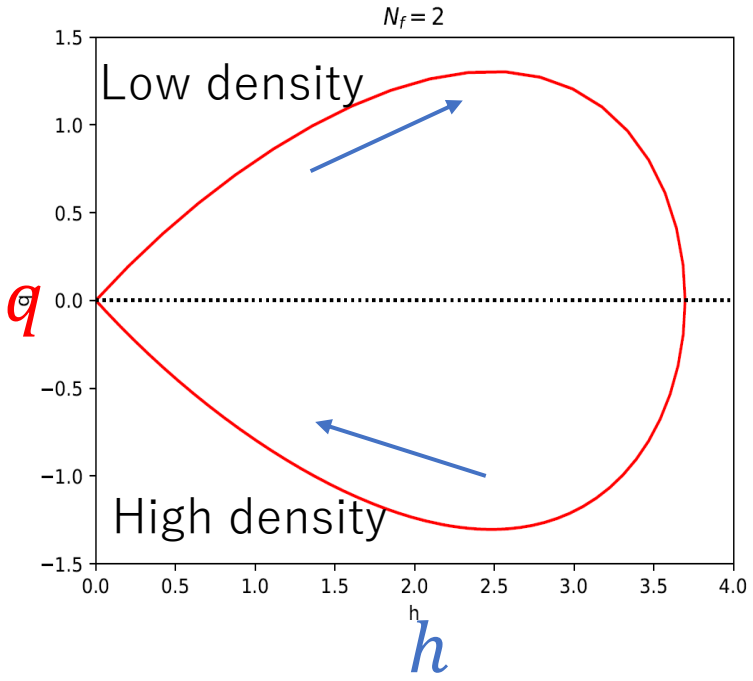
Heavy dense QCD and Three state Potts model with complex phase

Masanari Koiida, Hideya Hashizume, Shinji Ejiri (Niigata Univ.)

Heavy dense QCD is similar to the three states Potts model with complex phase (Ω is Polyakov loop)

$$Z_{heavy\ dense} = \int \mathcal{D}U\, e^{-S_{gauge}(U)} \prod_{\vec{x}} \{1 + 3C\Omega(\vec{x}) + 3C^2\Omega^*(\vec{x}) + C^3\}^{2N_f}$$

$$Z_{\mathrm{potts}} = \sum_{\{s\}} \exp\left\{\beta \sum_{x,j} \mathrm{Re}\left(s_x s^*_{x+\hat{j}}\right) + h \sum_x \mathrm{Re}(s_x) + iq \sum_x \mathrm{Im}(s_x)\right\}$$



The symmetry between high density and low density suggests the first order phase transition in the high-density region. The parameter space is finite.

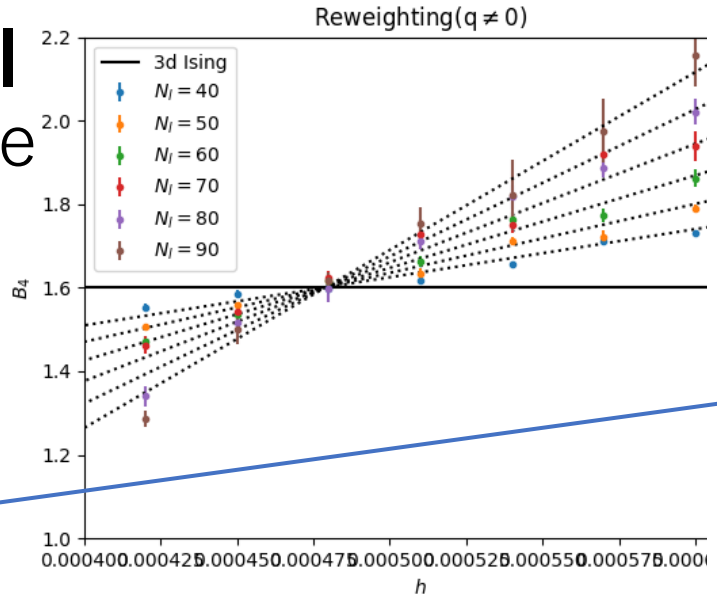
S.Ejiri, M.Koiida, Phys. Rev. D 113, 074515 (2026)

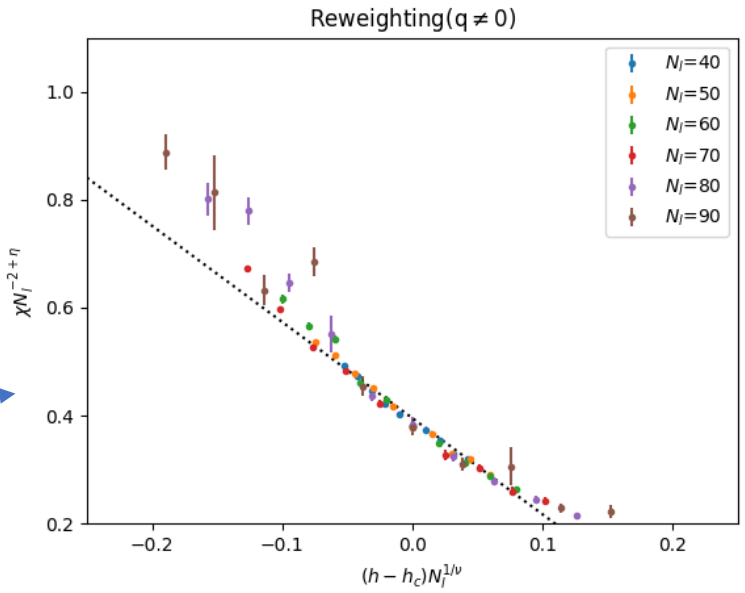
The Critical Point & Singularities

We determine the critical point of this model by finite size scaling.

Binder cumulant

Susceptibility

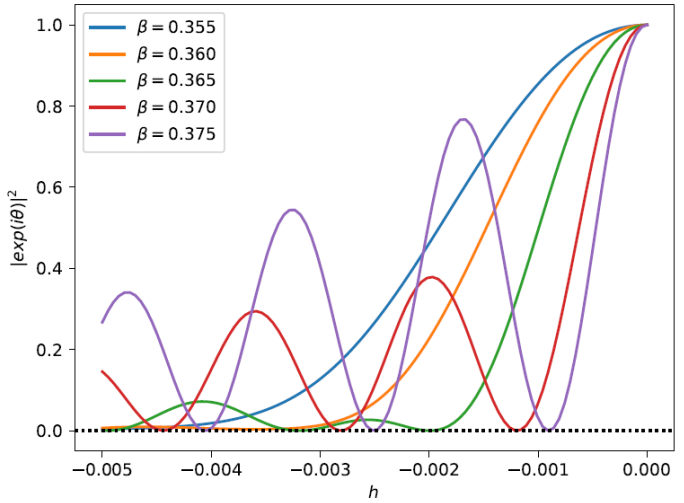




There are singularities at h<0 (negative external field)

They correspond to the Roberge-Weiss singularity.

$$\left|\frac{Z_{potts}(\beta, h, q)}{Z_{potts}(\beta, h, 0)}\right|^2$$



2-color QCD with 1-flavor (Canonical Approach)

In the heavy dense effective theory, 2-color QCD with 1-flavor fermion, the canonical partition function $Z_c(T,N)$ is defined by

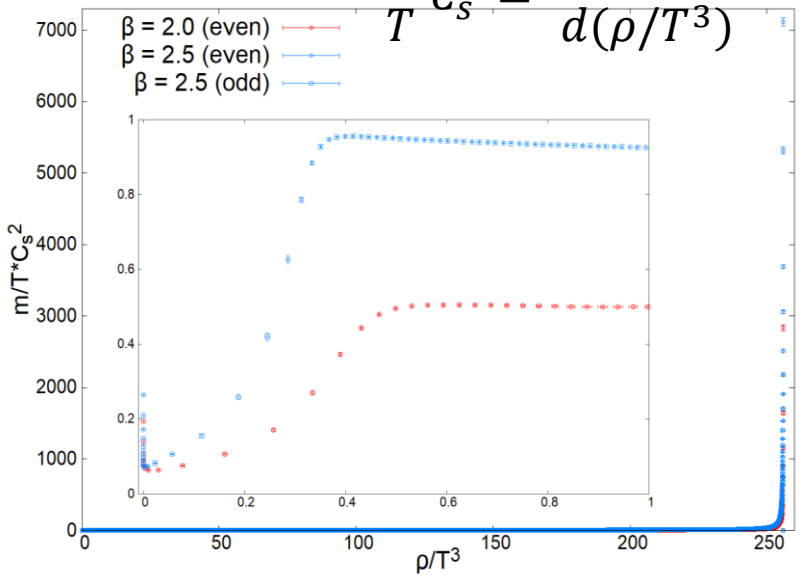
$$Z_{GC}(T,\mu) = \sum_N Z_c(T,N)\exp\left(\frac{\mu N}{T}\right) \quad (N: the\ number\ of\ particle)$$

The sign problem (N=odd) is removed. We also discuss the overlap problem.

Results:

Speed of sound,
Chiral condensate etc.

$$\frac{m}{T}C_s^2 = \frac{d(\Delta p/T^4)}{d(\rho/T^3)}$$



$$(\langle\bar{\psi}\psi\rangle/N_s^3N_t)/4N_c$$