QCD regimes/phases at small μ, their symmetries and N_c scaling

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Encyclopedia of Nuclear Physics [2510.14084]

Chiral spin symmetry and its flavor extension

The chromoelectric field of QCD interacts with the the color charge of quarks

$$\boldsymbol{F} = Q^a \boldsymbol{E}^a; \qquad Q^a = g \int d^3x \; q^\dagger(x) T^a q(x), \qquad a = 1, ..., 8$$

The interaction is invariant under $SU(2)_{CS}$ (L.Ya.G., 2015):

$$\begin{pmatrix} R \\ L \end{pmatrix} \to \begin{pmatrix} R' \\ L' \end{pmatrix} = \exp\left(i\frac{\varepsilon^n \sigma^n}{2}\right) \begin{pmatrix} R \\ L \end{pmatrix}$$

or, equivalently:

$$q \to q' = \exp\left(i\frac{\varepsilon^n \Sigma^n}{2}\right) q, \qquad \Sigma = \{\gamma_0, -i\gamma_5\gamma_0, \gamma_5\}$$

The Dirac Lagrangian prohibits such transformation.

The transformation can be local: $\varepsilon^n = \varepsilon^n(x)$.

In a given Lorentz frame interaction of quarks with the electric part of the gluonic field is chiral spin invariant.

$SU(2)_{CS} \times SU(N_F) \subset SU(2N_F)$; $SU(2N_F)$ is also a symmetry of the color charge.

$SU(N_F)_L \times SU(N_F)_R \times U(1)_A \subset SU(2N_F)$

The color charge and electric part of the theory have a $SU(2N_F)$ symmetry that is larger than the chiral symmetry of QCD as a whole.

The fundamental vector of $SU(2N_F)$ at $N_F = 2$

$$\Psi = \begin{pmatrix} u_{\rm R} \\ u_{\rm L} \\ d_{\rm R} \\ d_{\rm L} \end{pmatrix} .$$

L.Ya.G., EPJA 51 (2015) 27
L.Ya.G., M.Pak, PRD 92 (2015) 016001

The $SU(2)_{CS}$ and $SU(2N_F)$ are broken by the magnetic interaction, by the quark kinetic term, by the $U(1)_A$ anomaly and by the quark condensate. They can be seen as approximate symmetries if, and only if the breaking effect is small, i.e., when the physics is dominated by the confining chromoelectric field.

We can consider the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries as symmetries of the confining interaction with ultrarelativistic light quarks.

Compare with the center symmetry.

Full symmetry of confinement

Minkowski QCD Hamiltonian in Coulomb gauge:

$$H_{QCD} = H_{E,B} + \int d^3x \Psi^\dagger(\boldsymbol{x})[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}]\Psi(\boldsymbol{x}) + H_T + H_C,$$

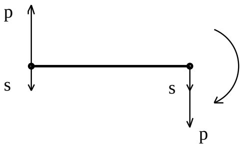
with the transverse and instantaneous "Coulombic" interactions to be:

$$H_T = -g \int d^3x\, \Psi^\dagger(\boldsymbol{x})\boldsymbol{\alpha}\cdot t^a \boldsymbol{A}^a(\boldsymbol{x})\,\Psi(\boldsymbol{x})\ ,$$

$$H_C = \frac{g^2}{2}\int d^3x\, d^3y\, J^{-1}\ \rho^a(\boldsymbol{x}) F^{ab}(\boldsymbol{x},\boldsymbol{y})\, J\, \rho^b(\mathbf{y})\ .$$

The confining "Coulombic" part is $SU(2N_F)\times SU(2N_F)$-symmetric.

A symmetry of confinement in QCD with light quarks is $SU(2N_F)\times SU(2N_F)$

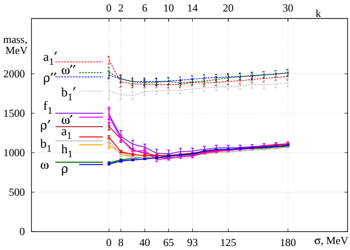


Observation of the $SU(4) \times SU(4)$ and its implications for hadrons

Banks-Casher: $i\gamma_\mu D_\mu \psi_n(x) = \lambda_n \psi_n(x), \qquad < \bar{q}q >= -\pi\rho(0).$

Low mode truncation, M.Denissenya, L.Ya.G., C.B.Lang, 2014-2015:

$$S = S_{Full} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|.$$



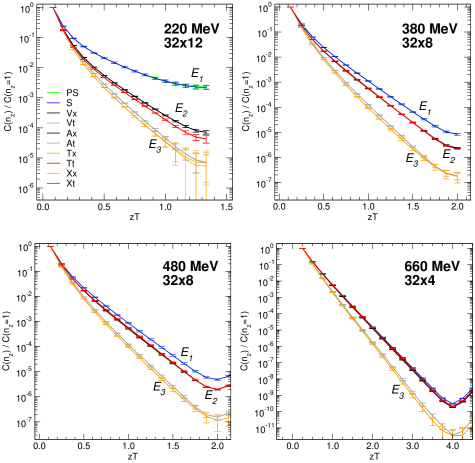
$SU(2)_{CS}$, $SU(4)$ and $SU(4) \times SU(4)$ symmetries.

- It would be incorrect to say that the hadron mass comes from the condensate. The role of the chiral symmetry breaking is to lift the $SU(4) \times SU(4)$ degeneracy of the confining electric interaction.
- Confinement and spontaneous breaking of chiral symmetry are not directly related phenomena. The elimination of the chiral symmetry breaking, e.g. in a medium at high temperatures, does not require deconfinement.

Spatial correlators above T_{ch}

C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, C. Gattringer, L.Ya.G., S. Hashimoto, C.B. Lang, S. Prelovsek , 2017 - 2019

$N_f = 2$ QCD with the chirally symmetric Dirac operator.



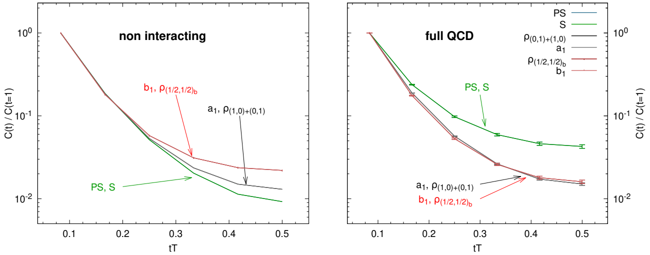
$E1$: $I = 1$ pseudoscalar and scalar propagators - $U(1)_A$ symmetry; $E2$: $I = 1, J = 1$ propagators - $SU(2)_{CS}$; $SU(4)$; $E3$: conserved charges that can propagate in z-direction; consistent with chiral symmetry.

The approximate $E2$ multiplet structure persists up to $T \sim 500$ MeV.

Temporal correlators above T_{ch}

C. Rohrhofer, Y. Aoki, L.Ya.G., S. Hashimoto, 2020

$N_F = 2$ QCD at $T = 220$ MeV



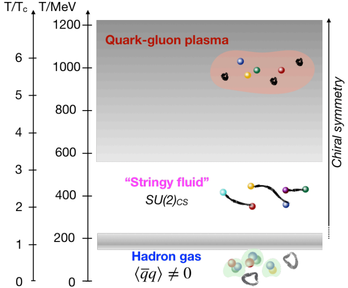
Free quarks (QGP): $SU(2)_L \times SU(2)_R$ and $U(1)_A$ multiplets.

Full QCD at $T = 220$ MeV: $U(1)_A$, $SU(2)_L \times SU(2)_R$, $SU(2)_{CS}$ and $SU(2N_F)$ multiplets.

Above T_{ch} QCD is approximately $SU(2)_{CS}$ and $SU(4)$ symmetric.

Free quark gas is qualitatively inconsistent with QCD!

Three regimes of QCD. C. Rohrhofer et al, PRD 100 (2019) 014502



$0 - T_{ch}$ - Hadron Gas (broken chiral symmetry);

$T_{ch} - 3T_{ch}$ - Stringy Fluid (chiral, $SU(2)_{CS}$ and $SU(4)$ symmetries; electric confinement). The deconfinement crossover is very smooth, which is centered around $T_d \sim 300$ MeV - the deconfinement phase transition in pure glue theory. See also center vortices percolation in QCD J. A. Mickley et al, PRD 111 (2025) 034508 and Hagedorn temperature $T_H = 300$ MeV with string description M. Marczenko et al, PRD 112 (2025) 096010

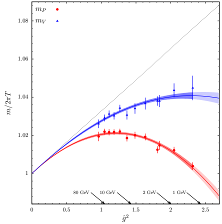
Stringy fluid is a densely packed system of the overlaping color-singlet clusters that interact strongly.

$T > 3T_{ch}$ - a QGP (chiral symmetry)

Screening masses and stringy fluid. L.Ya.G., O. Philipsen, R. Pisarski, 2022

$$
\begin{aligned}
e^{pV/T} = Z &= \mathrm{Tr}(e^{-aHN_\tau}) \\
&= \mathrm{Tr}(e^{-aH_z N_z}) = \sum_{n_z} e^{-E_{n_z} N_z} \ ,
\end{aligned}
$$

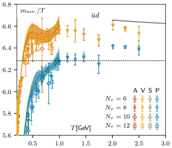
Lattice at $T \sim 1$ - 160 GeV (M.D. Brida et al, JHEP 04 (2022) 034) :



$$
\begin{aligned}
\frac{m_{PS}}{2\pi T} &= 1 + p_2\,\hat{g}^2(T) + p_3\,\hat{g}^3(T) + p_4\,\hat{g}^4(T) \ , \\
\frac{m_V}{2\pi T} &= \frac{m_{PS}}{2\pi T} + s_4\,\hat{g}^4(T) \ ,
\end{aligned}
$$

A perturbative description suggests partonic degrees of freedom and is a signal of QGP.

From A. Bazavov et al, PRD 100 (2019) 094510:



The rapid bending at $T \sim 500$ MeV. The screening masses at $T < 500$ MeV cannot be described with the perturbative series fixed at $T > 600 - 1000$ MeV.

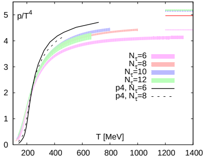
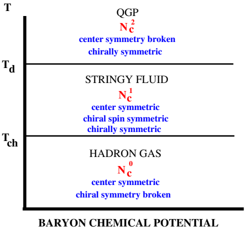


Figure: From A. Bazavov, P. Petreczky, J. Weber, PRD 97 (2018) 014510.

The Stefan-Boltzmann behavior is provided mostly by deconfined gluons. The deconfined gluons are in the game only above 500 MeV !

An independent demonstration of the existence of a temperature window $T_{ch} < T < 3T_{ch}$, in which chiral symmetry is restored but the dynamics is inconsistent with a (quasi)partonic description.

N_c scaling of three regimes: T.D. Cohen, L.Ya.G., EPJA 60(2024)171



$$\epsilon_{\rm HG} \sim N_c^0 \ , \ \ P_{\rm HG} \sim N_c^0 \ , \ s_{\rm HG} \sim N_c^0 \ ,$$

$$\epsilon_{\rm str} \sim N_c^1 \ , \ \ P_{\rm str} \sim N_c^1 \ , \ s_{\rm str} \sim N_c^1 \ ,$$

$$\epsilon_{\rm QGP} \sim N_c^2 \ , \ \ P_{\rm QGP} \sim N_c^2 \ , \ s_{\rm QGP} \sim N_c^2 \ .$$

It is a challenge for lattice simulations!

The N_c^0 scaling in the hadron gas was understood long ago.

$< \bar{q}q > \sim N_c$. When chiral symmetry is restored $< \bar{q}q >= 0$. I.e. a shift only by one power of N_c is required at the chiral transition.

The N_c^2 scaling in the QGP is because there are $N_c^2 - 1$ gluon quasiparticles and only N_c^1 quark quasiparticles.

See also Fujimoto, Fukushima, Hidaka, McLerran, PRD 112(2025)074006

N_c^1 scaling of energy density in stringy fluid: L.Ya.G., EPJC, 85 (2025) 1358

The hadron gas: At large N_c baryons decouple and mesons do not interact

$$n_k(T) = (2S_k+1)(2I_k+1)\int \frac{\mathrm{d}^3 p}{(2\pi)} \frac{1}{e^{\sqrt{p^2+m_k^2}/T}-1} .$$

$$m_k \sim N_c^0 \longrightarrow n_k(T) \sim N_c^0.$$

$$The\ energy\ density \sim n_k(T)\sqrt{p^2+m_k^2} \sim N_c^0.$$

What happens above T_{ch} in the confining regime? The medium is strongly interacting. The Bose-Einstein distribution is not valid. The meson excitation energies are still $E_k \sim N_c^0$. What should one use as the number density $n_k(T)$?

At vanishing chemical potential all expectation values of the color-singlet quark-antiquark bilinears with not vacuum quantum numbers automatically vanish:

$$< \bar{q}\Gamma_k q >= 0$$

But their fluctuations do not !

$$n_k \sim \sqrt{\int d^3x_1 d^3x_2 < \bar{q}(x_1)\Gamma_k q(x_1)\bar{q}(x_2)\Gamma_k q(x_2) >} \sim N_c^1$$

$$energy\ \ density \sim \sum_k n_k E_k \sim N_c^1$$

A simple pedagogical example

Consider an ideal gas at rest.

For velocity of molecules we have a vanishing expectation value:

$$< \boldsymbol{v} >= 0.$$

But obviously the molecule's velocity is not zero, but is actually large!

We evaluate the typical velocity via fluctuations:

$$\sqrt{< \boldsymbol{v}^2 >} = \sqrt{< \boldsymbol{v} \cdot \boldsymbol{v} >}.$$

So there is obvious analogy:

$$< \boldsymbol{v} >= 0 \longrightarrow < \bar{q}\Gamma_k q >= 0$$

$$\sqrt{< \boldsymbol{v} \cdot \boldsymbol{v} >} \longrightarrow \sqrt{\int d^3x_1 d^3x_2 < \bar{q}(x_1)\Gamma_k q(x_1)\bar{q}(x_2)\Gamma_k q(x_2) >}.$$

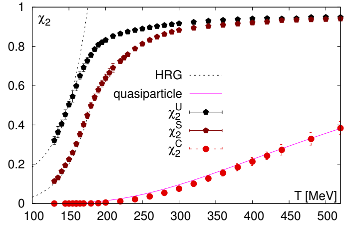
Fluctuations of conserved charges as evidence for the stringy fluid regime/phase. T.D.Cohen, L.Ya.G., EPJA 60(2024)170

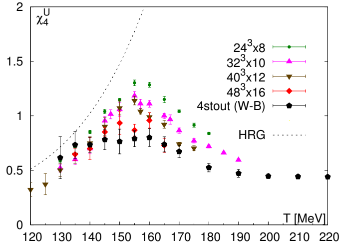
$$N_q \equiv \int d^3x \; n_q(x) \quad \text{with} \quad n_q(x) = \bar{q}(x)\gamma^0 q(x) \sim N_c^1, \quad q = u, d, s$$

The expectation value of the operator N_q at vanishing chemical potential vanishes. But its fluctuations do not and scale as N_c^1:

$$< \bar{q}(x)\gamma^0 q(x) >= 0; \quad \sqrt{\int d^3x_1 d^3x_2 < \bar{q}(x_1)\gamma^0 q(x_1)\bar{q}(x_2)\gamma^0 q(x_2) >} \sim N_c^1$$

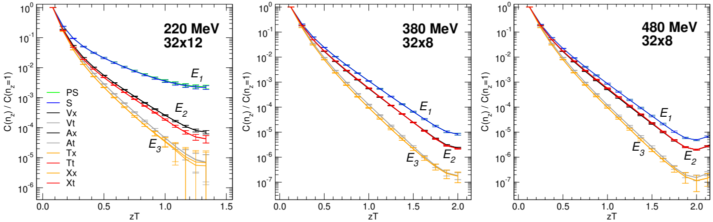
This is in contrast to the fluctuations of the conserved charges in the hadron gas below T_{ch} that scale as N_c^0.
The regime change $N_c^0 \longrightarrow N_c^1$ across the chiral transition is very clearly seen on the lattice:





The figures for the cumulants χ_2, χ_4 taken from R. Bellwied et al, PRD92 (2015) 114505

Why conserved charges behave in the confining regime as if quarks were free? L.Ya.G., in preparation



$E1: I=1$ pseudoscalar and scalar propagators; $E2: I=1, J=1$ propagators;
$E3:$ conserved quark number density that propagates in z-direction
The mesonic propagators are qualitatively different from the free quark loop. It indicates confinement and absence of the quark-hadron duality. The conserved quark numbers propagators in QCD coincite with the free quark loop: THE QUARK HADRON DUALITY. $\Longrightarrow$ The conserved charges in the confining regime behave as if quarks were free.

This explains why the fluctuations of conserved charges above T_{ch} behave in the confining regime as if the quarks were free.

CONCLUSIONS

• In QCD there is a chiral spin symmetry that is a symmetry of the confining interaction with light quarks.

• This symmetry is well seen in hadrons at T=0 when the effects of spontaneous breaking of chiral symmetry are artificially removed. This means that confinement and chiral symmetry breaking are not directly related. It also means that the hadron mass generation is not related to the quark condensate.

• This symmetry is well seen on the lattice above T_{ch}, which implies that the theory is still in the confining regime.

• The QCD phase diagram at vanishing chemical potential has three regimes/phases: the hadron gas at $T < T_{ch}$ with the scaling N_c^0, the stringy fluid at $T_{ch} < T < T_d$ with the scaling N_c^1, and the QGP at $T > T_d$ with the scaling N_c^2.

• The origin of the N_c^1 scaling of the energy density in the stringy fluid is fluctuations of the color-singlet quark-antiquark systems.

• The transition $N_c^0 \longrightarrow N_c^1$ between the hadron gas and the stringy fluid is clearly seen on the lattice via the fluctuations of conserved charges.

• The conserved charges above T_{ch} are sensitive only to small spatial distances and do not see confinement. This explains why the fluctuations of conserved charges behave as if the quarks were quasifree.

L.Ya.G., A. Nefediev, R. Wagenbrunn, PLB 854 (2024) 138707; EPJC 85 (2025) 462

Coulomb gauge QCD at large N_c; only a confining linear potential is retained:

$$H = \int d^3x\, \psi^\dagger(\boldsymbol{x},t)\left(-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m_q\right)\psi(\boldsymbol{x},t) + \frac{1}{2}\int d^3x\, d^3y\, \rho^a(\boldsymbol{x}) V_{conf}(|\boldsymbol{x}-\boldsymbol{y}|)\rho^a(\boldsymbol{y})$$

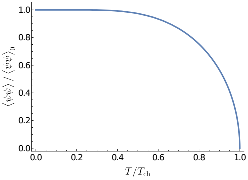
$$V_{conf}(r) = \sigma r$$

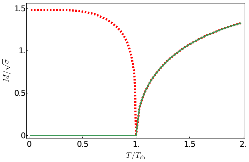
It is a 3+1 dimensinal generalization of the large N_c 1+1 dim 't Hooft model.
The gap (Schwinger-Dyson) eq. was solved in the vacuum in : S.L. Adler, A. C. Davis NPB 244 (1984) 469

$$< \bar{\psi}\psi >_{T=0} \approx -(0.23\sqrt{\sigma})^3$$

Pion and other mesons were obtained from the Bethe-Salpeter eq.: P. Bicudo and H. Ribeiro, PRD (1990) 1625

We have solved the model at finite temperatures. At T_{ch} the chiral symmetry gets restored and the chiral condensate vanishes; pion and sigma get degenerate:





With the phenomenological value for the chiral condensate $< \bar{\psi}\psi >_0 = -(250 \text{ MeV})^3$, it predicts $T_{ch} \approx 90$ MeV (lattice $T_{ch} \simeq 130$ MeV).

Above T_{ch} the spectrum of the quark-antiquark excitations reveals restored chiral symmetry and approximate $SU(4) \times SU(4)$ symmetry of confinement.

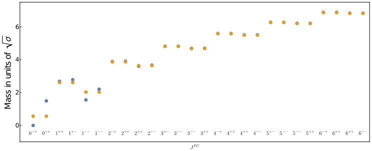
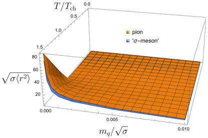


Figure: Symmetries of bound quark-antiquark states $T = 0$ (blue points) and $T = 1.1\,T_{ch}$ (yellow points).

At $T < T_{ch}$ the meson wave functions of low spin mesons are localized in a small space volume. At T_{ch} the thermal excitations of quarks and antiquarks block the levels required for the existence of a non-vanishing quark condensate. This Pauli blocking leads to chiral restoration and weakens effectively the confining potential: the color-singlet quark-antiquark system gets delocalized. This does not mean that the confinement property is lost: V_{conf} is assumed to be the same both below and above T_{ch}: Only the color-singlet states are allowed.



A very natural explanation of a collectivity and of a small mean free path of the effective constituents above T_{ch}.