Low-Energy Expansion from Operator Product Expansion in asymptotically free theories with a mass gap

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OPE at high-energy $\rightarrow$ low-energy expansion(LEE)

$$S_{\mathrm{OPE}}(p^2) = \sum_{n=0}^{\infty} C_n(p^2) \frac{\langle O_n \rangle}{(p^2)^n}$$

$$S_{\mathrm{LEE}}(p^2) = \sum_{m=0}^{\infty} c_m \left(\frac{p^2}{M^2} \right)^m$$

Takaura, JHEP 10 (2024) 085

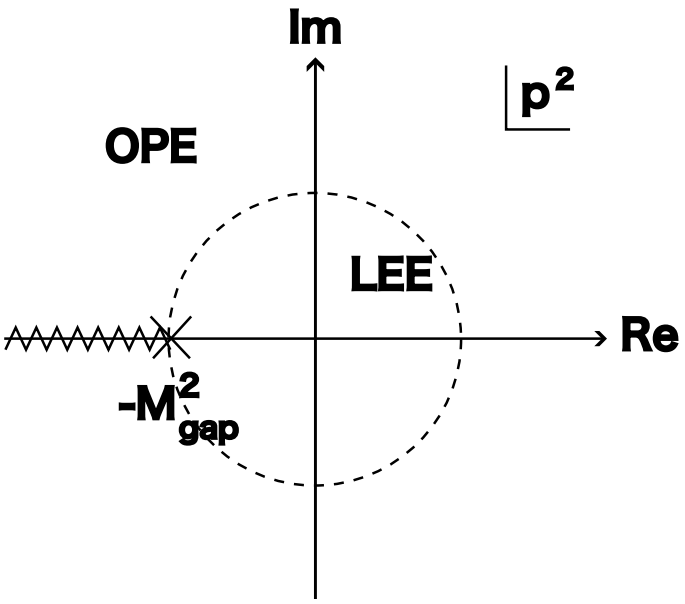
· OPE-LEE connection
· Extraction of the low-energy limit c_0

This work: Extension

· Extraction of general-order coefficients c_m
· Improvement of the method

Key idea of Takaura, JHEP 10 (2024) 085

$$S_{\text{LEE}} = \sum_{m=0}^{\infty} c_m \left(\frac{p^2}{M^2}\right)^m$$



Because of Mass gap,

OPE is reliable in $|p^2| > M_{\text{gap}}^2$;

LEE converges in $|p^2| < M_{\text{gap}}^2$

→ Borel transform of LEE, $\tilde{S}_{\text{LEE}}$

Relates LEE and OPE, $\tilde{S}_{\text{LEE}} \overset{?}{=} \tilde{S}_{\text{OPE}}$

Extract LEE coefficient c_m $= \oint \frac{dz\, m!}{2\pi i z^{m+1}} \tilde{S}_{\text{LEE}}$

But, Duality violation appears in the matching, $\tilde{S}_{\text{LEE}} = \tilde{S}_{\text{OPE}} + \tilde{S}_{\text{DV}}$

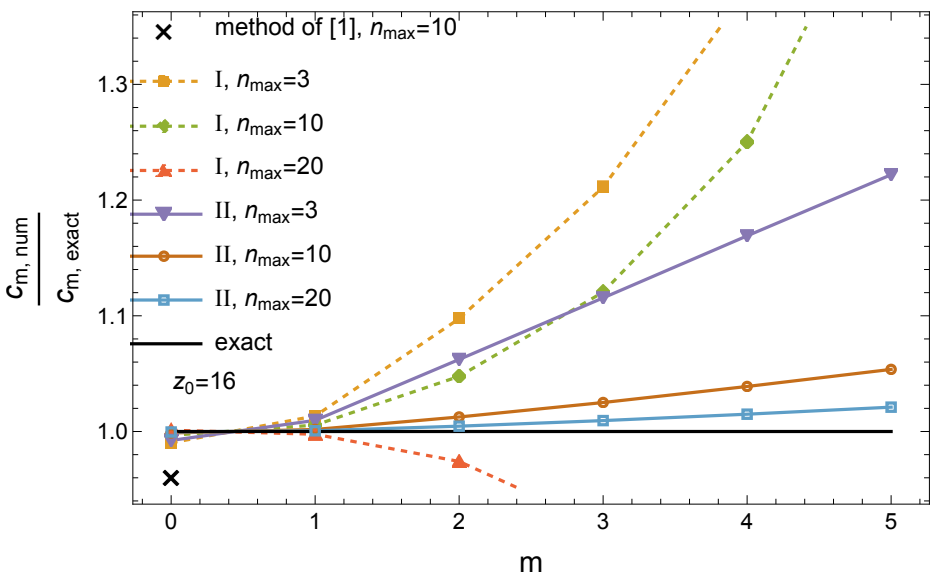
A weight function to suppress DV was found **only for c_0**.

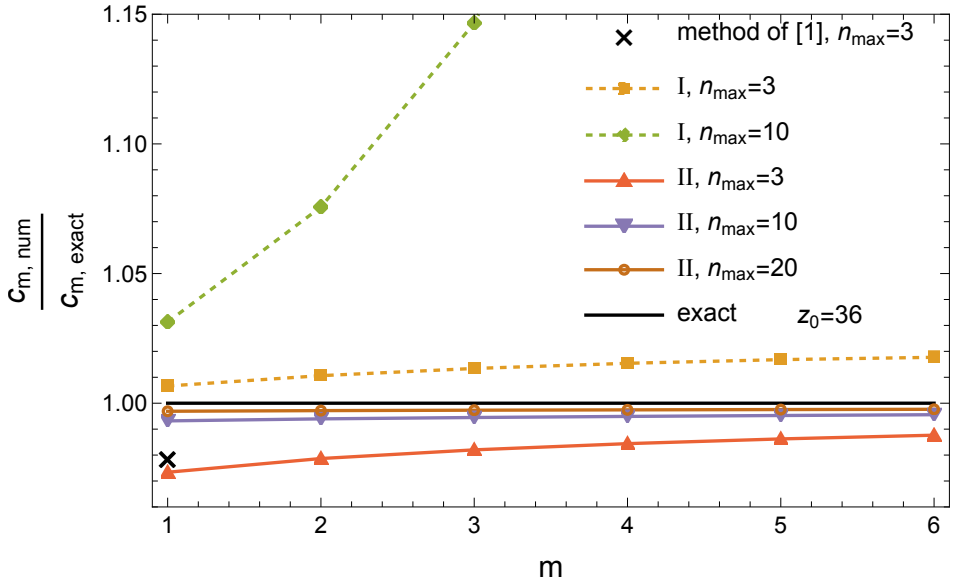
Our Extension

- Method Ⅰ : Weight function for general order
- Method Ⅱ : Including $\tilde{S}_{\text{DV}}$ explicitly

→ extract **general order c_m**

General-order coefficients c_m are extracted





O(N) Nonlinear Sigma Model (toy model) $D_\alpha(p^2)$ (left), $\rho(p^2)$ (right)

- Method I and Method II extract general-order LEE coefficients c_m.
- Method II remains stable even for large OPE truncation order $n_{\max}$.

These results reinforce viewpoint of Takaura:
high-energy OPE can provide access to low-energy information.