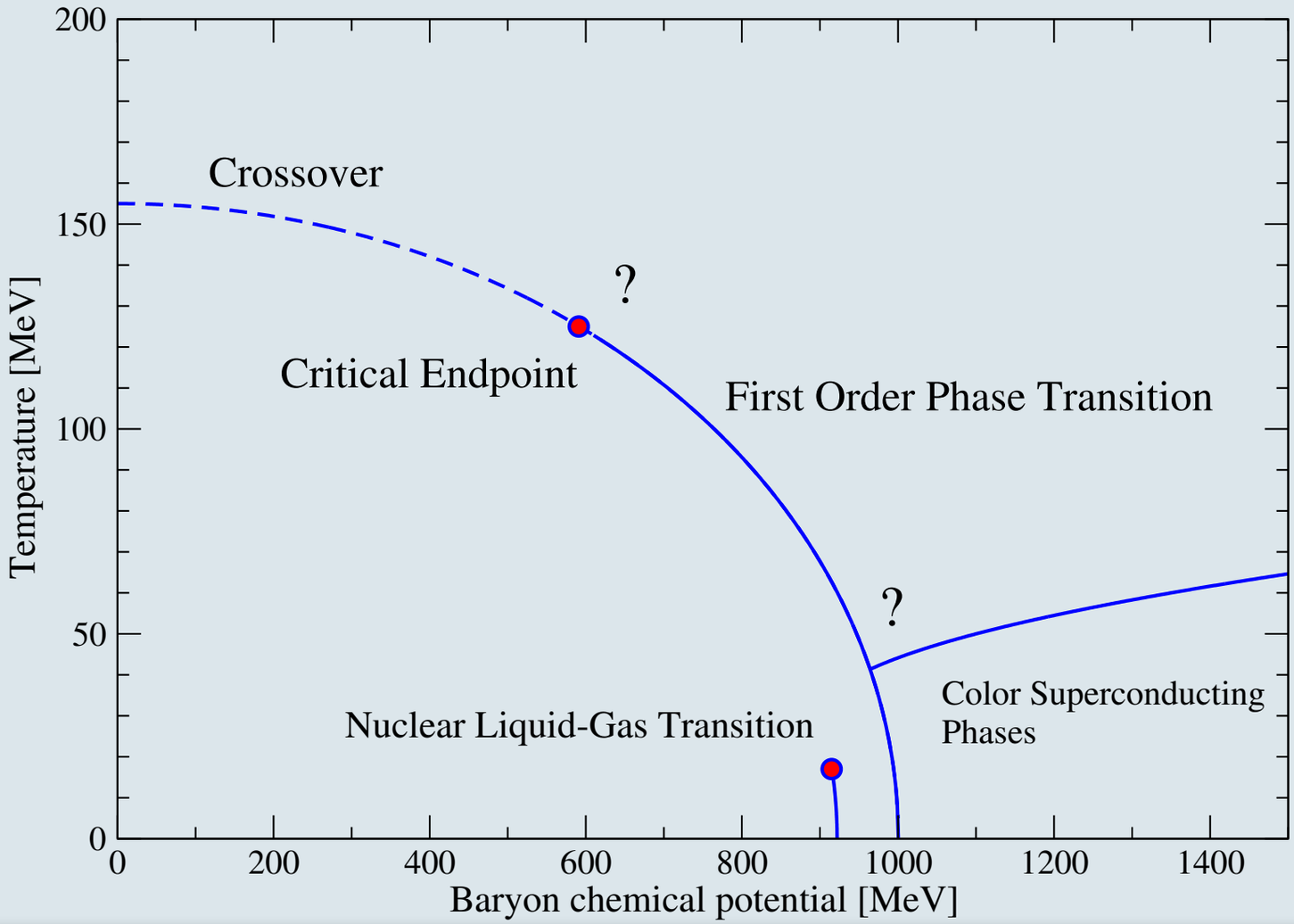

Columbia plot, critical point and bound states in dense QCD

Review: CF, PPNP 105 (2019) [1810.12938]

CF and Pawlowski, EPJST [2603.11135]
CF and Pawlowski, in preparation

Overview

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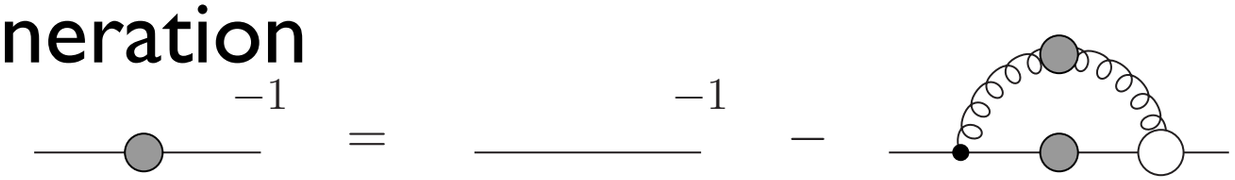

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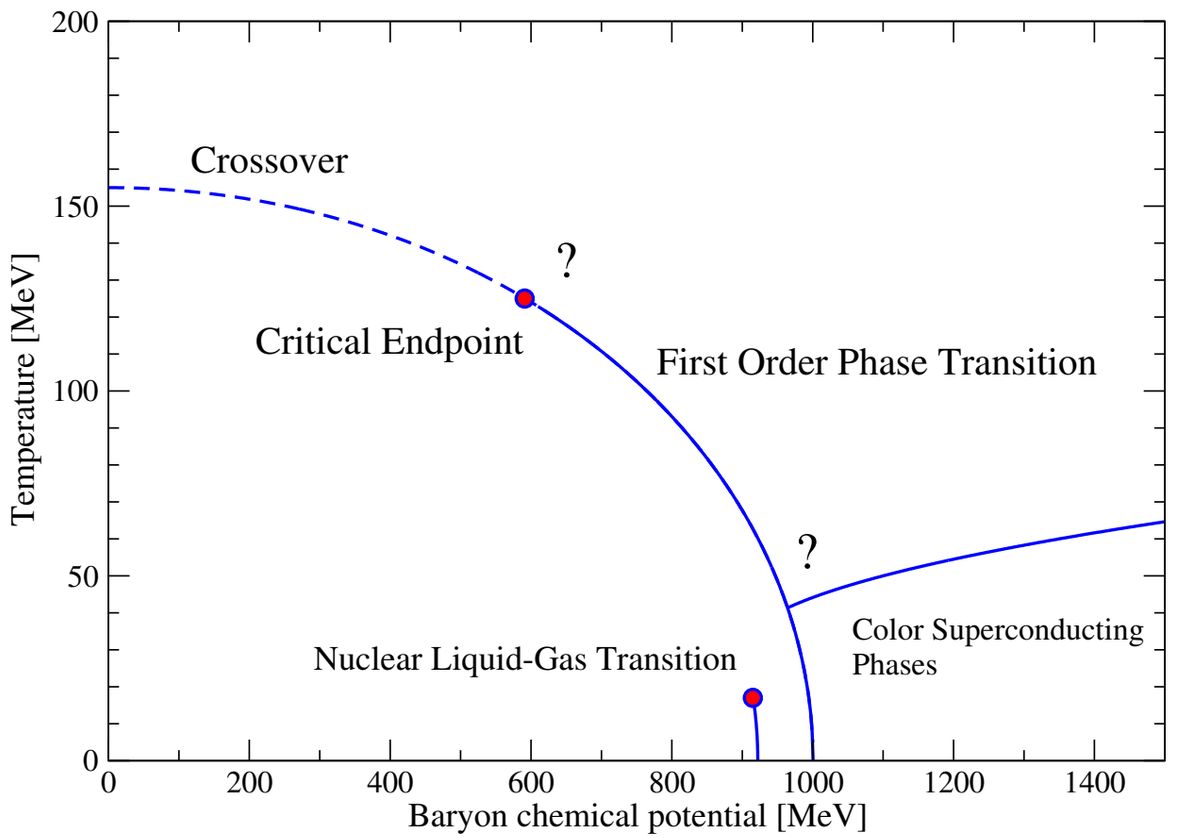
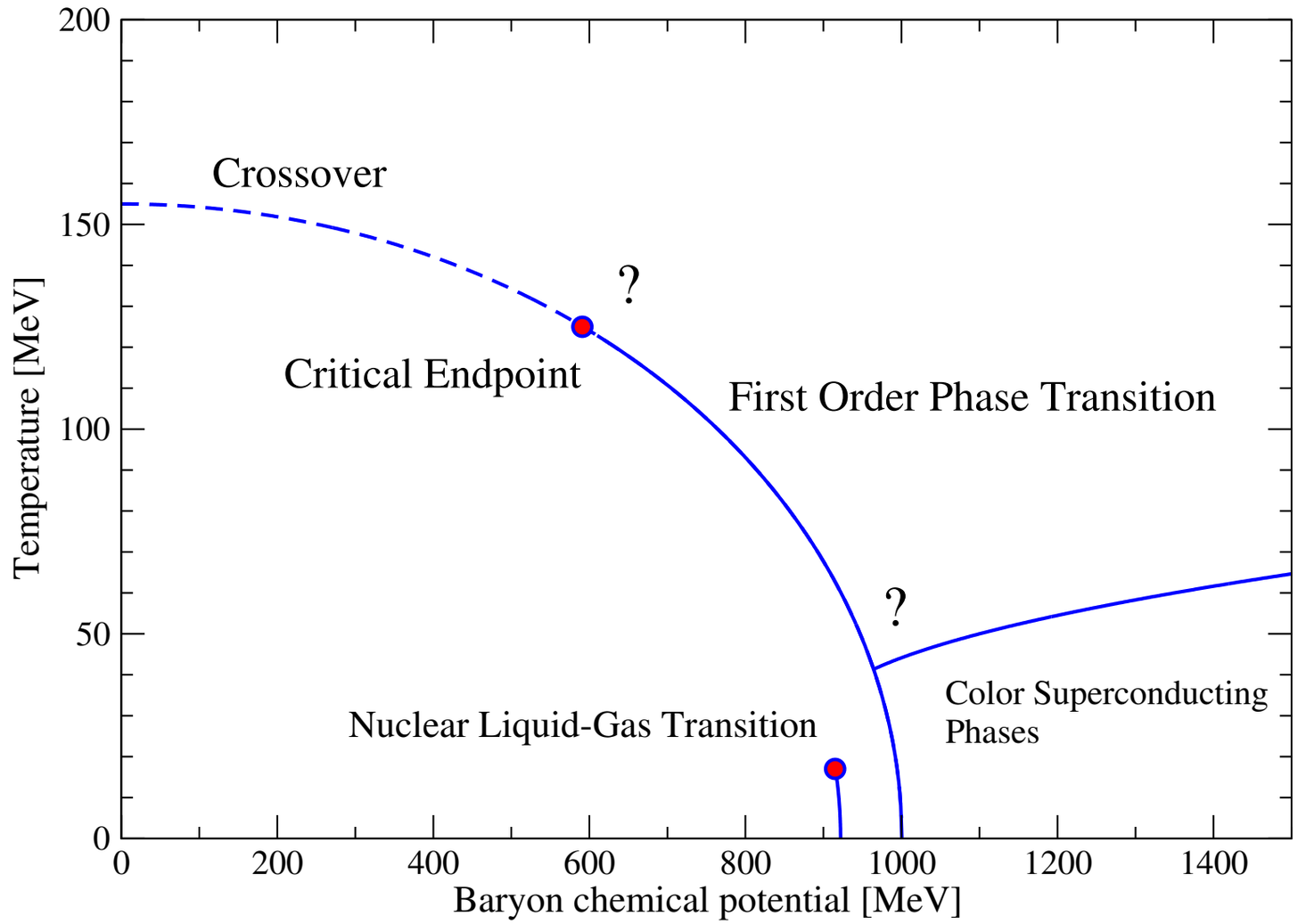


Phase diagram of quark matter: QCD



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- Details of phase transitions ??
- Consequences for early universe and physics of neutron stars

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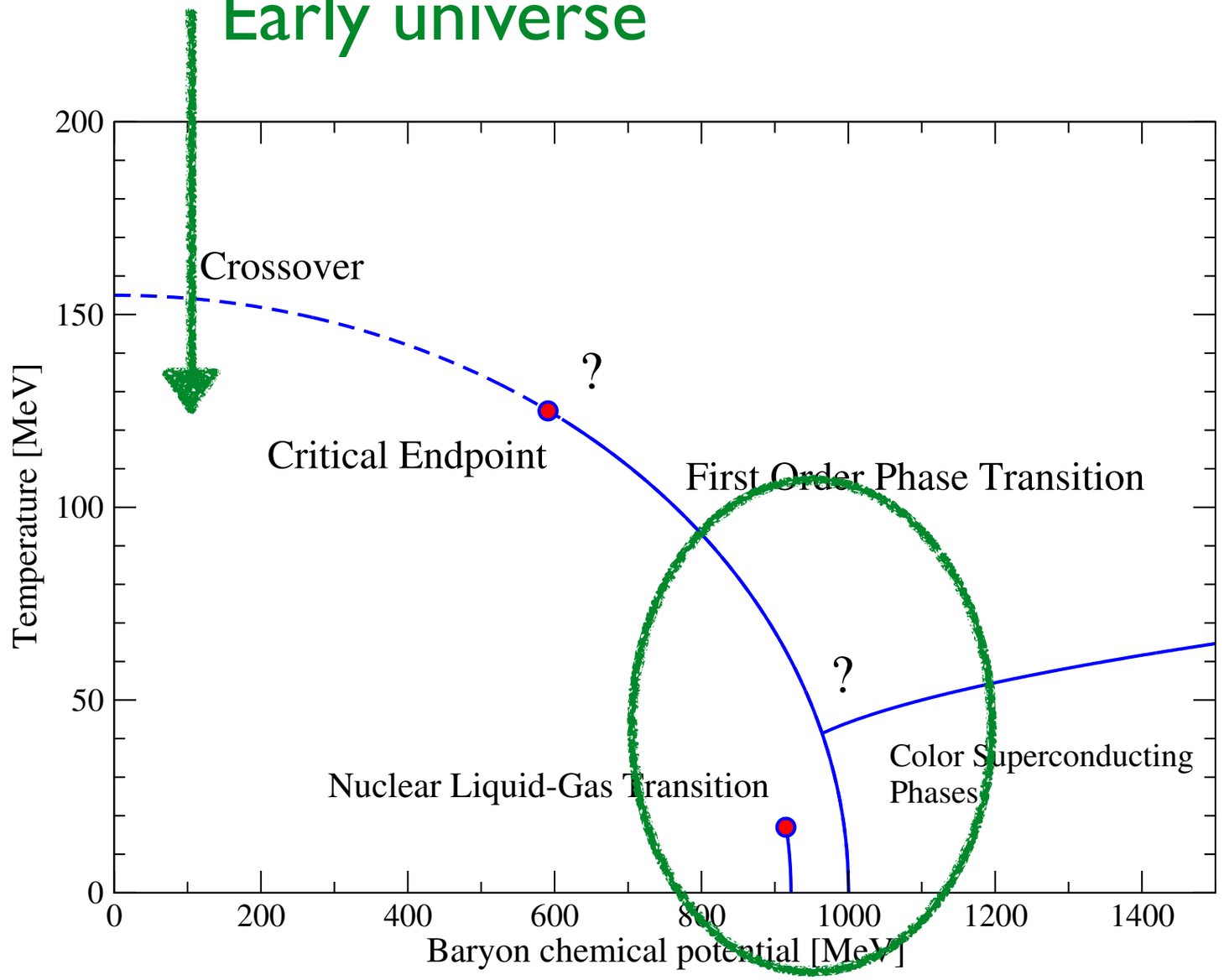

Neutron star (mergers)

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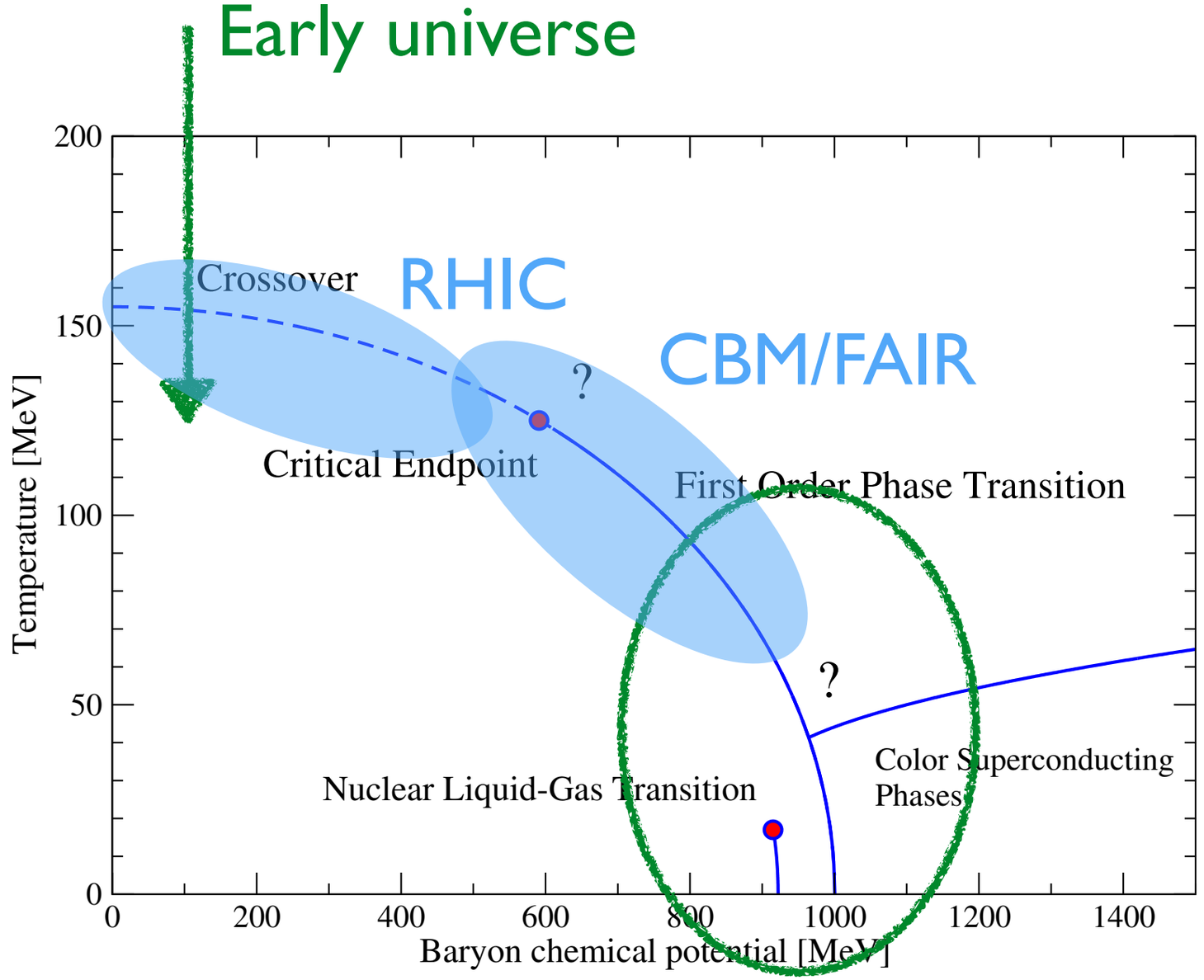

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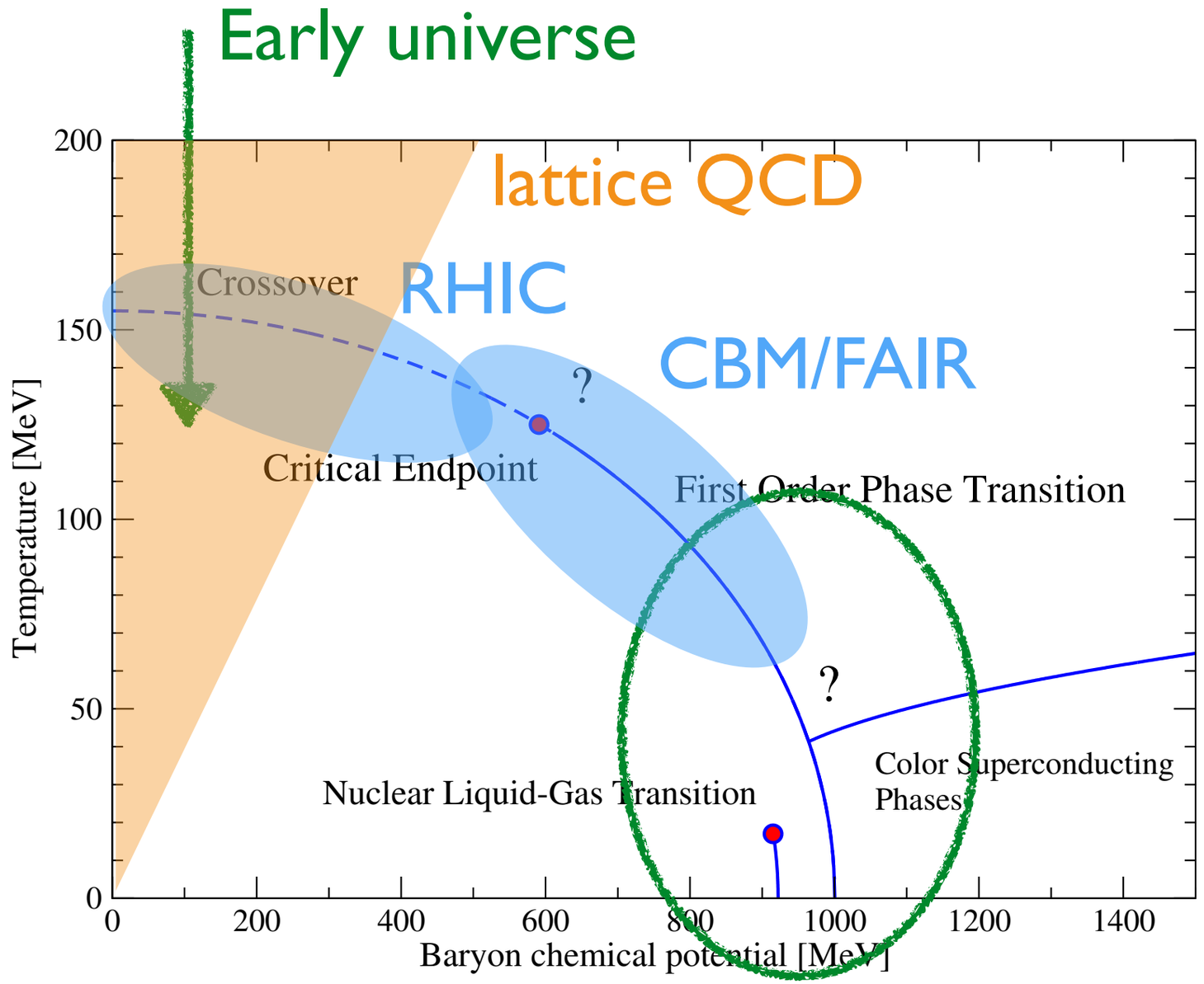

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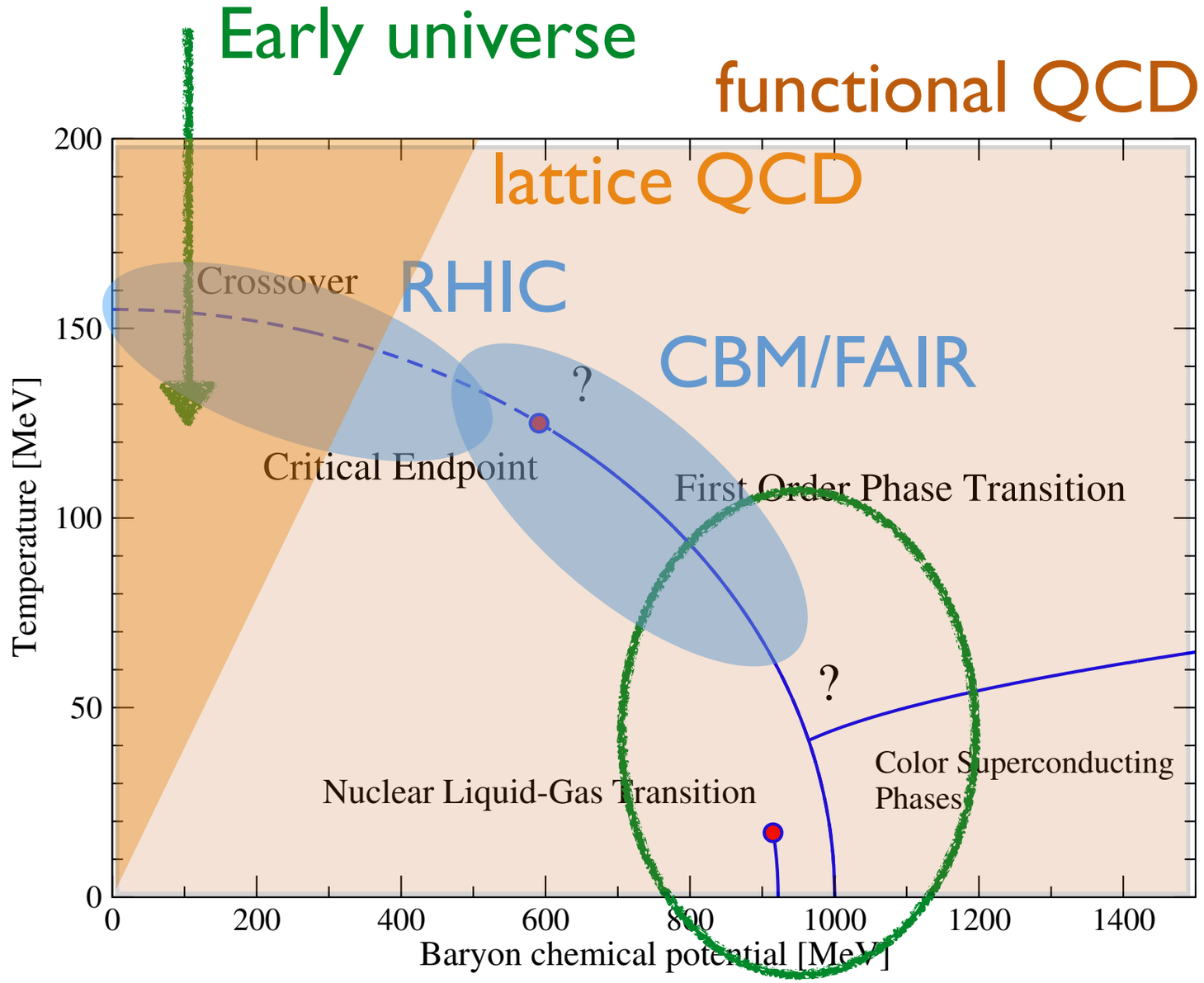

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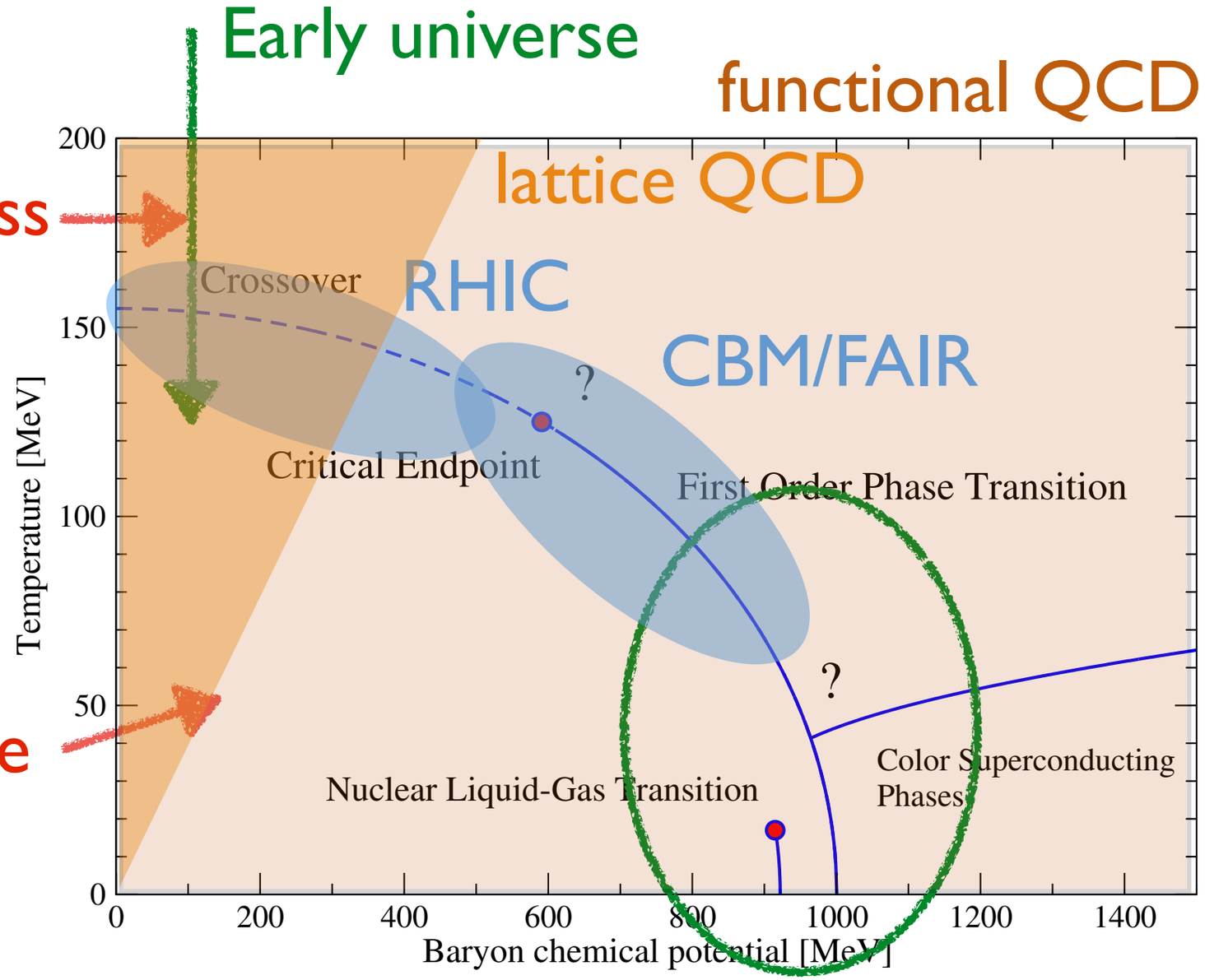
Phase diagram of quark matter: QCD

Quarks (almost) massless

Quarks massive



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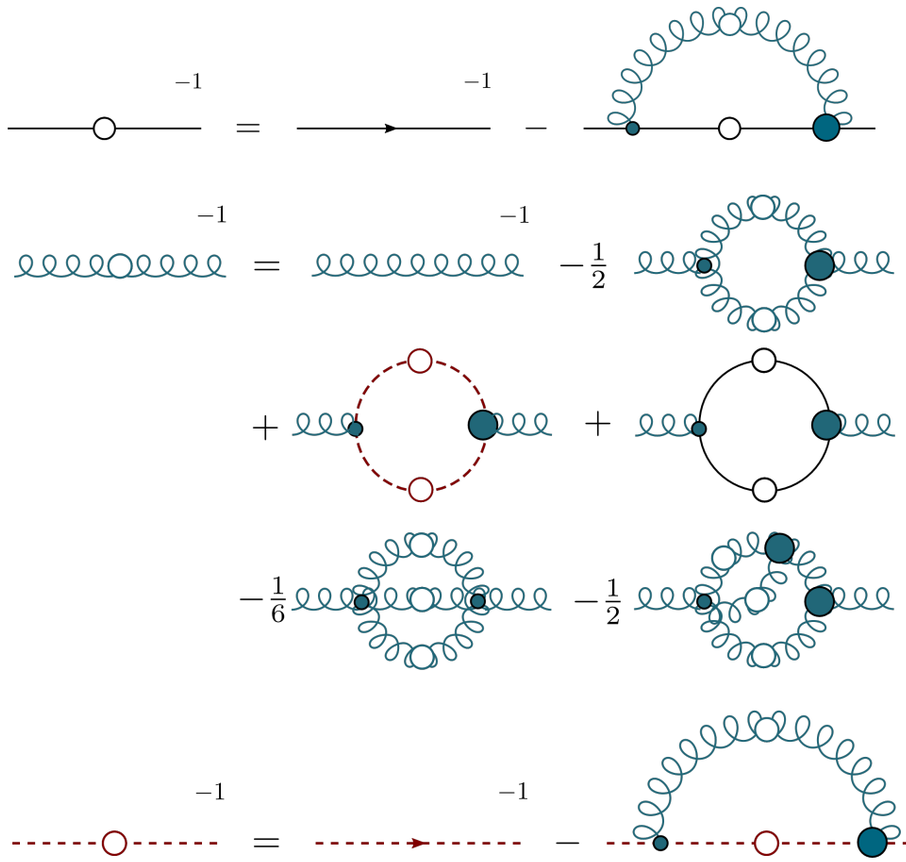
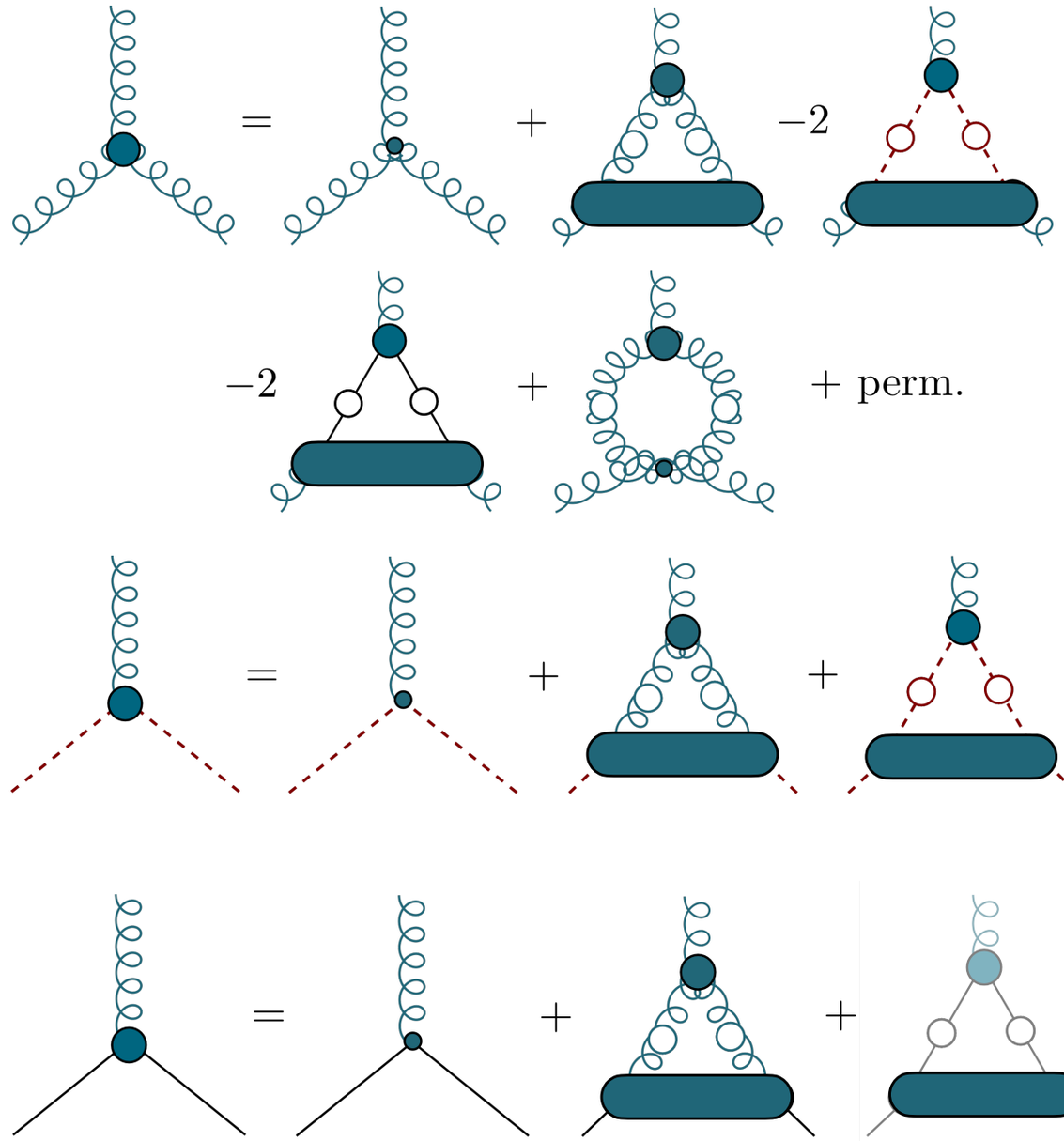
QCD with functional methods (T=0, μ=0)

propagators



for different BRL approaches see work of
Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister, Huber, Maas, Mitter, Papavassiliou, Pawlowski, Roberts, Smekal, Strodthoff, Vujinovic, Watson, Williams…

vertices



Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

CF, Alkofer, PRD67 (2003) 094020
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Huber, PRD 101 (2020) 114009

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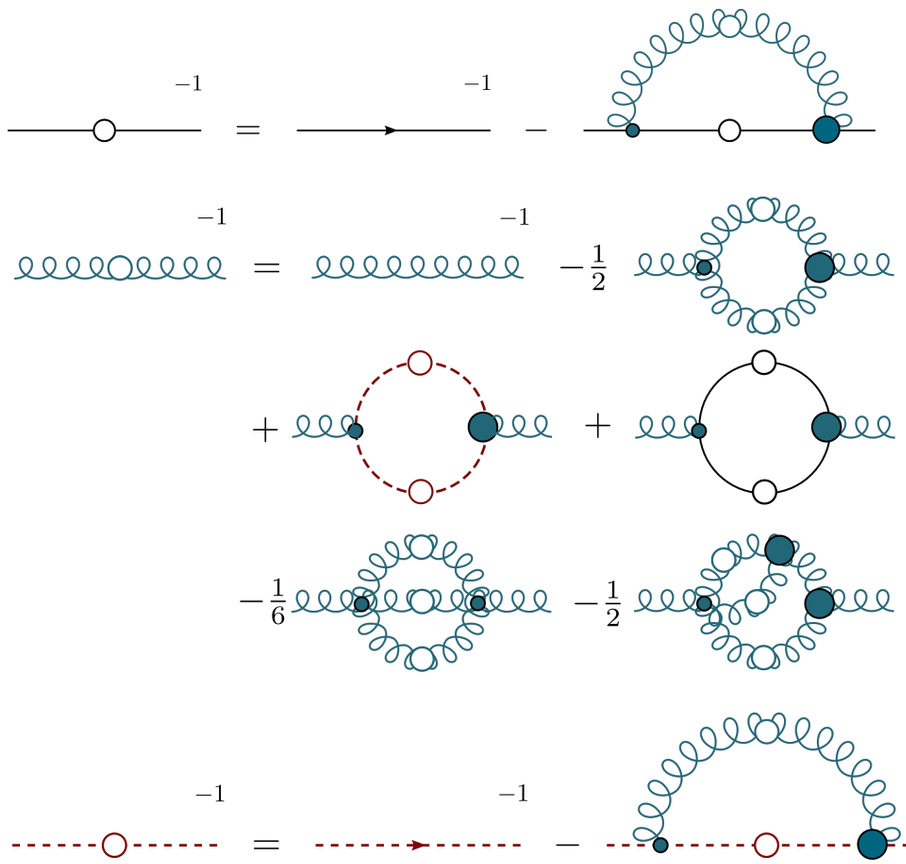

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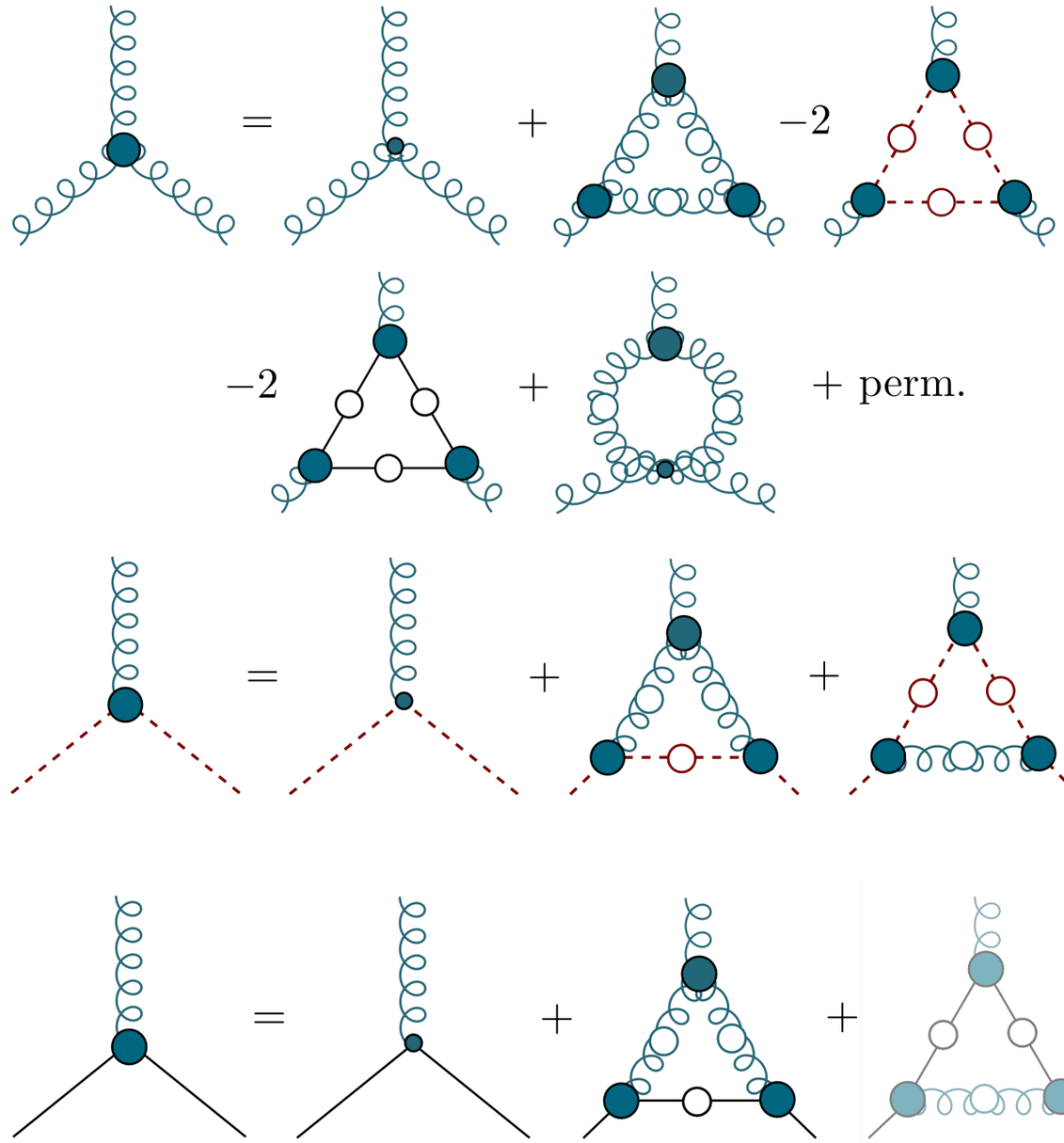

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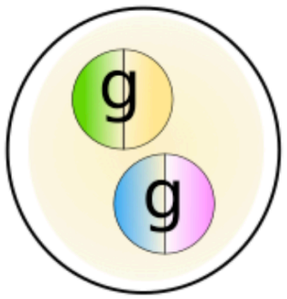
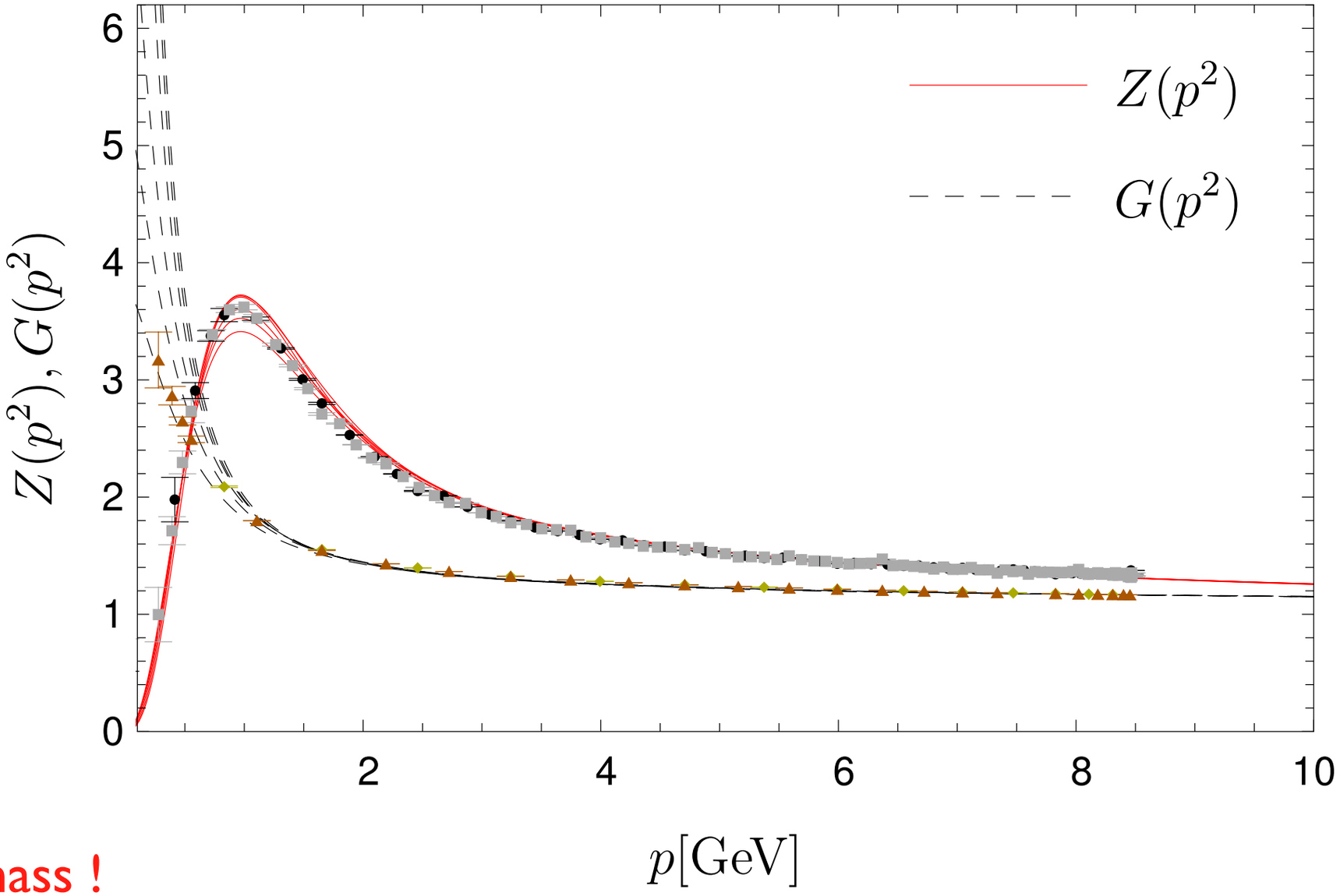
Glueballs from pure glue I



Glueball

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$



- dynamical gluon mass !

Cornwall PRD 26 (1982);
Cucchieri, Mendes PoS Lat2007 297
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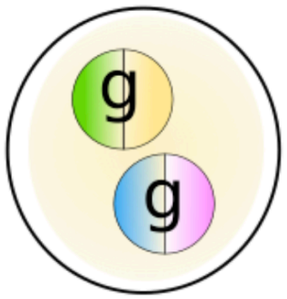

Glueball

QED: photon

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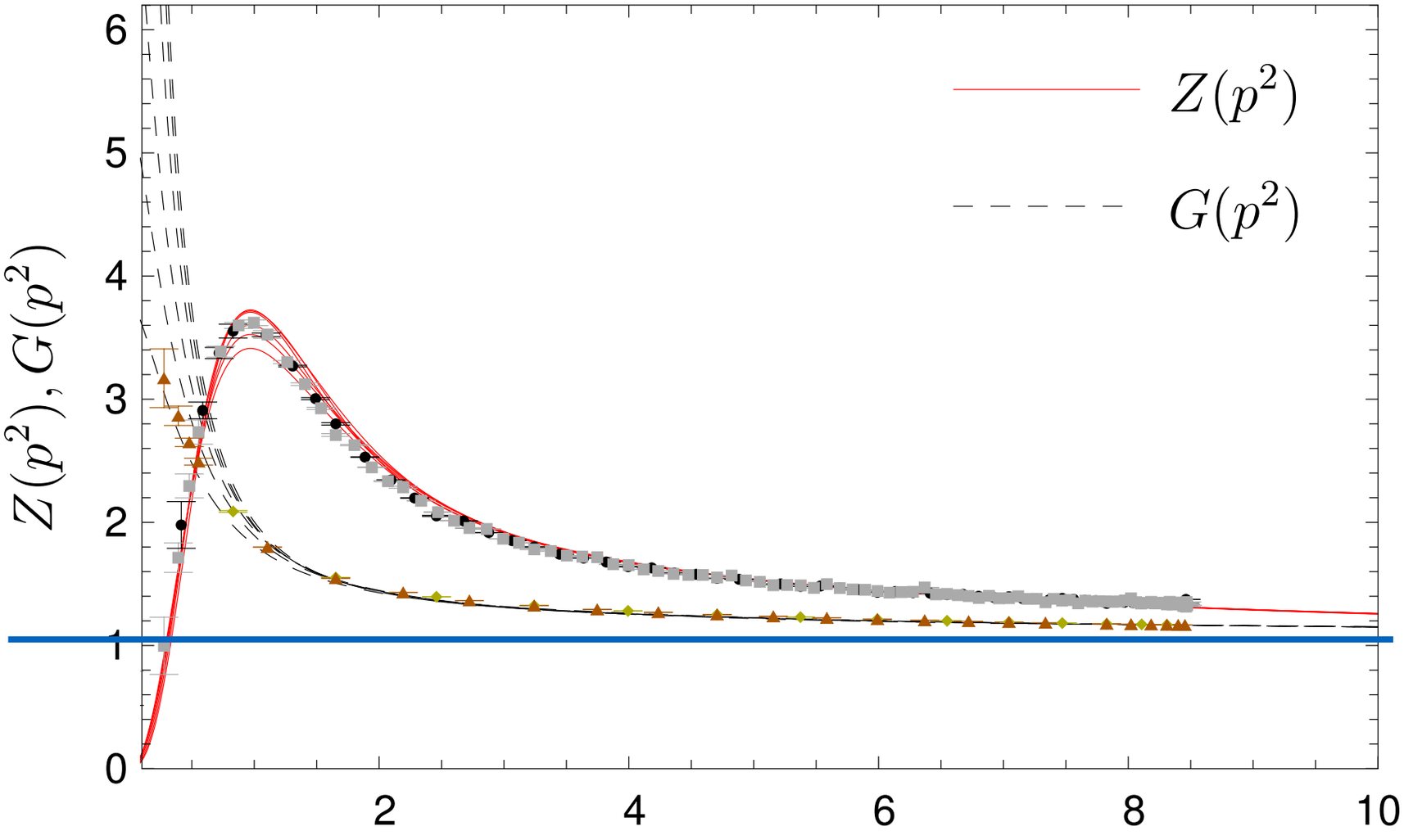

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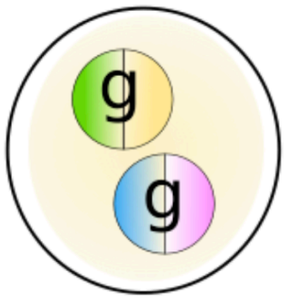

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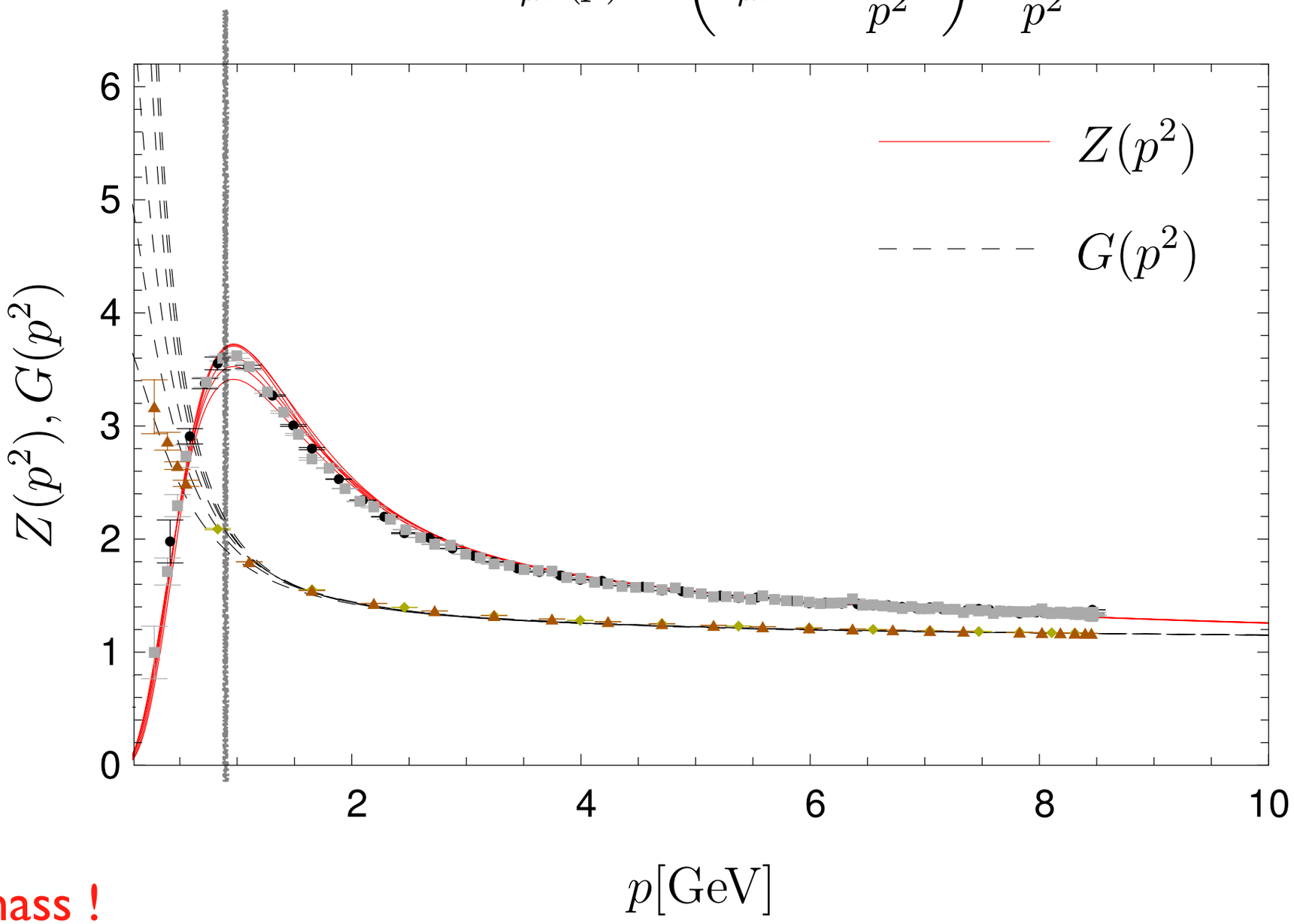

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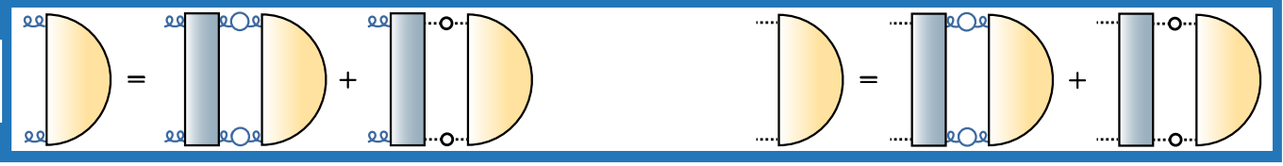

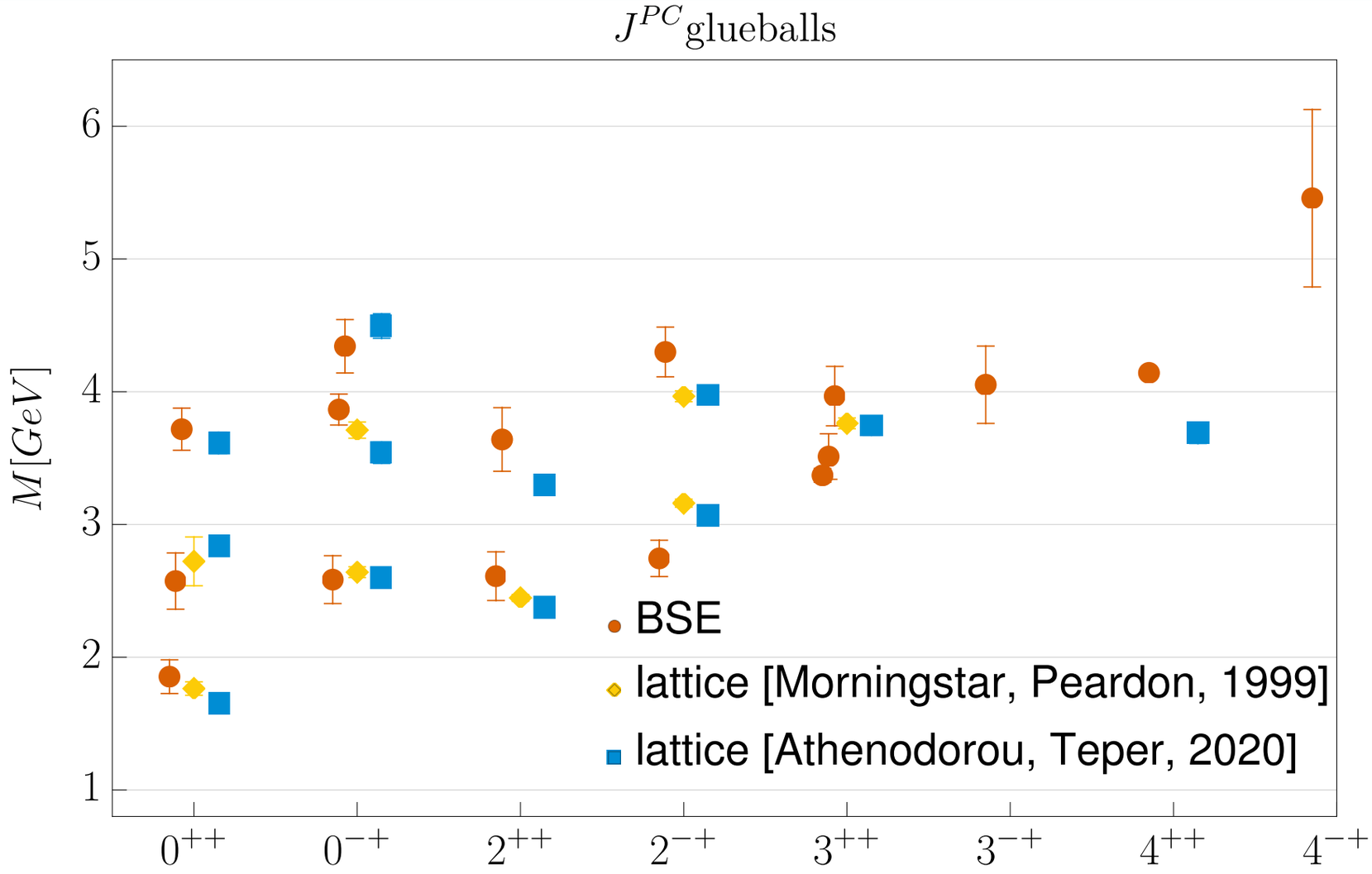

- confirmation of results from lattice YM-theory
- predictions for some channels

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
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To do:
chart the mixing of glueballs with conventional meson states…

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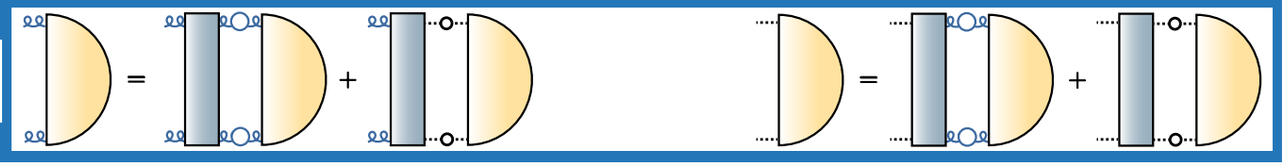

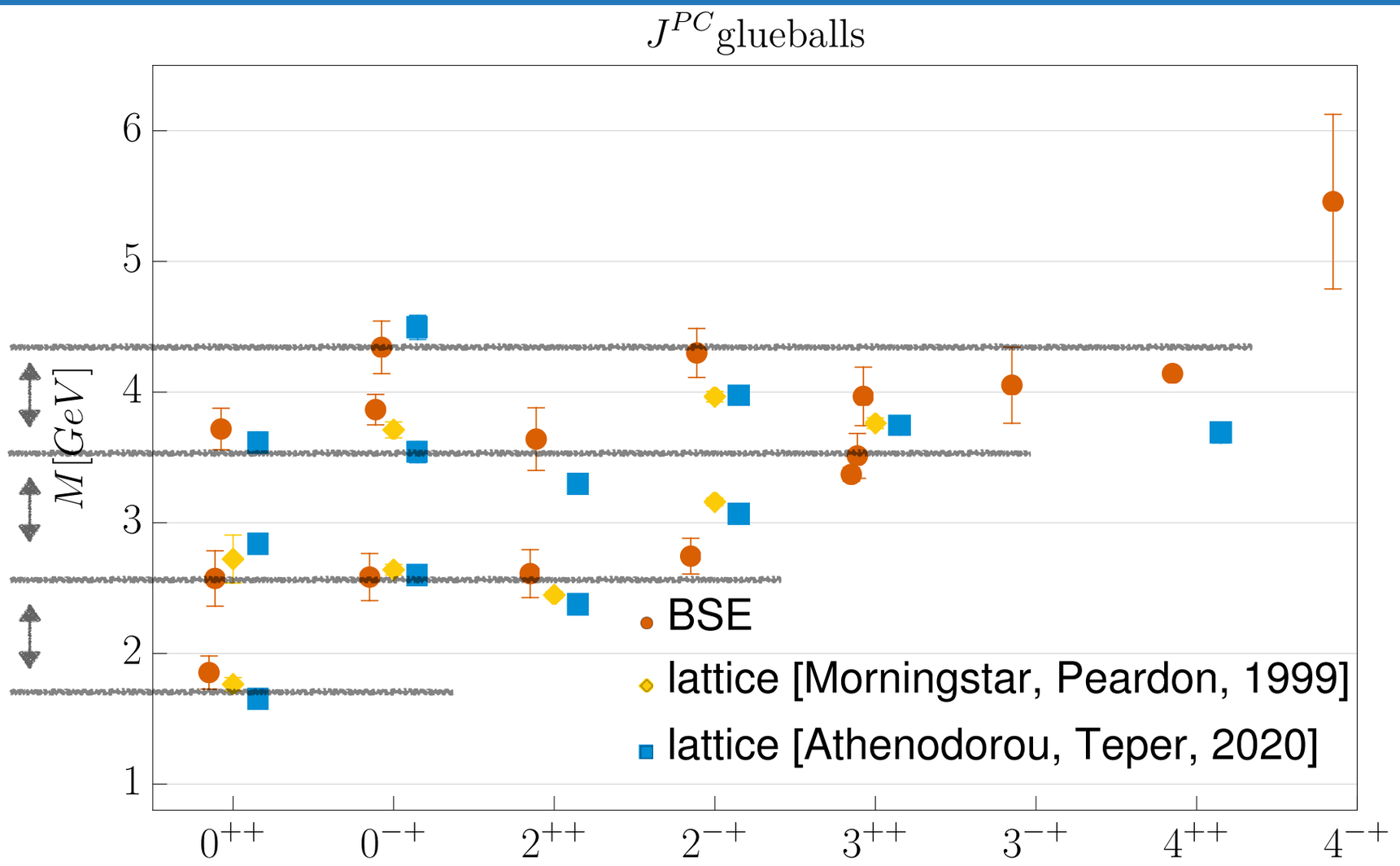

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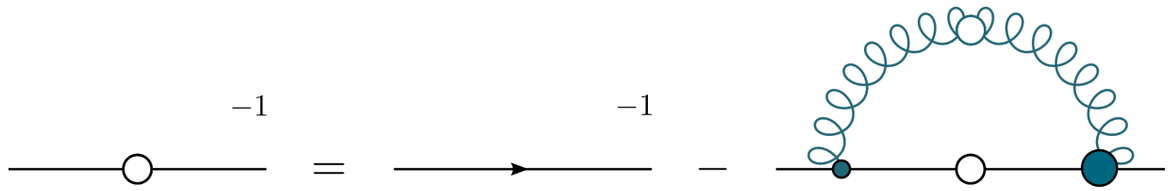
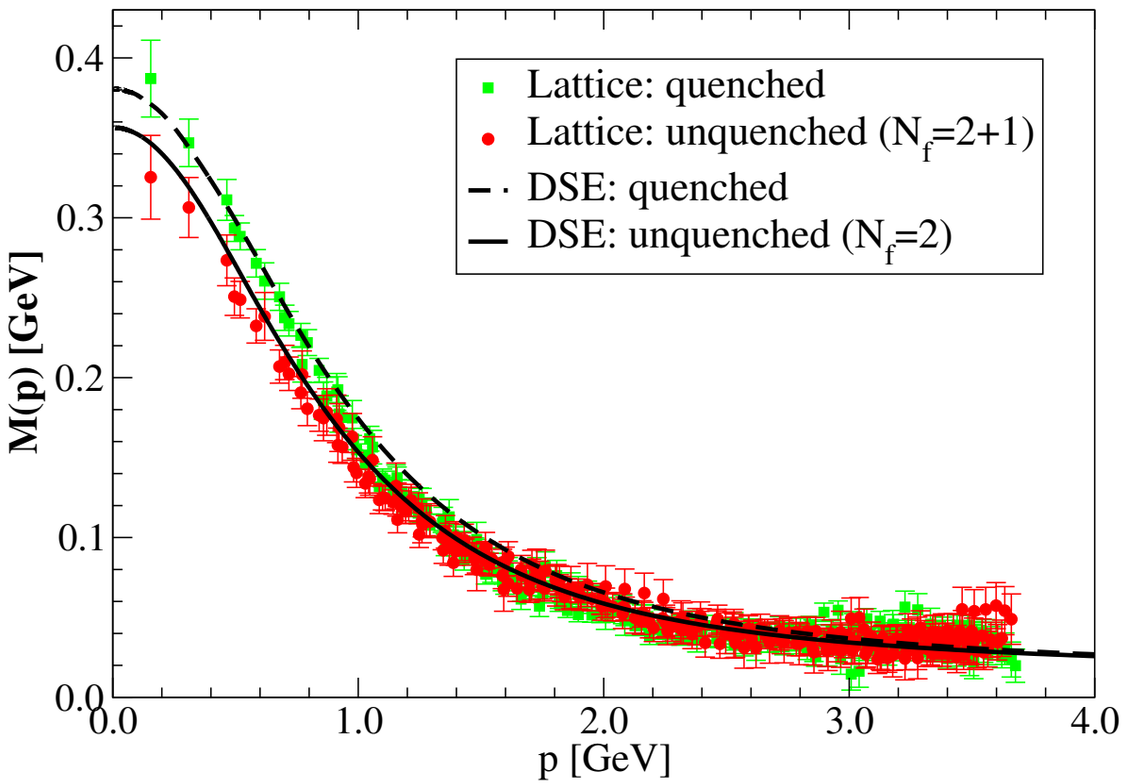
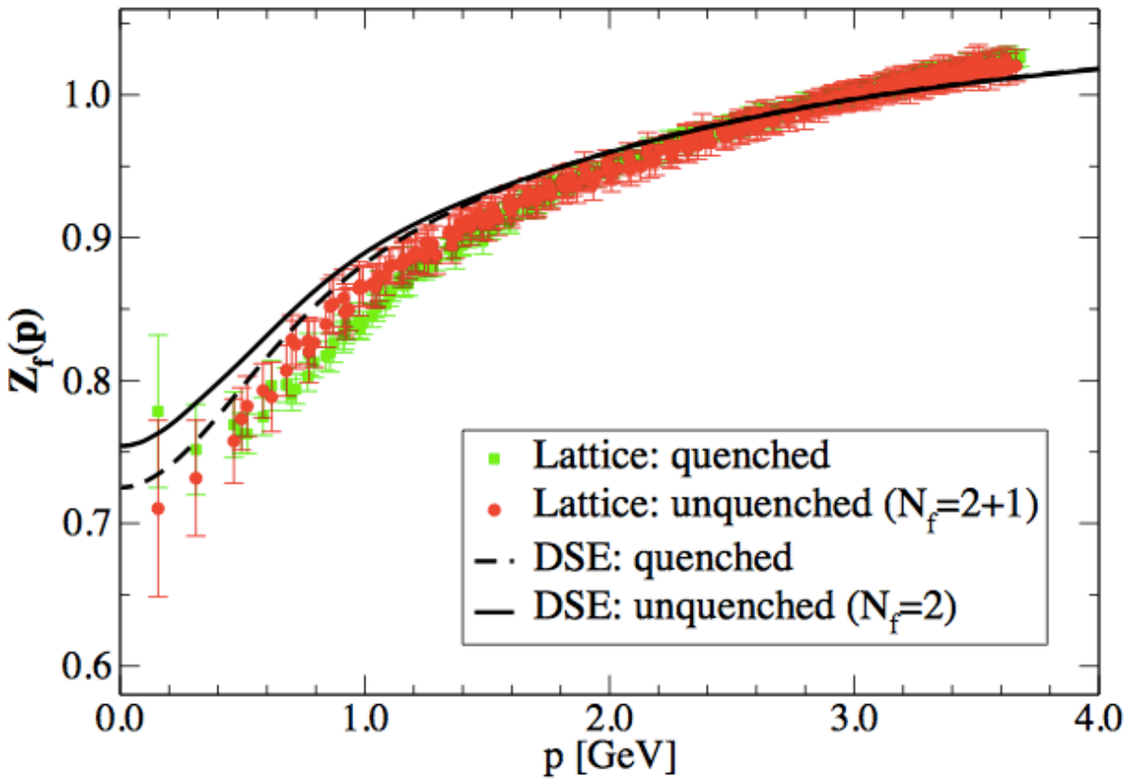


DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507





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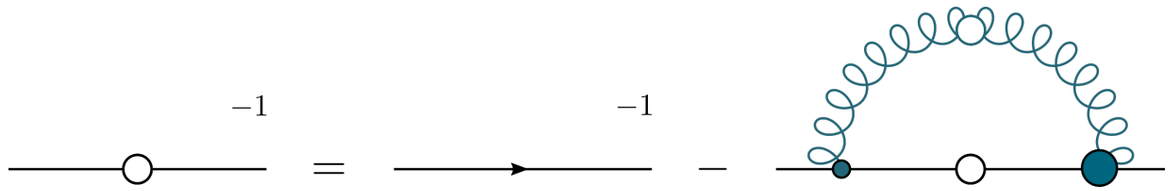

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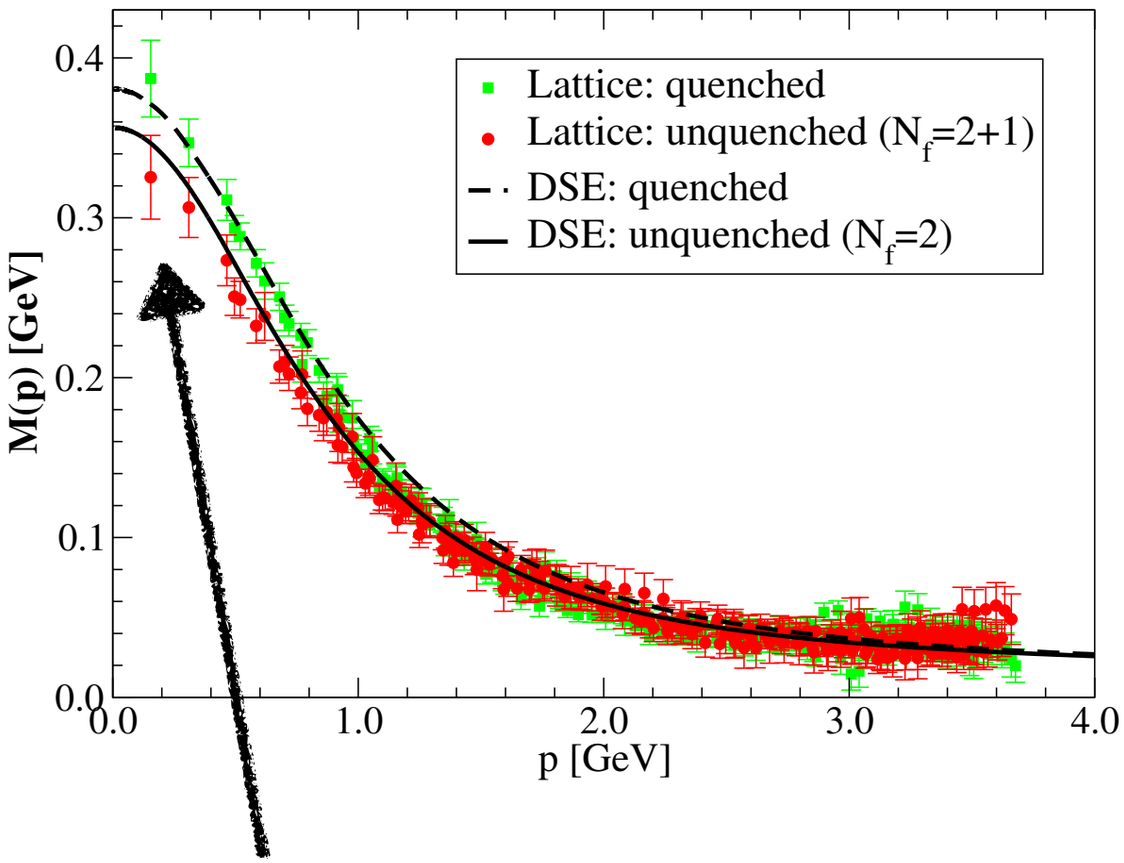

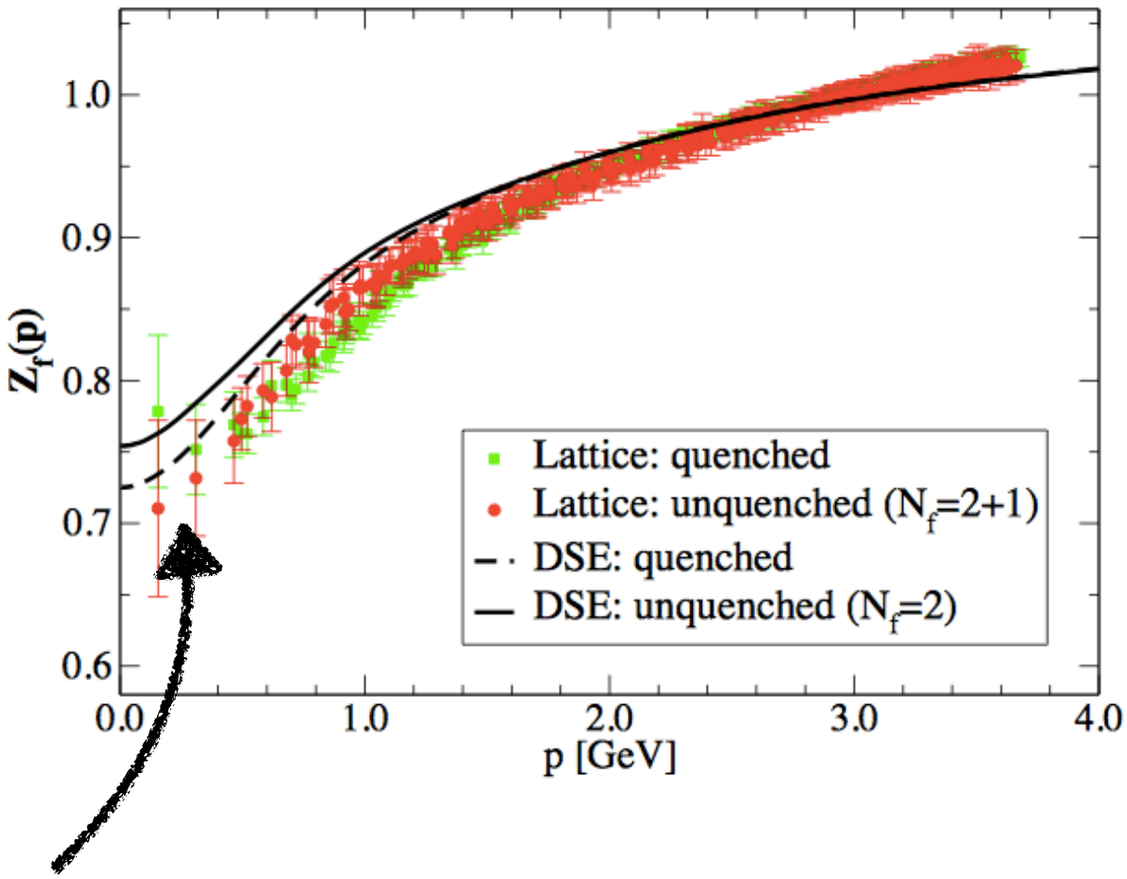

‘constituent quark’: large mass - very composite

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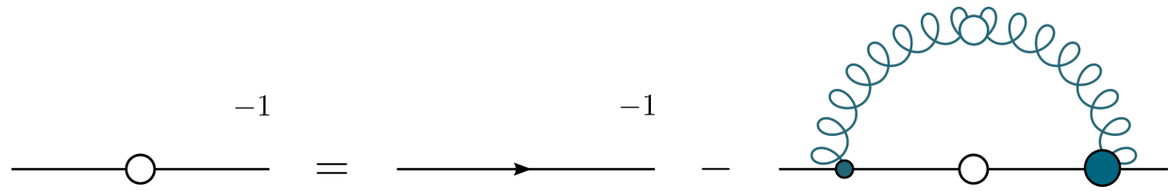

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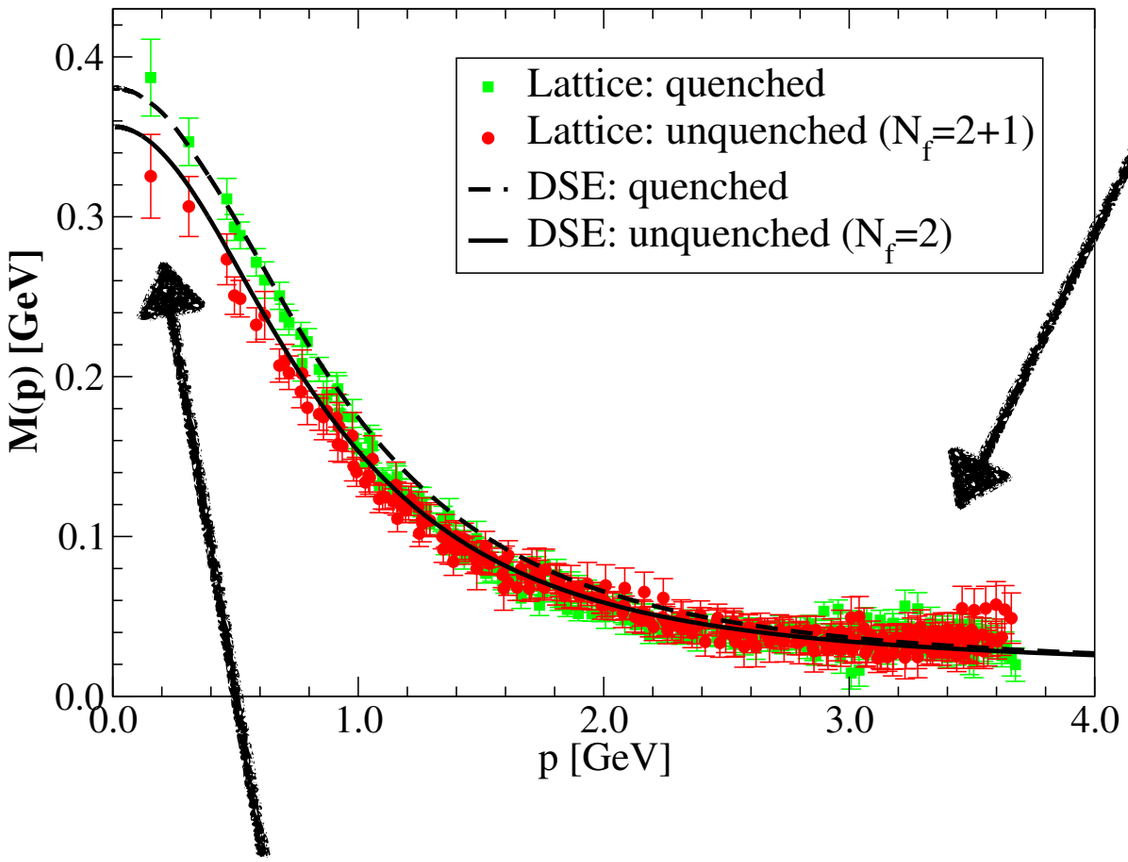
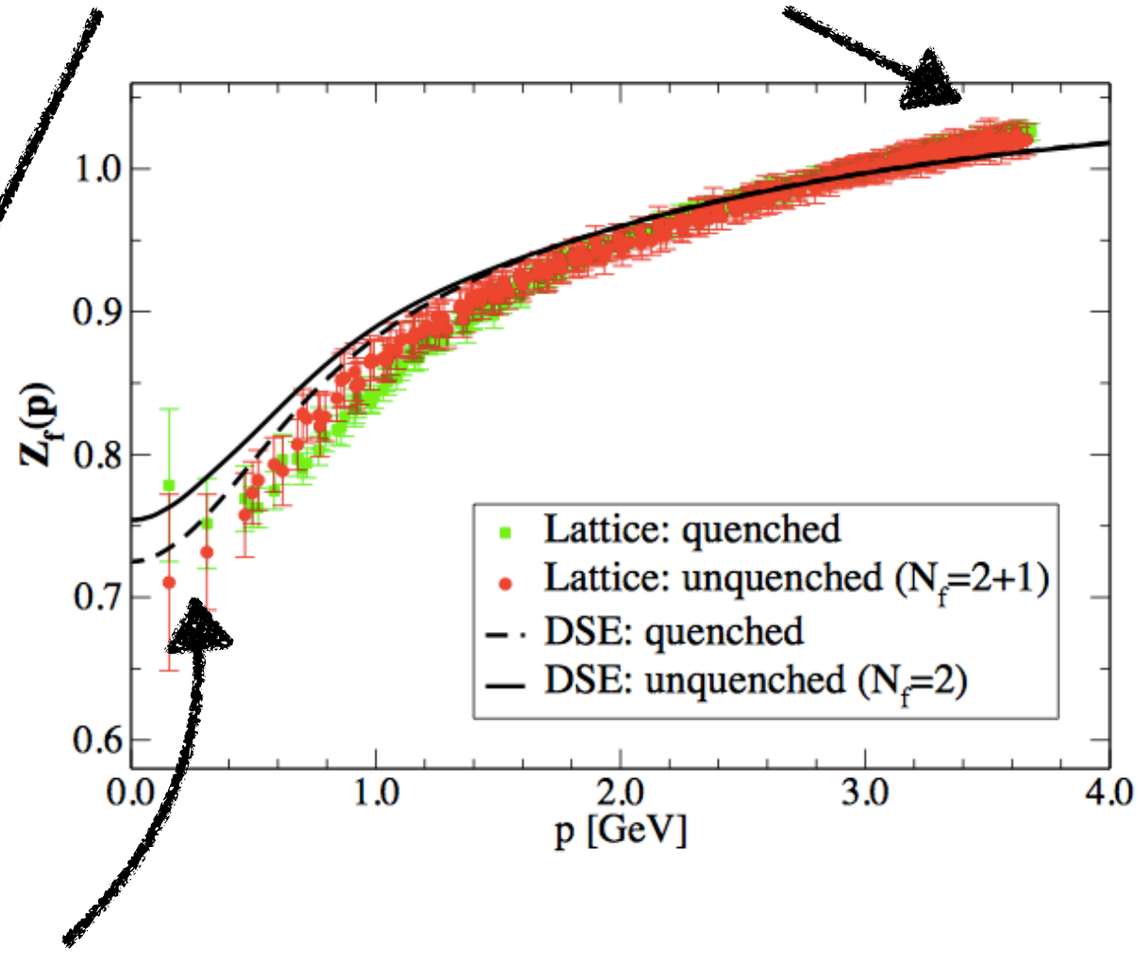
‘current quark’: small mass; non-composite





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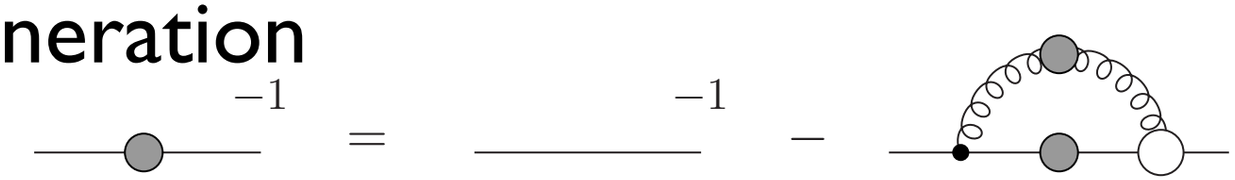

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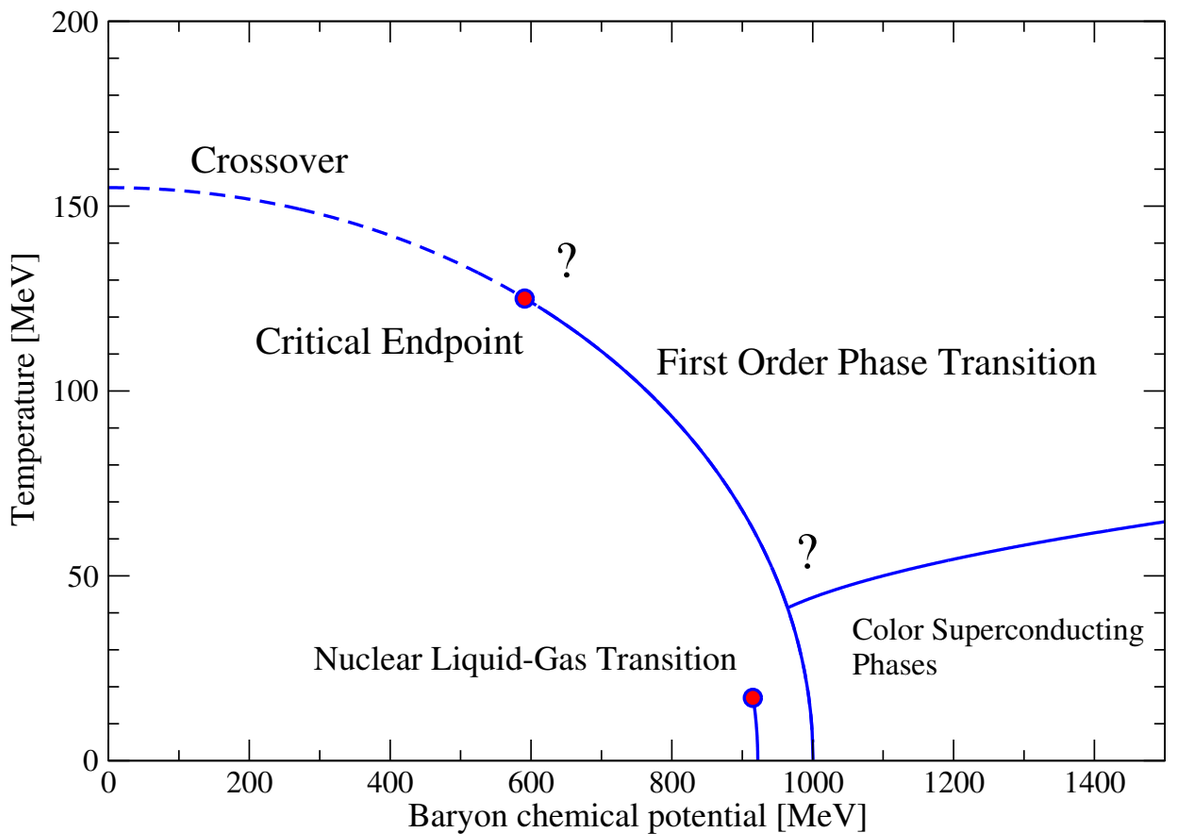
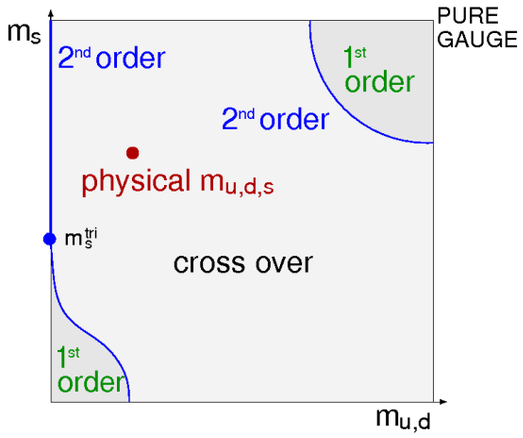
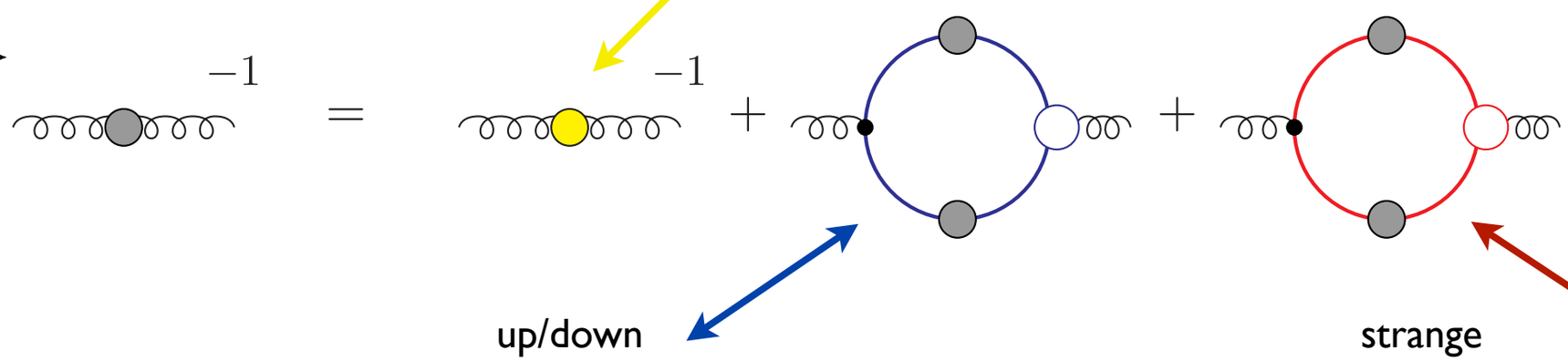
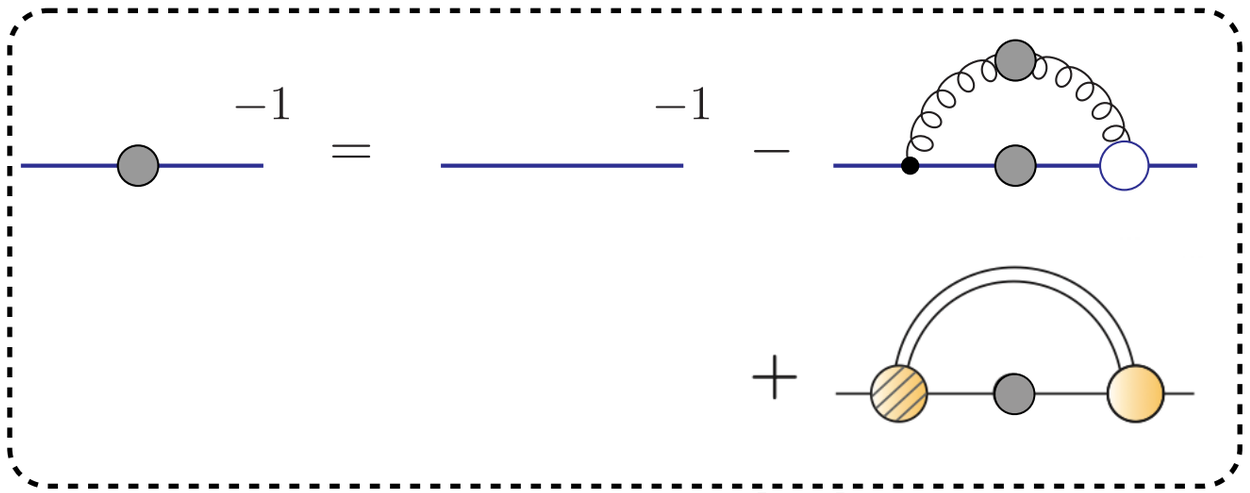


N_f=2+1-QCD with functional methods (DSEs)

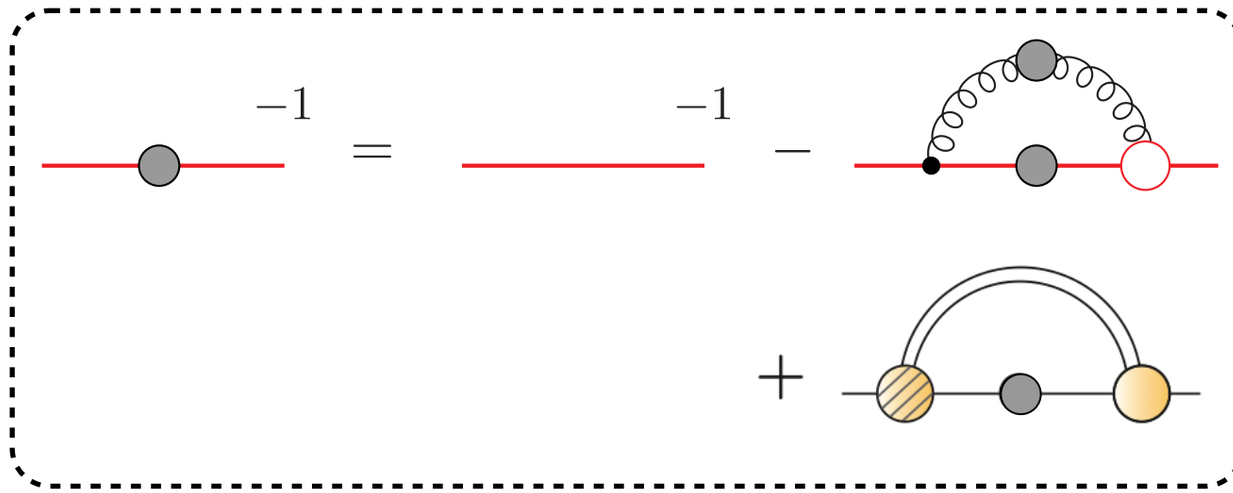


Yang-Mills theory





light mesons



strange mesons

$$S^{-1}(p) = \frac{(i\not{p} + M(p^2))}{Z_f(p^2)}$$

CF, Luecker, Welzbacher, PRD 90 (2014) 034022
Gunkel, CF, PRD 104 (2021) [2106.08356]

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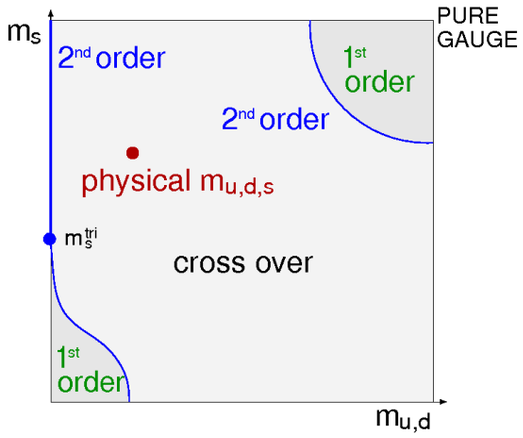

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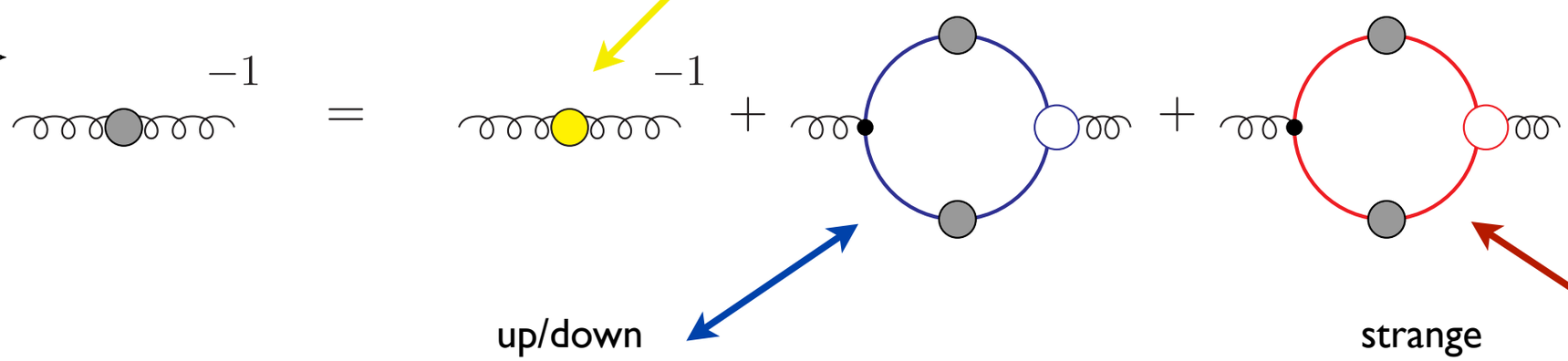
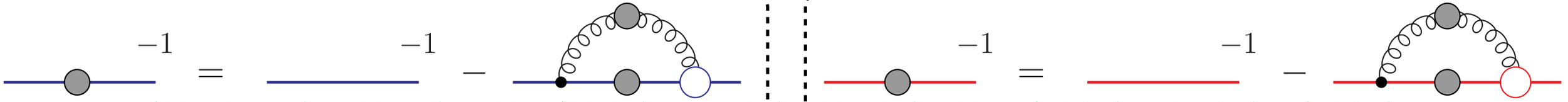


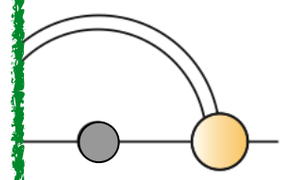


FRG: Fu, Rennecke, Pawlowski, PRD 101 (2020)

FRG/DSE: Gao, Pawlowski, PLB 820 (2021)

DSE: Gunkel, CF, PRD 104 (2021)

quality of truncations on same level Pawlowski, CF, in prep.



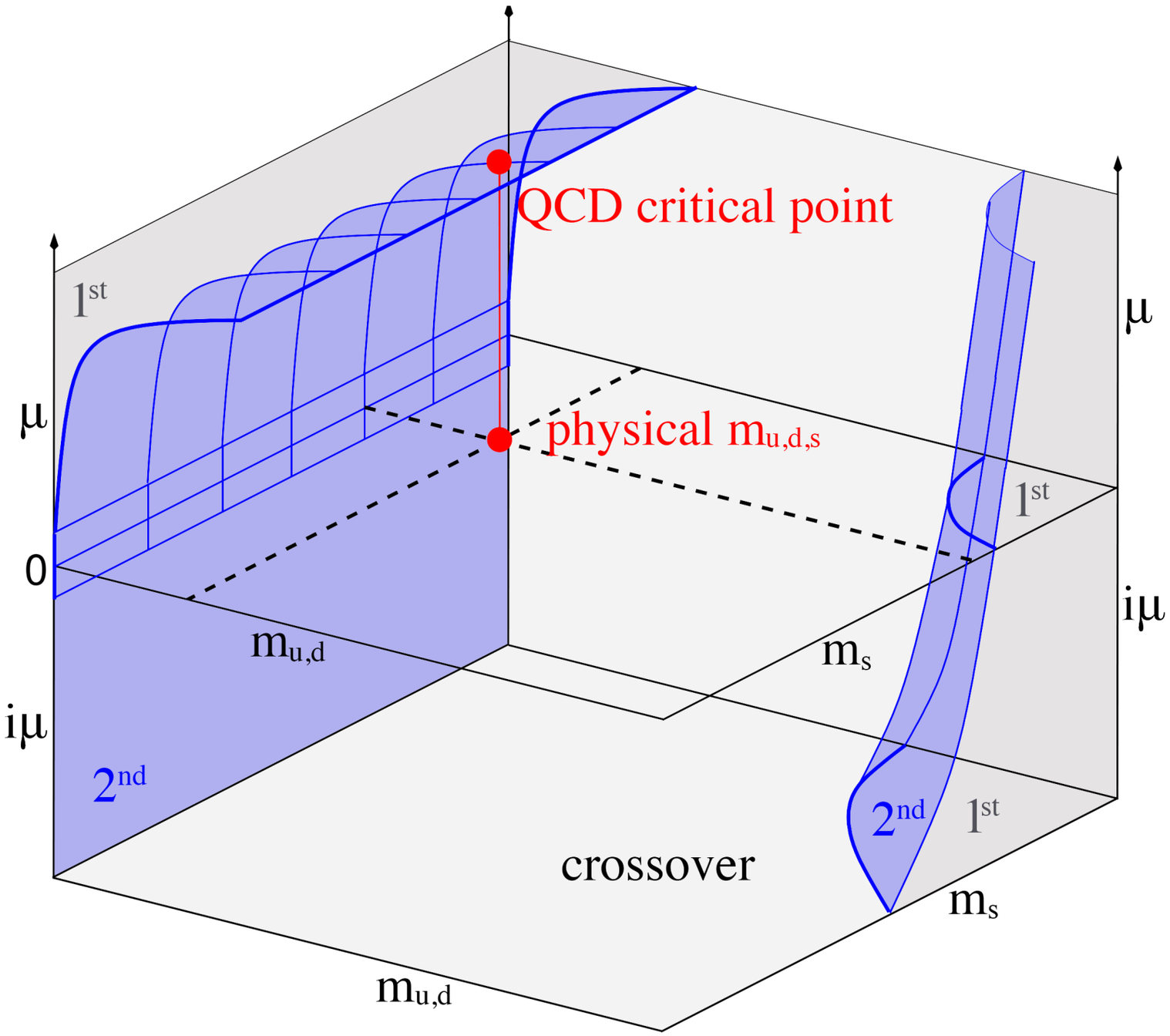
mesons

$S^{-1}(p) =$

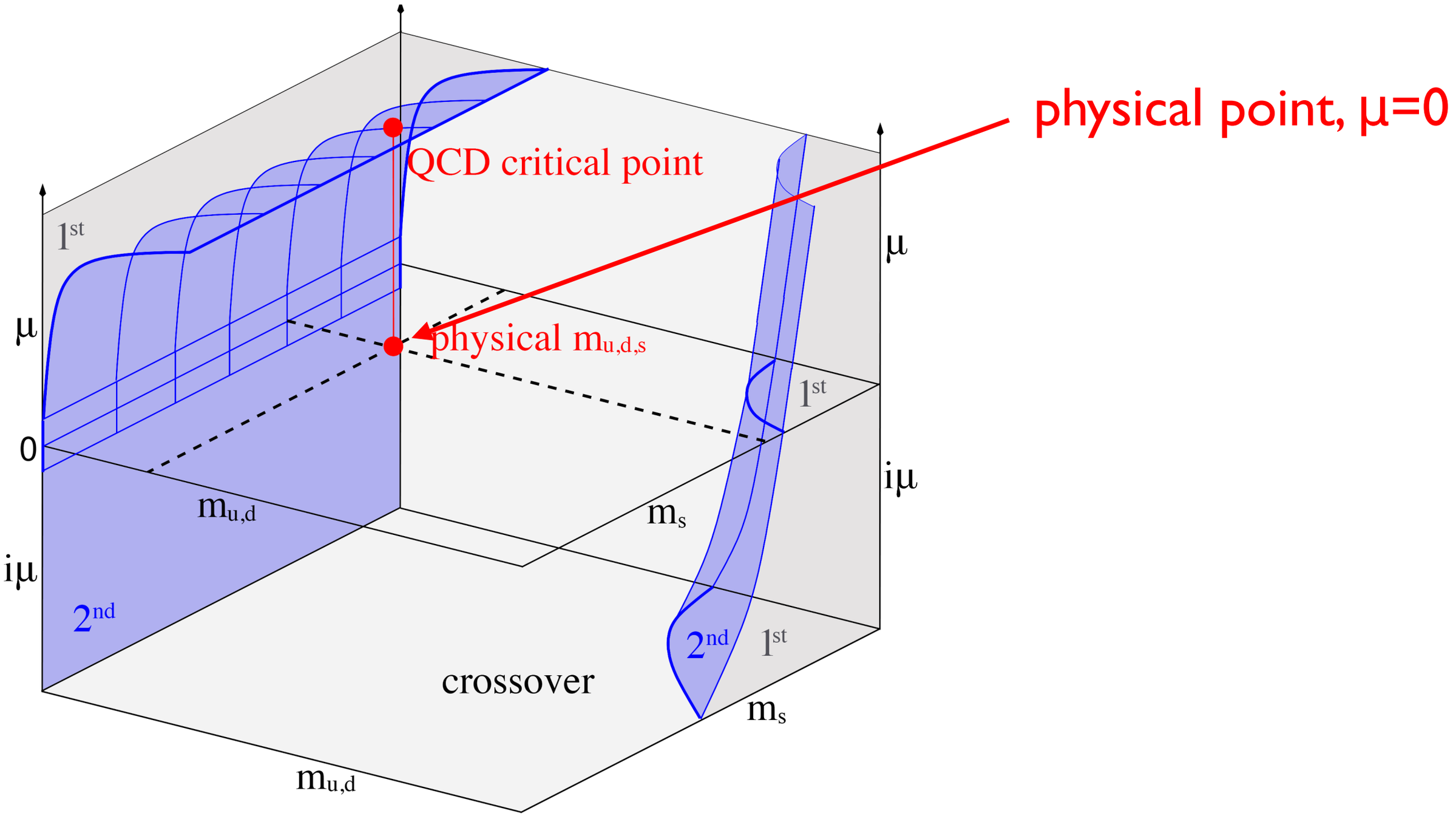
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Gunkel, CF, PRD 104 (2021) [2106.08356]

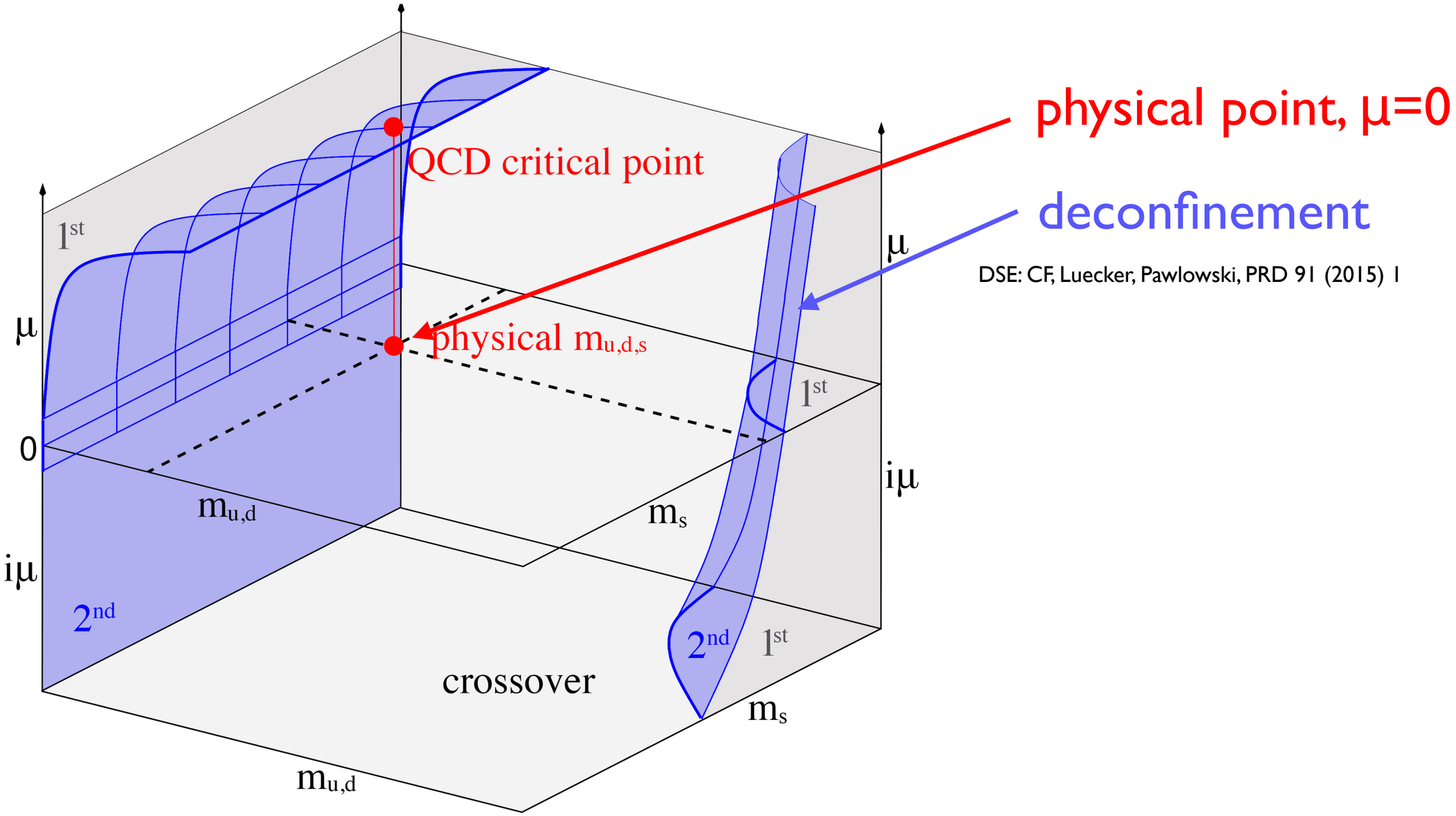
Roadmap


QCD critical point
physical $m_{u,d,s}$
1st
μ
0
$m_{u,d}$
m_s
iμ
2nd
crossover
1st
μ
1st
iμ
2nd
1st
m_s
$m_{u,d}$

Roadmap


QCD critical point
physical point, μ=0
physical $m_{u,d,s}$
1st
μ
0
$m_{u,d}$
m_s
1st
μ
iμ
2nd
2nd
1st
crossover
m_s
$m_{u,d}$

Roadmap

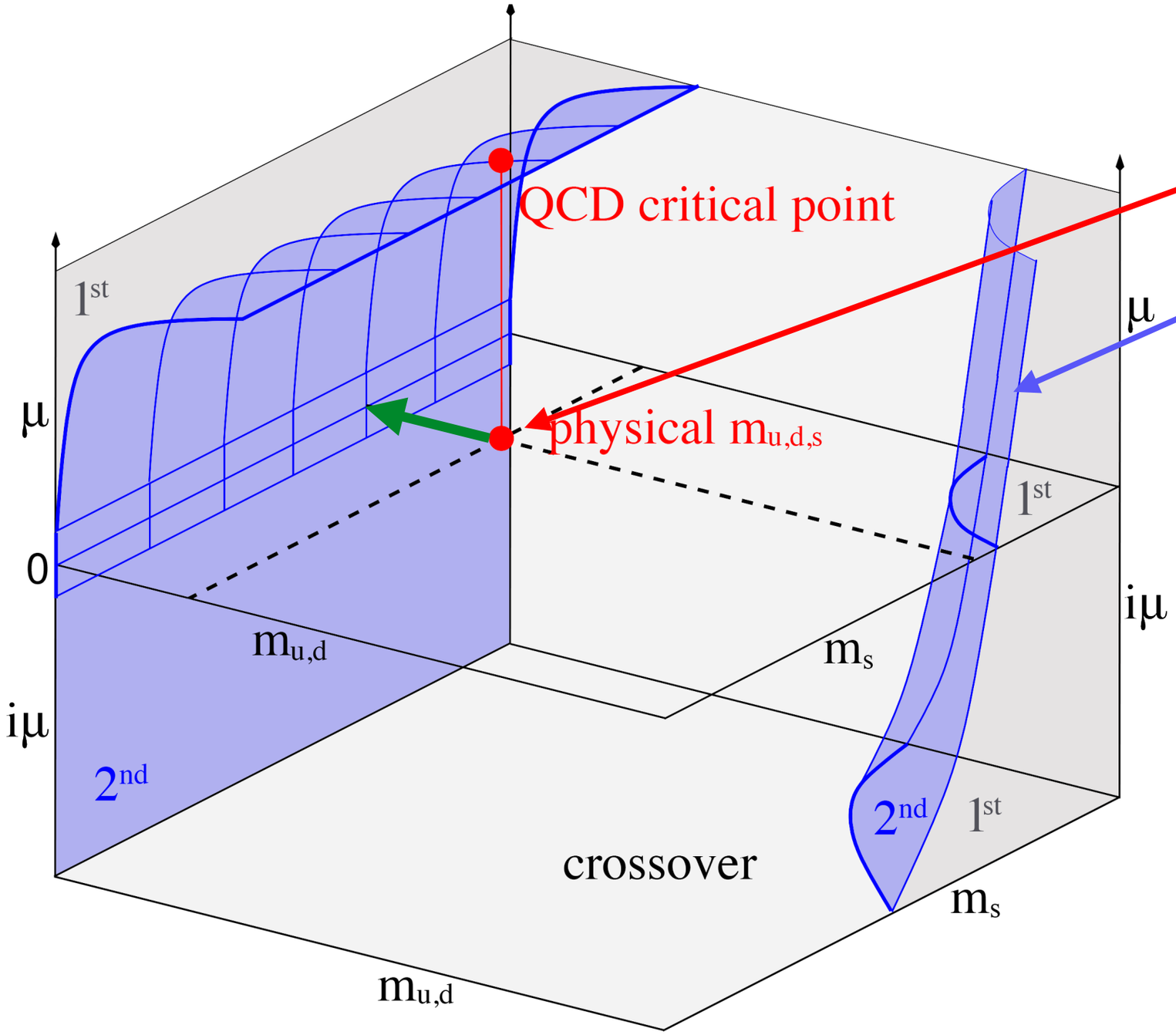


physical point, μ=0

deconfinement

DSE: CF, Luecker, Pawlowski, PRD 91 (2015) 1

Roadmap



physical point, μ=0

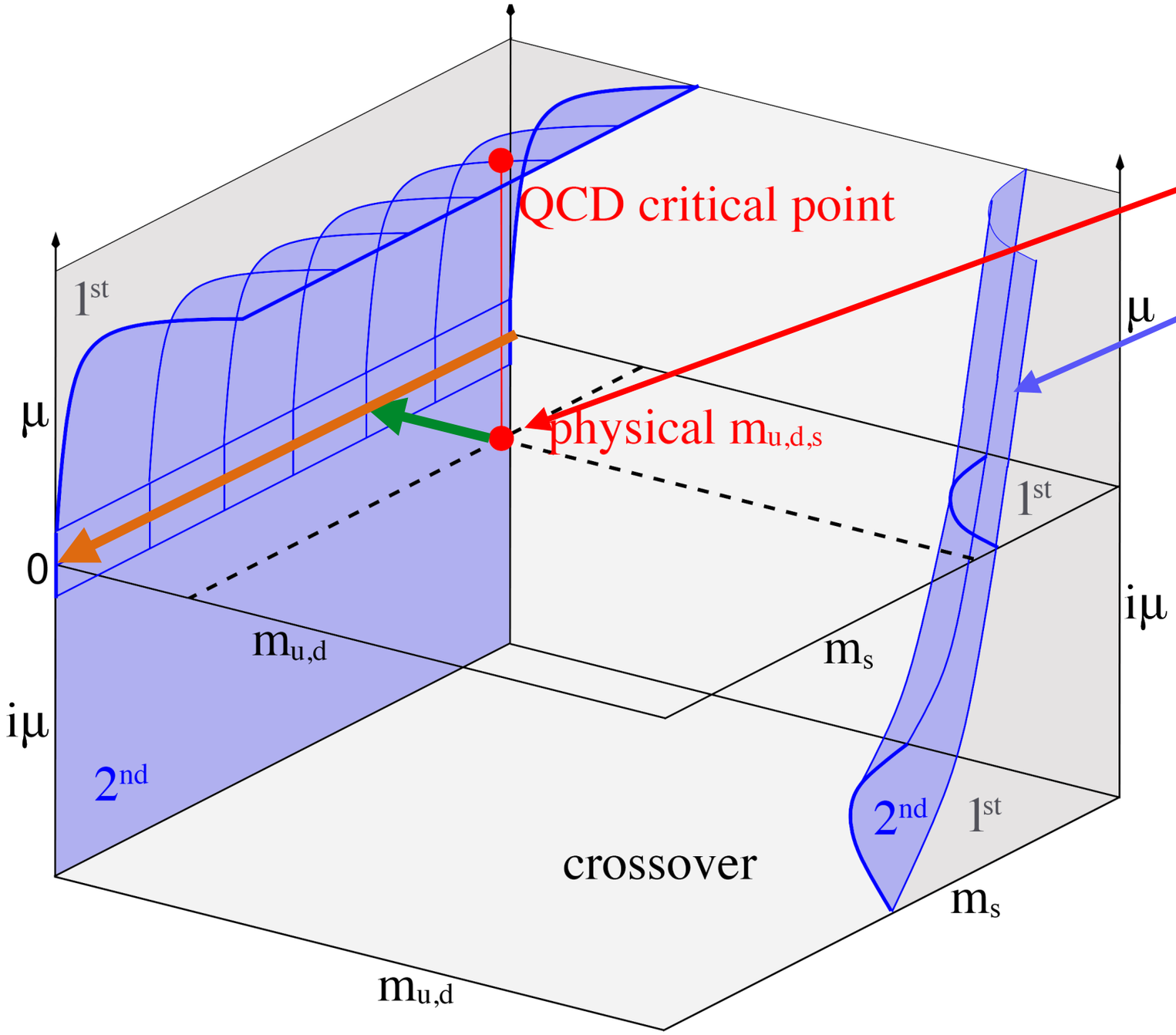
deconfinement

DSE: CF, Luecker, Pawlowski, PRD 91 (2015) 1

towards chiral limit

FRG: Braun et al, PRD 102 (2020) 5, 056010
FRG/DSE: Gao and Pawlowski PRD 105(2022) 094020
DSE: Bernhardt and CF, PRD 108 (2023) 114018

Roadmap



physical point, μ=0

deconfinement

DSE: CF, Luecker, Pawlowski, PRD 91 (2015) 1

towards chiral limit

FRG: Braun et al, PRD 102 (2020) 5, 056010
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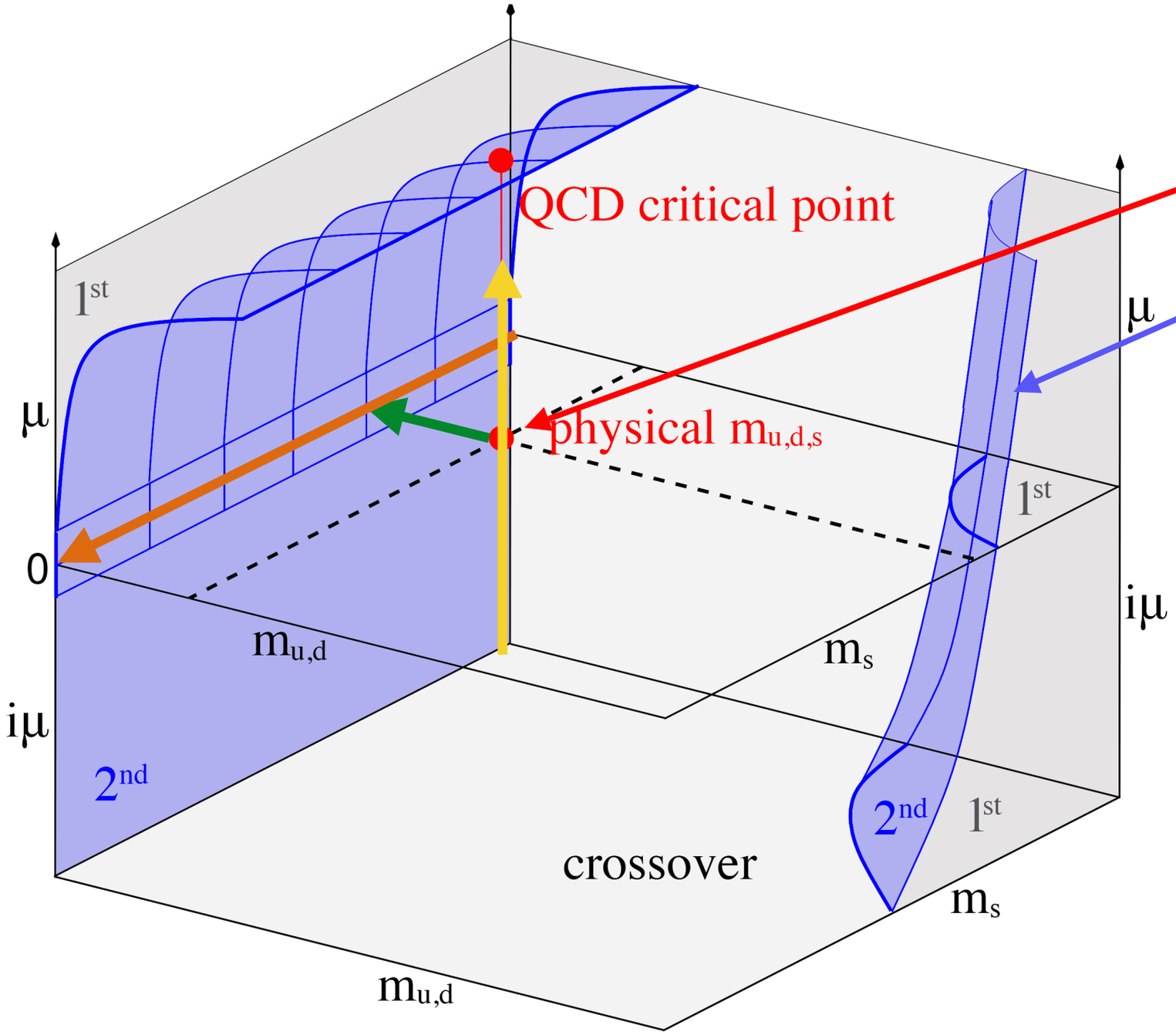
chiral limit

DSE: Bernhardt and CF, PRD 108 (2023) 114018

Roadmap



physical point, μ=0

deconfinement

DSE: CF, Luecker, Pawlowski, PRD 91 (2015) 1

towards chiral limit

FRG: Braun et al, PRD 102 (2020) 5, 056010
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DSE: Bernhardt and CF, PRD 108 (2023) 114018

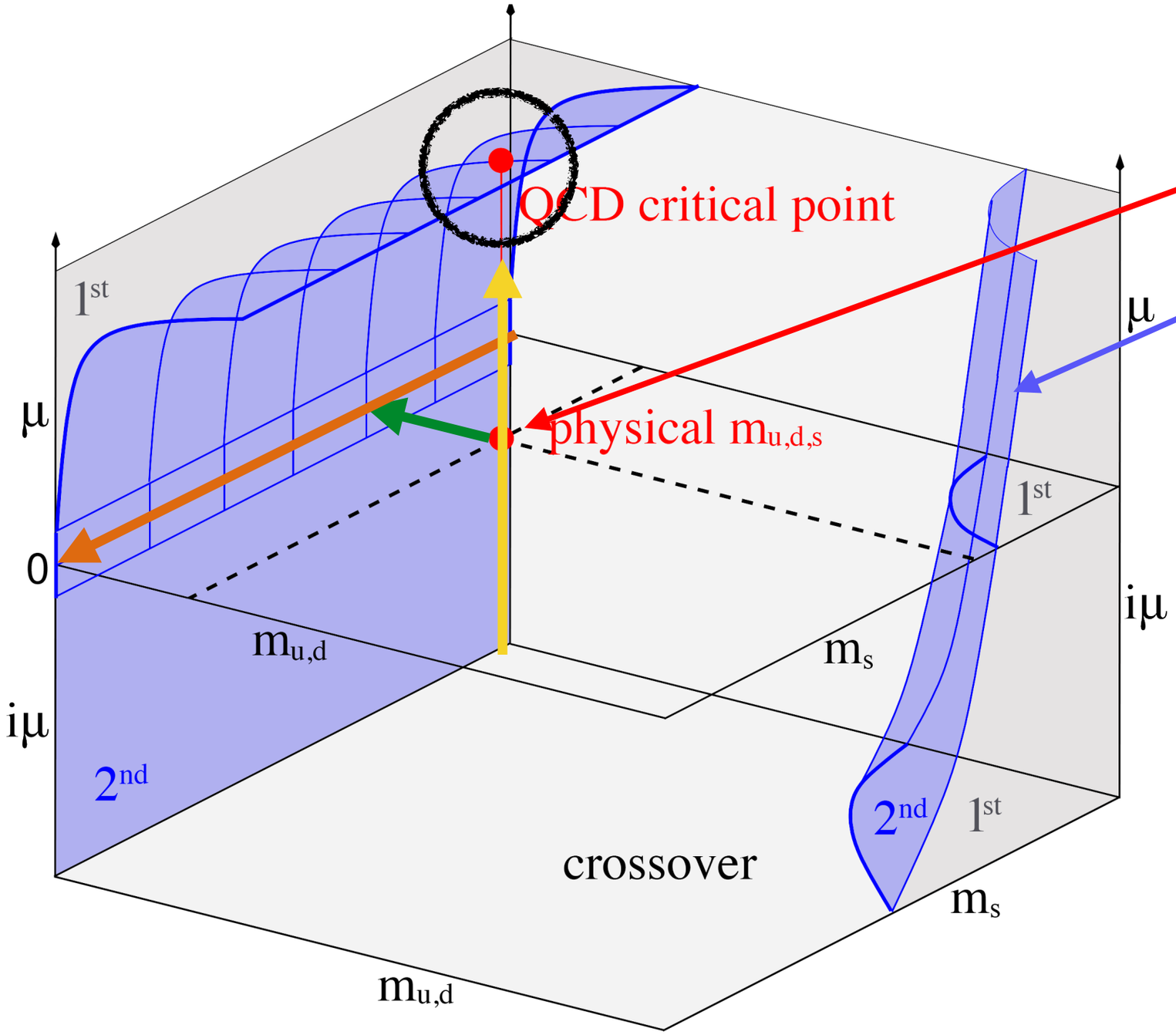
chiral limit

DSE: Bernhardt and CF, PRD 108 (2023) 114018

imaginary μ

FRG (Nf=2): Braun et. al. PRL 106 (2011)
DSE: Bernhardt, CF, EPJA 59 (2023) 8, 181

Roadmap



physical point, μ=0

deconfinement

DSE: CF, Luecker, Pawlowski, PRD 91 (2015) 1

towards chiral limit

FRG: Braun et al, PRD 102 (2020) 5, 056010
FRG/DSE: Gao and Pawlowski PRD 105(2022) 094020
DSE: Bernhardt and CF, PRD 108 (2023) 114018

chiral limit

DSE: Bernhardt and CF, PRD 108 (2023) 114018

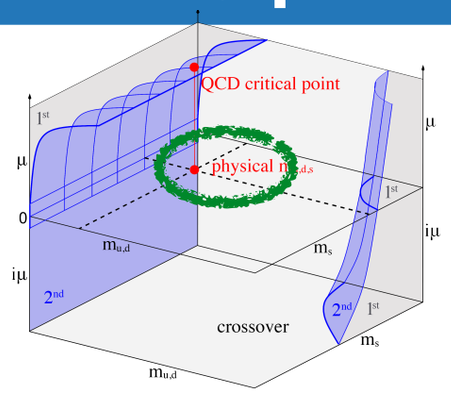
imaginary μ

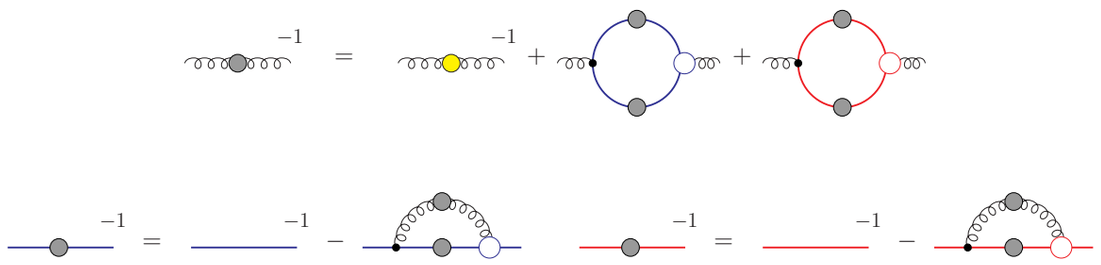
FRG (Nf=2): Braun et. al. PRL 106 (2011)
DSE: Bernhardt, CF, EPJA 59 (2023) 8, 181

location of CEP

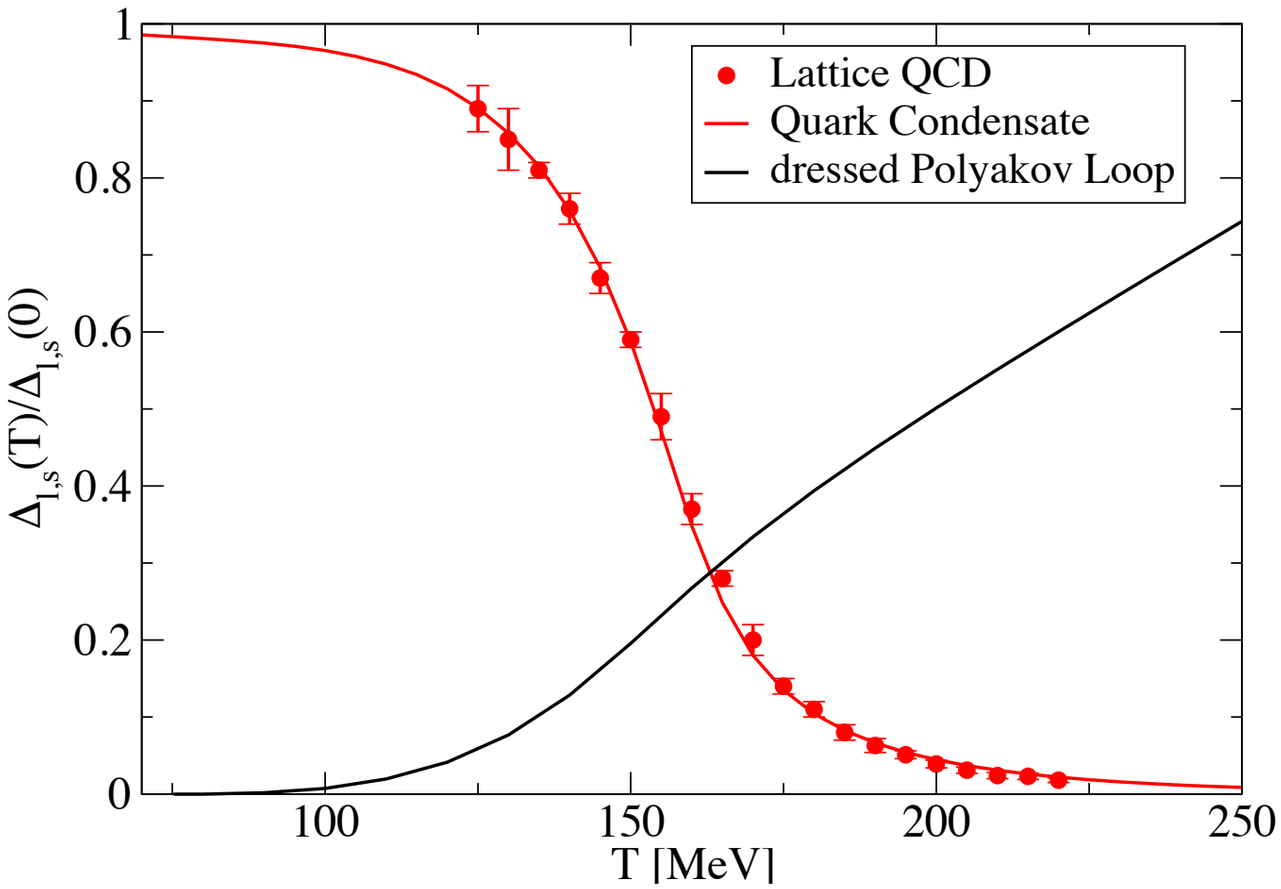
FRG: Fu, Rennecke, Pawlowski, PRD 101 (2020)
FRG/DSE: Gao, Pawlowski, PLB 820 (2021)
DSE: Gunkel, CF, PRD 104 (2021)

N_f=2+1, μ=0, physical point



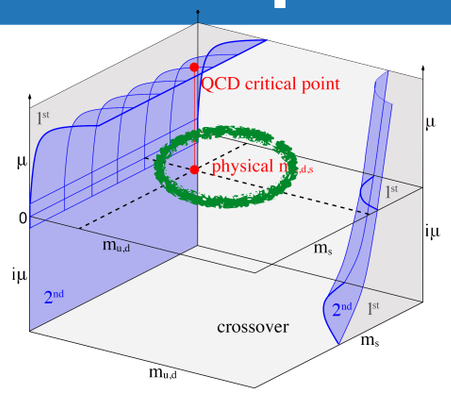


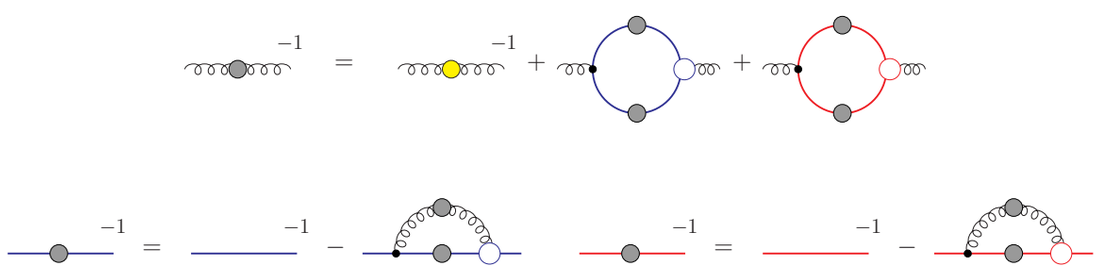
$$\Delta_{u,d,s} \sim Tr(S_{u,d,s})$$



Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

N_f=2+1, μ=0, physical point





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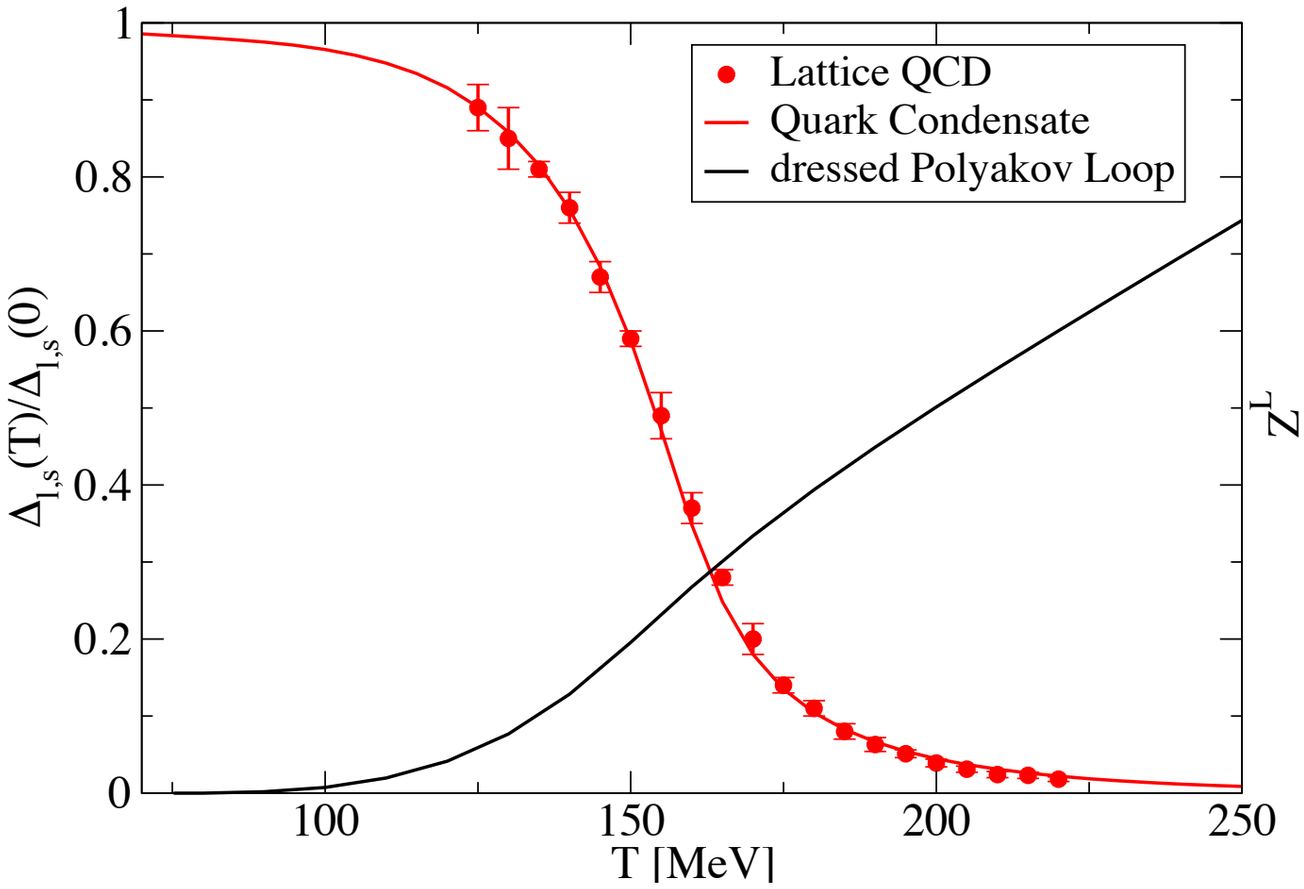

Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
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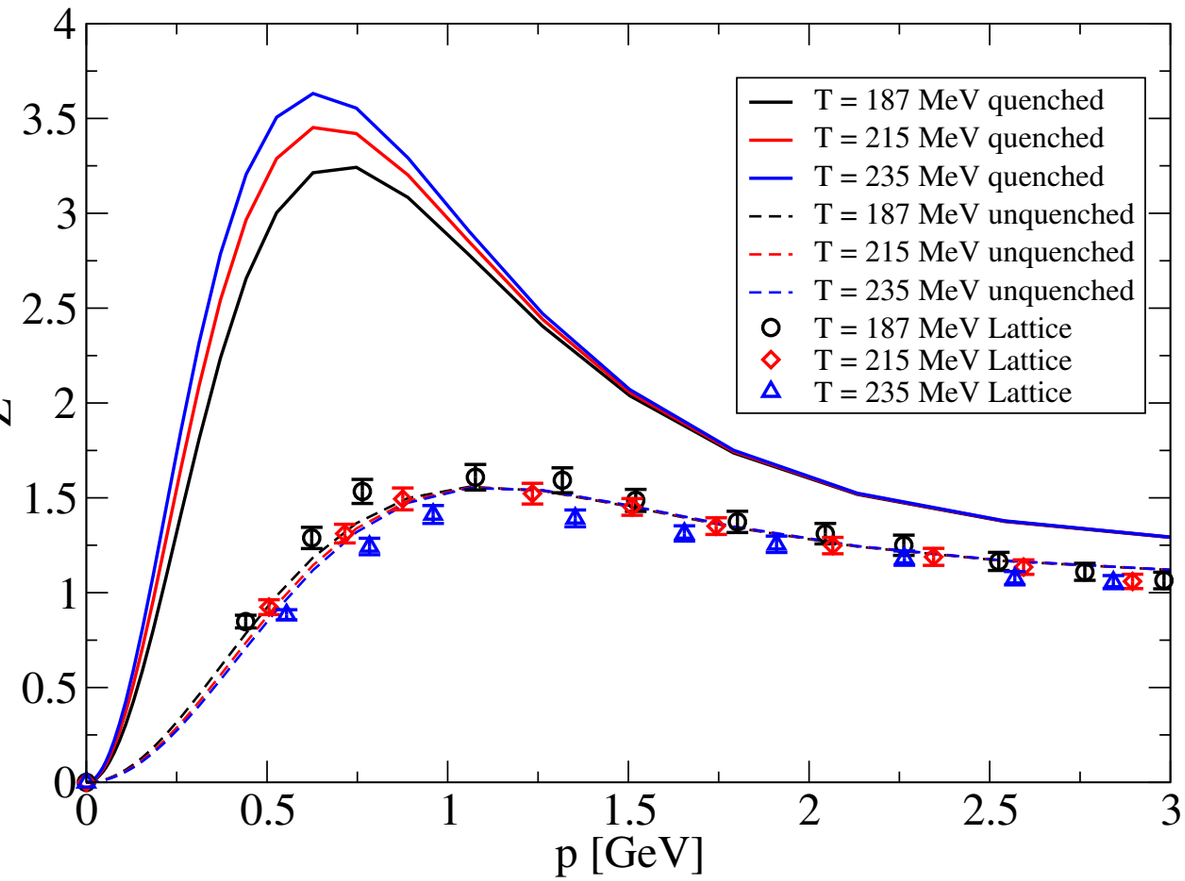
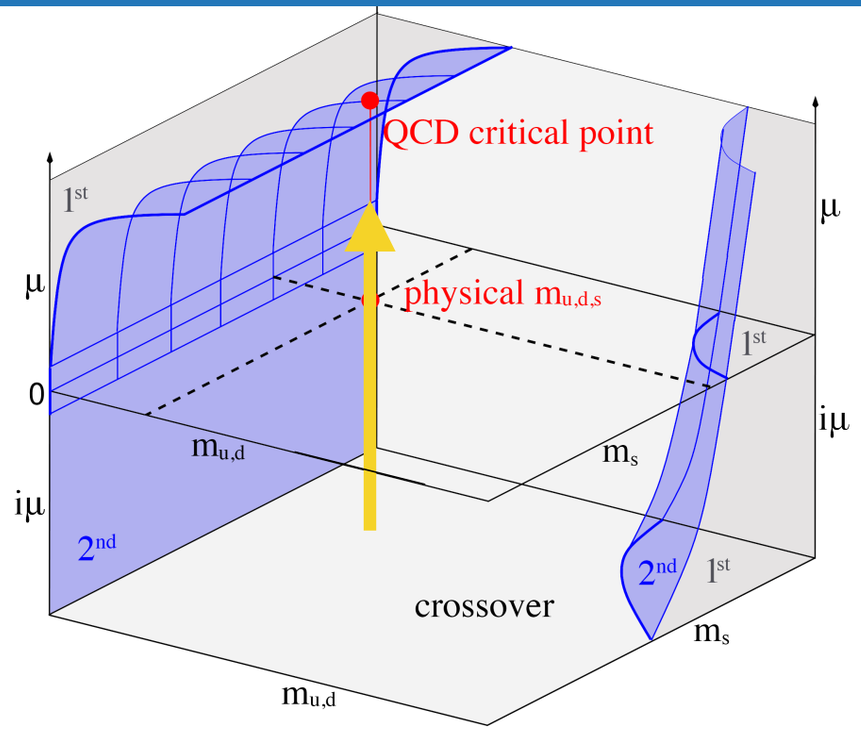
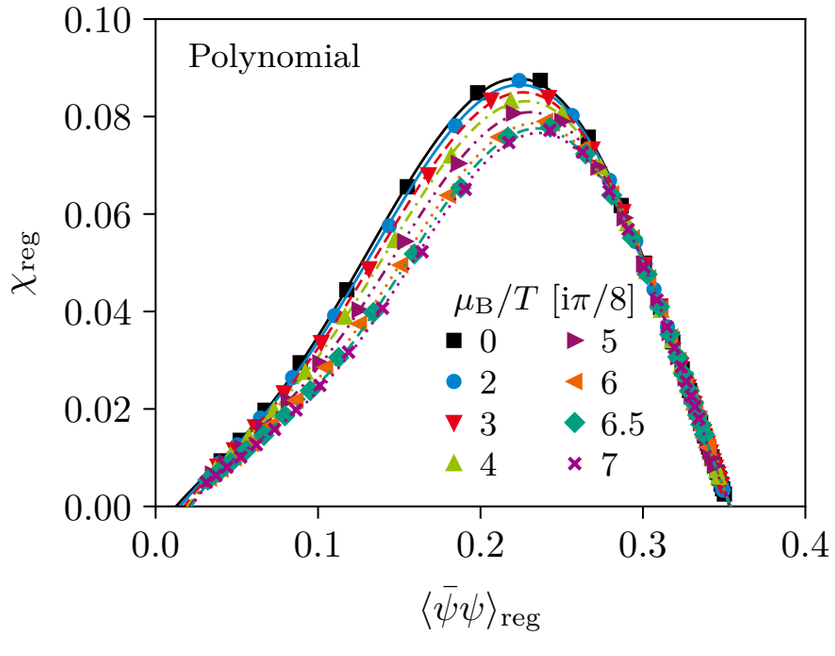


Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

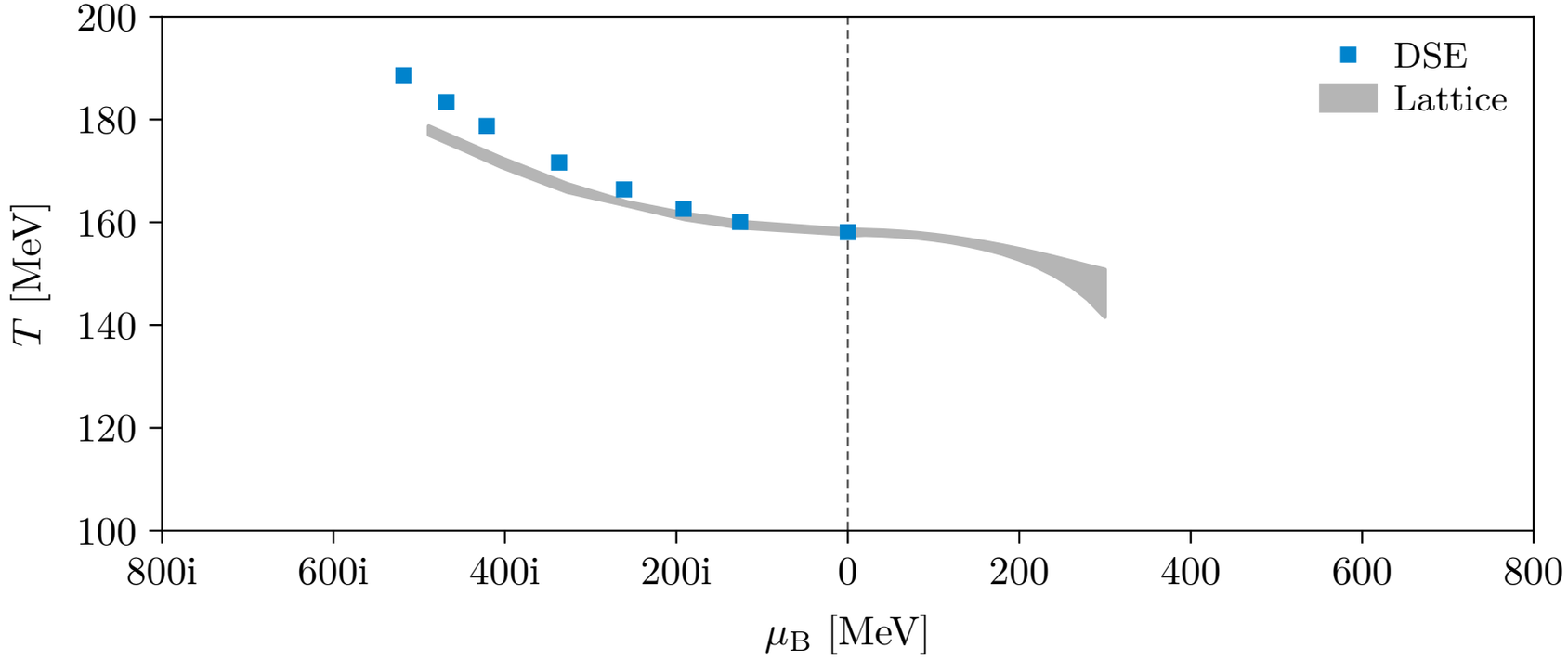
- quantitative agreement: DSE prediction verified by lattice
  FRG: similar results Fu, Rennecke, Pawlowski, PRD 101 (2020)

Extrapolation from imaginary chemical potential





$$\chi(T) = \frac{\partial\langle\bar{\psi}\psi\rangle(T)}{\partial m_u}$$



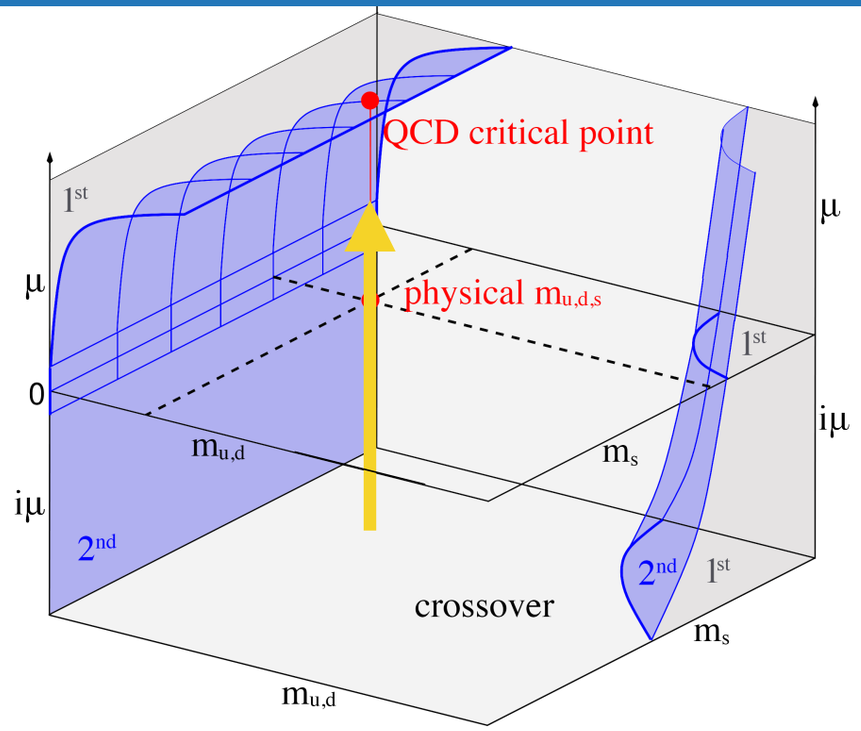
Lattice: Borsanyi et al. PRL 125 052001 (2020)
DSE: Bernhardt, CF, EPJA 59 (2023) 8, 181

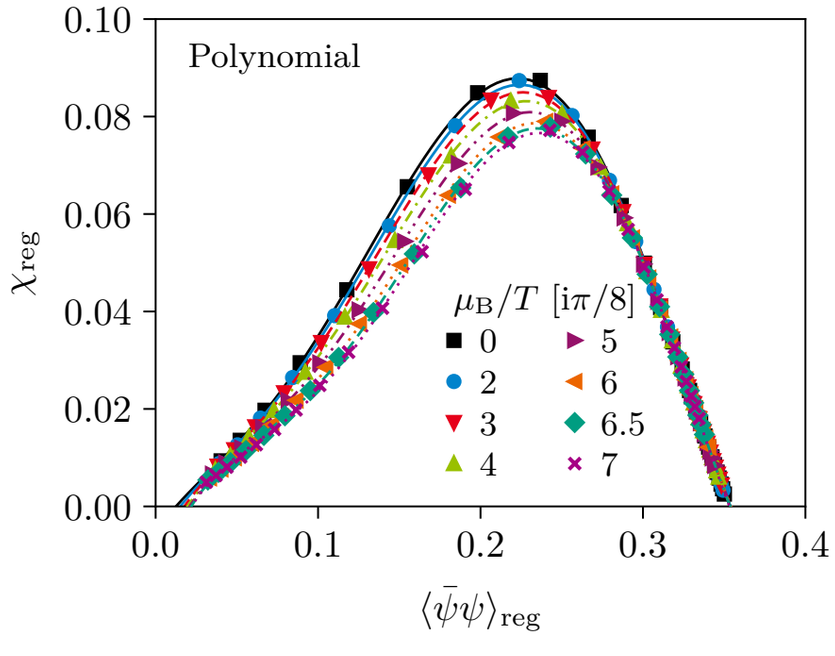
see also FRG (Nf=2): Braun et. al. PRL 106 (2011)

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2\left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_c}\right)^4$$

$$\kappa_2^{\rm poly} = 0.0196\,, \quad \kappa_4^{\rm poly} = 0.00015\,,$$

Extrapolation from imaginary chemical potential





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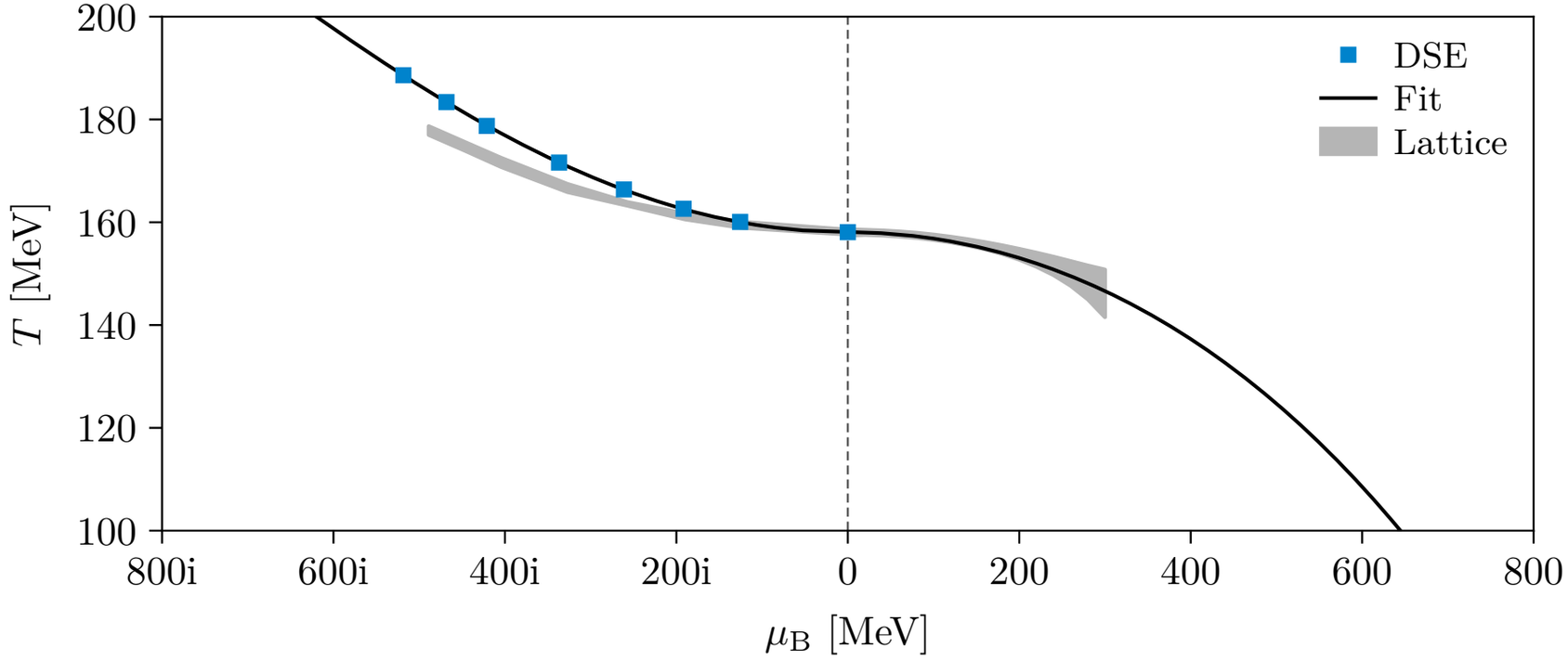

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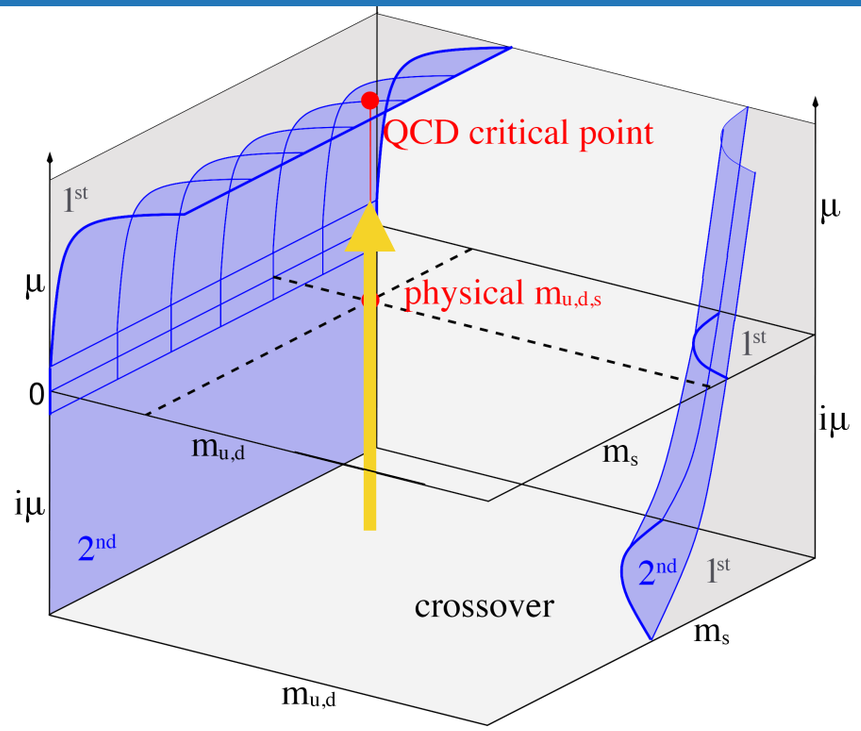

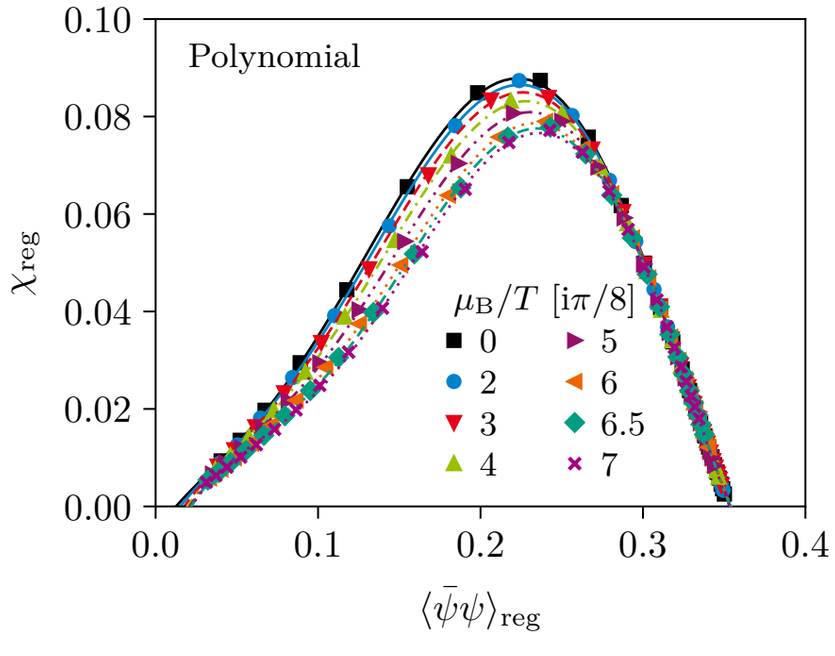

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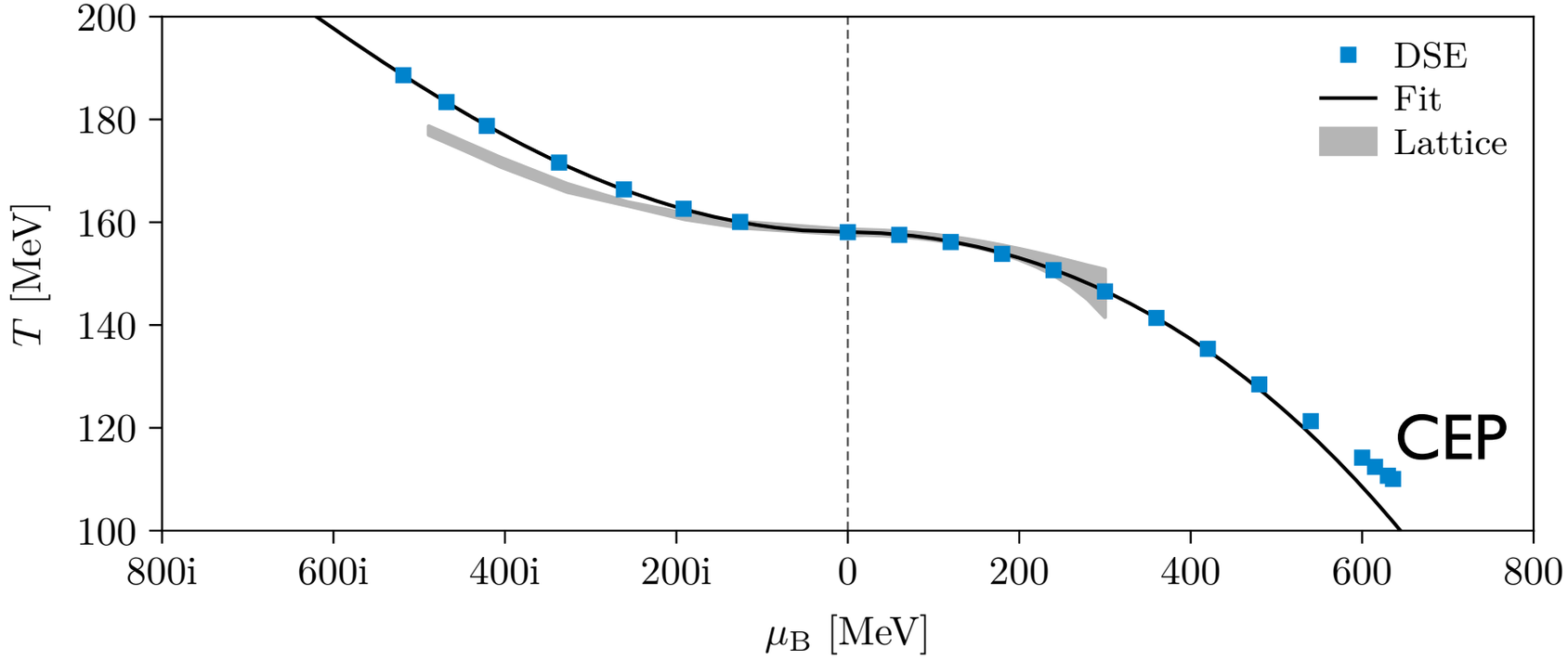

- Extrapolation works very well!

see also FRG (Nf=2): Braun et. al. PRL 106 (2011)

Lattice: Borsanyi et al. PRL 125 052001 (2020)
DSE: Bernhardt, CF, EPJA 59 (2023) 8, 181

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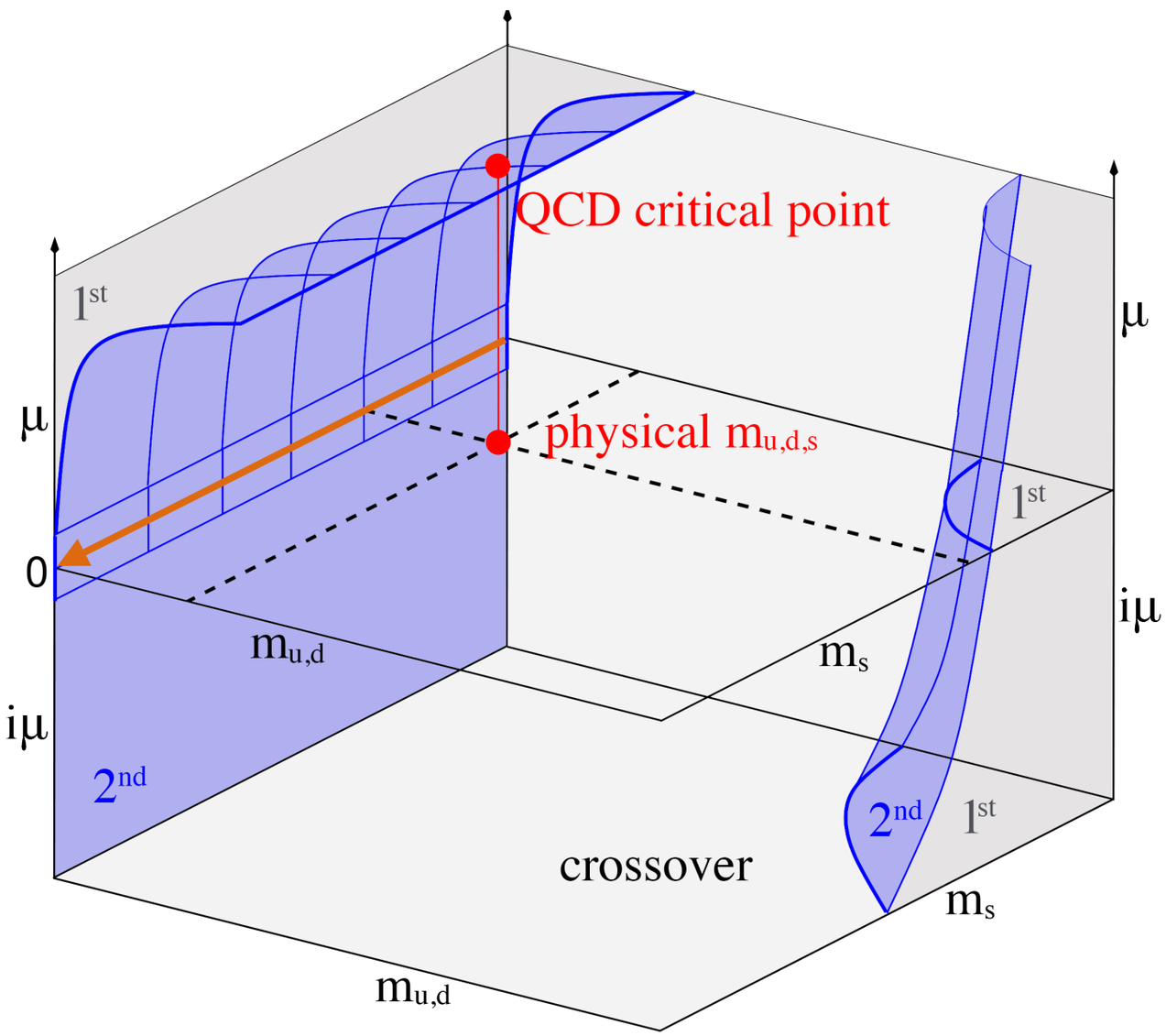
Columbia plot - chiral critical surface



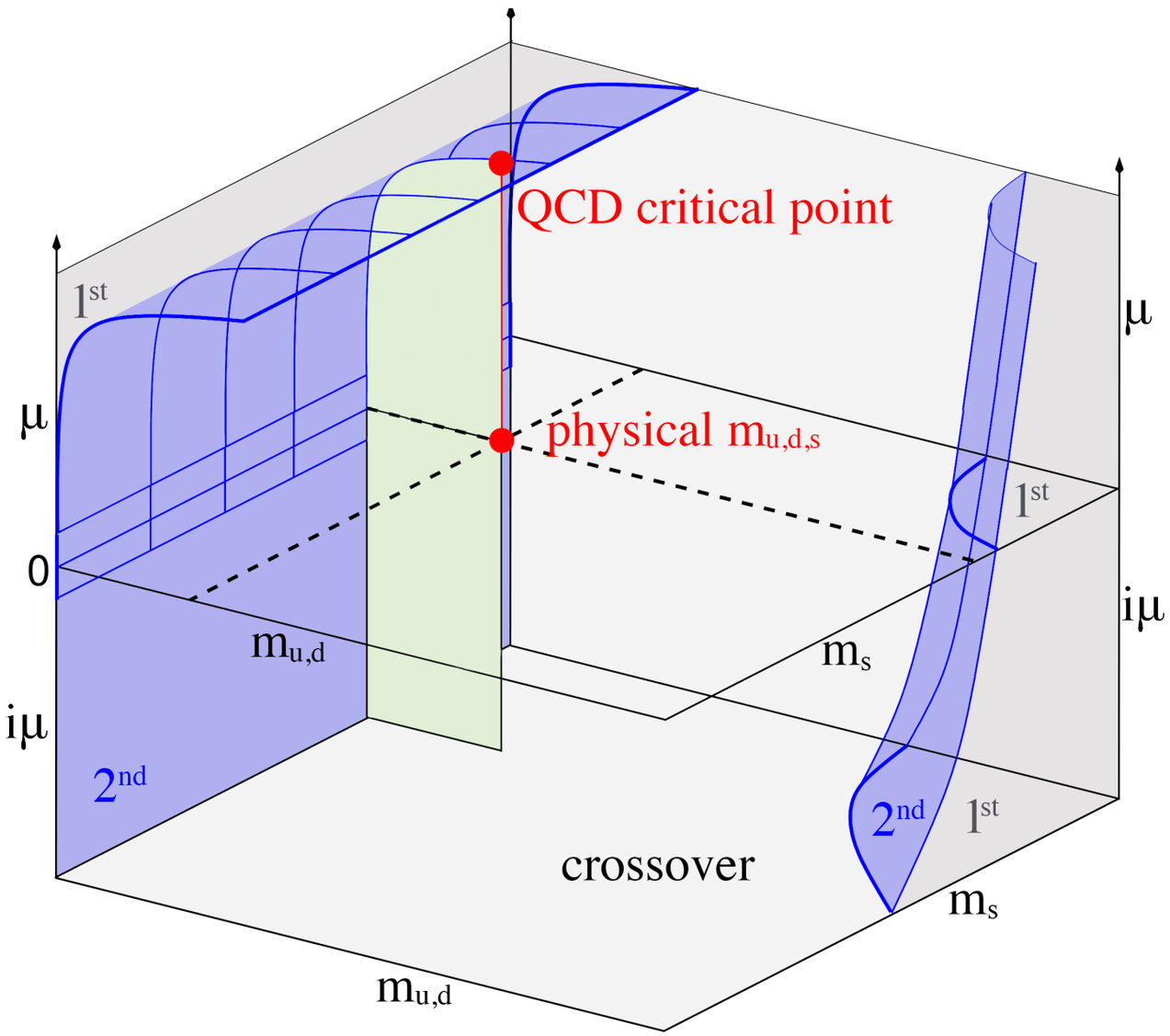
no first order region found

DSE: Bernhardt and CF, PRD 108 (2023) 114018

Lattice: Cuteri, Philipsen and Sciarra, JHEP 11 (2021), 141
Dini, et al, PRD 105 (2022) no.3, 034510

Ding et al. PRL 123, 062002 (2019)
Bornyakov et al. PRD 82, 014504 (2010)

Columbia plot - chiral critical surface



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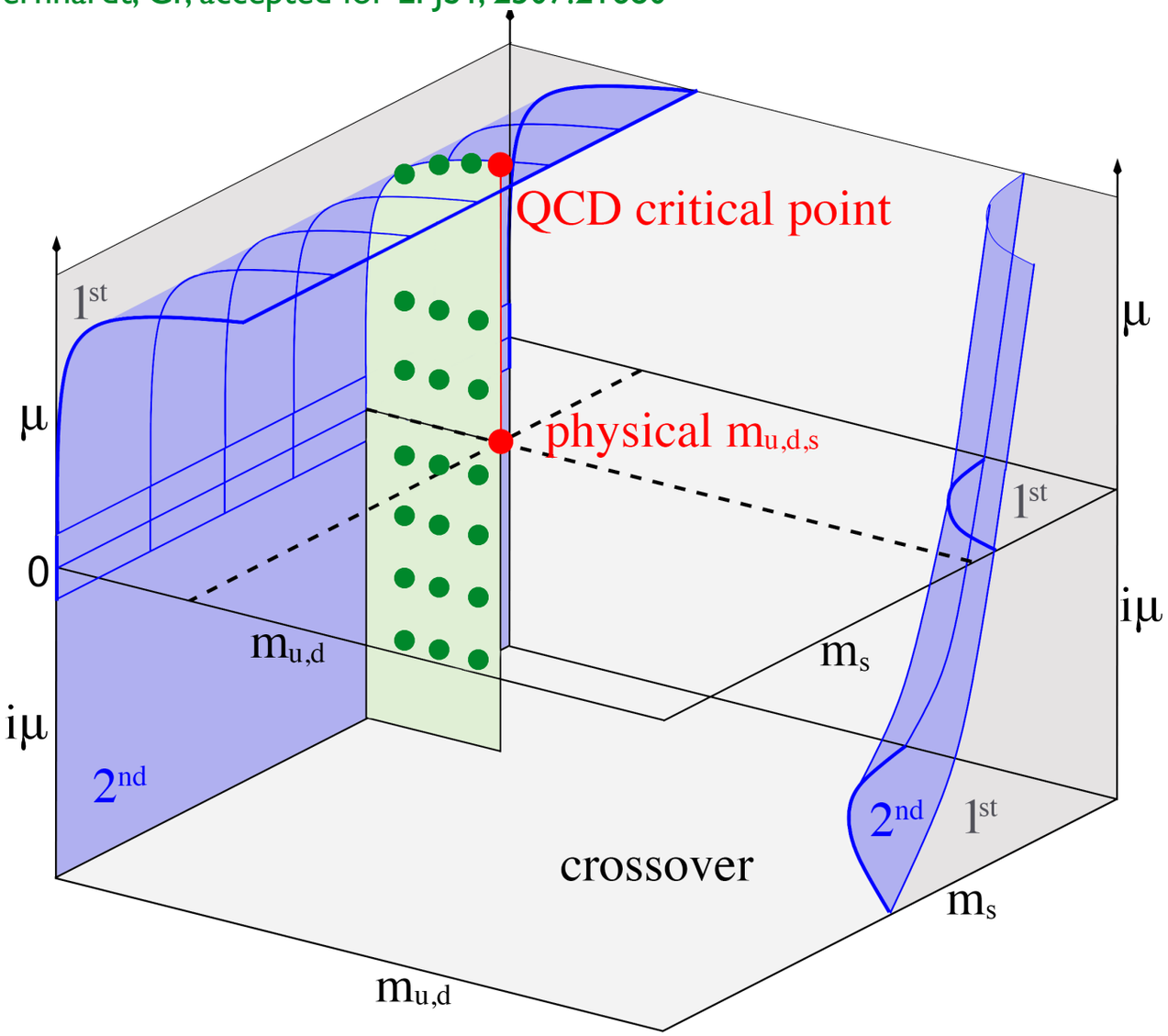
Columbia plot - chiral critical surface

Bernhardt, CF, accepted for EPJST, 2507.21680



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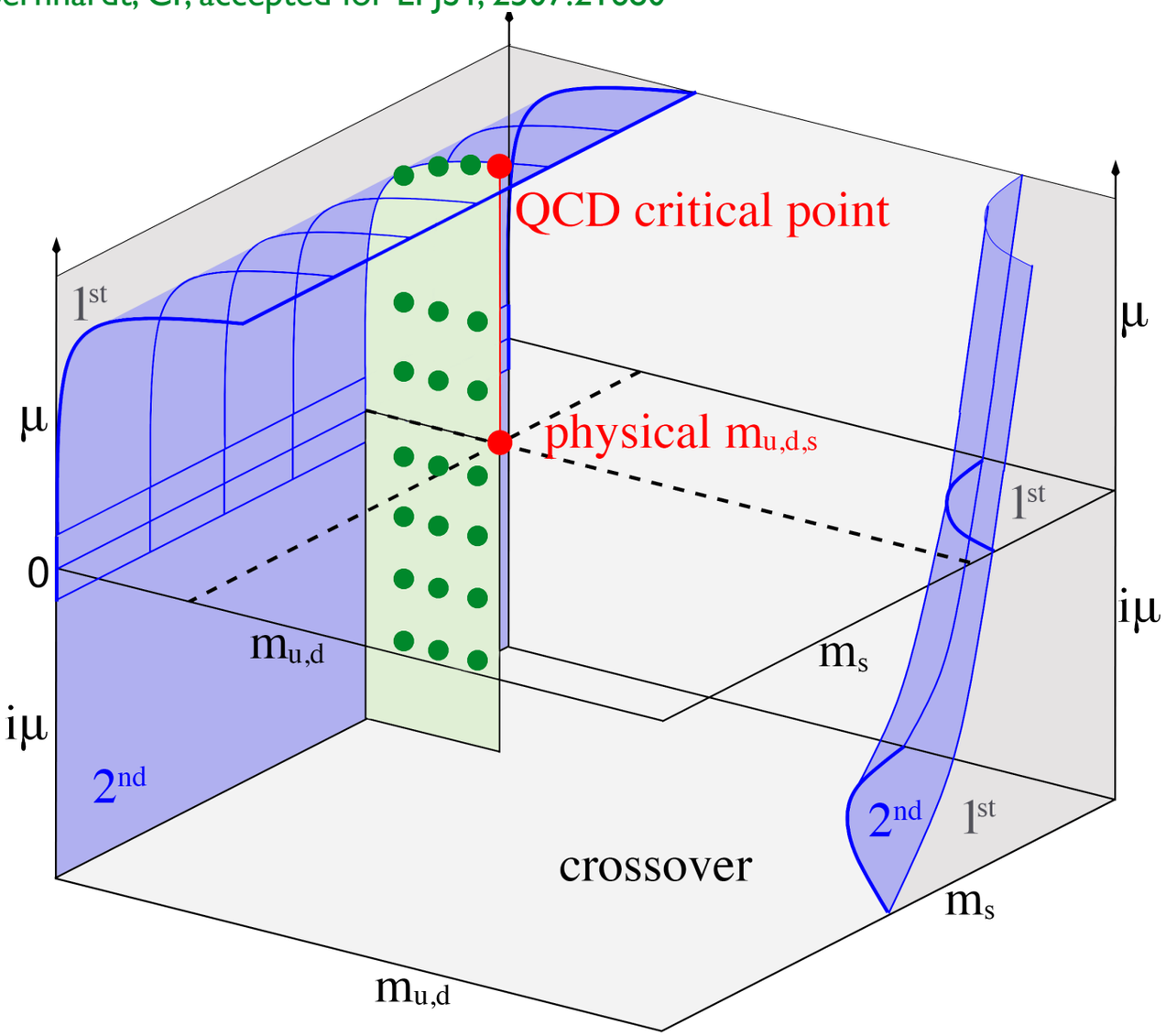

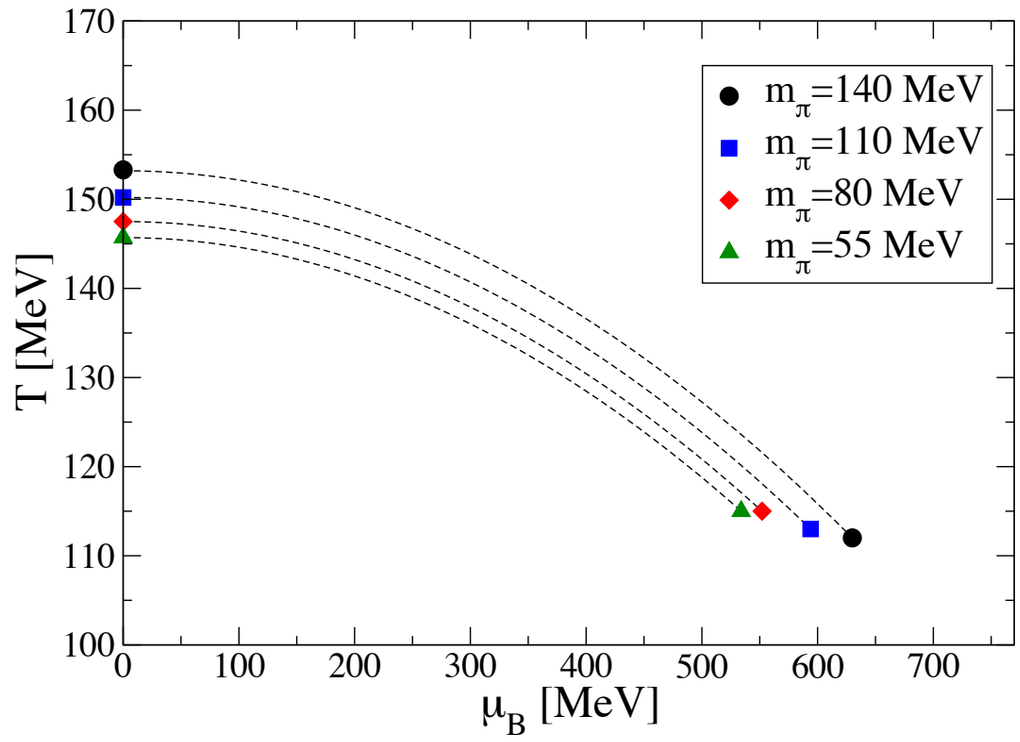

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Columbia plot - chiral critical surface

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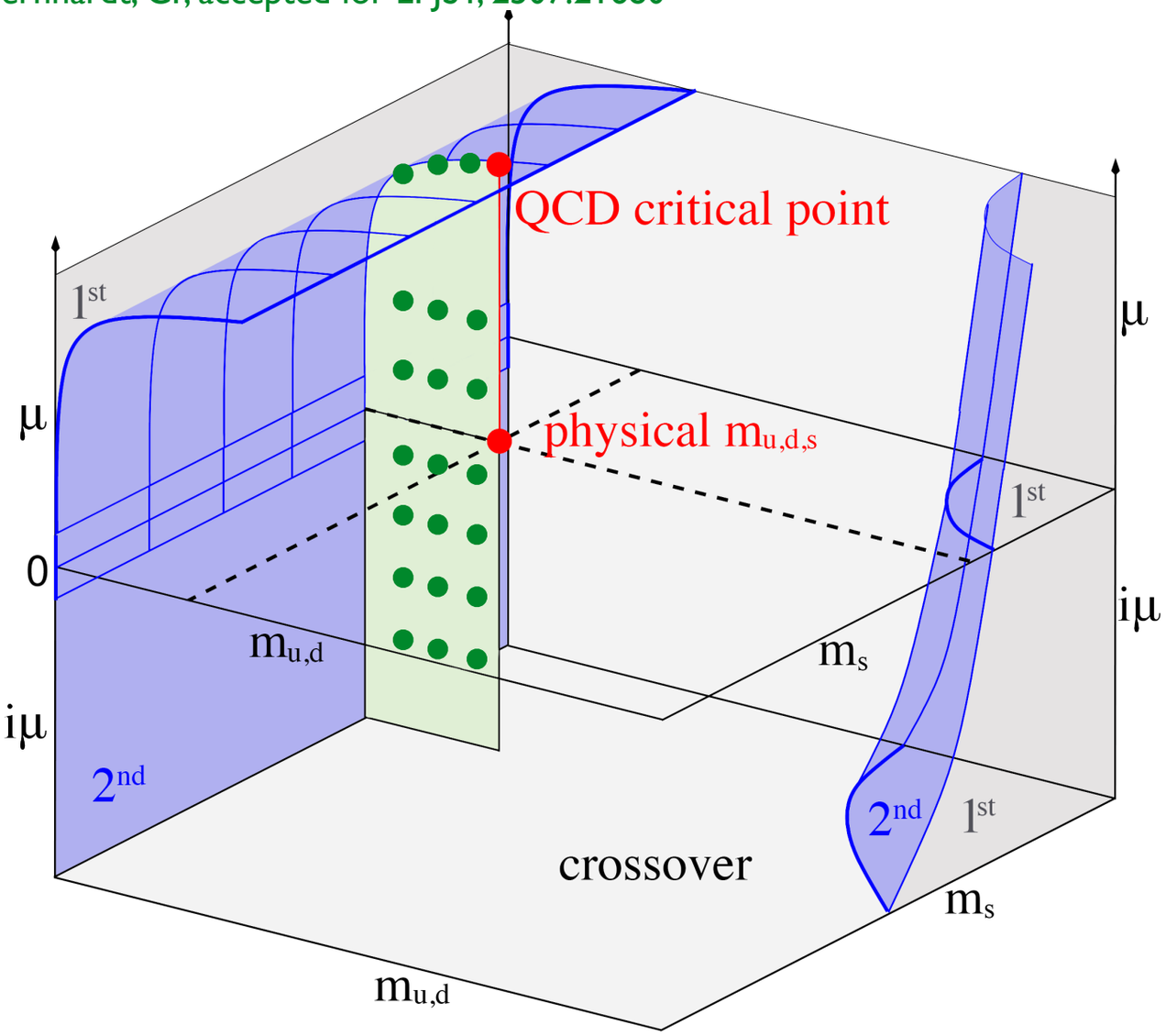

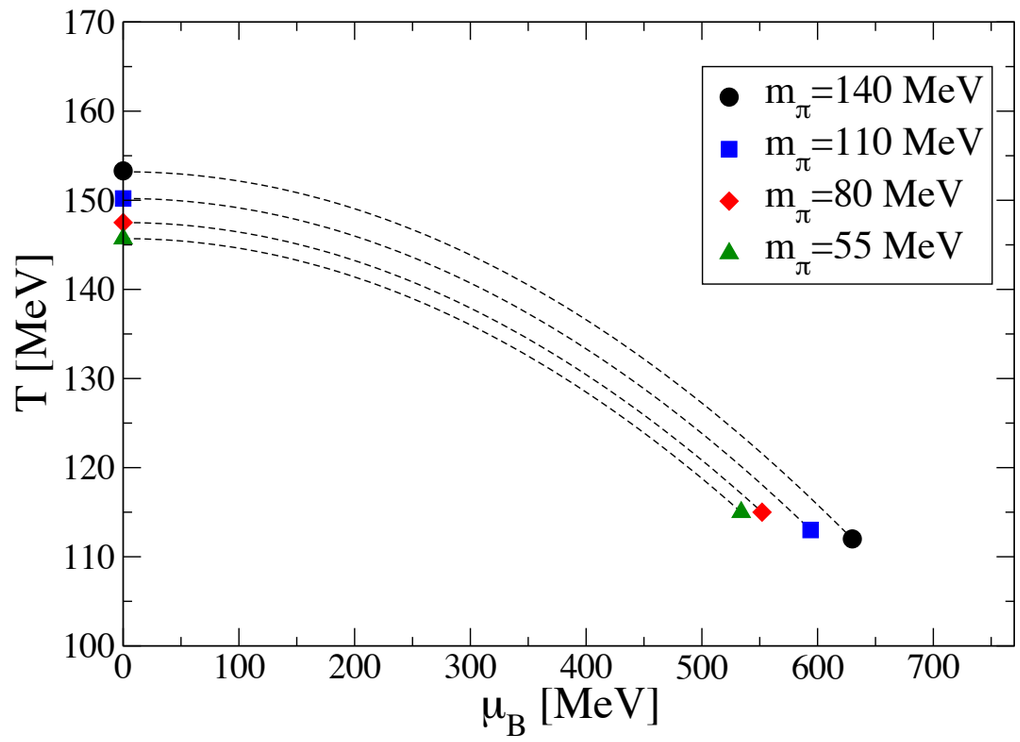

no first order region found

DSE: Bernhardt and CF, PRD 108 (2023) 114018

Lattice: Cuteri, Philipsen and Sciarra, JHEP 11 (2021), 141
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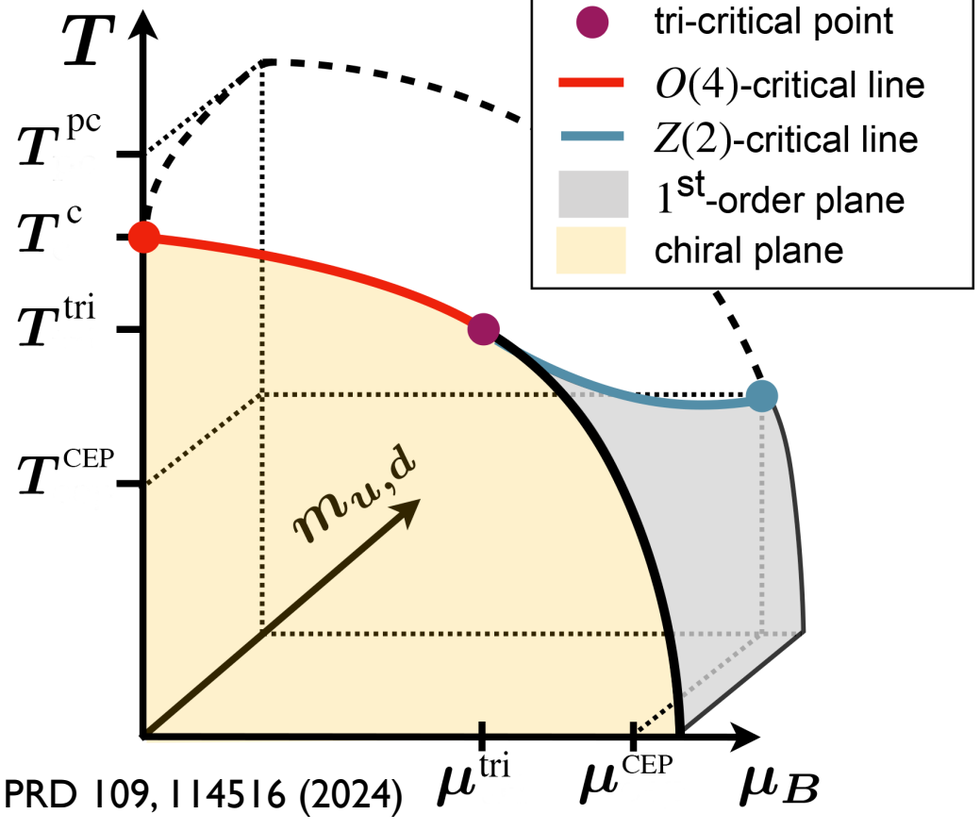
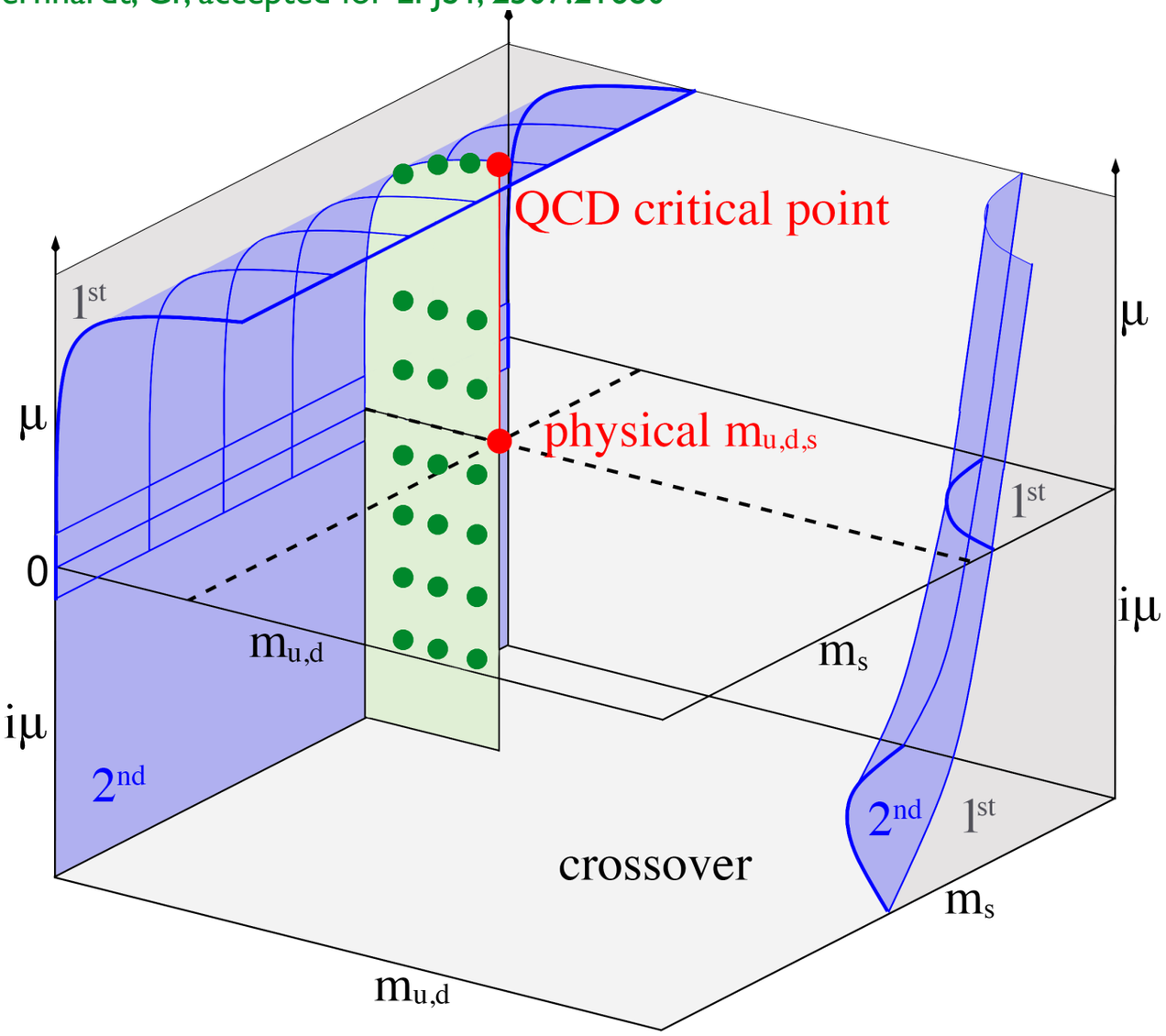
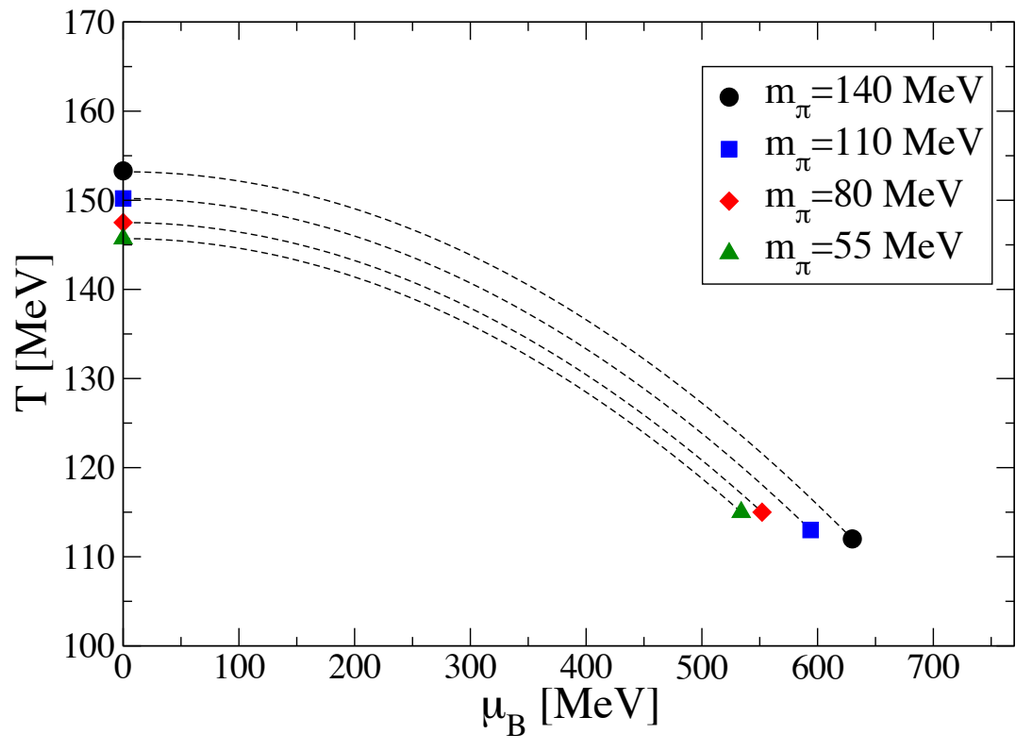


Ding, et al. PRD 109, 114516 (2024)

Columbia plot - chiral critical surface

Bernhardt, CF, accepted for EPJST, 2507.21680





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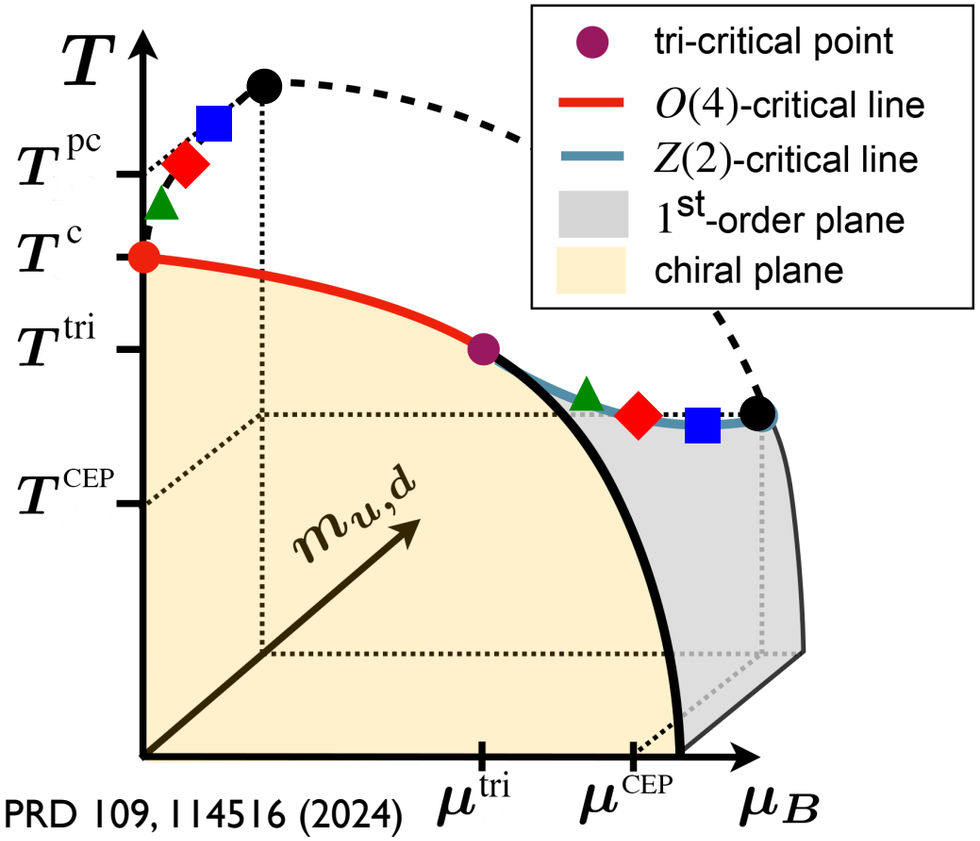
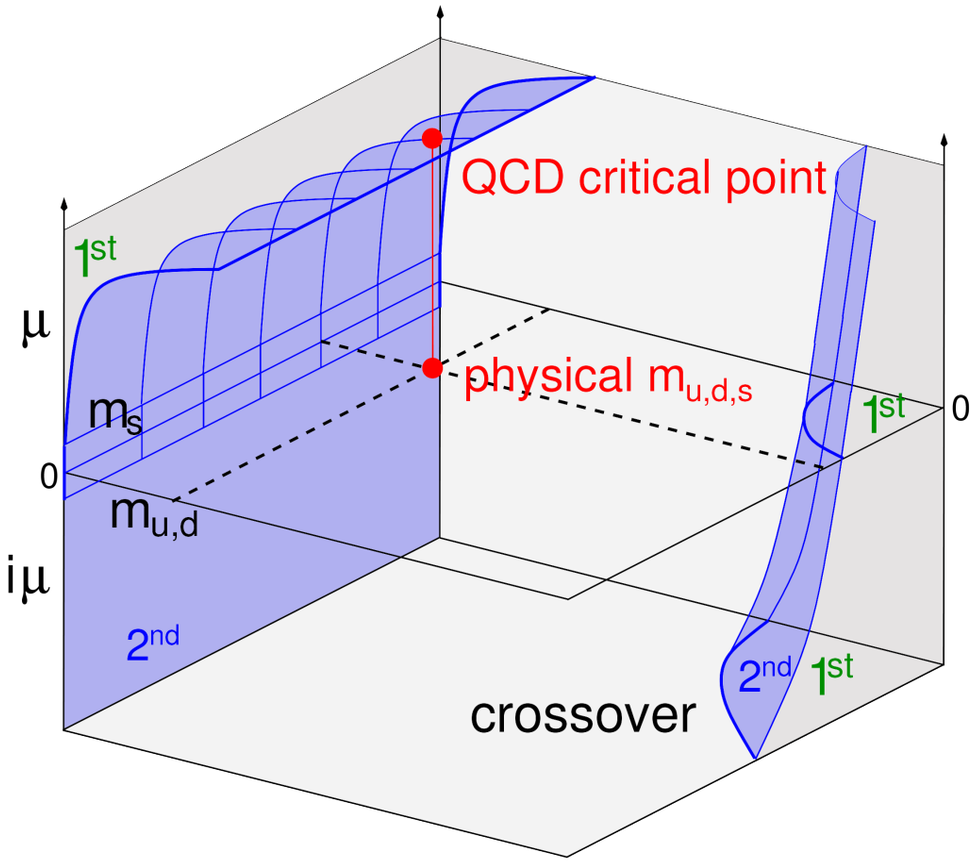
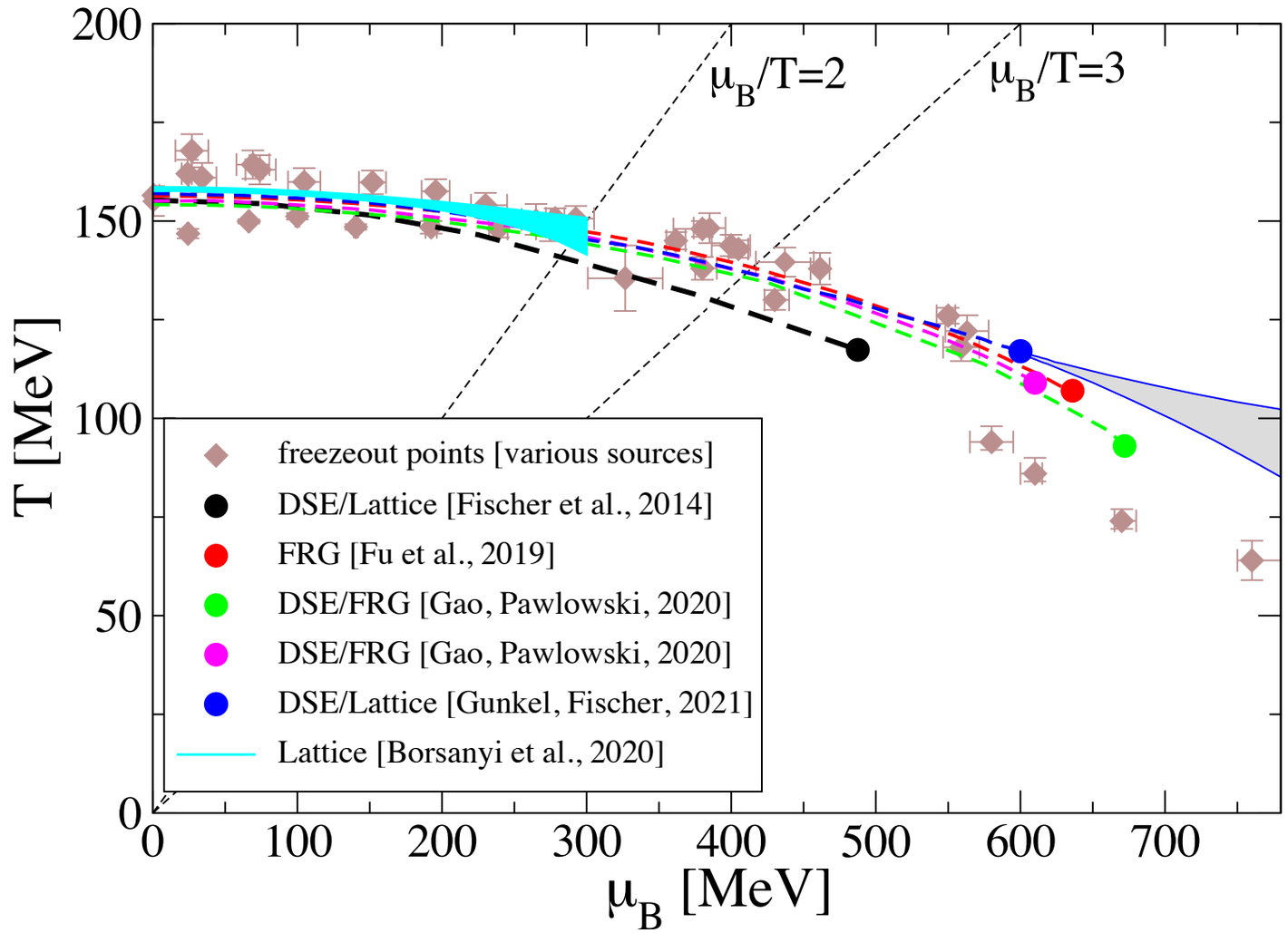


Ding, et al. PRD 109, 114516 (2024)

Location of CEP





- how stable is this result ??
  - crosscheck DSE-FRG ✔

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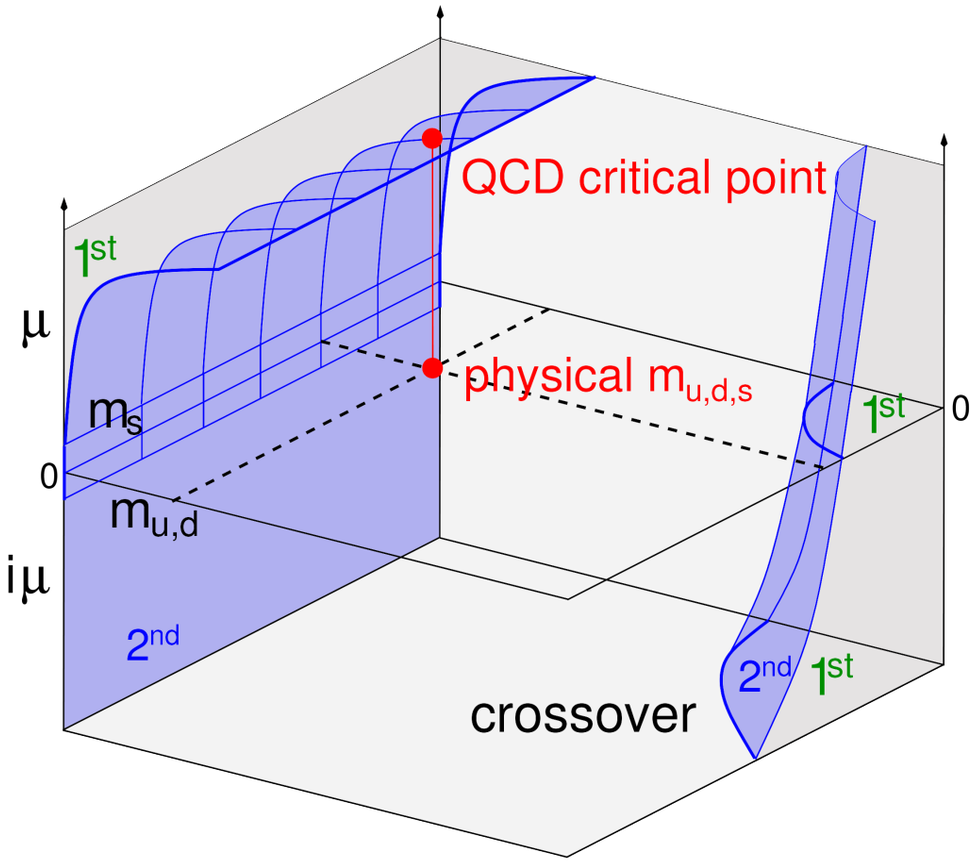

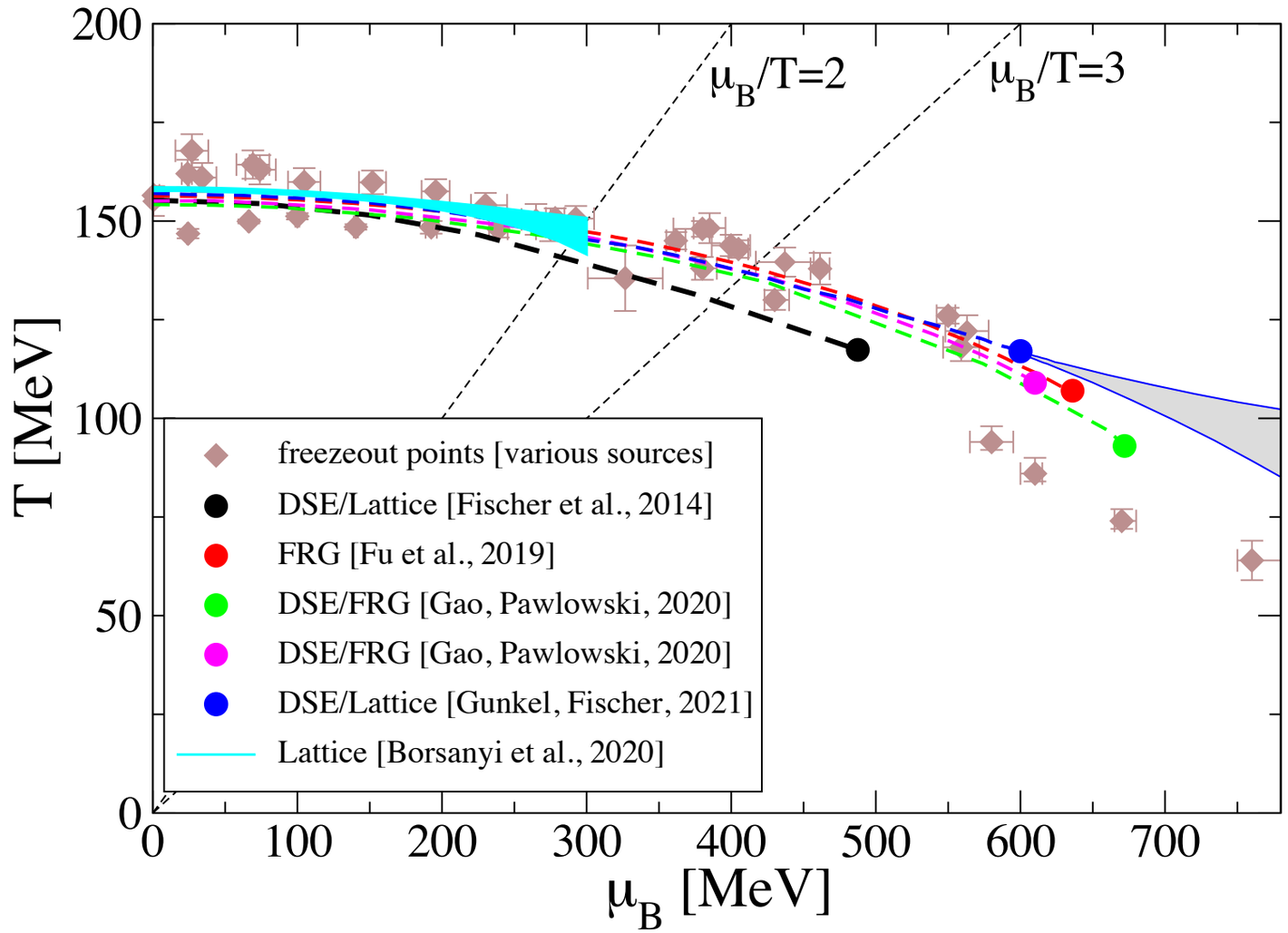

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CF, Luecker, Welzbacher, PRD 90 (2014) 034022

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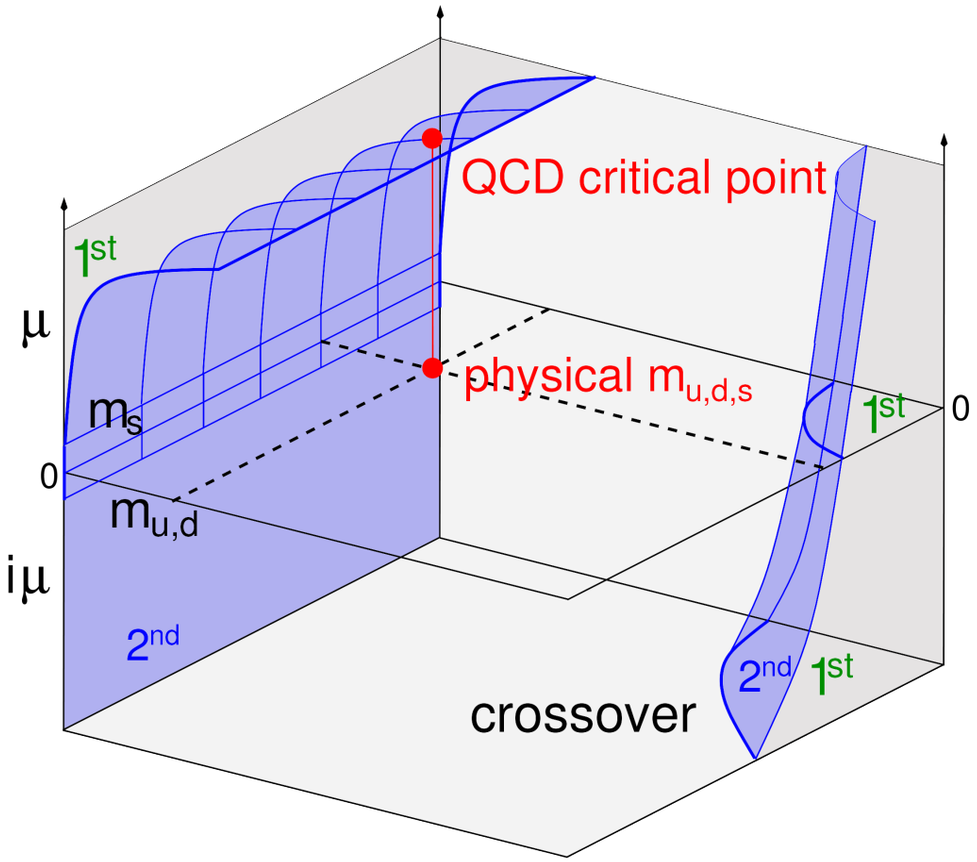
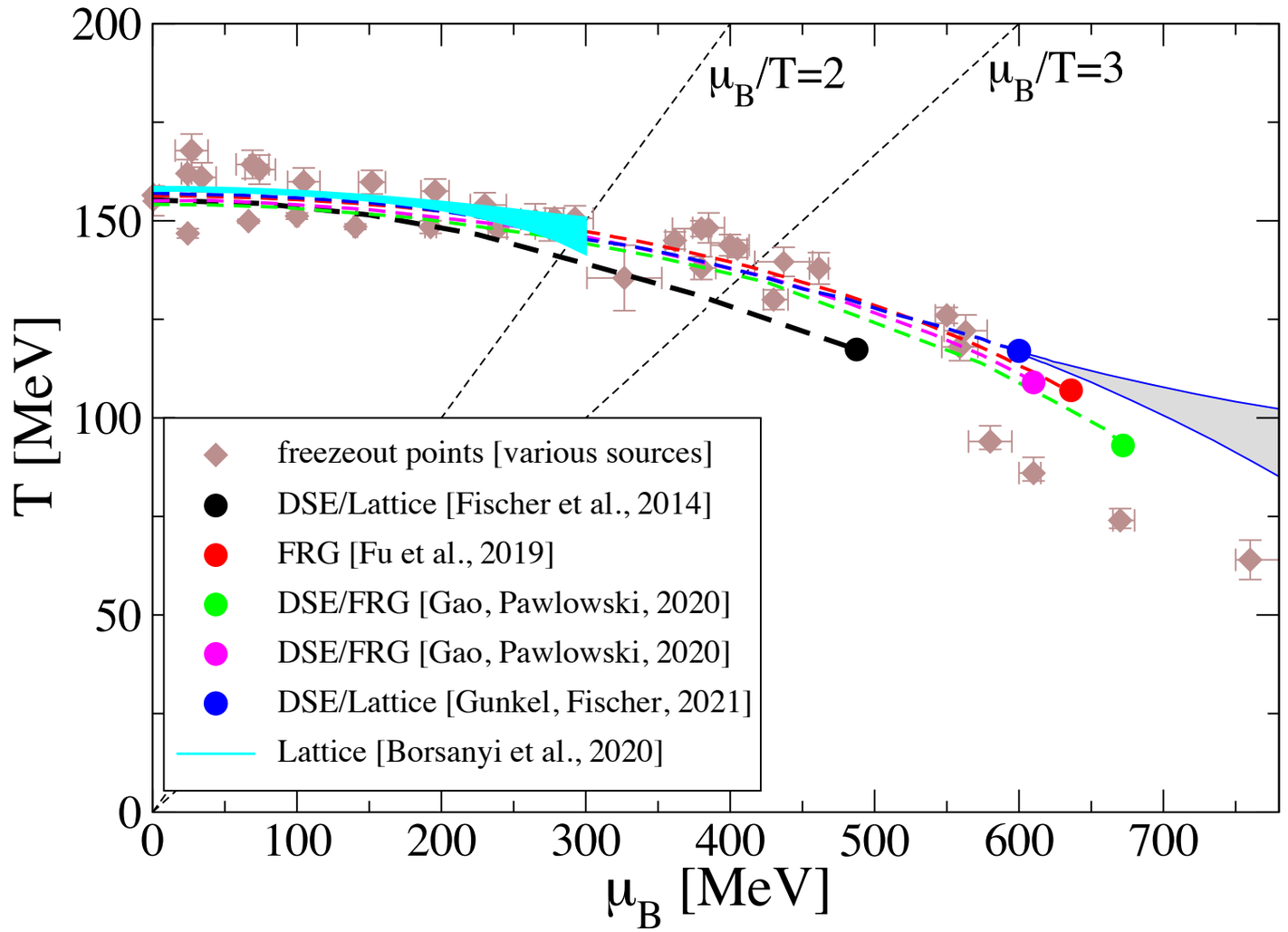
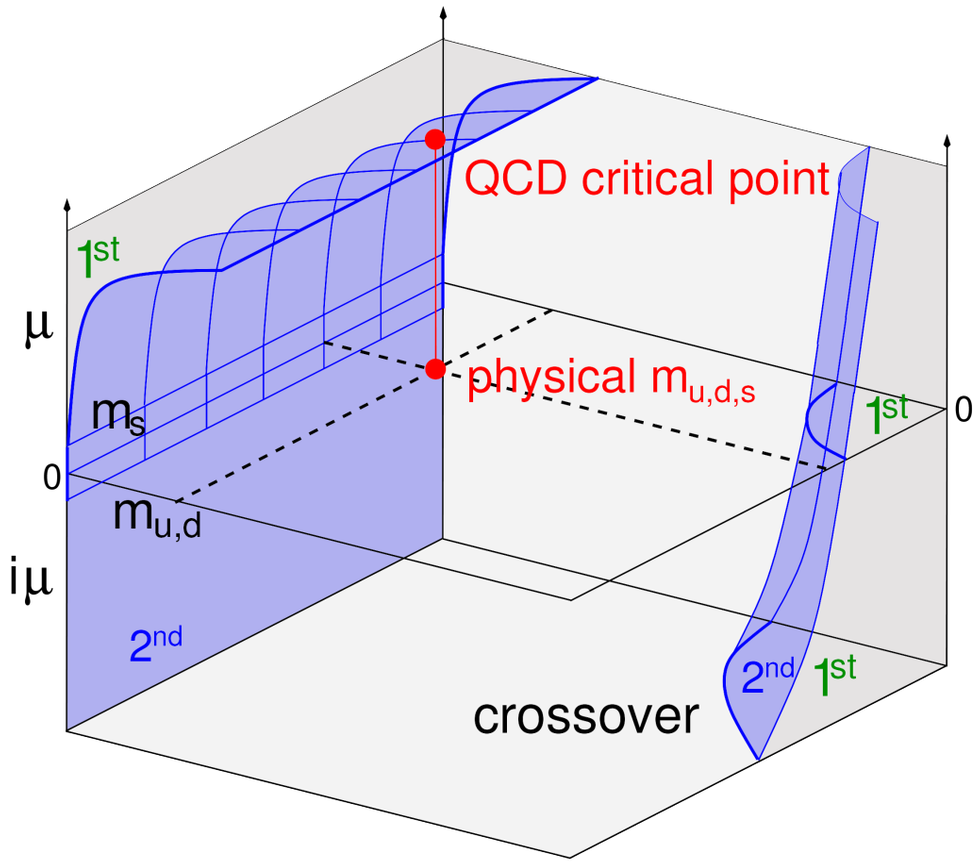
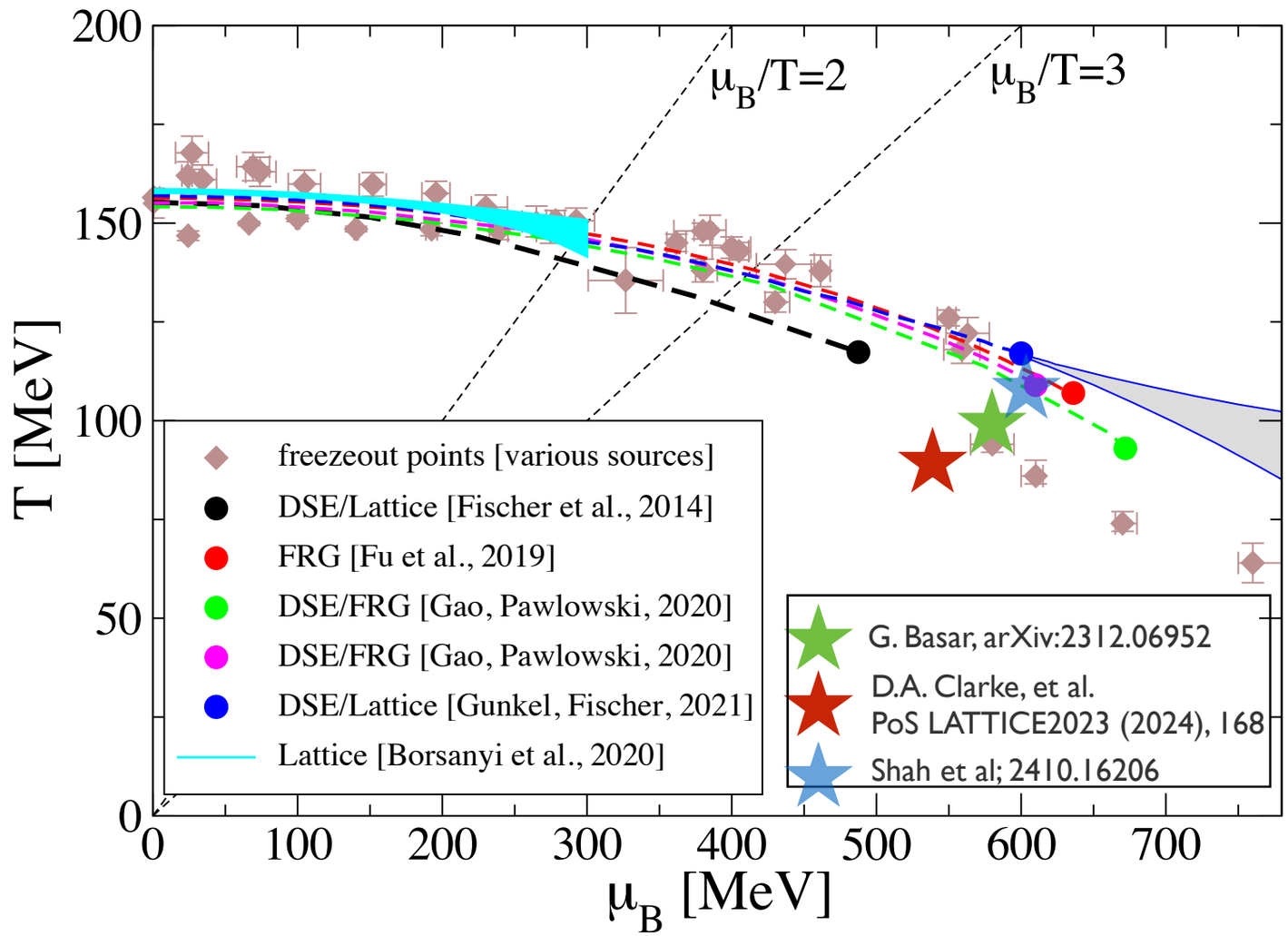




- how stable is this result ??
  - crosscheck DSE-FRG ✔
  - N_f=2+1+1 ✔ CF, Luecker, Welzbacher, PRD 90 (2014) 034022
  - baryon effects ✔ Eichmann, CF, Welzbacher, PRD93 (2016); Braun et al. PRD 101 (2020)

Location of CEP



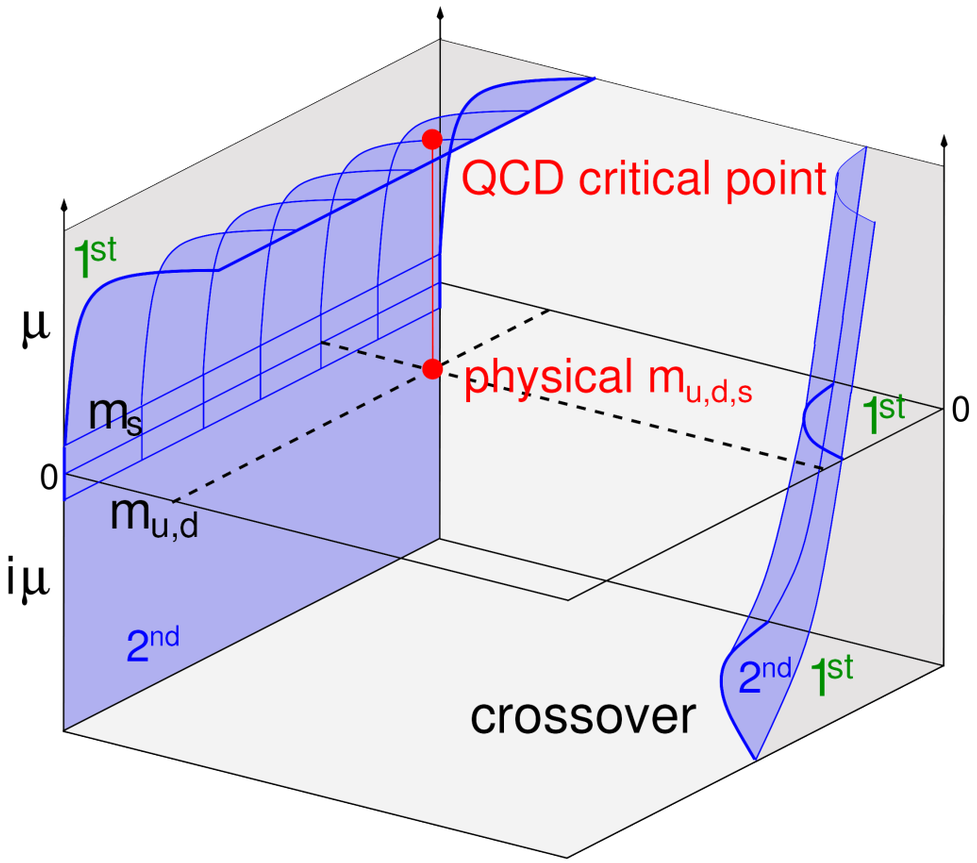


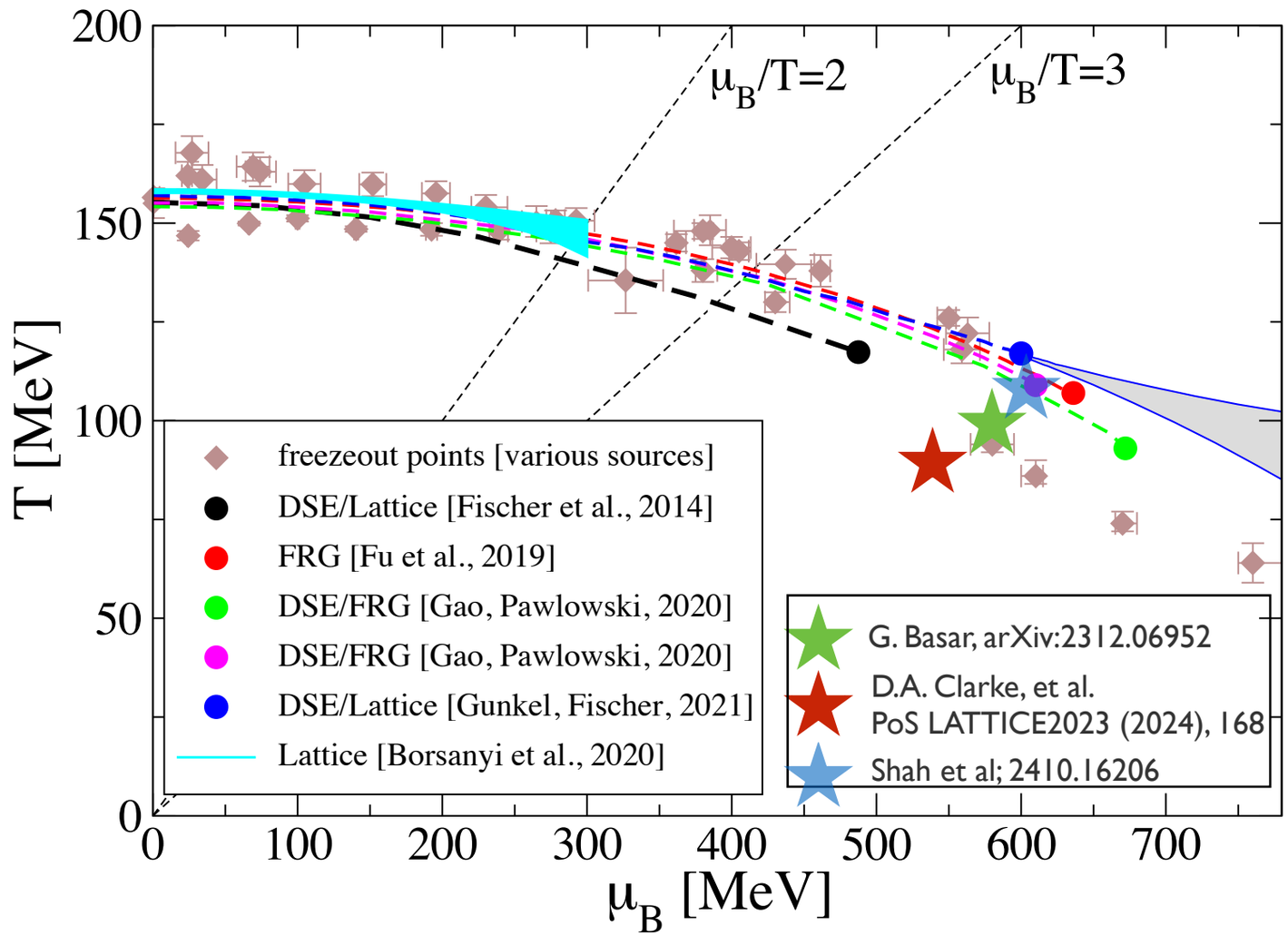
- how stable is this result ??
  - crosscheck DSE-FRG ✔
  - N_f=2+1+1 ✔
  - baryon effects ✔
  - 'cross-check' with lattice (✔)

CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Eichmann, CF, Welzbacher, PRD93 (2016)
Braun et al. PRD 101 (2020)

Location of CEP





- how stable is this result ??
  - crosscheck DSE-FRG ✔
  - N_f=2+1+1 ✔
  - baryon effects ✔
  - 'cross-check' with lattice (✔)
  - moat and inhom. phases

CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Eichmann, CF, Welzbacher, PRD93 (2016)
Braun et al. PRD 101 (2020)

T. F. Motta, J. Bernhardt, M. Buballa and CF, PRD 108 (2023)
T. F. Motta, J. Bernhardt, M. Buballa and CF, acc. for PRD, [2411.02285]

Connecting with experiment: fluctuations

X.~Luo and N.~Xu, Nucl. Sci. Tech. 28 (2017) no.8, 112 [arXiv:1701.02105 [nucl-ex]].

Quark chemical potentials related to those of conserved charges:

$$\mu_u = \mu_B/3 + 2\mu_Q/3$$
$$\mu_d = \mu_B/3 - \mu_Q/3$$
$$\mu_s = \mu_B/3 - \mu_Q/3 - \mu_S$$

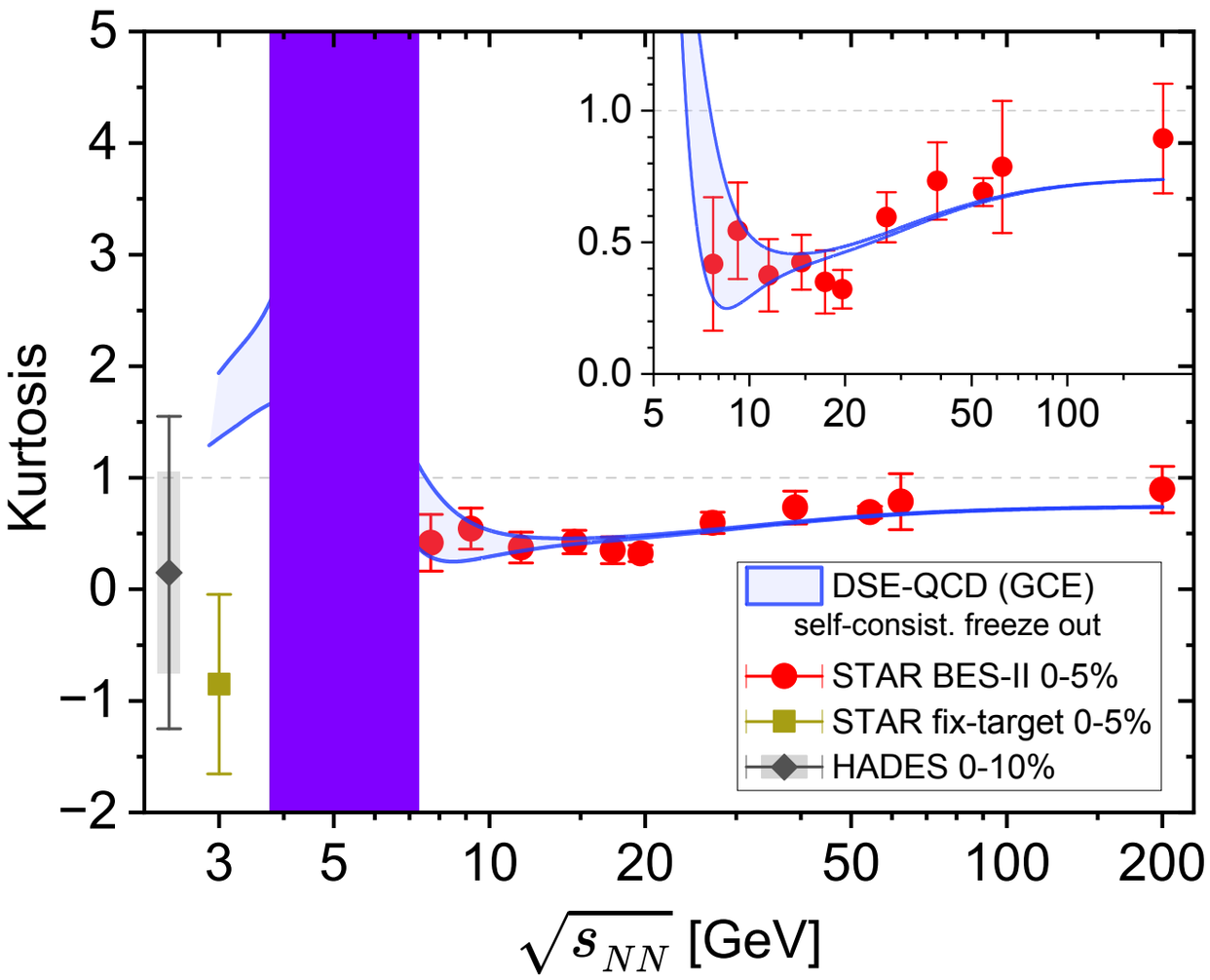
Serve to calculate susceptibilities:

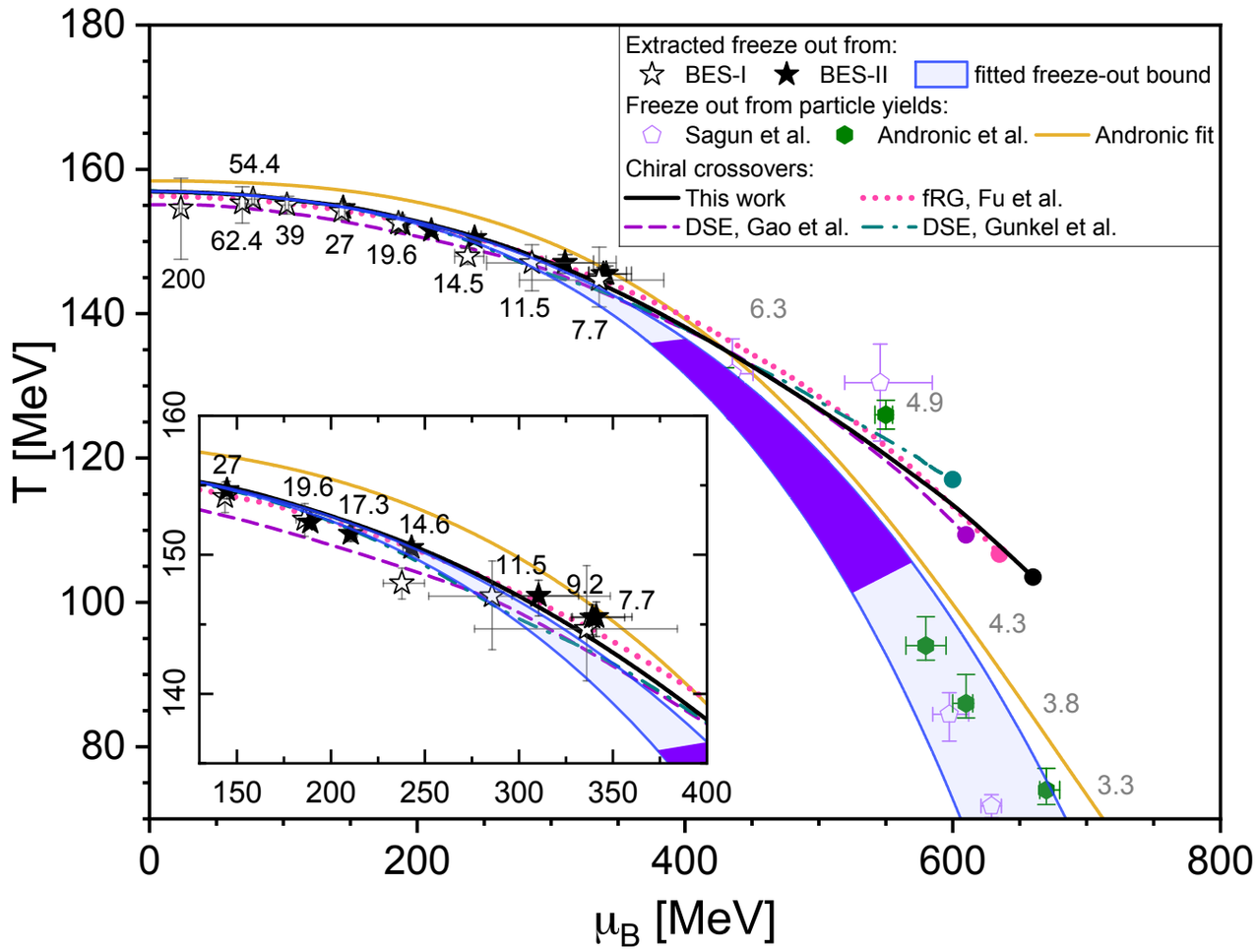
$$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}$$

Related to cumulants, which can be extracted from experiment:

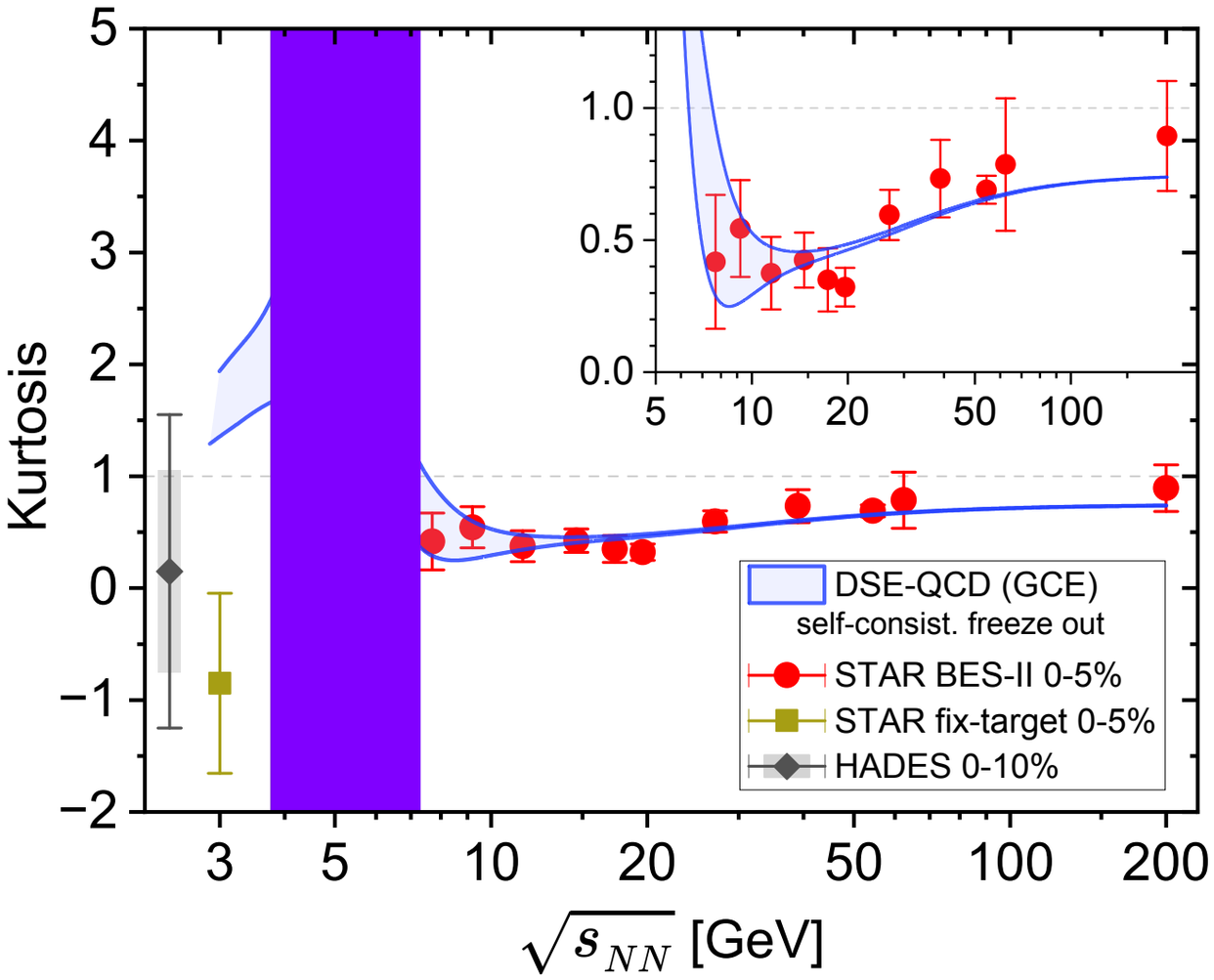
$$C^{BSQ}_{lmn} = VT^3 \chi^{BSQ}_{lmn}$$

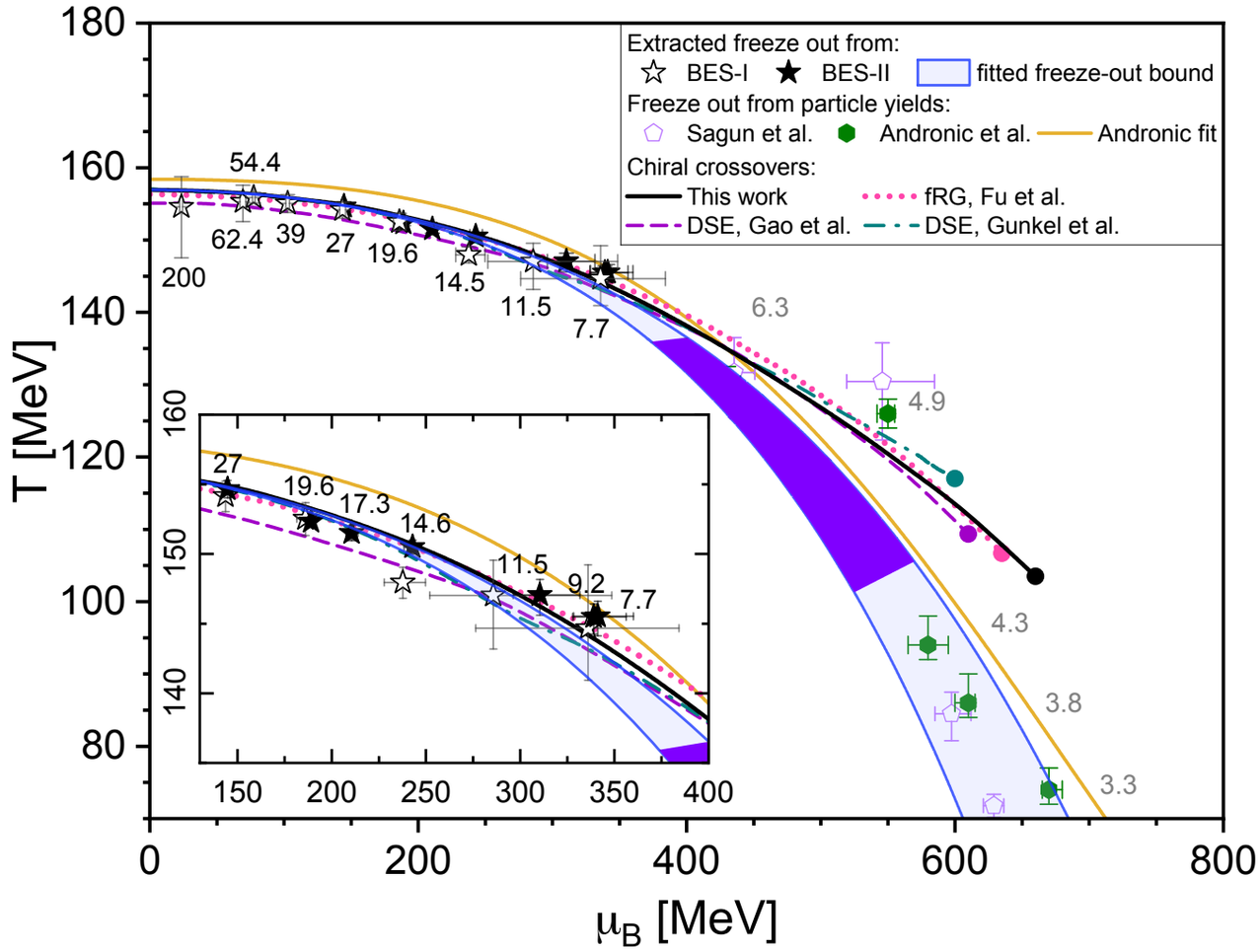
Connecting with experiment: fluctuations

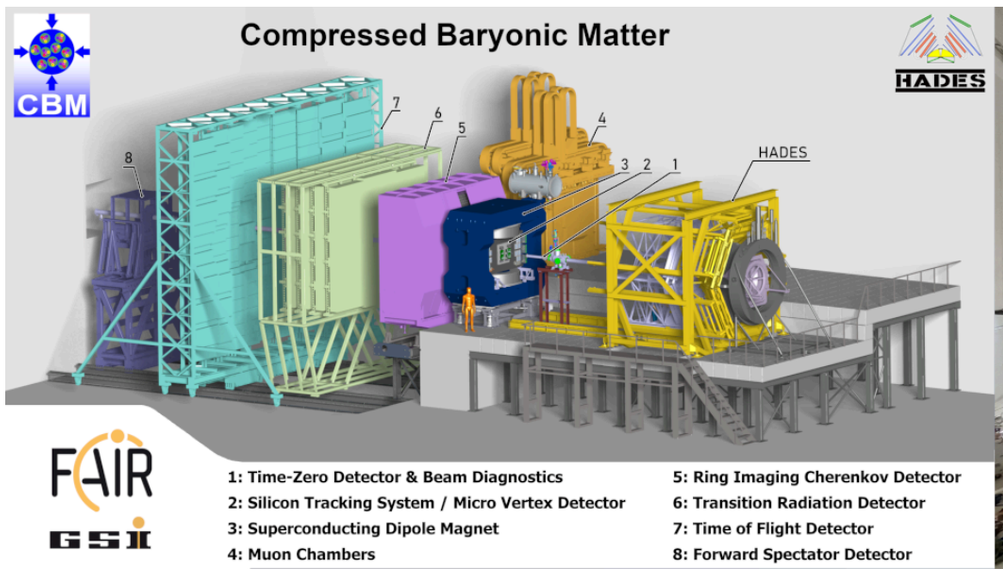




Connecting with experiment: fluctuations

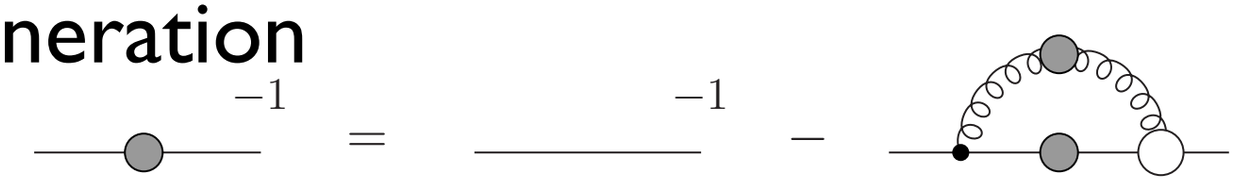






Overview

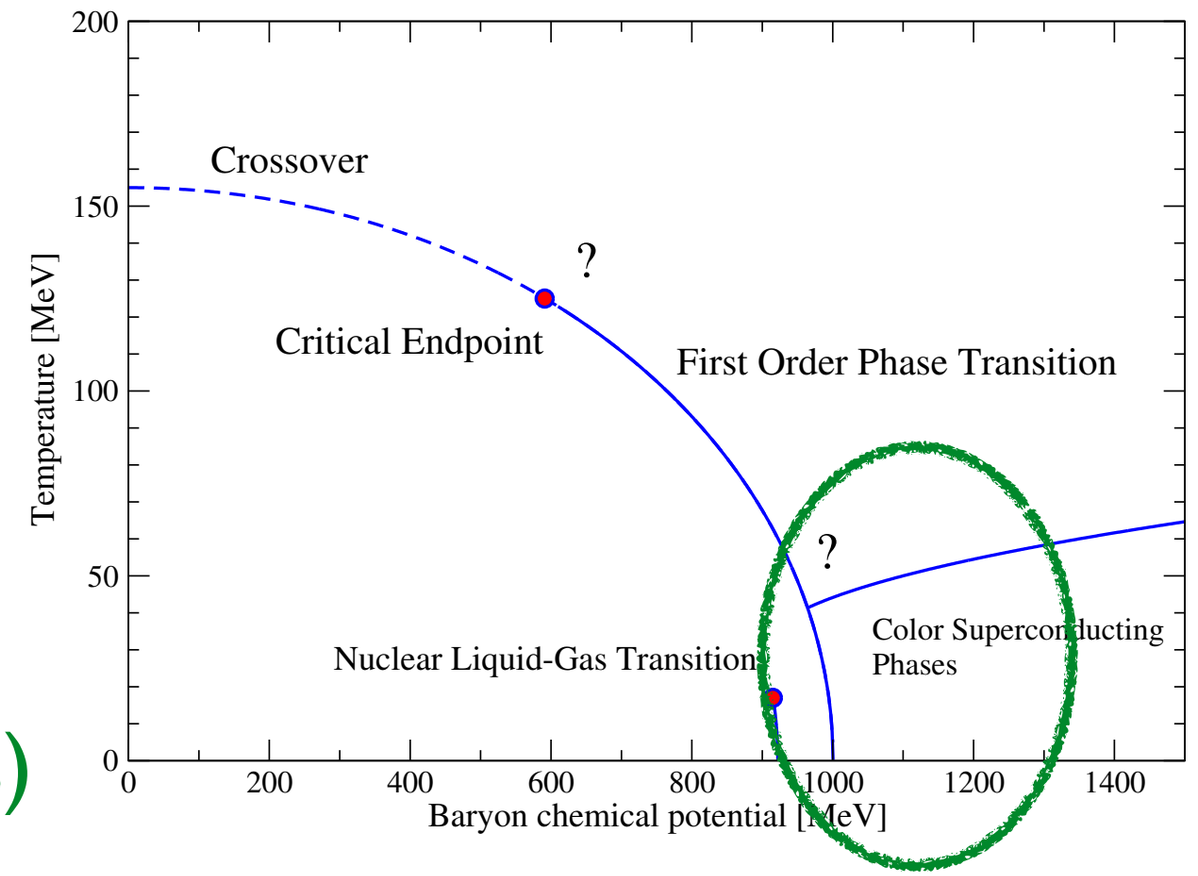
1. Introduction: dynamical mass generation



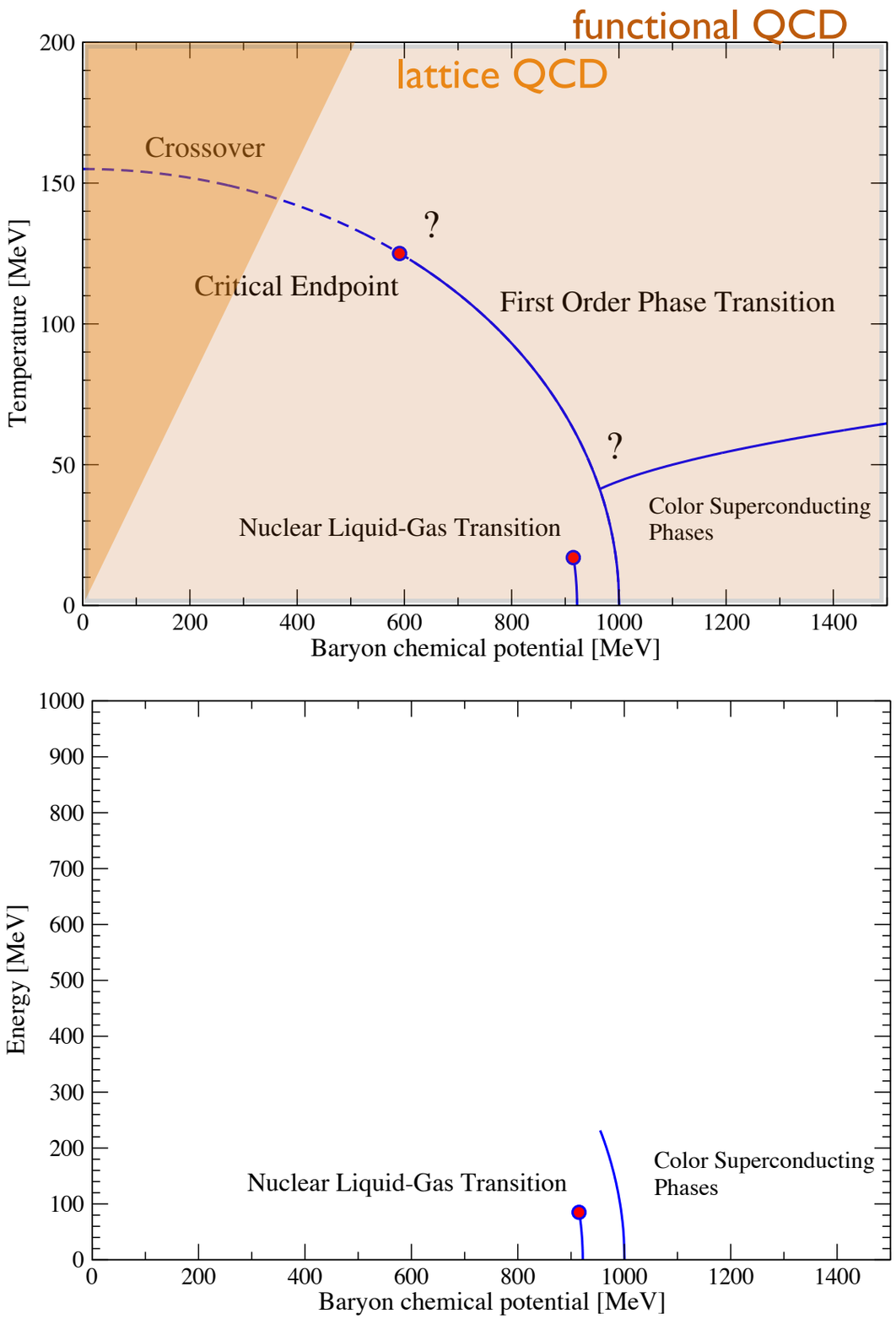
2. The quest for the critical end point

3. The quest for the EoS

Neutron star (mergers)



Phase transitions at zero T: what to expect…



"silver blaze": no excitations until

T. D. Cohen, PRL 91 , 222001 (2003)

$$\mu_B \leq m_N^*$$

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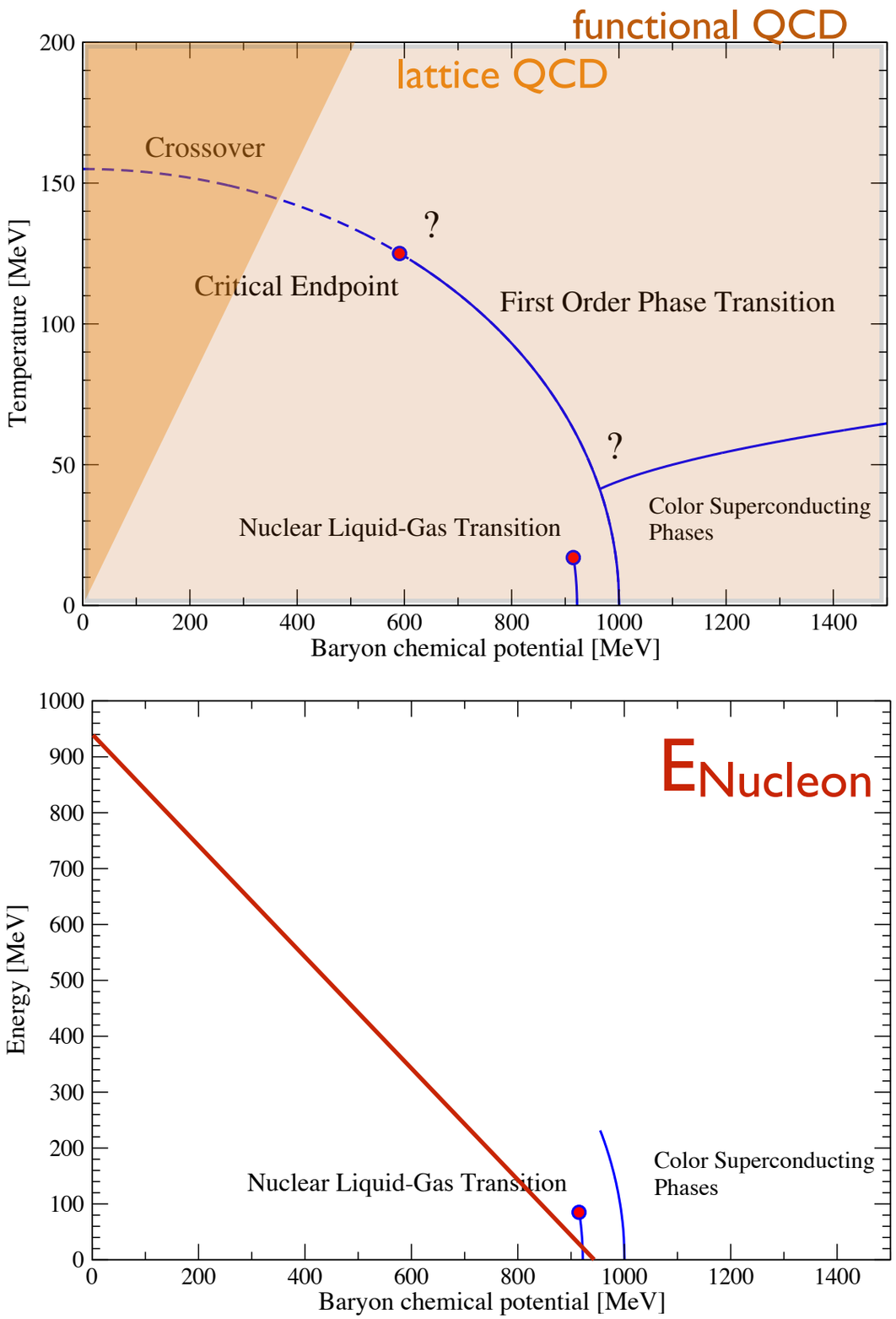

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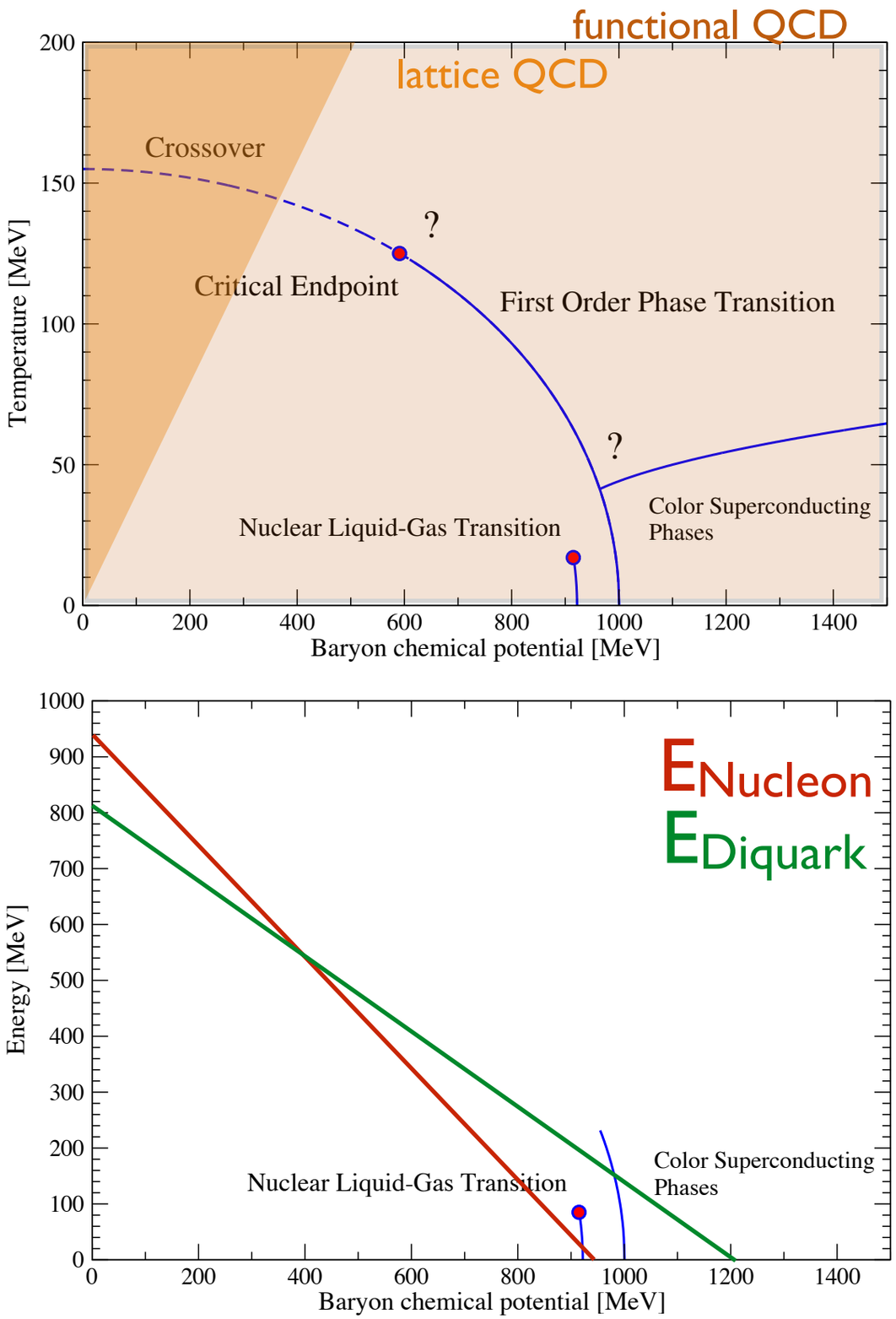

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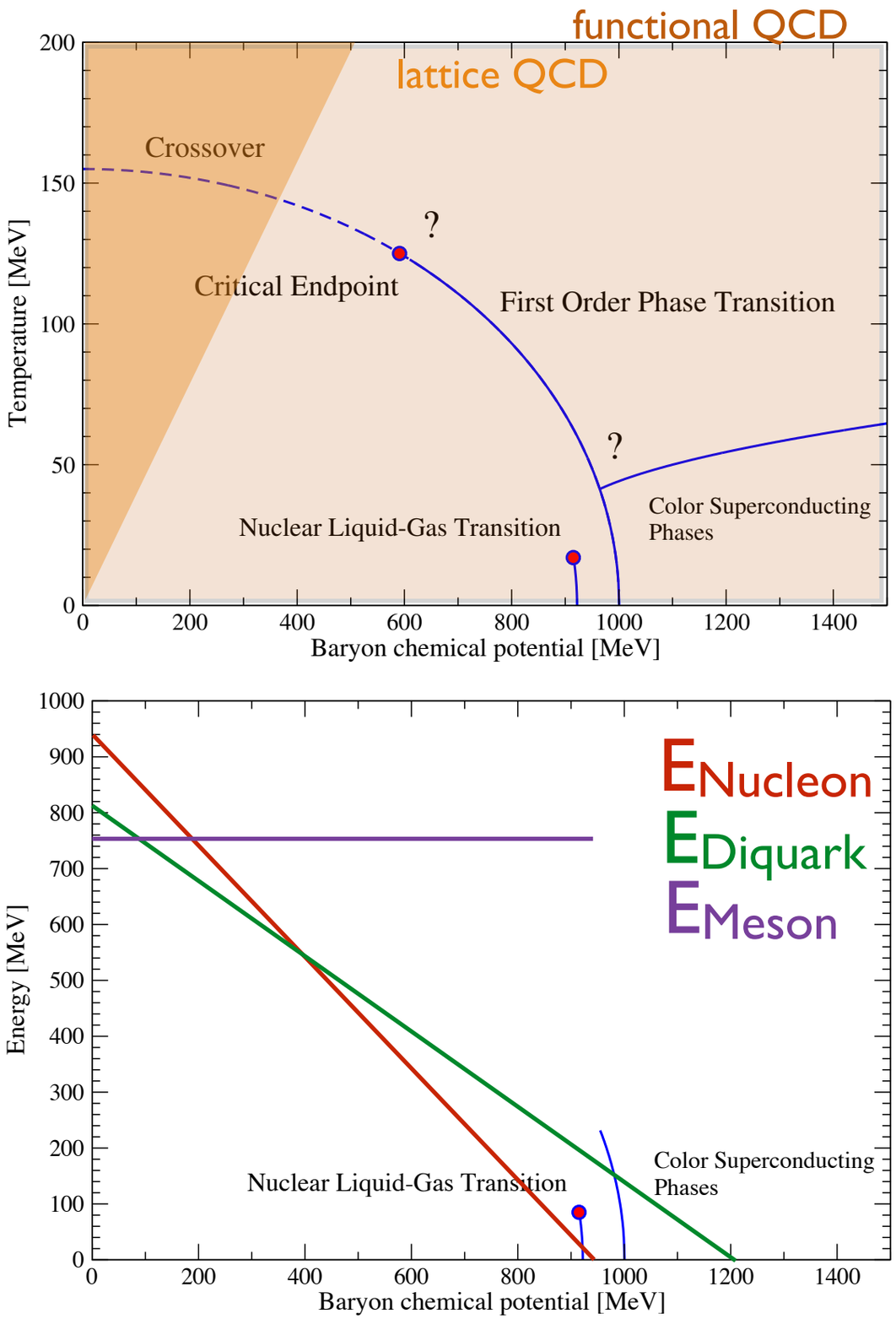

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$$\mu_B \leq m_N^*$$

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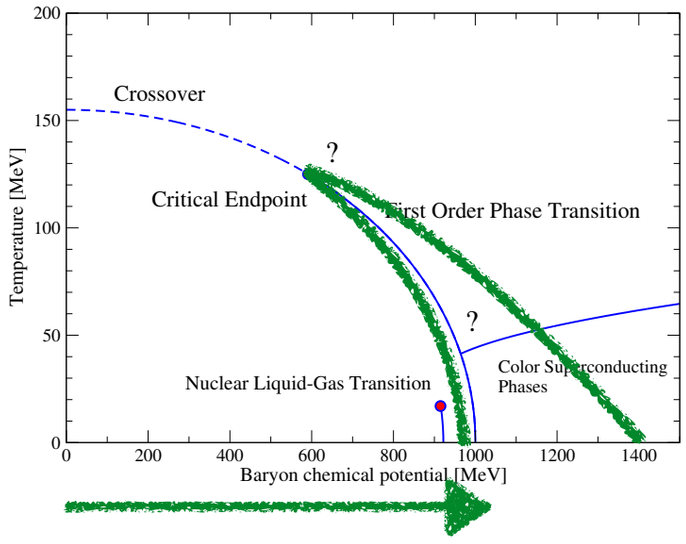
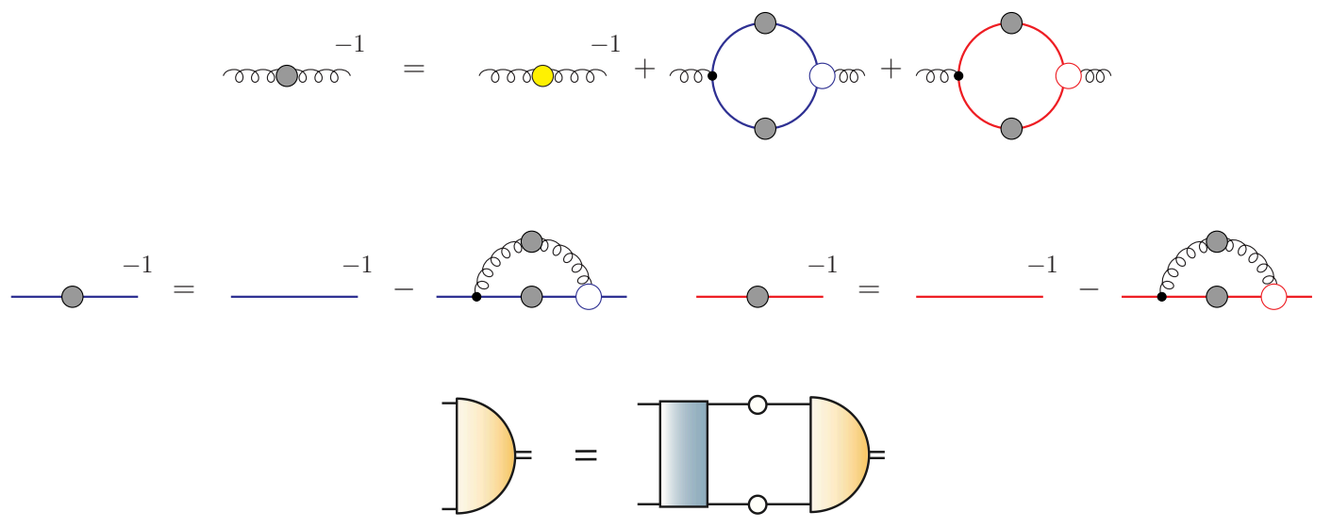
$$E_{\rm Diquark} = m_D - 2/3\,\mu_B$$

$$E_{\rm Meson} = m_M$$

Meson properties at finite chemical potential

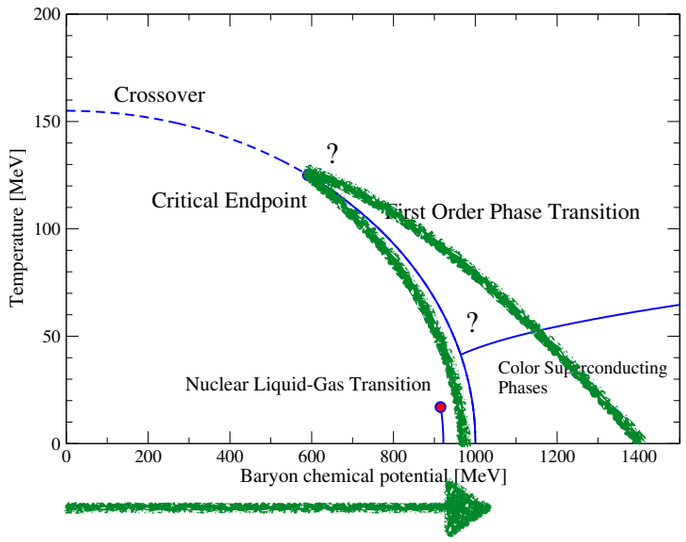


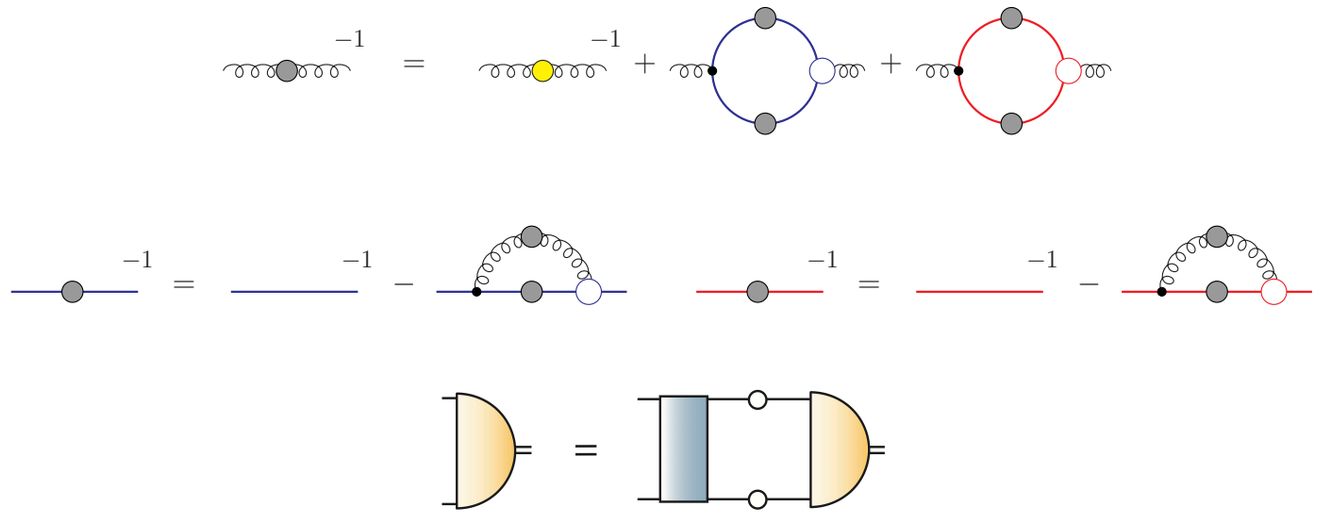


- Quarks/meson wave functions do change !

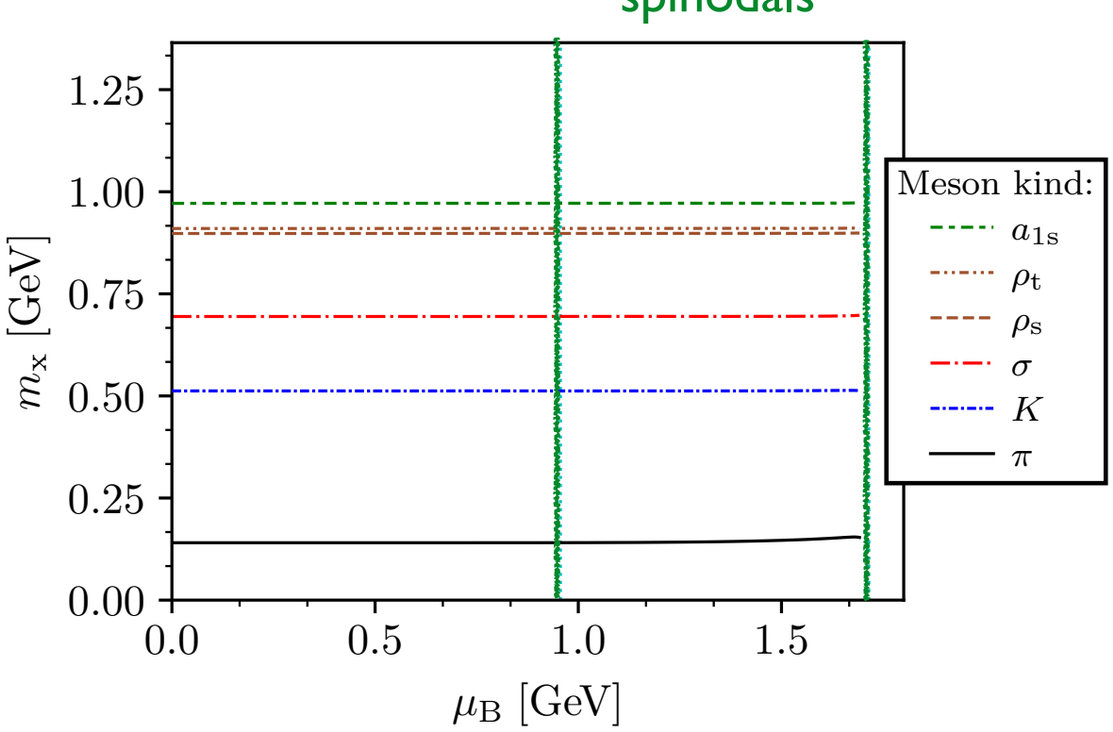
Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147

Meson properties at finite chemical potential

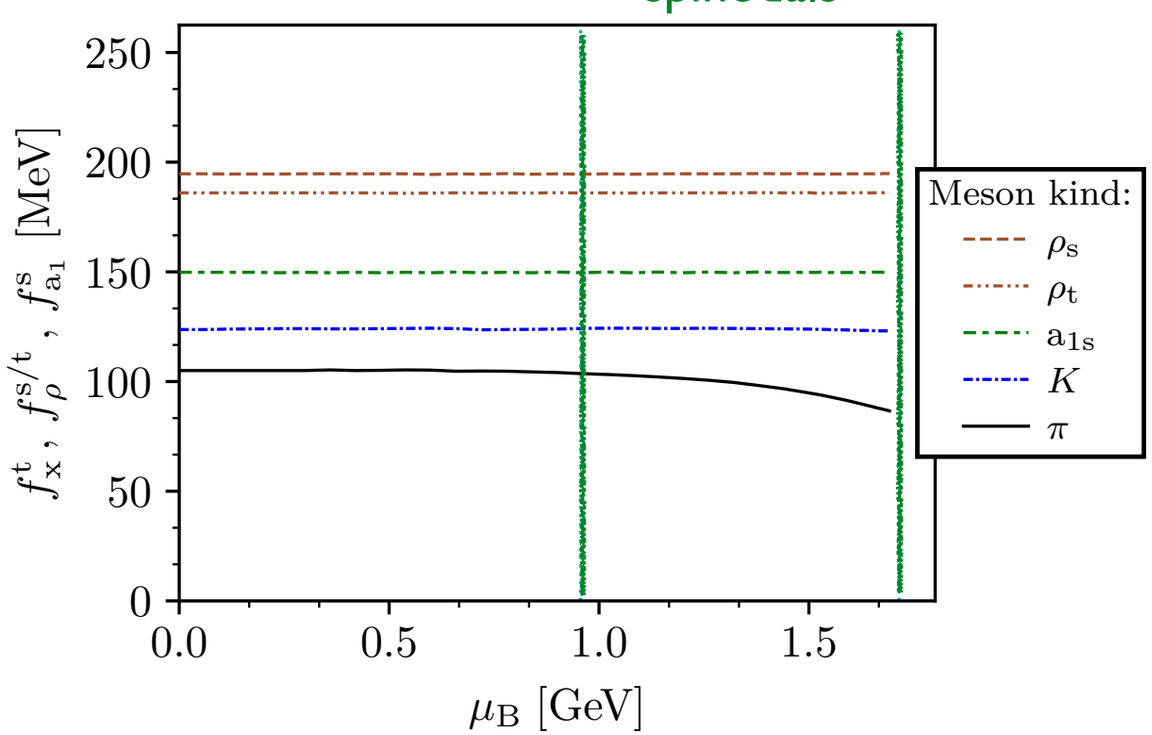




spinodals



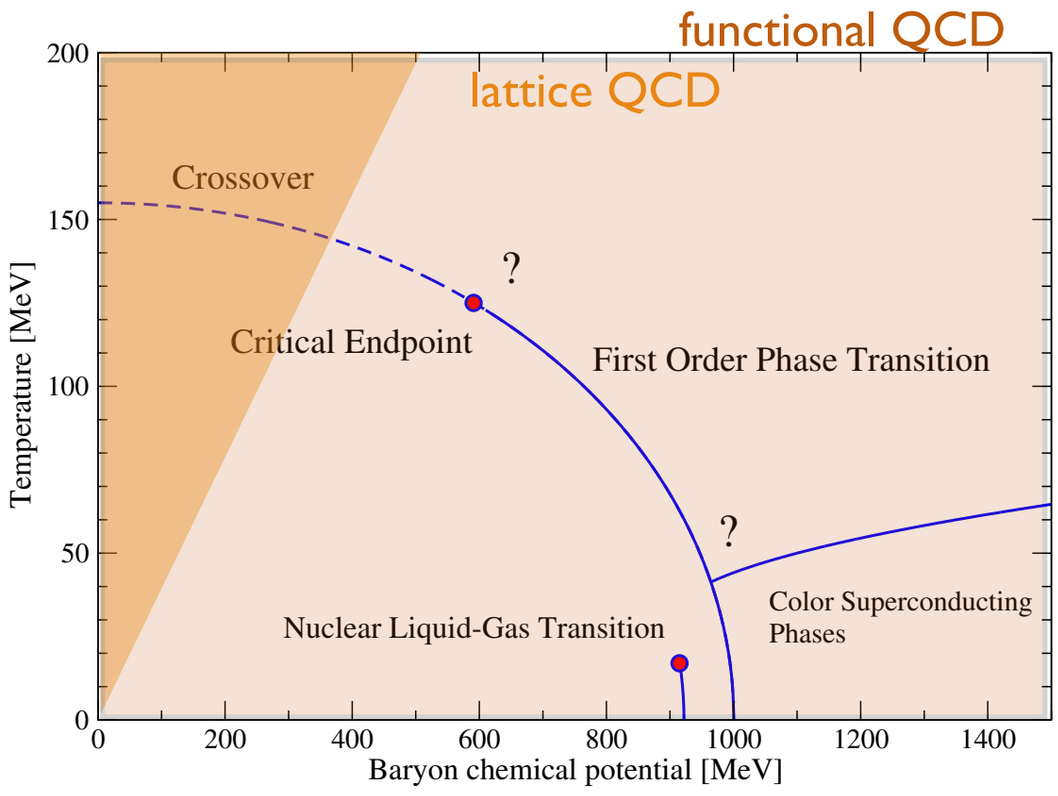
spinodals



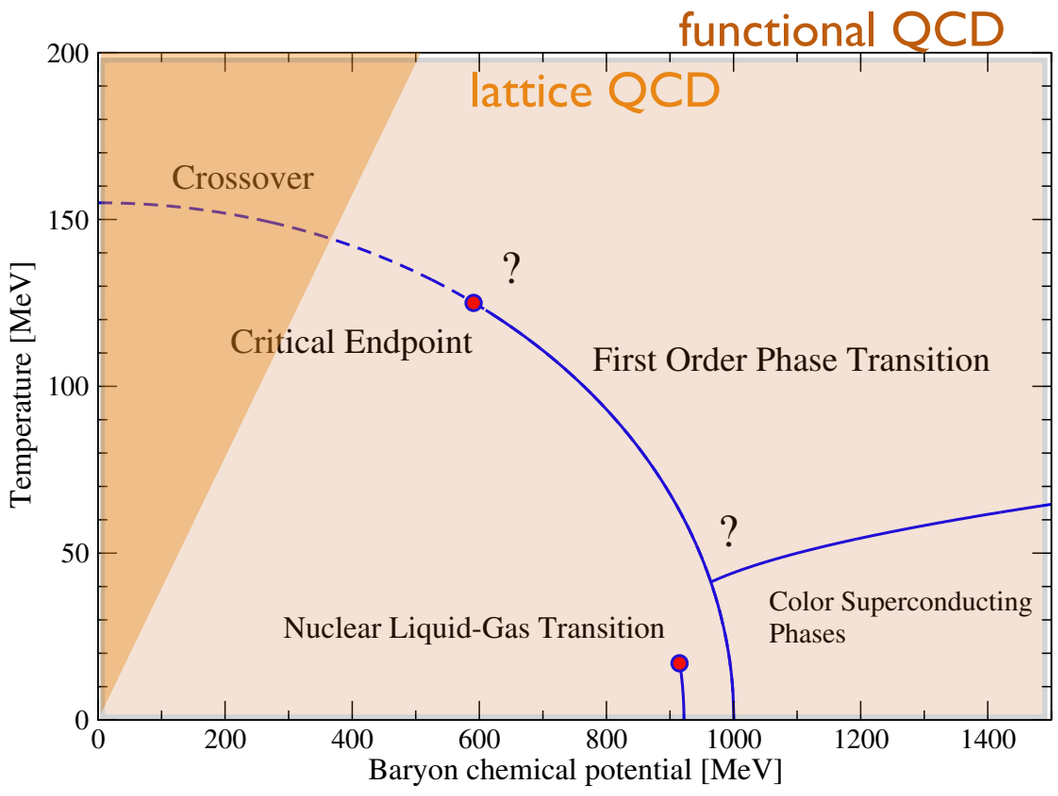
- Quarks/meson wave functions do change !
- But: Silver blaze satisfied

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147

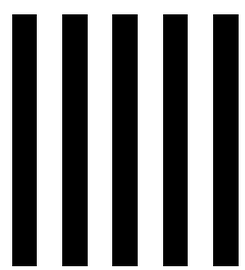
Phase transitions at non-zero T: what to expect…



Phase transitions at non-zero T: what to expect…

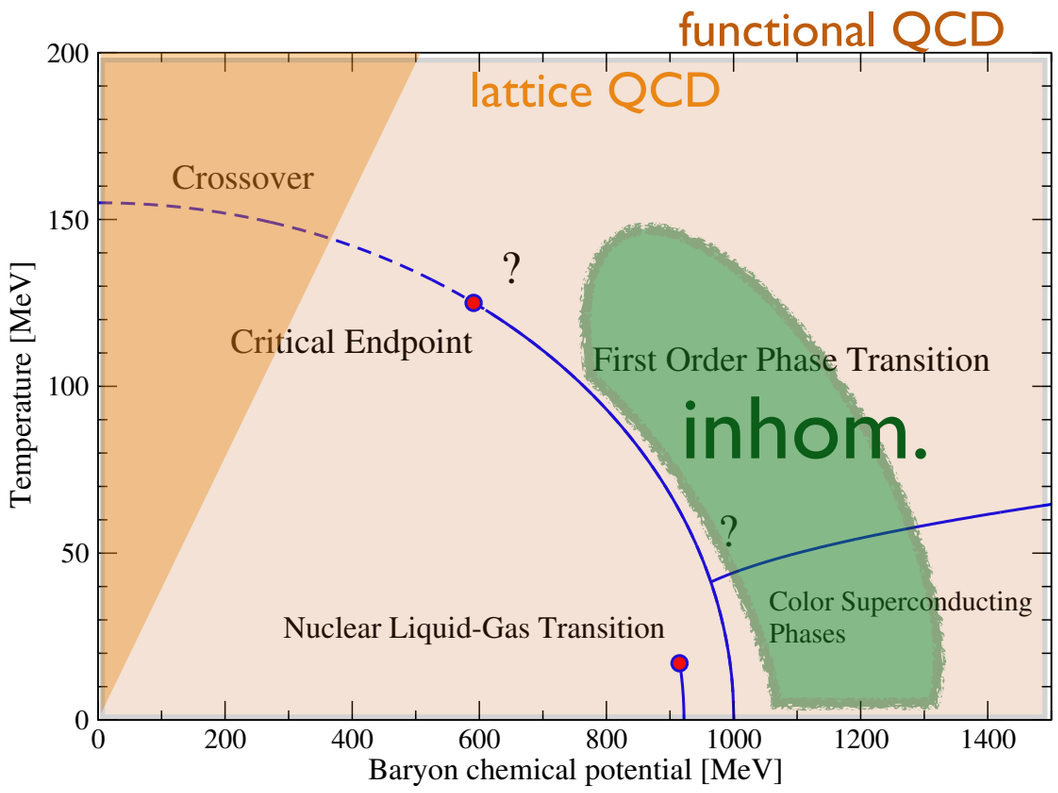


$\langle \bar{\Psi}\Psi \rangle$



$\langle \bar{\Psi}\Psi \rangle(x)$

Phase transitions at non-zero T: what to expect…



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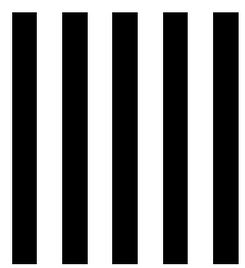
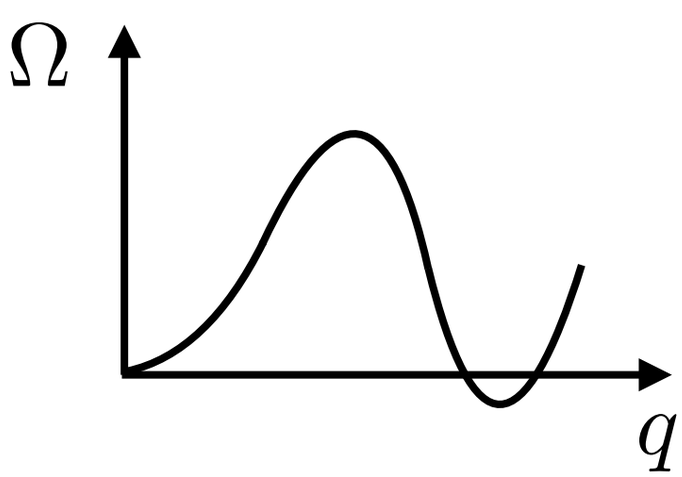


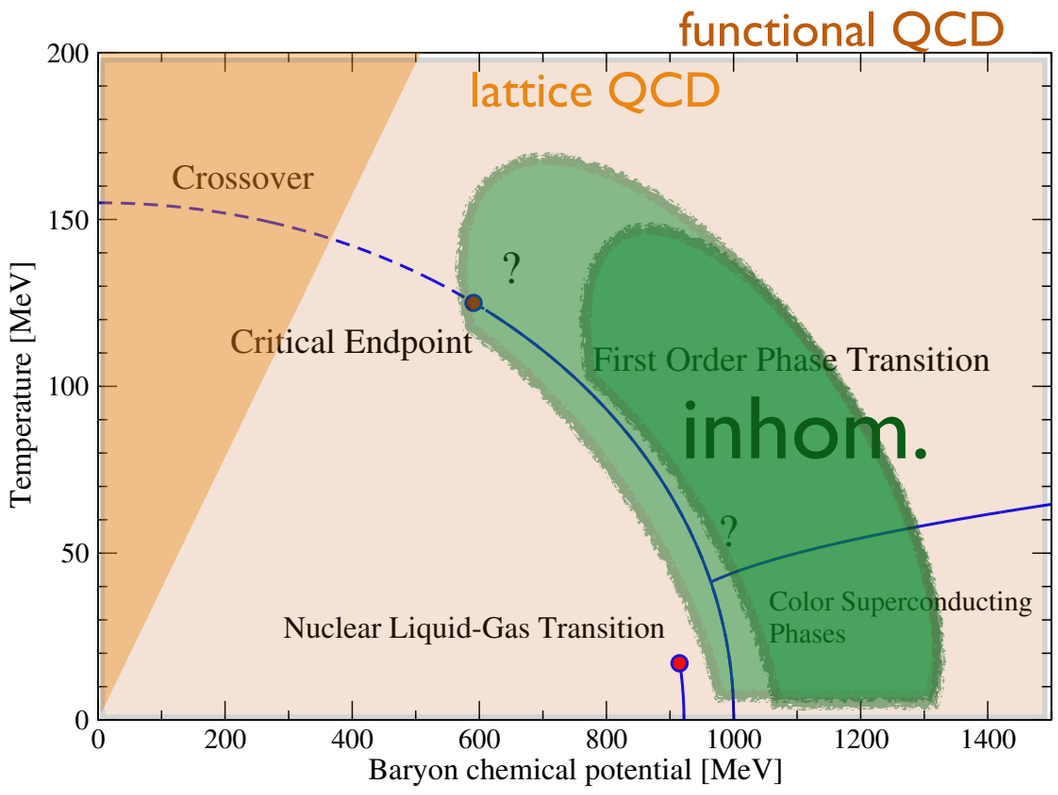
$\langle \bar{\Psi}\Psi \rangle(x)$



inhom. phase

Buballa and Carignano, PPNP 81 (2015), 39-96

Phase transitions at non-zero T: what to expect…



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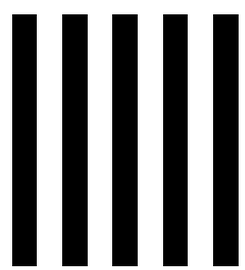

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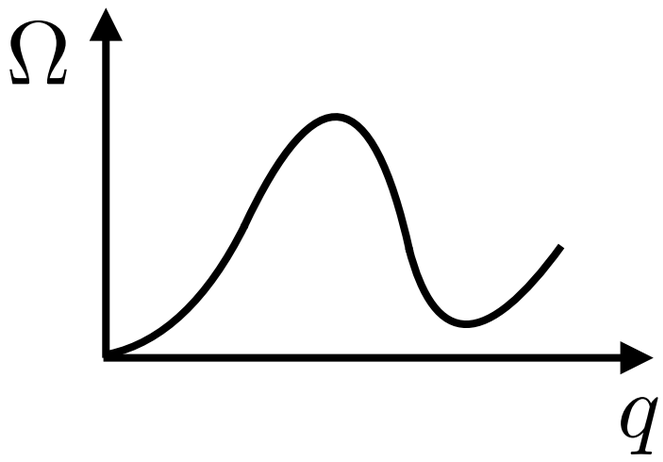
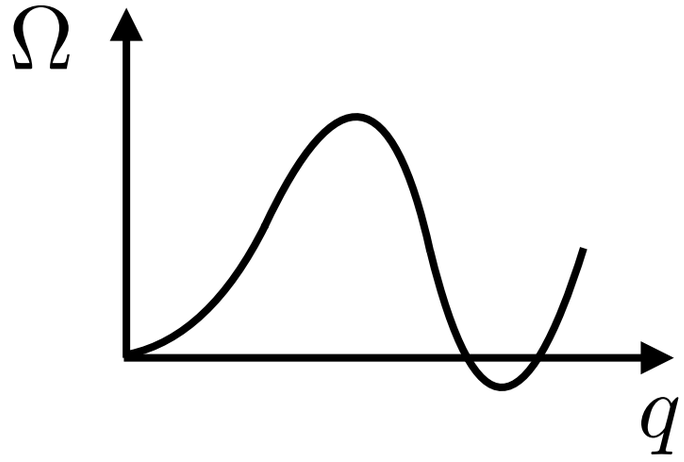


moat regime

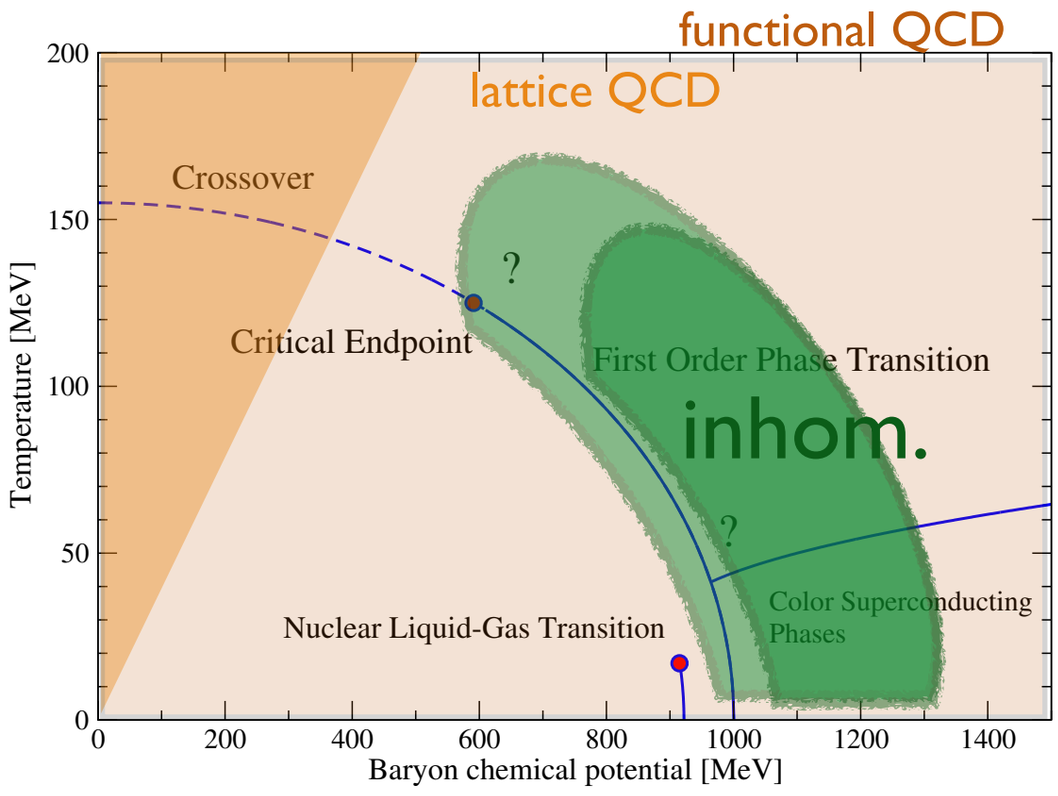


inhom. phase

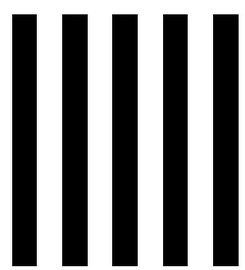
Fu, Pawlowski, Rennecke, PRD 101, (2020) 5 054032
Fu, Pawlowski, Pisarski, Rennecke, Wen, Yin, PRD 111 (2025) 9 094026

Buballa and Carignano, PPNP 81 (2015), 39-96

Phase transitions at non-zero T: what to expect…



$\langle \bar{\Psi}\Psi \rangle$



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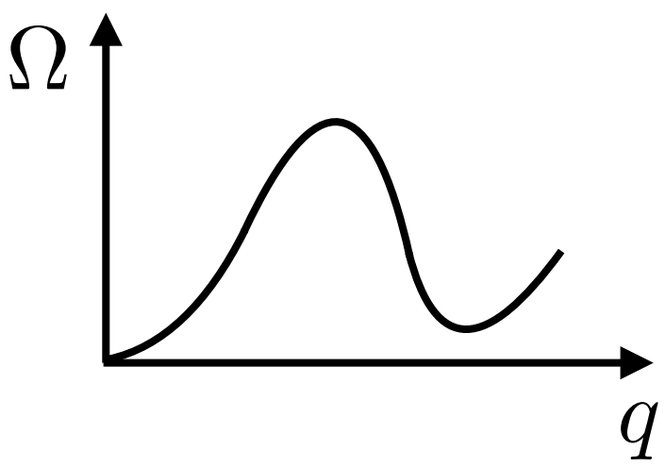

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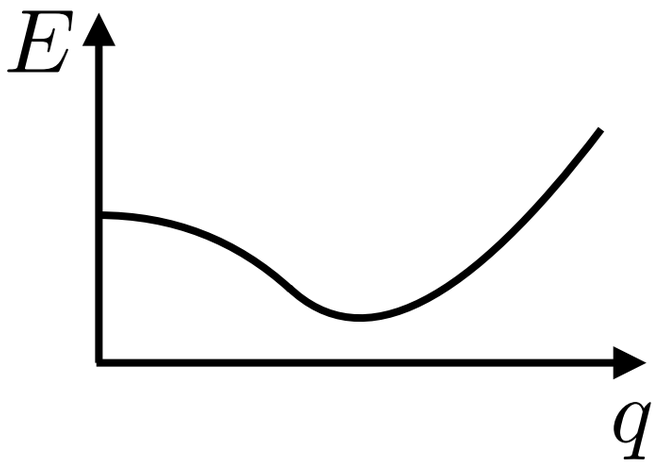
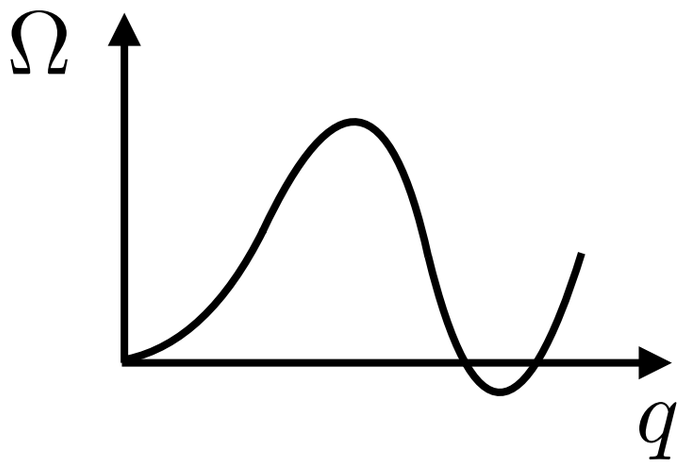


nontrivial static energy

Pisarski, Rennecke, PRL 127 (2021), 15 152302



inhom. phase

Buballa and Carignano, PPNP 81 (2015), 39-96

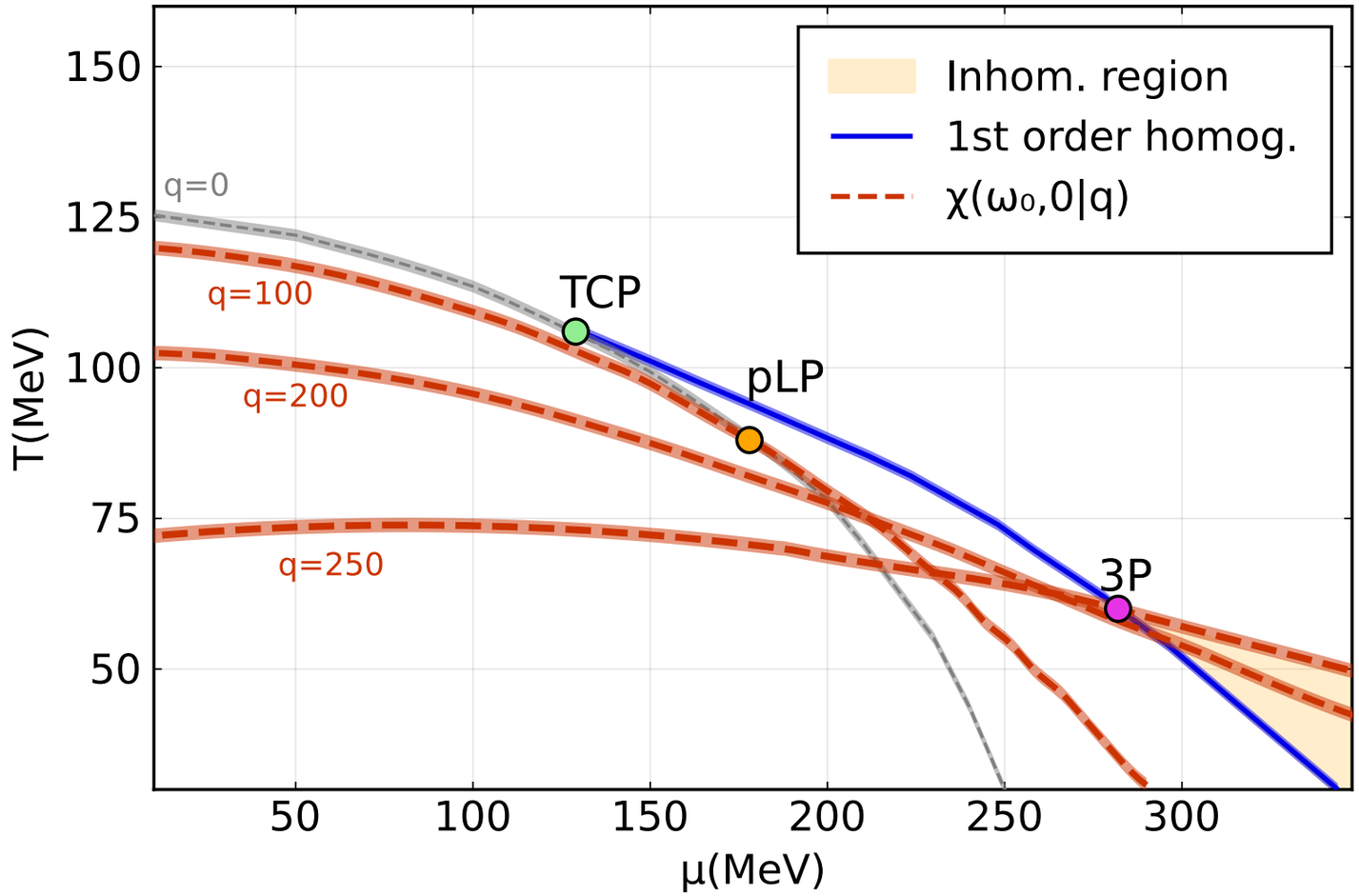
Inhomogeneous phases (simple truncation)

$$\xi = e^{i\gamma_5\tau_3 q\cdot x}\psi$$

$$\langle\bar{\psi}(x)\psi(x)\rangle = \cos(2q\cdot x)\,\langle\bar{\xi}\xi\rangle$$
$$\langle\bar{\psi}(x)i\gamma_5\tau_3\psi(x)\rangle = \sin(2q\cdot x)\,\langle\bar{\xi}\xi\rangle$$

stability analysis

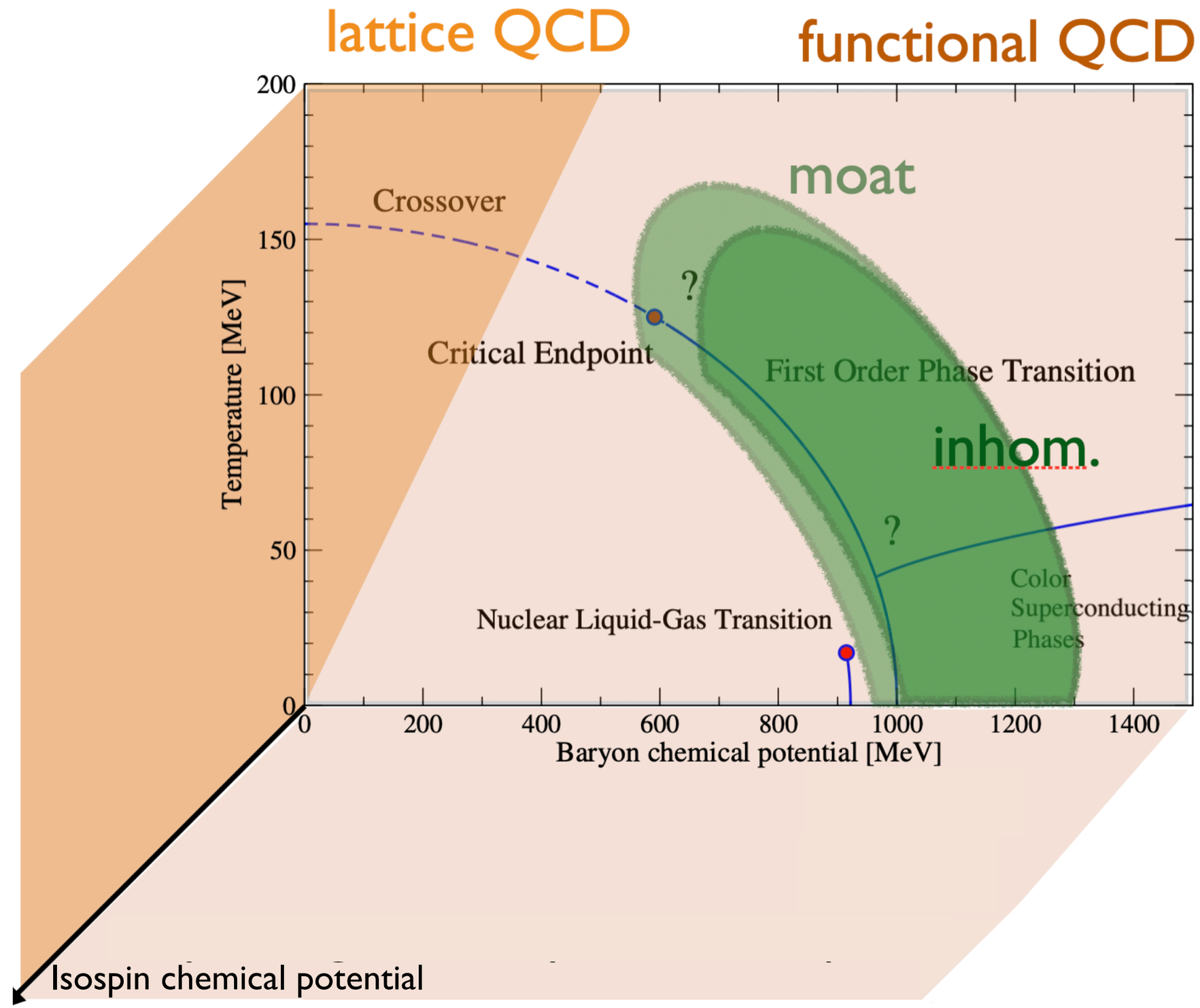
$$\chi(q) = \frac{\partial M}{\partial m} < 0$$



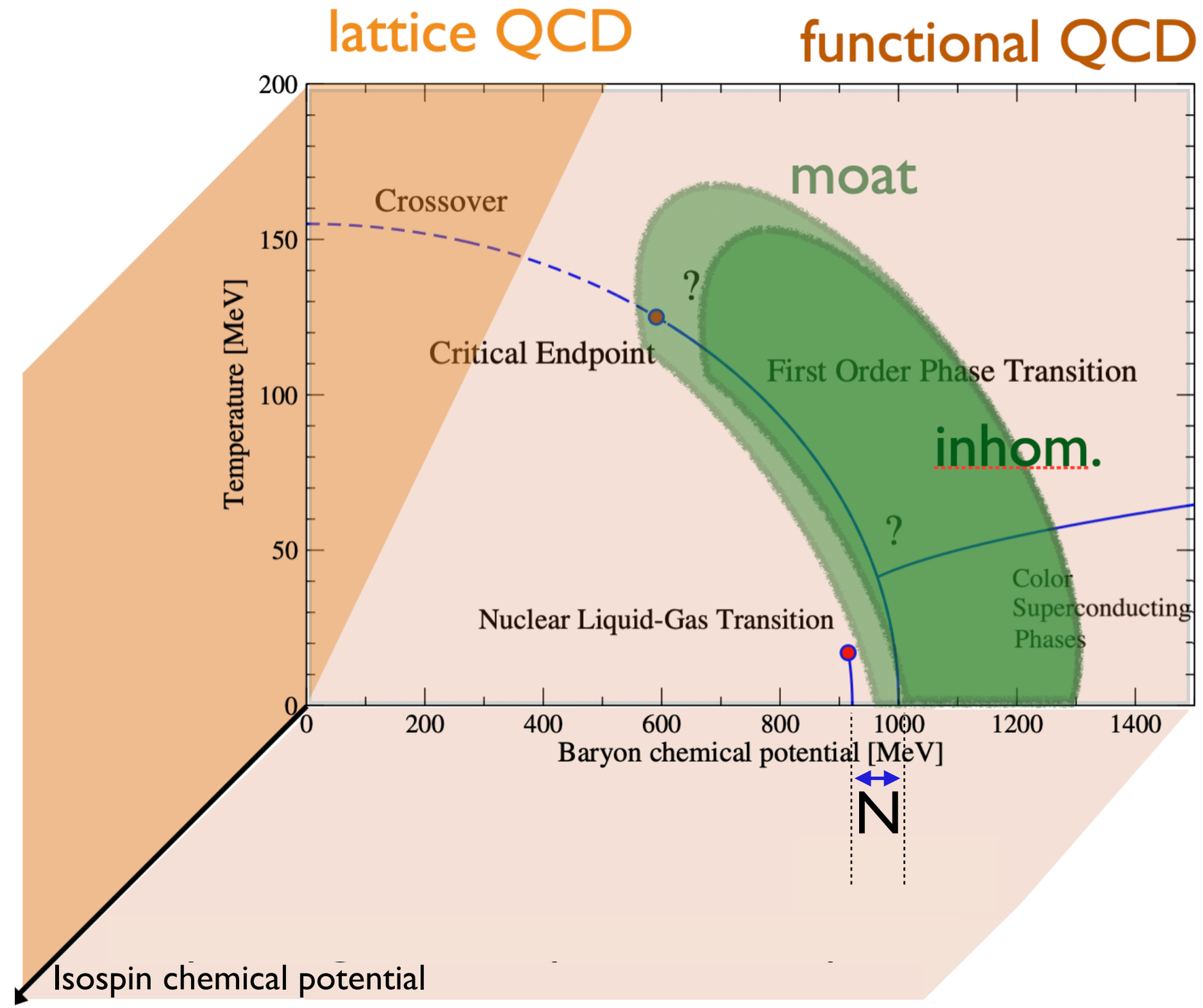
- unambiguous identification of inhomogeneous region
- locations of CEP and 3P are far apart… (in this model…)

T. F. Motta, J. Bernhardt, M. Buballa and CF, PRD 108 (2023), PRD 110 (2024), PRD 111 (2025)

Dense QCD: relevant degrees of freedom in EoS?



Dense QCD: relevant degrees of freedom in EoS?



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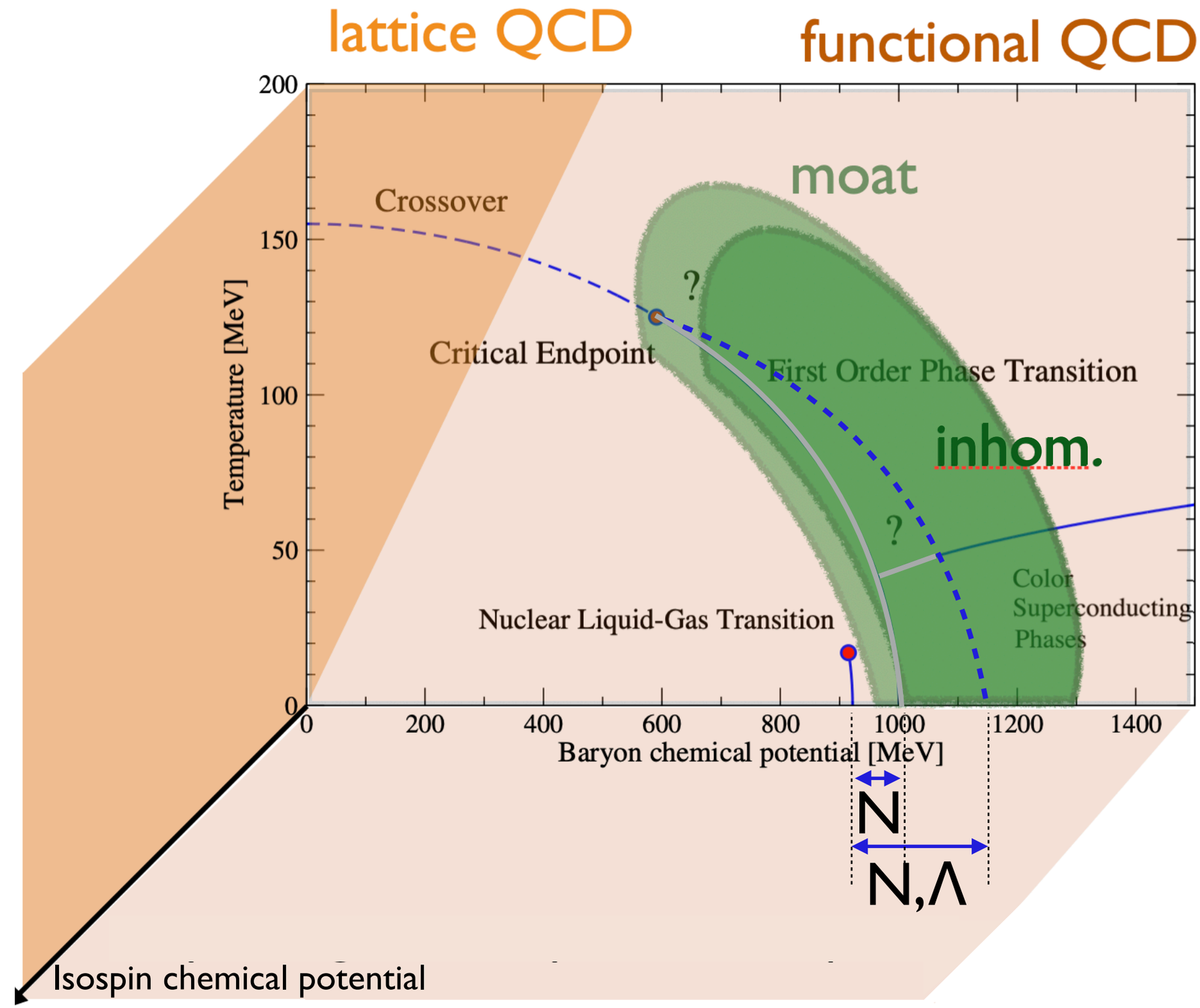


Dense QCD: relevant degrees of freedom in EoS?

EoS from microscopic QCD
(functional approach):

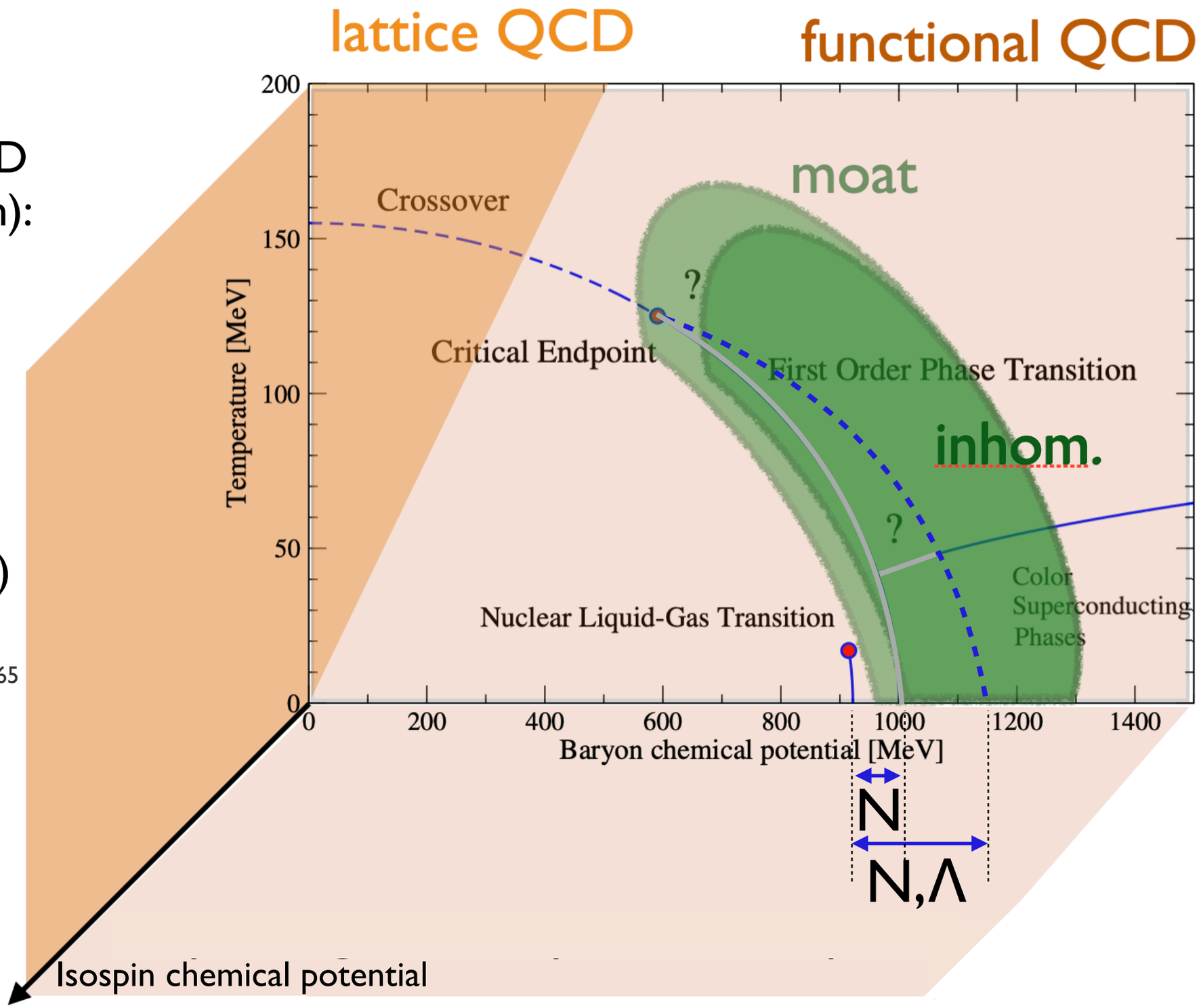
- chirally broken phase
  - quarks, mesons ✔
  - baryons
    work in progress

- superconducting phase(s)

  ✔ Buballa et al.
  Müller, Buballa, Wambach, arXiv:1603.02865
  work in progress

- inhomogenuous broken ('cristaline') phase(s)

  work in progress
  Buballa, Motta, CF, Rennecke, Pawlowski, Pisarski, …



Summary: QCD with functional methods

Main goals:

- **one** framework for all areas of hadron physics:
  mesons, baryons, 'exotic states', form factors,
  hadronic contributions to precision observables (g-2)
- **same** framework for QCD phase diagram

Main challenge:

- systematic control over error budget:
  —> DSE vs. FRG, —> func. QCD vs lattice QCD

CF and Pawlowski, EPJST [2603.11135]

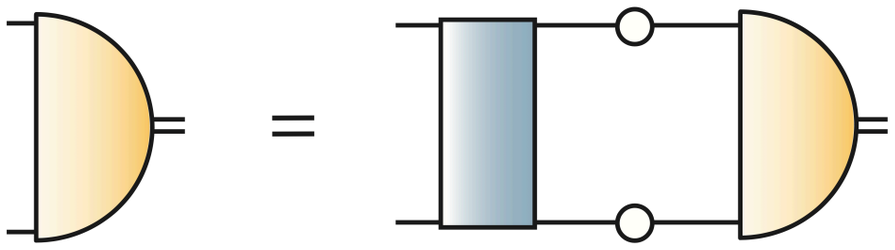
Main results:

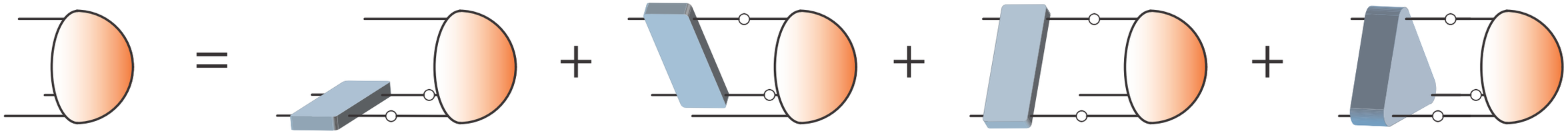
- not high precision physics
  but competitive contributions in many areas
- QCD-based tool to explore phase diagram at large μ
  at physical quark masses

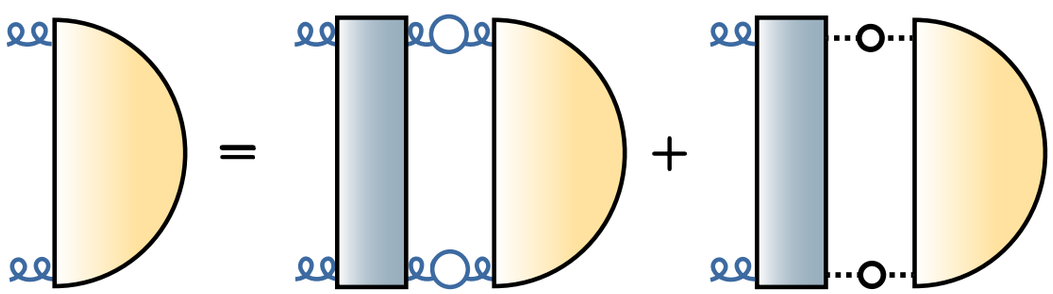
Backup

Bound states and Bethe-Salpeter equations

BSEs:



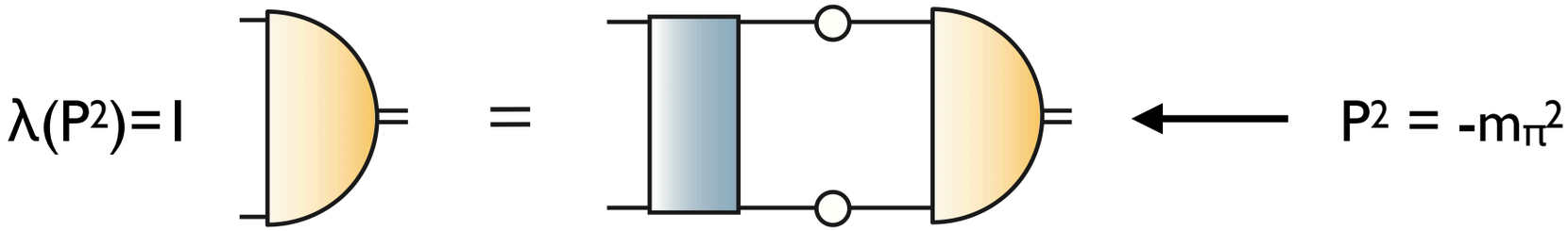




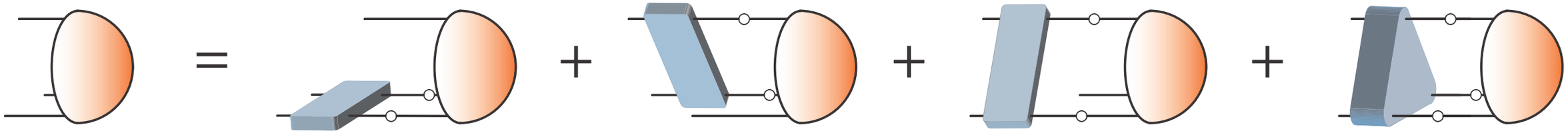
Eigenvalue equations: masses and wave functions

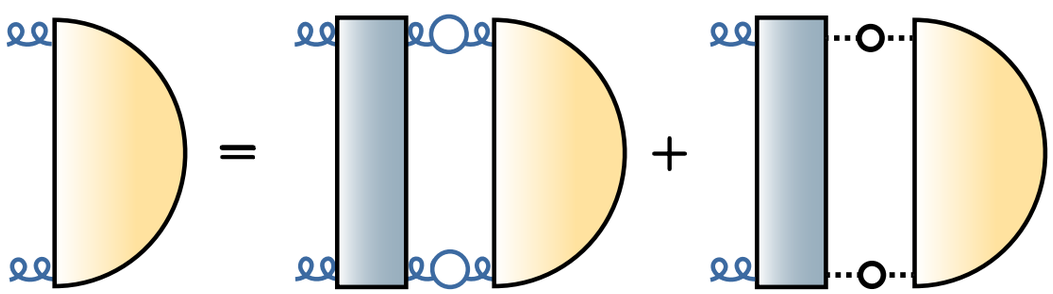
Bound states and Bethe-Salpeter equations

BSEs:



Bethe-Salpeter Amplitude



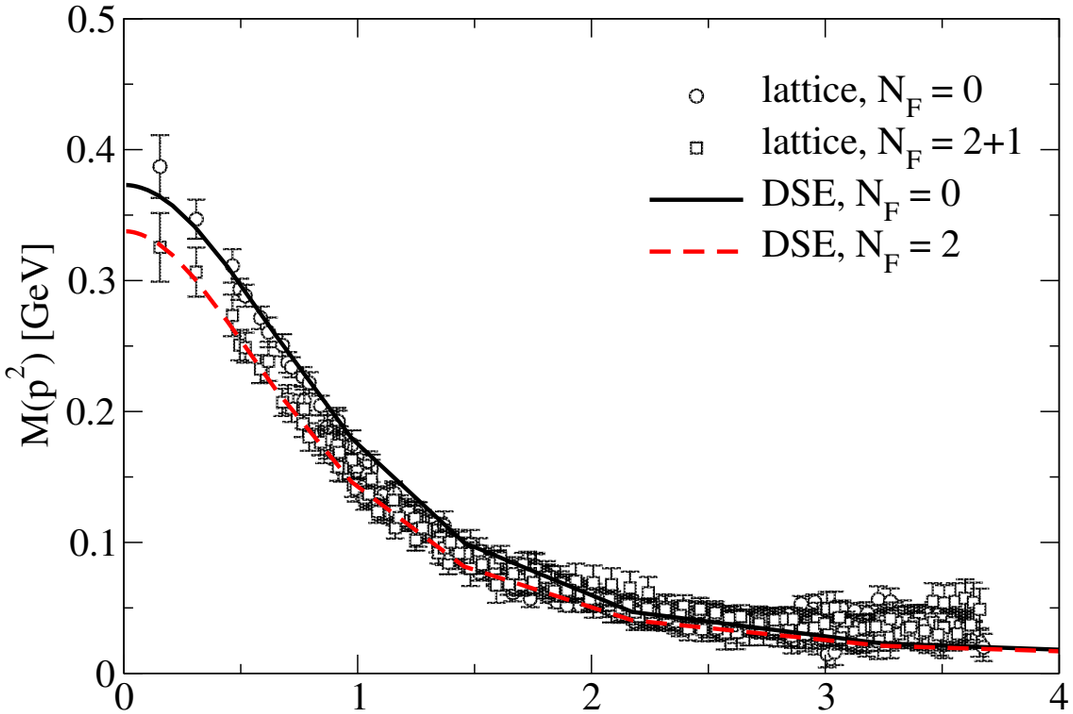


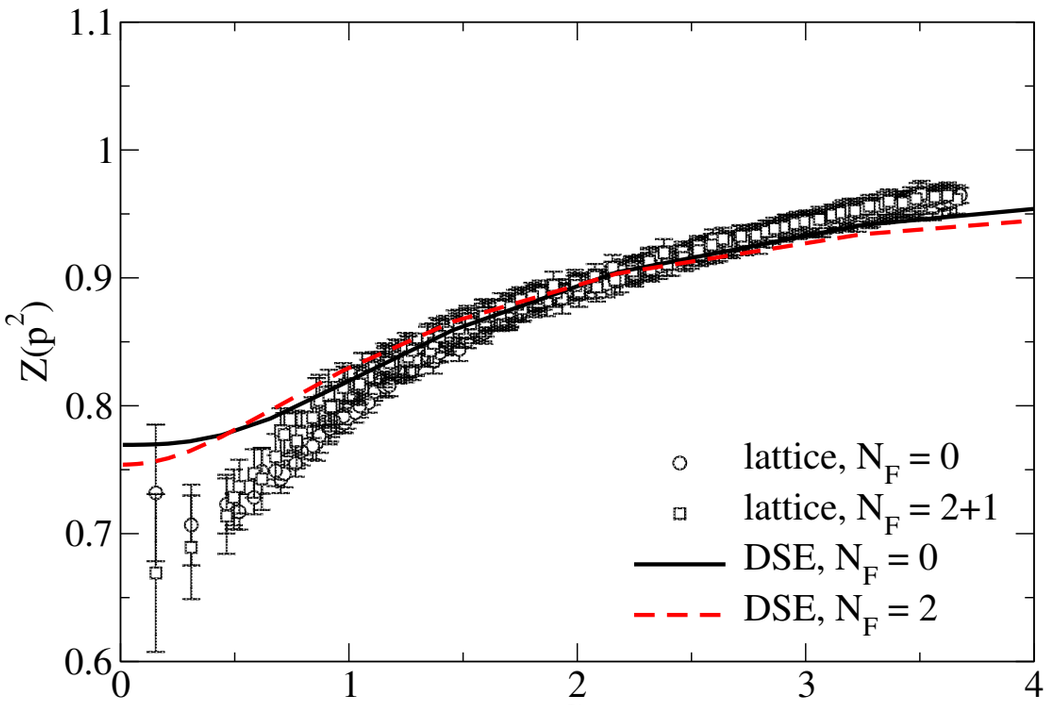
Eigenvalue equations: masses and wave functions

Selected results for Green's functions

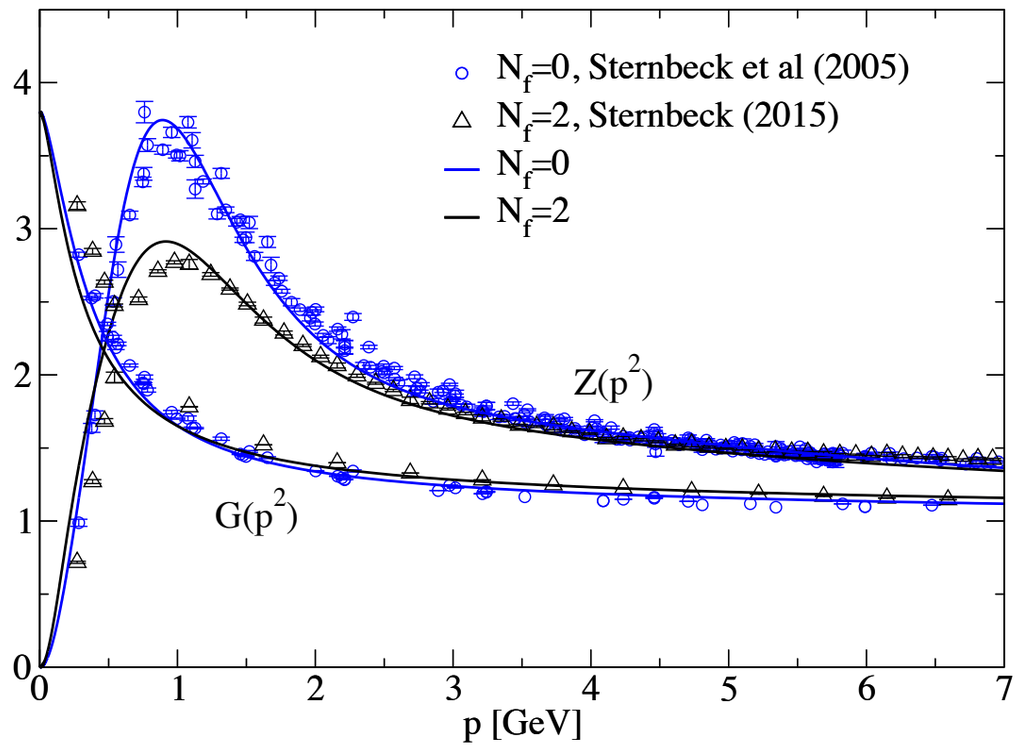
Quark

Williams, CF, Heupel, PRD 93 (2016) 034026

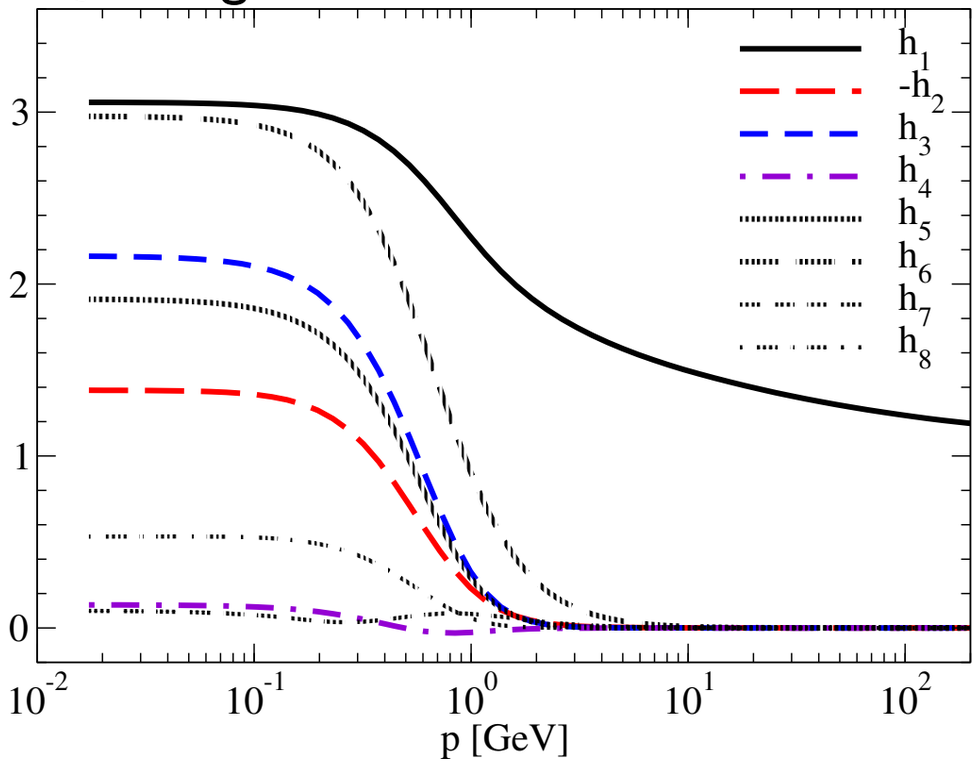




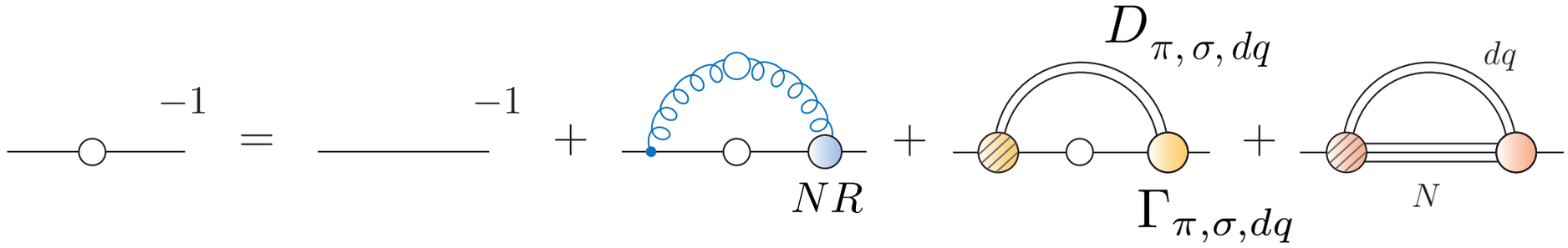
Gluon



Quark-gluon-vertex

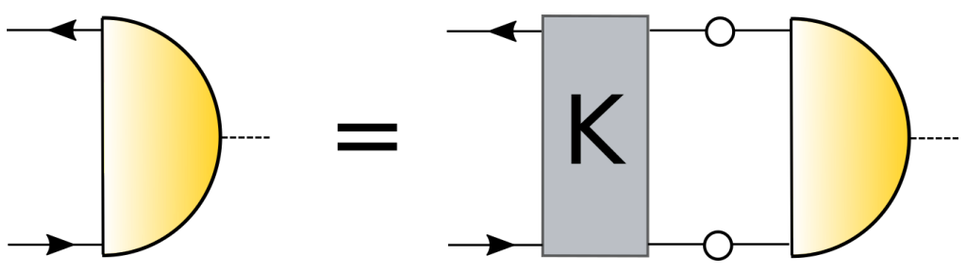


Meson effects at finite T and μ



$$D_\pi(p) = \frac{1}{p_4^2 + u^2(\vec{p}^{\,2} + m_\pi(T,\mu)^2)} \qquad u = \frac{f_s}{f_t}$$

Son, Stephanov, PRD 66 (2002) 7

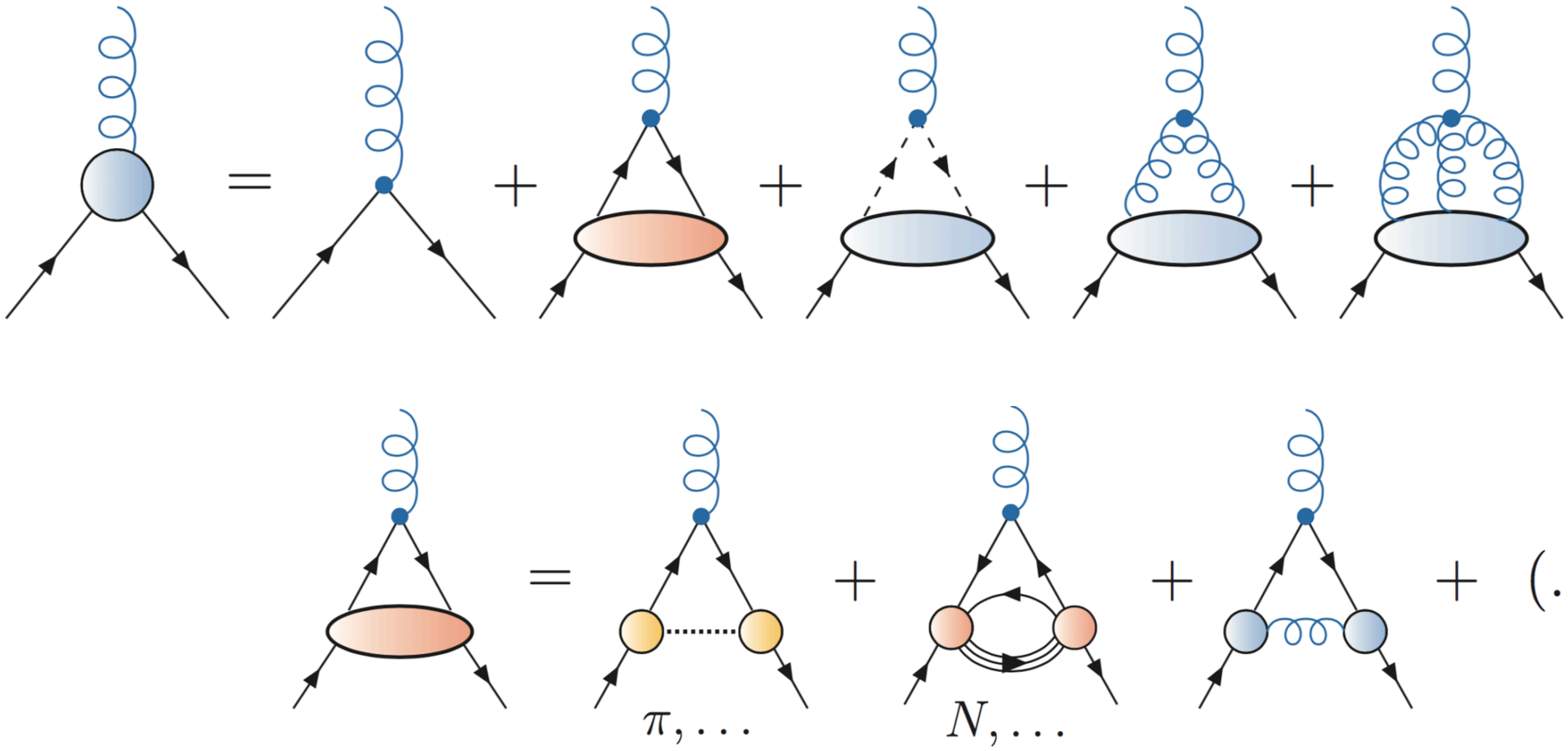


$$\Gamma_\pi(P,q) = \gamma_5\, E(P,q,T,\mu) + \dots$$

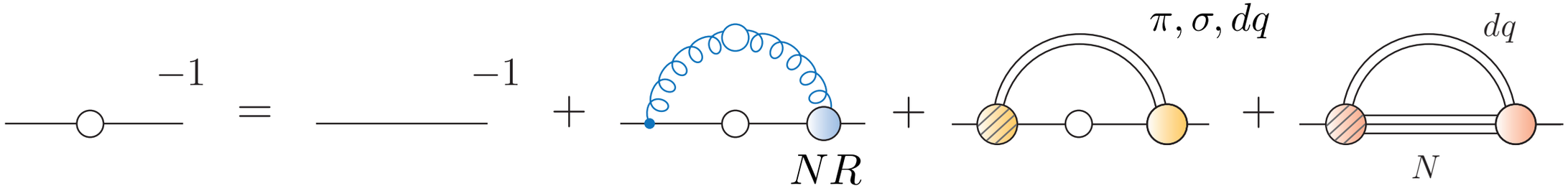
chiral limit: $\Gamma_\pi = \gamma_5 \dfrac{B}{f_t}$

Hadron effects in quark-gluon interaction

quark-gluon vertex:



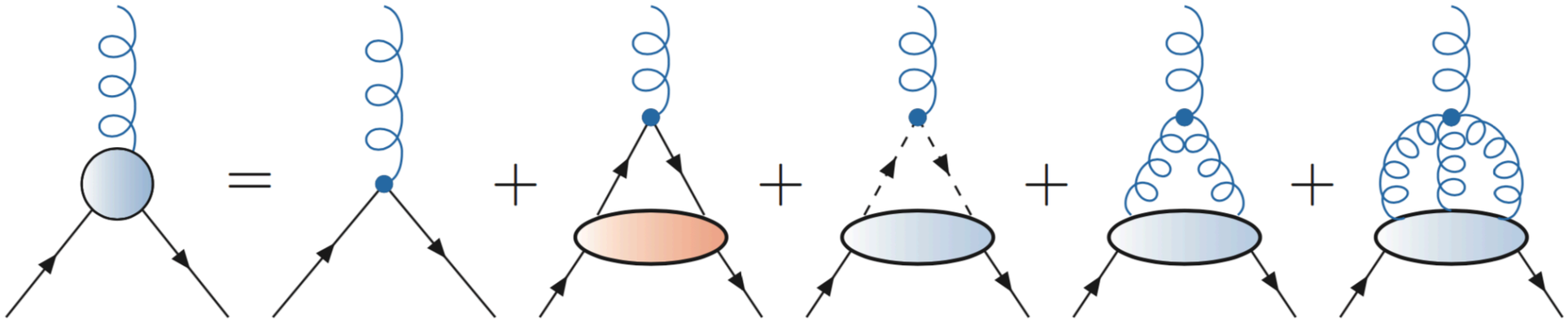
quark:

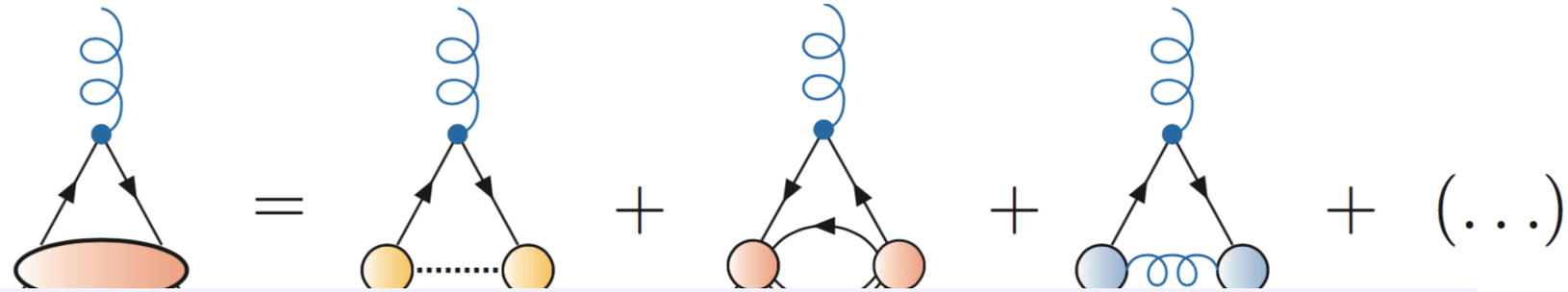


Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

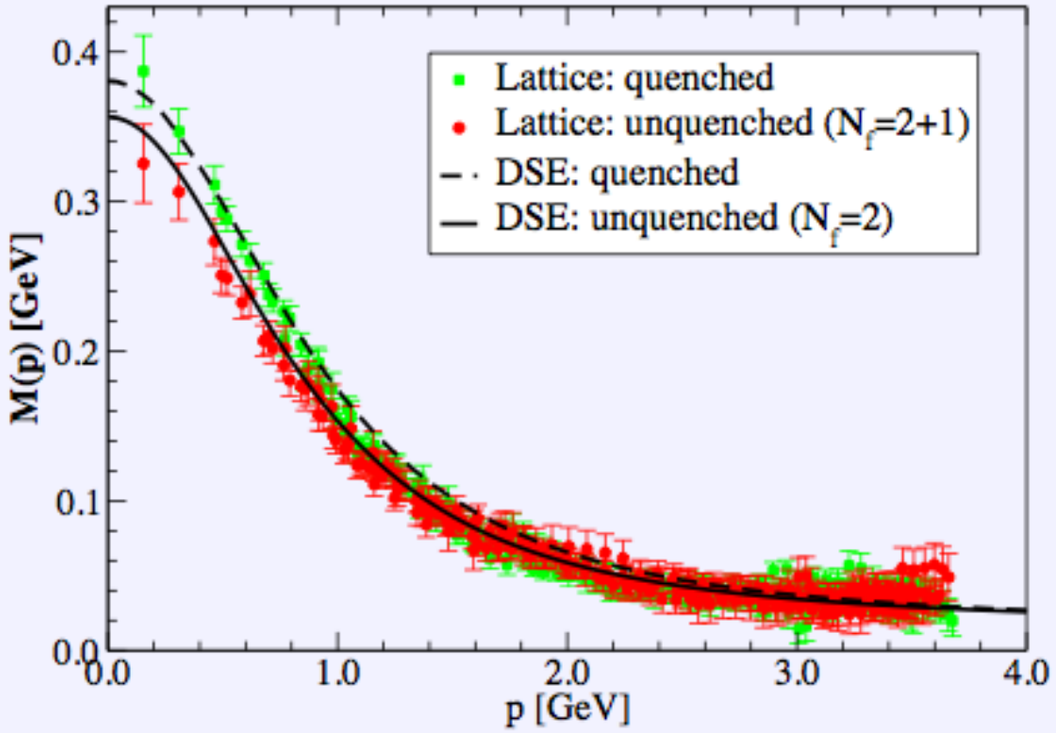
Hadron effects in quark-gluon interaction

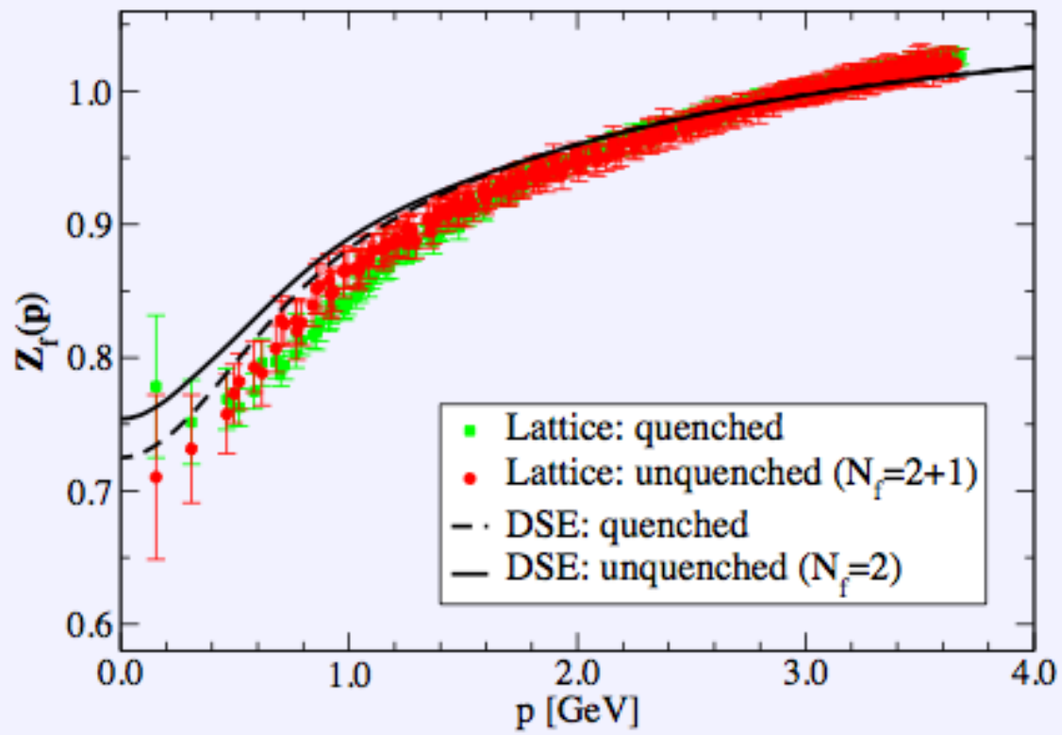
quark-gluon vertex:





quarl





CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

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