Anomalous current fluctuations in the Hubbard model with infinite interaction

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(Based on a work with K. Fujimoto, T. Ishiyama, T. Kurose, T. Yoshimura)

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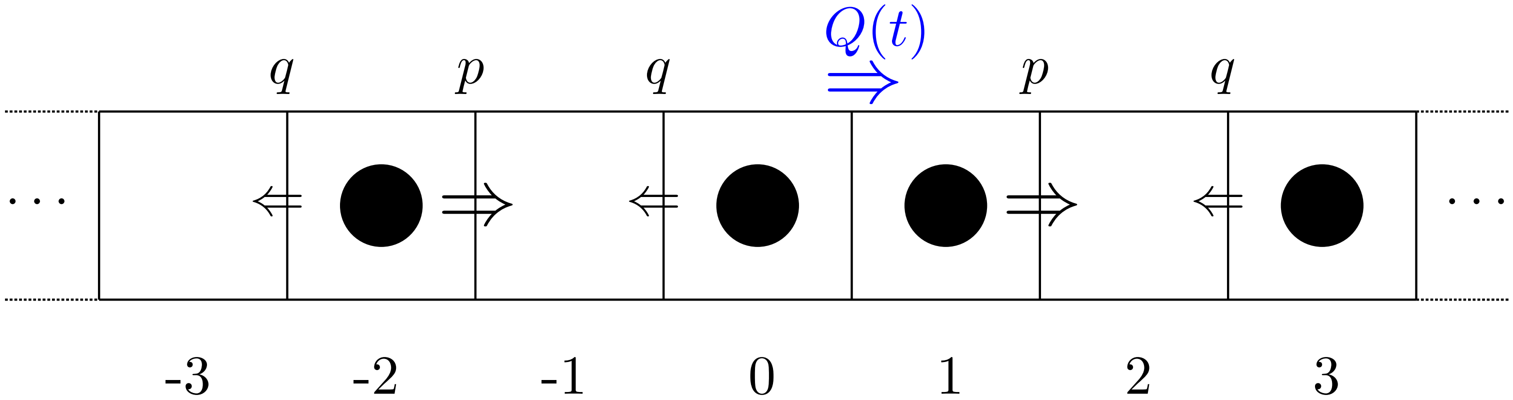
Outline

1. Introduction and results
2. Anomalous fluctuations for cellular automaton models
3. Derivation for the Hubbard model with infinite interaction
4. Hydrodynamic approach

1. Introduction

- Fluctuations of currents are an fundamental and important subject in nonequilibrium statistical mechanics.
- Examples are KPZ and MFT(macroscopic fluctuation theory). They have been considered mainly for classical systems.

ASEP (= asymmetric simple exclusion process)



Universal limiting current distribution (2000 Johansson, 2009 Tracy-Widom, 2010 Sasamoto-Spohn,...)

$$\lim_{t\to\infty} \mathbb{P}[Q(t) \leq vt + ct^{1/3}s] = F_{\mathrm{TW}}(s)$$

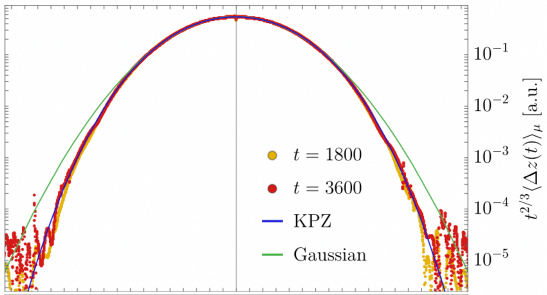
KPZ in spin chain (?)

- Recently there has been increasing interest in current fluctuations for quantum systems.

- An outstanding case is the 1D Heisenberg spin chain at $T > 0$.

$$H_{\rm XXX} = J \sum_j [\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \sigma_j^z \sigma_{j+1}^z]$$

  2019 Ljubotina Znidaric Prosen DMRG calculation strongly suggests that $\langle \sigma_j^z(t) \sigma_0^z(0) \rangle$ shows the universal KPZ two point function (2004 Prähofer-Spohn).



But spin current does not show TW type fluctuations.

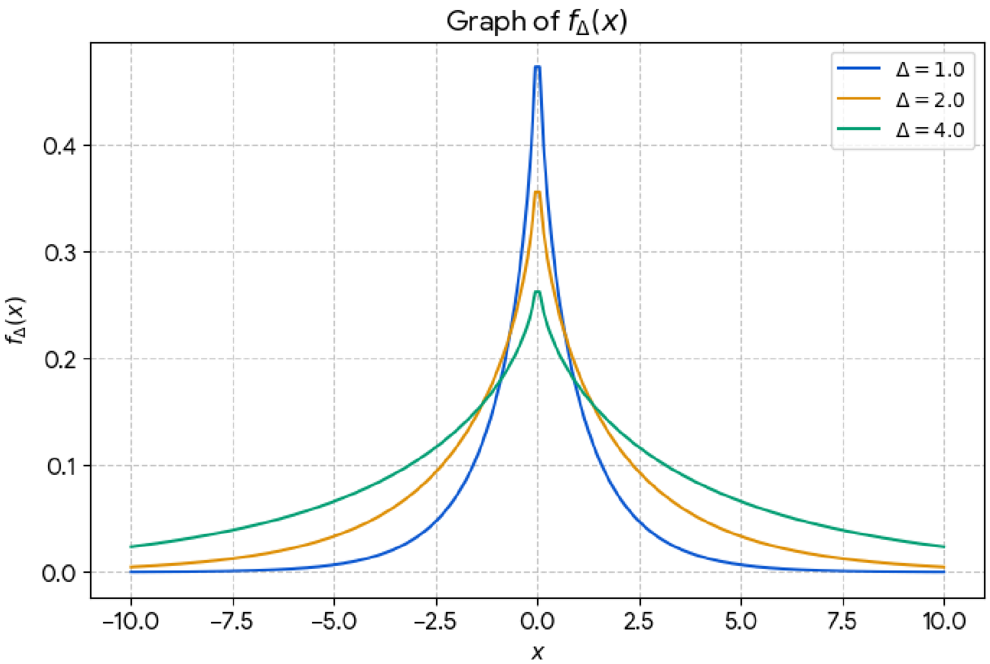
An easier anomalous fluctuation

- For the XXZ spin chain the spin current is expected (again mainly by numerics) to be given by an anomalous fluctuation described by the **M-Wright function**.
- The distribution has been first established for a simple automaton model.
- Quite recently we have found that the same distribution describes the limiting spin current distribution of the Hubbard model with infinite strength of interaction (sometimes called the $t0$ model).
- In this talk we explain these.

M-Wright distribution

The pdf of the M-Wright distribution f_Δ is given by

$$f_\Delta(x) = \frac{1}{\pi\Delta} \int_0^\infty \frac{du}{\sqrt{u}} \exp\left[-\frac{u^2}{2\Delta^2} - \frac{x^2}{2u}\right]$$



This distribution also appears in fractional diffusion processes.

M-Wright function

The following function is often called the M-Wright function.

$$M_\nu(x) = \sum_{k=0}^{\infty} \frac{(-x)^k}{k!\Gamma((1-\nu)-\nu k)}$$

The pdf of our M-Wright distribution is written as

$$f_\Delta(x) = \frac{1}{2^{\frac{1}{4}}\sqrt{\Delta}} M_{1/4}\left(\frac{1}{\sqrt{\Delta}}|x|\right)$$

M-Wright distribution for current fluctuations

Recently, for a few models (with 0 mean current), it has been shown that a certain integrated current $Q(t)$ is of $O(t^{1/4})$ and on this scale the limiting distribution of the current is described by the M-Wright distribution,

$$\lim_{t \to \infty} \mathbb{P}[Q(t) \leq c t^{1/4} s] = F_\Delta(s)$$

for a certain Δ (and c) depending on parameters of each model. F_Δ is the distribution function corresponding to f_Δ.

The $t0$ Model

The Hubbard model

$$H_{\text{Hub}} = -\sum_{x\in\mathbb{Z}}\sum_{\sigma=\uparrow,\downarrow}\left(\hat{c}^{\dagger}_{x,\sigma}\hat{c}_{x+1,\sigma} + \hat{c}^{\dagger}_{x+1,\sigma}\hat{c}_{x,\sigma}\right) + u\sum_{x\in\mathbb{Z}}\hat{n}_{x,\downarrow}\hat{n}_{x,\uparrow}$$

where $\hat{c}_{x,\sigma}$ is the fermion operator on x with spin σ and $\hat{n}_{\sigma,j} = \hat{c}^{\dagger}_{\sigma,j}\hat{c}_{\sigma,j}$ is the number operator. The interaction is such that, if there are two particles of spins $\uparrow$ and $\downarrow$ on the same site, the system has extra energy u.

2023 Gamayun Hutsalyuk Pozsgay Zvonarev

In the limit of strong interaction $u \to +\infty$, this reduces to

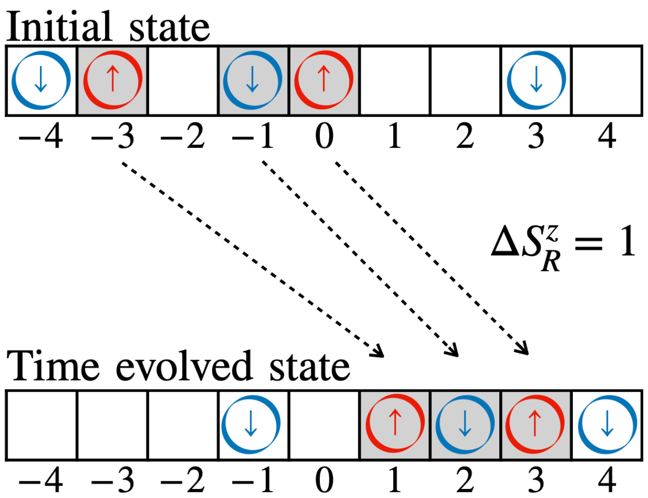
$$H_{t0} = -\hat{P}\sum_{x\in\mathbb{Z}}\sum_{\sigma=\uparrow,\downarrow}\left(\hat{c}^{\dagger}_{x,\sigma}\hat{c}_{x+1,\sigma,} + \hat{c}^{\dagger}_{x+1,\sigma}\hat{c}_{x,\sigma}\right)\hat{P},$$

where $\hat{P} = \prod_{j\in\mathbb{Z}}(1 - \hat{n}_{\uparrow,j}\hat{n}_{\downarrow,j})$ is the projector to Hilbert space without any double occupancy. We call this the $t0$ model.

Result: M-Wright dist. for spin current

Initial condition: each site is occupied by a fermion with probability 1/2 and its spin is up or down with probability 1/2 independently.

$Q(t)$: Integrated current of spins:



Result (Fujimoto Ishiyama Kurose Yoshimura S 2026)

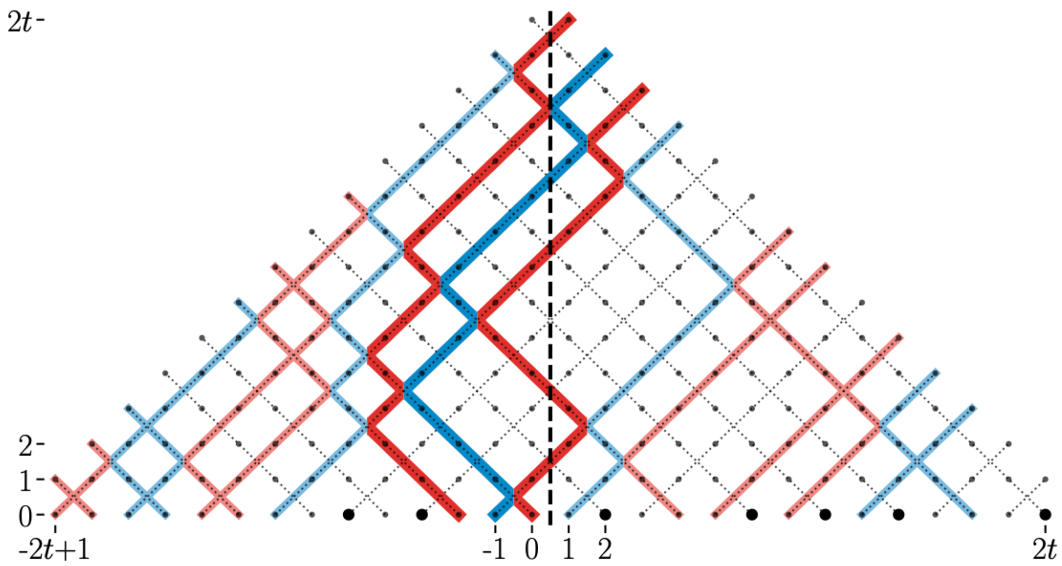
$$\lim_{t\to\infty} \mathbb{P}[Q(t) \le s t^{1/4}] = F_{\frac{1}{\sqrt{\pi}}}(s)$$

2. Deterministic automaton

Krajnik Schmidt Pasquier Ilievski Prosen (arXiv: 2201.05126)

Two types of particles, + and −, move ballistically with velocities 1. They collide ellastically.

Initial conditions are random and product with particle density ρ. Each particle is + or − with prob. 1/2.



Combinatorics

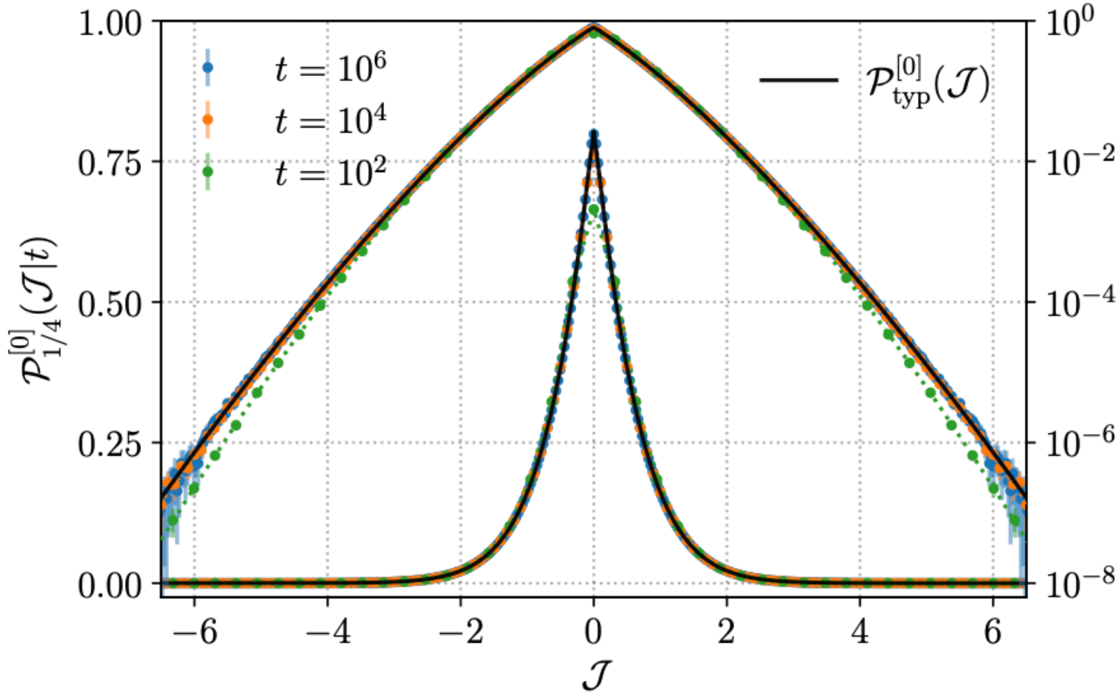
We are interested in the integrated current of charge $Q(t)$ at the origin.

By combinatorial arguments, its generating function is written as

$$\langle e^{\lambda Q(t)} \rangle = \rho^{2t} \sum_{l=0}^{t} \sum_{r=0}^{t} \binom{t}{l} \binom{t}{r} \nu^{l+r} \cosh(\lambda)^{|\Lambda_+|+|\Lambda_-|}$$

where $\nu = \rho/(1-\rho)$, $|\Lambda_\pm| = (|l-r| \pm (l-r))/2$.

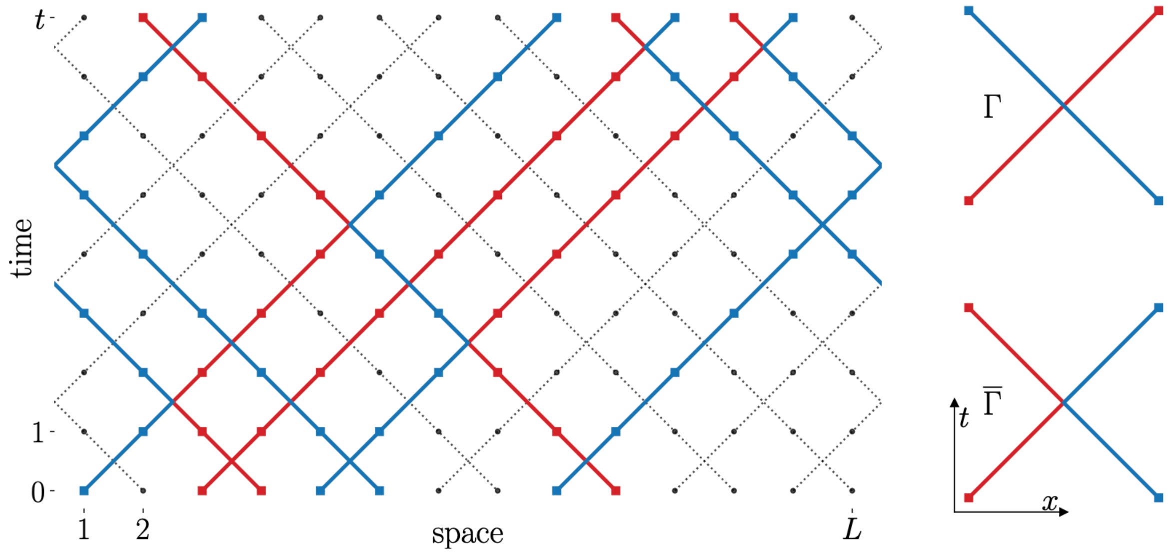
By doing asymptotics, they showed that the current distributions obeys the M-Wright distribution with $\Delta = \sqrt{\rho(1-\rho)}$.



Stochastic automaton

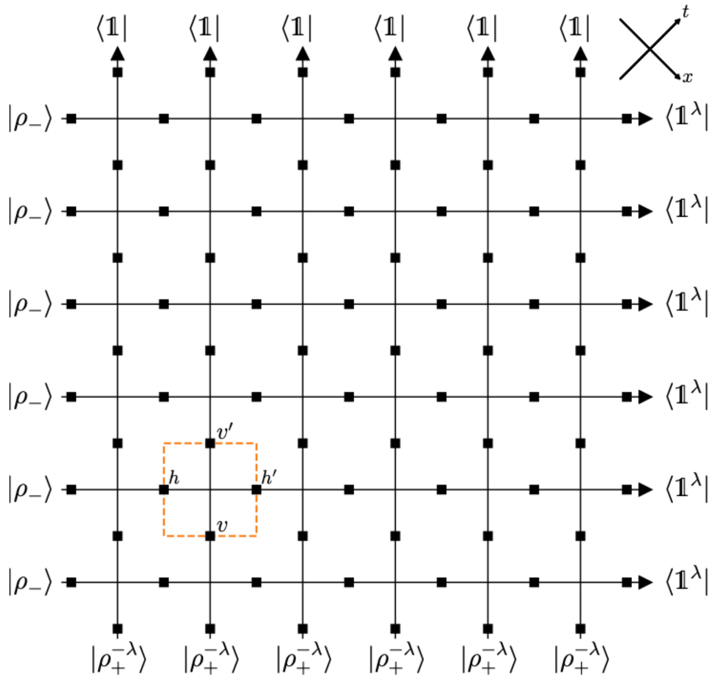
Krajnik Klobas Bertini Prosen (arXiv:2502.02509)

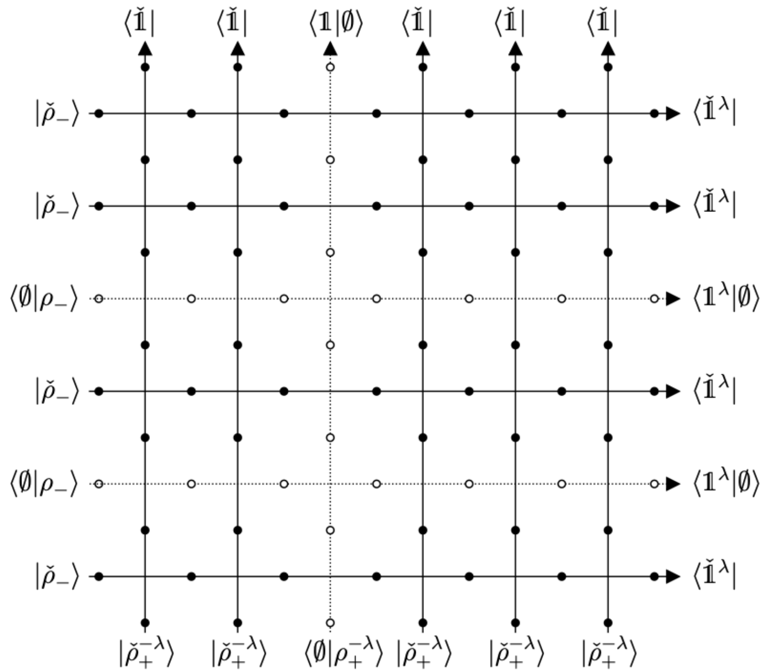
There is a generalization of the automaton model, in which collisions occur with probabilitiy Γ. When $\Gamma = 0$, this reduces to the deterministic model.



Relation to SEP

Some part of calcuations are reduced to the one of the symmetric exclusion process (SEP).





A formula for $\langle e^{\lambda Q(t)} \rangle$

The generating function can be written in terms of certain expectation for the SEP with step initial condition.

$$\langle e^{\lambda Q(t)} \rangle = \sum_{n_-,n_+=0}^{t} P(n_-|t)P(n_+|t)\mu^{(n_+-n_-)/2}\mathbb{E}^{\mathrm{SEP}}_{\mathrm{step}}(\mu^{N_{t-n_+}}|t-n_-)$$

Here $P(n_\pm|t) = \binom{t}{n_\pm}\rho_\pm^{t-n_\pm}(1-\rho_\pm)^{n_\pm}$, $\mu = \cosh(\lambda)$ and

$$\mathbb{E}^{\mathrm{SEP}}_{\mathrm{step}}(\mu^{N_x}|t) = \sum_{k=0}^{\infty} \frac{(\mu-1)^k}{k!} \oint_{c_r} \prod_{i=1}^{k} \frac{dz_i}{2\pi i} \det\left(\frac{z_i^{t-x} h^t(z_i)}{1-2z_i+z_i z_j} \right)_{1\le i,j\le k}$$

with $h(z) = (1+(z^{-1}-2)\Gamma)/(1-\Gamma z)$. In the continuous limit this is the same as the quantity studied by 2021 Mallick Moriya Sasamoto related to the fluctuation of the tagged particle in SEP.

Generalized M-Wright dist.

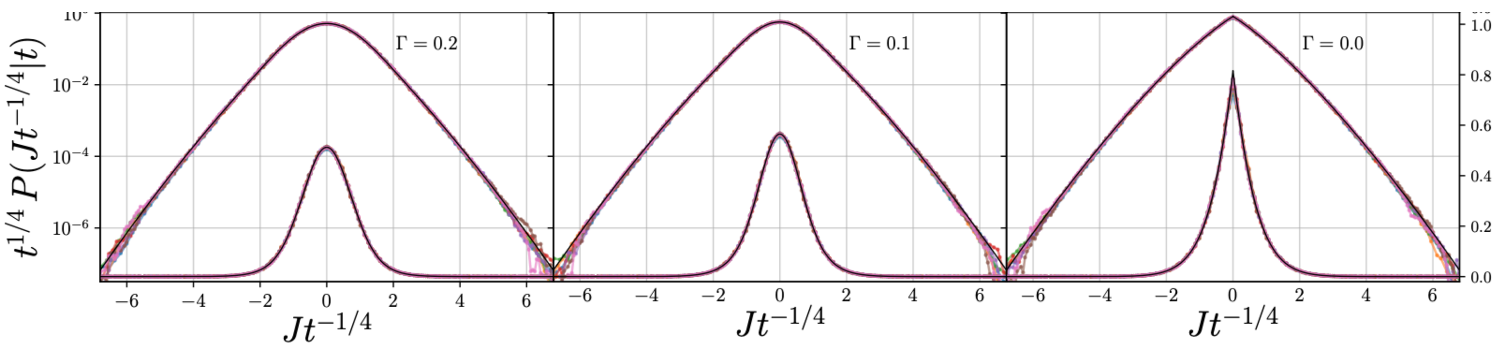
They showed that the the current distributions obeys a generalization of the M-Wright distribution defined by

$$f_{\Delta,a}(x) = \frac{1}{\pi\Delta}\int_0^\infty \frac{du}{\sqrt{u(1+s(u/a))}} \exp\left[-\frac{u^2}{2\Delta^2} - \frac{x^2}{2u(1+s(u/a))}\right]$$

where

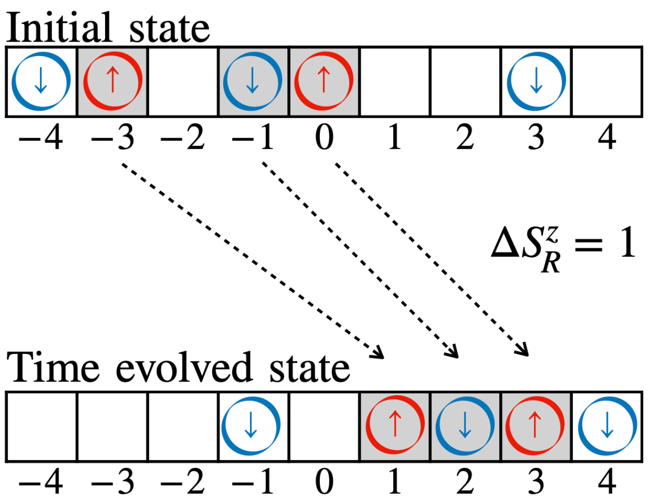
$$s(z) = \pi^{-1/2} z^{-1} e^{-z^2} - \mathrm{erfc} z$$

and $a = 2\sqrt{\rho/\gamma}$ with $\gamma = \bar{\Gamma}/\Gamma$.



3. M-Wright dist. for the $t0$ model

Particle movements in the $t0$ model



The order of spins never change and the dynamics of particles do not depend on spins (spin-particle separation or spin-charge separation).

For the $t0$ model, the particles are free fermions. We can study their dynamics in terms of determinants!

Consequence of the spin-particle sepration

Recall the initial condition is s.t. each site is occupied by a fermion with probability 1/2 and its spin is up or down with probability 1/2.

Thanks to the spin-particle separation, its generating function is written in terms of pdf of the particle current Q_p:

$$\langle e^{\lambda Q(t)} \rangle = \mathbb{P}_p[Q_p(t) = 0] + 2 \sum_{k=1}^{\infty} (\cosh(\lambda))^k \, \mathbb{P}_p[Q_p(t) = k].$$

The remaining task is to study the distribution of the particle current.

Particle current fluctuation

Using the free fermion (determinantal) nature of the particle dynamics, generating function of $Q_p(t)$ is written in the form of a determinant

$$\mathbb{E}_p[e^{\lambda Q_p(t)}] = \det\left(\delta_{m,n} + \sinh\left(\frac{\lambda}{2}\right)^2 D_{m,n}(t)\right)_{m,n\in\mathbb{N}}$$

where

$$D_{m,n}(t) = \mathrm{i}^{n-m} \sum_{l=0}^{\infty} J_{n+l}(2t) J_{m+l}(2t)$$

with $J_n(x)$ the Bessel function of the first kind.

By doing asymptotics, we can see that the fluctuation of the particle current is simply a Gaussian:

$$\sqrt{t}\mathbb{P}_p[Q_p(t) = \sqrt{t}v] \sim \frac{1}{\sqrt{2}} \exp\left(-\pi v^2/2\right).$$

One can also see that randomness of the i.c. gives another Gaussian fluctuation. Finally we find

$$\lim_{t\to\infty} \mathbb{P}[Q(t) \leq st^{1/4}] = F_{\frac{1}{\sqrt{\pi}}}(s).$$

That is, the integrated spin current of the $t0$ model obeys the M-Wright distribution with parameter $\Delta = 1/\sqrt{\pi}$ in the long time limit.

Remark: The automaton model can be studied in a parallel way.

4. Hydrodynamic approach

- The M-Wright distribution for these models can also be found based on hydrodynamic arguments.
- For the automaton model, Yoshimura Krajnik 2024, 2025 did this by assuming that initial randomness are propagated according to deterministic ballistic hydrodynamic (GHD) flows.

  Establishing this rigorously would be an interesting problem.
- Similar arguments can be employed for the $t0$ model (Fujimoto Ishiyama Kurose Yoshimura S 2026)
- Yoshimura Zrajnik Bastianello Ilievski 2026 applied the arguments to the XXZ spin chain.

Deterministic automaton case

Yoshimura Krajnik 2024

- 3 conserved densities $\rho_{l,r,c}$ of left- right- movers and charge
- They satisfy the Euler equations

$$\partial_t \rho_l - \partial_x \rho_l = 0, \quad \partial_t \rho_r + \partial_x \rho_r = 0, \quad \partial_t \rho_c - \partial_x (v \rho_c) = 0$$

  with $v = (\rho_+ - \rho_-)/(\rho_+ + \rho_-)$.

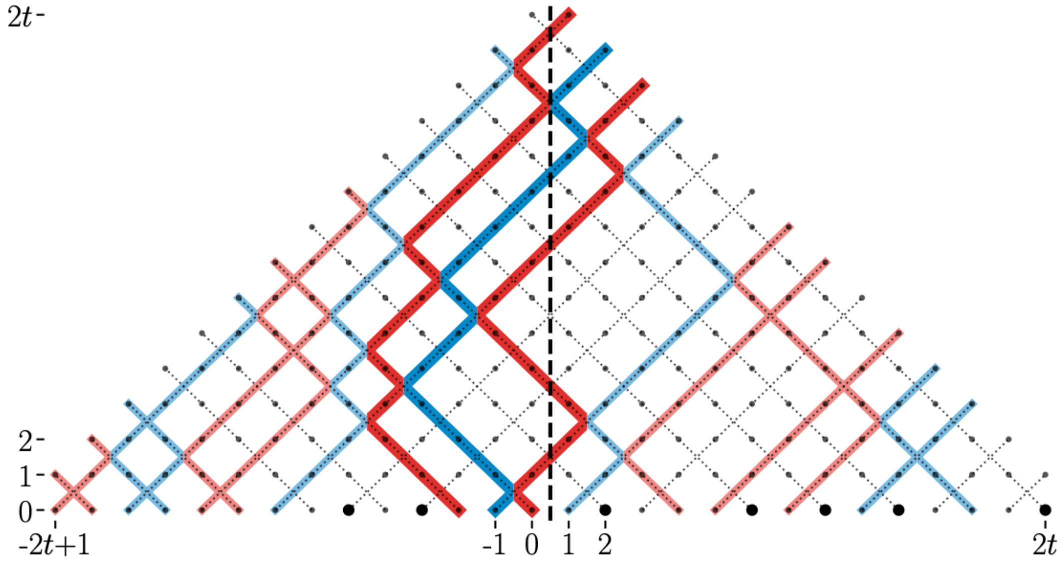
- Current of charge is written as

$$Q_c(\tau) \simeq \tau^{1/4} \int_0^{X_{t=1}} dx\, \delta \varrho_c(-x, 0),$$

  where X_t is the characteristics that satisfies $dX_t/dt = \sqrt{\tau} v(X_t, t)$.

- Fluctuations of X_t and $\int dx \rho_c(-x, 0)$ are Gaussians, giving the M-Wright distribution.

Deterministic automaton (again)



Two types of particles, + and −, move ballistically with velocities 1. They collide ellastically.

Initial conditions are random and product with particle density ρ. Each particle is + or − with prob. $1/2$.

$t0$ **case**

Fujimoto Ishiyama Kurose Yoshimura S 2026

- Infinite conserved densities $\rho^{(c)}, \rho^{(s)}$ with a parameter λ.
- They satisfy the Euler equations

$$\frac{\partial}{\partial t}\rho^{(\mathrm{c})}(\lambda, x, t) + 2\sin(\lambda)\frac{\partial}{\partial x}\left(\rho^{(\mathrm{c})}(\lambda, x, t)\right) = 0,$$
$$\frac{\partial}{\partial t}\rho^{(\mathrm{s})}(\lambda, x, t) + \frac{\partial}{\partial x}\left(v(x,t)\rho^{(\mathrm{s})}(\lambda, x, t)\right) = 0$$

with $v(x,t) = \int d\lambda 2\sin(\lambda)\rho^{(\mathrm{c})}(\lambda, x, t)/\int d\lambda\rho^{(\mathrm{c})}(\lambda, x, t)$

- Spin current is written as

$$Q(\tau) = \tau^{1/4}\int_0^{X_{t=1}} dx\delta S^z(-x, 0),$$

giving the M-Wright distribution from 2 Gaussians.

- Can be equally applied to general temperature.

5. Summary

- We established that the fluctuation of the spin current for the Hubbard model with infinite interaction starting from a product initial condition is described by the M-Wright distribution, which is written as a nested Gaussians
- The same distribution has shown up in a few models, including a deterministic automaton model with ballistic particles. Our result is the first confirmation for a quantum mechanical model.
- We also explained that the same fluctuation can be understood by hydrodynamic arguments (but with some assumptions).
- Establishing the same distribution for the XXZ model by microscopic calculations would be challenging.
- A hope is that this line of results are helpful for understanding KPZ behaviors of the Heisenberg spin chain.