First-Principles Lattice Study of Dense QCD-like Theories

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iTHEMS Interdisciplinary Theoretical & Mathematical Sciences

CREST

Recent Review:
EI "Lattice results for the equation of state in dense QCD-like theories"
arXiv: 2508.03090
K. Iida, EI, K. Murakami, D. Suenaga, to appear on arXiv

Buenas Ideas on the QCD Phase Diagram, YITP, Kyoto University, 2026/05/28

Our 2color QCD projects

- K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
  Phase diagram by Lattice simulation (T=80MeV)
- T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253
  Phase diagram by 't Hooft anomaly matching
- K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
  Scale setting of Lattice simulation
- K.Ishiguro, K.Iida, EI, PoS, Lattice 2021
  Flux tube and quark confinement by Lattice simulation
- K.Iida, EI, PTEP 2022 (2022) 11, 111B01
  Velocity of sound by Lattice simulation (T=80MeV)
- D. Suenaga, K.Murakami, EI, K.Iida, PRD 107, 054001 (2023) and
  Mass spectrum using effective model
- K.Murakami, D.Suenaga, K.Iida, EI, PoS, Lattice 2022
  Mass spectrum by Lattice simulation
- K.Murakami, K.Iida, EI, JHEP 02 (2024) 152
  Hadron potential w/ finite μ by Lattice simulation
- K.Iida, EI, K.Murakami, D. Suenaga, JHEP 10 (2024) 022
  Phase diagram and EoS by Lattice simulation (T=40MeV)

Two problems in finite-density QCD simulations

K.Nagata, Finite-density lattice QCD and sign problem:
Current status and open problems
Prog.Part.Nucl.Phys. 127 (2022) 103991

(1) sign problem

$$Z = \int \mathcal{D}U [\det D] e^{-S_g[U]}$$

e^{-S_E} must be real-positive

$\mu = 0 \quad D^\dagger = \gamma_5 D \gamma_5 \qquad \det D$ real

$\mu \neq 0 \quad \Delta(-\mu)^\dagger = \gamma_5 \Delta(\mu) \gamma_5 \qquad \det \Delta(\mu)$ complex

In two-color QCD $\det \Delta(\mu)$ is real,

since the fundamental reps. of SU(2) takes a pseudo-real reps.

(2) numerical instability (onset problem)

in the low-T and high-density regime:

$\mu/m_{PS} \geq 1/2$ in low-T $\qquad m_{PS}$: pseudo-scalar (pion) mass at $\mu = 0$

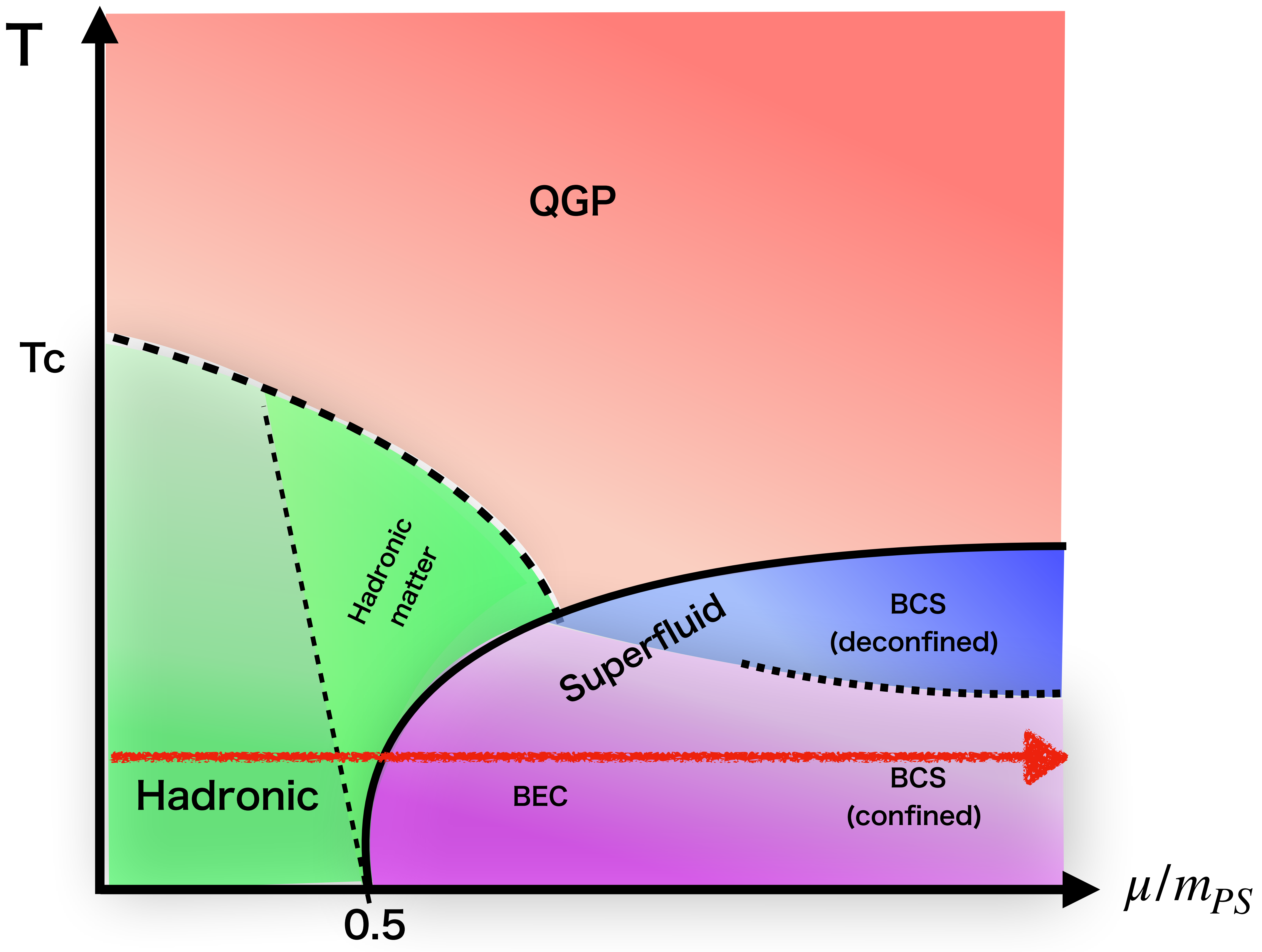
Dynamical pair-creation and annihilation frequently occur,
then system becomes unstable

2color QCD phase diagram

(1) K.Iida, EI, K.Murakami, D.Suenage arXiv: 2405.20566 [hep-lat]
(2) K.Iida, K.Ishiguro , EI, arXiv: 2111.13067
(3) K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
(4) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
(5) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Our model: 2color 2flavor dense-QCD

2color QCD Phase diagram



40MeV (32^4 lattices)

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
K.Iida, EI, K.Murakami, D.Suenage, JHEP 10 (2024) 022

- Lagrangian

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

$$-J[\bar{\psi}_1(C\gamma_5)\tau_2\bar{\psi}_2^T - \psi_2^T(C\gamma_5)\tau_2\psi_1]$$

diquark source term ($J \ll 1$)

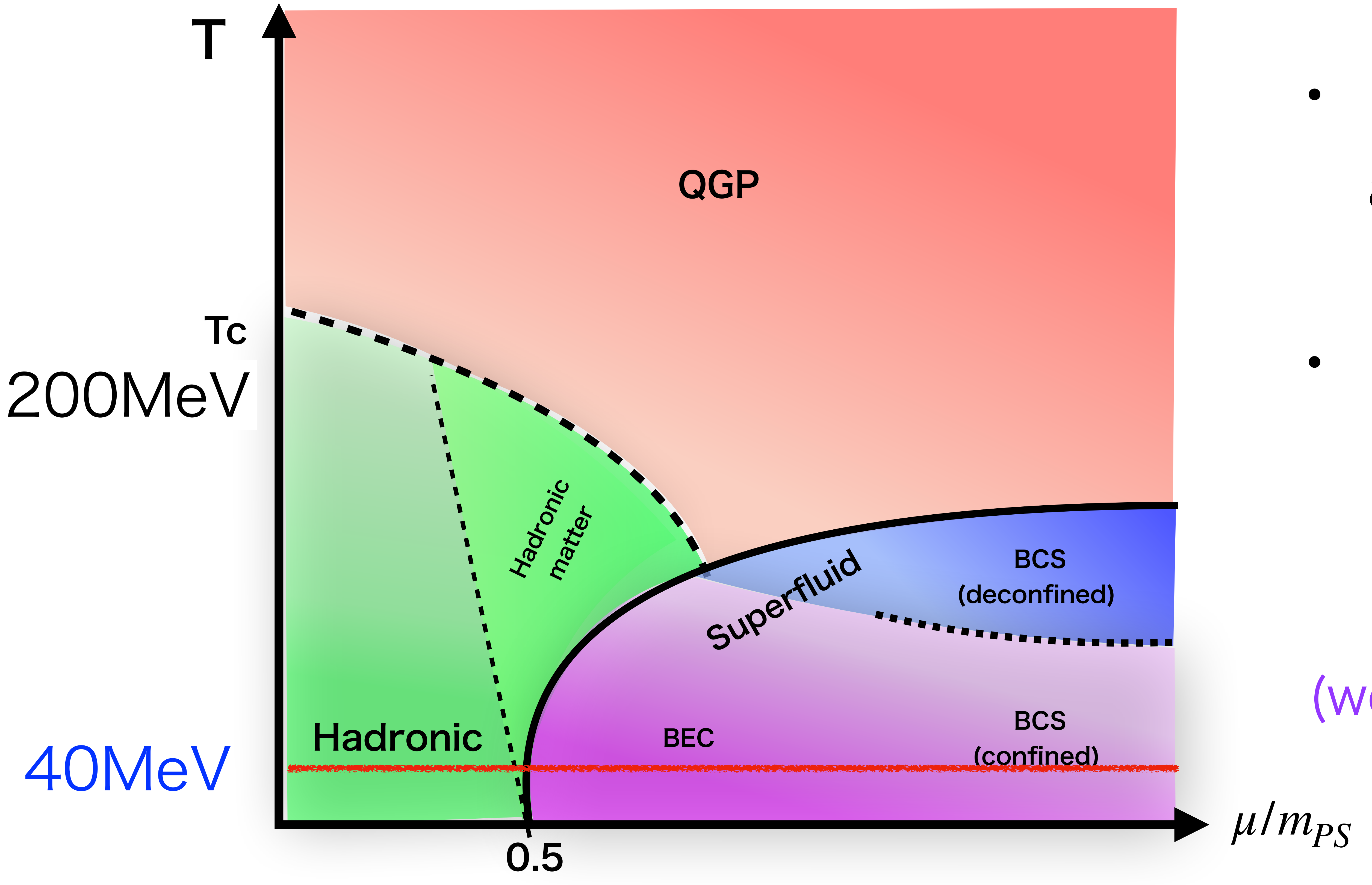
- Physical obs. is calculated in $J \neq 0$ for each μ,

  and then take the $J \to 0$ limit

- In Superfluid phase, diquark cond.$\langle qq \rangle \neq 0$

  (color-singlet in 2color QCD)

Three phases in low temperature regions

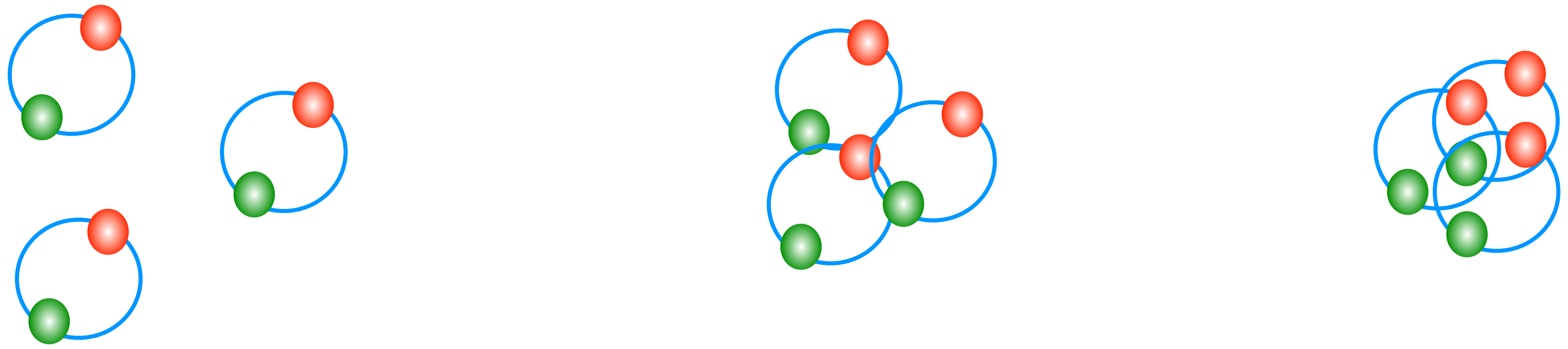


K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
K.Iida, EI, K.Murakami, D.Suenage, JHEP 10 (2024) 022

- We mainly investigated T=40MeV
- Hadronic / Superfluidity phase transition around $\mu \approx m_{\rm PS}/2$
- BEC/BCS crossover in SF phase

BEC phase
strong coupled
(well-described by ChPT)

BCS phase
weakly coupled



Distance between quarks $\gg \Delta^{-1}$

Distance between quarks $\ll \Delta^{-1}$
Quarks behave free particles

Δ^{-1} Δ: gap energy

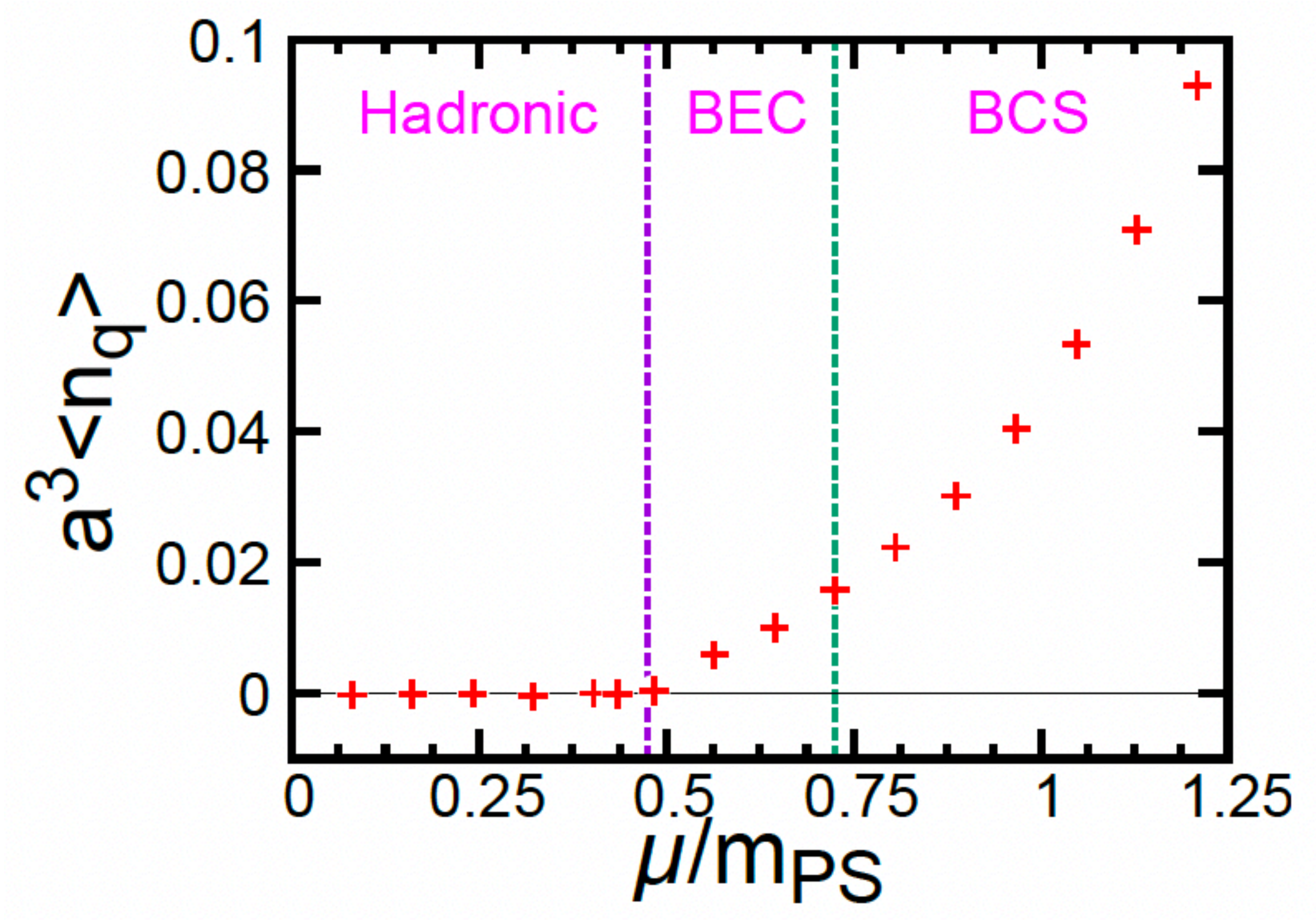
density

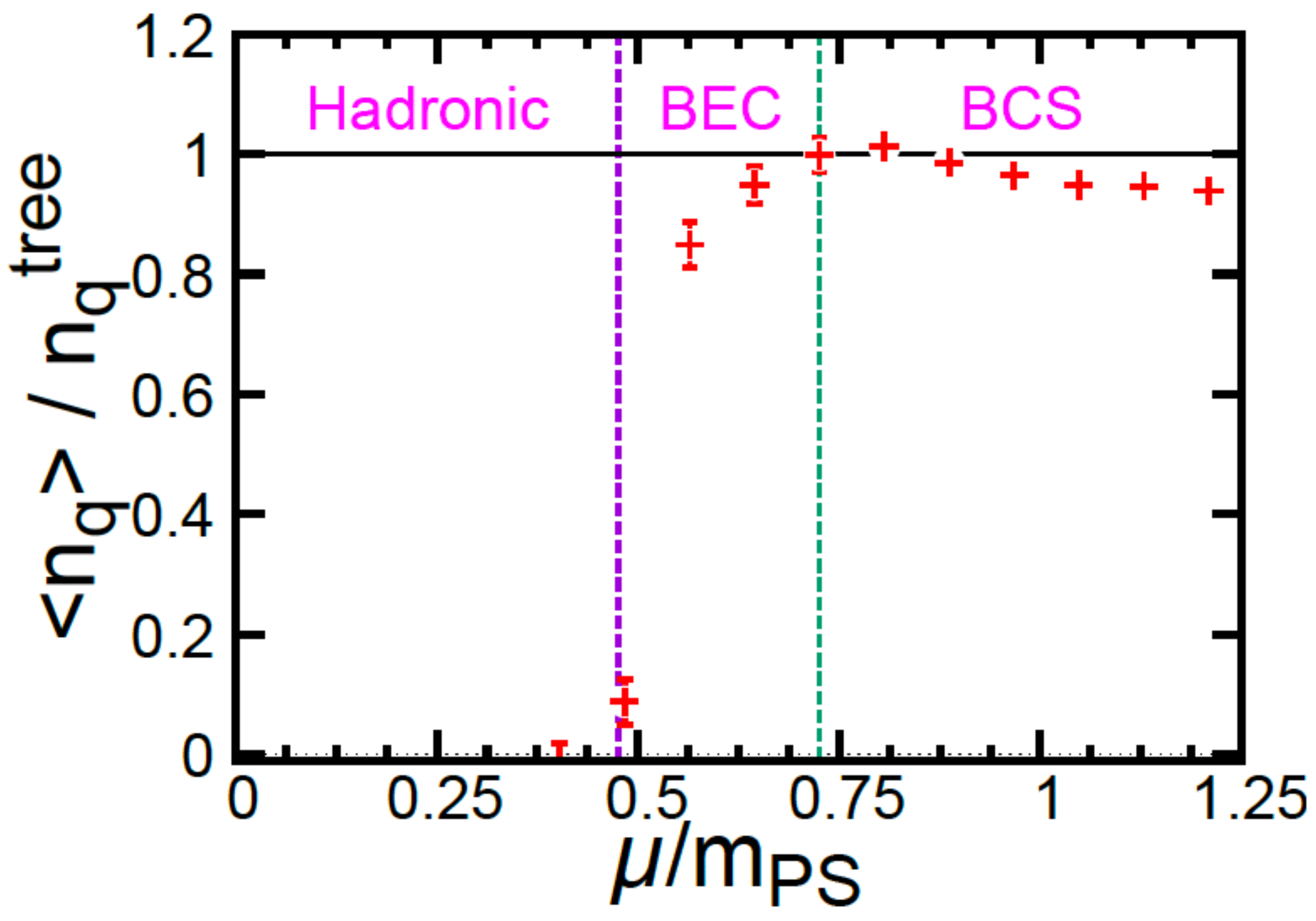
“definition” of BEC/BCS crossover

S. Hands, S. Kim and J.-I. Skullerud(2006)

- If $\langle n_q \rangle \approx n_q^{\text{tree}}$, then the region is identified as BCS phase

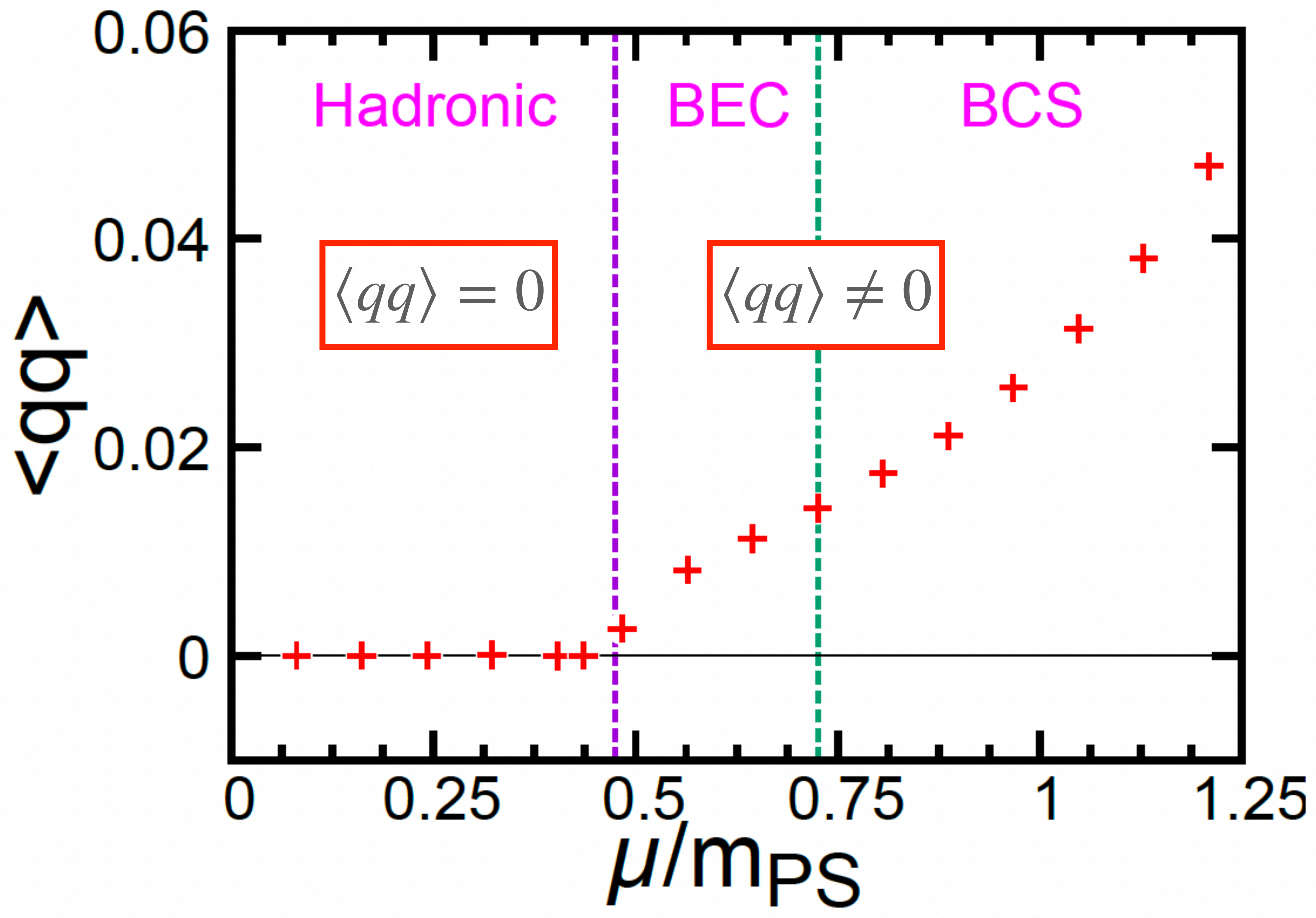
  $\langle n_q \rangle$ denotes a net quark number density





Order parameter of Superfluidity : $\langle qq \rangle$

Diquark condensate



Order parameter of Superfluidity : $\langle qq \rangle$

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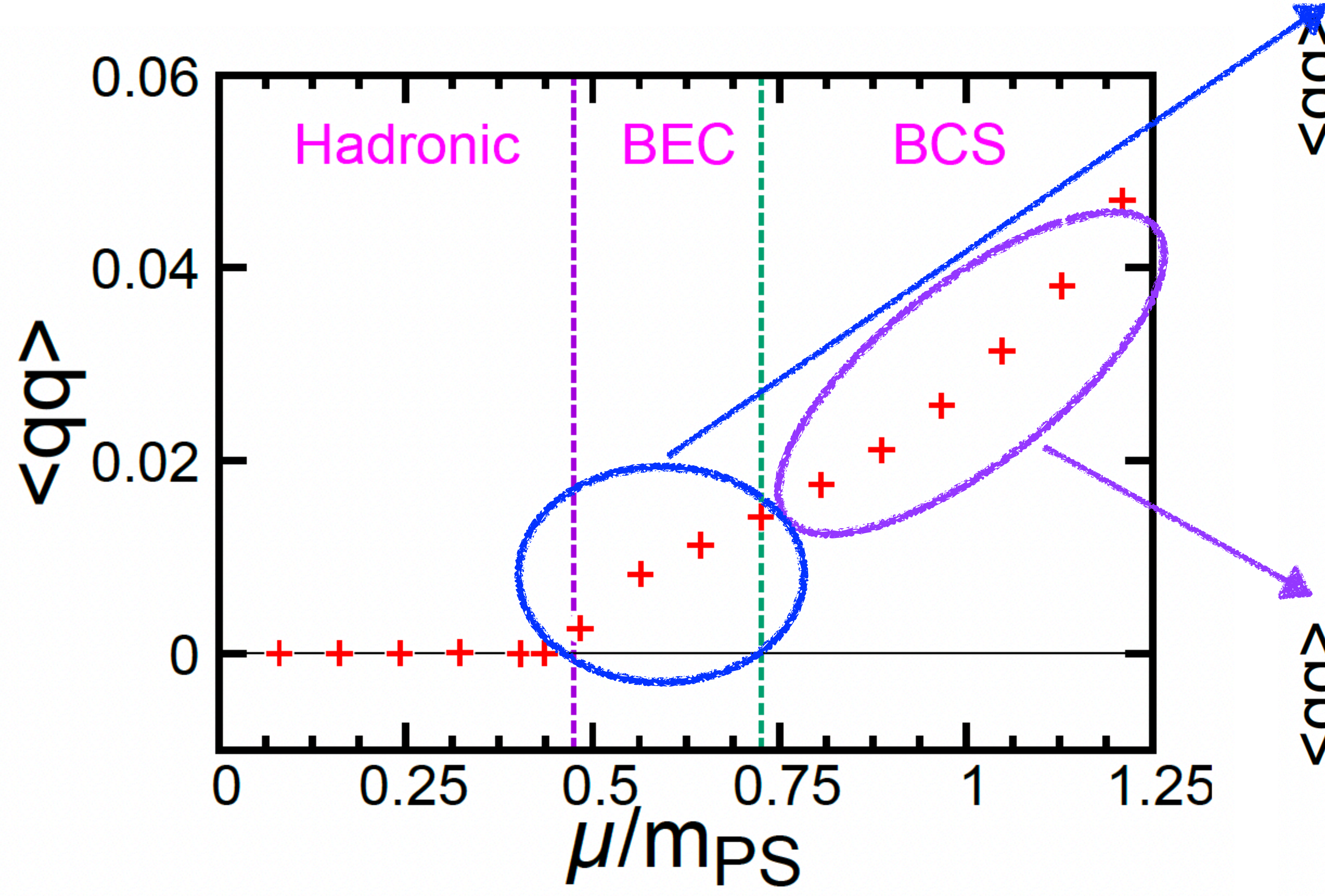
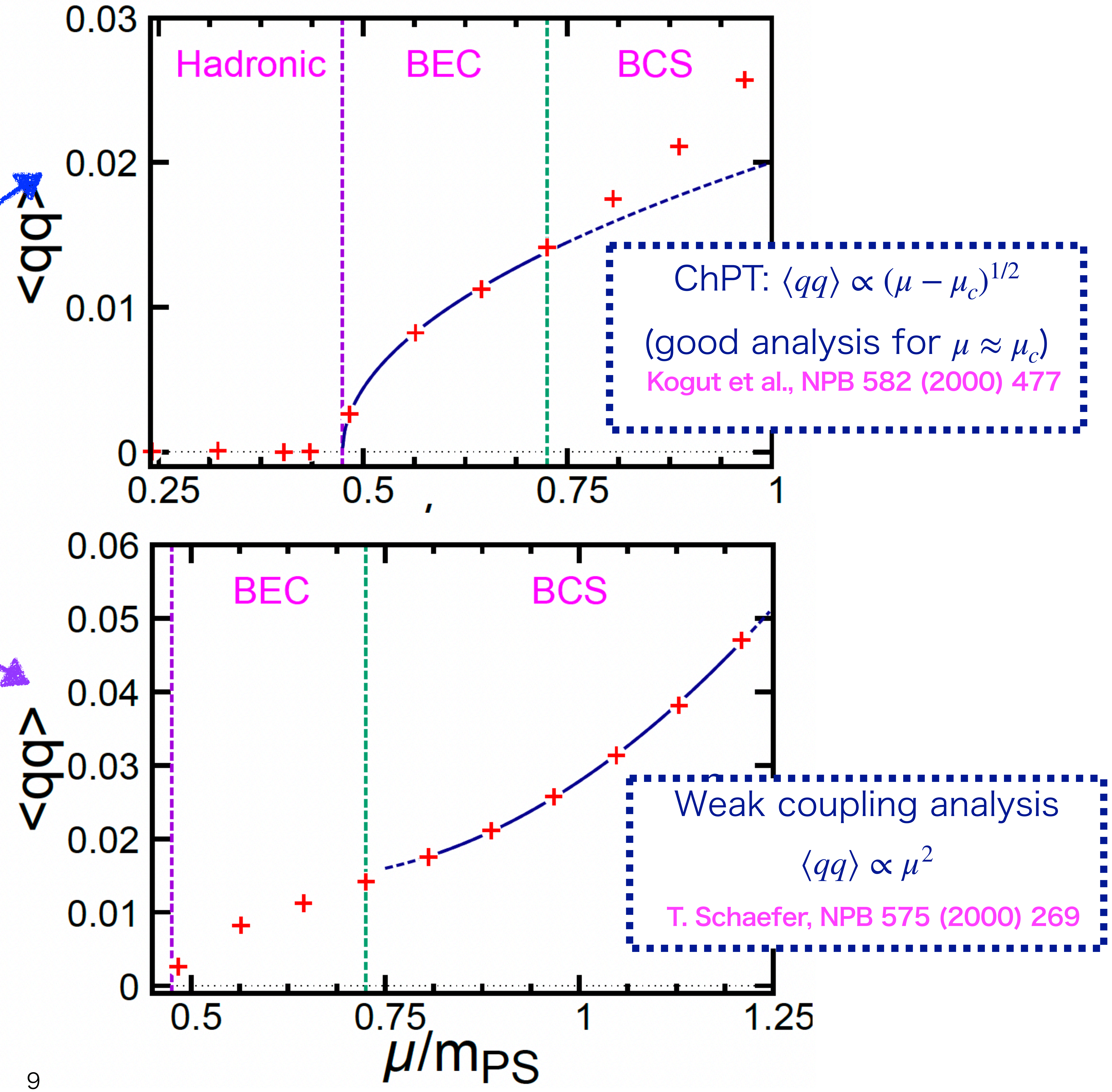




Equation of state

K.Iida and EI, PTEP 2022 (2022) 11, 111B01
K.Iida, EI, K.Murakami, D.Suenaga, JHEP 10 (2024) 022

Equation of state

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left(a\frac{d\beta}{da}|_{LCP}\langle\frac{\partial S}{\partial\beta}\rangle_{sub.} + a\frac{d\kappa}{da}|_{LCP}\langle\frac{\partial S}{\partial\kappa}\rangle_{sub.} + a\frac{\partial j}{\partial a}\langle\frac{\partial S}{\partial j}\rangle \right)$

No renormalization for μ

$\langle\cdot\rangle_{sub.} = \langle\cdot\rangle_{\mu,T} - \langle\cdot\rangle_{\mu=0,T}$

Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Early works for EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

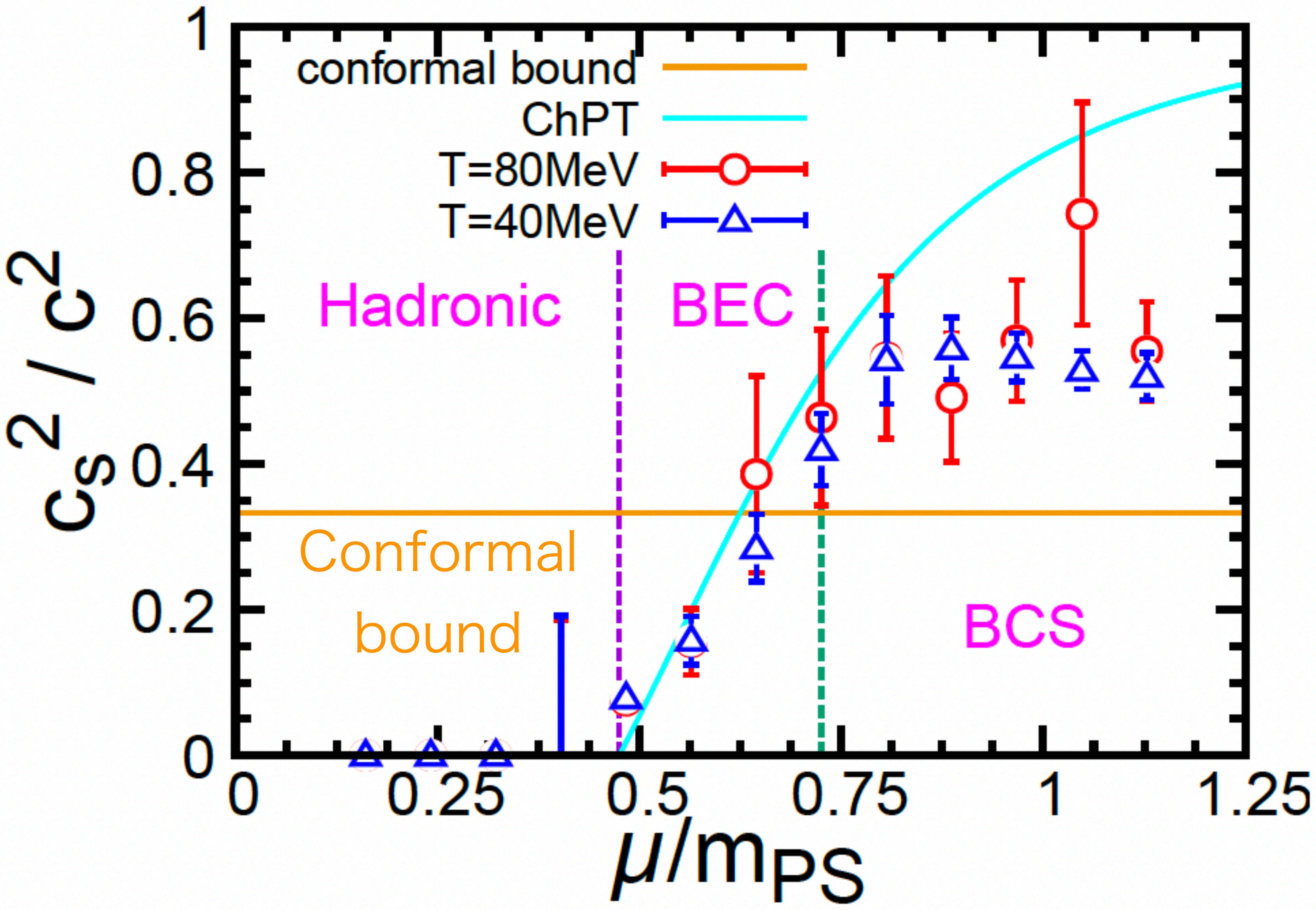
Our work

Nonperturbatively calculate beta fn.

$a\frac{d\beta}{da} = -0.3521, \quad a\frac{d\kappa}{da} = 0.02817$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

Square of sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$)



- T-dependence of the sound velocity is negligible!

- In BEC phase, result is consistent with ChPT

  Chiral Perturbation Theory (ChPT)

  $$c_s^2/c^2 = \frac{1 - \mu_c^4/\mu^4}{1 + 3\mu_c^4/\mu^4}$$: no free parameter!

  Son and Stephanov (2001) : 3color QCD with isospin μ

  Hands, Kim, Skullerud (2006) : 2color QCD with real μ

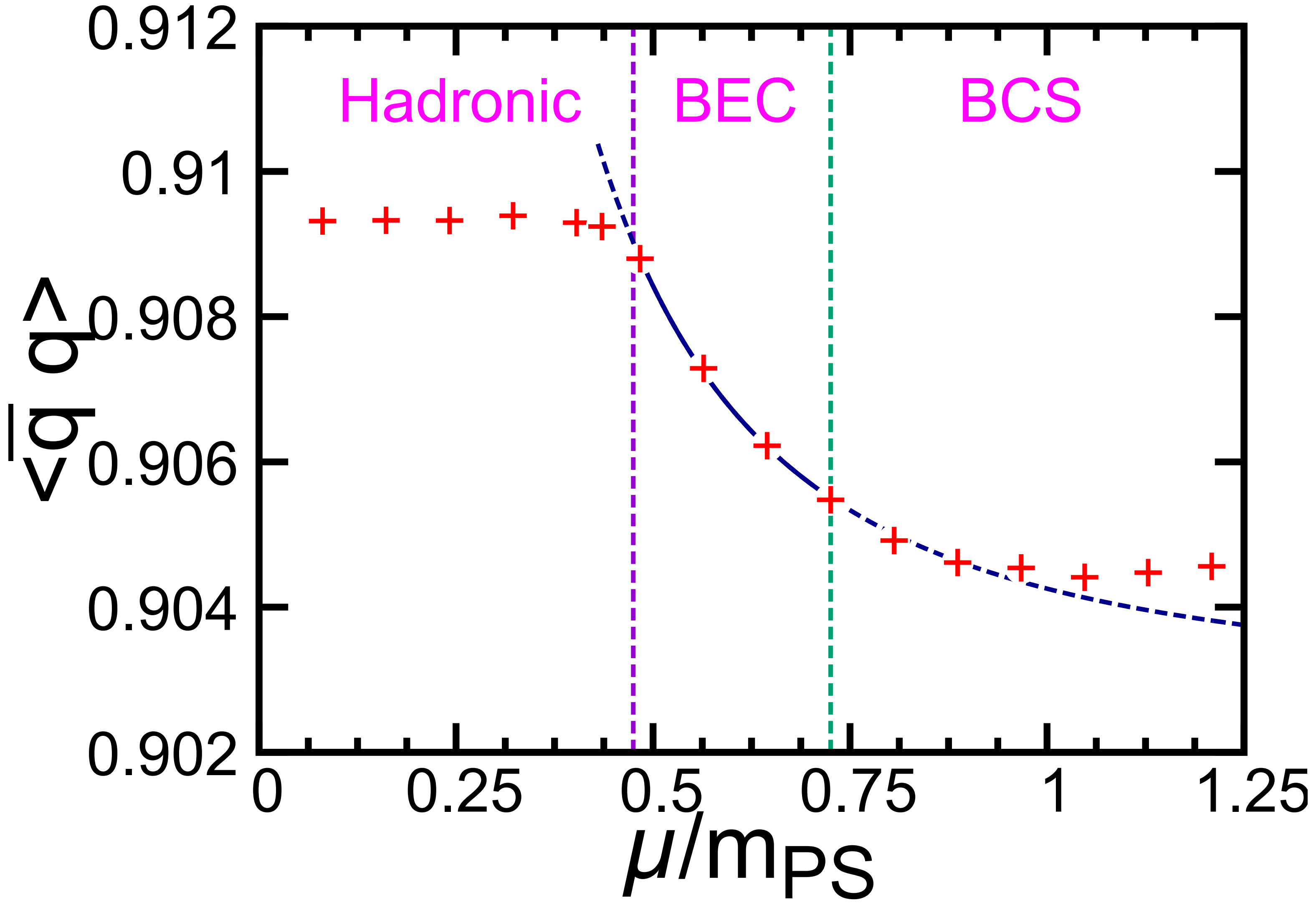
- c_s^2/c^2 exceeds the conformal bound

mass spectrum

K. Iida, EI, K. Murakami, D. Suenaga, work in progress

Chiral symmetry and hadron spectra

K.Iida et al., JHEP 10 (2024) 022

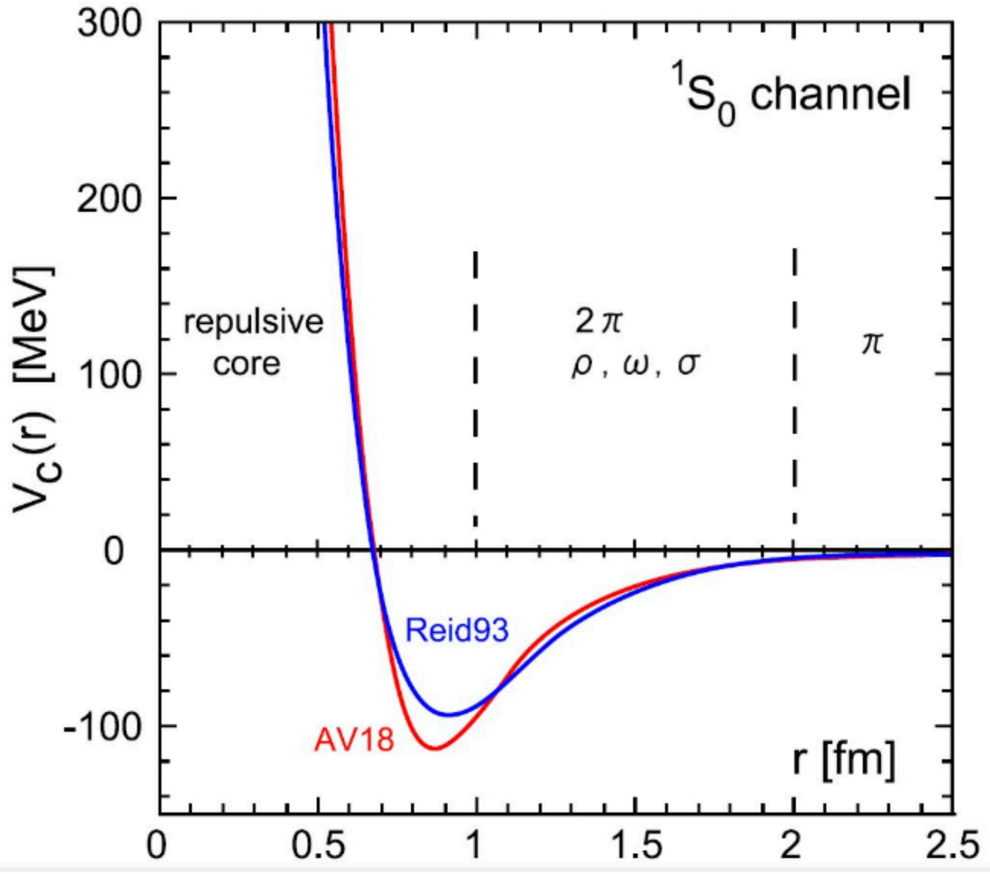


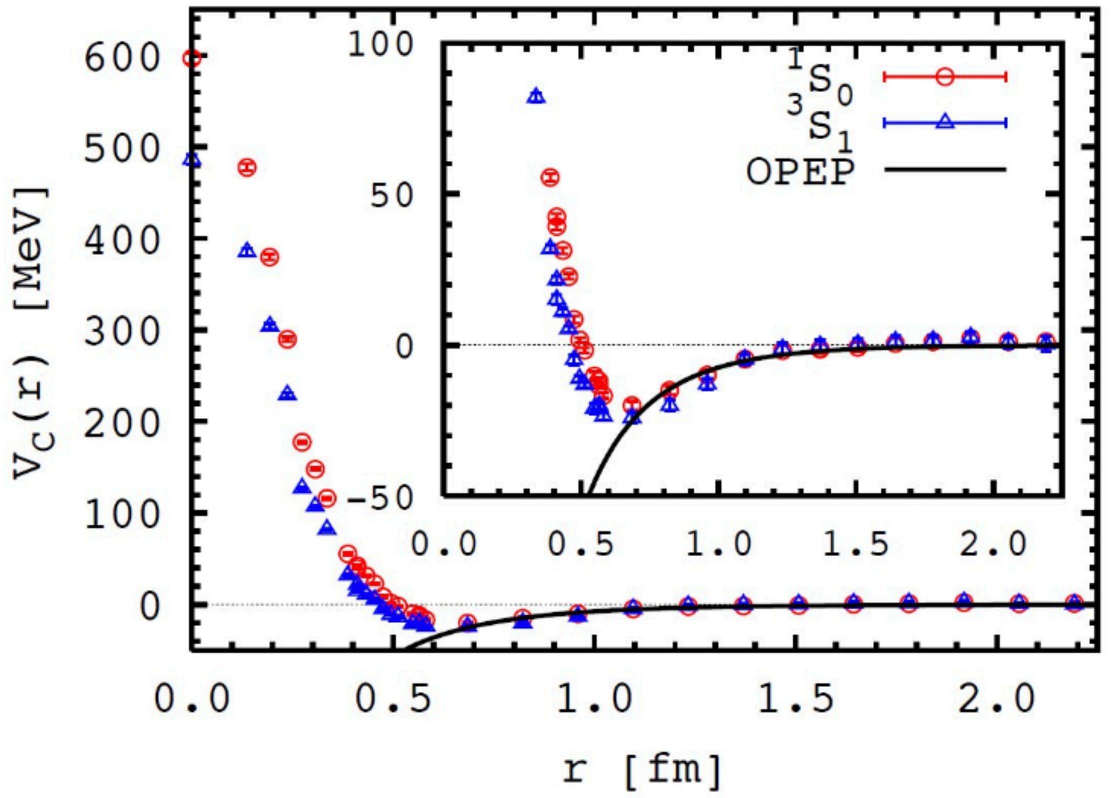
- In the superfluid phase, the chiral condensate decreases.
  Some hadron masses should change.

Brown-Rho (1991)

- The NN potential may also be modified at finite density if meson masses change.

- At $\mu = 0$, the method for calculating mass spectra and the NN potential is well established.





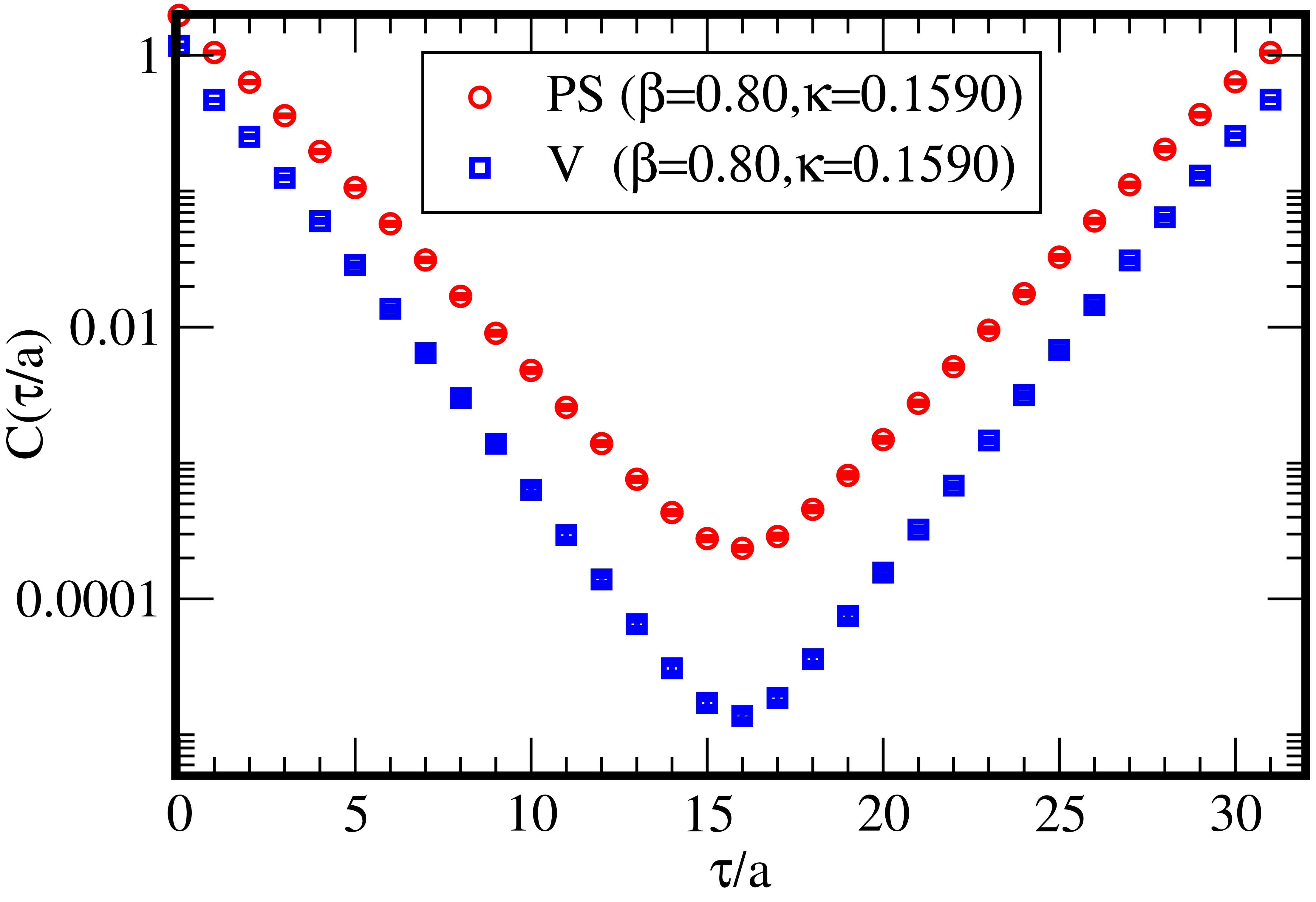
(left) Examples of phenomenological nuclear potential, (right) The lattice QCD result of nuclear potential

Aoki, Ishii, Hatsuda
HAL QCD coll.(2007 -)

Lightest hadron and the QCD inequality

2color 2flavor QCD

$m_\pi/m_\rho \approx 0.8$



- At $\mu = 0$

(1) γ_5-hermiticity: $\gamma_5 D \gamma_5 = D^\dagger$

(2) No disconnected diagram

Schwarz's inequality for ${}^\forall\Gamma$ mesons ($\bar{q}\Gamma q$)

$\mathrm{Tr} S(x,0)\Gamma S(0,x)\Gamma = \mathrm{Tr} S(x,0)\Gamma\gamma_5 S(x,0)^\dagger \gamma_5 \Gamma \leq \mathrm{Tr} S(x,0) S(x,0)^\dagger$

Here, $S(x,0)$ is quark propagator

- From this inequality, we obtain

$C_{PS}(\tau) \geq C_M(\tau)$

On PBC lattice, 2-pt fn. $C(\tau) = A_1(e^{-m\tau} + e^{-m(N_\tau - \tau)})$

PS meson (pion, $\Gamma = \gamma_5$) must be the lightest!

At $\mu \neq 0$, what is happened?

- $\mu \neq 0$: γ_5-hermiticity is broken : $\gamma_5 D(\mu)\gamma_5 = D^\dagger(-\mu) \neq D^\dagger(\mu)$

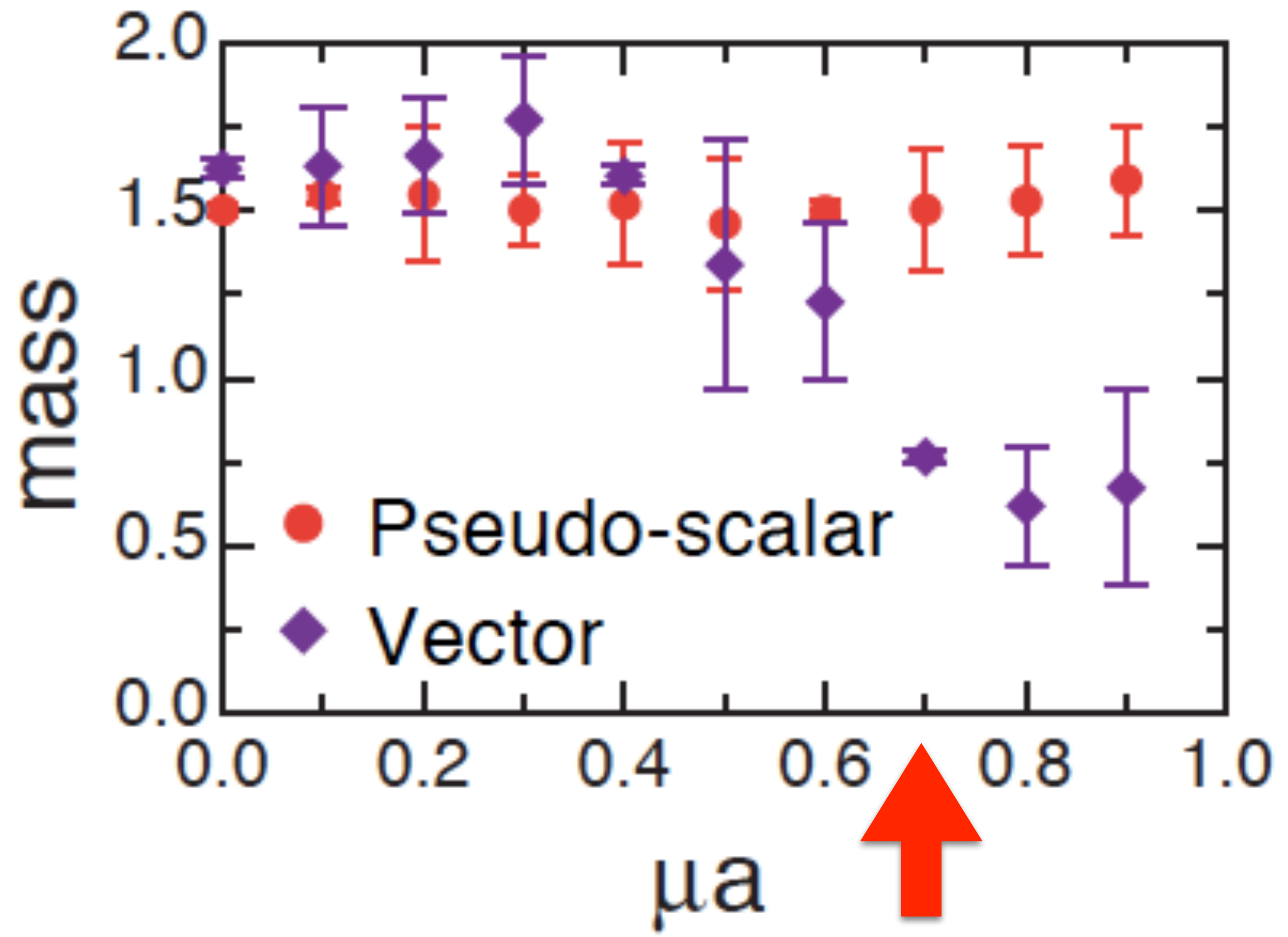
  In dense-QCD, neither positivity nor inequality holds

  (cf:in high-density, color-flavor-locking phase, $m_\pi > m_{\eta'}$?) Son-Stepanov (1999)

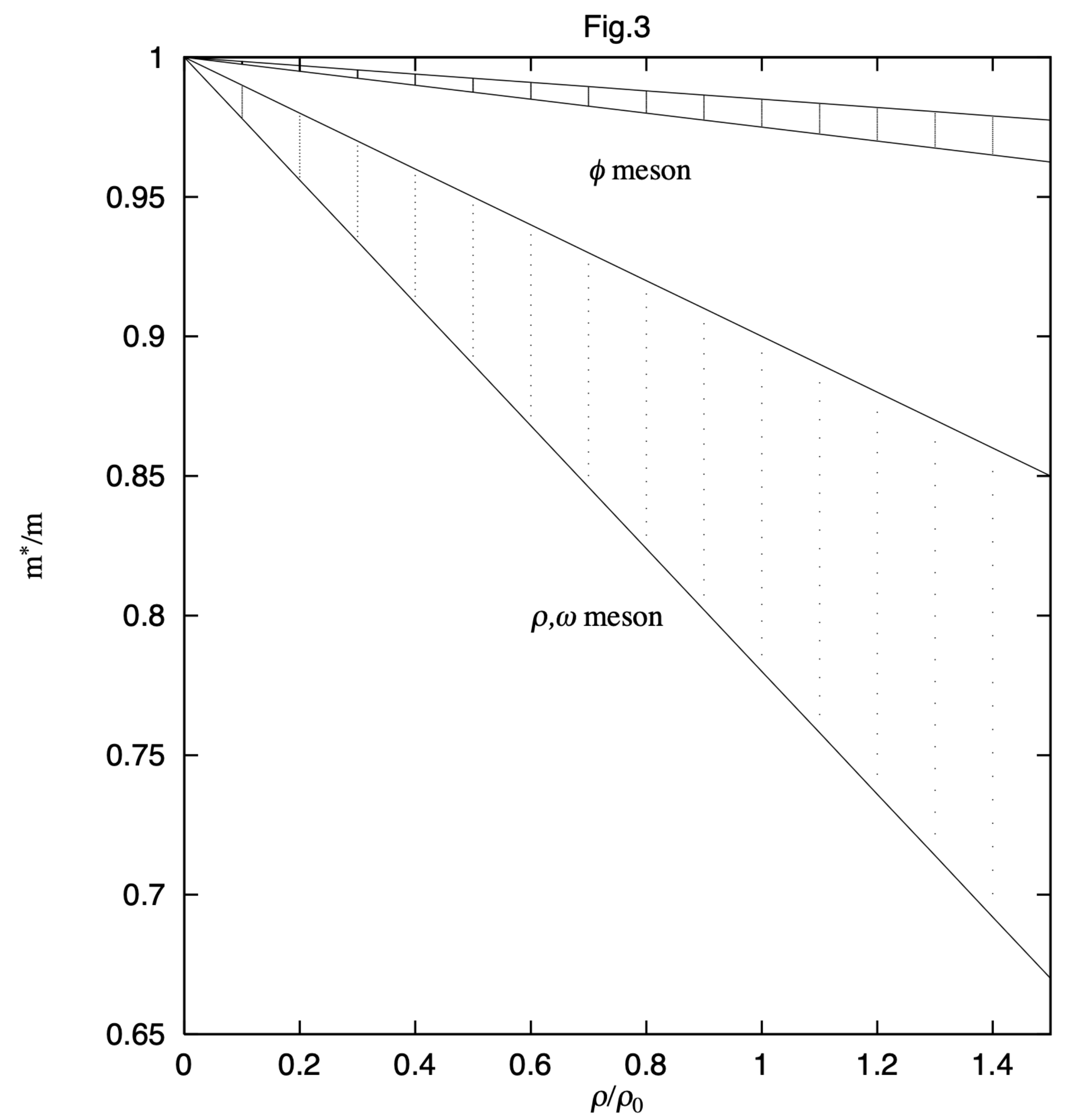
Muroya et al. Phys.Lett. B551 (2003) 305

Hatsuda and Lee, PRC 46(92)R34, PRC 52(95)3364
arXiv:nucl-th/9608037

- 2color QCD, Lattice size 4^3 x 8



$\mu/m_{PS} = 0.5$



Vector mesons becomes lighter?

But, in QCD sum rule, ρ and ω masses do not necessarily become lighter.

(ϕ can be said to do so).

S. Zschocke, O.P. Pavlenko, B. Kämpfer, Eur. Phys. J. A 15 (2002) 529–534

Also, E16 experiment at J-PARC

Flavor symmetry and its breaking in Nc=Nf=2

$SU(4)$ at $m = \mu = 0$

↓ $m > 0$

$Sp(4) \simeq SO(5)$

5 pseudo-Goldstone bosons
$3\pi, qq, \bar{q}\bar{q}$

↓ $0 < \mu < \mu_c$

$SU(2)_V \times U(1)_B$
meson-baryon sym.

diquark condensation ↓ $\mu_c < \mu$

$Sp(1)_V \simeq SU(2)_V$

- Extended multiplet: $\Psi \equiv \begin{pmatrix} \psi \\ K\bar{\psi}^T \end{pmatrix}$

  Lagrangian is invariant under $U \in SU(4)$;

  $\Psi \to U\Psi$

- pion $P^a(x)$ and I=0 S diquark $qq(x)$

  $\delta P^a(x) = i\epsilon[X, P^a(x)] = i\epsilon c^a(qq)$

  for broken generator $X^\alpha \in SU(4)/Sp(4)$,

  $\alpha = 1, \cdots 5$ => 5-dim. multiplet $\{P^a(x),\ qq(x),\ \bar{q}\bar{q}(x)\}$

- Rho $V_i^a(x)$ and I=1 AV diquark $D_{AV}^{a,i}(x)$

  same multiplet (3dim.) of unbroken $Sp(4)$

Special properties in 2color QCD superfluid phase

- $\mu \neq 0$: γ_5-hermiticity is broken : $\gamma_5 D(\mu)\gamma_5 = D^\dagger(-\mu) \neq D^\dagger(\mu)$

- In dense 2color QCD, quarks take a pseudo-real reps.$\gamma_5 C\tau_2 D(\mu)\gamma_5 C\tau_2 = D^*(\mu)$, here C is charge conjugation

  $C = i\gamma_0\gamma_2$

  From the QCD inequality, the iso-singlet scalar diquark (baryon) $M_{qq} = \psi^T C\tau_2\gamma_5\psi$ must be the lightest hadron

- In hadronic phase, violation of time-reversal symmetry ($\tau \leftrightarrow (N_\tau - \tau)$) of C(tau)

  $C(T,\mu;\tau) = \frac{1}{Z}\mathrm{Tr}[e^{-\frac{1}{T}(\hat{H}-\mu\hat{N})}\hat{O}(\tau)\hat{O}^\dagger(0)]$ -> $C(\mu;\tau) = \sum_n |\langle 0|\hat{O}(0)|n\rangle|^2 e^{-(E_n-\mu n_O)\tau}$

  -> $E(\vec{p},\mu) = \sqrt{\vec{p}^2+m^2} - \mu n_O$, it can be regarded as a mass shift: $m(\mu) = m(\mu=0) - n_O\mu$

$|0\rangle$: vacuum state at $\mu = 0$ $(\hat{N}|0\rangle = 0)$

(This is valid only for Hadronic phase)

On lattice, $\langle\Omega_i|\hat{N}|\Omega_i\rangle = 0$ for each conf. in $\mu < m_\pi/2$

K. Nagata, PPNP(2022), A.Alexandru et al., JHEP(2016)

- In the SuperFluid (SF) phase, meson-baryon mixing

  $U(1)_B$ is broken, no difference between mesons and baryons

  $|n\rangle_{SF} \propto c_0|$meson-like state $(\bar{q}\Gamma q)\rangle + c_1|$baryon-like state$(q^T\Gamma q)\rangle$

  Technically, consider all possible contractions of $\bar{q}$-q and q-q.

2pt fn. formula using quark propagators

Mesons: $\bar{\psi}\Gamma\psi$

- iso-single (I=0) or iso-triplet (I=1)
- $\Gamma = \{1, \gamma^5, \gamma^1, i\gamma^5\gamma^1\}$={S, P, V, AV}

$$\begin{aligned}
\langle M^0(x)M^{0\dagger}(0)\rangle_F &= -2\mathrm{tr}\langle t,\vec{x};c,\beta|S_N\Gamma|t,\vec{x};c,\beta\rangle\cdot\mathrm{tr}\langle 0,\vec{0};c,\beta|S_N\bar{\Gamma}|0,\vec{0};c,\beta\rangle \\
&+\mathrm{tr}\langle t,\vec{x};a,\rho|S_N\bar{\Gamma}|0,\vec{0},c;\beta\rangle\left(\langle t,\vec{x};a,\rho|\Gamma^\dagger S_N^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
&+\mathrm{tr}\langle t,\vec{x};a,\rho|S_A\bar{\Gamma}^T|0,\vec{0},c;\beta\rangle\left(\langle t,\vec{x};a,\rho|\Gamma^\dagger \bar{S}_A^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
\langle M^1(x)M^{1\dagger}(0)\rangle_F &= \mathrm{tr}\langle t,\vec{x};a,\rho|S_N\bar{\Gamma}|0,\vec{0},c;\beta\rangle\left(\langle t,\vec{x};a,\rho|\Gamma^\dagger S_N^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
&-\mathrm{tr}\langle t,\vec{x};a,\rho|S_A\bar{\Gamma}^T|0,\vec{0},c;\beta\rangle\left(\langle t,\vec{x};a,\rho|\Gamma^\dagger \bar{S}_A^\dagger|0,\vec{0};c,\beta\rangle\right)^*
\end{aligned}$$

Diquarks: $\psi_1^T K\Gamma\psi_2$, $K \equiv C\gamma_5\tau_2$

- iso-single (I=0) or iso-triplet (I=1)
- $\Gamma = \{1, \gamma^5, \gamma^1, i\gamma^5\gamma^1\}$={S, P, V, AV}

$$\begin{aligned}
\langle D^0(x)D^{0\dagger}(0)\rangle_F &= -2\mathrm{tr}\langle t,\vec{x};c,\beta|\bar{S}_A\Gamma K|t,\vec{x};c,\beta\rangle\cdot\mathrm{tr}\langle 0,\vec{0};c,\beta|S_A K\bar{\Gamma}|0,\vec{0};c,\beta\rangle \\
&+\mathrm{tr}\langle t,\vec{x};a,\rho|S_N\Gamma K|0,\vec{0},c;\beta\rangle\left(\langle t,\vec{x};a,\rho|\bar{\Gamma}^\dagger K\bar{S}_N^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
&+\mathrm{tr}\langle t,\vec{x};a,\rho|S_N K\Gamma^T|0,\vec{0};c,\beta\rangle\left(\langle t,\vec{x};a,\rho|\bar{\Gamma}^\dagger K\bar{S}_N^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
\langle D^1(x)D^{1\dagger}(0)\rangle_F &= -\mathrm{tr}\langle t,\vec{x};a,\rho|S_N\Gamma K|0,\vec{0},c;\beta\rangle\left(\langle t,\vec{x};a,\rho|\bar{\Gamma}^\dagger K\bar{S}_N^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
&+\mathrm{tr}\langle t,\vec{x};a,\rho|S_N K\Gamma^T|0,\vec{0};c,\beta\rangle\left(\langle t,\vec{x};a,\rho|\bar{\Gamma}^\dagger K\bar{S}_N^\dagger|0,\vec{0};c,\beta\rangle\right)^*
\end{aligned}$$

Updated points from Murakami et al. (2023, Proceeding of Lattice conference)

Disconnected contribution is included

$\mathrm{Tr}_{\vec{x},\mathrm{color},\mathrm{spinor}}[O\langle\tau,\vec{x}\,|\,\tau,\vec{x}\rangle] - \langle\sum_\tau \mathrm{Tr}_{\vec{x},\mathrm{color},\mathrm{spinor}}O(\tau,\vec{x})/N_\tau\rangle$ and its at $(\tau = 0, \vec{x} = \vec{0})$

Mesons has 1pt fn. of $\mathrm{Tr}[S_N\Gamma]$, which are relatively large.

Diquark has the one of $\mathrm{Tr}[S_A\Gamma]$, which are proportional to $J^2 \ll 1$ and it is negrigible

All hadrons and its qualities of signals

Note that to satisfy Fermi statistics, diquarks satisfy $\psi_1^T(C\gamma_5\Gamma)(\tau^a/2)\psi_2 = -\psi_2^T(C\gamma_5\Gamma)(\tau^a/2)\psi_1$.

I=1 channel, spin structure $C\gamma_5\Gamma$ should be symmetric => AV is allowed

I=0 channel, spin structure should be anti-symmetric => S, P, V are allowed

In the words in meson sector, $a_1, \sigma, \eta, \omega$ mesons have a baryon-mixing in the SF phase.

(To disentangle the mixing, we have to solve a generalized eigenvalue problem (GEVP))

	Meson (Hadronic)	Diquark (Hadronic)	Higgs/NG (Hadronic)	Meson(SF)	Diquark(SF)	Higgs/NG (SF)	Disconn.
I=0, 0+ (sigma)	Noisy	OK	OK	OK	OK	Noisy	Yes
I=1, O+ (a0)	Noisy	-	OK	Heavy?	-	OK (massless?)	-
I=0, 0- (eta)	OK	OK	-	Noisy	OK	-	YES
I=1, 0- (pion)	OK	-	-	OK	-	-	-
I=0, 1- (omega)	OK	OK	-	Heavy?	OK ?	-	YES
I=1, 1- (rho)	OK	-	-	OK	-	-	-
I=0, 1+ (f1)	Noisy	-	-	Noisy	-	-	YES
I=1, 1+ (a1)	Noisy	OK	-	OK	OK	-	-

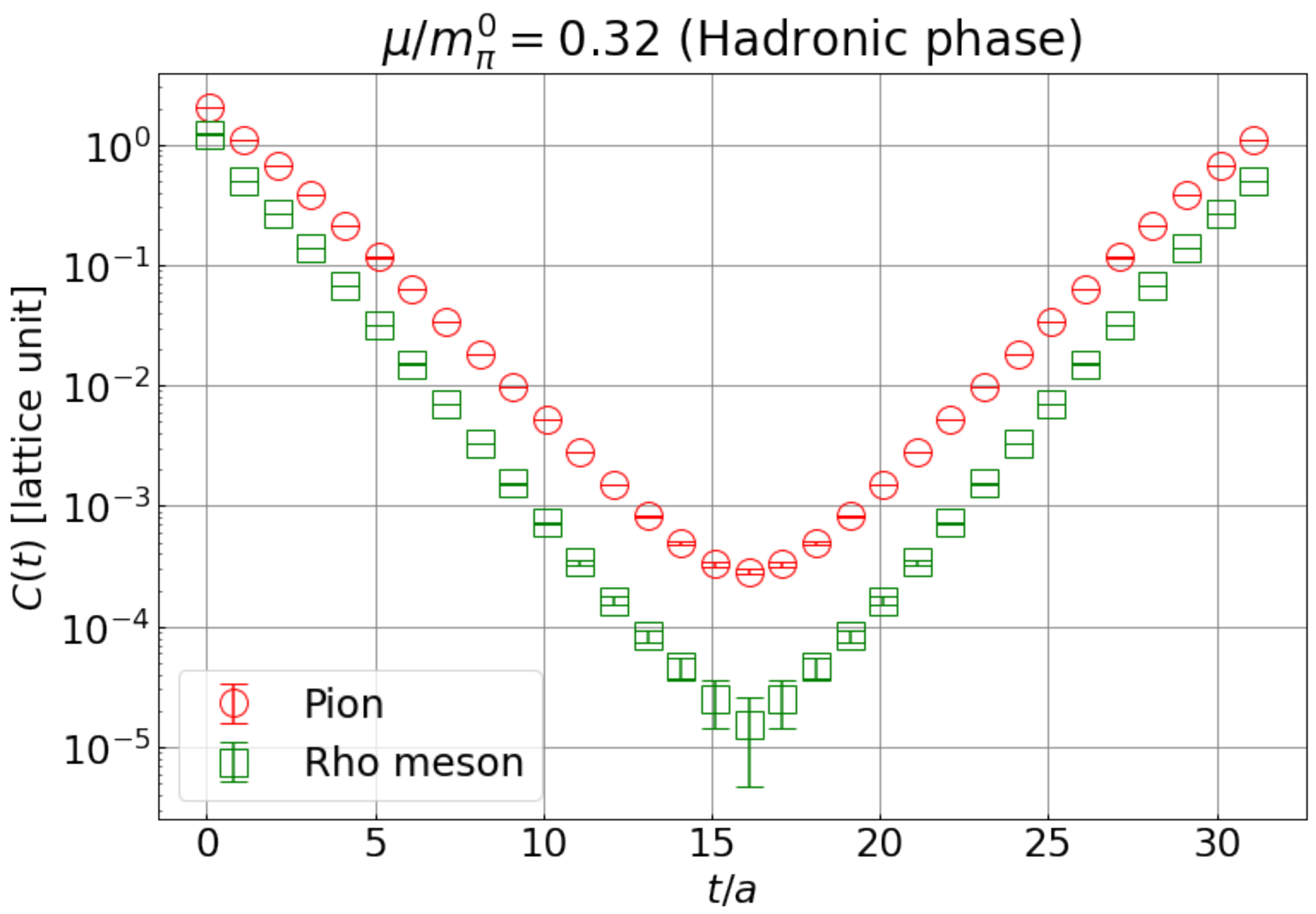
Results!

K. Iida, EI, K. Murakami, D. Suenaga, to appear on arXiv

Correlation fn.: pion and rho meson

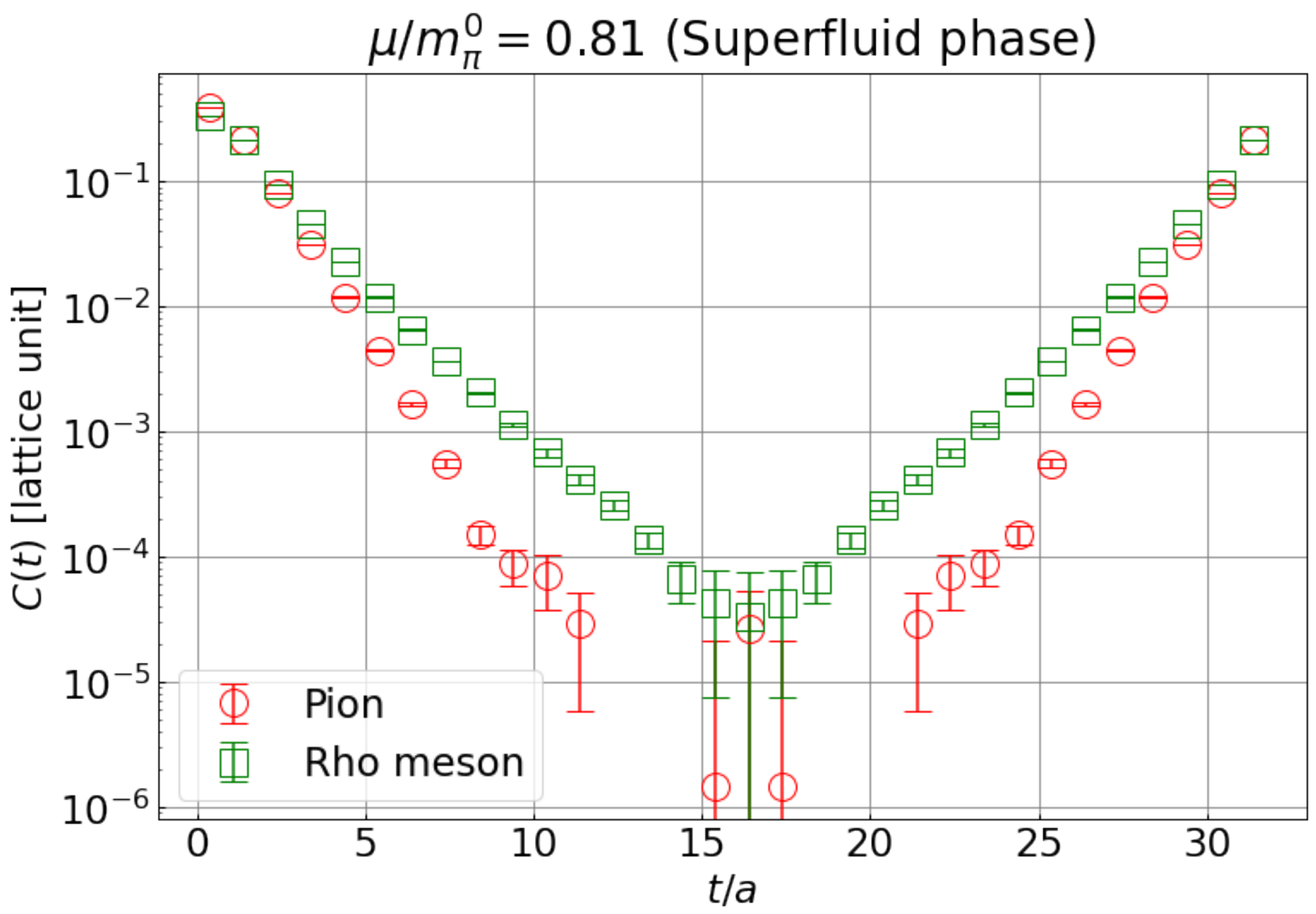
K. Murakami et al., PoS LATTICE2022 (2023) 154

Hadronic phase (low-density)



$C_\pi(\tau) > C_\rho(\tau)$ <=> $m_\pi < m_\rho$

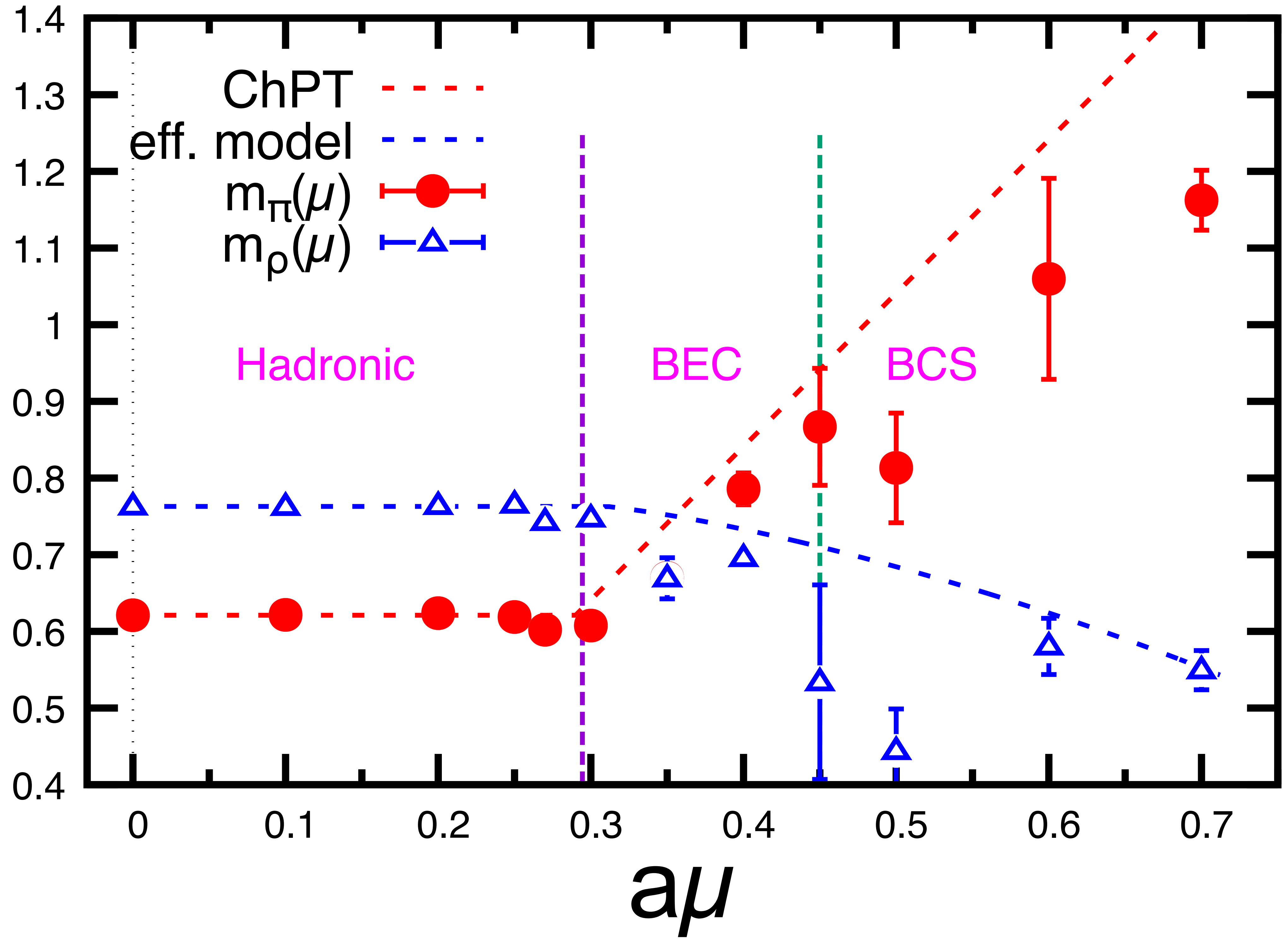
Superfluid phase (high-density)



$C_\pi(\tau) < C_\rho(\tau)$ <=> $m_\pi > m_\rho$

Furthermore, pion may decay to something?

μ-deps of mass: pion and rho meson



- In hadronic phase,
  the pion is lighter than the rho
  (as usual)
  In SF phase, that is opposite.

- ChPT prediction for pion

  $$m_\pi^{\mathrm{ChPT}}(\mu) = m_\pi(\mu = 0) + 2(\mu - \mu_c)\theta(\mu - \mu_c)$$

  Kogut et al., NPB(2000)

  Effective model prediction for rho

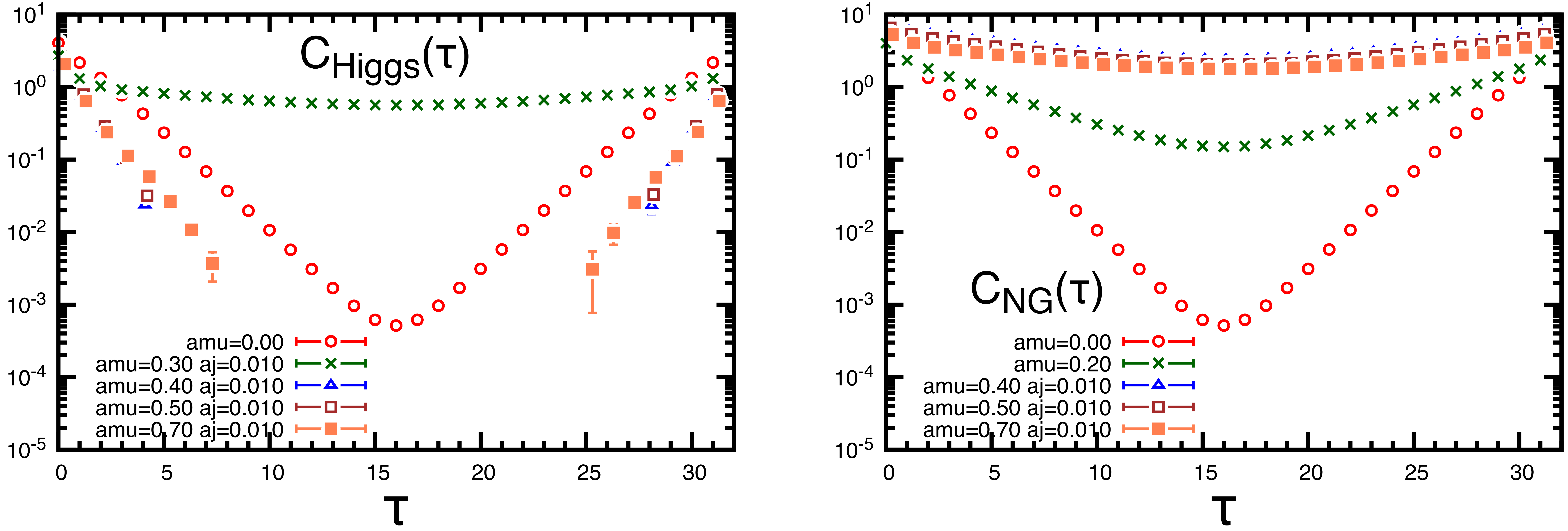
  Suenaga et al., PRD (2024)

- In SF phase, both signals are noisy.
  It indicates an existence of further lighter hadron?

Correlation fn.: Higgs and NG mode

Phase fluctuations (NG mode) of diquark cond. and amplitude fluctuations (Higgs mode)

operator: $D^{NG} = \bar{\psi}_1 K \bar{\psi}_2^T - \psi_1^T K \psi_2$ (same w/ diquark source in action), $D^{Higgs} = \bar{\psi}_1 K \bar{\psi}_2^T + \psi_1^T K \psi_2$

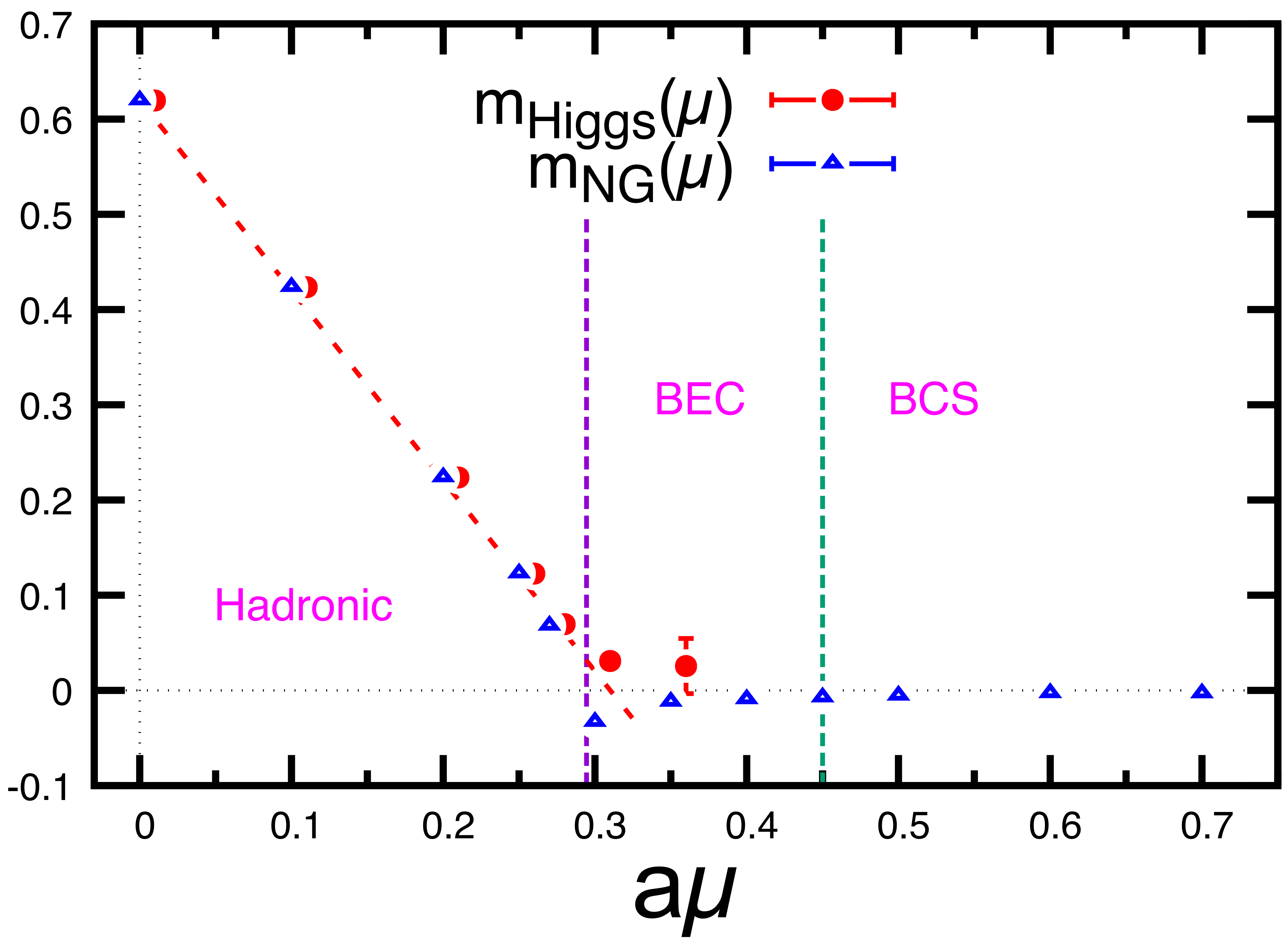


In hadronic phase, both modes are degenerated.

In SF phase, the signal of NG mode is clear, while Higgs mode becomes noisy.

It indicates that NG mode has a light mass, while the Higgs mode is heavy.

Mass spectra: Higgs and NG mode



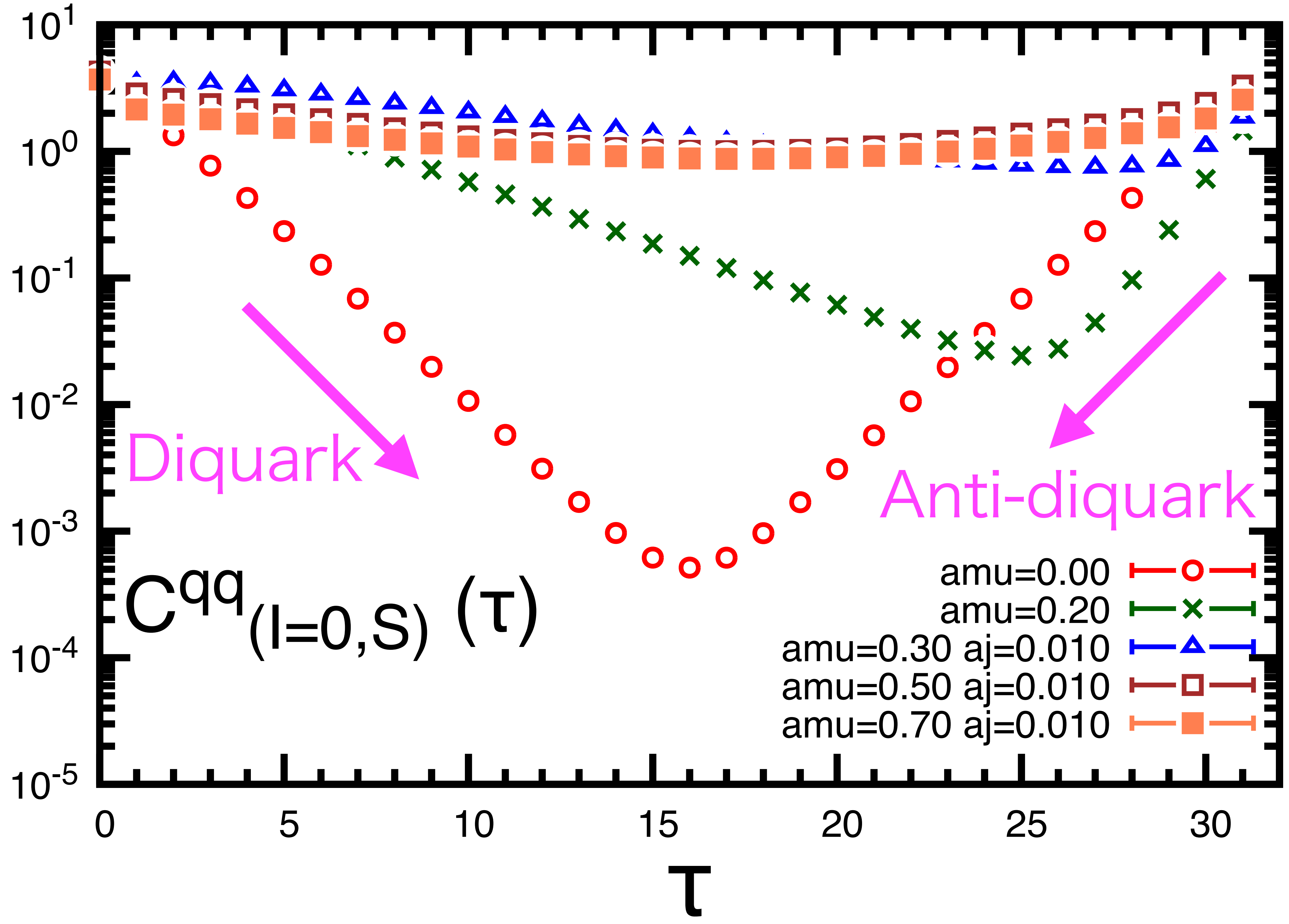
- In hadronic phase, both modes are degenerated.
  They follow the prediction of mass shift
  $m(\mu) = m(\mu = 0) - 2\mu$

- In SF phase, NG mode has a light mass (nearly zero).

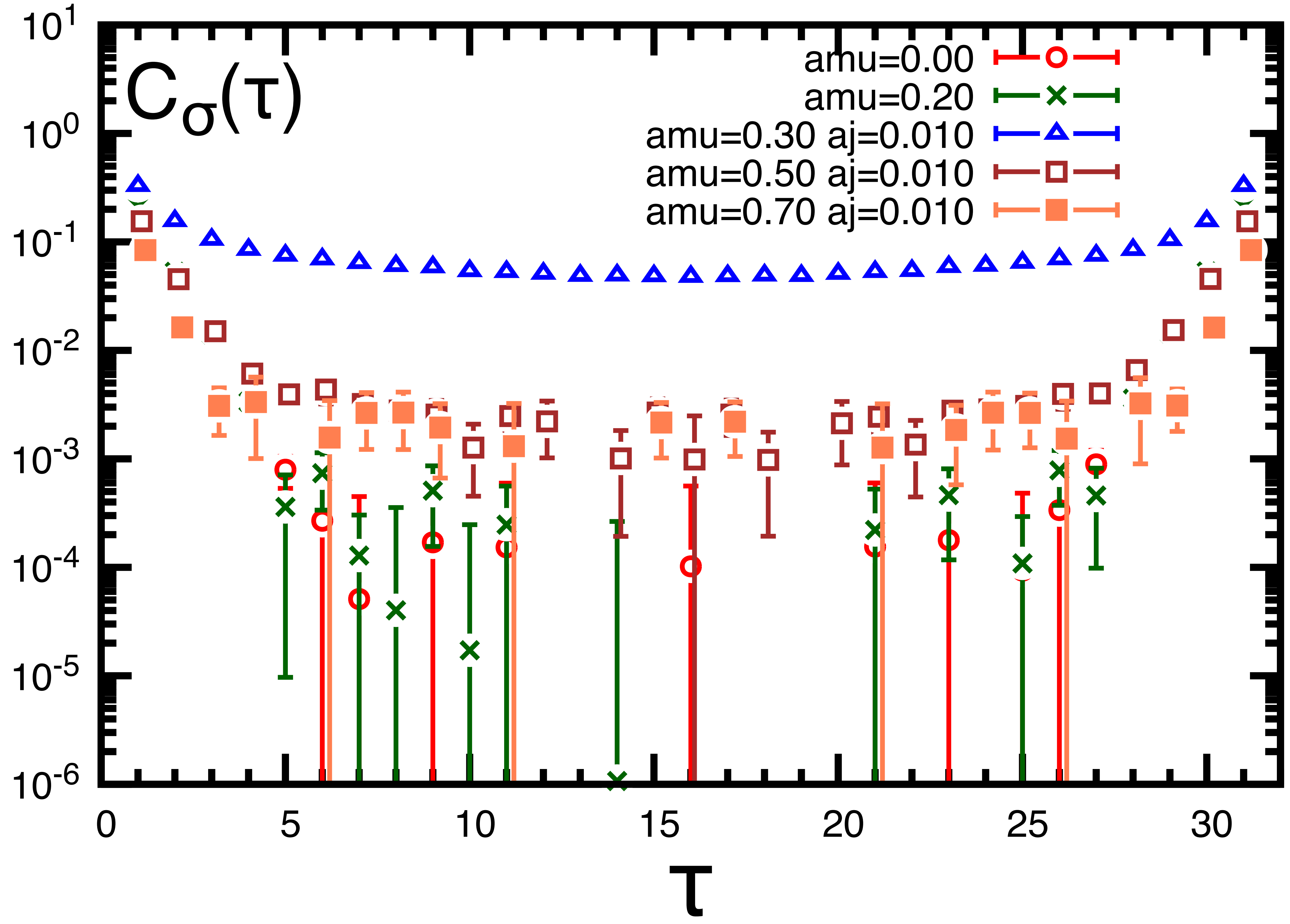
- As for the Higgs mode, the signal becomes very noisy, so that we cannot calculate its mass.

Correlation fn.: I=0 scalar diquark and meson

Diquark/Anti-diquark



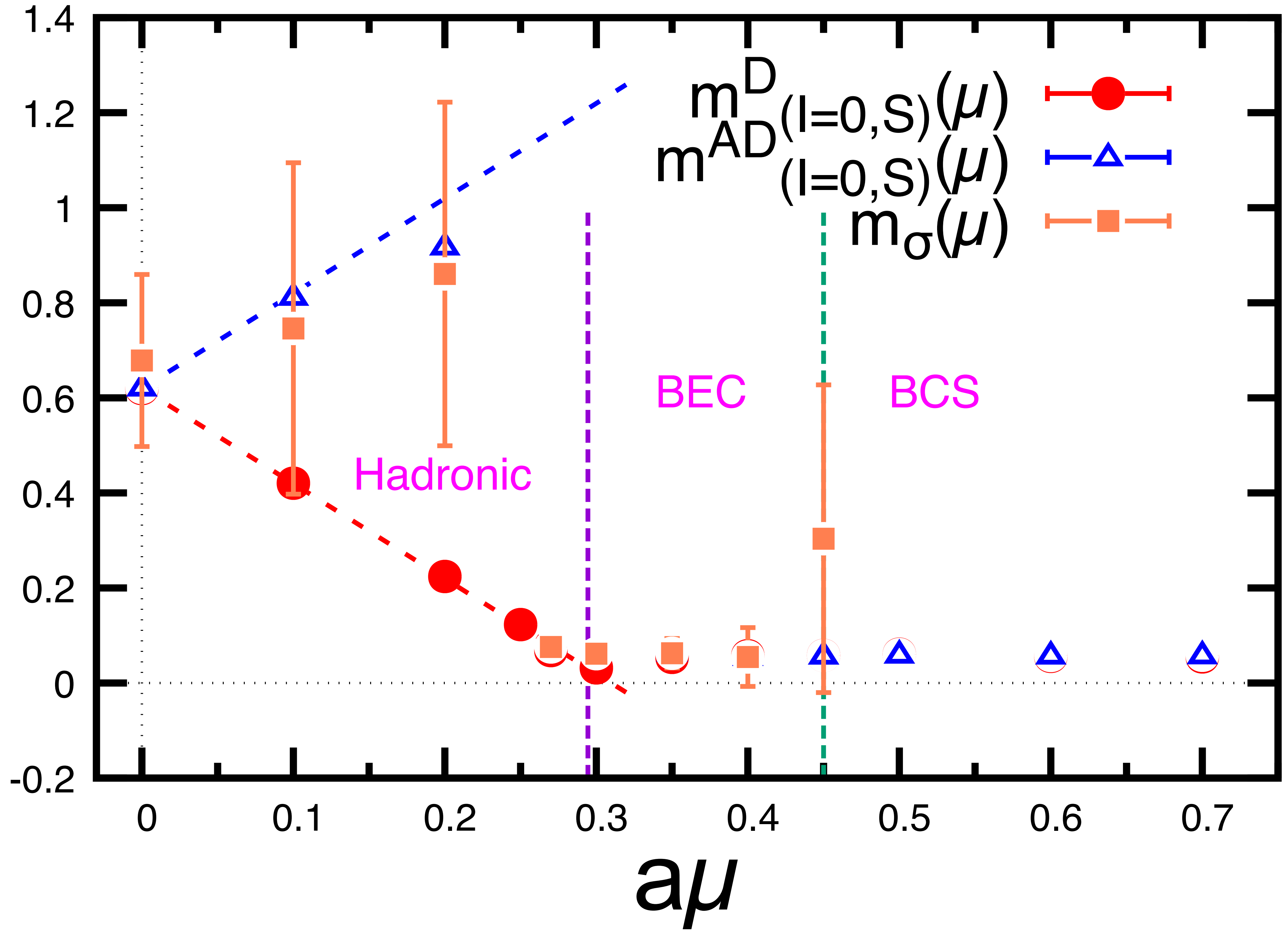
Meson (sigma meson)



At $\mu \neq 0$ in hadronic phase (green), time-reversal sym. of C(tau) is broken. In SF phase, C(tau) takes larger values.

Meson channel has a large contribution from disconnected terms. Noisy in the both phases

Mass spectra: I=0 scalar diquark and meson



- In hadronic phase, the prediction of mass shift

  $m(\mu) = m(\mu = 0) - n_O \mu$

  is reproduced.

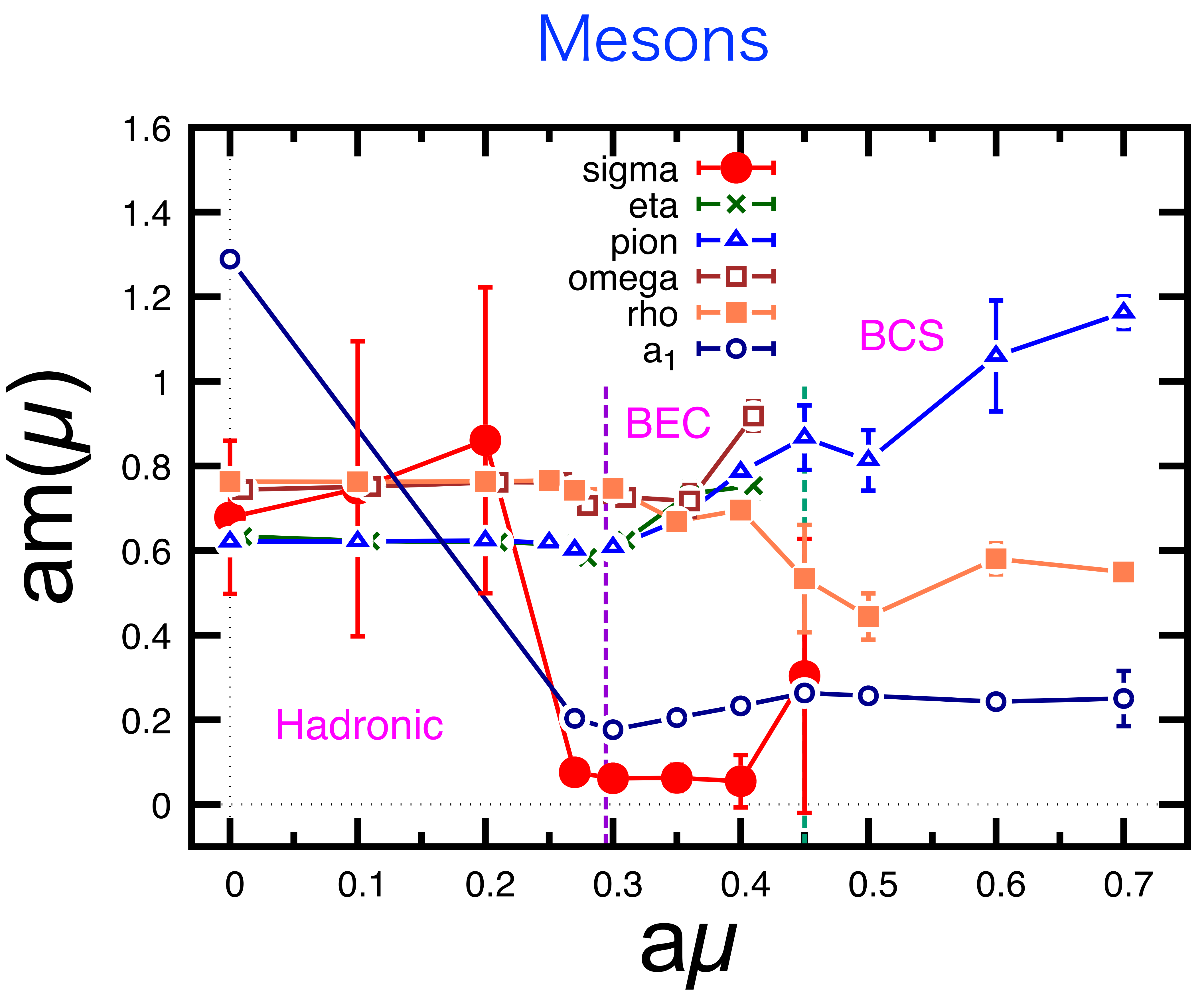
  $m(\mu) = m(\mu = 0) - 2\mu$:diquark

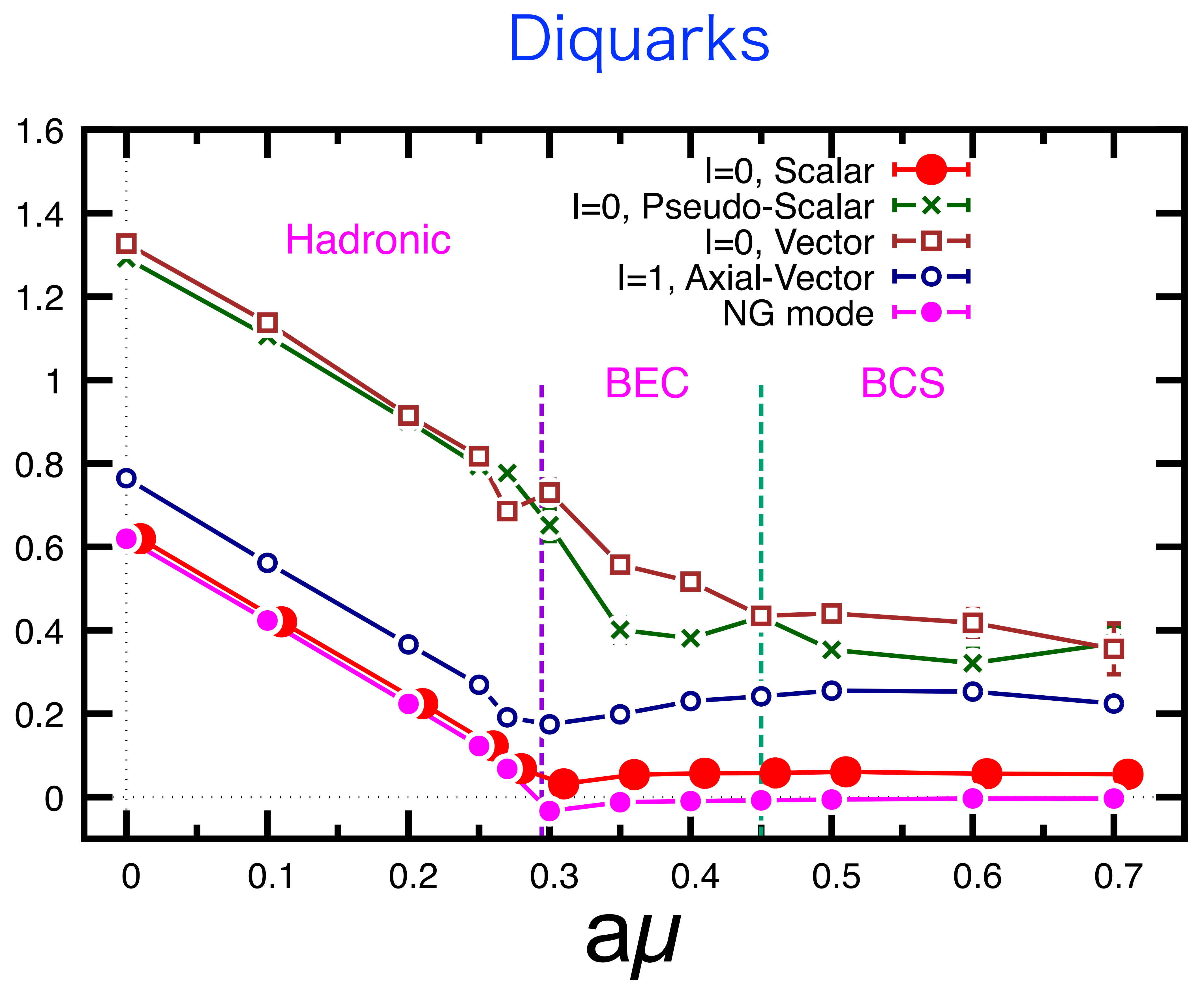
  $m(\mu) = m(\mu = 0) + 2\mu$: anti-diquark

  $m(\mu) = m(\mu = 0)$: sigma meson

- In SF phase, all three modes are degenerated.
  To resolve the meson-baryon mixing, we will need the variational method.

Summary of mass spectra





In Hadronic phase, $m_\pi \lesssim m_\eta < m_\sigma(\text{noisy}) < m_\rho \sim m_\omega \ll m_{a_1}$

(same with 3color QCD vacua)

In SF phase, $m_\sigma(\text{noisy}) < m_{a_1} < m_\rho < m_\pi \sim m_\eta(\text{noisy}) \ll m_\omega(\text{noisy})$

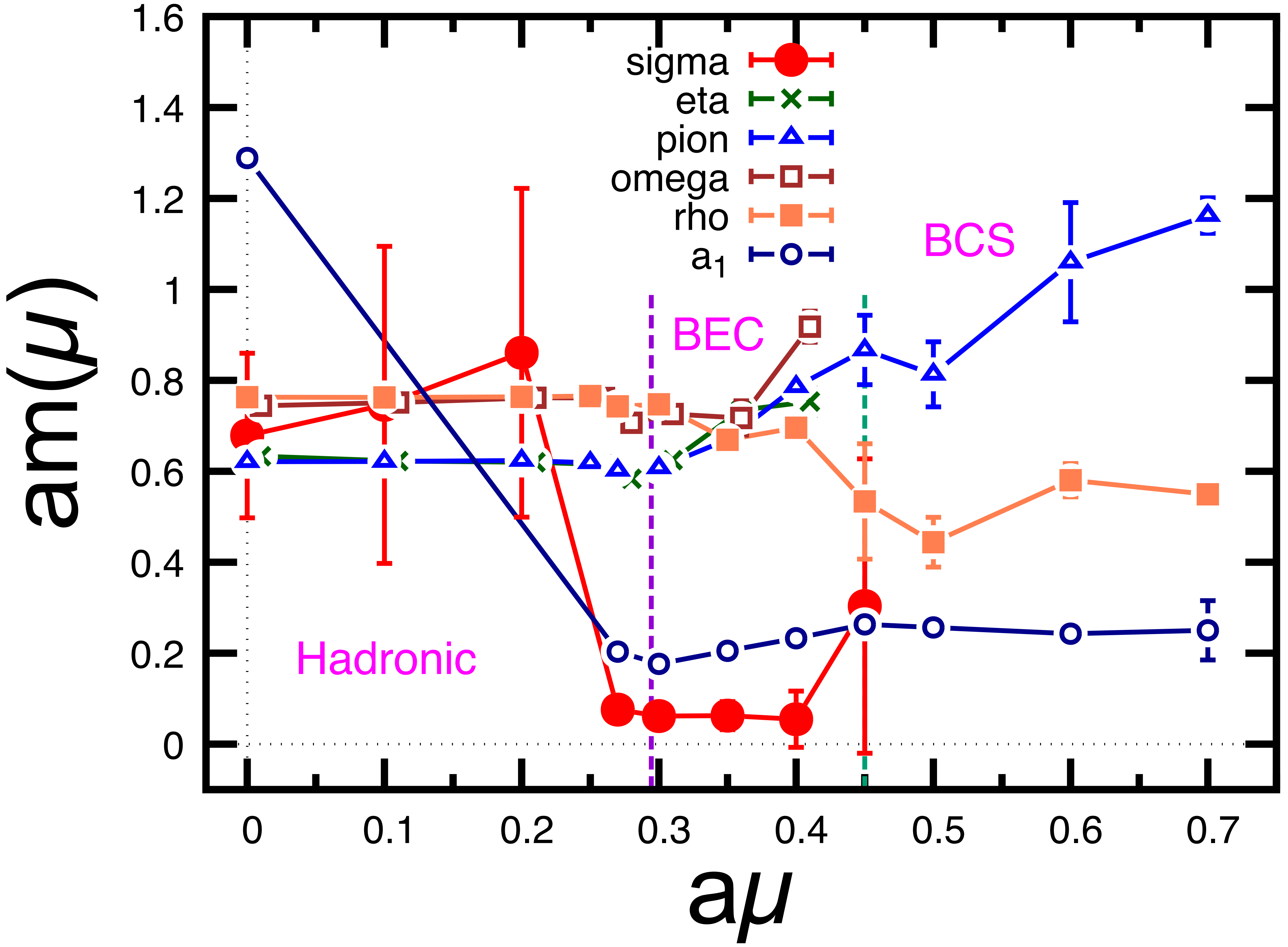
In both phases, $m_{NG} \lesssim m_{I=0,S} < m_{I=1,AV} < m_{I=0,PS} \lesssim m_{I=0,V}$

(In our notation($\psi_1^T K \Gamma \psi_2$, $K \equiv C\gamma_5\tau_2$), at $\mu = 0$

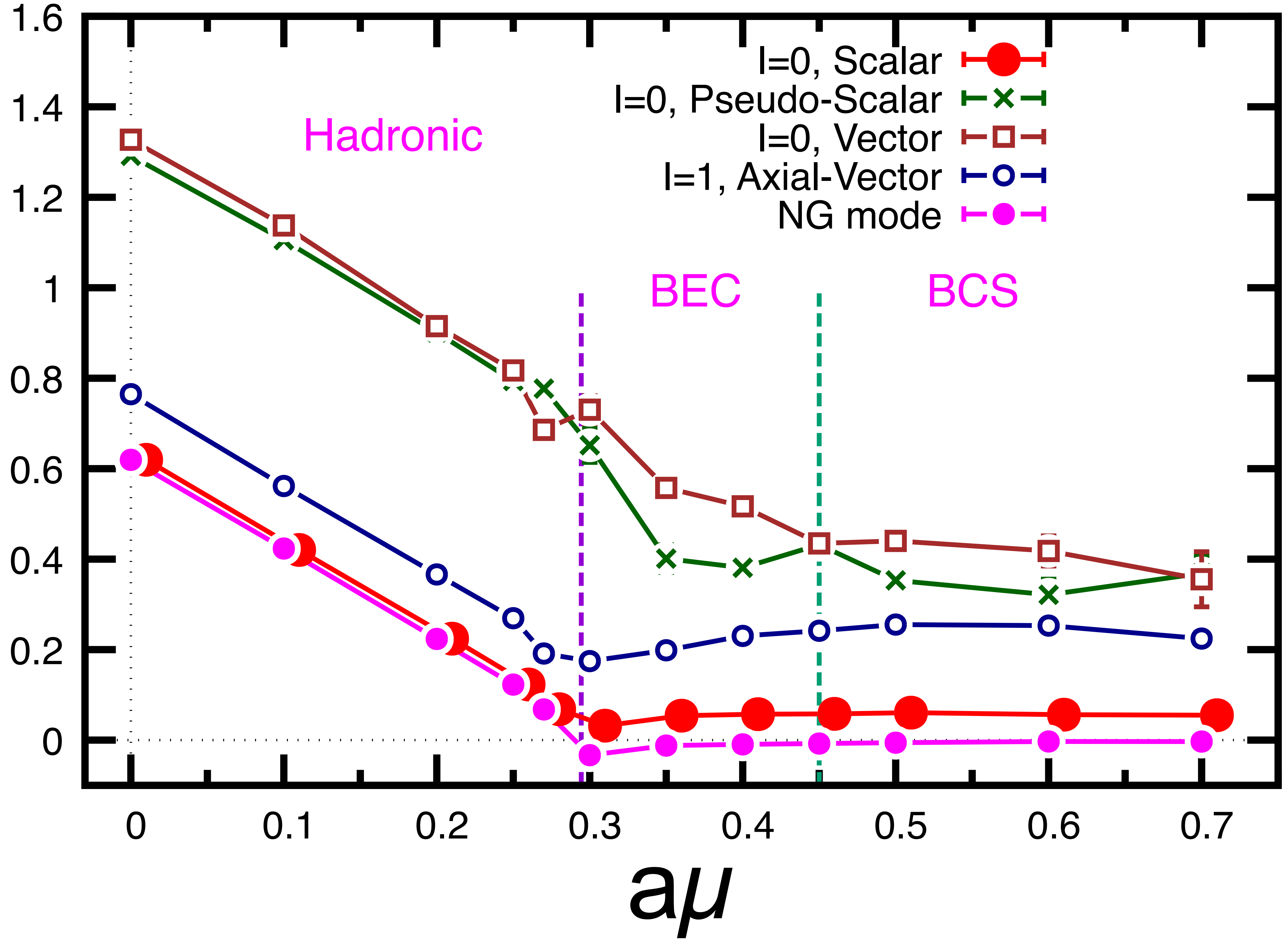
$\pi \leftrightarrow D_{I=0,S}$, $\rho \leftrightarrow D_{I=1,AV}$ are same multiplets)

Summary of mass spectra

Mesons



Diquarks



In Hadronic phase, $m_\pi \lesssim m_\eta < m_\sigma(\text{noisy}) < m_\rho \sim m_\omega \ll m_{a_1}$

(same with 3color QCD vacua)

In SF phase, $m_\sigma(\text{noisy}) < m_{a_1} < m_\rho < m_\pi \sim m_\eta(\text{noisy}) \ll m_\omega(\text{noisy})$

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(In our notation($\psi_1^T K \Gamma \psi_2$, $K \equiv C\gamma_5\tau_2$), at $\mu = 0$

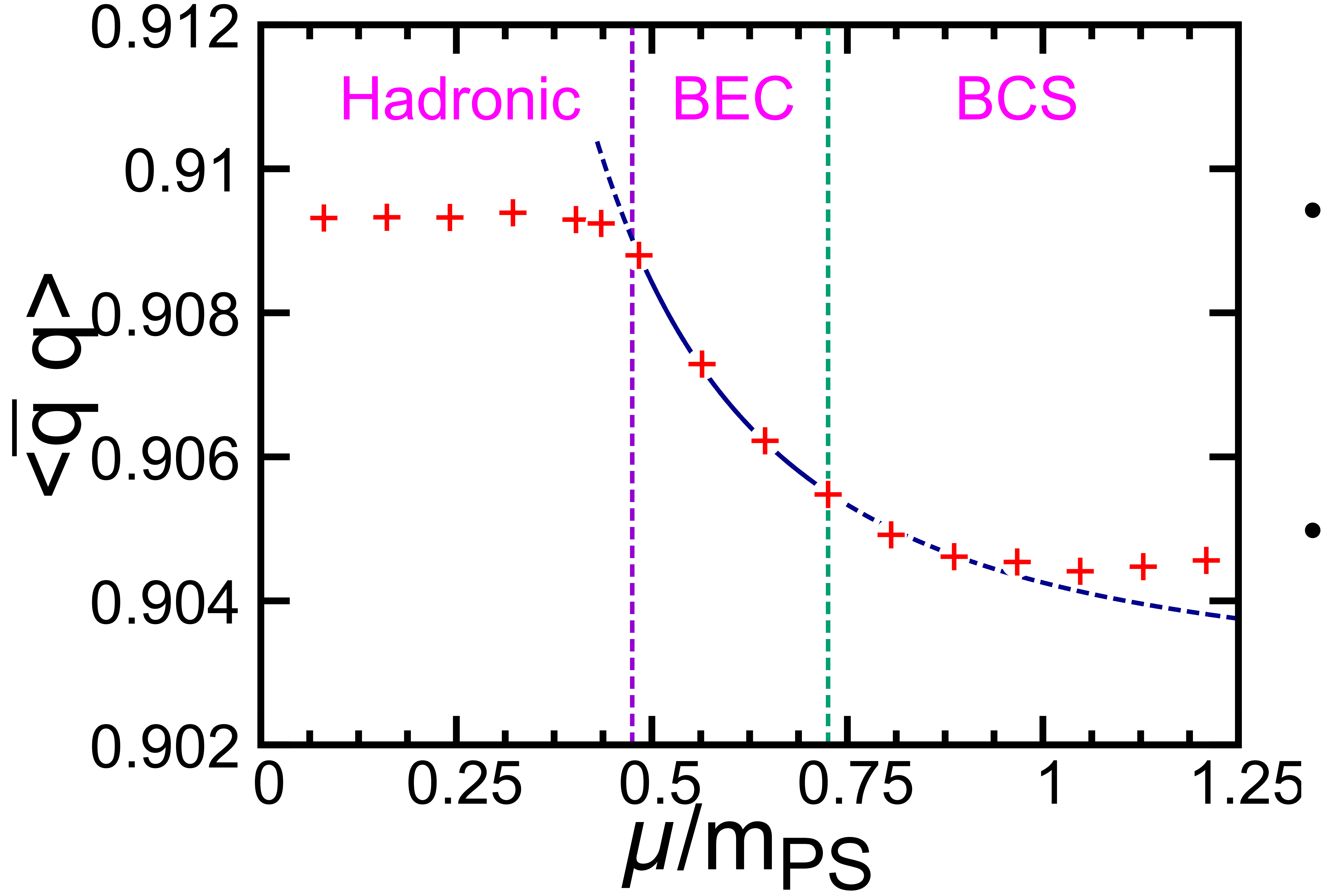
$\pi \leftrightarrow D_{I=0,S}$, $\rho \leftrightarrow D_{I=1,AV}$ are same multiplets)

Chiral symmetry and 2pt fn.

K. Iida, EI, K. Murakami, D. Suenaga, to appear on arXiv

μ-dependence of chiral condensate: our previous work

K.Iida et al., JHEP 10 (2024) 022

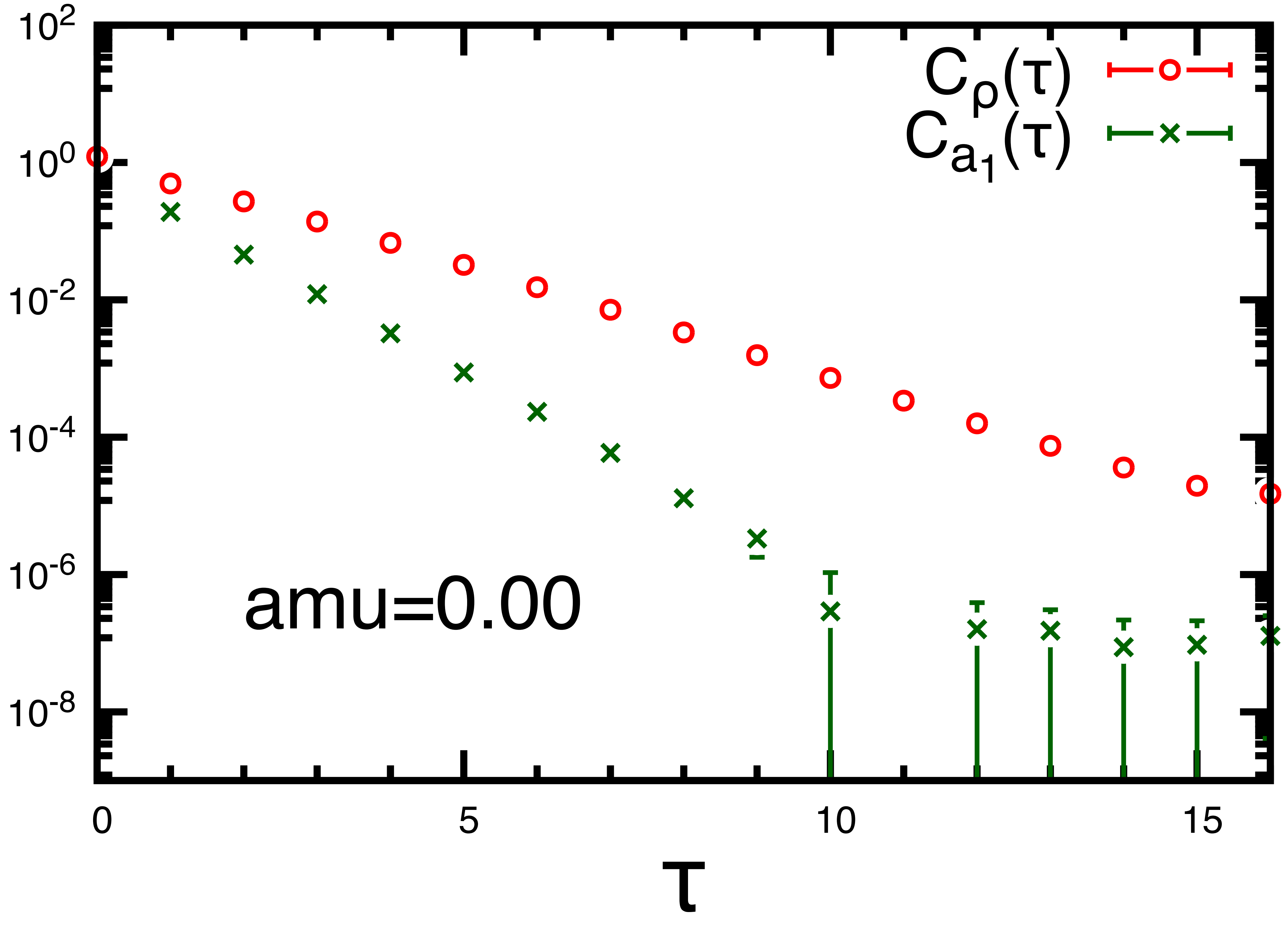


- Is chiral symmetry restored in the superfluid phase?
- This calculation uses (hearvy) Wilson fermions
- The chiral cond. decreases in SF phase.
  There must be an additive renormalization for this calc. setup, so that it does not go to zero.

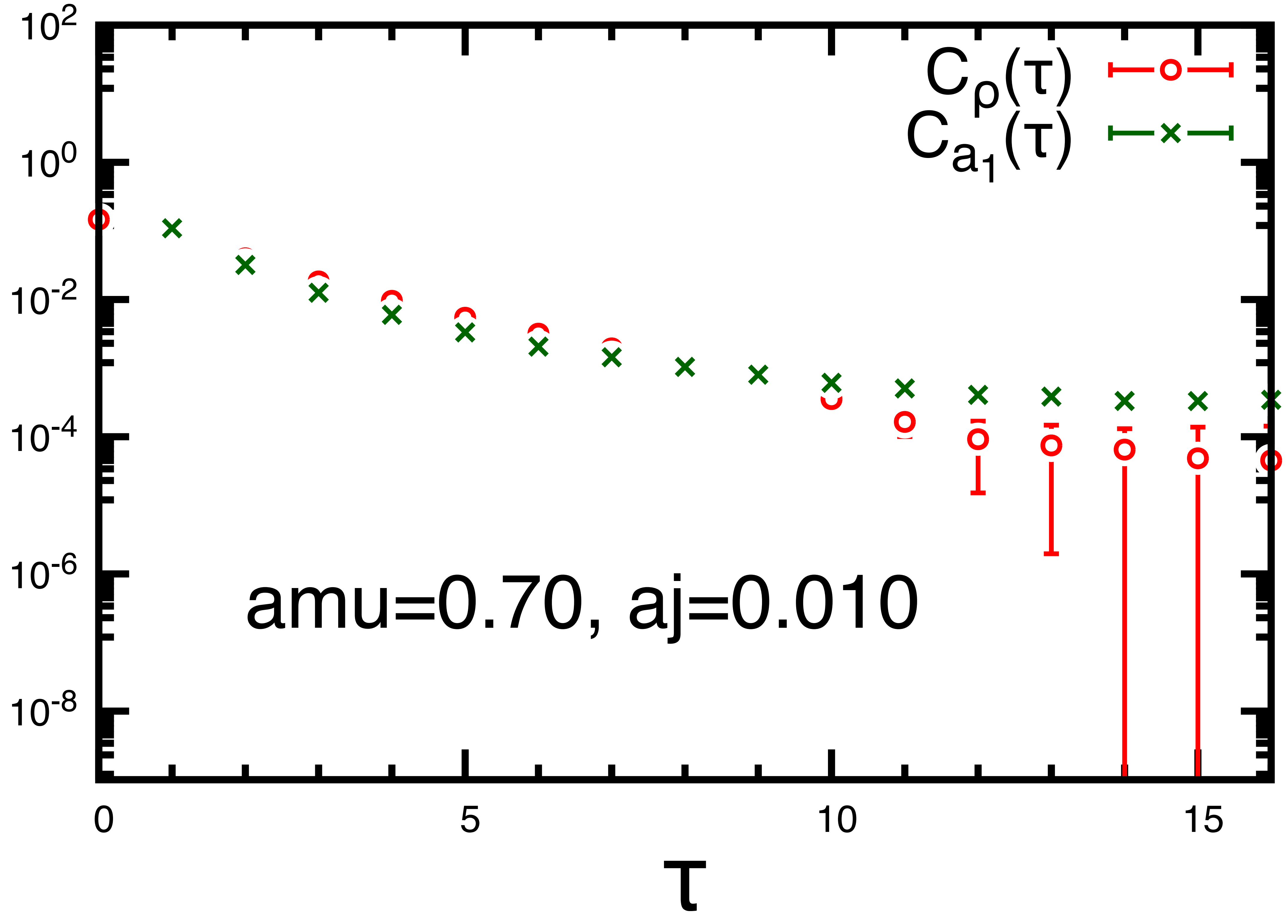
2pt fn. of rho and a1 mesons

We take $\mu = 1$ for V_μ^a and A_μ^a current op. to extract J=1 component.

Hadronic phase (low-density)



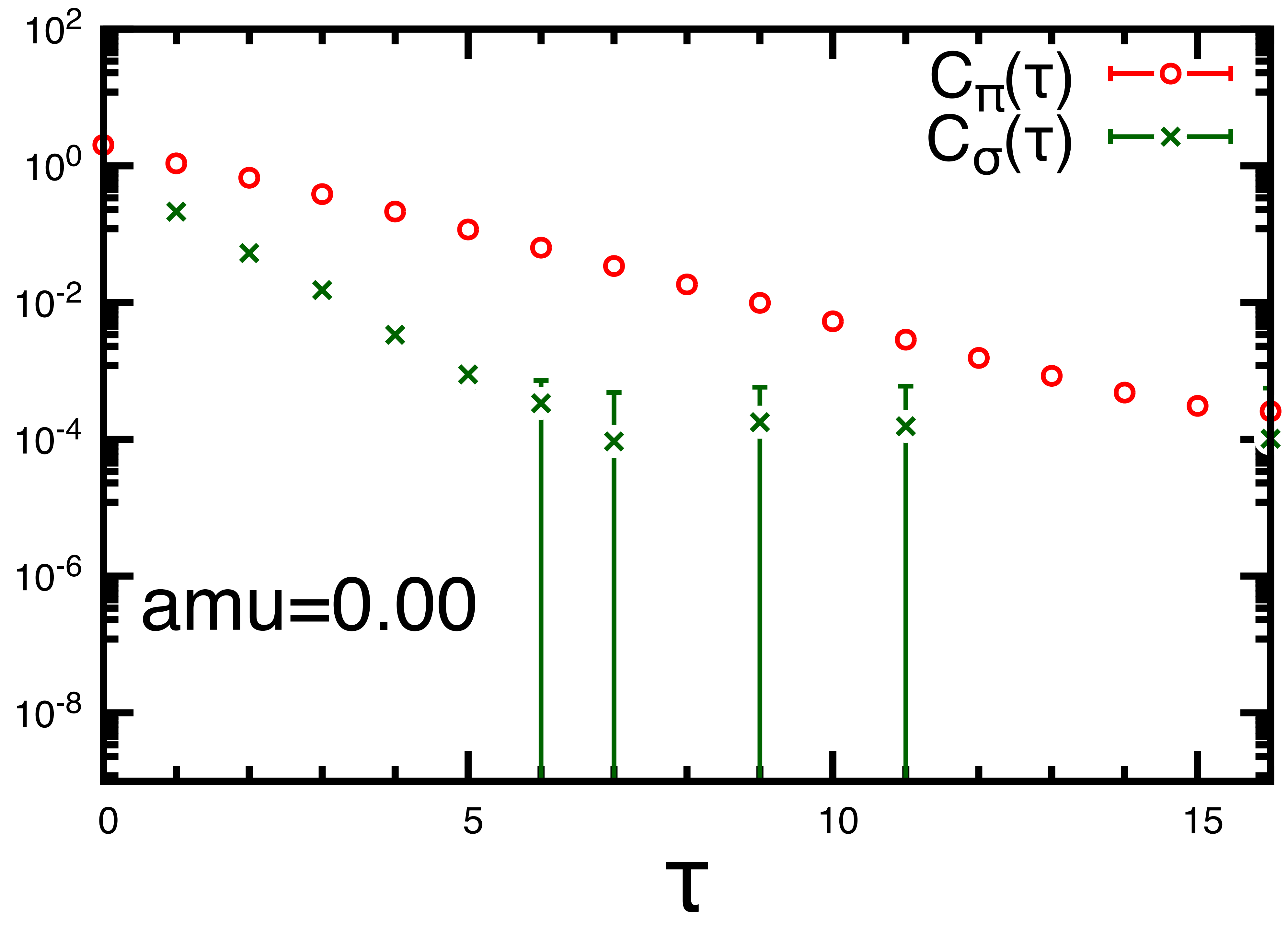
Superfluid phase (high-density)



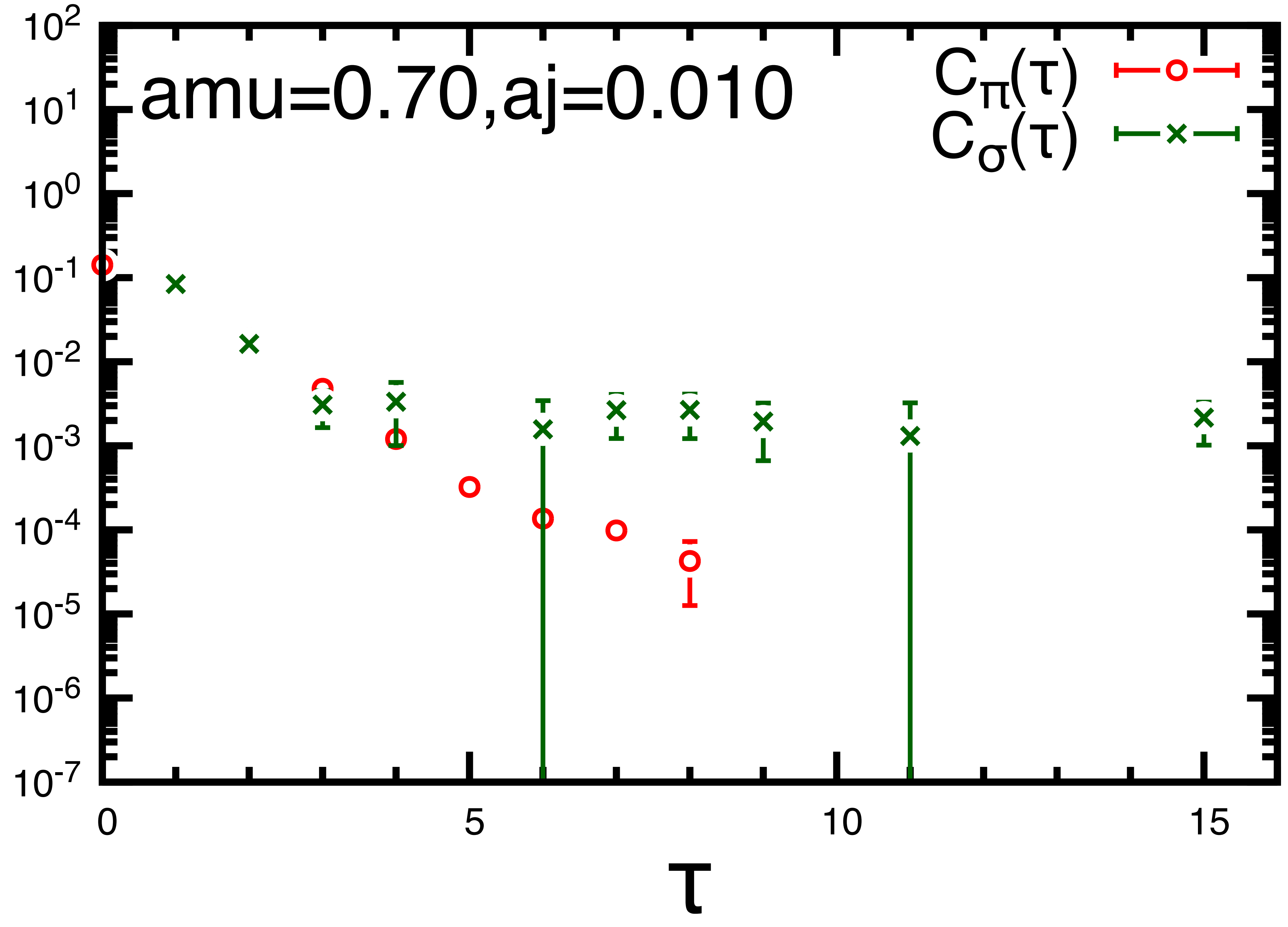
At high density, two 2pt-fns. appear to degenerate.

2pt fn. of pion and sigma meson

Hadronic phase (low-density)



Superfluid phase (high-density)

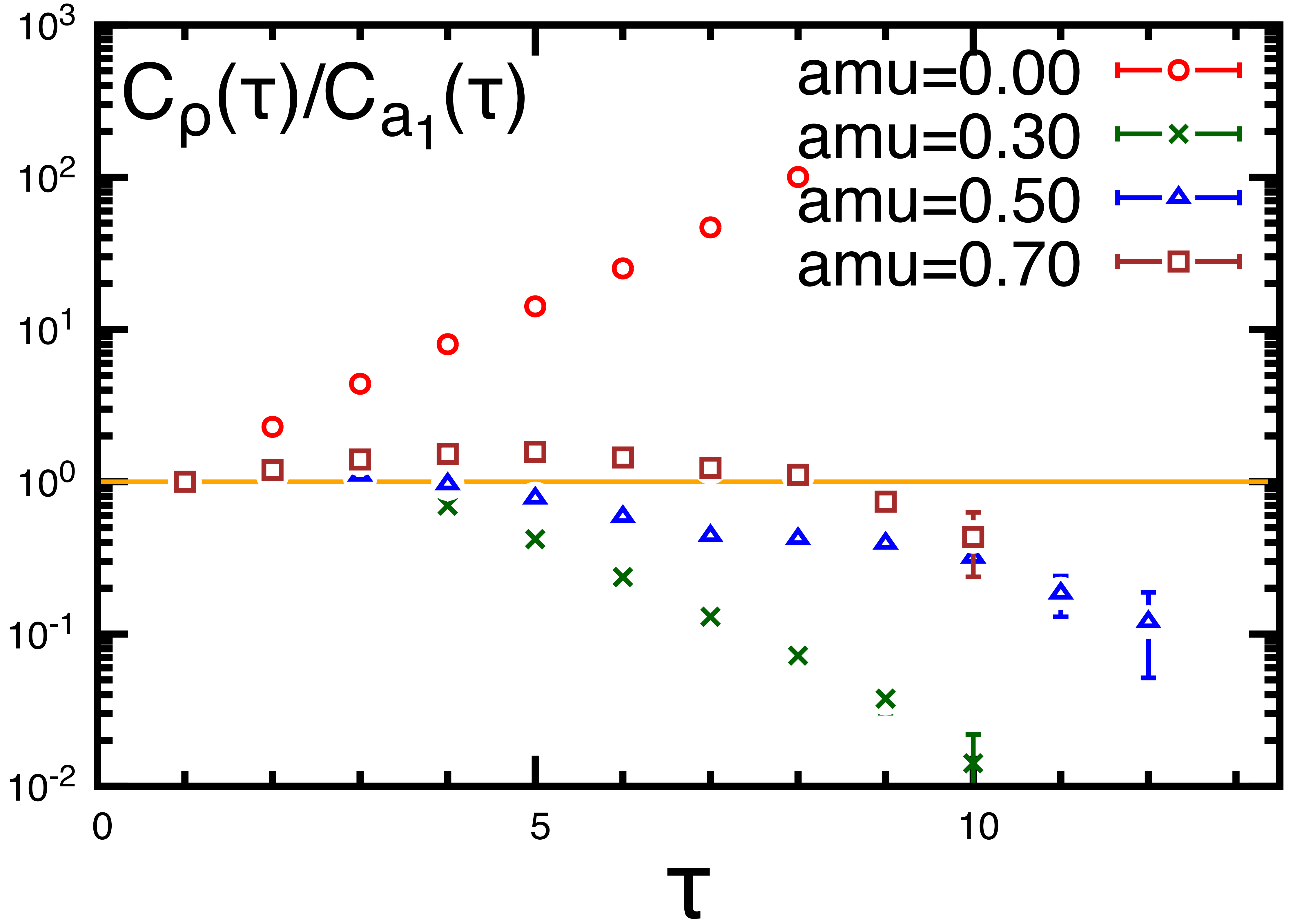


At high density, two 2pt-fns. appear to degenerate.

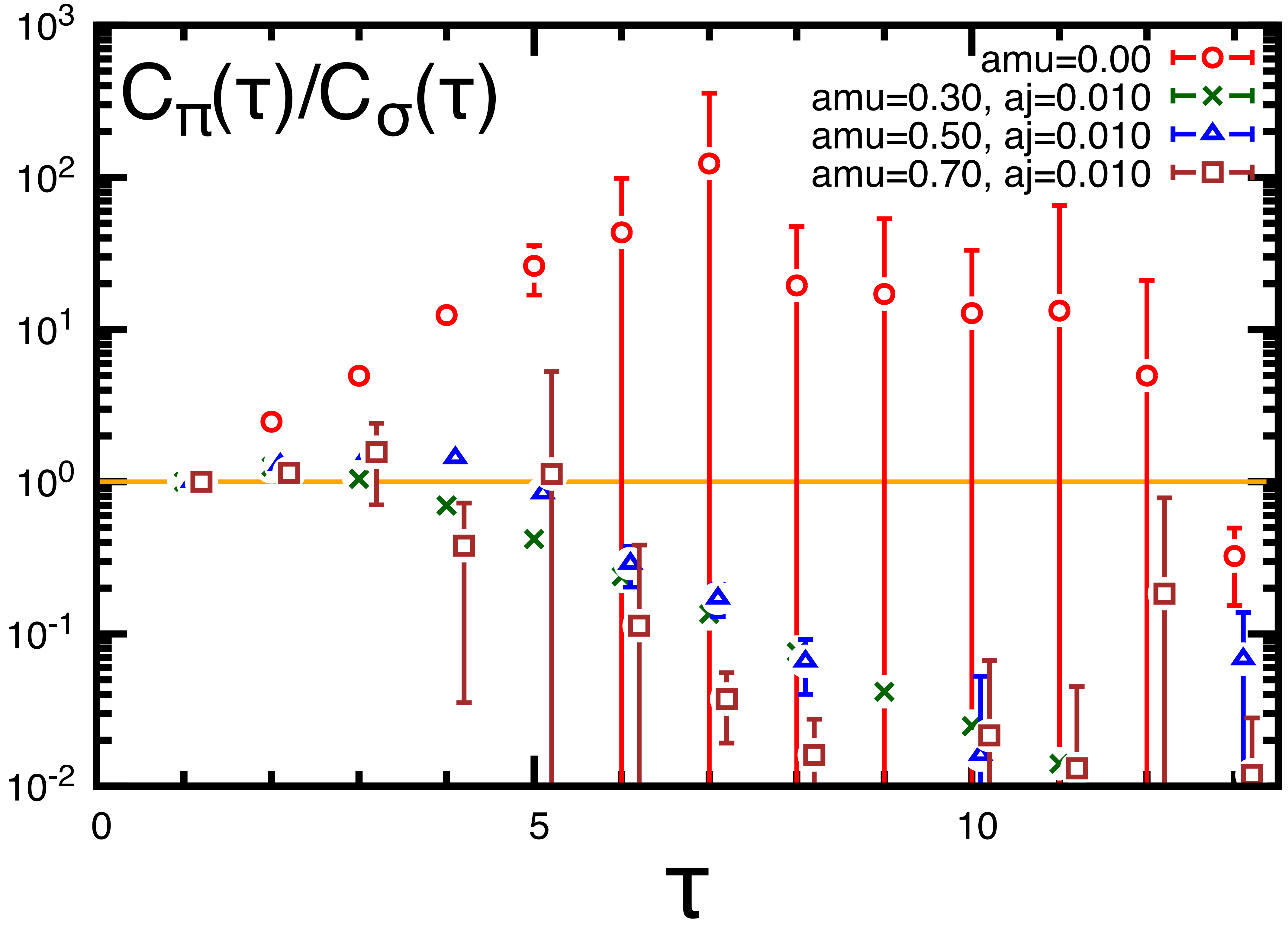
(but noisy…)

Ratio of 2pt fn. bw chiral partners

Rho and a1



Pion and sigma



Here, the ratio is normalized to 1 with $\tau = 1$.

A region close to 1 extends in higher density.

It suggests the recovery of chiral symmetry.

Summary

Summary and future direction

- In two-color QCD, the mass spectra even in the superfluid phase can be calculated using first-principles calculations

- Meson channels may be similar to 3-color QCD.
  In particular, mesons without diquark-mixing, as a0(noisy), pion, rho, f1(noisy)

- The 2pt fn. of diquark/anti-diquark has time-reversal asymmetry in the hadronic phase.

- NG mode for $U(1)_B$ breaking becomes massless in the SF phase.
  It is a special property for 2color QCD.

- A tendency towards the restoration of chiral symmetry is observed from the degeneracy of the 2-pt fn. of chiral partners

- Technically, more detailed analysis to solve the meson-baryon mixing in superfluid phase by a generalized eigenvalue problem (GEVP) is important future work.

- Next, extend the HAL QCD method to the SF phase!

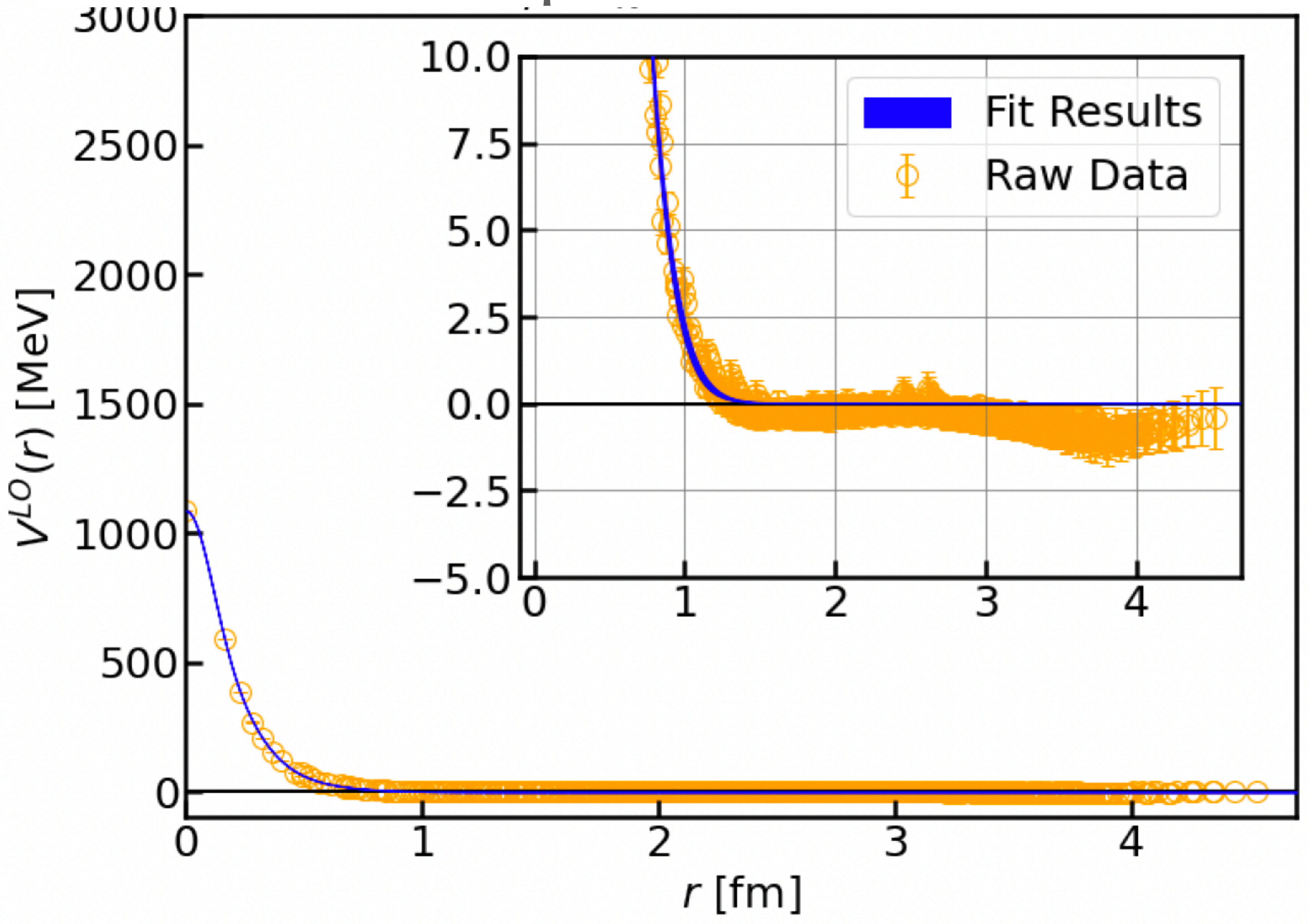
For the hadronic phase: K. Murakami et al., JHEP 02 (2024) 152

Hadron potential by HAL QCD method

- In hadronic phase, pion and diquark potential are equivalent because of extended flavor symmetry.

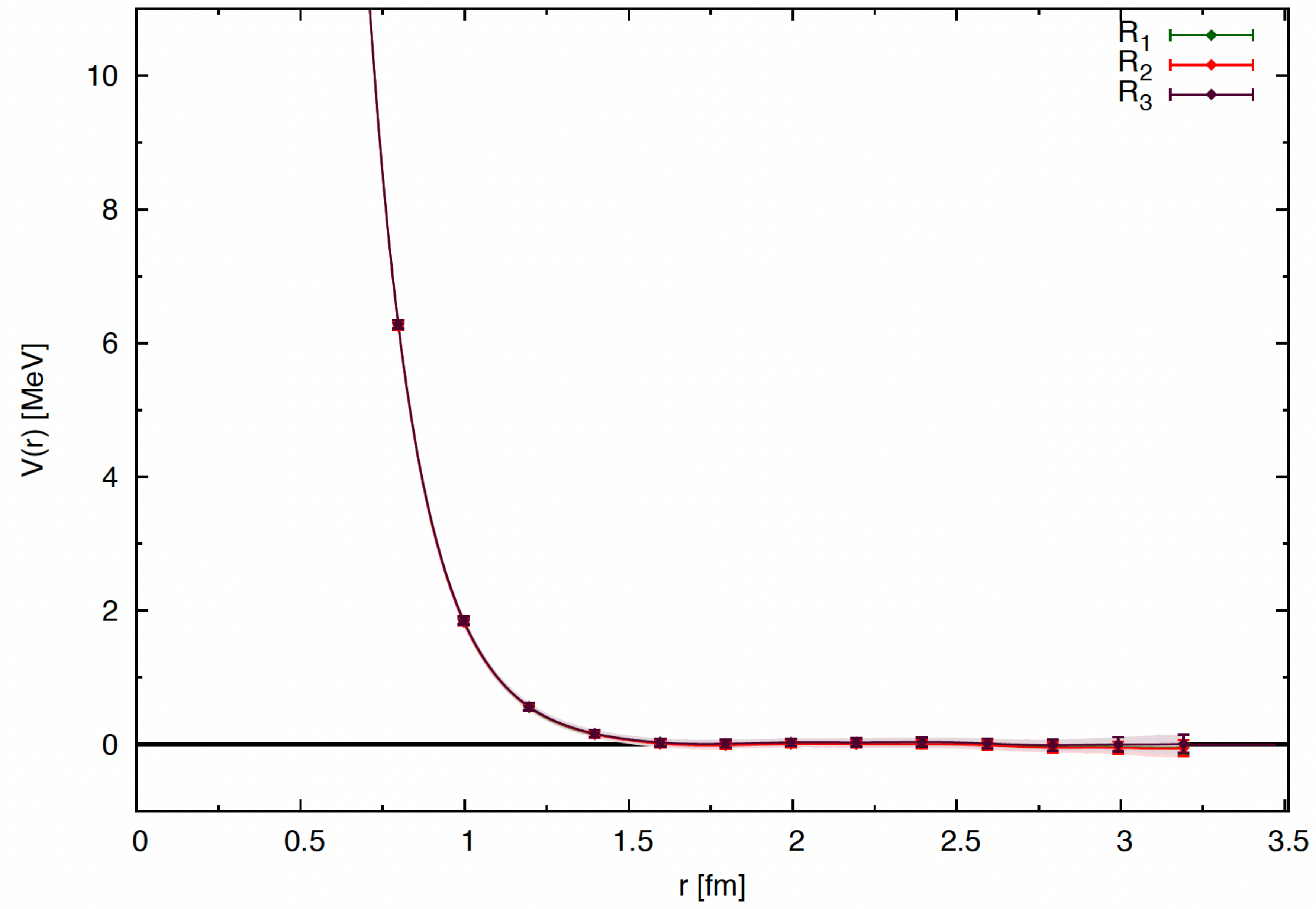
- Pion potential for 2color and 3color QCD are qualitatively same

Diquark-diquark potential
in hadronic phase of 2color QCD



K.Murakami, K.Iida, EI, JHEP 11 (2023) 231

I=2 $\pi\pi$ potential of 3color QCD



T.Kurth et al.(HAL QCD coll.), JHEP12(2013)015

Backup

Meson-baryon mixing in SF phase

- In SF phase, $U(1)_B$ is broken, no difference bw mesons and baryons.

  2pt fn. $C(\mu;\tau) = \sum_n |\langle 0|\hat{O}(0)|n\rangle|^2 e^{-(E_n-\mu n_O)\tau}$

  here $|n\rangle_{SF} \propto c_0|$meson-like state $(\bar{q}\Gamma q)\rangle + c_1|$baryon-like state$(q^T\Gamma q)\rangle$

  Thus, consider all possible contractions of $\bar{q}$-q and q-q.

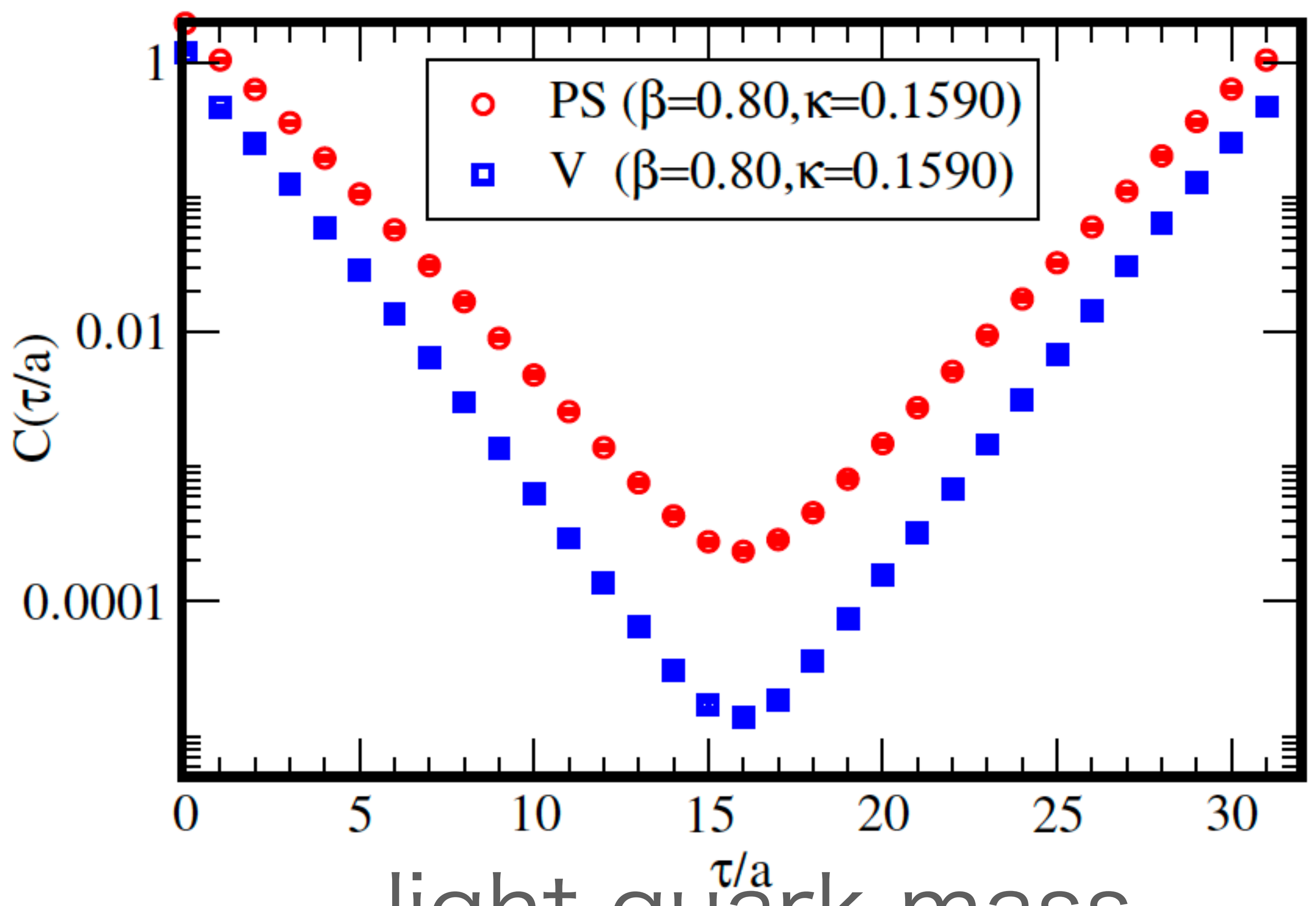
- Normal quark propagator ($\bar{q}$-q): $S_N = Q^{-1}(\mu)\Delta^\dagger(\mu)$

  Anomalous quark propagator (q-q):$S_A = JQ^{-1}(\mu)K$

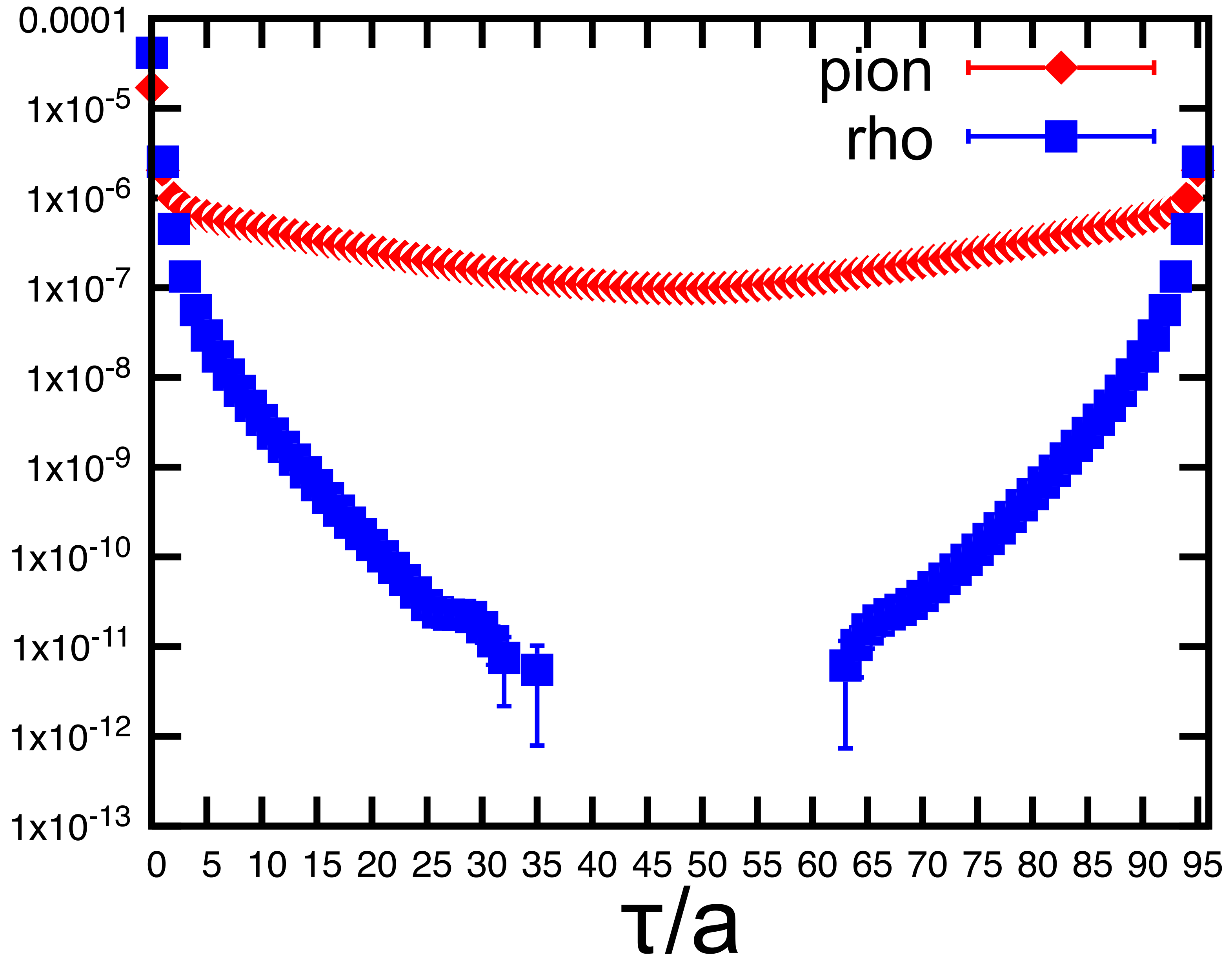
  $Q(\mu) = \Delta^\dagger(\mu)\Delta(\mu) + J^2$ (this is $DD^\dagger$ in standard QCD vacua)

Quark mass dependence of correlation fns.

heavy quark mass



light quark mass



- Top panel: $m_\pi/m_\rho \sim 0.8$ (heavy quark mass)

  Bottom panel: $m_\pi/m_\rho \sim 0.2$ (light quark mass)

  (physical point)

  By changing lattice bare mass (κ), this ratio is a result of.

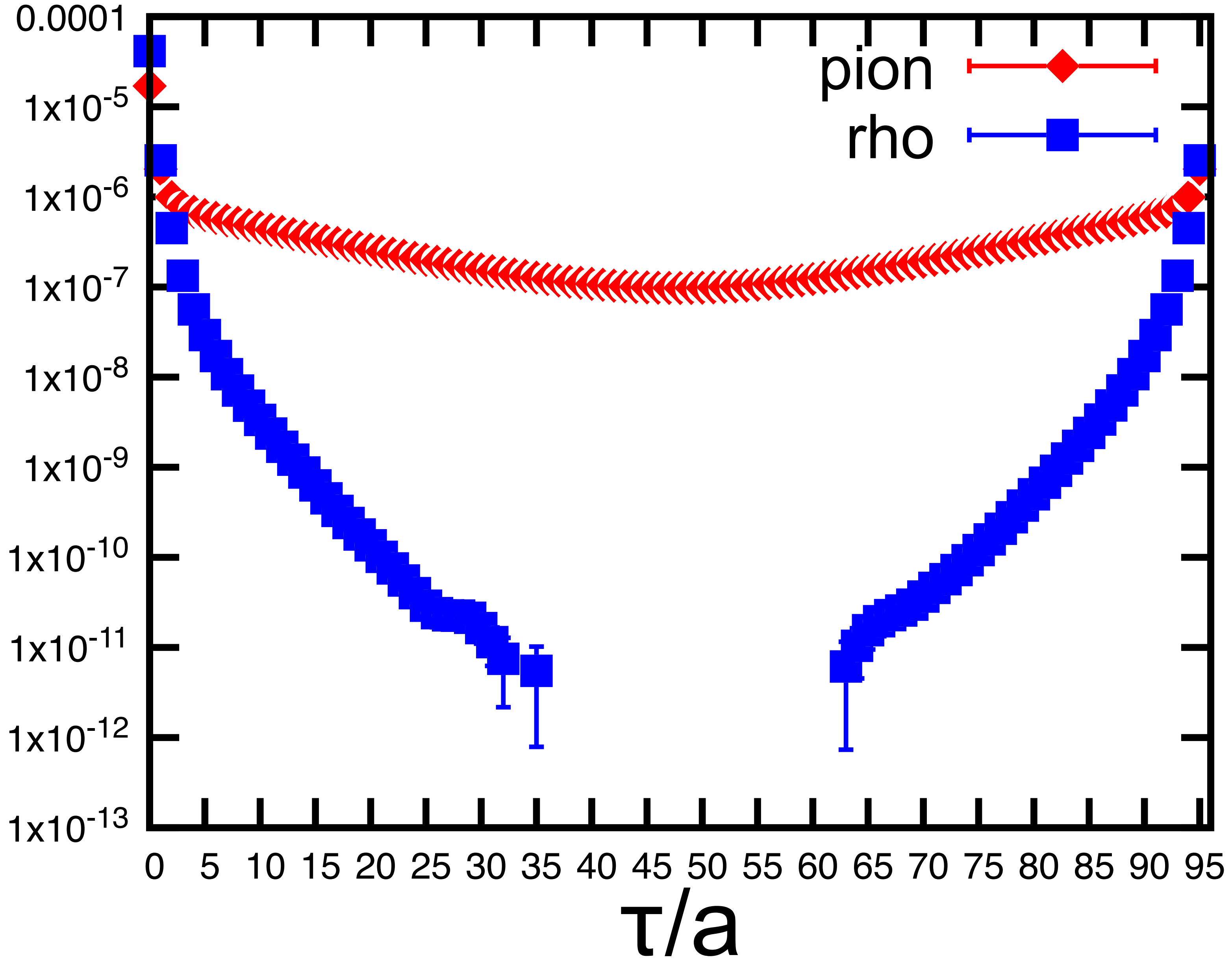
- In lighter quark mass, ρ can decay to 2 pion.

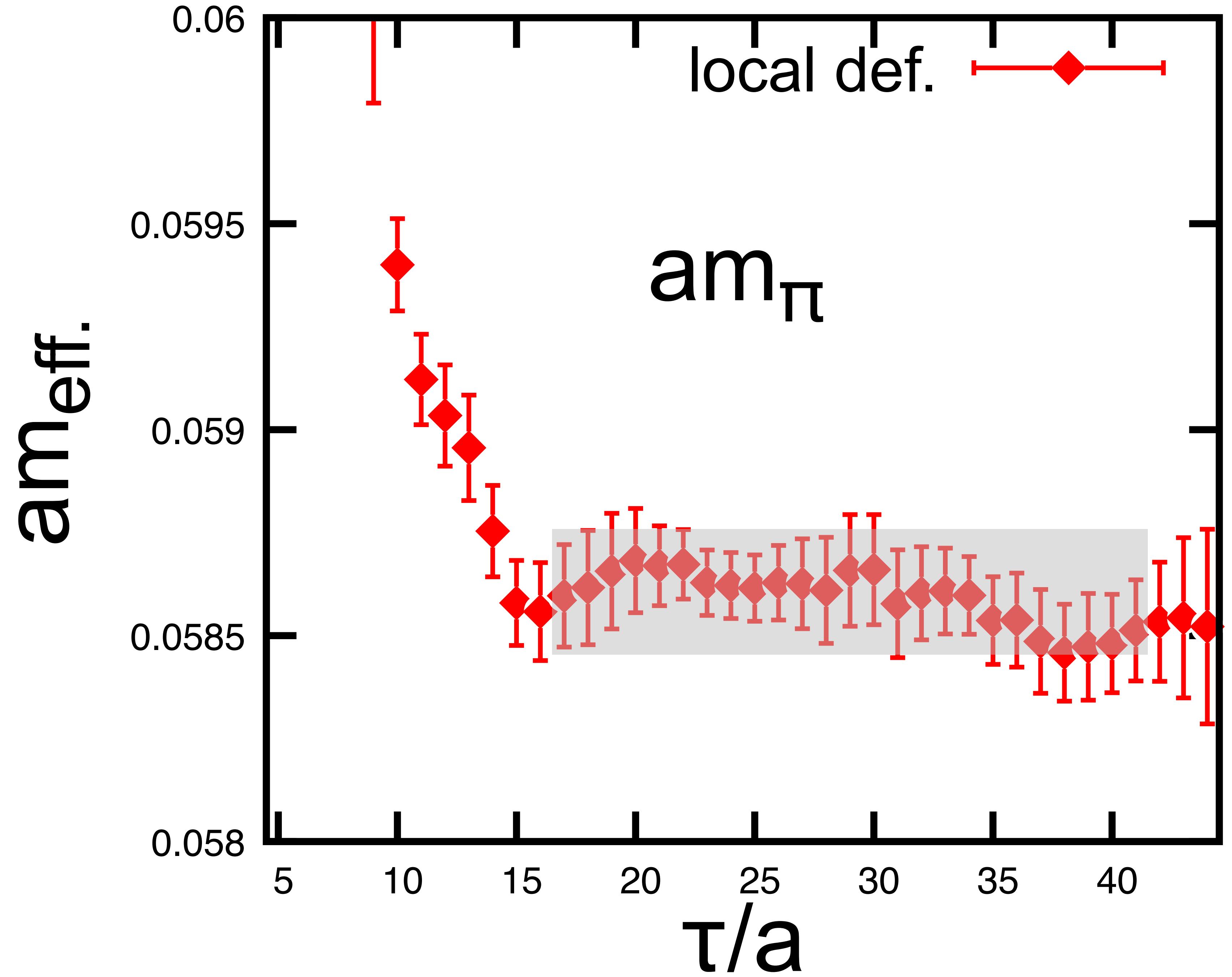
  ρ cannot propagator long time.

  The signal of the correlation fn. becomes noisy in long τ regime.

  Hard to obtain precise data for heavier hadron mass

Calculate mass from the long-time range of correlations

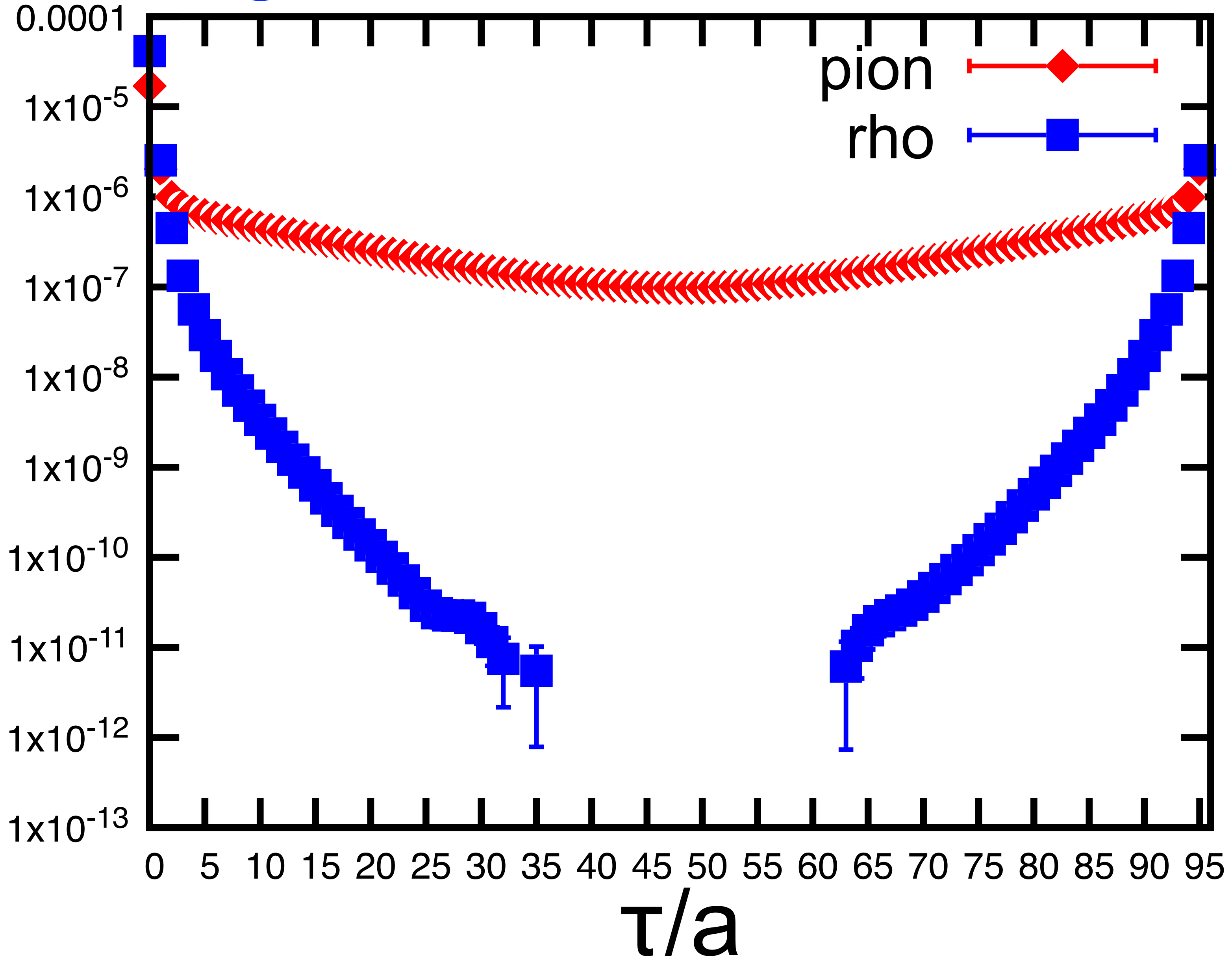


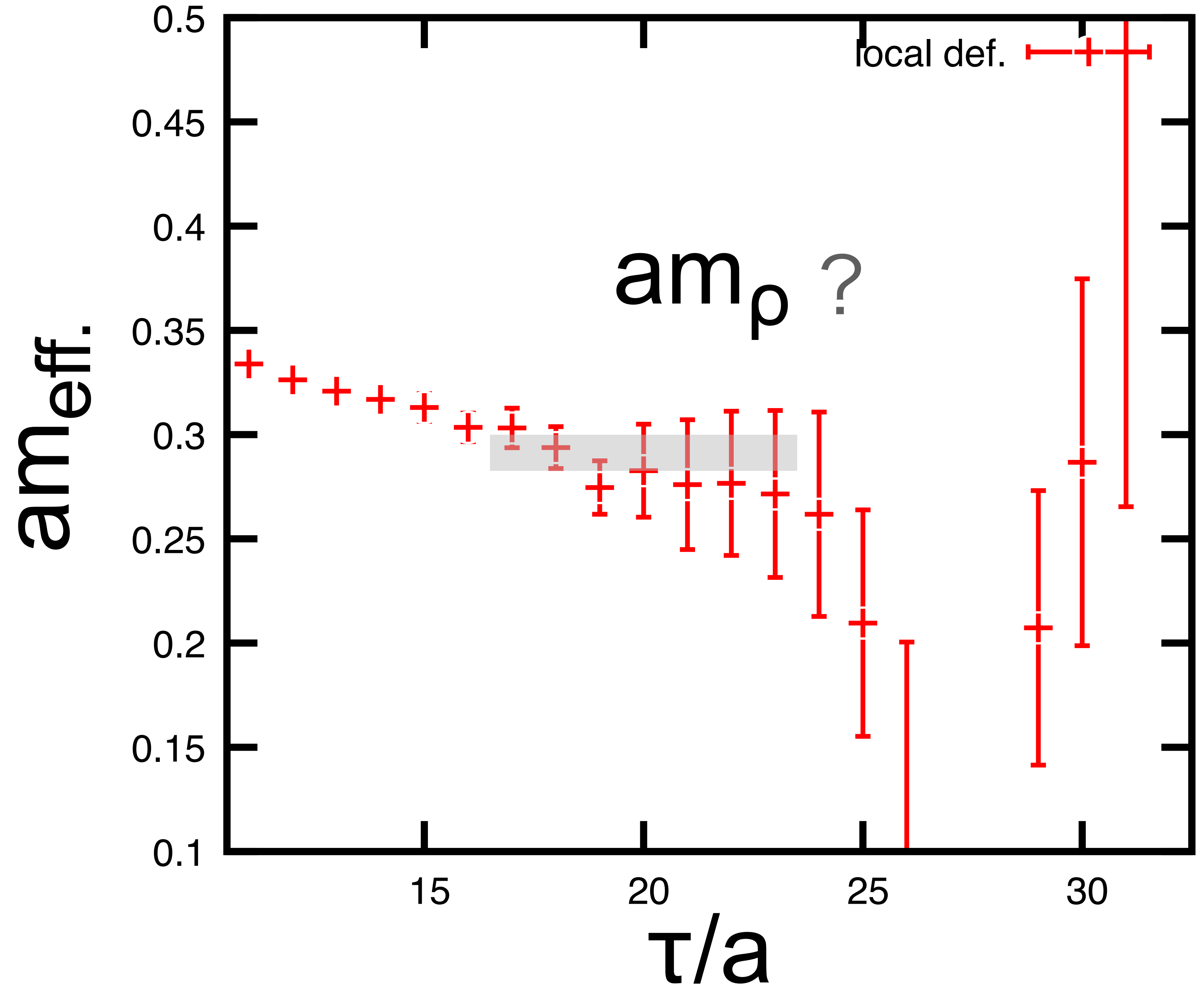


1. Fit the data in the appropriate τ region with $f(\tau) = c_0 + c_1 e^{-c_2 \tau}$ the best-fit value of $c_2 = m$

2. Calc. effective mass $m_{eff}(\tau) = -\log[C(\tau+1)/C(\tau)]$ In long τ, it should converge with the lowest state mass Find a plateau of m_{eff}

Heavy (unstable) meson case





- Calc. effective mass

  $$m_{eff}(\tau) = -\log[C(\tau+1)/C(\tau)]$$

  In long τ, it should converge with the lowest state mass
  Find a plateau of m_{eff}

- It is difficult to find a plateau for heavy (unstable) case