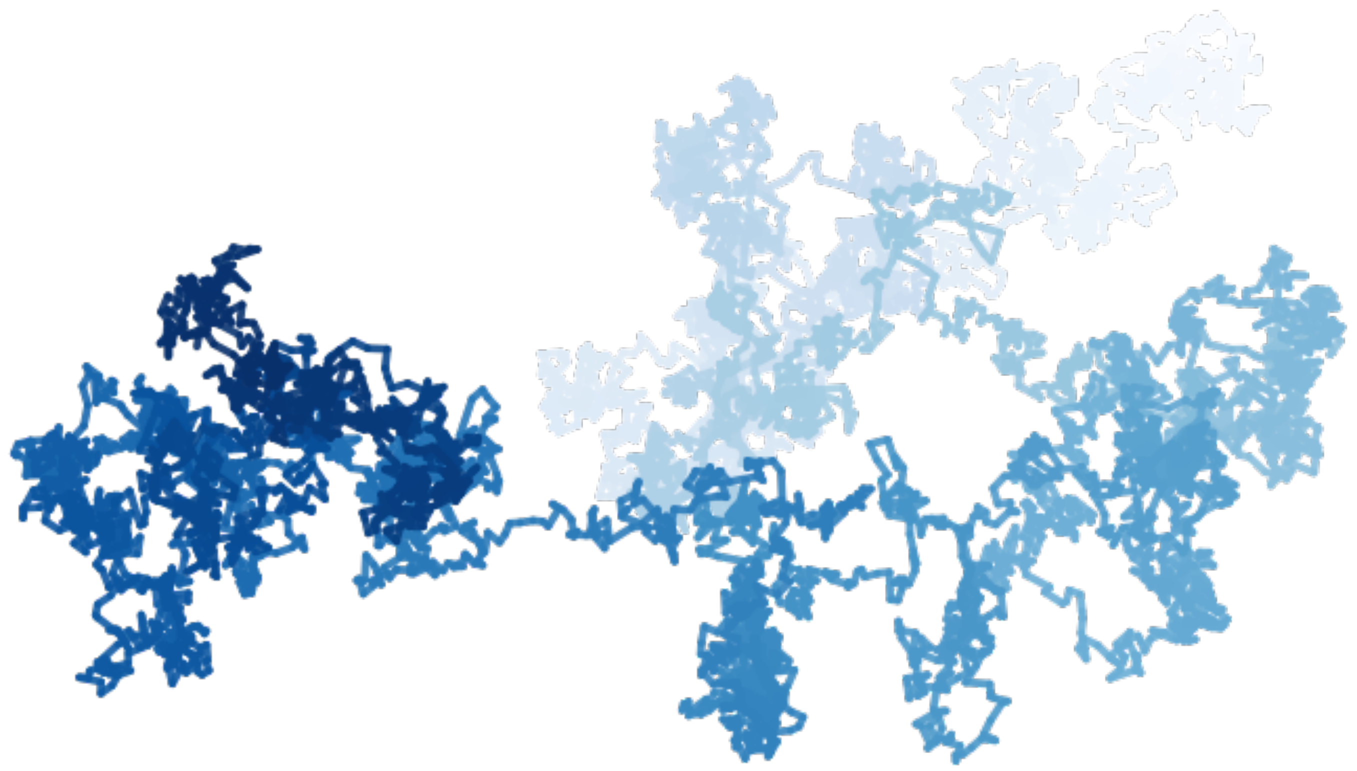

Abhishek Dhar

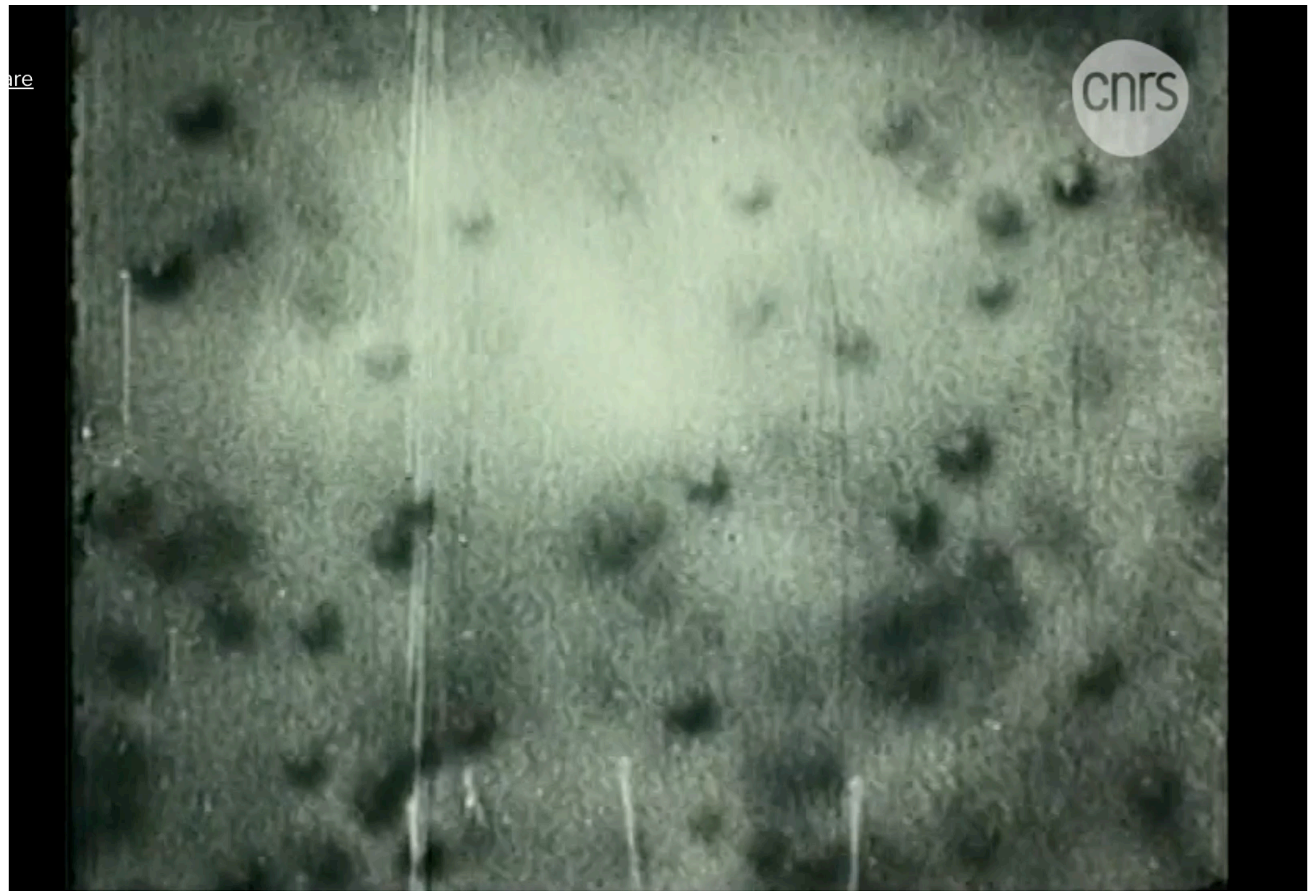

Sanjib Sabhapandit


Jacopo de Nardis

First: Active matter

Brownian Motion



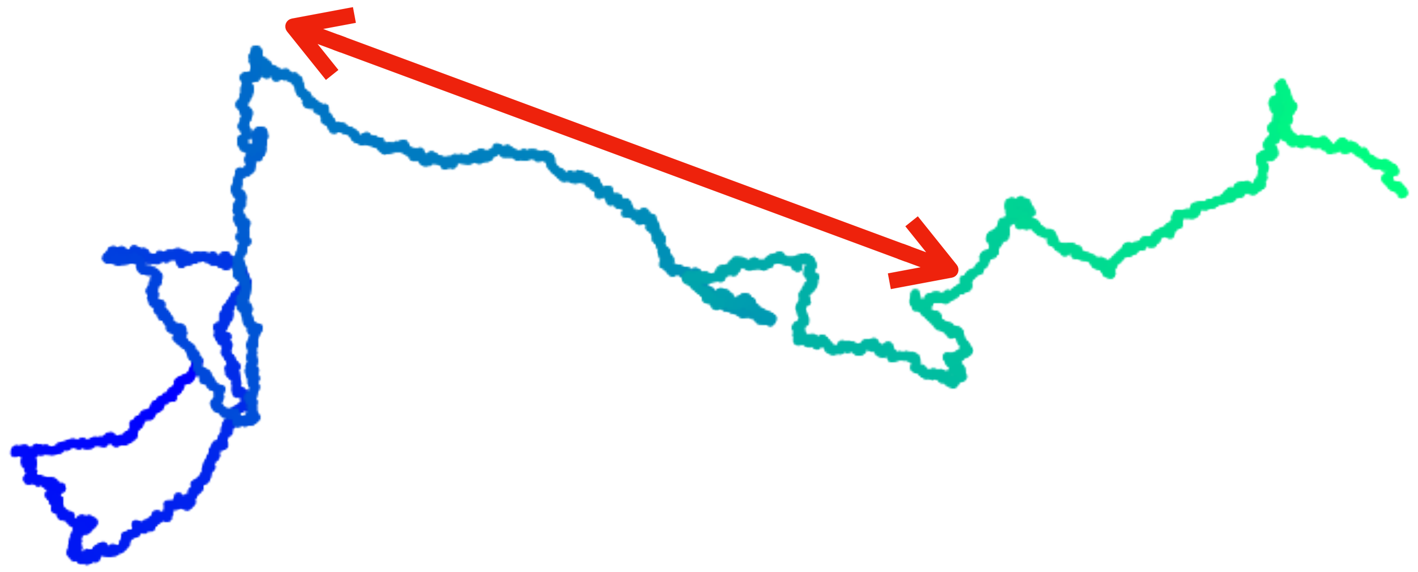


Expt of Paul Langevin (CNRS archive)

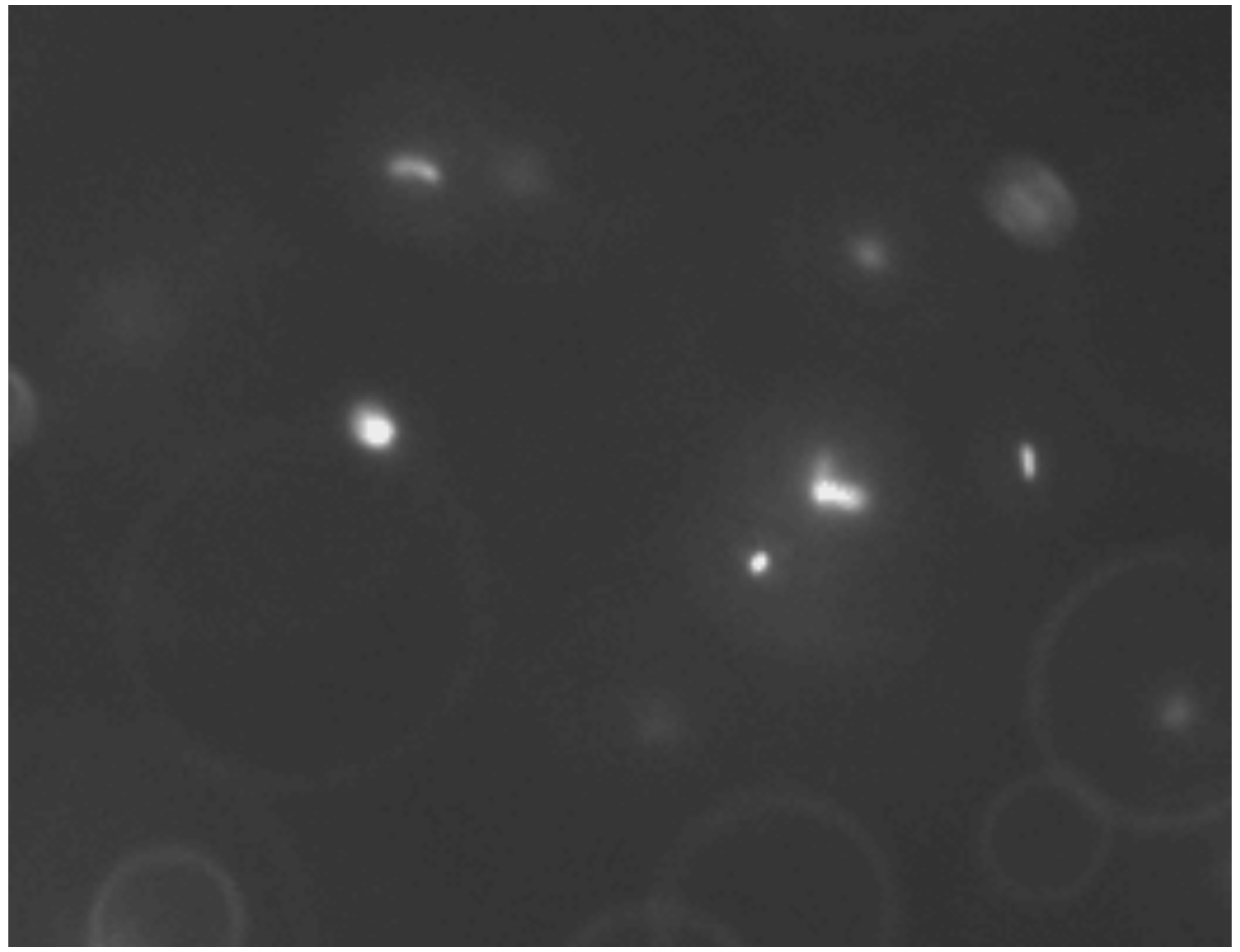
$$\dot{\mathbf{r}} = \boldsymbol{v}(t)$$

$$\langle v_i(t)\, v_j(t')\rangle = \delta_{ij}\,\delta(t - t')$$

Active particle



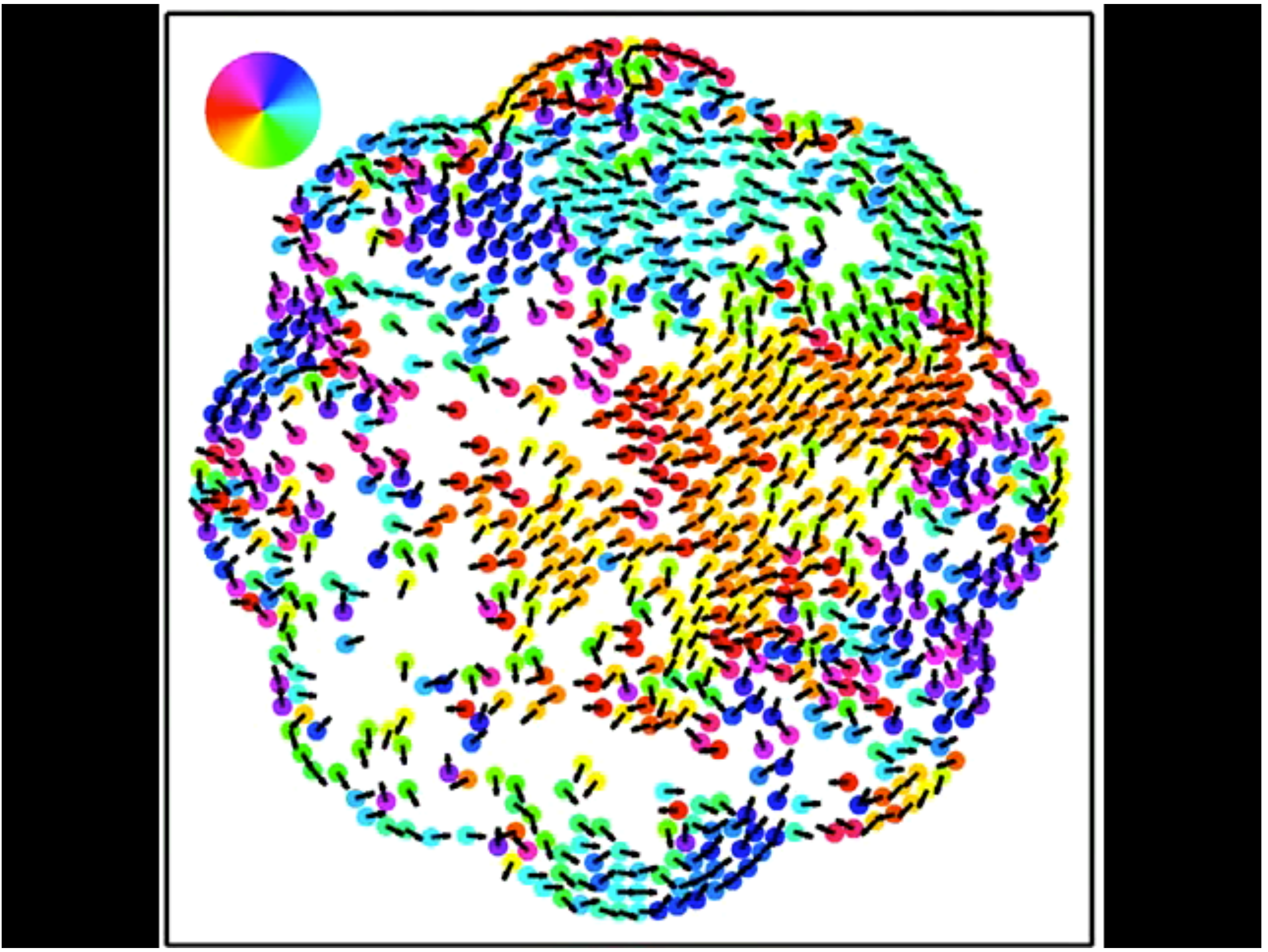
[Bechinger et al, RMP 88, 045006 (2016)]



Courtesy: Mingming Wu, Cornell.

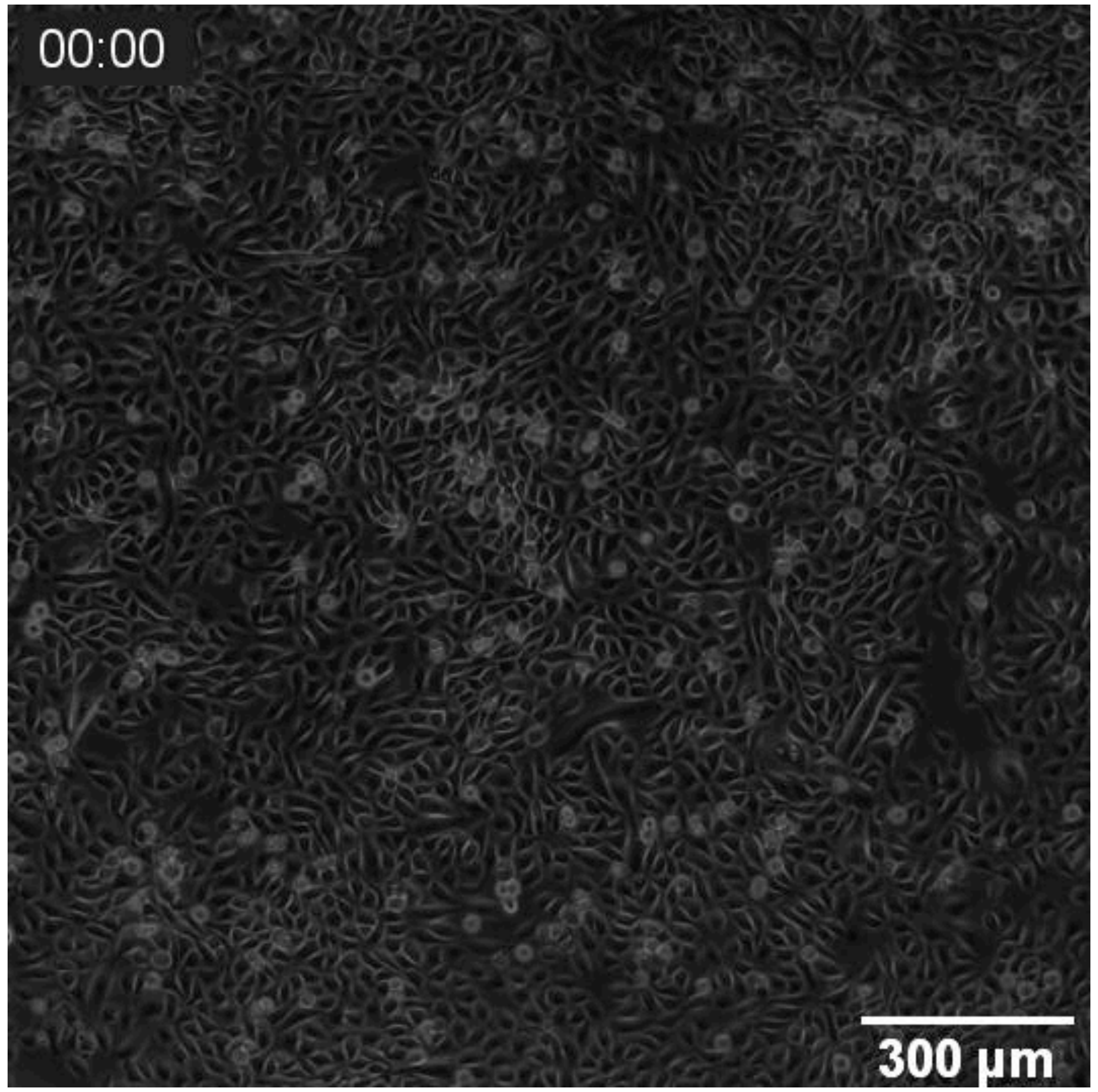
$$\langle v_i(t)\, v_j(t')\rangle = v_0 \delta_{ij}\, e^{-\frac{|t-t'|}{\tau}}$$

Collective phenomena

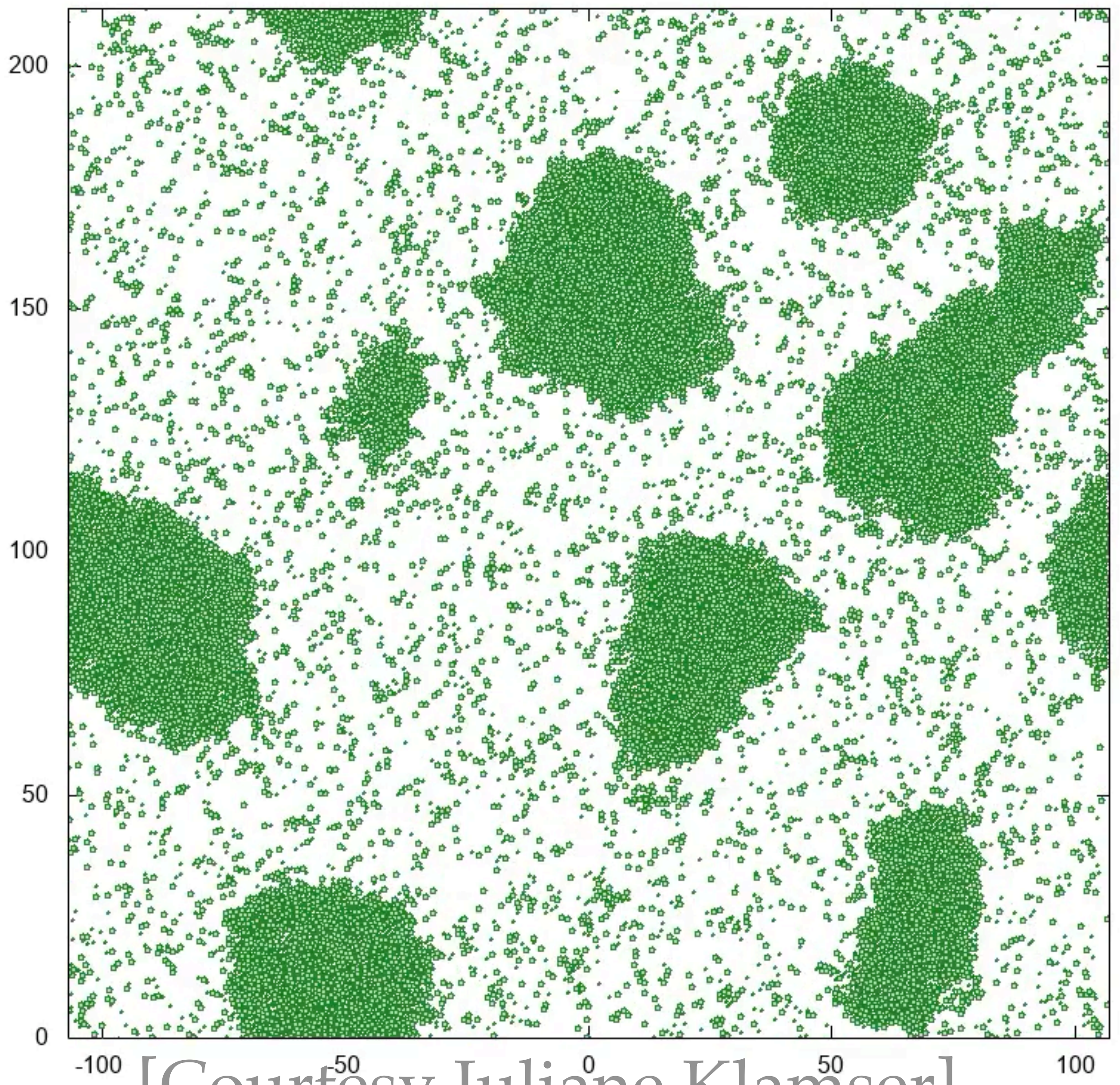


[Deseigne, Dauchot, Chaté, PRL 105, 098001 (2010)]

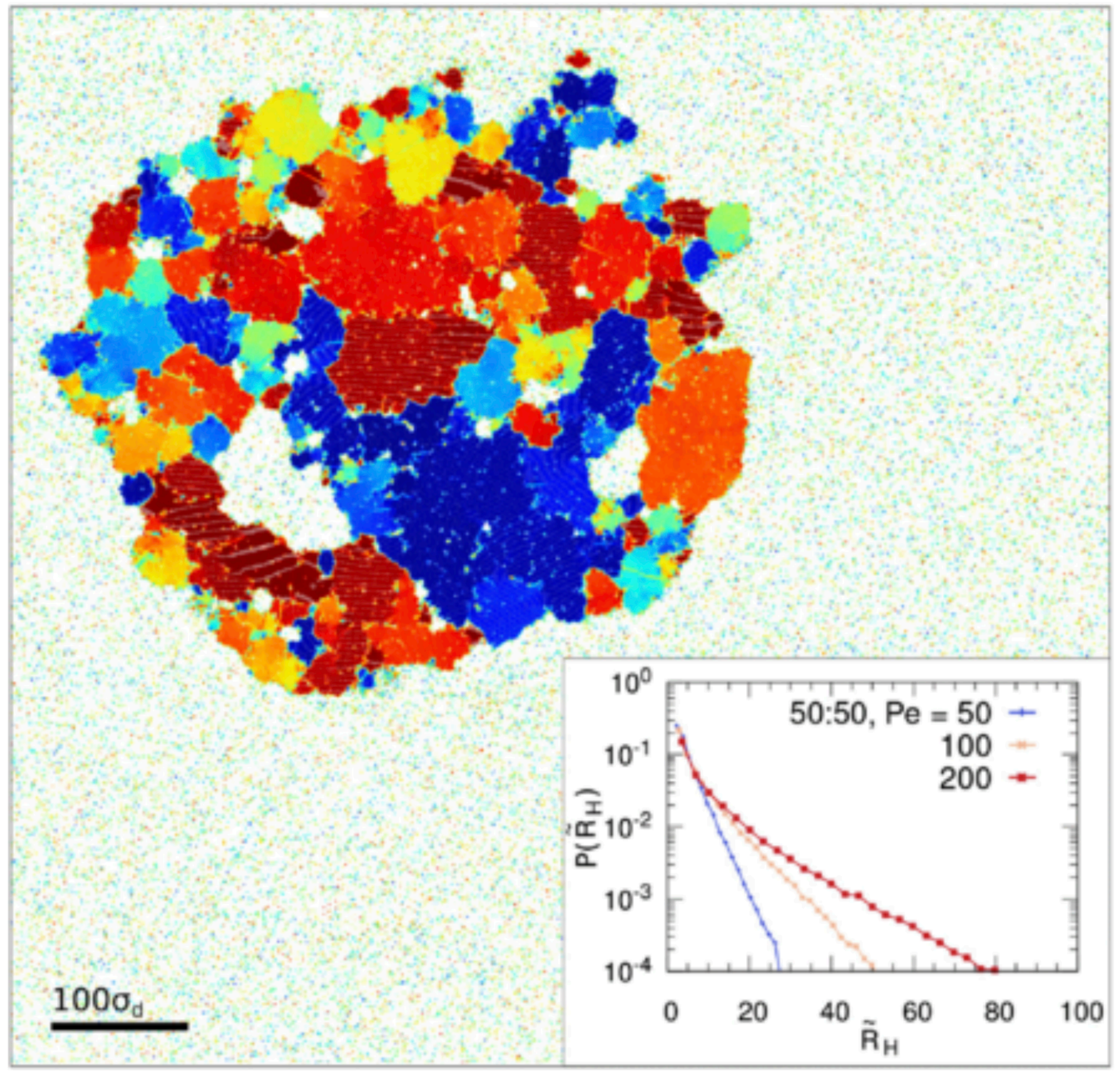
[Deseigne, Léonard, Dauchot, Chaté, Soft Matter (2012)]



[Blanch-Mercader, Yashunsky, Garcia, Duclos, Giomi, Silberzan, PRL 120, 208101 (2018)]

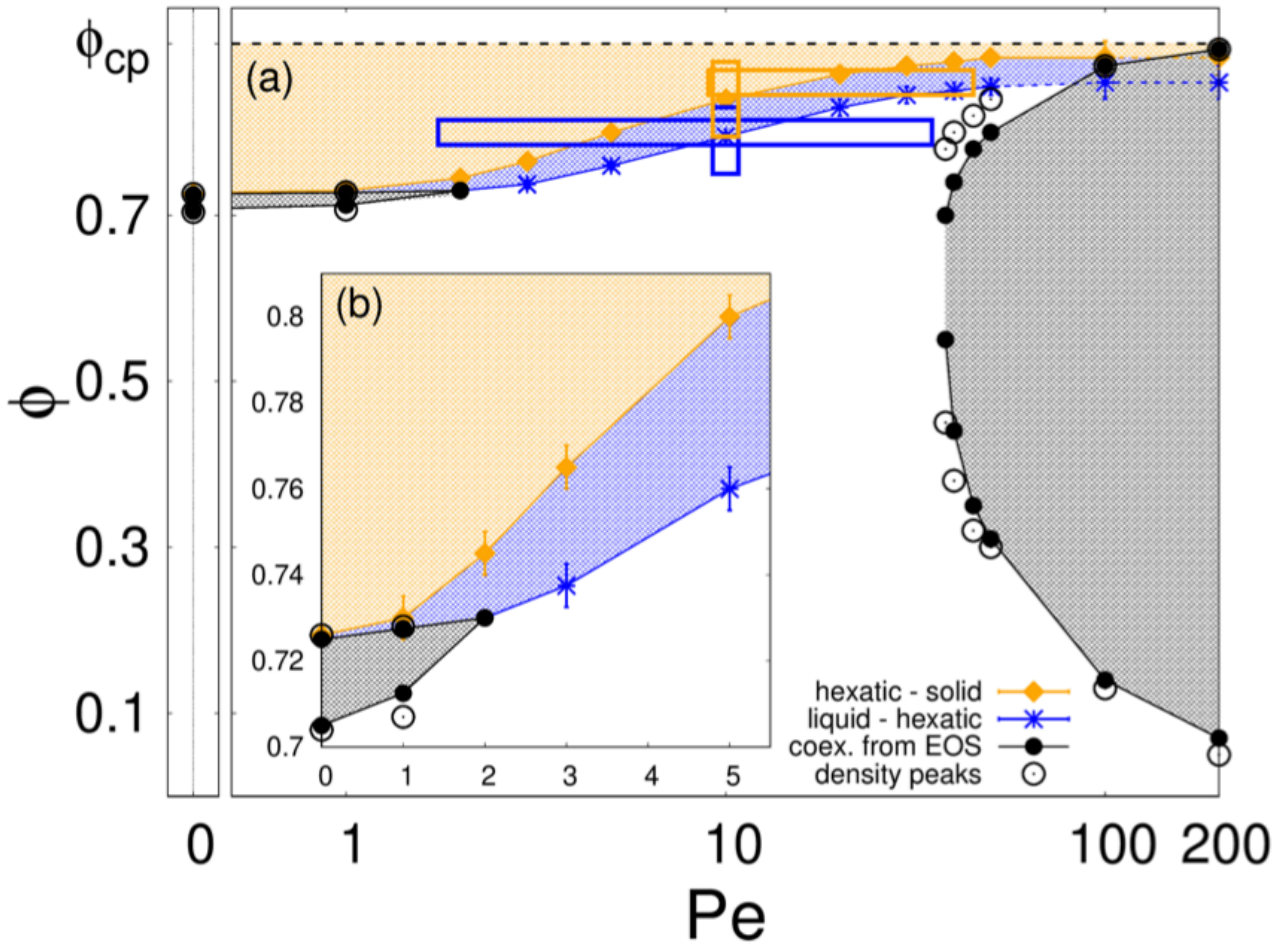


[Courtesy Juliane Klamser]



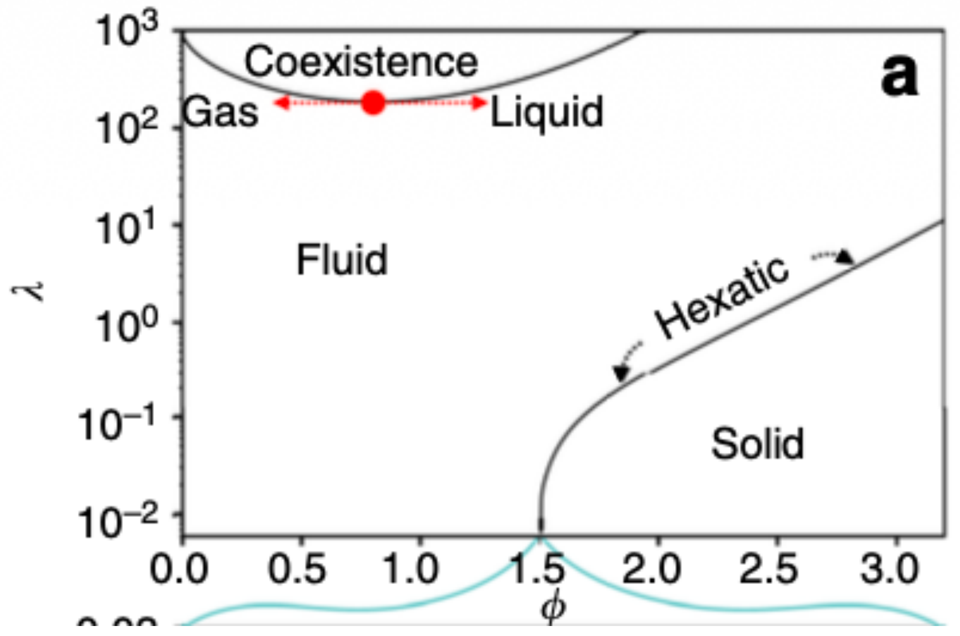
[Caporusso, Digregorio, Levis, Cugliandolo, Gonnella PRL 125, 178004 (2020)]

[Cugliandolo and Gonnella, Les Houches Lectures 2018]



Full phase diagram of Active Brownain discs

[Digregorio, Levis, Suma, Cugliandolo, Gonnella, Pagonabarrga PRL 121, 098003 (2018)]



[Klamser, Kapfer, Krauth Nat Com 9, 5045 (2018)]

Most theories are non-equilibrium extensions of Landau-Ginsberg type effective descriptions.

[Active field theories, Cates, Les Houches (2018)]

[Jülicher, Grill, Salbreux, Rep Prog Phys 81, 076601 (2018)]

[Bowick, Fakhri, Marchetti, Ramaswamy, PRX 12, 010501 (2022)]

[Marchetti, Joanny, Ramaswamy, Liverpool, Prost, Rao, Simha RMP 85, 1143 (2013)]

[Dinelli, O'Byrne, Tailleur, J Phys A 57, 395002 (2024)]

[Barré, Chetrité, Muratori, Peruani JSP 158, 589 (2015)]

[Bebon, Robinson, Speck, PRX 15, 021050 (2025)]

[Burekovic, Luca, Cates, Nardini, arXiv:2601.16539 (2026)]

[Supekar, Song, Hastewell, Dunkel, PNAS 120 (2023)]

*Is there a **quantitative** macro-scale theory for Active matter, describing large fluctuations?*

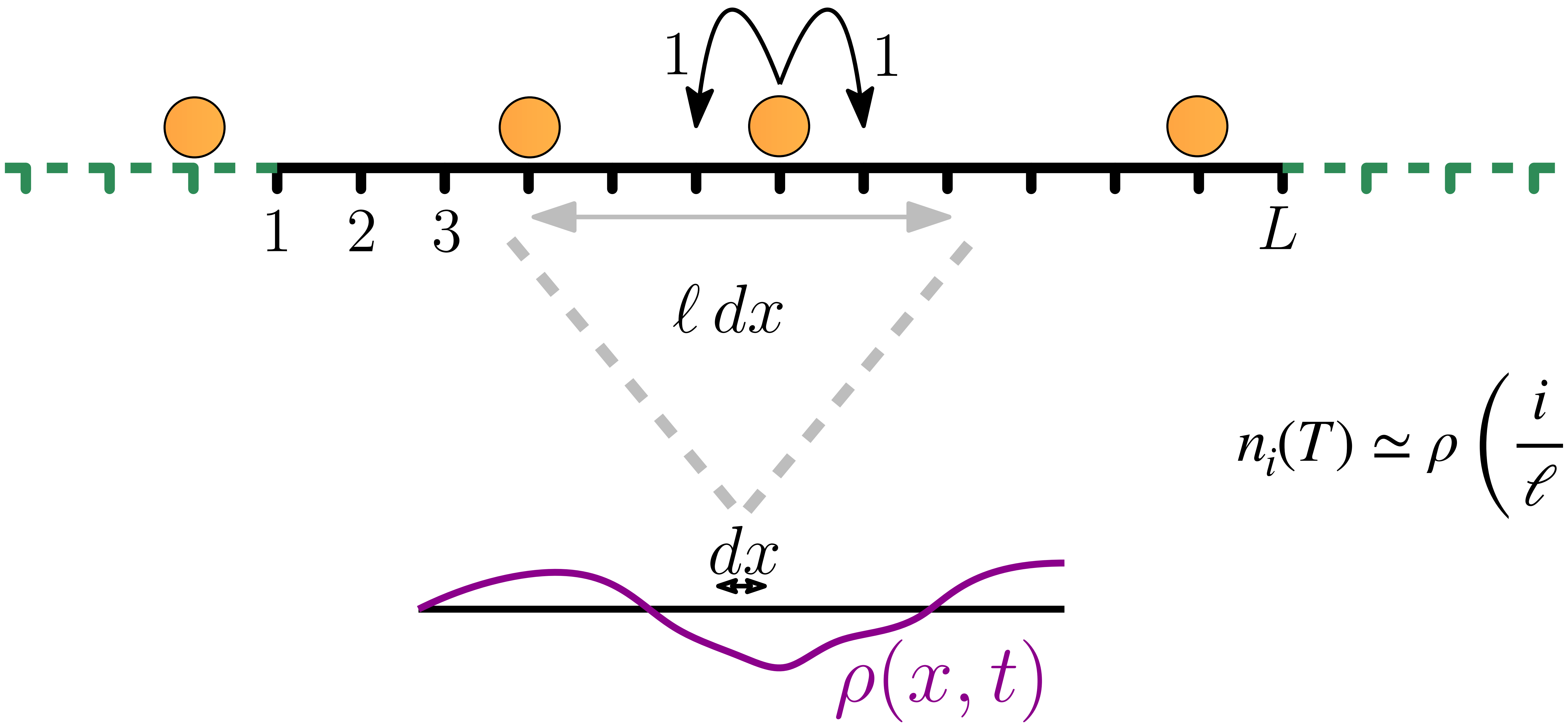
Knowledge from Passive systems:

Fluctuating hydrodynamics

&

Macroscopic fluctuation theory

Symmetric Simple Exclusion Process: Continuous time random walkers with hardcore exclusion



$$n_i(T) \simeq \rho\left(\frac{i}{\ell}, \frac{T}{\ell^2}\right)$$

$$\partial_t \rho(x,t) = -\,\partial_x j(x,t); \qquad j(x,t) = -\,\partial_x \rho(x,t) + \frac{1}{\sqrt{\ell}}\sqrt{2\rho(1-\rho)}\;\xi(x,t)$$

Fluctuating hydrodynamics

[Spohn (1991)]

[Kipnis, Landim (1999)]

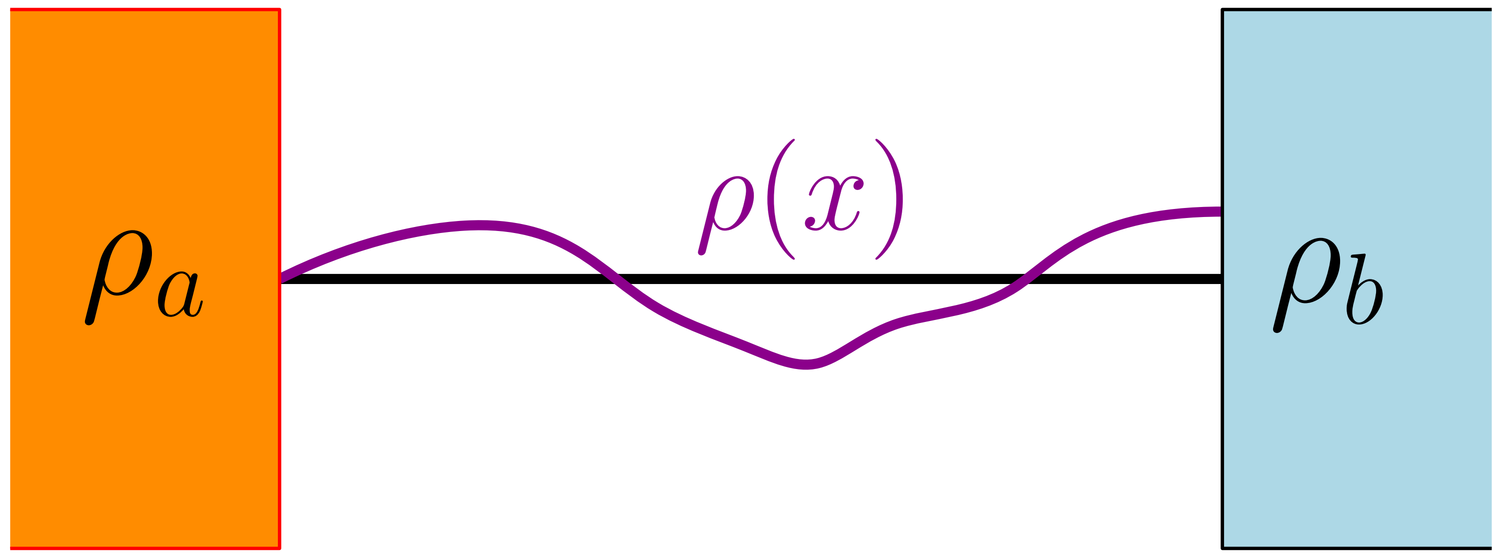
[Bertini, De Sole, Gabrielli, Jona-Lassinio, Landim (PRL, 2000) (JSP 2002) (RMP 2015)]

[Tailleur, Kurchan, Lecomte, (PRL 2007) (JPA 2008)]

What can the theory describe?

All macroscopic correlations including large deviations!

Non-equilibrium stationary state



$$\mathrm{P}[\rho(x)] \sim e^{-\ell\, \mathscr{F}[\rho(x)]}$$

Exact microscopic solution

[Derrida, Lebowitz, Speer (PRL, 2000) (JSP 2002)]

[Derrida, Hirschberg, TS (JSP 182, 15 (2020)]

Fluctuating Hydrodynamics

[Bertini, De Sole, Gabrielli, Jona-Lassinio, Landim (PRL, 2000) (JSP 2002) (RMP 2015)]

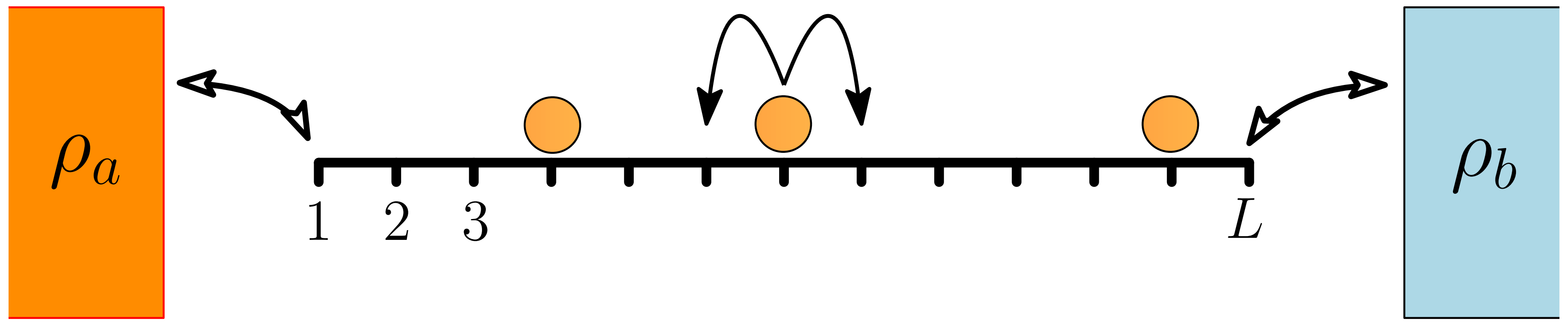
[Tailleur, Kurchan, Lecomte, (PRL 2007) (JPA 2008)]

[Saha, TS (Scipost Phys 17, 033 (2024)]

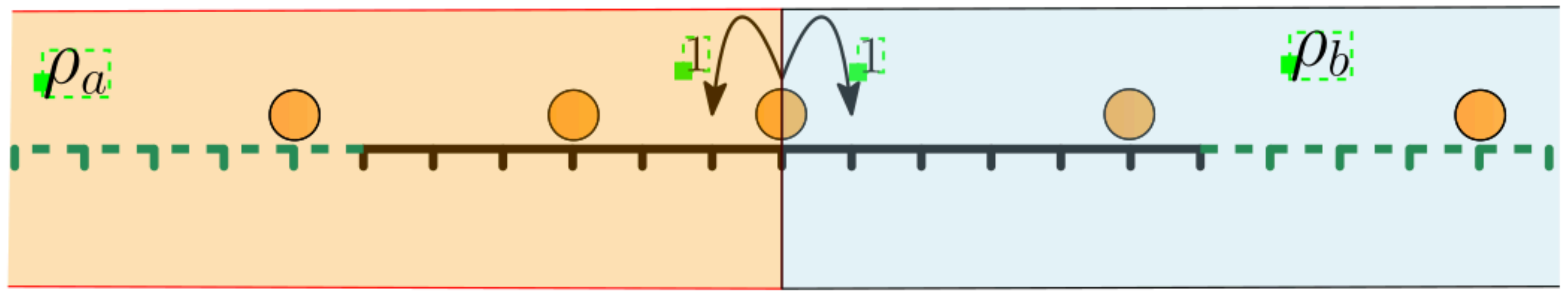
Fully characterises the Macroscopic state

Transport properties

Stationary state



Transient state



$$\langle e^{\lambda Q_t} \rangle \sim e^{-t^{\alpha} \mu(\lambda)}$$ at large t

[Derrida, Doucot, Roche (JSP, 2004)]

[Derrida, Bodineau (PRL, 2004)]

[Bertini, De Sole, Gabrielli, Jona-Lassinio, Landim (JSP, 2005)]

[Derrida, Hirschberg, TS (JSP 182, 15 (2020)]

[Saha, TS (Scipost Phys 17, 033 (2024)]

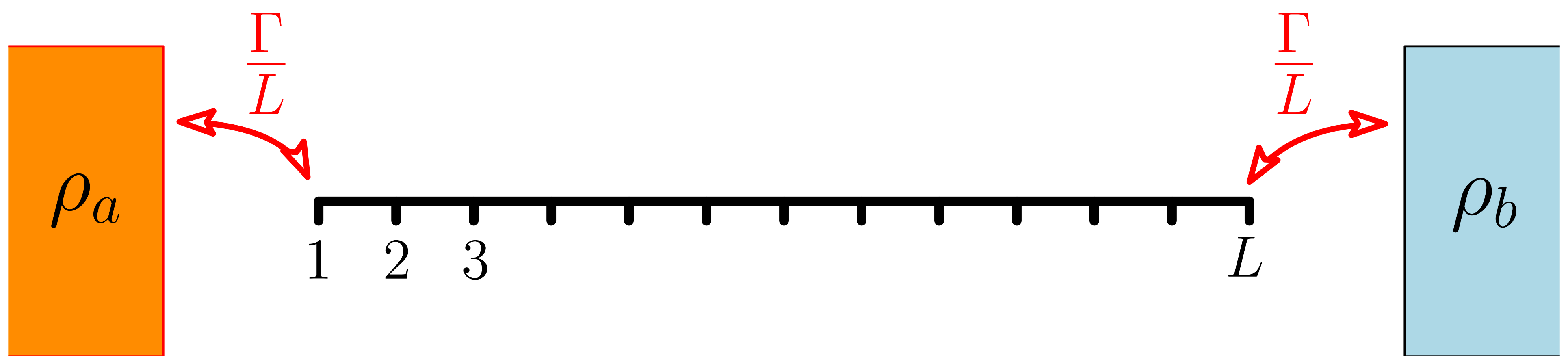
[Derrida & Gershenfeld (JSP 2009)]

[Mallick, Moriya, Sasamoto (PRL 2022)]

The Macroscopic approach has the advantage of tractability

Transport properties

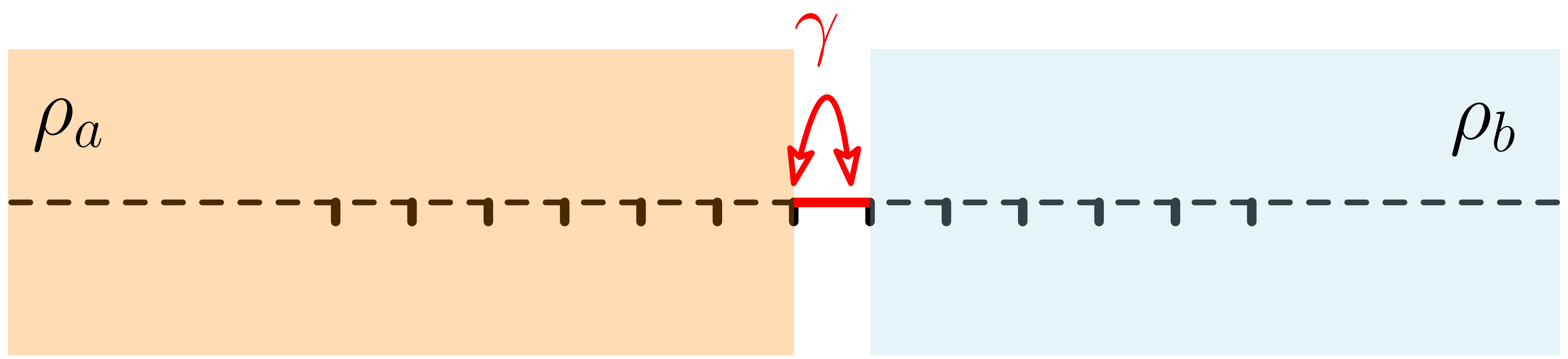
Slowly coupled SEP



[Saha, TS (Scipost Phys 17, 033 , 2024)]

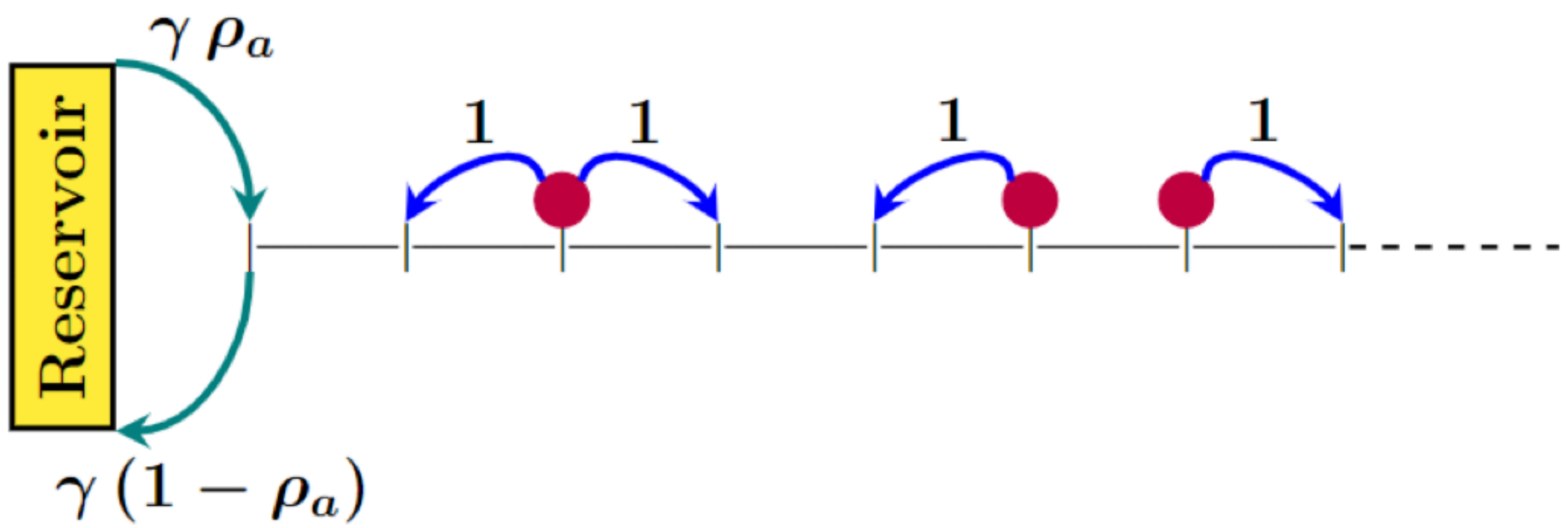
[Lazarescu (J Phys A 48, 503001 , 2015)]

Infinite SEP with slow region



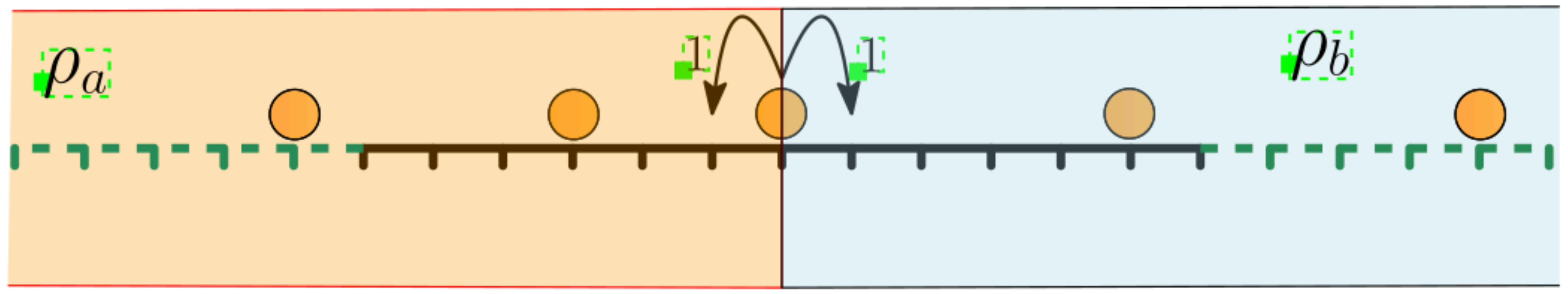
[Sharma, Saha, Jangid, TS (PRE 2026)]

Semi-infinite SEP

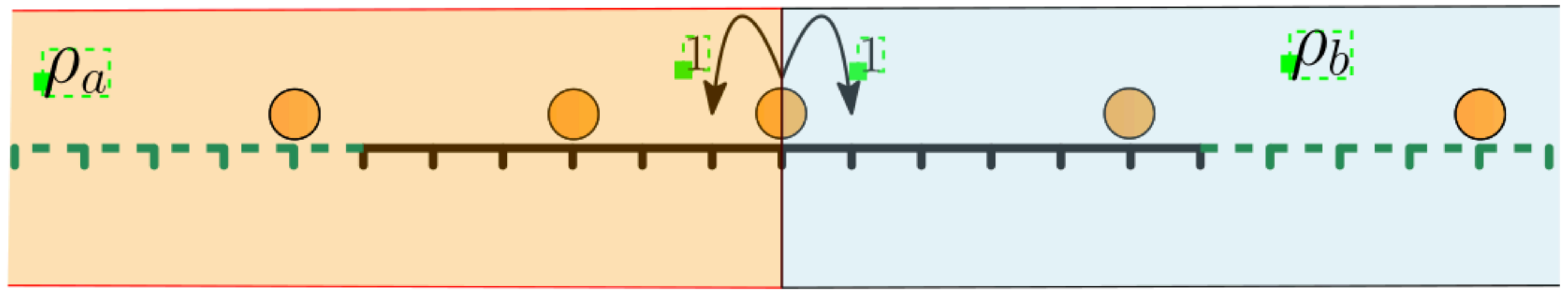


[Sharma, Saha, Jangid, TS (PRE 113, L052101 2026)]

Multi-time statistics in SEP



[Grabsch, Moriya, Mallick, Sasamoto, Benichou (PRL 2024)]



$$\langle e^{\lambda Q_t} \rangle \simeq e^{\sqrt{t}\,\chi(\lambda)}$$

[Derrida, Gershenfeld (JSP 2009)]

[Mallick, Moriya, Sasamoto (PRL 2022)]

$$\langle e^{\int_0^T \mathrm{d}t\, \lambda(t)\, Q_t} \rangle \simeq e^{\mathcal{O}[\lambda(t)\partial_\kappa]\, \chi(\kappa)|_{\kappa=0}}$$

$$\mathcal{O}[z_t] = \sqrt{\pi} \int_{-\infty}^{\infty} \mathrm{d}x \left(P(x,0) \mathrm{e}^{-\Theta(x) \int_0^T \mathrm{d}t\, z_t} - 1 \right)$$

$$\partial_t P(x,t) + \partial_x^2 P(x,t) = -z_t\, \Theta(x)\, P(x,t)$$

$$P[x,T] = 1$$

[Sharma, Chowdhury, Jangid, TS (upcoming)]

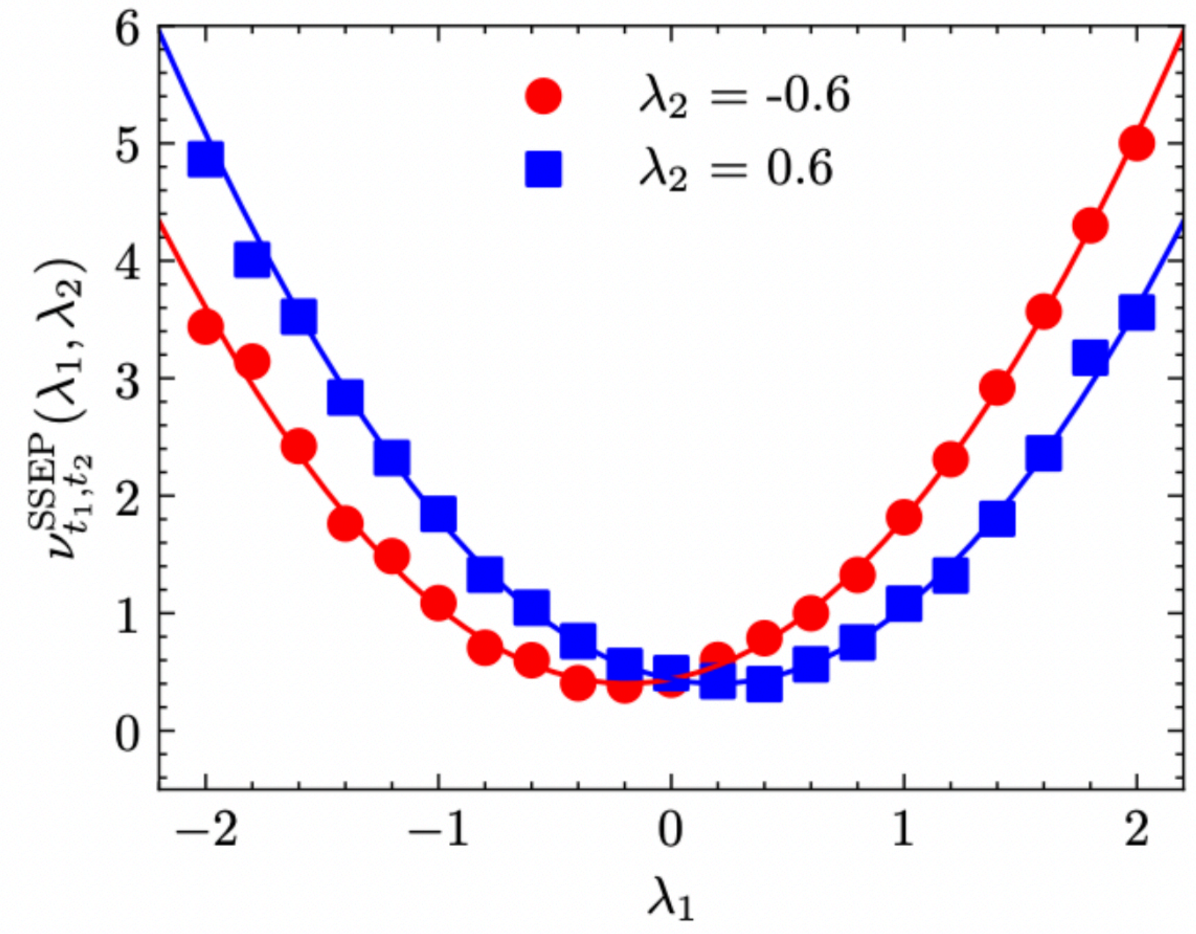


FIG. 1. **Two time SSEP for** $\rho_a = \rho_b = \frac{1}{2}$, parameters: N_c (clone populations) = 20000 , $t_1 = 50$ and $t_2 = 125$

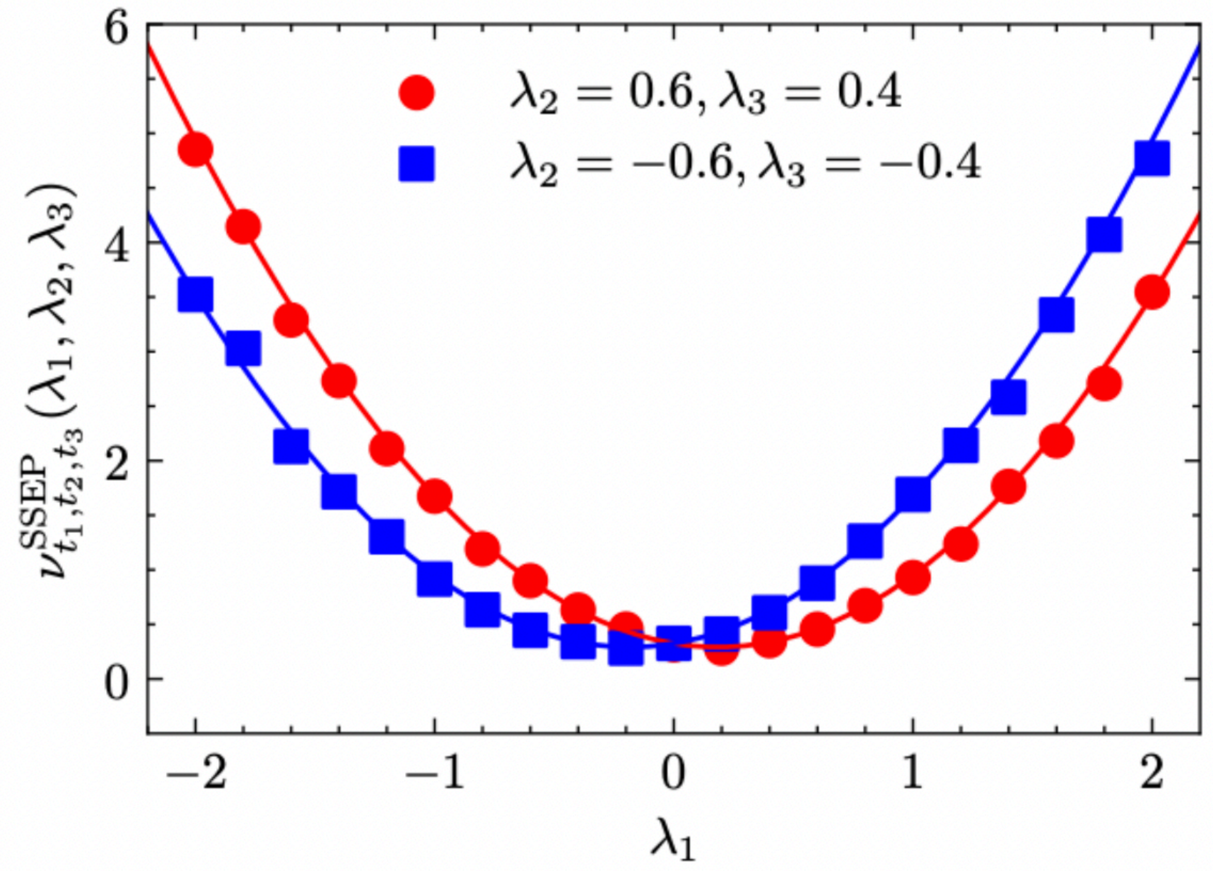
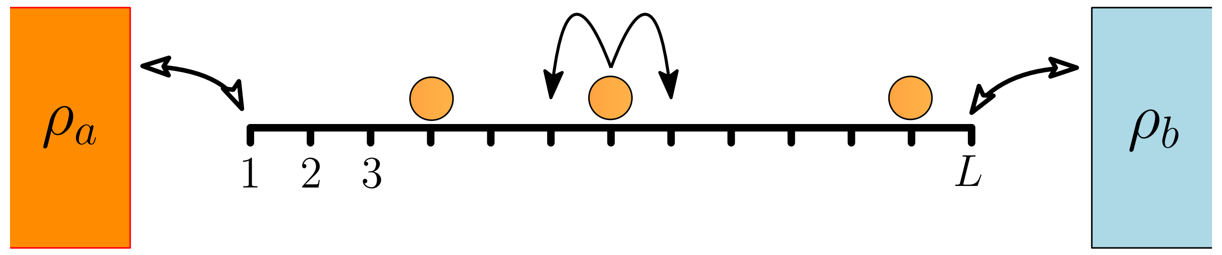


FIG. 2. **Three time SSEP for** $\rho_a = \rho_b = \frac{1}{2}$, parameters: N_c (clone populations) = 20000 , $t_1 = 50$, $t_2 = 75$ and $t_3 = 150$

[Giardina, Kurchan, Peliti (PRL 2006)]

Applications

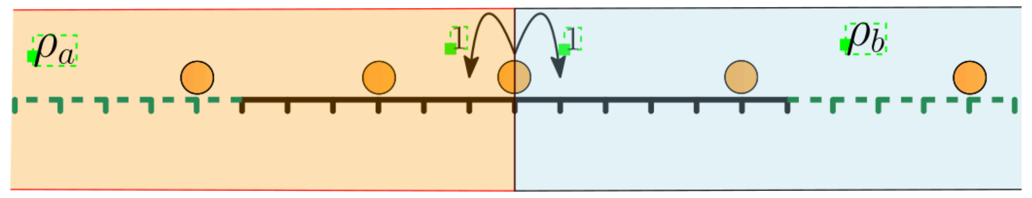
Finite SEP



Quantum Transport through disordered conductors.

[Lee, Levitov, Yakovets, (PRB 1995)]

Infinite SEP



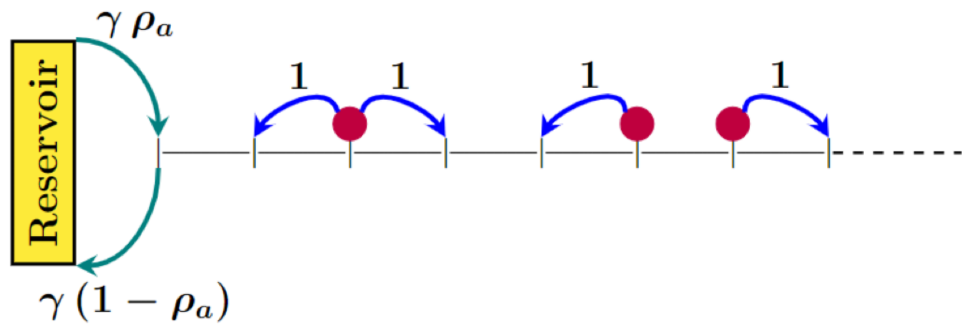
Random unitary quantum circuit

[McCulloch, Nardis, Gopalakrishnana, Vasseur (PRL 2023)]

Chaotic quantum dynamics

[Wienand, Vasseur, Gopalakrishnana, Bloch (Nat Phys 2024)]

Semi-Infinite SEP



Quantum Mpemba effect

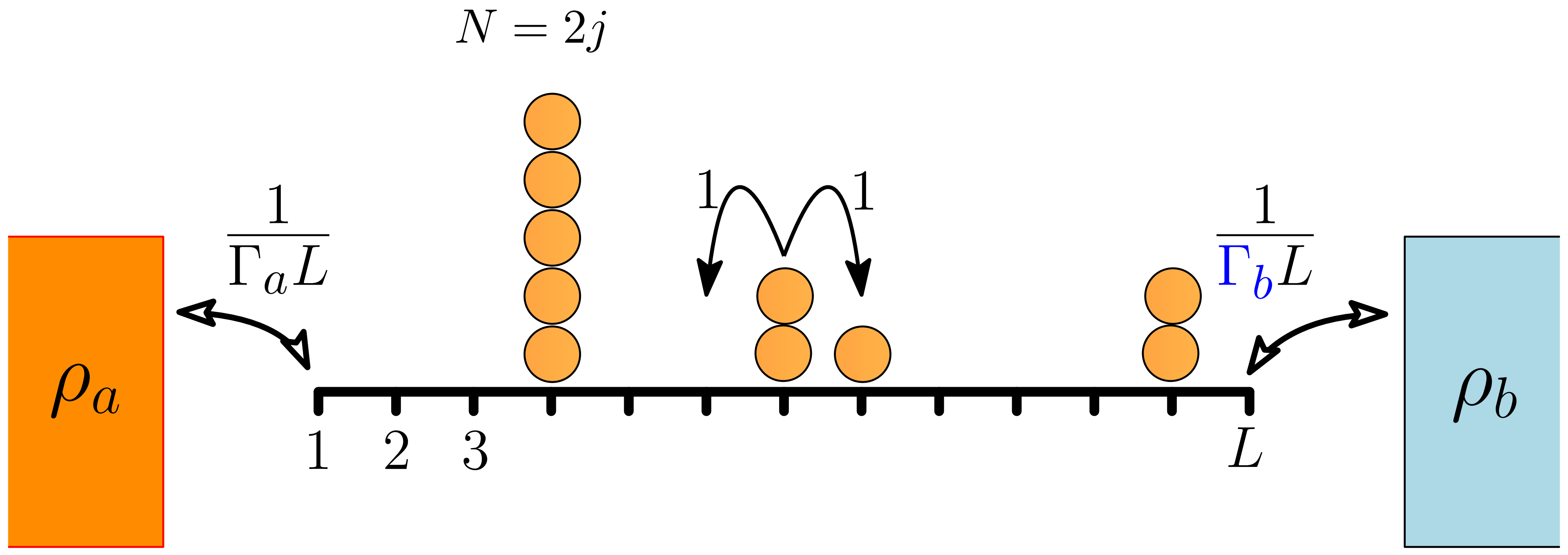
[Turkeshi, Calabrese, de Luca (PRL 2025)]

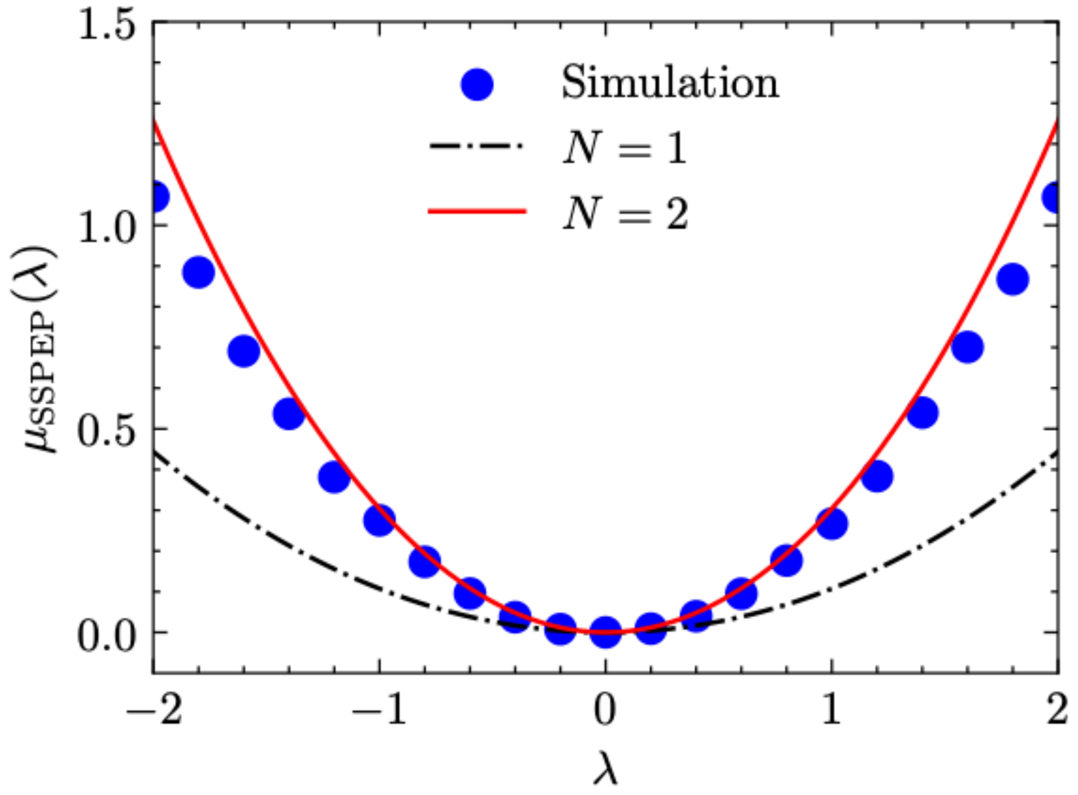
Evolution of Quantum coherence

[McCulloch, Jacoby, Gopalakrishnan (arXiv: 2604.27074)]

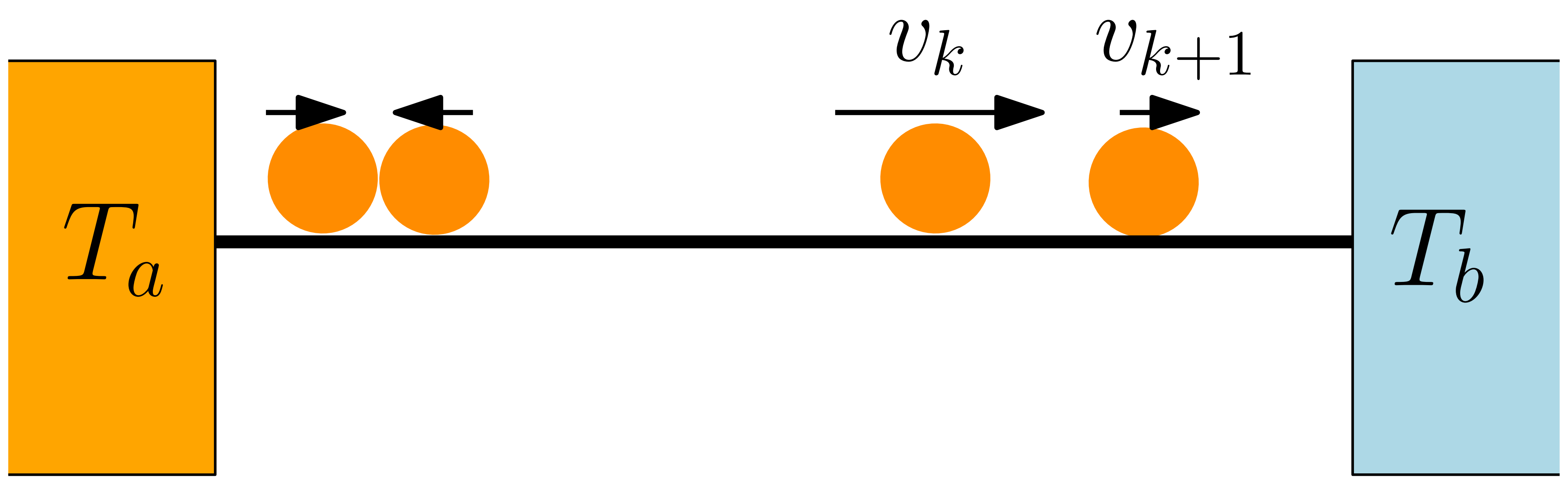
Non-Integrable dynamics

Simple symmetric Partial Exclusion Process





Kipnis Marchiro Pressutti model



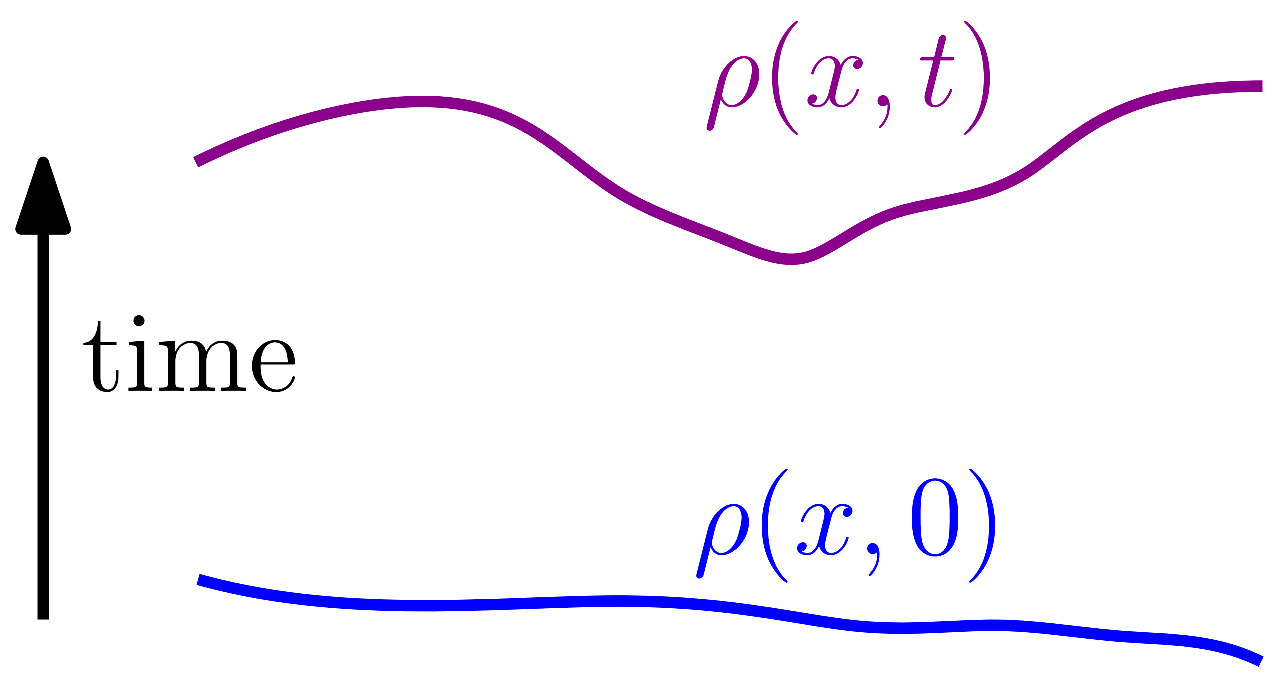
[Tailleur, Kurchan, Lecomte 2007]

[Frassek, Giardina, Kurchan 2020]

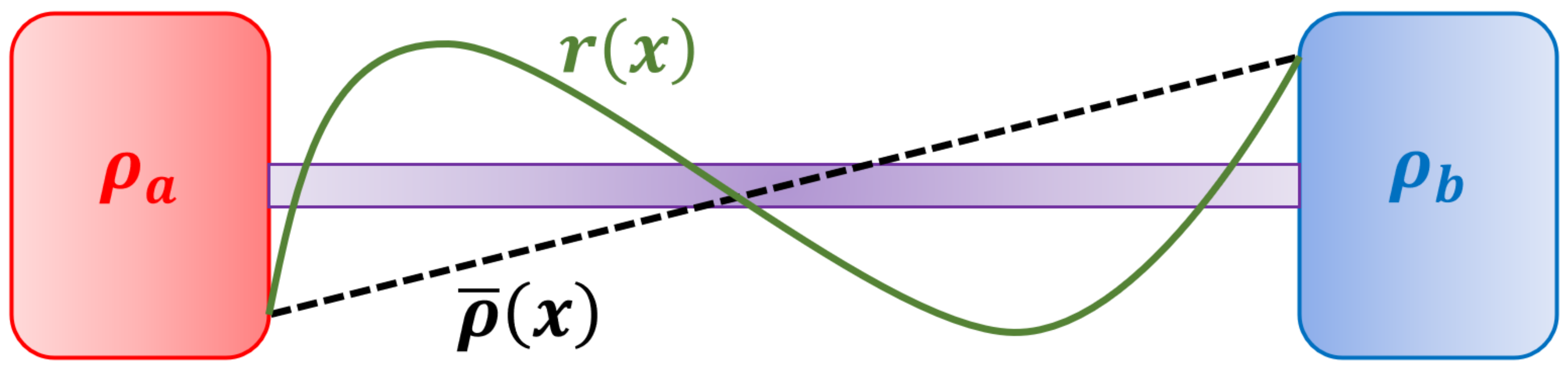
Integrability emerges in large scale!

Generic diffusive systems

Coarse-grained description at a large length ℓ and time ℓ^2



$$\partial_t \rho = \nabla \cdot \left(D(\rho)\nabla\rho\right) + \ell^{-\frac{d}{2}} \nabla \cdot \left(\sqrt{\sigma(\rho)}\,\xi\right)$$

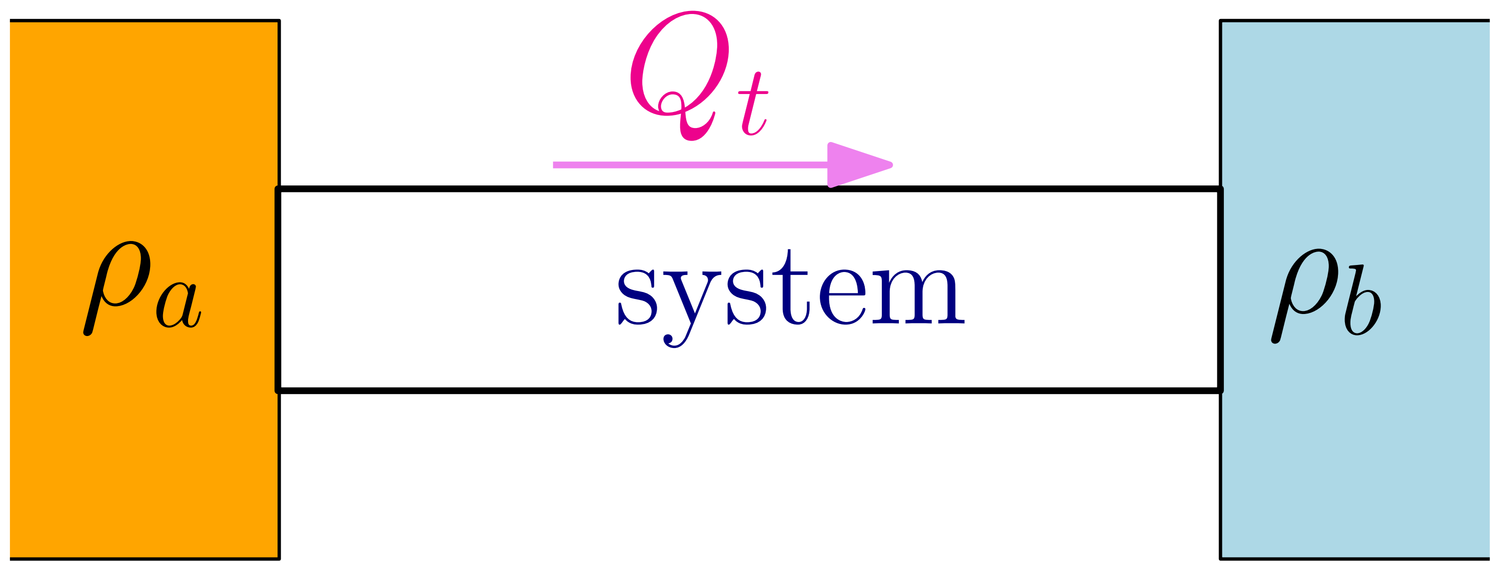


Model	$D(\rho)$	$\sigma(\rho)$
GL model	arbitrary	constant
ZRP	$g'(\rho)$	$2g(\rho)$
RAP	$\frac{\mu_1}{2}\rho^{-2}$	$\frac{\mu_1\mu_2}{\mu_1-\mu_2}\rho^{-1}$
SSPEP	N	$2\rho(N-\rho)$
KMP	1	$2\rho^2$
SSPIP	K	$2\rho(K+\rho)$
DPT model	1	$1+\rho^2$
SSMEP	$\frac{1}{(1-(M-1)\rho)^2}$	$\frac{2\rho(1-M\rho)}{1-(M-1)\rho}$
Brownian hard rods	$\frac{1}{(1-a\rho)^2}$	2ρ
Brownian gas	$\beta P'(\rho)$	2ρ

[Saha, TS (arXiv:2501.0316 (2025)]

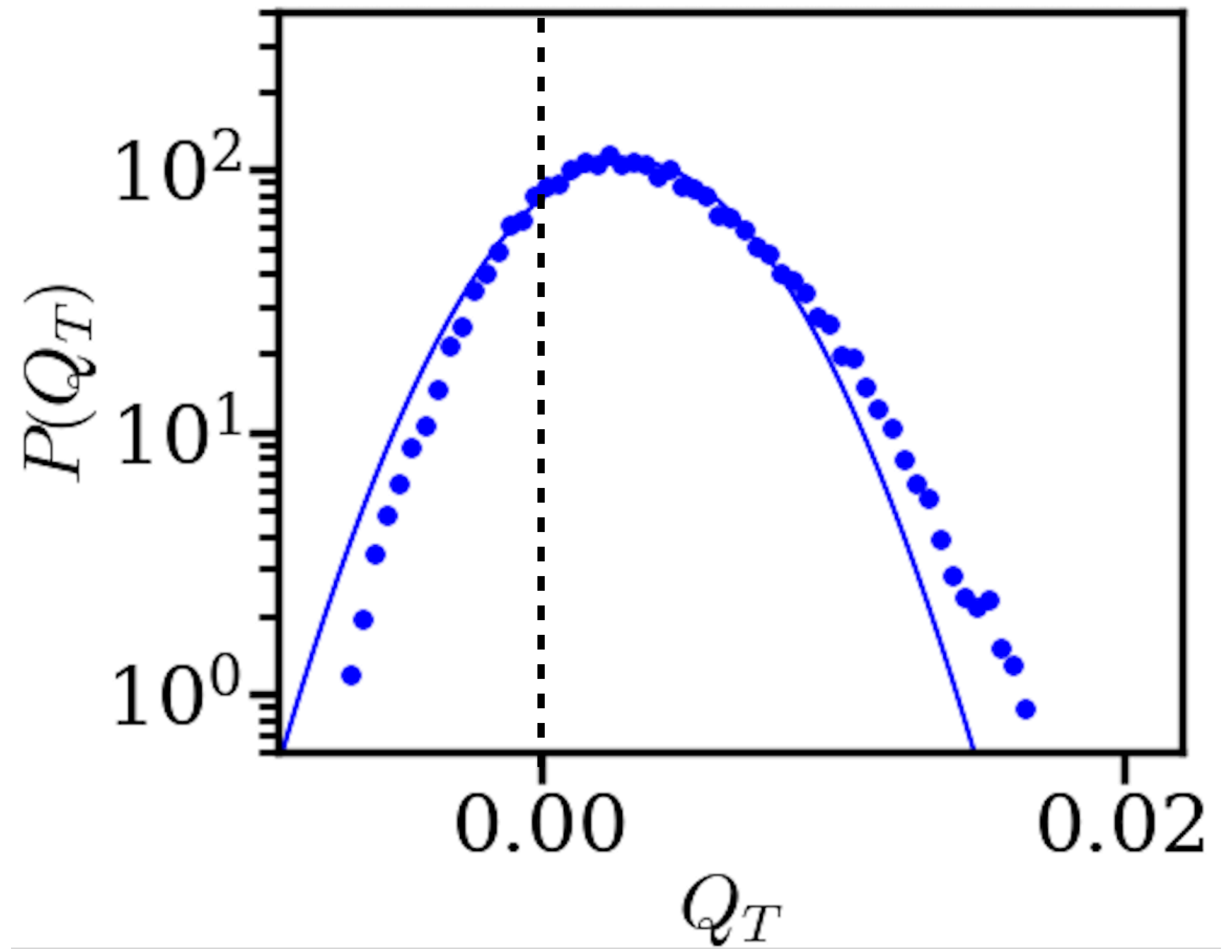
Infer generic properties

Fluctuation symmetry



$$\frac{P(-Q_t)}{P(Q_t)} \simeq e^{-c\,Q_t}$$

$$c = \int_{\rho_a}^{\rho_b} dx\, \frac{2D(x)}{\sigma(x)}$$



[Evans, Cohen, Morris, Searles' 1993, 1994] [Gallavotti, Cohen' 1995]
[Kurchan' 1998][Lebowitz, Spohn' 1998] [Derrida, Roche, Douchot 2004]

[Das, Ghosh, TS, Klamser, PNAS Nexus (2025)]

$$\langle e^{\lambda\,Q_t}\rangle \simeq e^{\sqrt{t}\,\chi(\lambda)}, \qquad \chi(\lambda) = \chi(-\lambda + c)$$

$$For \quad \langle e^{\int_0^T \lambda(t)\dot{Q}_t}\rangle \sim e^{\mu[\lambda(t)]}, \qquad \mu[\lambda(t)] = \mu[-\lambda(1-t) + c]$$

[Sharma, Chowdhury, Jangid, TS (upcoming)]

Simple consequences of Multi-time GCEMS

Consequence 1:

$$\langle Q_t \rangle = \left[\frac{\langle Q_t^2 \rangle_{eq}}{K_B T \rho^2 \kappa(\rho)} \right] (\rho_a - \rho_b)$$

$$\langle Q_{t_1} Q_{t_2} Q_{t_3} \rangle = \left[\frac{\sum_{perm} \langle Q_{t_i}^2 Q_{t_j} Q_{t_k} \rangle_{eq}}{K_B T \rho^2 \kappa(\rho)} \right] (\rho_a - \rho_b)$$

Consequence 2:

$$\langle Q(t_2) Q(t_1) \rangle_{eq} \propto t_1^{2H} + t_2^{2H} - (t_2 - t_1)^{2H}$$

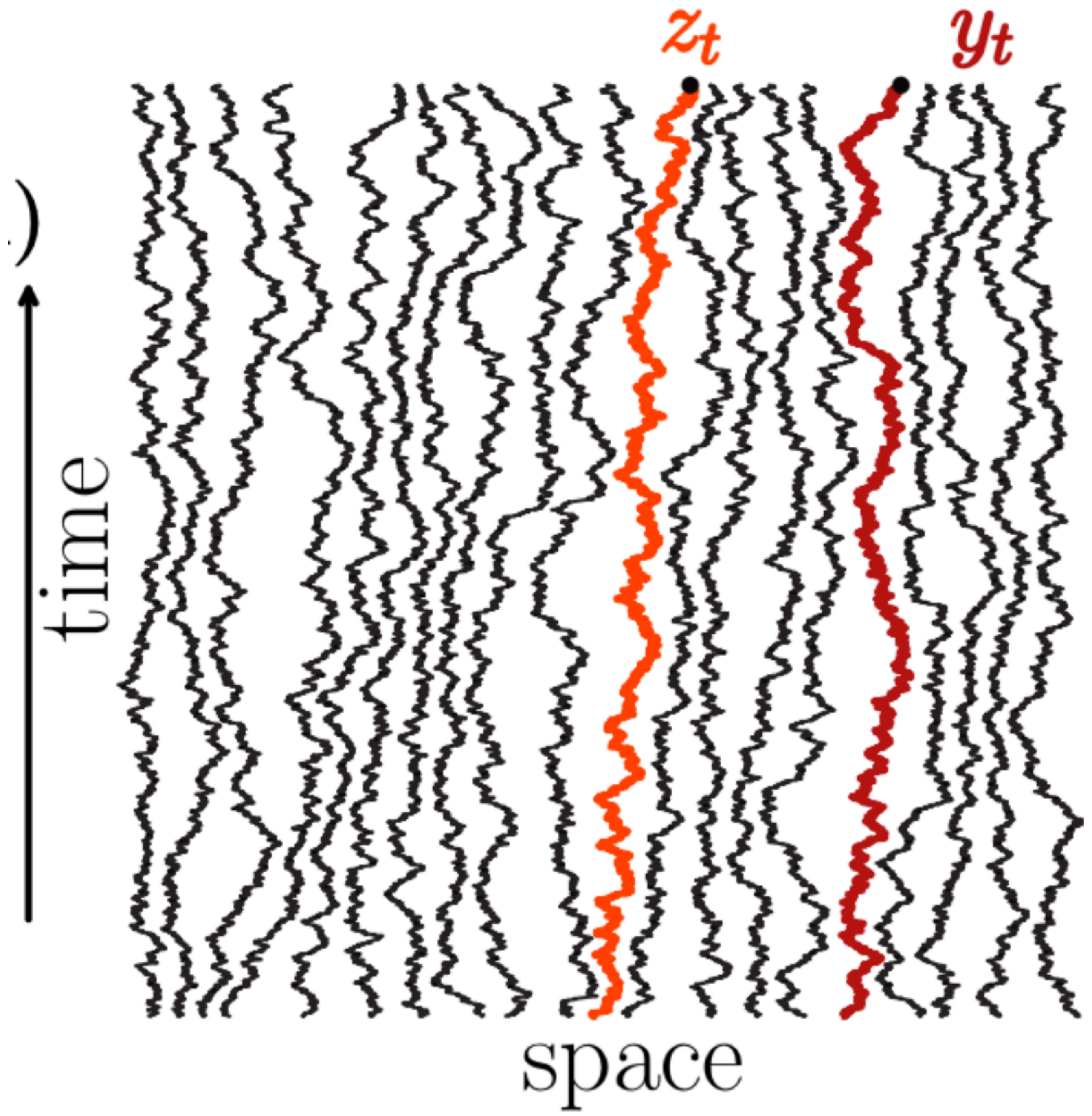
Fractional Brownian motion with Hurst exponent H

[Sharma, Chowdhury, Jangid, TS (upcoming)]

Another extreme:

Integrable Ballistic transport

Brownian hard rods



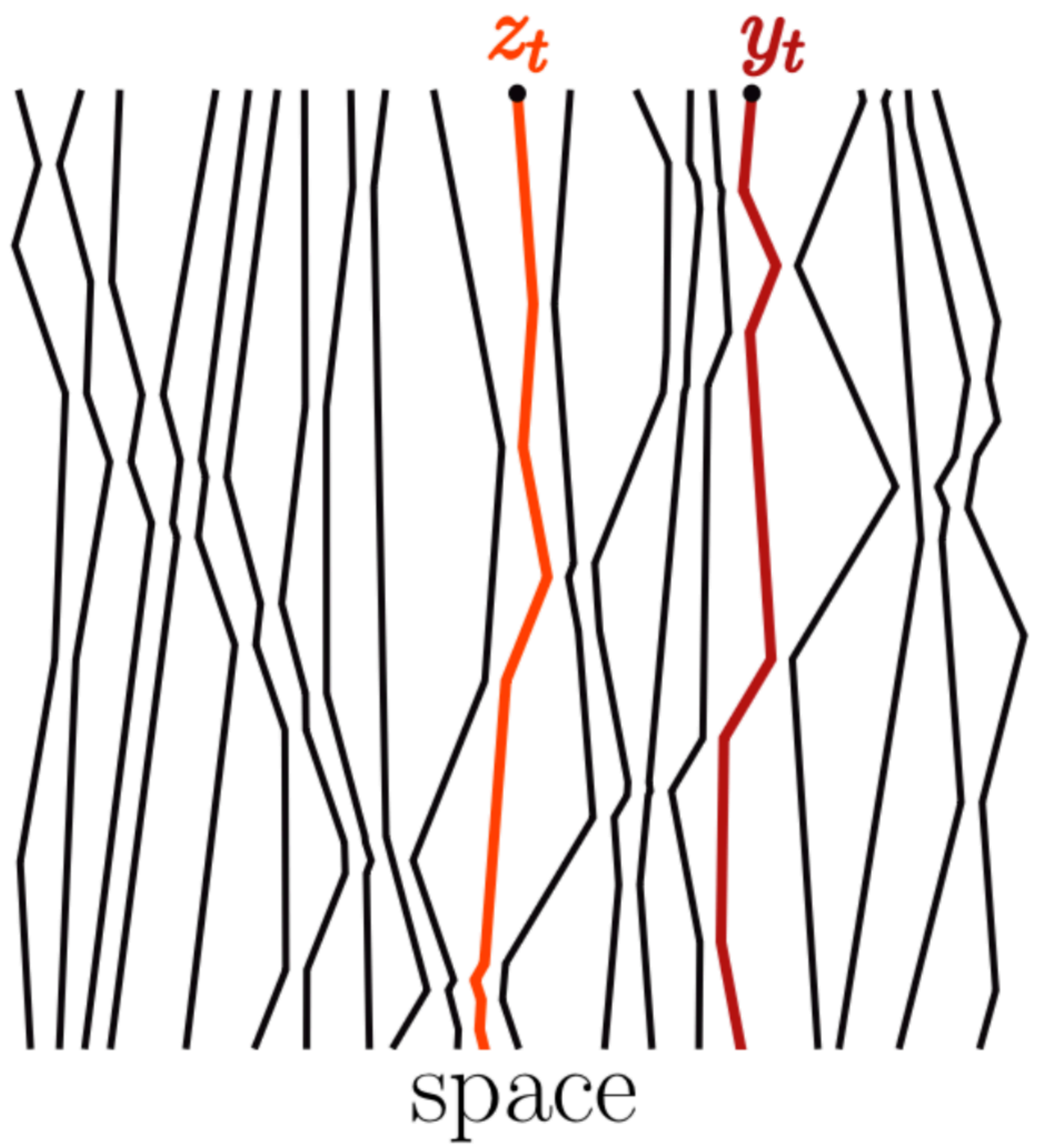
FHD

$$\partial_t \rho_t(x) = -\,\partial_x \left(-\frac{1}{(1-a\rho)^2}\partial_x \rho + \sqrt{\frac{2\rho}{\ell}}\,\xi \right)$$

[Grabsch, Ventruelli, Benichou (PRL 2025)]

[Saha, Jangid, Arnold de Pirey, Klamser, TS (arXiv:2601.02319)]

Ballistic hard rods



GHD

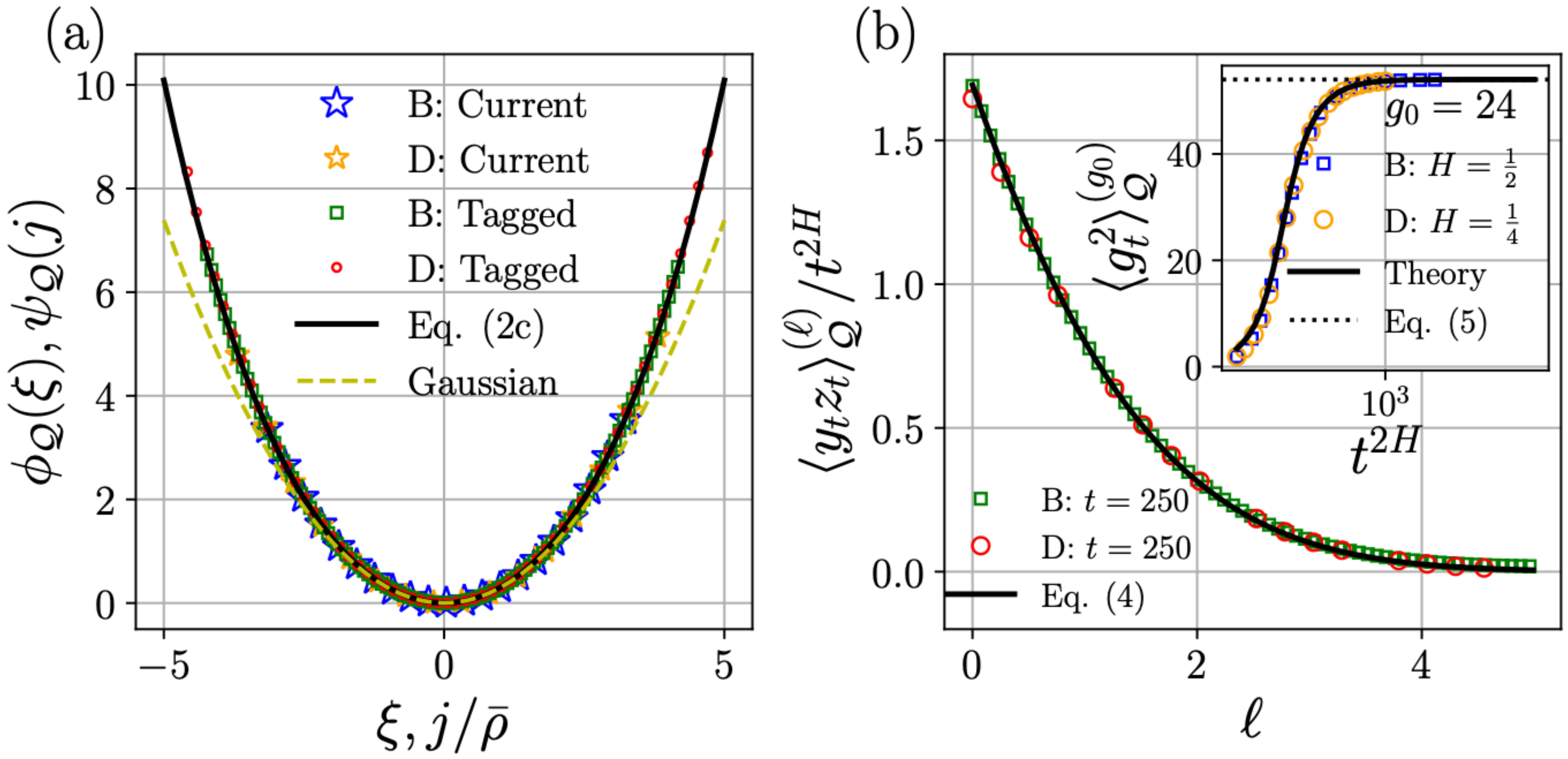
$$\partial_t \varrho_t(x,v) = -\,\partial_x \left(\frac{v - a\int \mathrm{d}v\, v\varrho_t(x,v)}{1 - a\int \mathrm{d}v\,\varrho_t(x,v)}\varrho_t + 0 \right)$$

[Bertini, Collura, de Nardis, Fagotti (PRL 2016)]

[Castro-Alvaredo, Doyon, Yoshimura (PRX 2016)]

Initial fluctuations propagate

$$\Pr\left(\frac{z_t}{t^{2H}} = \xi\right) \asymp e^{-t^{2H}\phi(\xi)} \qquad \text{with } H = \begin{cases} 1/4 & \text{diffusive} \\ 1/2 & \text{ballistic} \end{cases}$$



[Saha, Ketepalli, Guiselin, de Nardis, TS (arXiv:2604.24741)]

MFT

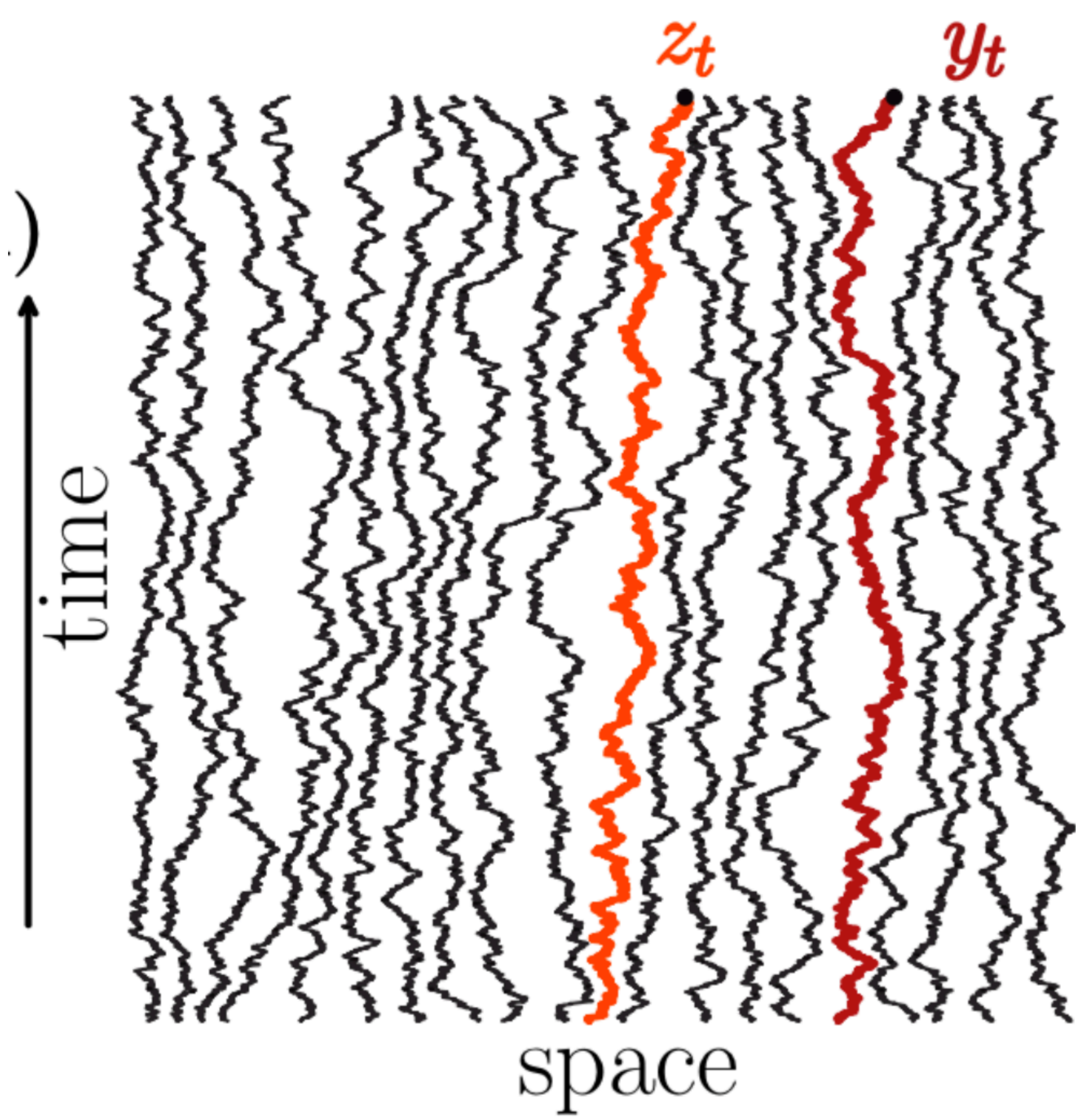
[Bertini, De Sole, Gabrielli, Jona-Lassinio, Landim (RMP, 2015)]

BMFT

[Doyon, Perfetto, Sasamoto, Yoshimura (Scipost Phys 2023)]

Is there analogous hydrodynamics possible for Active matter?

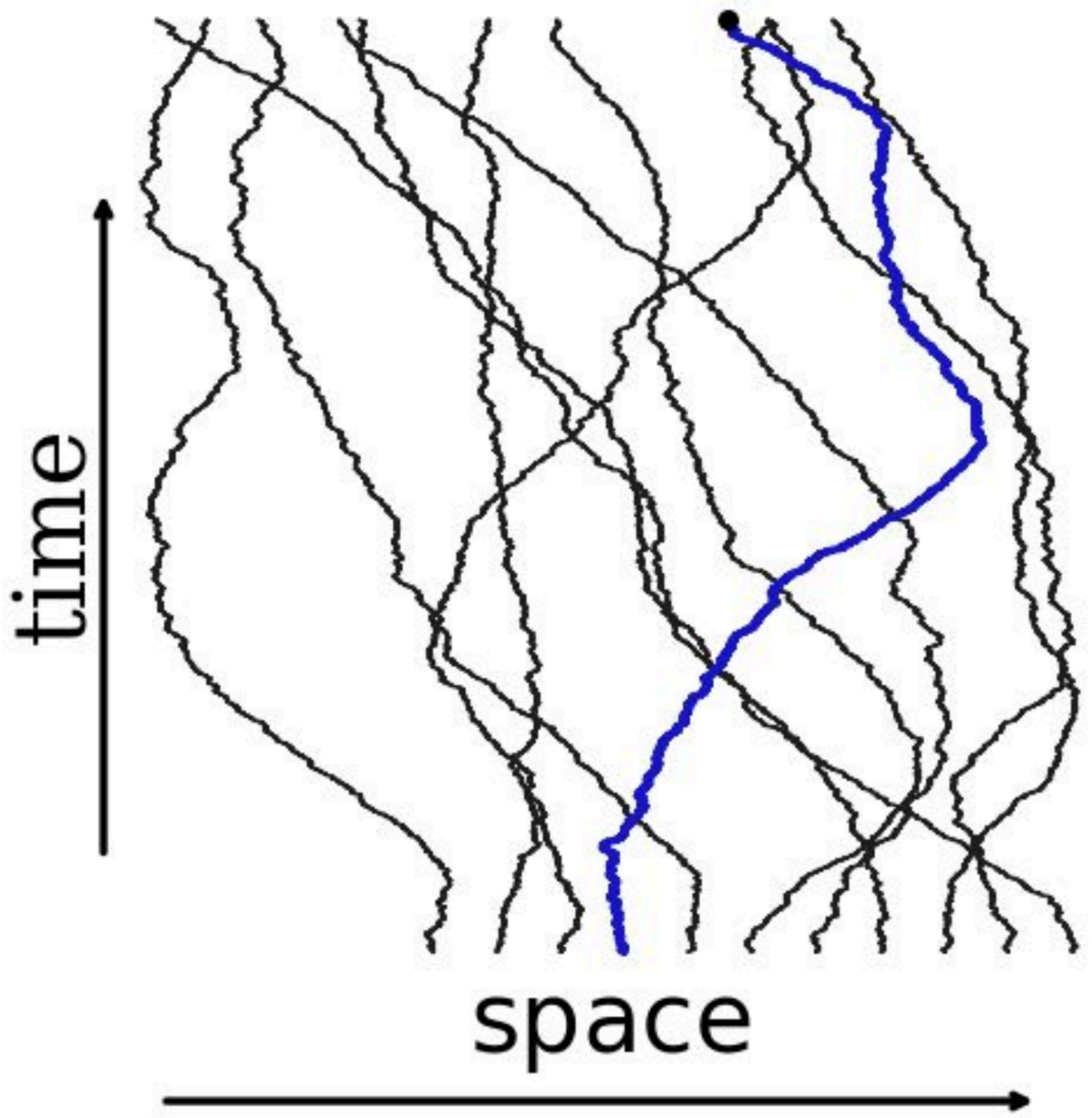
Diffusive



$$\partial_t \rho_t(x) = -\,\partial_x \left(J_{\text{euler}} + \text{noise} \right)$$

Initial randomness

Active

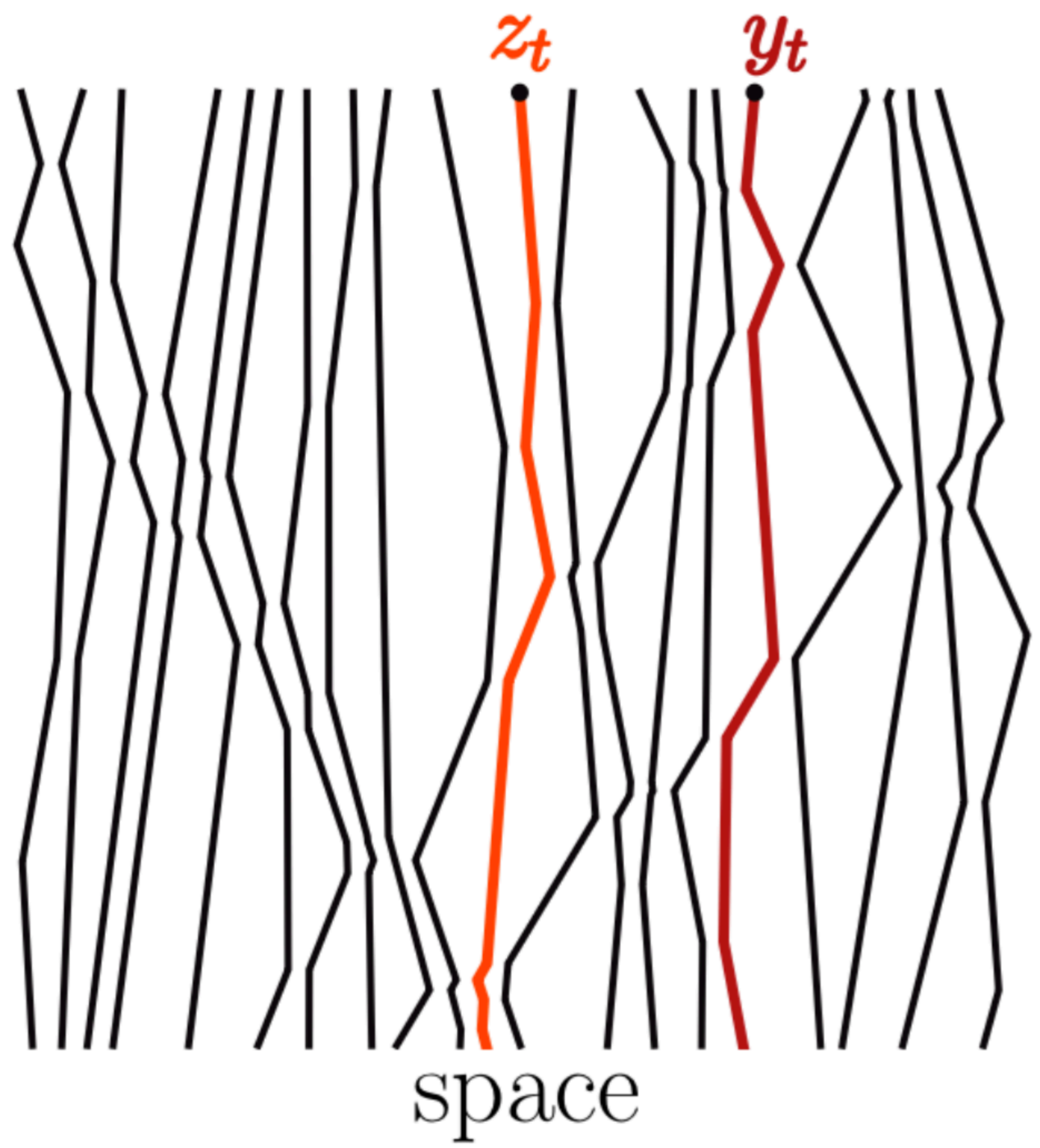


$$\partial_t \rho_t(x) = -\partial_x \left(J(\rho, m) + \text{noise} \right)$$

$$\partial_t m_t(x) = I(m, \rho) + \text{noise}$$

Initial randomness

Ballistic



$$\partial_t \varrho_t(x, v) = -\,\partial_x \left(J_{\text{euler}} + 0 \right)$$

Initial randomness

Active hydrodynamics

$$\partial_t \rho_t(x) = -\partial_x \left(J(\rho, m) + \text{noise} \right)$$
$$\partial_t m_t(x) = I(m, \rho) + \text{noise}$$

[Toner, Tu PRL 75, 4326 (1995)]

[Bertin, Droz, Grégoire, JPA 42, 445001 (2009)]

[Barré, Chetrité, Muratori, Peruani JSP 158, 589 (2015)]

[Partridge, Lee, PRL, 123, 068002 (2019)]

[Supekar, Song, Hastewell, Dunkel, PNAS 120 (2023)]

[Dinelli, O'Byrne, Tailleur, J Phys A 57, 395002 (2024)]

[Bebon, Robinson, Speck, PRX 15, 021050 (2025)]

[Burekovic, Luca, Cates, Nardini, arXiv:2601.16539 (2026)]

No proper Coarse-graining!

No multiplicative noise!

What should work?

- Work at the phase-space description, like in GHD.

Dean-Kawasaki equation.

[Kawasaki, Physica A, 208, 35 (1994)]
[Dean, J Phys A, 29, L613 (1996)]
[Illien, Rep Prog Phys, 88, 086681 (2025)]

- Do systematic coarse-graining.

Coarse-grain Dean-Kawasaki equation.

Coarse-graining passive Dean-Kawasaki

Brownian hard rods

$$\dot{X}_i(\tau) = -D_0\beta \sum_{j\neq i} \partial_{X_i} V\left(X_i(\tau) - X_j(\tau)\right) + \sqrt{2D_0}\eta_i(\tau)$$

$$\psi(X,\tau) = \sum_i \delta(X - X_i(\tau))$$

$$\partial_\tau \psi(X,\tau) = D_0\partial_X^2\psi(X,\tau) + D_0\beta\partial_X\left(\psi(X,\tau)\int_{-\infty}^{\infty} \mathrm{d}Y \partial_X V(X-Y)\psi(Y,\tau)\right) + \partial_X\sqrt{2D_0\psi(X,\tau)}\xi(X,\tau)$$

$$\psi(X,\tau) \sim \rho\left(\frac{X}{\ell}, \frac{\tau}{D_0\ell^2}\right)$$

[Saha, Jangid, arnoux de Pirey, Klamser, TS (arXiv:2601.02319)]

$$\partial_t\rho = \partial_X\left(\frac{1}{(1-a\rho)^2}\partial_x\rho\right) + \partial_x\left(\sqrt{\frac{2\rho}{\ell}}\,\xi\right) \longleftrightarrow \partial_t\rho = \partial_x\left(\beta P'(\rho)\partial_x\rho\right) + \partial_x\left(\sqrt{\frac{2\rho}{\ell}}\,\eta\right)$$

generic potential

Can we do this for active dynamics?

Non-Interacting Active matter models

Coarse-graining: $\psi_T(X, V) \sim \varrho_{\frac{T}{\tau}}\left(\frac{X}{\ell}, \frac{V}{v_0}\right)$

ℓ is diffusive length
τ is persistent time

Hydrodynamics: $\partial_t \varrho_t(x, v) = -\nabla_{x,v} \cdot \mathbf{J}[\varrho_t] + \frac{1}{\ell^{d/2}} \nabla_{x,v} \cdot \chi_t(x, v)$

1d AOUP

$$\dot{X}_i = V_i + \sqrt{2D}\,\eta_i$$
$$\tau \dot{V}_i = -V_i + \sqrt{2\mathrm{D_v}}\eta_i$$

with $\ell = \sqrt{D\tau}$ and $v_0 = \sqrt{\mathrm{D_v}/\tau}$

$$\partial_t \varrho_t(x, v) = \partial_x^2 \varrho - \mathrm{Pe}\, v\, \partial_x \varrho + \partial_v^2 \varrho + \partial_v(v\varrho)$$
$$+\ell^{-1/2}\, \partial_x\left(\sqrt{2\varrho}\,\eta\right)$$
$$+\ell^{-1/2}\, \partial_v\left(\sqrt{2\varrho}\,\xi\right)$$

2d ABP

$$\dot{X}_i = v_0(\cos\theta_i, \sin\theta_i) + \sqrt{2D}\,\eta_i$$
$$\dot{\theta}_i = \sqrt{\frac{2}{\tau}}\,\xi_i$$

with $\ell = \sqrt{D\tau}$

$$\partial_t \varrho_t(x, \theta) = \Delta\varrho - \mathrm{Pe}\ (\cos\theta, \sin\theta) \cdot \nabla\varrho + \partial_\theta^2 \varrho$$
$$+\ell^{-1}\nabla \cdot \left(\sqrt{2\varrho}\,\vec{\eta}\right)$$
$$+\ell^{-1}\partial_\theta\left(\sqrt{2\varrho}\,\xi\right)$$

1d RTP

$$\dot{X}_i = v_0\,\sigma_i + \sqrt{2D}\,\eta_i$$

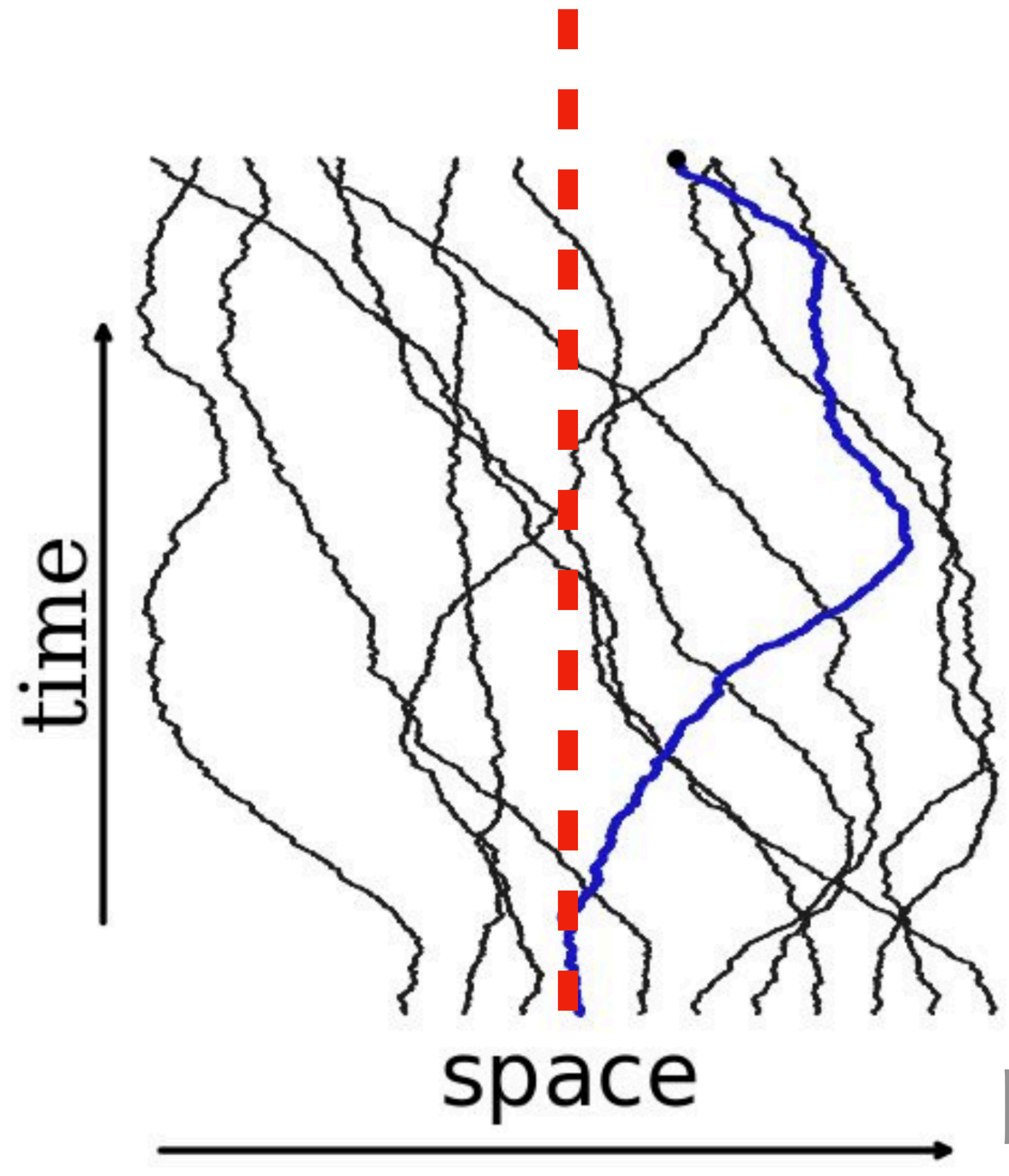
σ_i flips sign at rate $\frac{1}{\tau}$

with $\ell = \sqrt{D\tau}$

$$\partial_t \varrho_t(x, \sigma) = \partial_x^2 \varrho - \mathrm{Pe}\,\sigma\,\partial_x \varrho + (\varrho(x, -\sigma) - \varrho(x, \sigma))$$
$$+\ell^{-1/2}\,\partial_x\left(\sqrt{2\varrho}\,\eta\right)$$
$$+\frac{1}{\ell_d}\left(\xi_t(x, -\sigma) - \xi_t(x, \sigma)\right)$$

Does it live up to the promise?

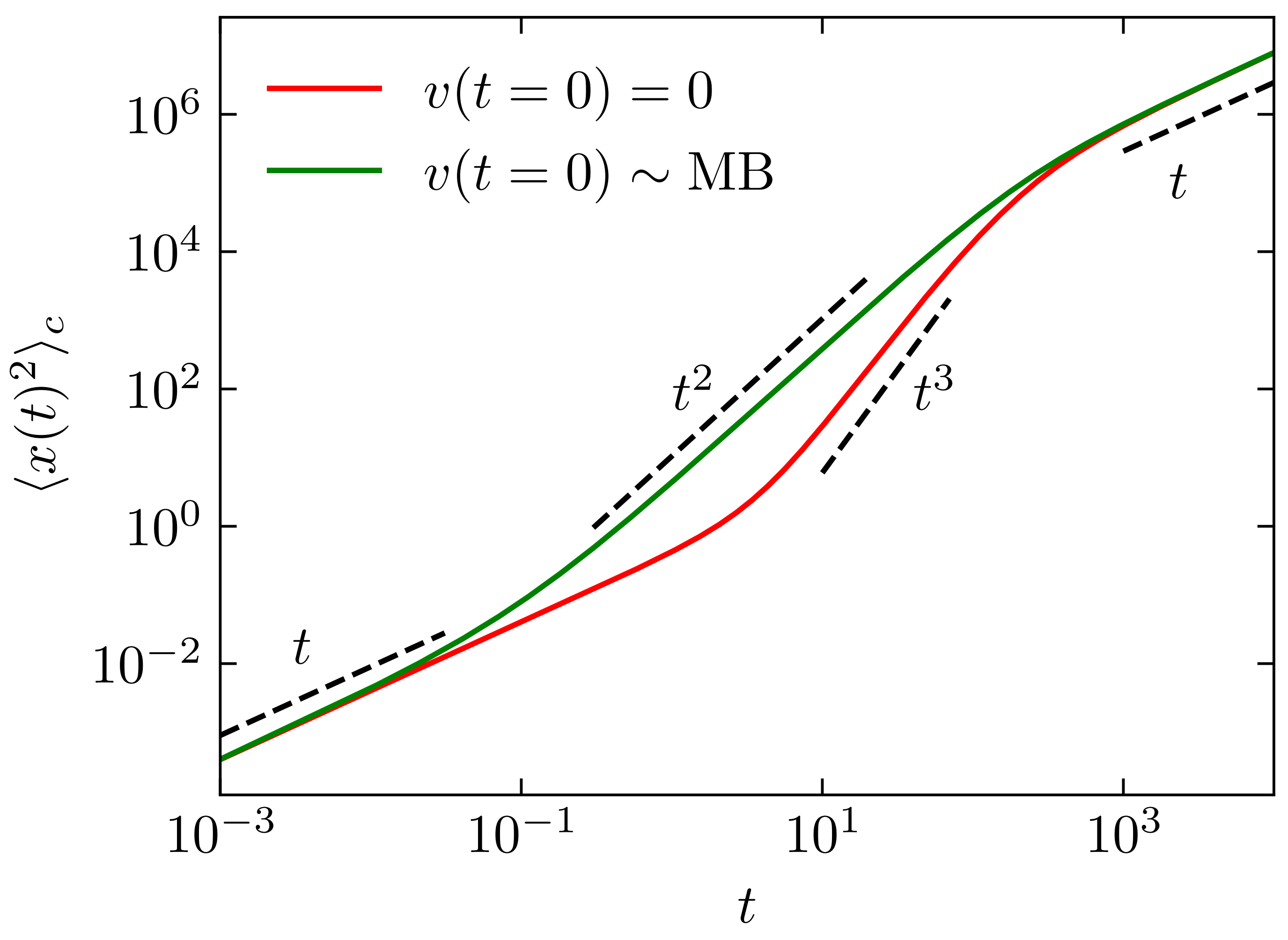
Current LDF in non-interacting AOUP



$$P(Q_t) \sim e^{-\sigma_t \, \phi\left(\frac{Q_t}{\sigma_t}\right)}$$

[Jangid, Kumbhakar, Klamser, TS (upcoming)]

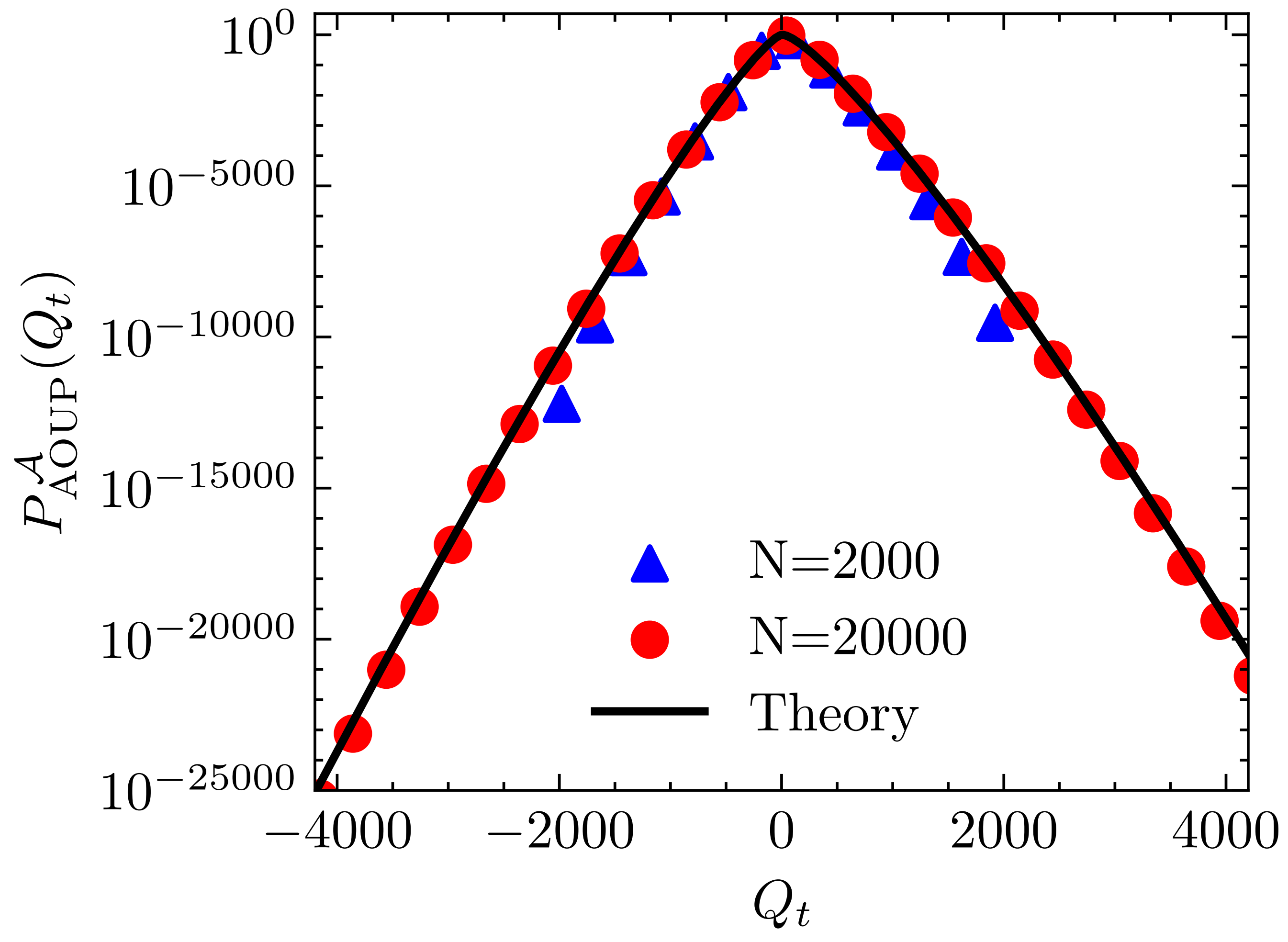
[Sharma, Jangid, Pattanayak, Klamser, Pirey, TS (upcoming)]

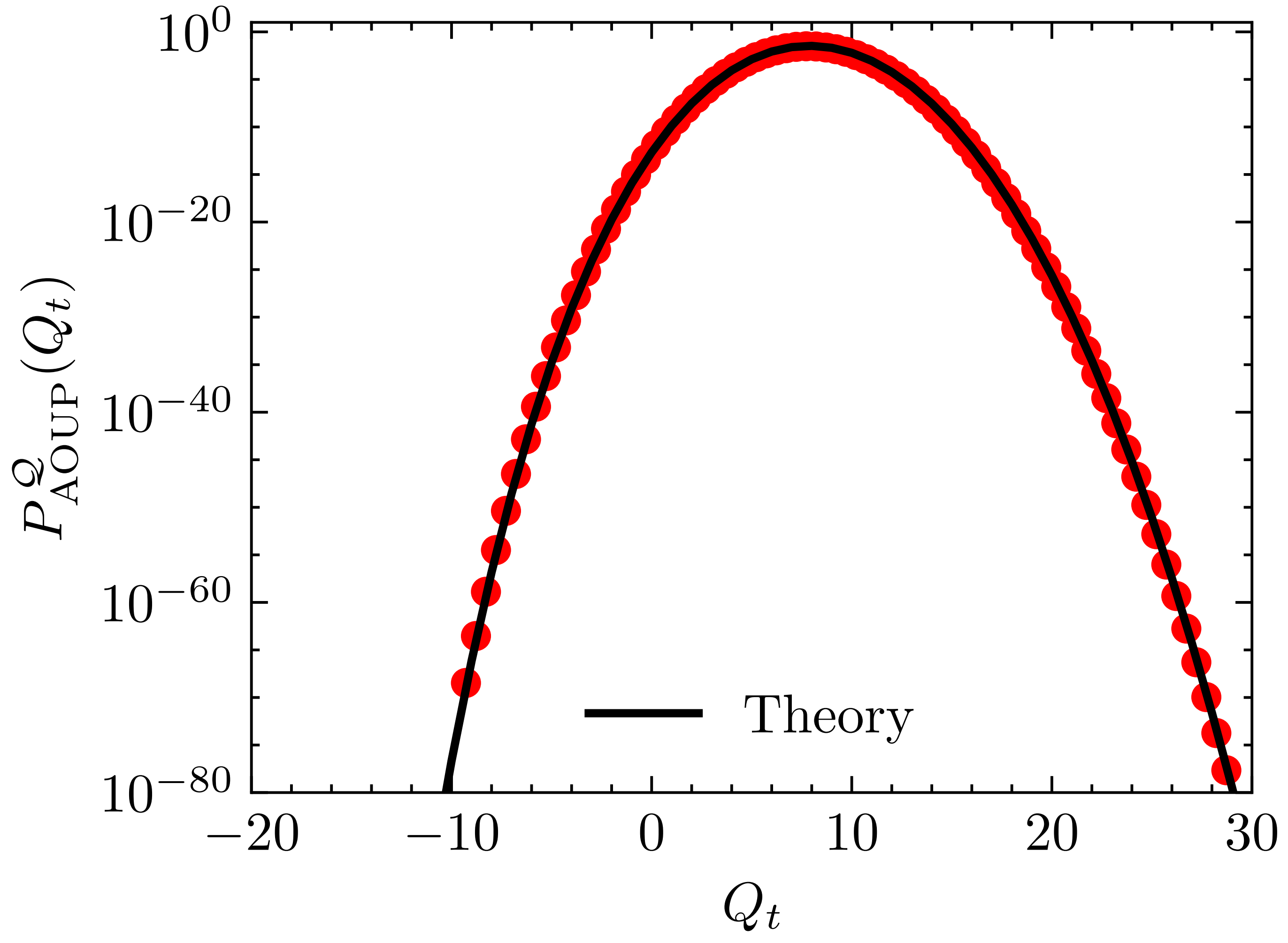


Annealed vs Quenched

$$\mu_{\mathscr{A}}(\lambda) = \ln \overline{\langle e^{\lambda Q_t} \rangle}$$

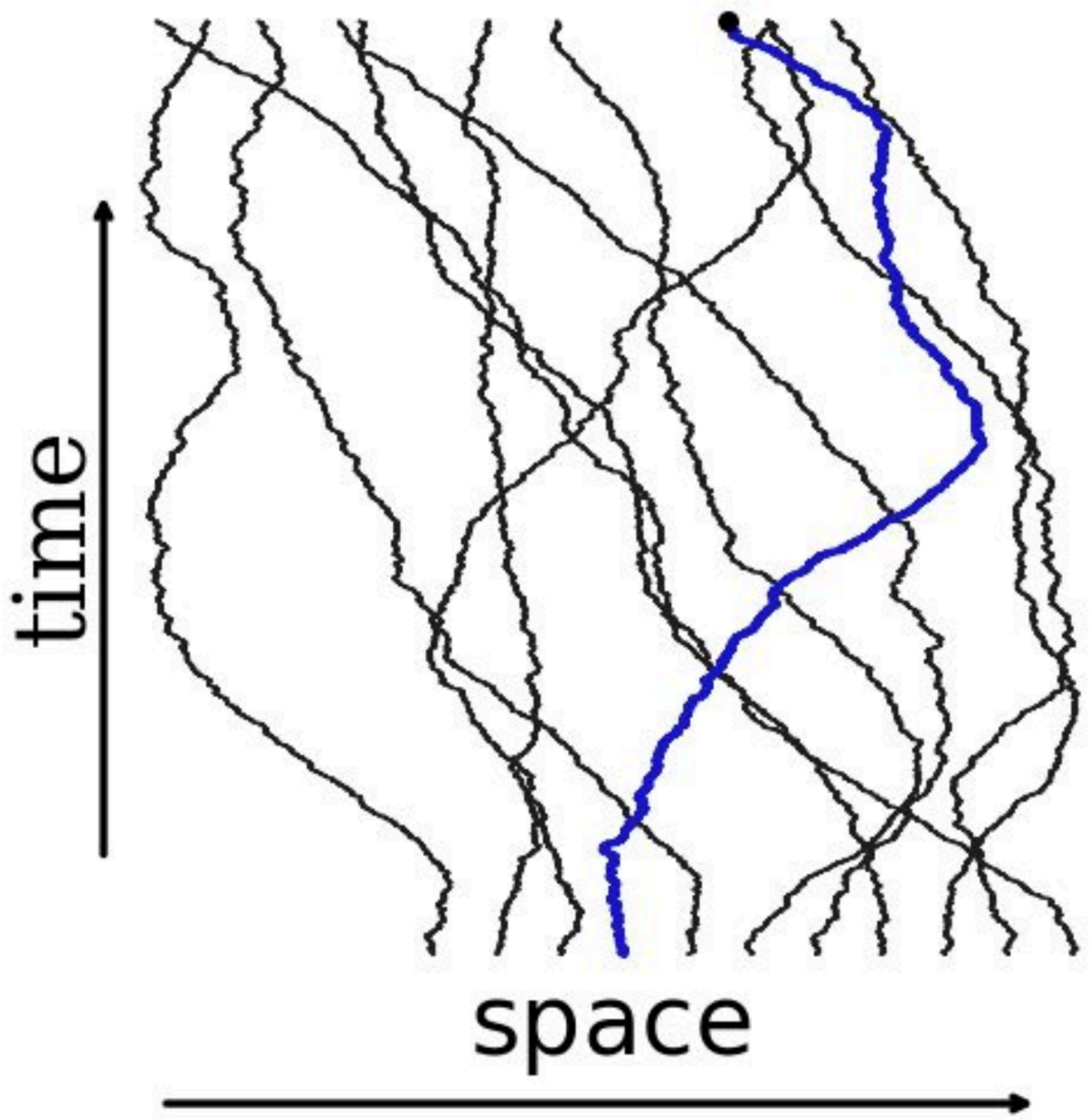
$$\mu_{\mathscr{Q}}(\lambda) = \overline{\ln \langle e^{\lambda Q_t} \rangle}$$





Current LDF in non-interacting RTP

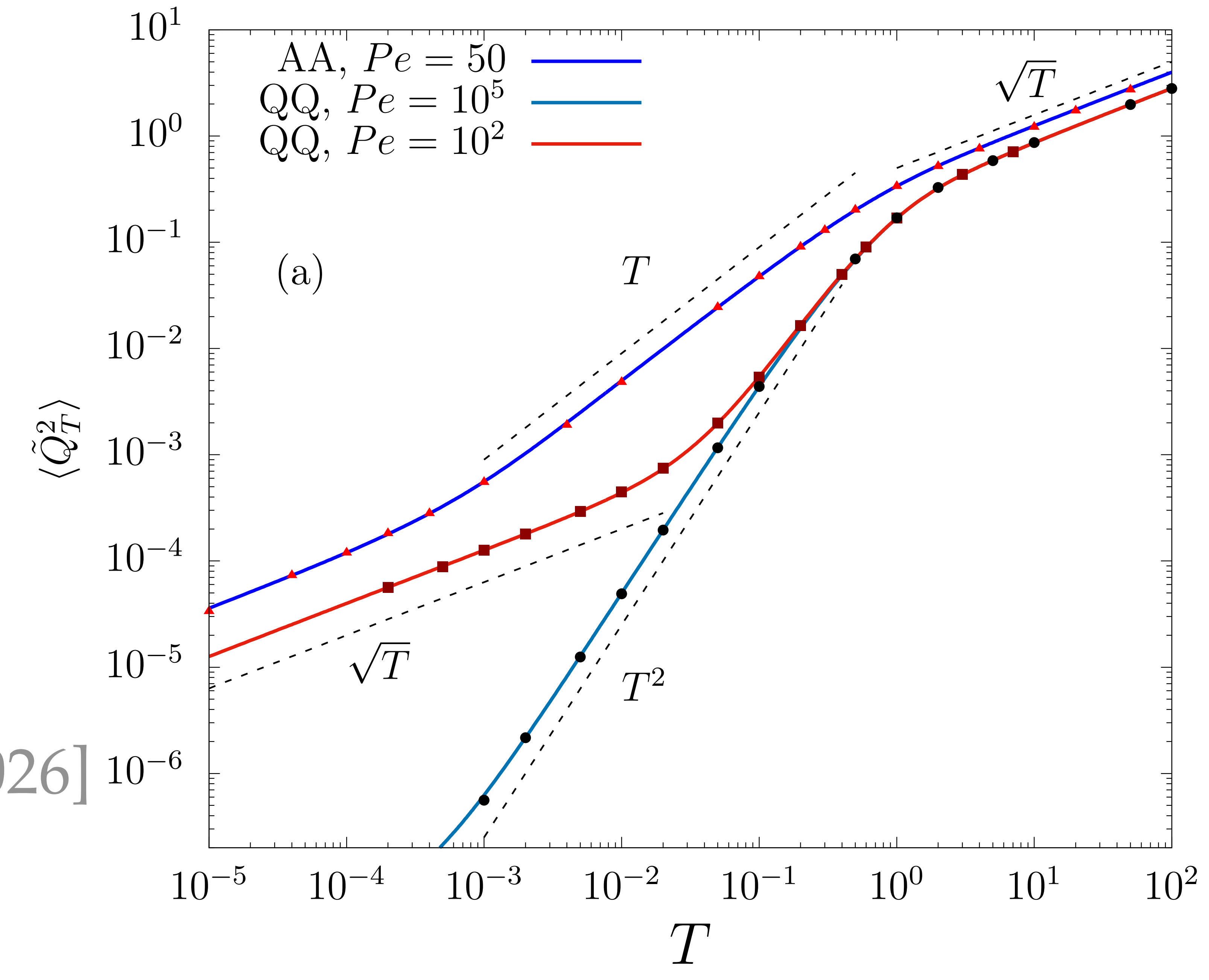
[Banerjee, Majumdar, Rosso, Schehr 2020]
[Jose, Rosso, Ramola 2023]

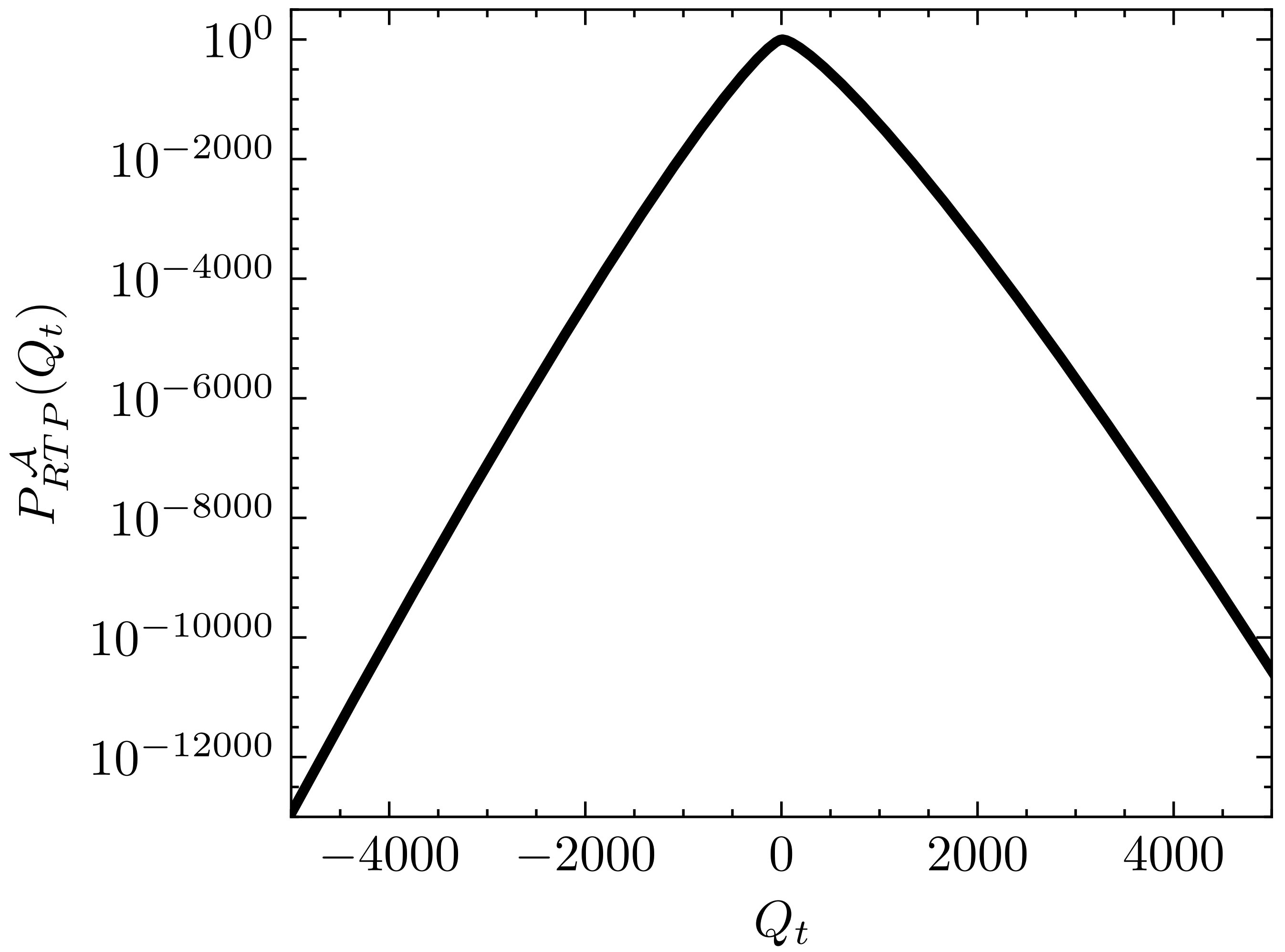


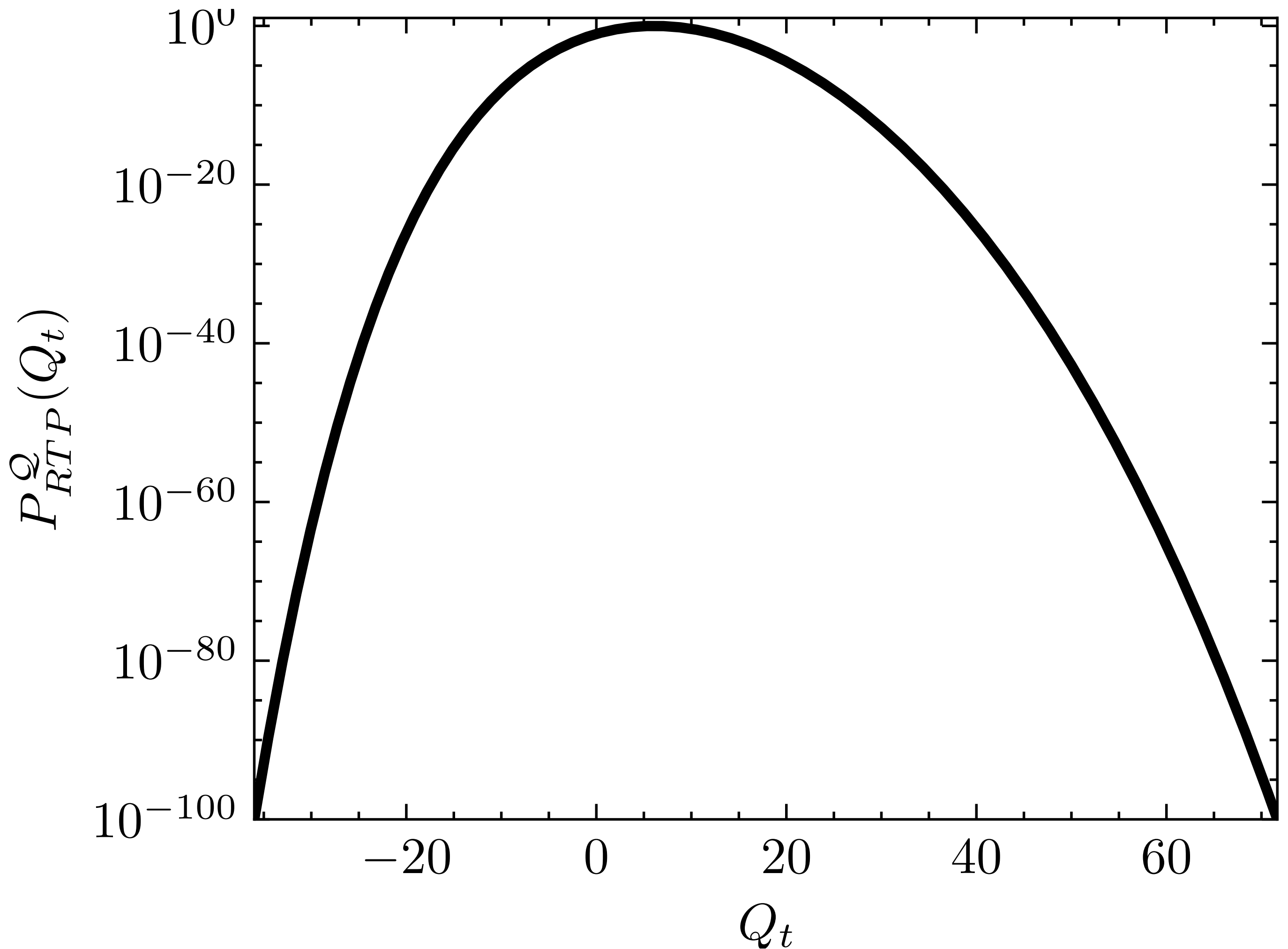
$$P(Q_t) \sim e^{-\sigma_t\,\phi\left(\frac{Q_t}{\sigma_t}\right)}$$

[Jangid, Kumbhakar, Klamser, TS 2026]

[Sharma, Jangid, Pattanayak, Klamser, Pirey, TS 2026]



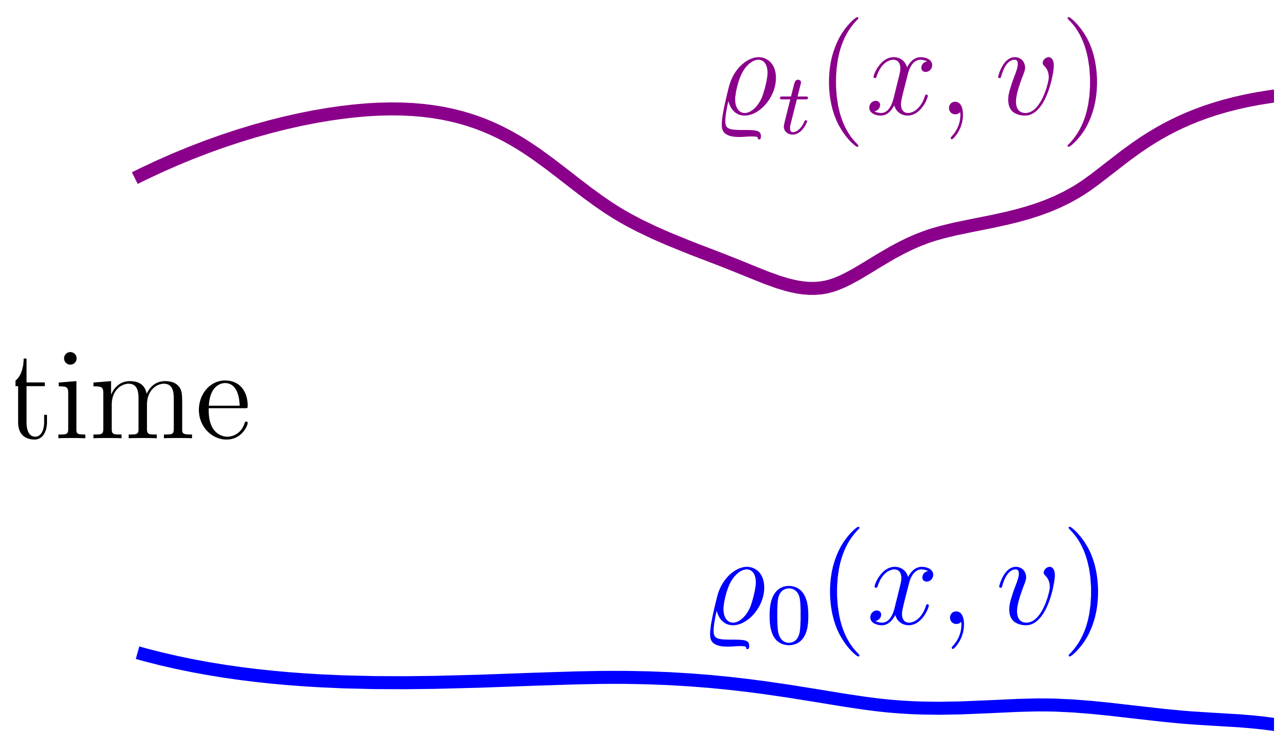




How was the computation done?

Active version of MFT / BMFT

$$\partial_t \varrho_t(x,v) = -\nabla_{x,v}\cdot \mathbf{J}[\varrho_t] + \frac{1}{\ell^{d/2}}\nabla_{x,v}\cdot \chi_t(x,v)$$



$$\mathscr{P}[\varrho_{\rm in}\,|\,\varrho_{\rm fi}] \sim \int_{\varrho_{\rm in}}^{\varrho_{\rm fi}} \mathscr{D}[\varrho,\hat{\varrho}]\mathrm{e}^{-\ell^d S[\varrho,\hat{\varrho}]}$$

$$\langle e^{\lambda Q}\rangle \sim \int_{\varrho_{\rm in}}^{\varrho_{\rm fi}} \mathscr{D}[\varrho,\hat{\varrho}]\mathrm{e}^{-\ell^d\{S[\varrho,\hat{\varrho}]-\lambda Q/\ell^d\}} \sim e^{-\ell^d \min\{S[\varrho,\hat{\varrho}]-\lambda Q/\ell^d\}}$$

[Doyon, Perfetto, Sasamoto, Yoshimura (Scipost Phys 2023)]

[Kethepalli, Urilyon, TS, de Nardis (Scipost Phys 2026)]

What about interacting particle?

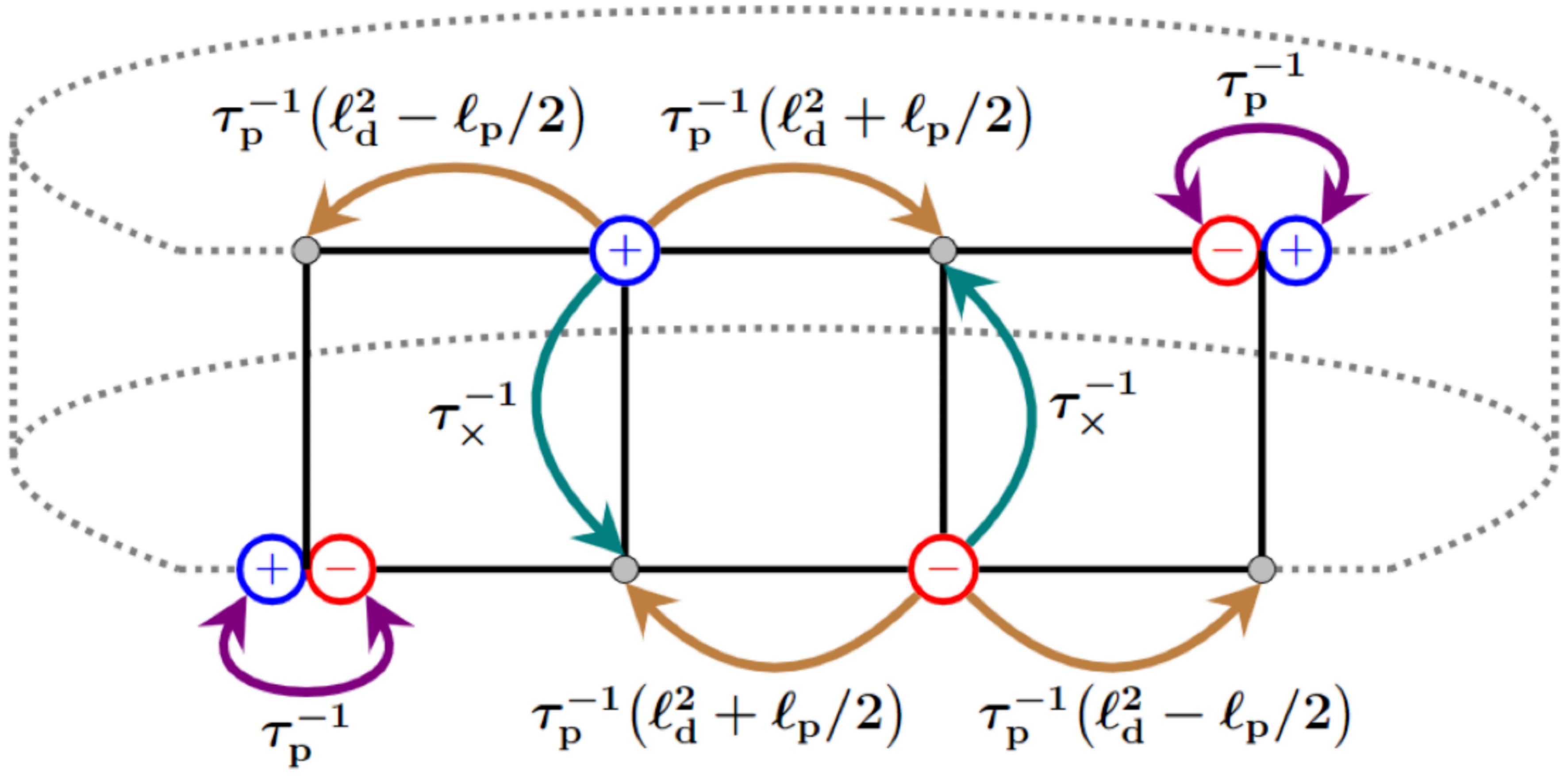
GHD of interacting ABP in 2d

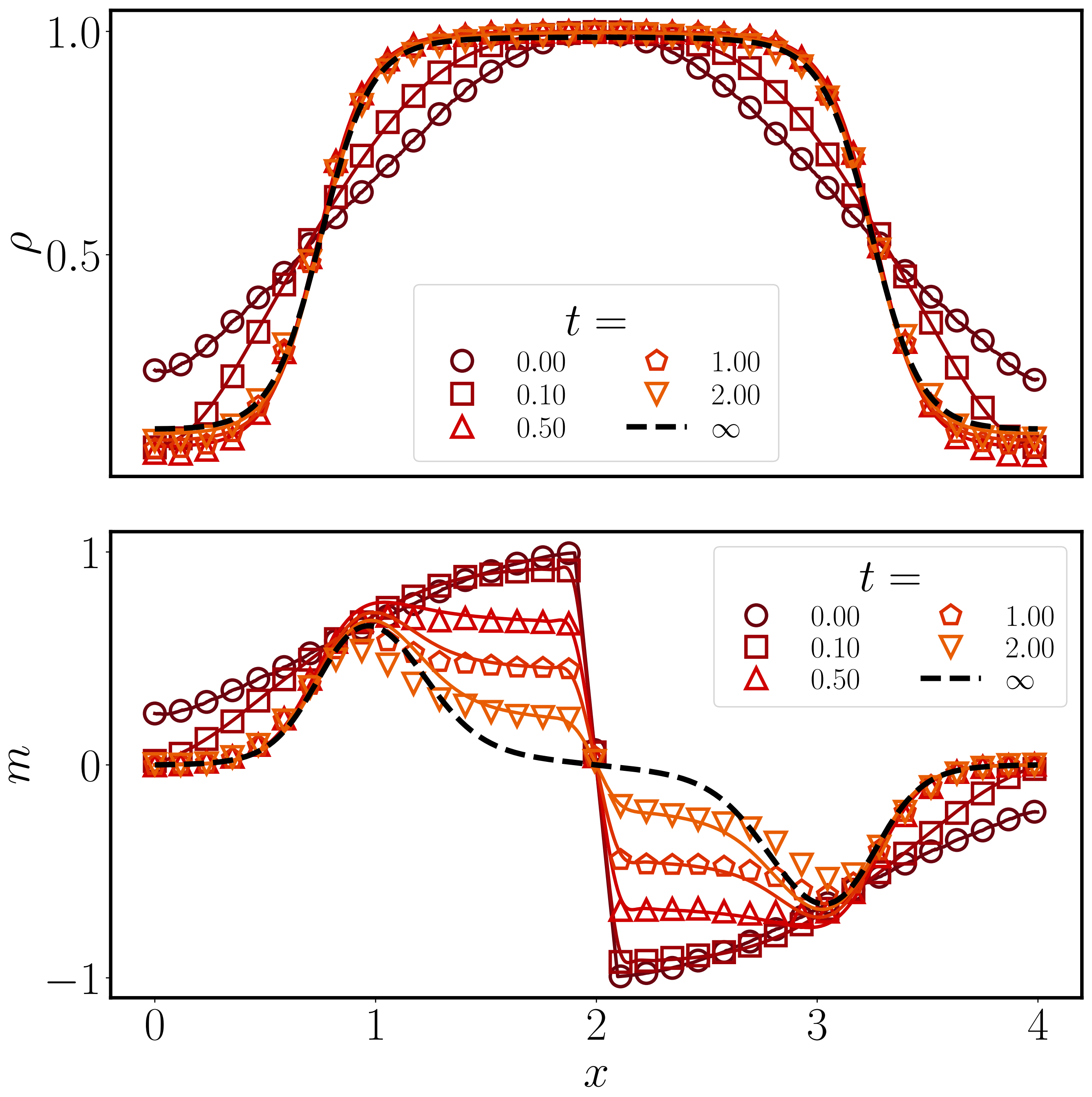
$$\partial_t \varrho_t(x,\theta) = -\nabla \cdot \Big[\mathrm{Pe}\, u\, \varrho_t + \text{[redacted]} \Big] + \partial_\theta^2 \varrho_t + \ell_d^{-d/2} \nabla \cdot \left(\sqrt{2\varrho_t}\, \hat{\eta}\right) + \ell_d^{-d/2} \partial_\theta \left(\sqrt{2\varrho_t}\, \hat{\xi}\right)$$

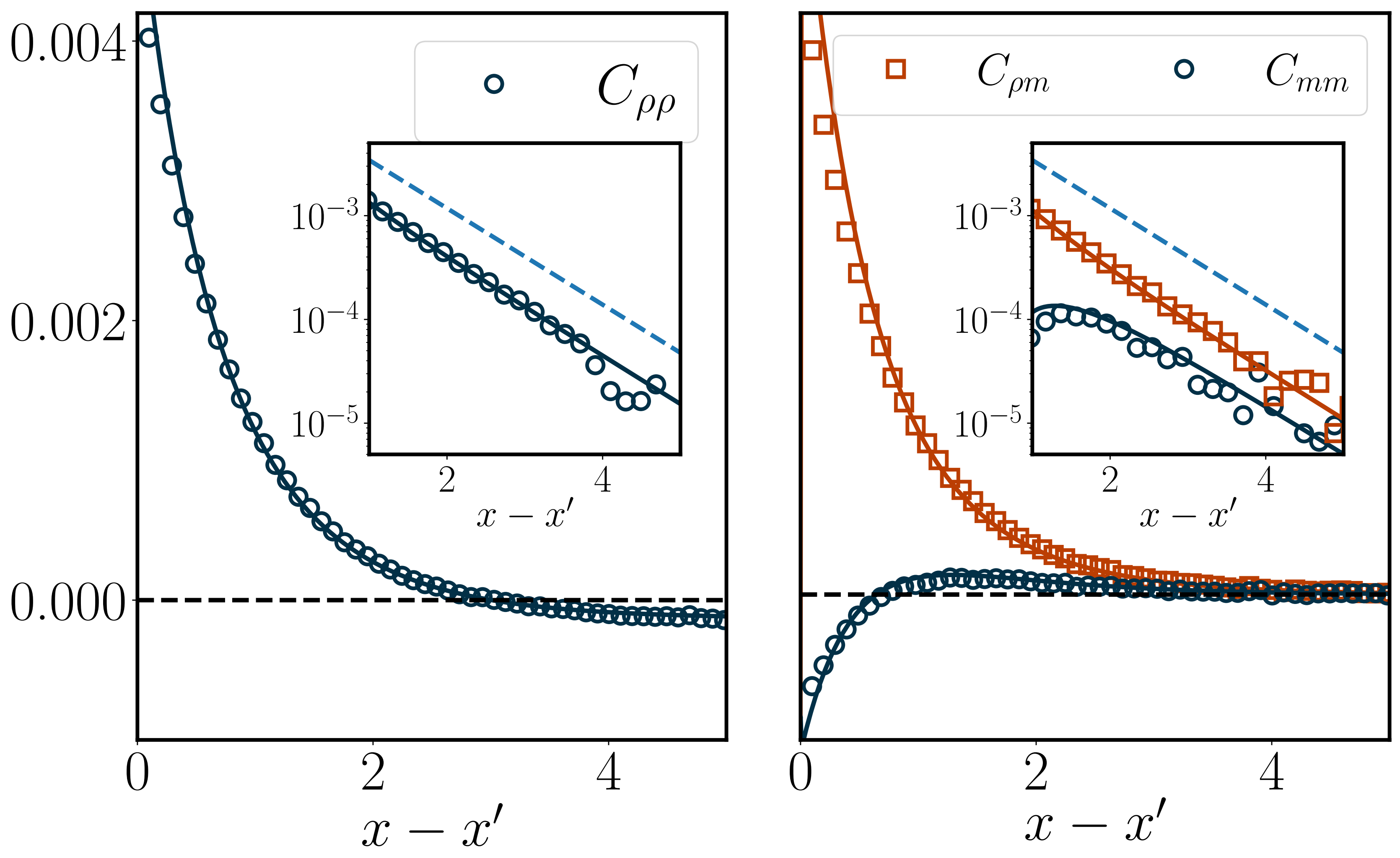
The next best option

Interacting Active matter on lattice

[Mukherjee, Saha, TS, Dhar, Sabhapandit, PRE 111,024128 (2025)]



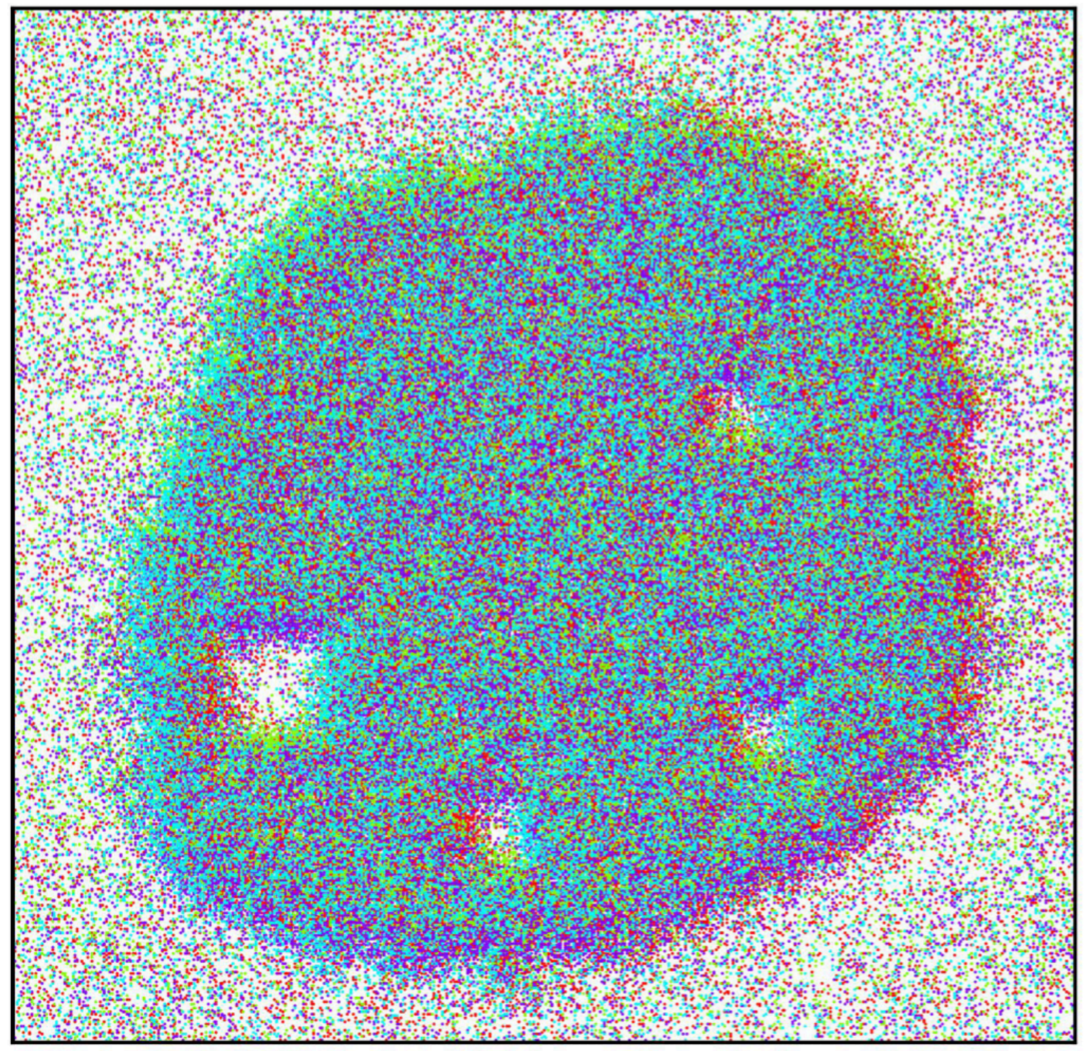


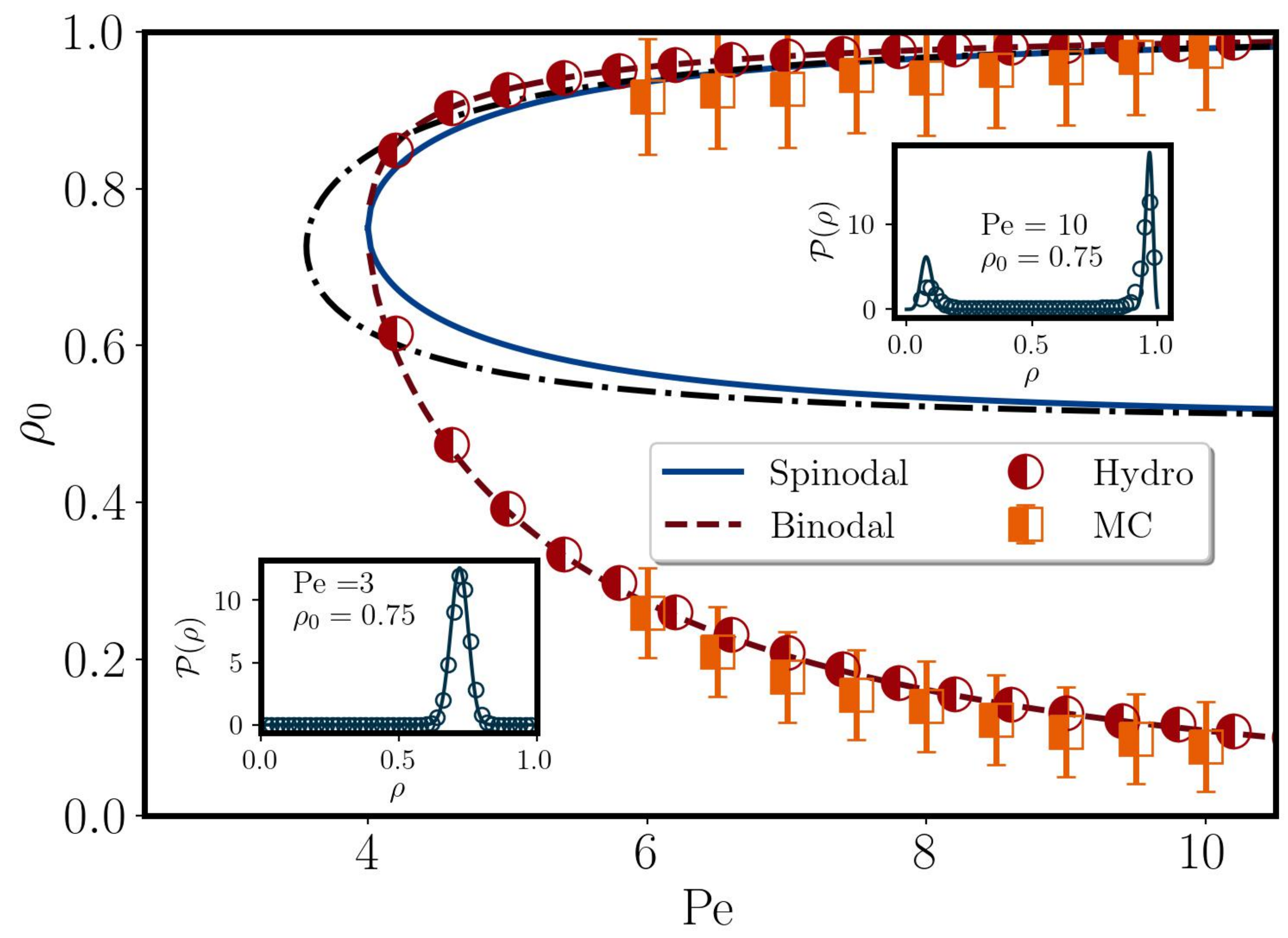


$$\psi_T(X, \pm) = \{0,1\} \qquad \psi_T(X, \pm) \simeq \varrho_{\frac{T}{\tau}}\left(\frac{X}{\ell_d}, \pm\right)$$

$$\rho_t(x) = \varrho_t(x, +) + \varrho_t(x, -)$$
$$m_t(x) = \varrho_t(x, +) - \varrho_t(x, -)$$

$$\partial_t \rho = \partial_x^2 \rho - \mathrm{Pe}\, \partial_\mathrm{x}\left[\mathrm{m}\,(1-\rho)\right] + \ell_\mathrm{d}^{-1/2}\, \partial_\mathrm{x} \eta_\rho$$
$$\partial_t m = (1-\rho)\, \partial_x^2 m + m\, \partial_x^2 \rho - \mathrm{Pe}\, \partial_\mathrm{x}\left[\rho\,(1-\rho)\right] - 2\mathrm{m}$$
$$+ \ell_d^{-1/2}\left(\partial_x \eta_m + 2\,\eta_f\right),$$





Large deviations? Interacting ABP/AOUP/RTP?

Ongoing ….

Take home message

For a quantitative macro-scale
description of Active matter,
Stay on phase space!

ICTS, Bengaluru.

Sep 28 - Oct 9, 2026

Registration open till May 31st