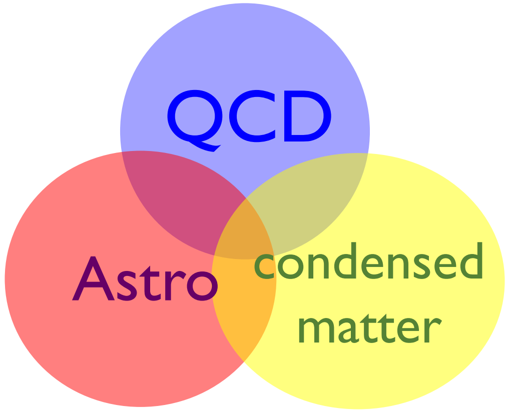
"Buenas Idea on the QCD Phase Diagram", YITP, May. 26-May. 29, 2026

Quarkyonic matter models for neutron stars



Toru Kojo

(KEK, Theory Center)

Refs) Baym-Hatsuda-TK-Powell-Song-Takatsuka, Review on QCD for neutron stars (2018)

TK, quark saturation & stiffening of matter in quark-hadron-continuity, PRD (2021)

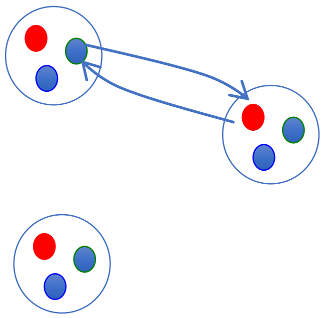
Fujimoto-TK-McLerran, on ideal quarkyonic matter model, PRL (2024)

Fujimoto-TK-McLerran, on mitigation of the hyperon puzzle, PRC (2026)

Tajima-Iida-TK-Liang, phase shift rep. for EOS, PRL (2025)

Three-window model (n_0 = 0.16 fm^{-3}) [Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

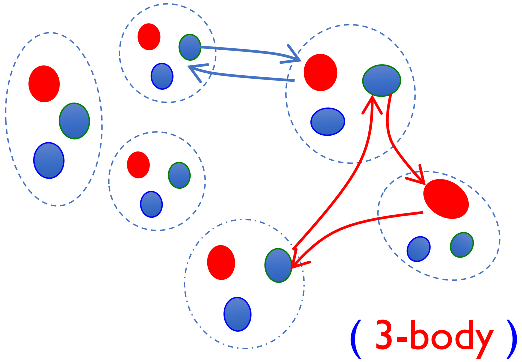


ab-initio nuclear cal.

laboratory experiments

steady progress

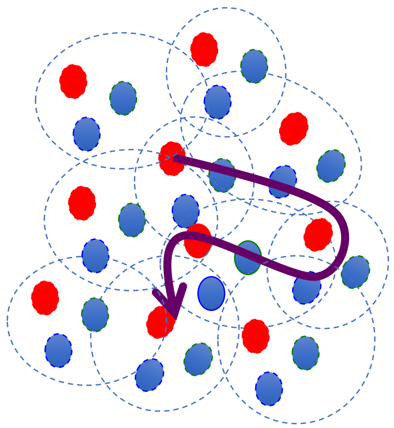
- many-quark exchange
- structural change,...
- hyperons, Δ, ...



most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

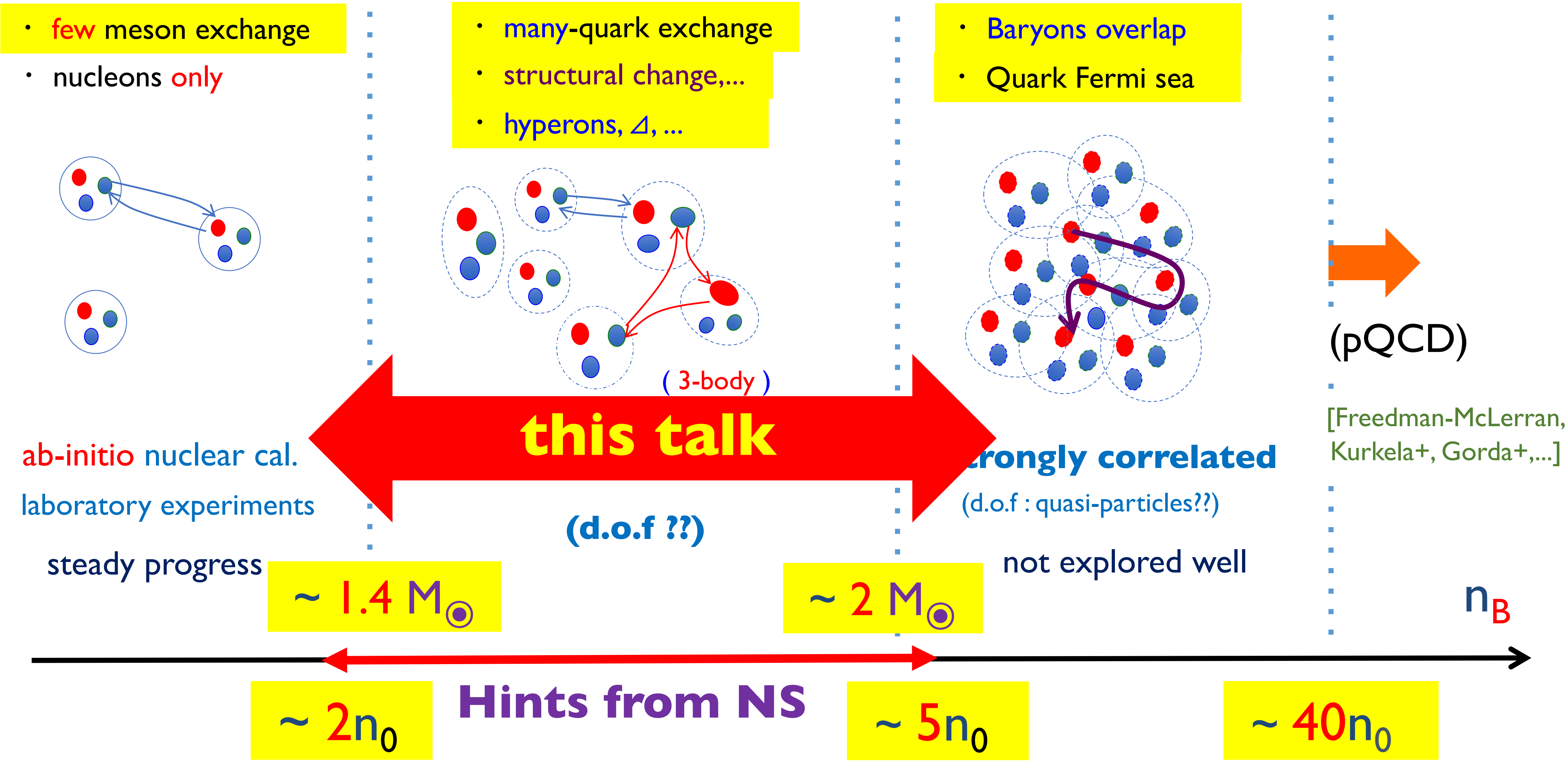
[Freedman-McLerran, Kurkela+, Gorda+,...]

~ 1.4 $M_\odot$ ~ 2 $M_\odot$

n_B

~ $2n_0$ **Hints from NS** ~ $5n_0$ ~ $40n_0$

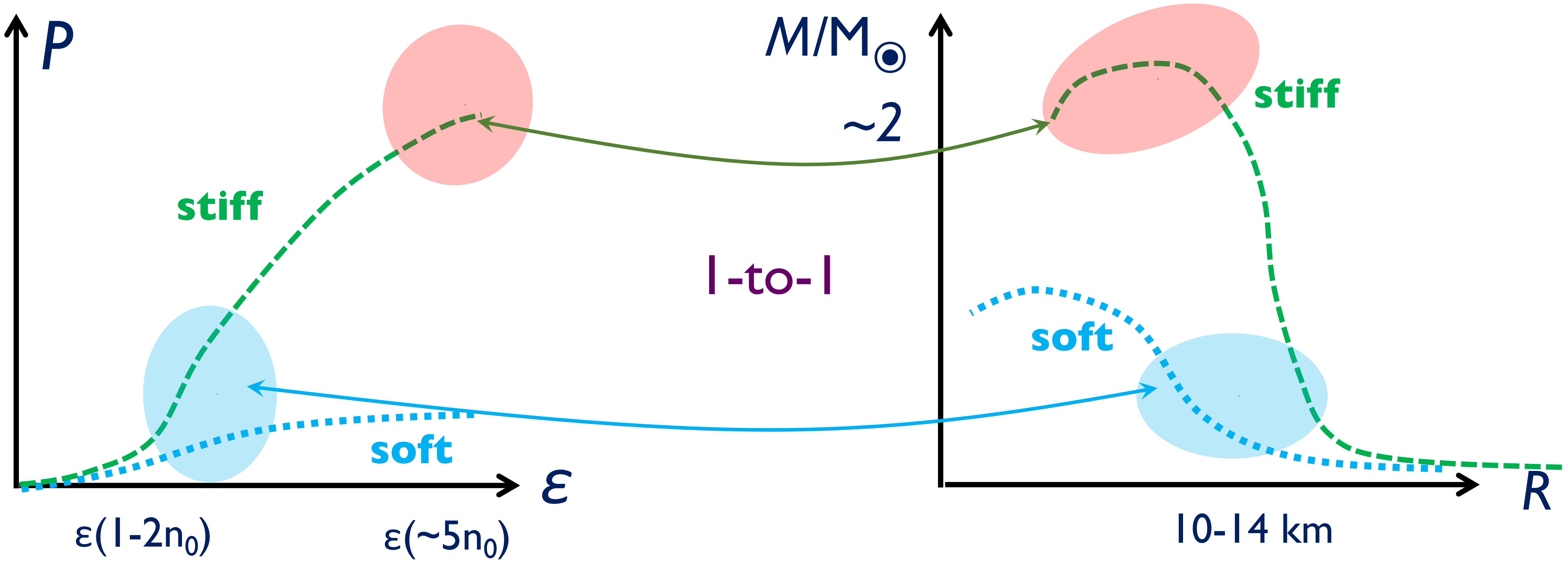
Three-window model (n_0 = 0.16 fm^{-3}) [Masuda+ '12; TK+ '14]



EoS stiffness & M-R

Ref) Lattimer & Prakash (2001)

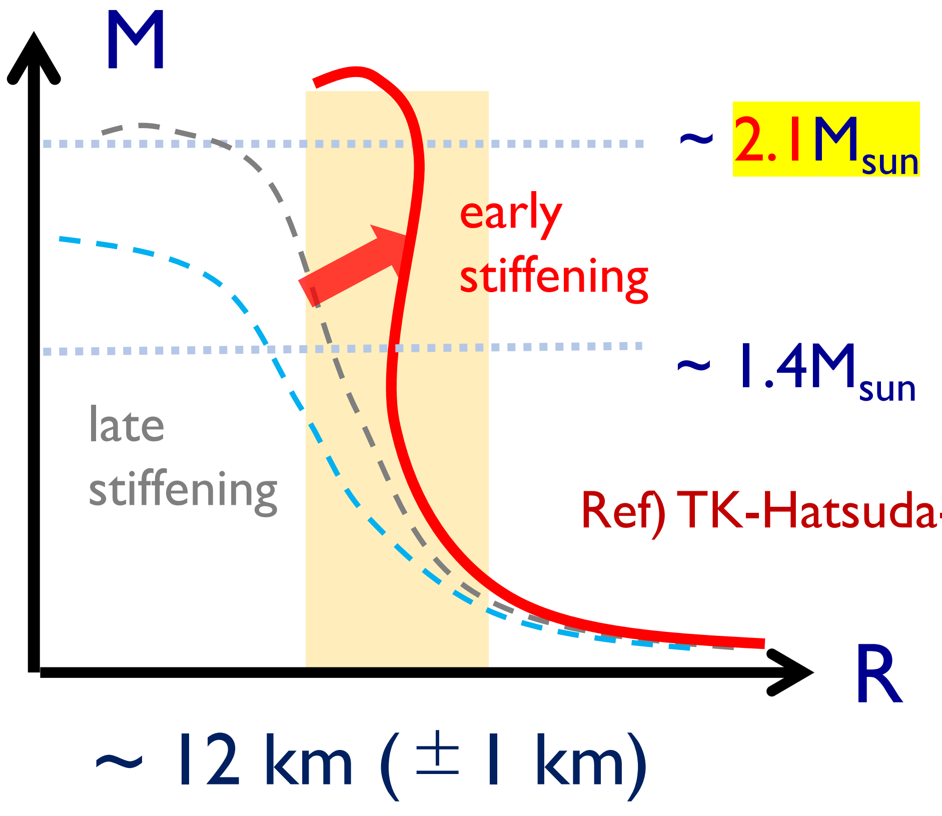
measure: **P vs ε**



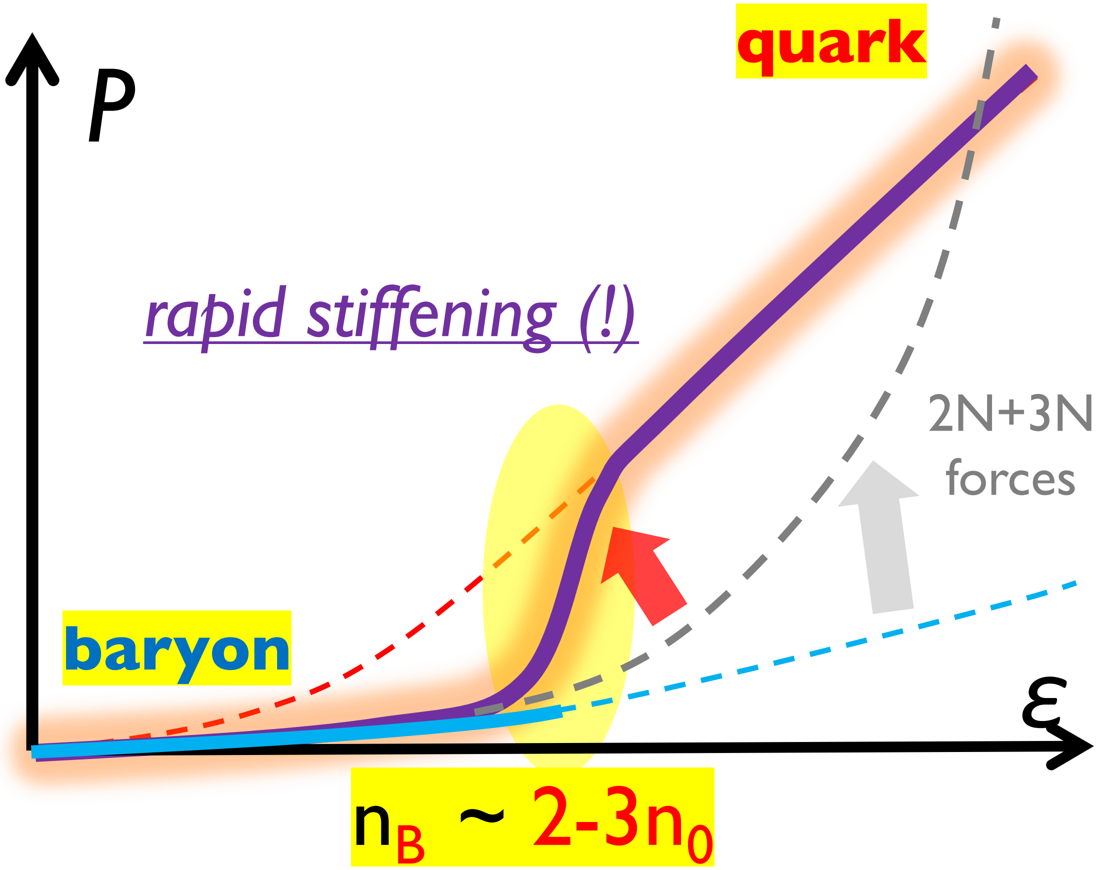
Implications from NS

NICER for 1.4 & 2.1 M_{sun} + **GW** + **nuclear** (< ~1.5n_0)

→ $R_{1.4} \sim R_{2.1}$ (!)



Ref) TK-Hatsuda-Baym ('21)

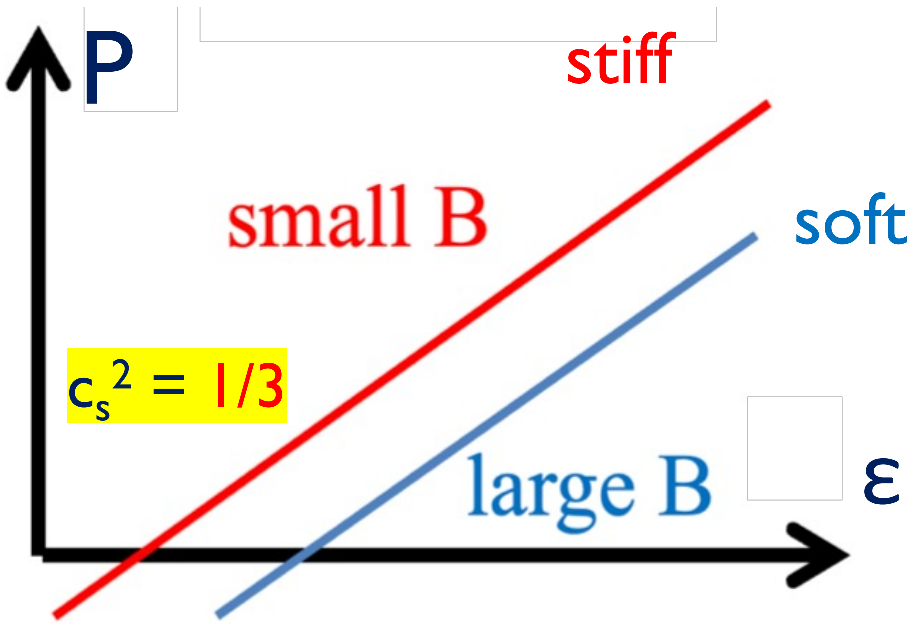


Quark EOS can be stiff

e.g.) free massless quarks

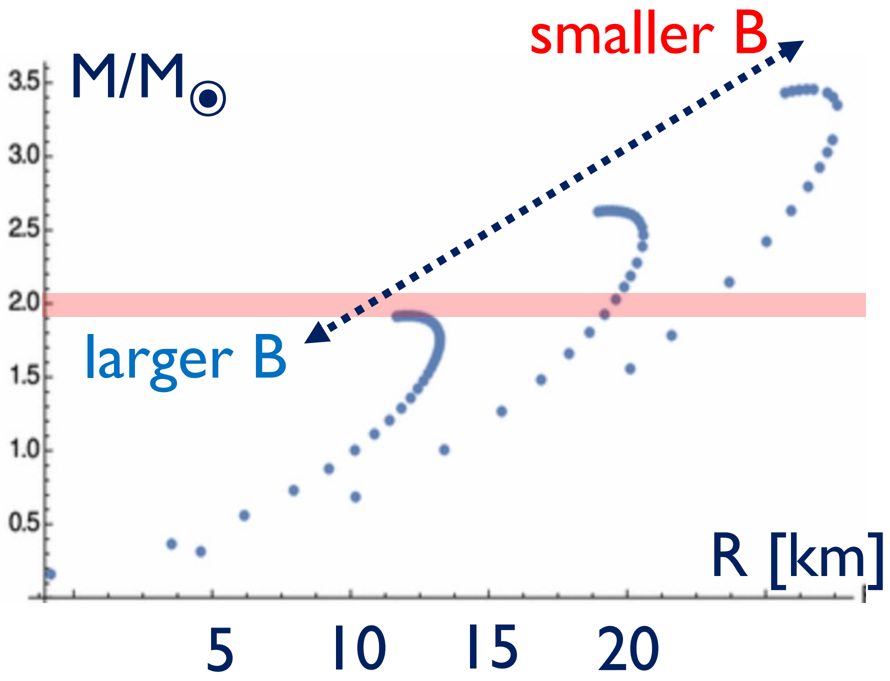
$$P = \frac{\varepsilon}{3} - B'$$

normalization



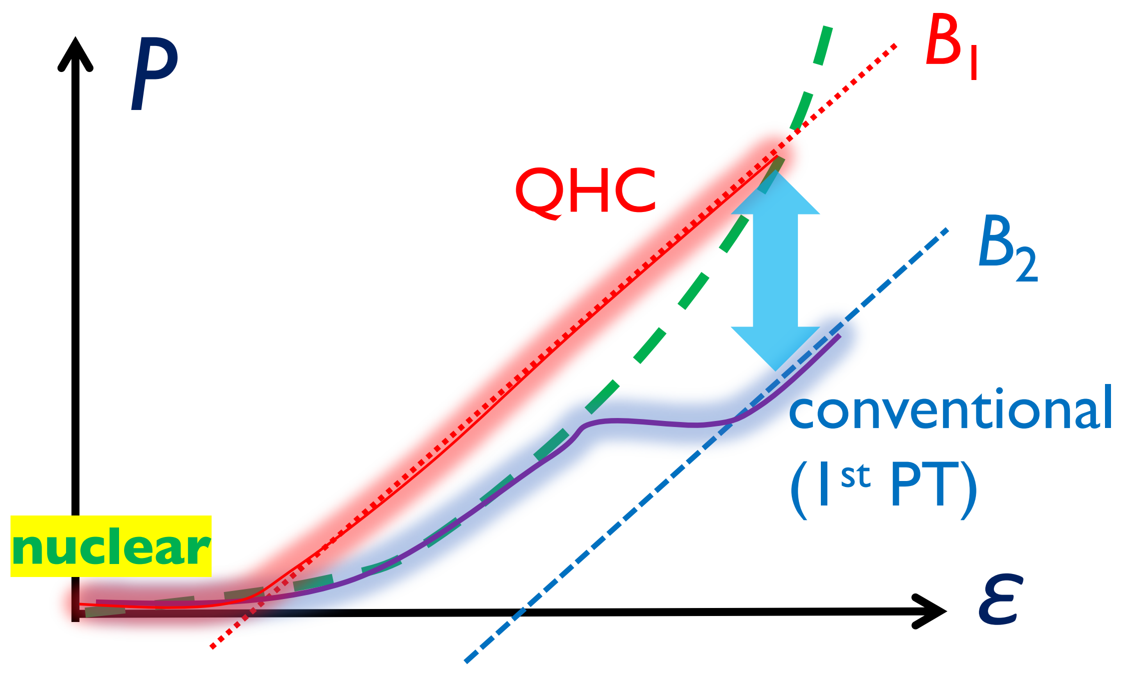
quark kin. pressure >> baryon kin. pressure

O(N_c) O(1/N_c)



Conformal c_s^2 = 1/3 can be stiff

hybrid EOS: normalization of EOS ??



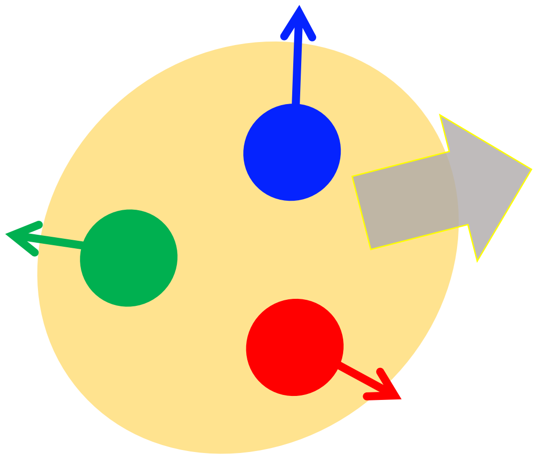
which B should be taken???

difficult to fix the relative normalization B of different models…

Need: *follow quark states from nuclear to quark matter*

Role of confinement

size ~1fm → p_q ~ 200-300 MeV



inner kin. energy $N_c\left(M_q + \frac{\langle \vec{p}^2 \rangle}{2M_q} + \cdots\right)$ ➡ M_B

center of mass energy $\frac{P_B^2}{2N_c M_q} \sim \frac{P_B^2}{2M_B}$ << M_B

➡ $\varepsilon \sim M_B n_B >> P$ soft baryonic matter

confinement allows quarks to appear in energy density

but NOT in pressure (!)

When do quarks begin to contribute to the thermodynamic pressure???

three topics to be discussed

1) how quarks contribute to the ***thermodynamic*** pressure already **at 2-3n_0**

quark saturation

2) how quark constraints mitigate ***hyperon softening problem***

statistical repulsion

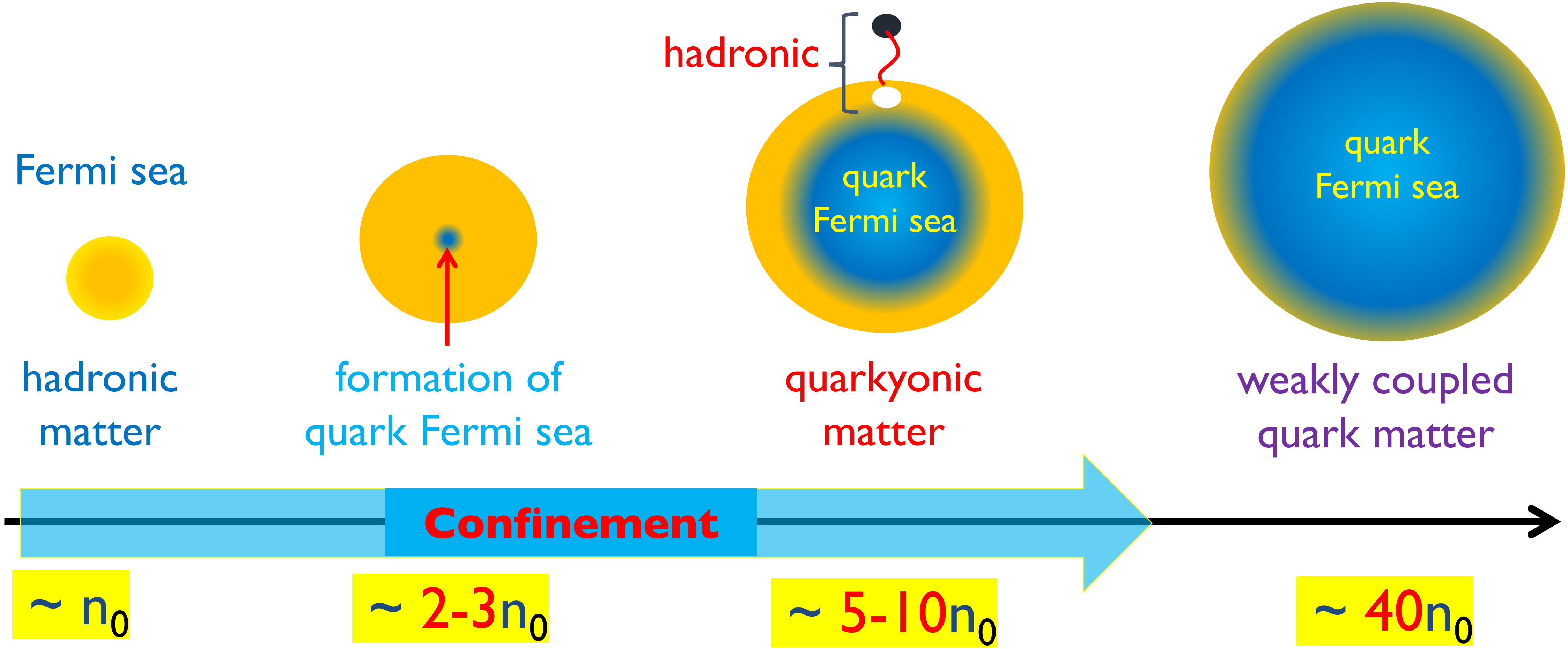
3) a **pragmatic** QFT framework to construct EOS

phase shift representation of EOS

A model of crossover

Quarkyonic matter := quark matter with confining gluons

McLerran-Pisarski ('07)



IdylliQ model

= **Ideal dual** Quarkyonic model

Describe **single** physics in **two** languages (baryon/quark)

Powerful in transient regimes (2-5n_0)

Sum rules for occupation probabilities

cf) [TK '21]

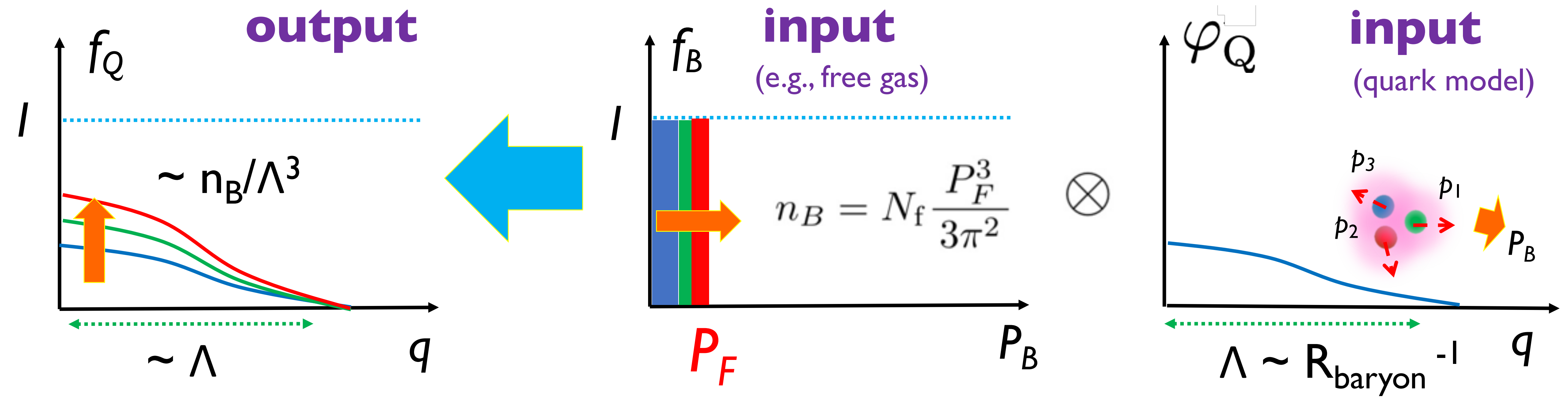
occupation **probability** of **quark** state with p

occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

$$f_Q(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B)\, \varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_c)$$

e.g.) in **ideal** baryonic matter



An ideal model

[Fujimoto-TK-McLerran, PRL'24]

1) neglect interactions *except* confining forces

e.g.) 2-flavor hamiltonian: $$\varepsilon_{\rm B}[f_{\rm B}] = 4\int_k E_{\rm B}(k) f_{\rm B}(k)$$

2) keep using the same $\varphi_{\rm Q}$ (quarkyonic)

3) use a special quark distribution → sum rules analytically *invertible*

$$\varphi_{\rm 3d}(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3}\frac{e^{-q/\Lambda}}{q/\Lambda} \qquad \hat{L} = -\boldsymbol{\nabla}^2 + \frac{1}{\Lambda^2} \qquad \hat{L}[\varphi(\boldsymbol{p}-\boldsymbol{q})] = \frac{(2\pi)^3}{\Lambda^2}\delta(\boldsymbol{p}-\boldsymbol{q})$$

nontrivial output

$$f_{\rm Q}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\rm B}(\boldsymbol{P}_B)\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_{\rm c})$$

↑ *natural at* ***low*** *density*

⇄

nontrivial output

$$f_{\rm B}(N_{\rm c}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\rm c}^3}\hat{L}[f_{\rm Q}(\boldsymbol{q})]$$

↑ *natural at* ***high*** *density*

An **ideal** model

[Fujimoto-TK-McLerran, PRL'24]

1) neglect interactions *except* confining forces

e.g.) 2-flavor hamiltonian: $\varepsilon_{\rm B}[f_{\rm B}] = 4\int_k E_{\rm B}(k) f_{\rm B}(k)$

2) keep using the ... (quarkyonic)

3)

To Do: solve variational problem of $f_B(k)$

with *quark substructure constraints* !

nontrivial output

$$f_{\rm Q}(\mathbf{q}) = \int_{\boldsymbol{P}_B} f_{\rm B}(\boldsymbol{P}_B)\varphi_Q^B(\mathbf{q} - \boldsymbol{P}_B/N_{\rm c})$$

natural at ***low*** *density*

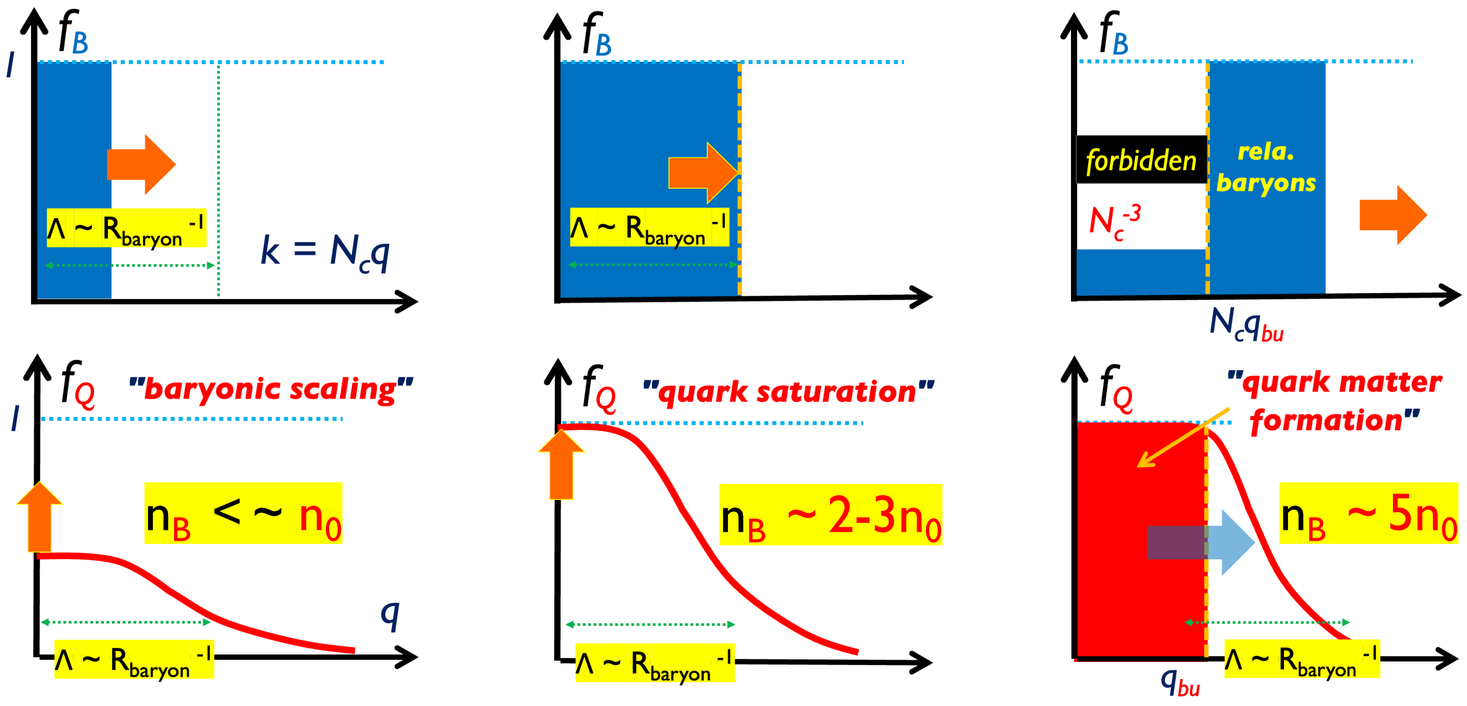
nontrivial output

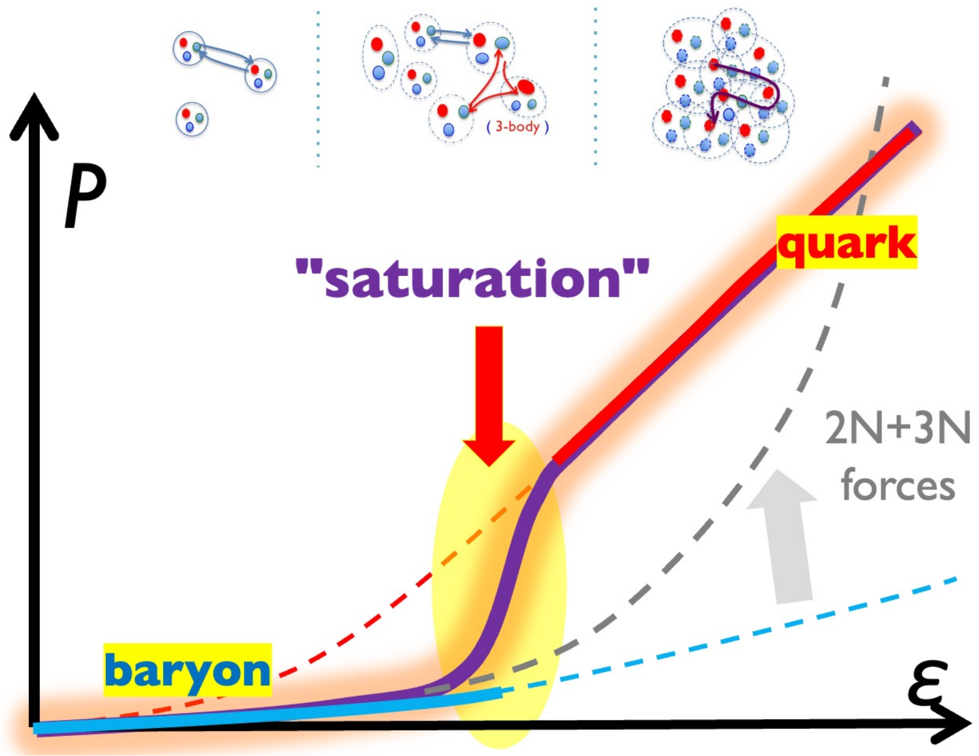
$$f_{\rm B}(N_{\rm c}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\rm c}^3}\hat{L}[f_{\rm Q}(\boldsymbol{q})]$$

natural at ***high*** *density*

Quark saturation & inevitable stiffening

$$f_{\rm Q}(\boldsymbol{q};n_{\rm B}) = \int_{\boldsymbol{P}_{\rm B}} f_{\rm B}(\boldsymbol{P}_{\rm B};n_{\rm B})\varphi_{\rm Q}^{\rm B}(\boldsymbol{q};\boldsymbol{P}_{\rm B})$$





n_B ~ 2-3n_0

low p baryons quenched

dual

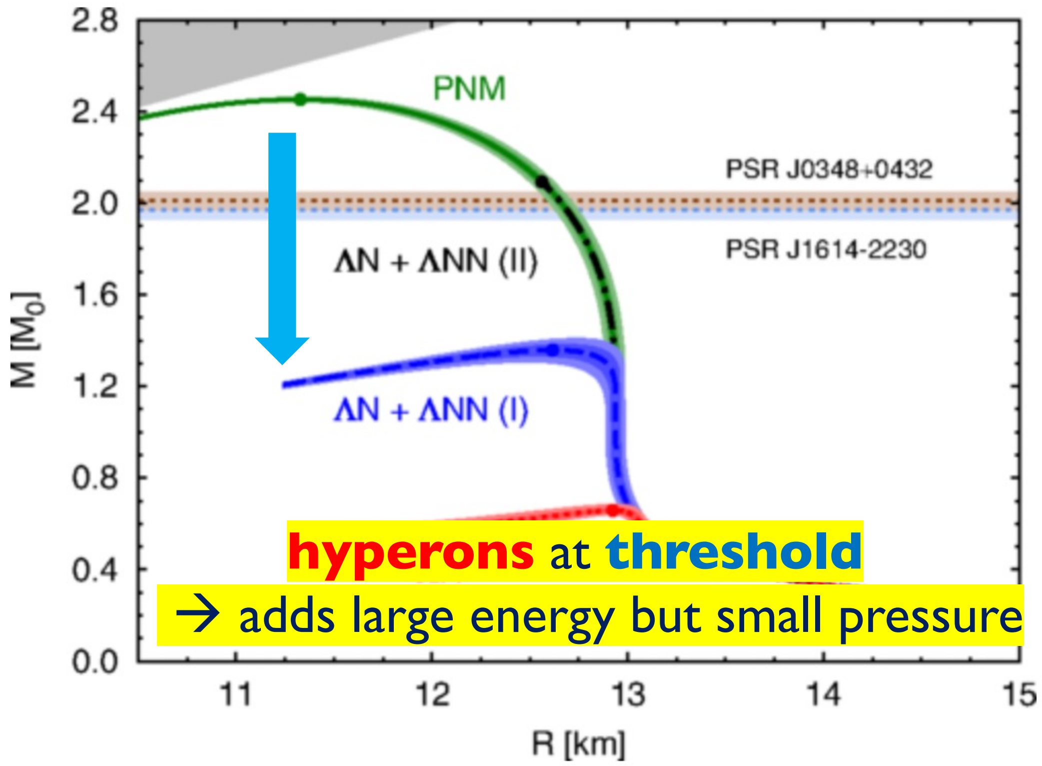
quark matter scaling

stiff EOS

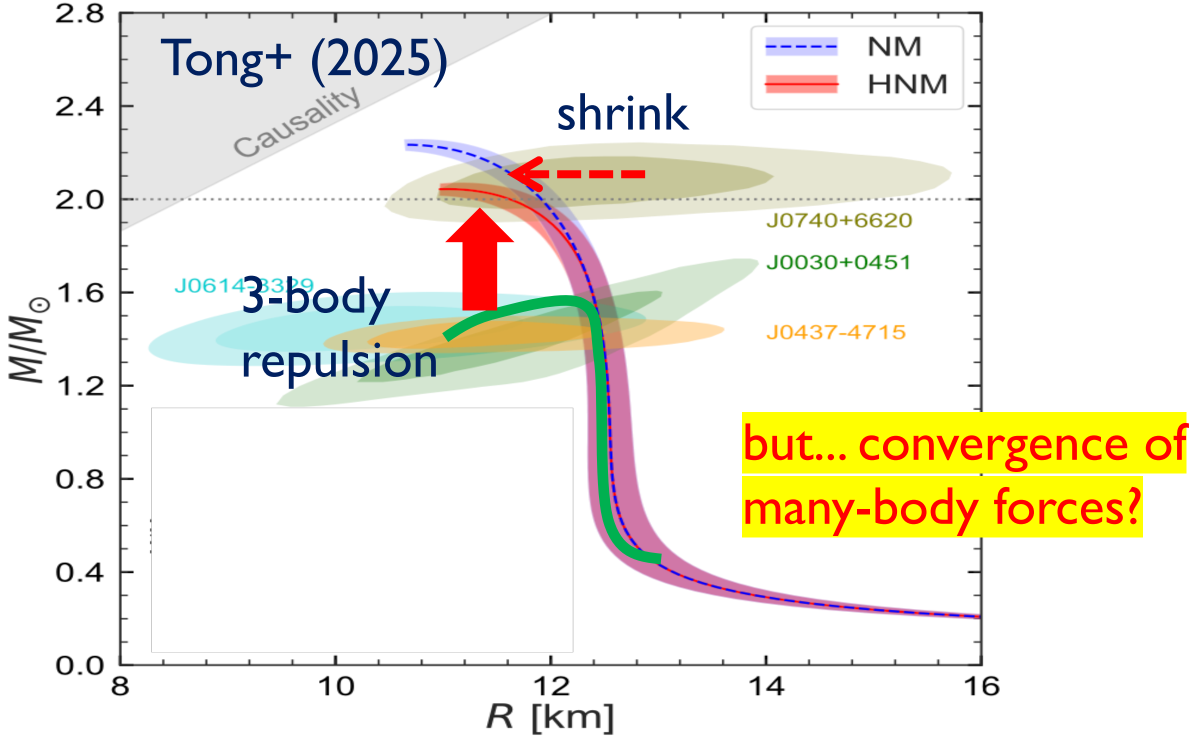
[see also McLerran,-Reddy PRL'19]

"*inevitable*" *stiffening*

Hyperon Puzzle ?



state-of-art, with empirical YN & YNN forces



supplemental idea:

quark saturation →

1) statistical repulsion (Pauli blocking)

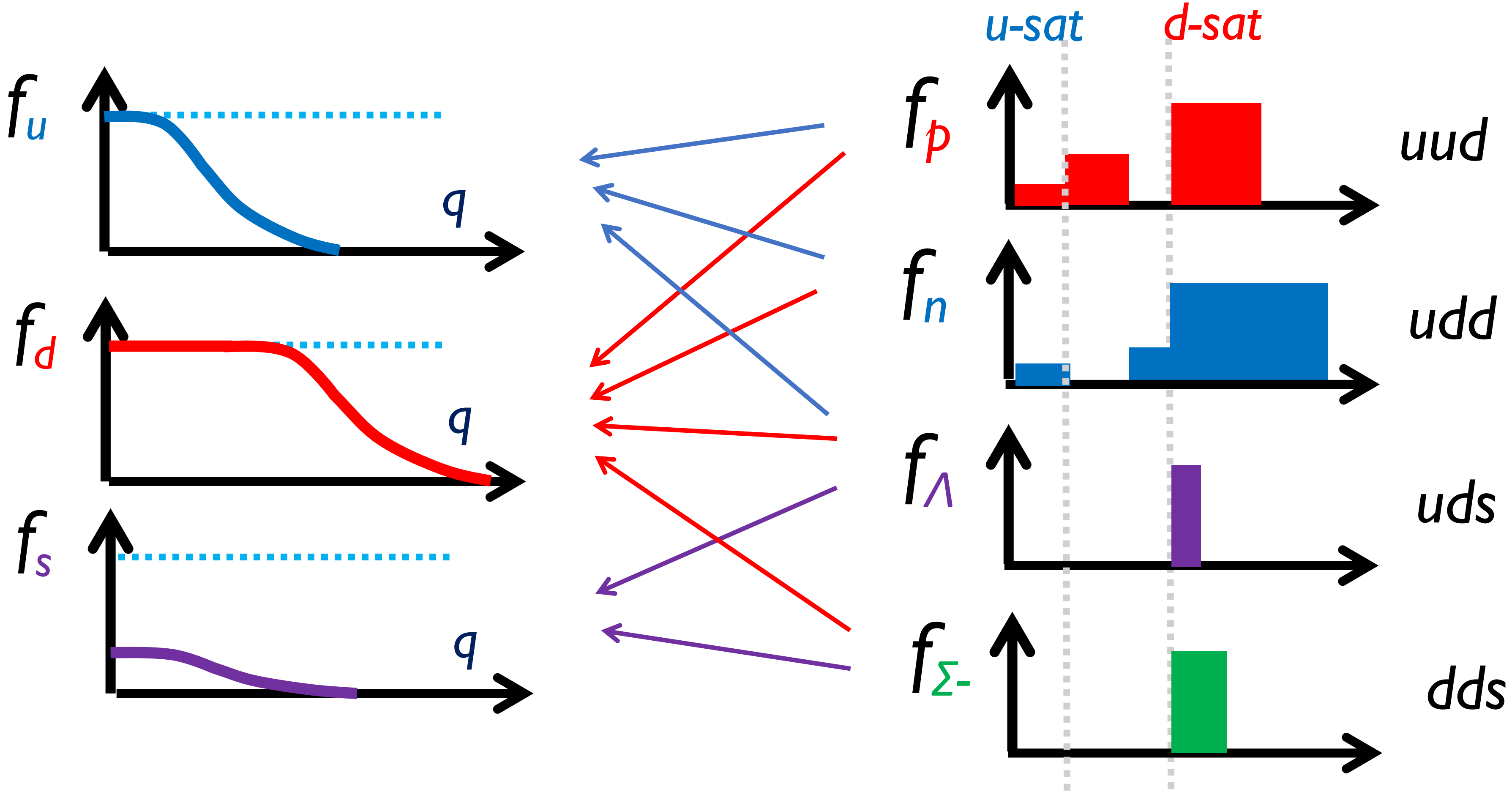
2) stronger repulsion at higher density

3) no double counting of quarks

Multi-flavor extension

$$f_{\mathrm{Q}}(\boldsymbol{q}) = \sum_{B=p,n,\Sigma,\cdots} N_{\mathrm{Q}}^{\mathrm{B}} \int_{\boldsymbol{k}} f_{\mathrm{B}}(\boldsymbol{k}) \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right)$$

Q = u, d, s



and so on...

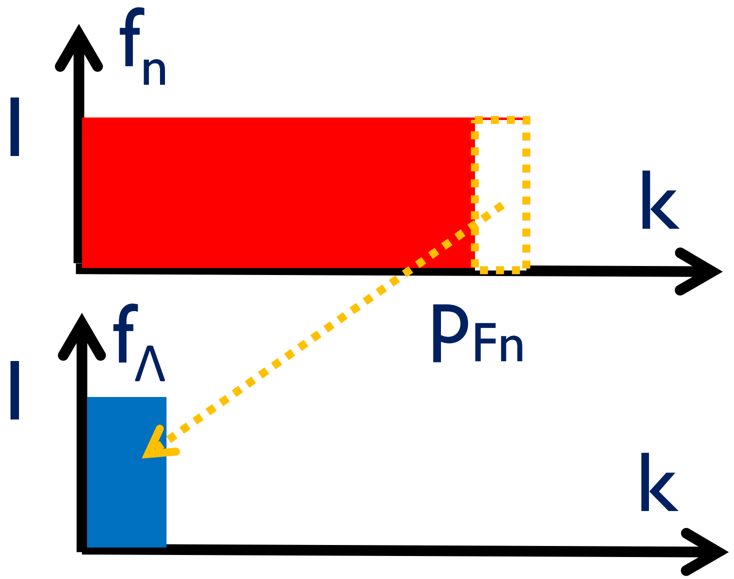
saturation constraints

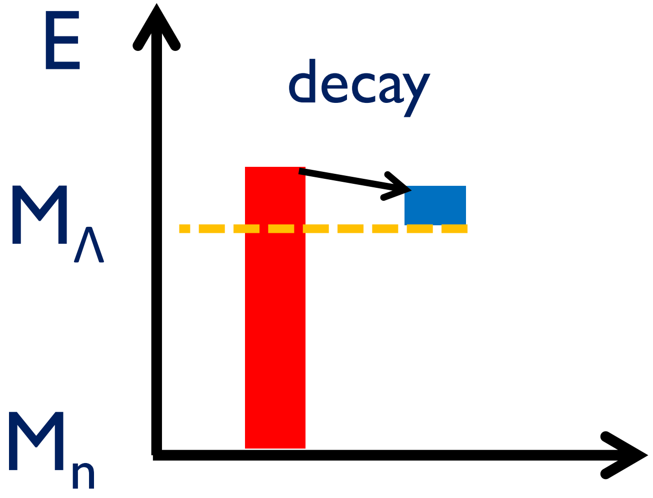
flavor correlations

n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]

· **conventional**





at $n_B \sim$ 2-3n_0

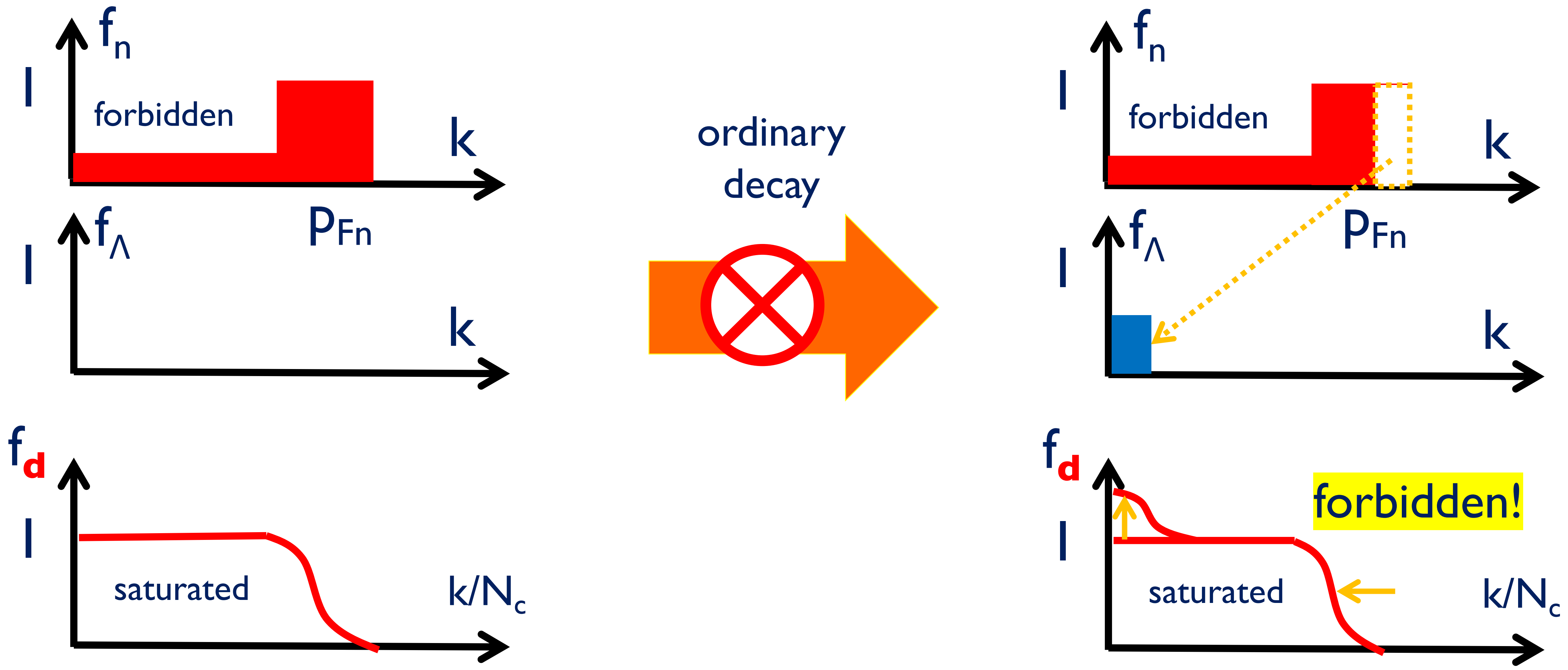
neutrons *at high momenta* → *non-relativistic* Λ

➡ ε increases much but pressure does not (**softening**)

n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]

- **with d-quark saturation**

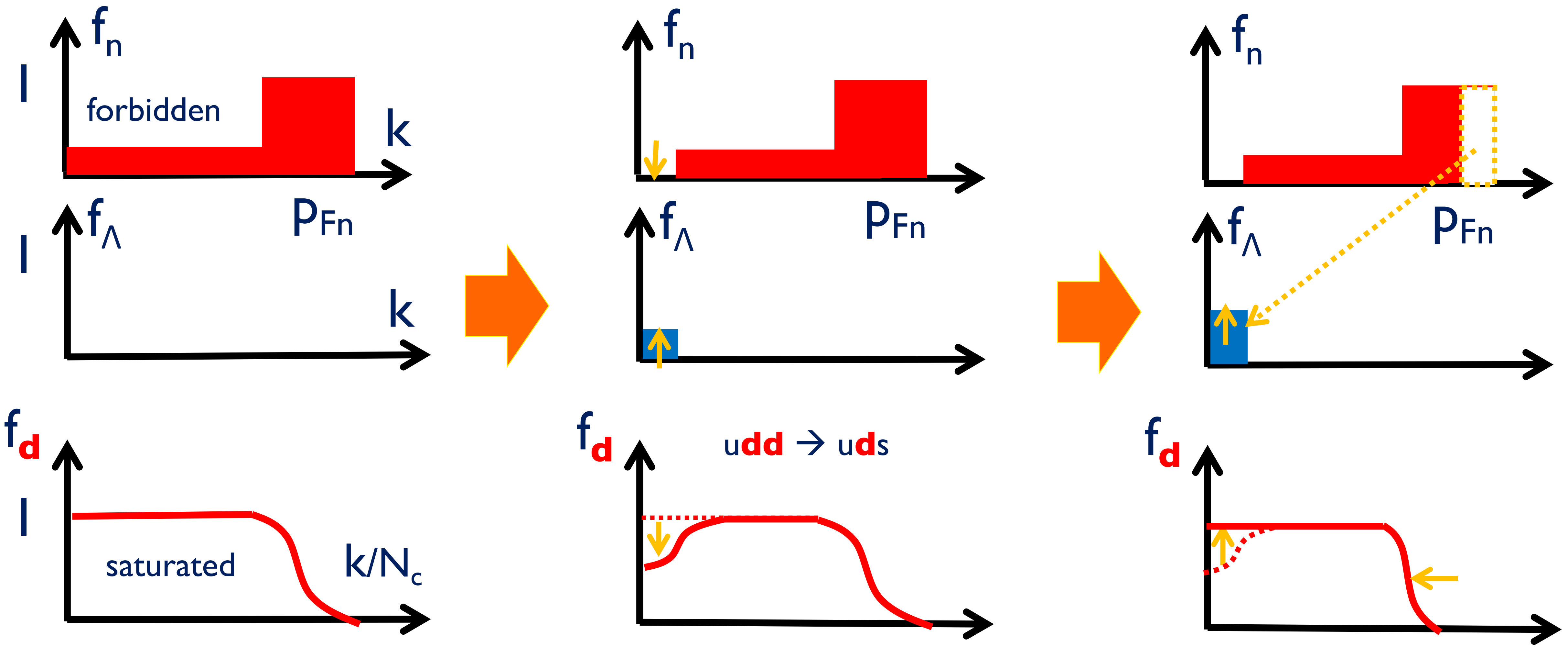


n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]

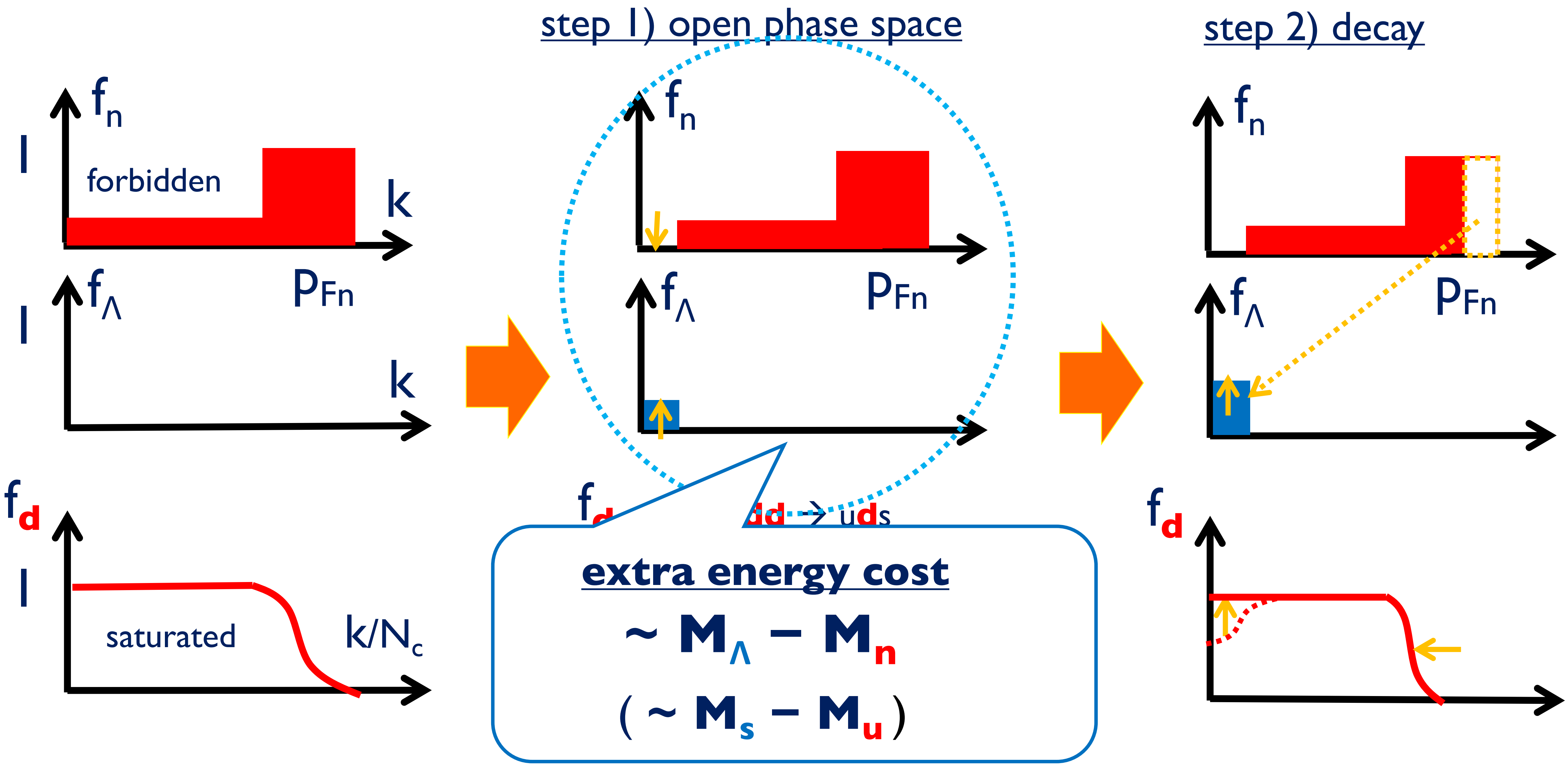
step 1) open phase space

step 2) decay



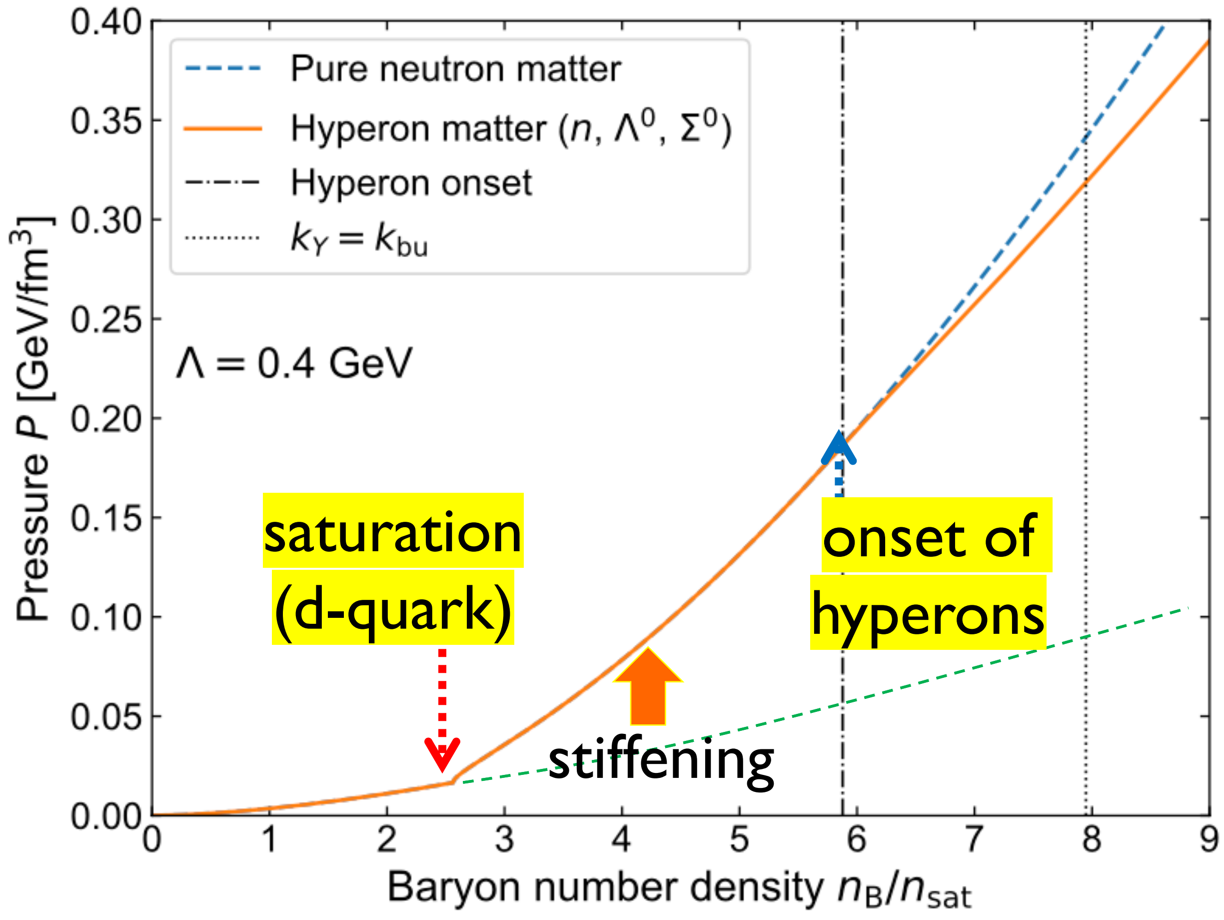
n-Λ_0 matter

[Fujimoto-TK-McLerran, '24]



EOS: n-Λ matter

[Fujimoto-TK-McLerran, ‘24]



see also
Nagatsuka-TK '26
for 2-color, 2+**2** flavor QCD

$n_B^{\Lambda\text{-onset}} \sim 2n_0 \rightarrow$ 5-6n_0

conventional with saturation

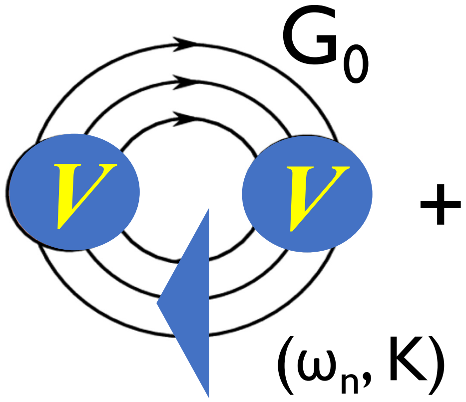
Phase shift representation of EOS

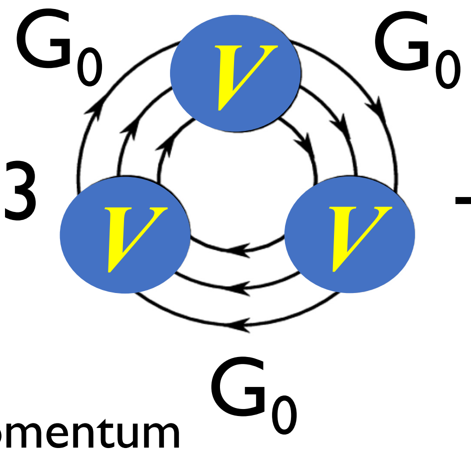
momentum shell in non-confining theories

Tajima-Iida-TK-Liang, PRL (2025)

work in progress with Jinno, Nagatsuka

Thermodynamics of composite particles

$\Omega_{3\text{-body}}$ = 1/2 

+ 1/3 

+ ....

total frequency & momentum

$$-V = G^{-1} - G_0^{-1}$$

full free

$$= T \sum_{\omega_n, \mathbf{K}} \mathrm{tr}_N \left[\mathrm{Ln}(G/G_0) - G_0 G^{-1} \right] + \text{const.}$$

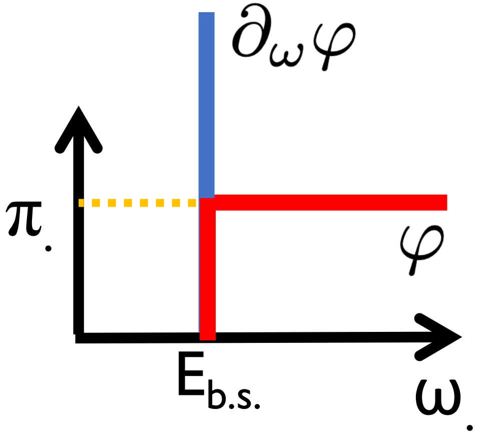
$$G/G_0 = |G/G_0| \mathrm{e}^{\mathrm{i}\varphi}$$ phase "shift"

$$= -T \sum_{\mathbf{K}} \int \frac{\mathrm{d}\omega}{\pi} \ln\left(1 + \mathrm{e}^{-\beta\omega}\right) \frac{\partial}{\partial\omega} \mathrm{tr}_N \left[\varphi - |G_0/G| \sin\varphi\right]$$

Dashen, Ma, and Bernstein, Phys. Rev. 187, 345–370 (1969) Pok Man Lo [arXiv: 1707.04490, review]

e.g. Hadron Resonance Gas

e.g.) stable bound particles $\varphi = \pi\Theta\left(\omega - E_{\text{b.s.}}(K)\right)$



$$\Omega_{\text{N-body}} = -T\sum_{\mathbf{K}}\int \frac{d\omega}{\pi}\ln\left(1+e^{-\beta\omega}\right)\boxed{\frac{\partial}{\partial\omega}\mathrm{tr}_N\left[\varphi - |G_0/G|\sin\varphi\right]}$$

$$\square = \pi\delta\left(\omega - E_{\text{b.s.}}\right)\left[1 - |G_0/G|\cos\varphi\right] - \sin\varphi\frac{\partial|G_0/G|}{\partial\omega}$$

→ 0 (G→∞ at pole) = 0

➔ HRG model: $\Omega_{\text{N-body}} = -T\sum_{\mathbf{K}}\ln\left(1+e^{-\beta E_{\text{b.s.}}}\right)$

Constraints on phase shifts

num. of states:

(int. cannot modify)

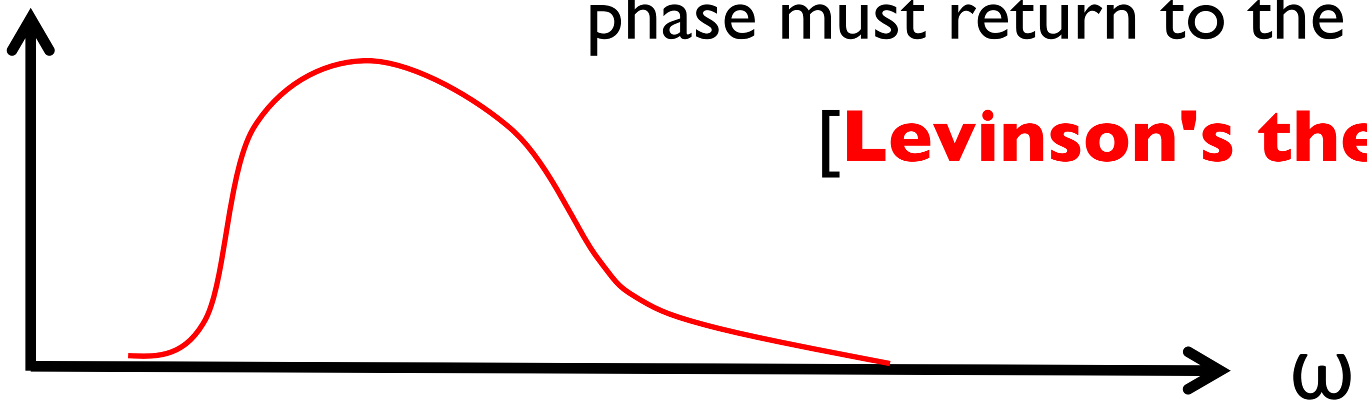
$$\int_\omega \mathrm{ImTr} G = \int_\omega \mathrm{ImTr} G_0$$

$$G = \frac{1}{\omega - H + \mathrm{i}\delta}$$

$$G_0 = \frac{1}{\omega - H_0 + \mathrm{i}\delta}$$

$$G = G \partial_\omega G^{-1} = \partial_\omega \ln G^{-1}$$

→ $$0 = -\int_\omega \mathrm{ImTr} \partial_\omega \ln\left(G/G_0\right) = \int_\omega \partial_\omega \mathrm{Tr}\varphi = \mathrm{Tr}\varphi(\infty) - \mathrm{Tr}\varphi(-\infty)$$

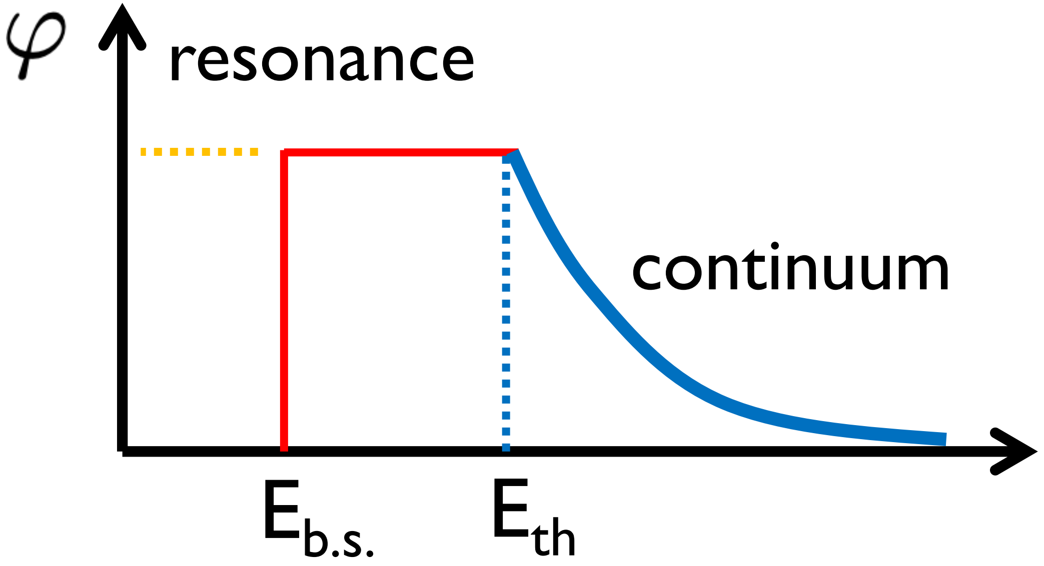


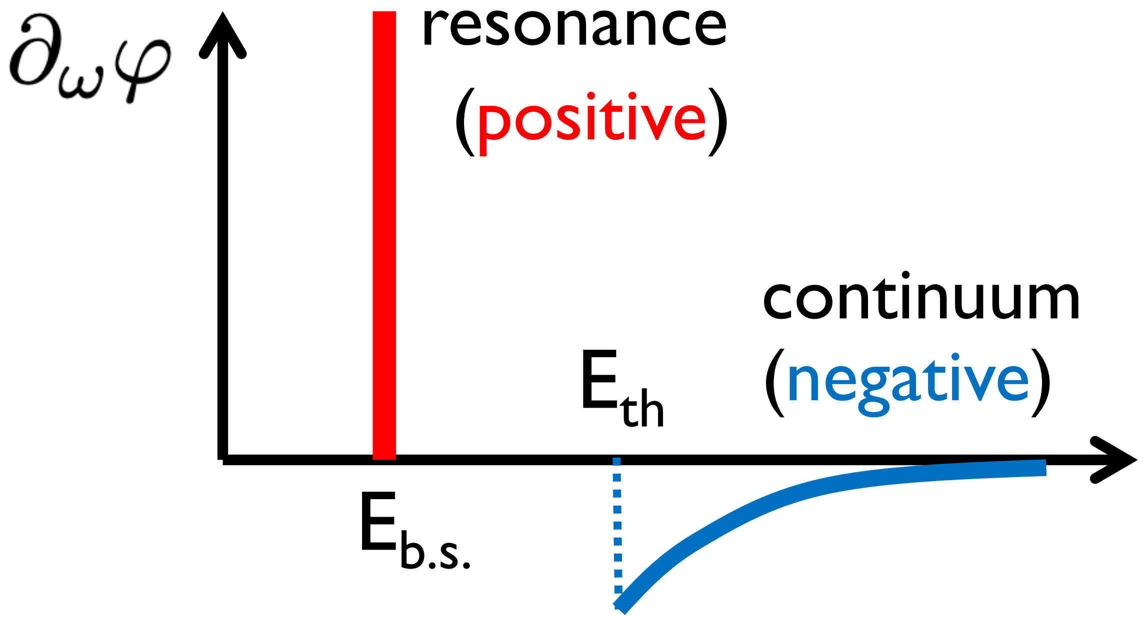
phase must return to the original value

[**Levinson's theorem**]

e.g.) Hadron Resonance Gas with the decay

$$\Omega_{\text{N-body}} = -T \sum_{\mathbf{K}} \int \frac{d\omega}{\pi} \ln\left(1 + e^{-\beta\omega}\right) \boxed{\frac{\partial}{\partial\omega} \mathrm{tr}_N \left[\varphi - |G_0/G| \sin\varphi\right]}$$





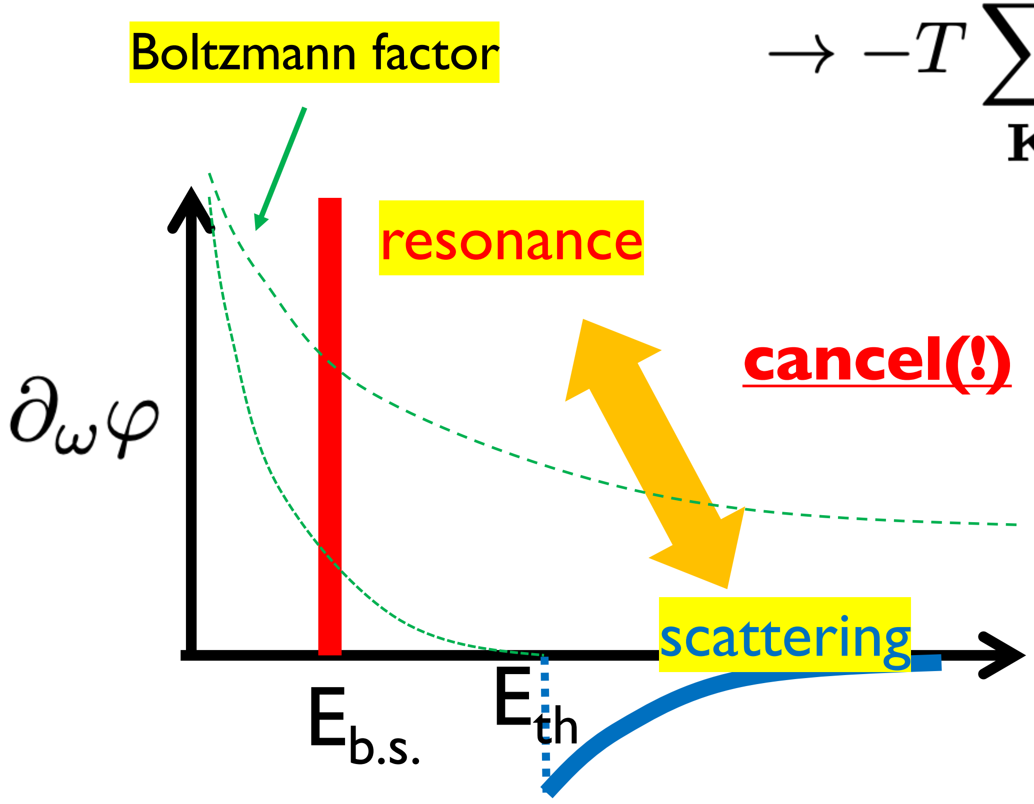
resonance and continuum contributions tend to cancel

High temperature limit

$$\Omega_{\text{N-body}} = -T\sum_{\mathbf{K}}\int\frac{\mathrm{d}\omega}{\pi}\ln\left(1+\mathrm{e}^{-\beta\omega}\right)\boxed{\frac{\partial}{\partial\omega}\mathrm{tr}_N\left[\varphi-|G_0/G|\sin\varphi\right]}$$

$$\to -T\sum_{\mathbf{K}}\ln 2\otimes\int\frac{\mathrm{d}\omega}{\pi}\boxed{\frac{\partial}{\partial\omega}\mathrm{tr}_N\left[\varphi-|G_0/G|\sin\varphi\right]}$$

$$\propto\ \varphi(\omega=\infty)-\varphi(\omega=0)$$ → 0



$$\Omega = \Omega_{q,g}+\Omega_{\text{meson}}+\Omega_{\text{baryon}}+\cdots$$

$$\to\ \Omega_{q,g}$$ at large T

saturated by elementary particles

At finite density: model study

$$H = \sum_{\mathbf{p}} (E_Q(\mathbf{p}) - \mu_q) c^\dagger_{\mathbf{p}} c_{\mathbf{p}} + V \sum_{\mathbf{K}} B^\dagger(\mathbf{K}) B(\mathbf{K})$$

(B ~ qqq operator)

3-to-3 vertex yields a 3-body bound state

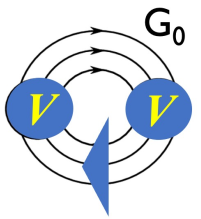
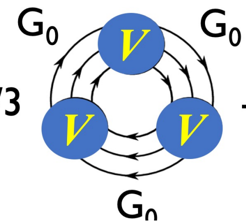
1-body 3-body

$$\Omega = \Omega_Q + \Omega_B$$

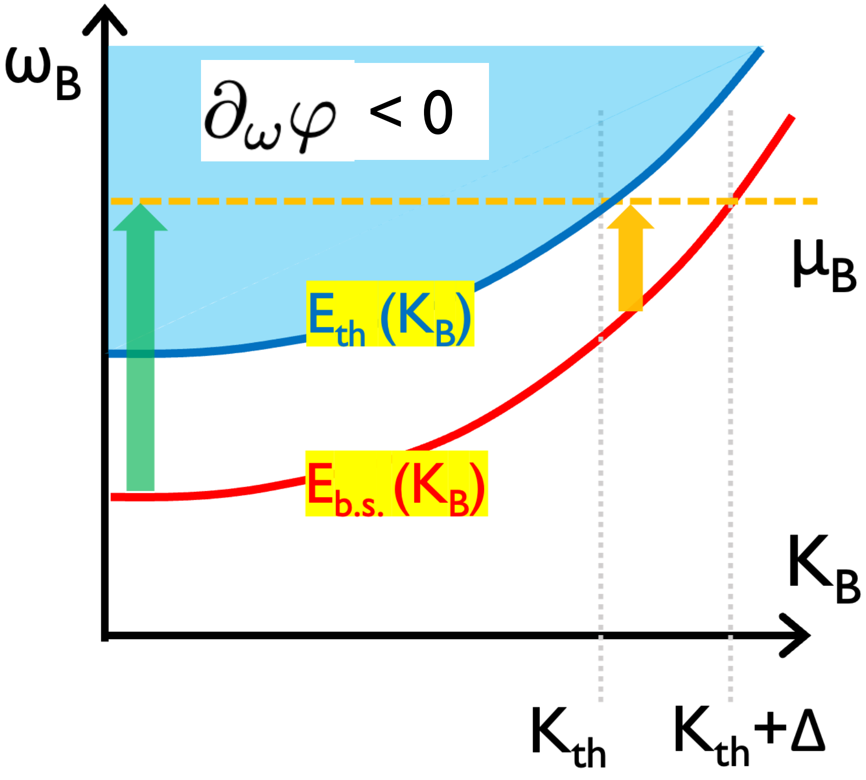
$$n^Q_B \equiv -\frac{\partial \Omega_Q}{\partial \mu_B} \equiv \sum_{\mathbf{p}} f_Q(\mathbf{p})$$

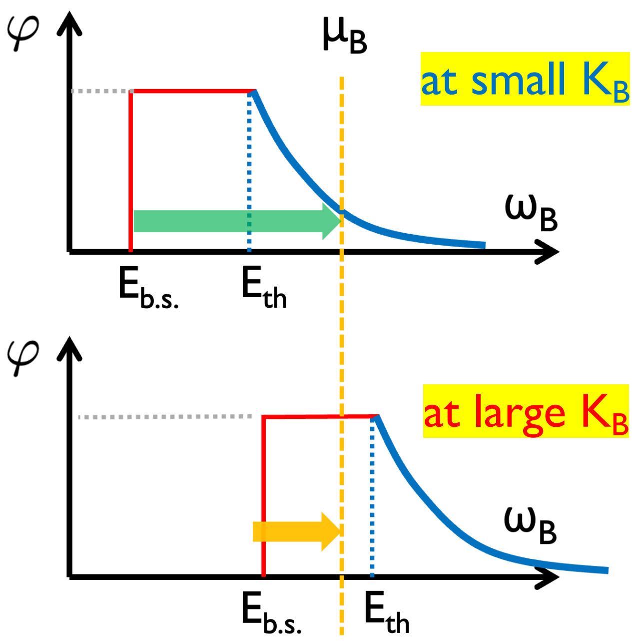
$$n^B_B \equiv -\frac{\partial \Omega_B}{\partial \mu_B} \equiv \sum_{\mathbf{K}} f_B(\mathbf{K})$$

Baryonic contributions

$\Omega_{3\text{-body}}$ = 1/2  + 1/3  + ...

$$n_{\mathrm{B}}^{B} = \sum_{\mathbf{K}} f_{\mathrm{B}}(\mathbf{K}) = \sum_{\mathbf{K}} \int \frac{\mathrm{d}\omega_B}{\pi} \frac{1}{\mathrm{e}^{\beta(\omega_B-\mu_B)}+1} \frac{\partial}{\partial\omega} \mathrm{tr}_N \left[\varphi - |G_0/G| \sin\varphi\right]$$



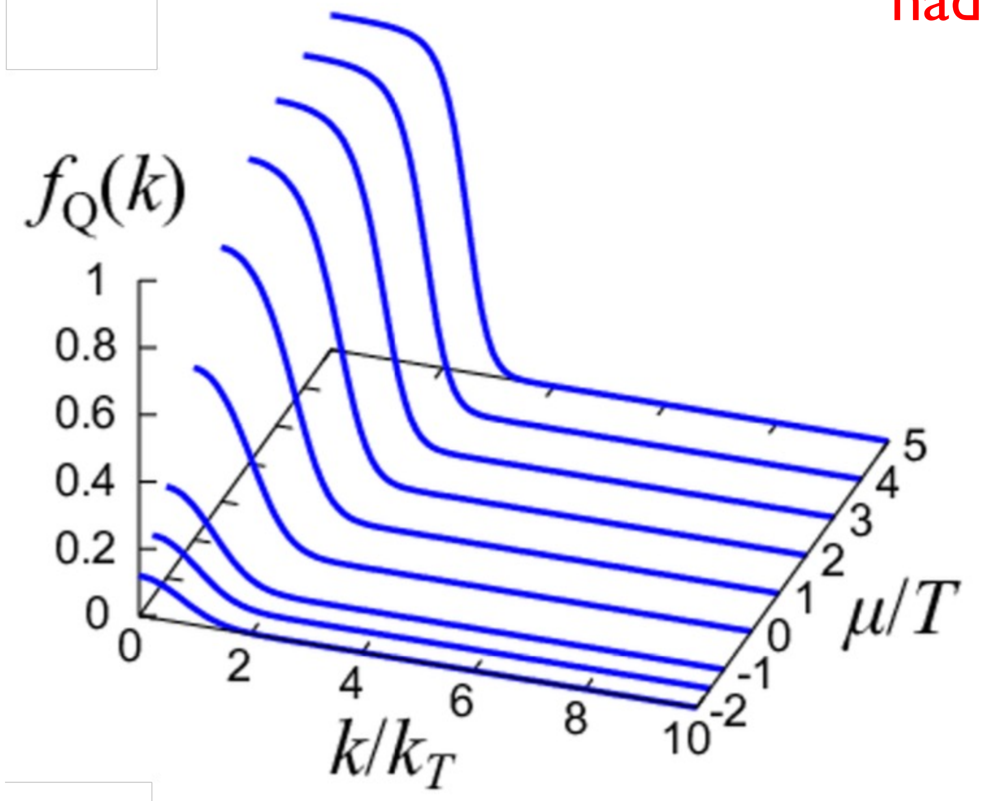


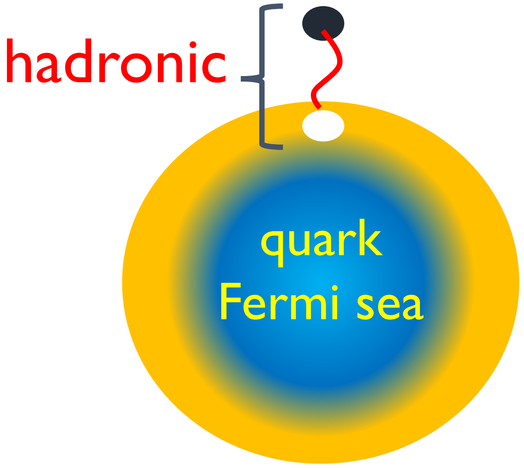
both bound & scattering states are picked up; **they cancel**

pick up only bound states
➔ **baryons survive**

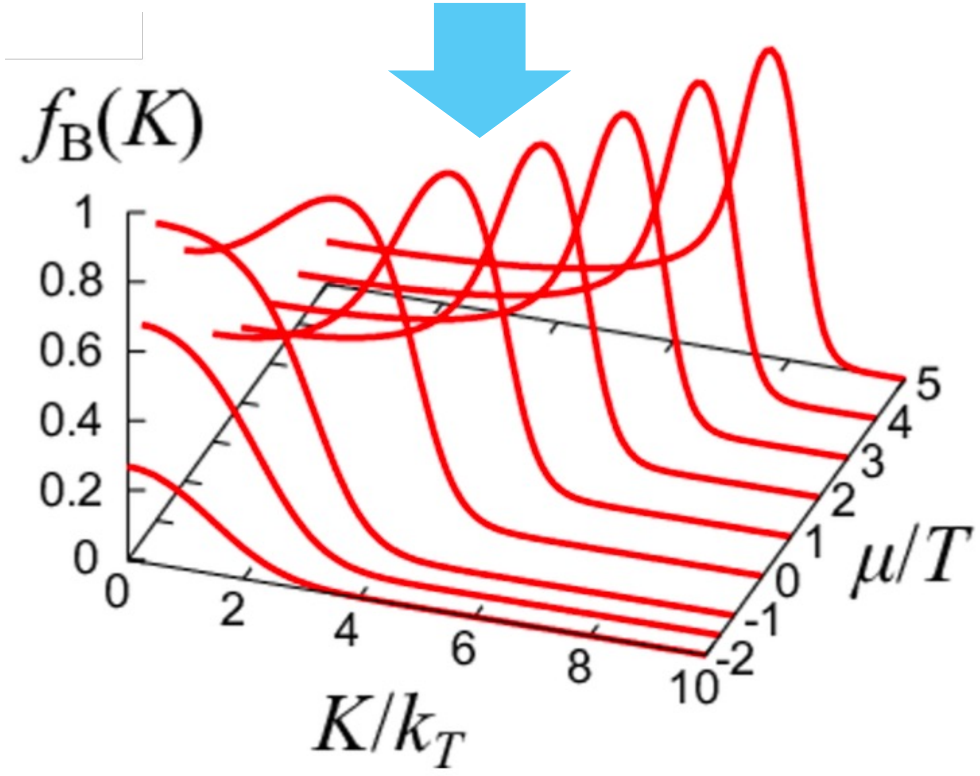
quark & baryon distributions

quark Fermi sea





suppression of bulk in f_B



quark Fermi sea + baryonic Fermi surface

Summary & Outlook

- Rapid stiffening → signature for the onset of quark(yonic) matter

- key observations for the **bulk**:

  1) **soft** baryonic matter is **forced** to transform into quark matter

  2) quark matter with $c_s^2 \sim 1/3$ is **stiff**; a good **baseline**

  (one should still consider the physics near the Fermi surface to explain NS)

- **Not discussed in this talk**: physics near the Fermi surface:

  - finite T extension of IdylliQ model [Bluhm-Fujimoto-Nahrgang, '26]
  - pairings; QHC at excitation levels

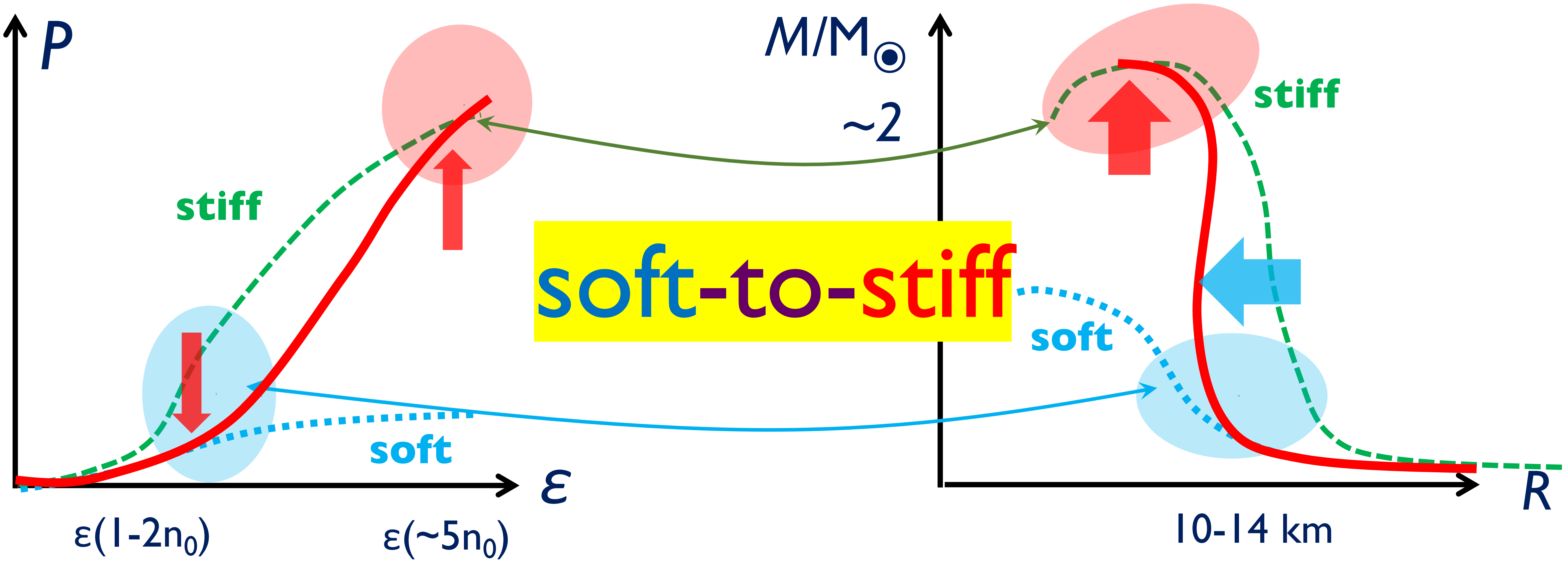
... and lot more

Back Up

EoS stiffness & M-R

Ref) Lattimer & Prakash (2001)

measure: **P vs ε**

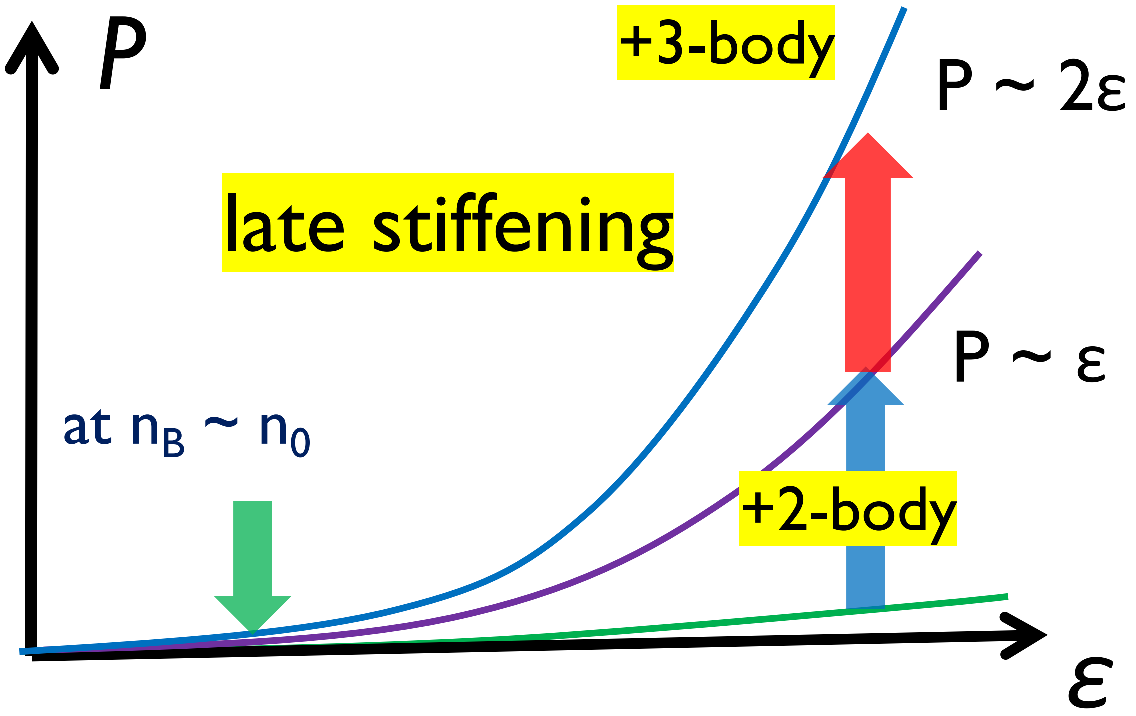


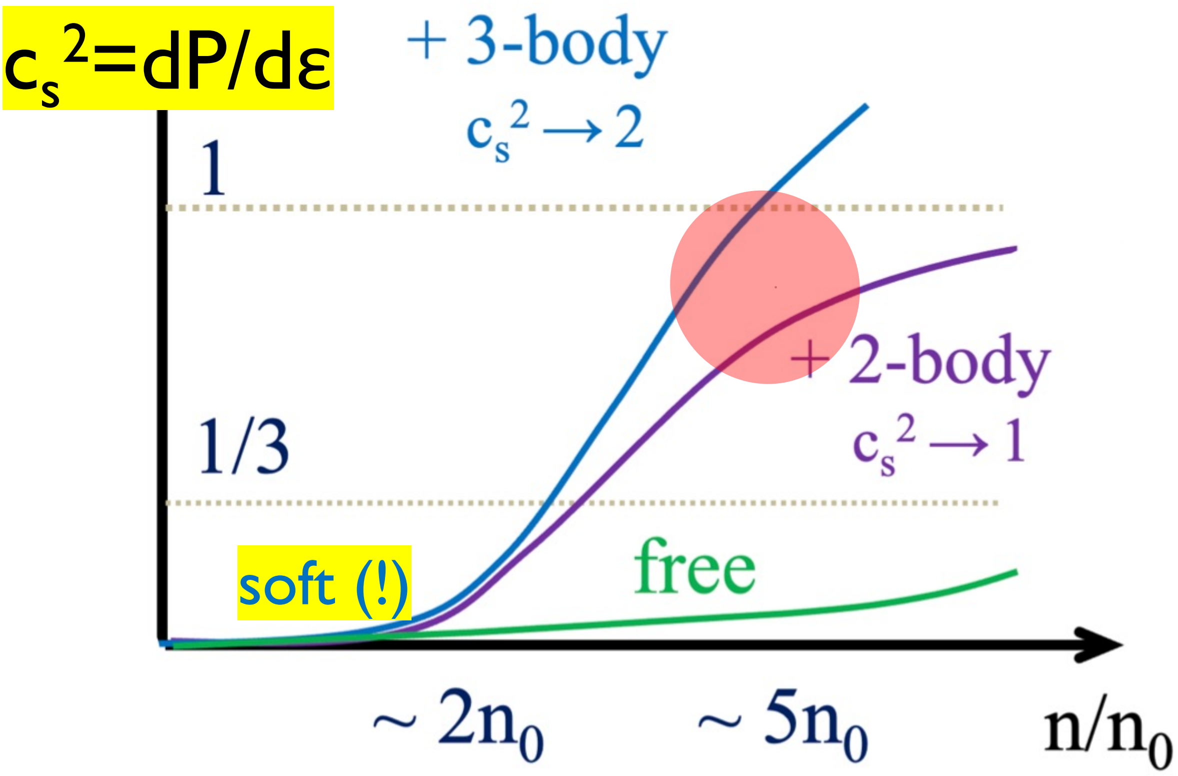
Remark 1: stiffening of nuclear EOS is slow

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

$$\varepsilon(n_B) = m_N n_B + a\frac{n_B^{5/3}}{m_N} + b n_B^{\alpha} \quad \Rightarrow \quad P = \frac{2}{3} a \frac{n_B^{5/3}}{m_N} + b(\alpha - 1) n_B^{\alpha}$$

soft (!)

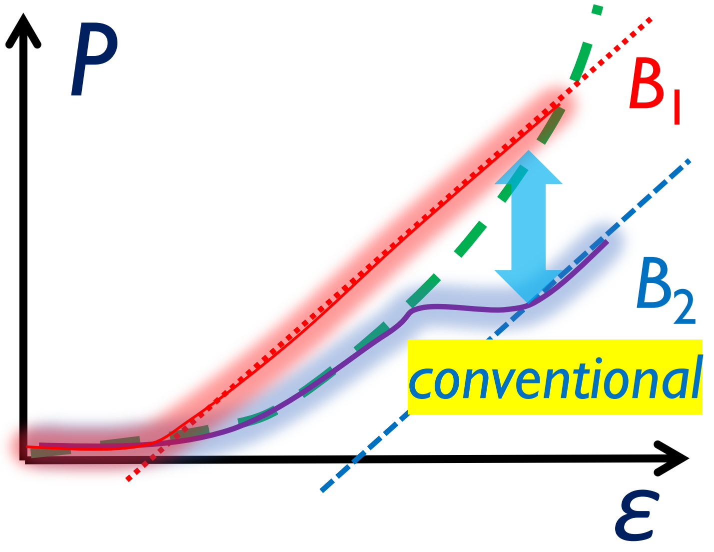




Remark: normalization of EOS is important

conventional hybrid EOS

prepare hadronic & quark EOS *as independent*,
and then choose the energetically favorable one



Difficulties:

fixing (relative) *normalization* for two independent EOSs is difficult

validity domain of *pure* hadronic and quark EOSs are assumed to overlap

transformation of hadronic matter to quark matter is a priori excluded

Need: *follow quark states from nuclear to quark matter*

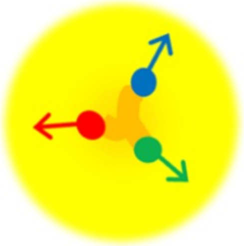
Quark matter can be stiff

For ideal gas

NR kinetic energy: quarks $\varepsilon_Q^{\text{kin}} \sim N_c \frac{n_B^{5/3}}{M_q}$ >> baryons $\varepsilon_B^{\text{kin}} \sim \frac{n_B^{5/3}}{N_c M_q}$

$$\Rightarrow \quad P_Q^{\text{ideal}} \sim N_c^2 \times P_B^{\text{ideal}}$$

already stiff at LO (i.e., without interactions)!



But in reality, confinement is important:

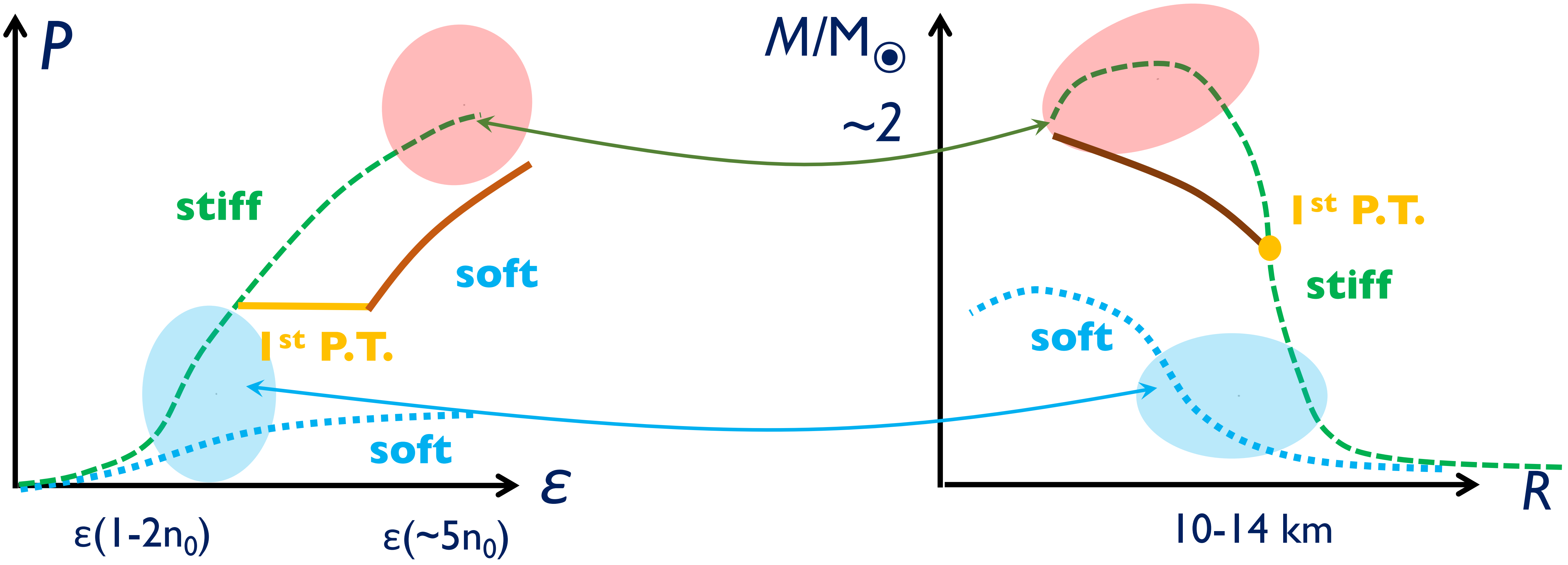
→ confinement allows quarks to make O(Nc) effects on the energy density, but little O(1/Nc) effects on the *thermodynamic* pressure !

When do quarks begin to contribute to the thermodynamic pressure ?

EoS stiffness & M-R Ref) Lattimer & Prakash (2001)

stiff-to-soft EOS

e.g.) hadron-to-quark phase transition (1^{st} order)



EoS stiffness & M-R

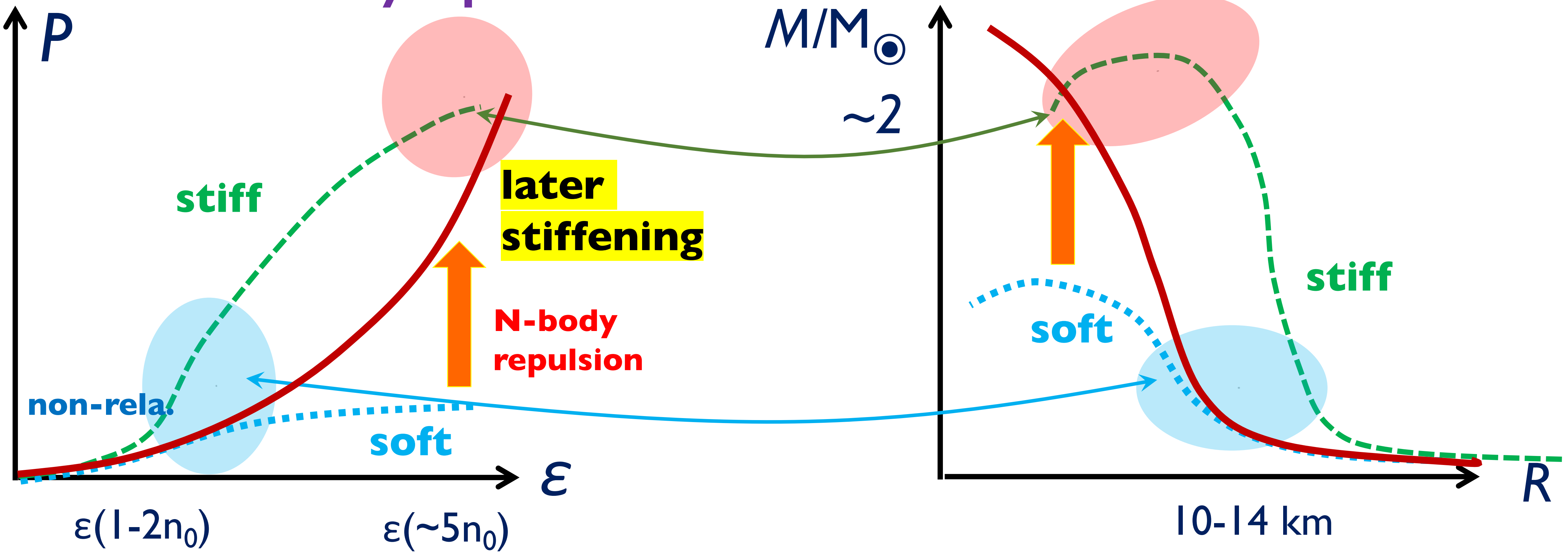
Ref) Lattimer & Prakash (2001)

soft-to-stiff EOS

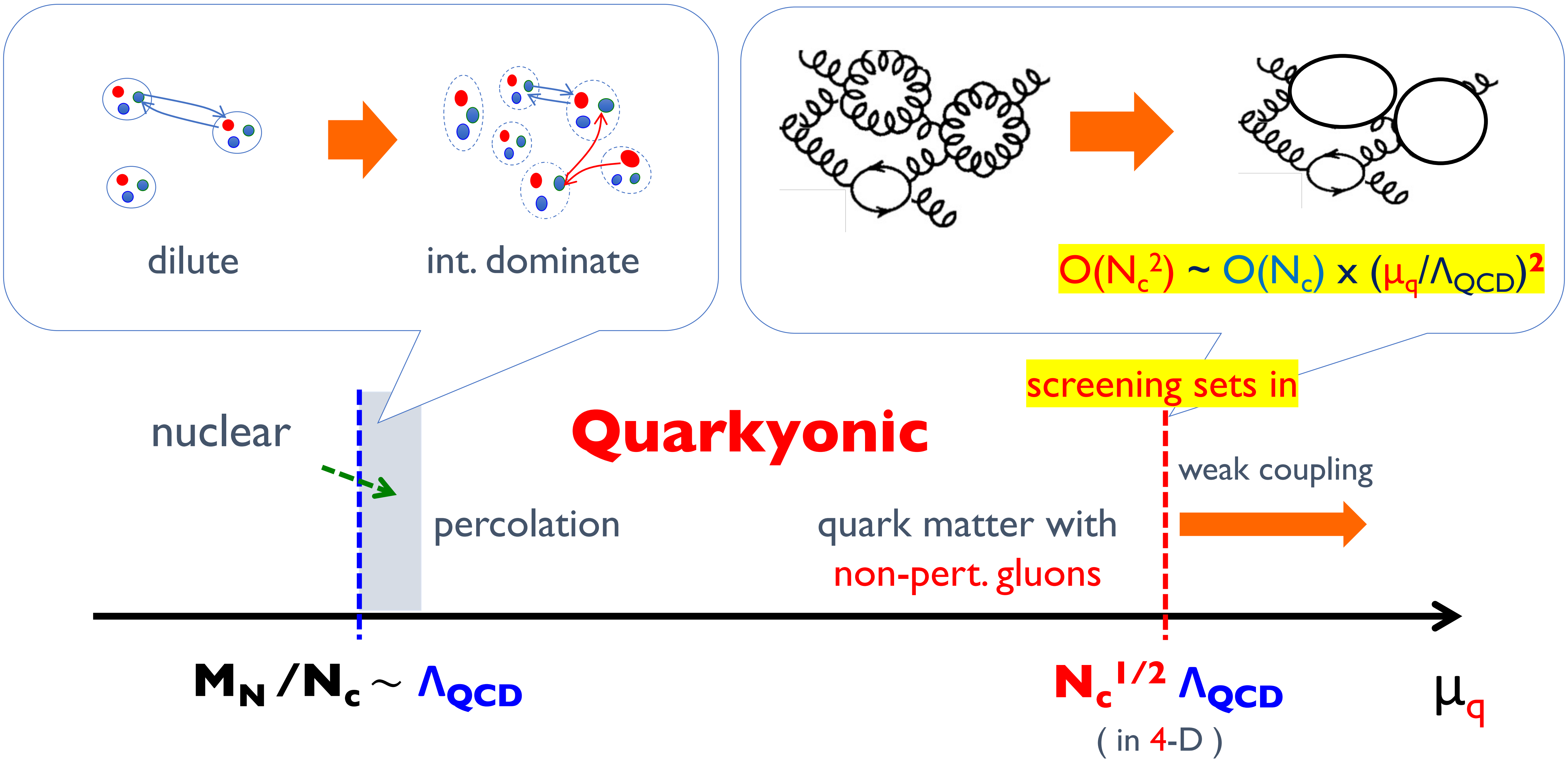
e.g.) nuclear matter with N-body repulsion

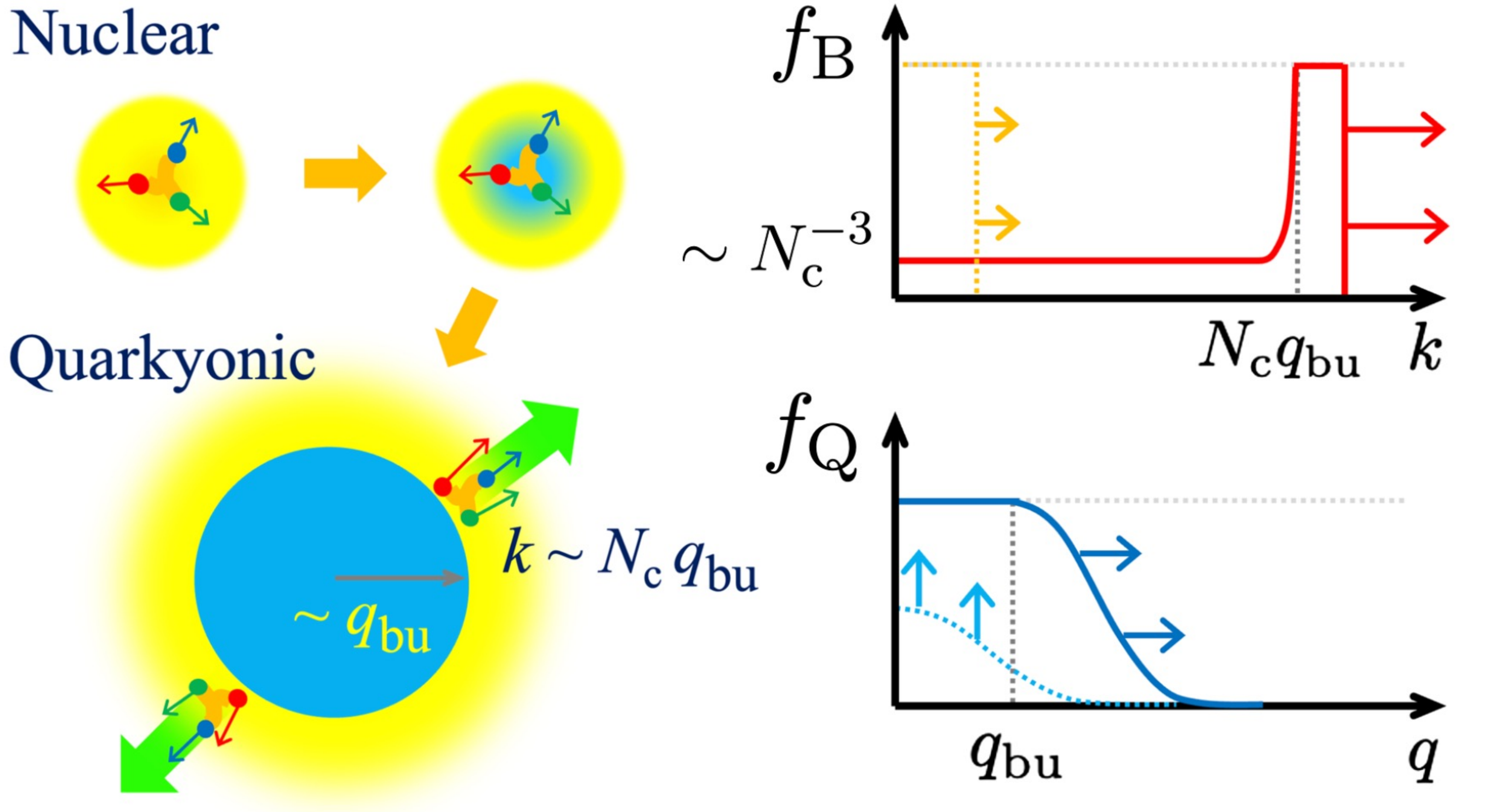
$$\varepsilon = m_N n_B + a\frac{n_B^{5/3}}{m_N} + b n_B^N$$

mass E — kin. E — N(>2)-body repulsion



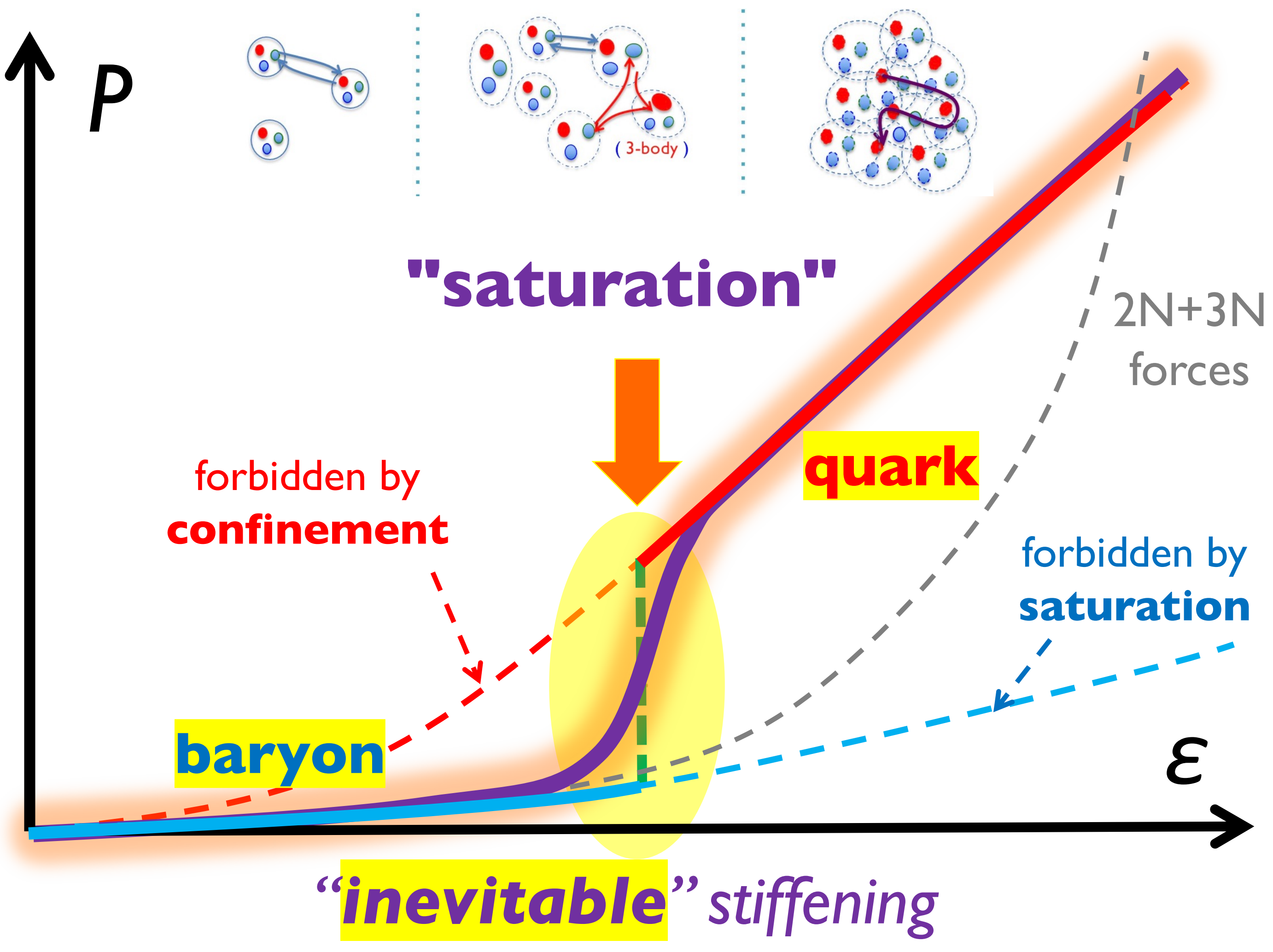
McLerran-Pisarski's "two-scale picture" [McLerran-Pisarski '07]

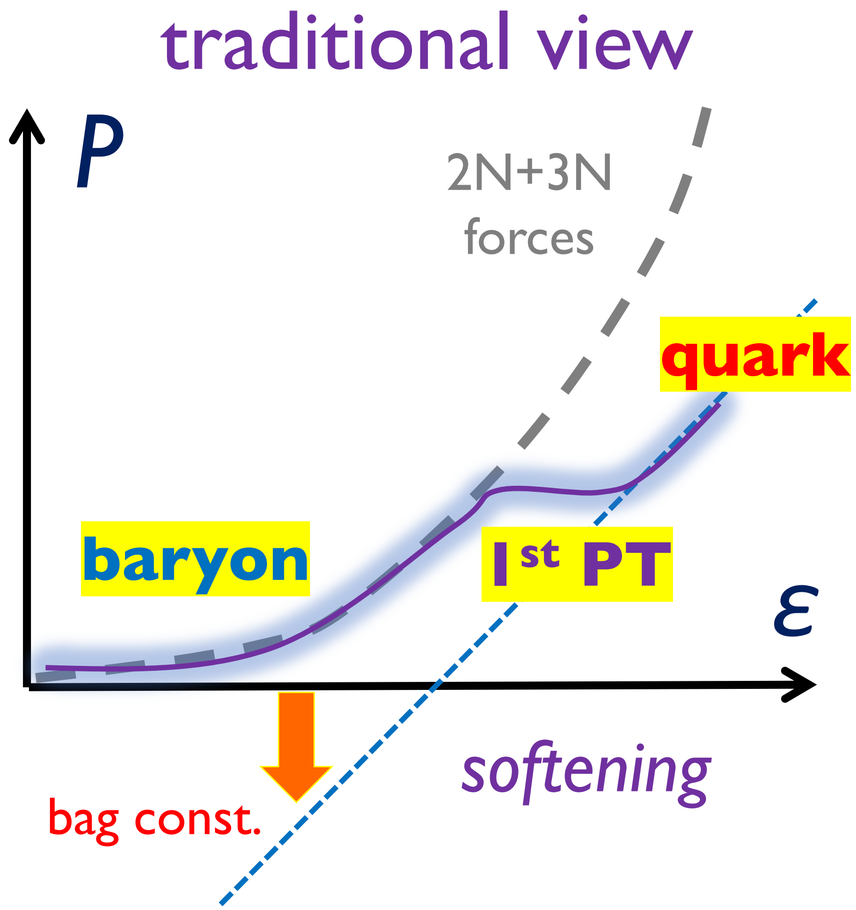




Nuclear
Quarkyonic
$k \sim N_c q_{bu}$
$\sim q_{bu}$
f_B
$\sim N_c^{-3}$
$N_c q_{bu}$
k
f_Q
q_{bu}
q

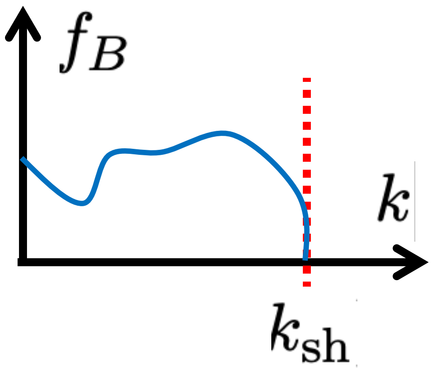
Inevitable stiffening





$n_{Q\text{-sat}} \sim 0.5\ n_{overlap}$ ➔ $n_{Q\text{-sat}} \sim 2\text{-}3n_0$

Number-conserving energy variation



$$\delta\varepsilon_{\rm B} = E_B(\mathbf{k})\delta f_B(\mathbf{k}) + E_B(\mathbf{k}_{\rm sh})\delta f_B(\mathbf{k}_{\rm sh})$$

$$= \left[E_B(\mathbf{k}) - E_B(\mathbf{k}_{\rm sh})\right]\delta f_B(\mathbf{k})$$

$$\equiv \lambda_{\rm B}$$

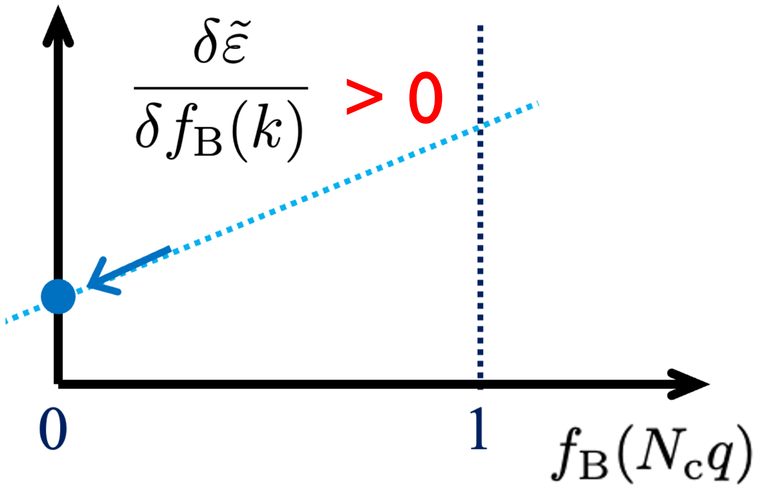
$$\delta f_B(\mathbf{k}) + \delta f_B(\mathbf{k}_{\rm sh}) = 0$$

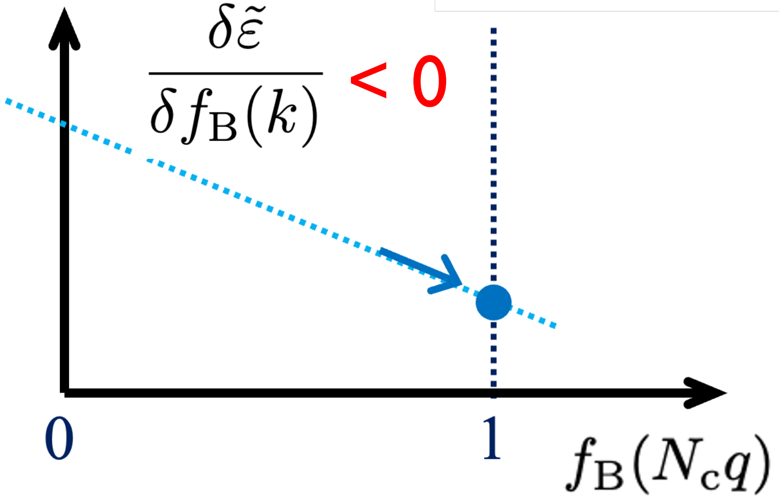
$E_B(k) > \lambda_B$

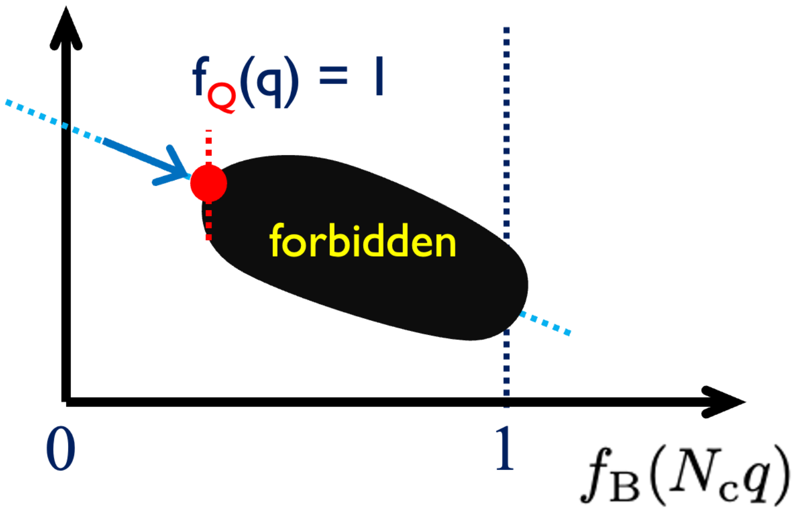
$E_B(k) < \lambda_B$

w.o. saturation

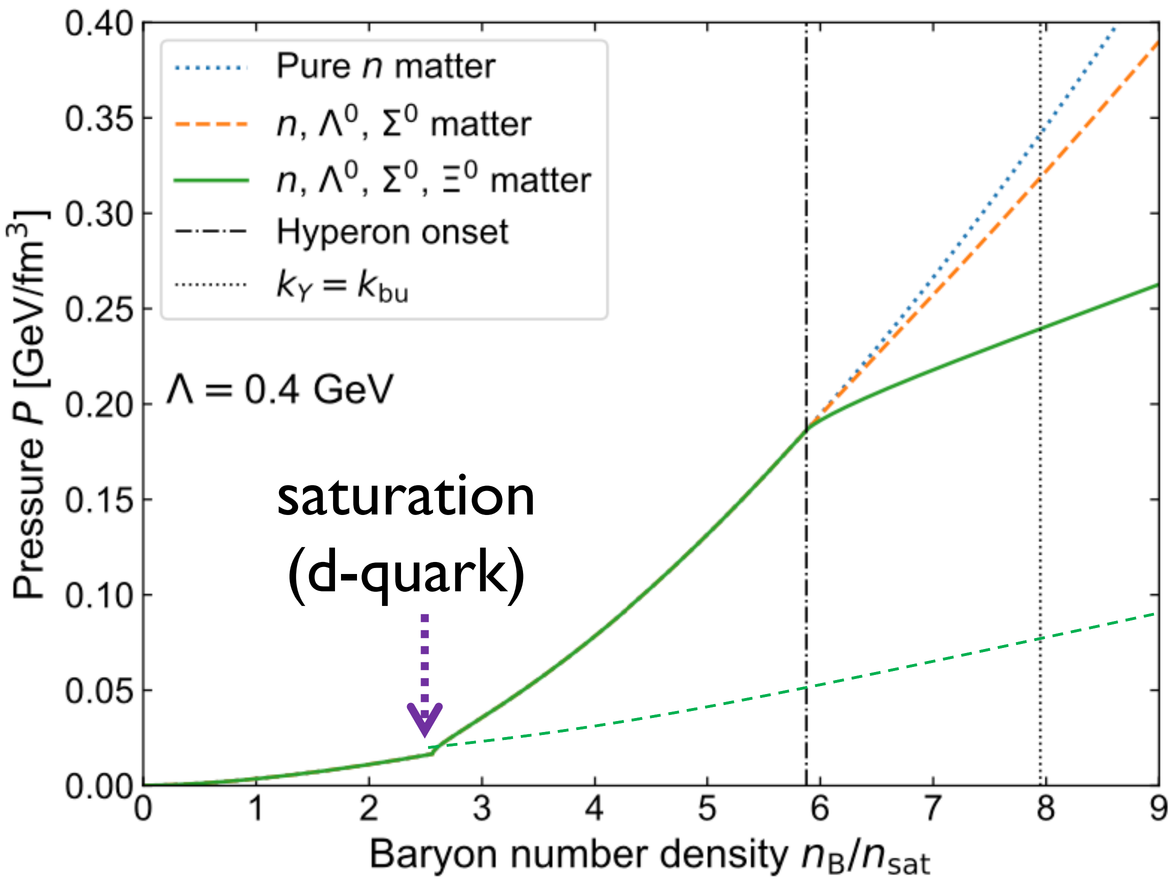
with saturation







EOS: n-Σ_0-Λ + Ξ_0 matter



Ξ_0 (uss) → free from d-quark sat.

$\mu_B^{\Xi\text{-onset}} = M_\Xi$ (as usual)

softening occurs,
but only at high density

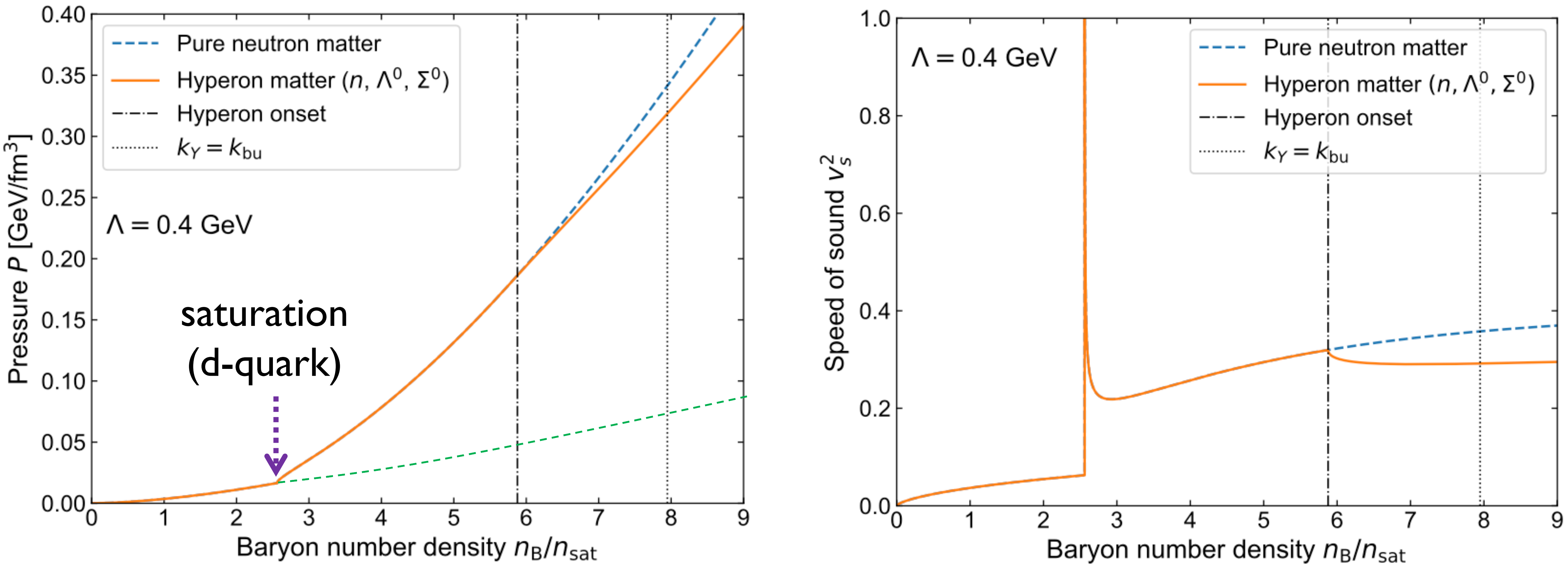
perhaps soon goes back to
quark scaling through u- or s- sat

(to be studied)

$n_B^{Y,\,\Xi\text{-onset}} \sim 6n_0$

EOS: n-Σ_0-Λ matter

[Fujimoto-TK-McLerran, '24]



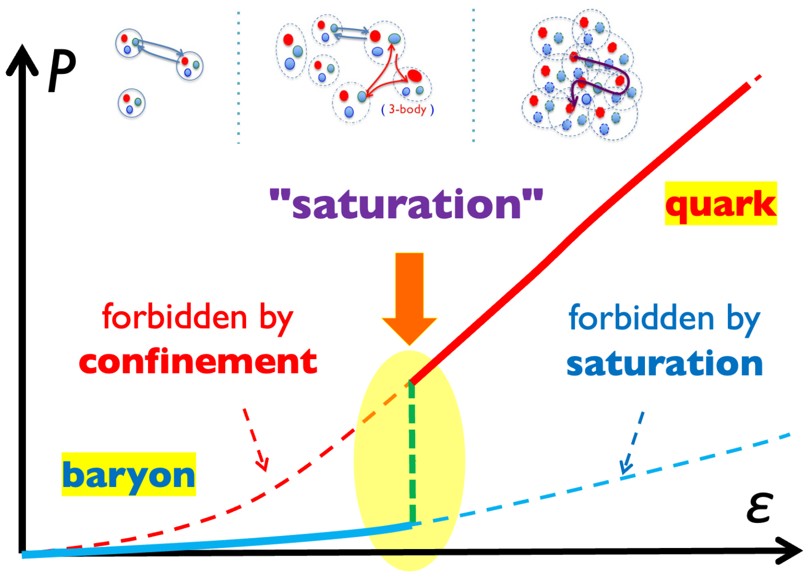
$n_B^{\text{Y-onset}} \sim 6n_0$

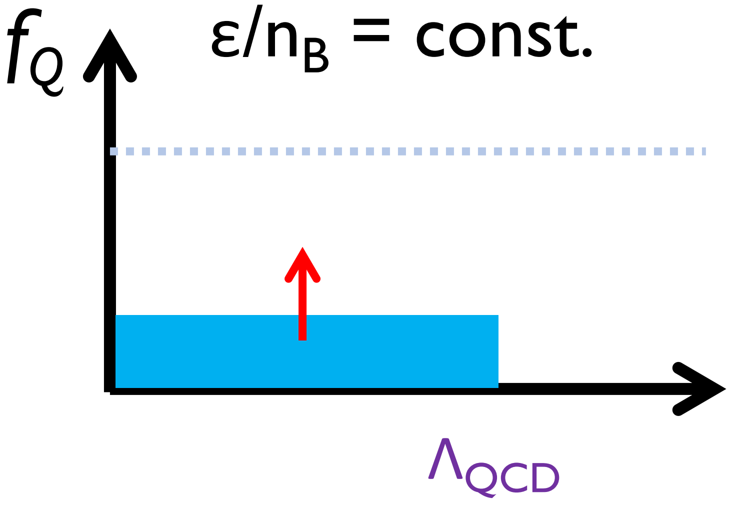
Stiffening in quark picture

(very schematic)

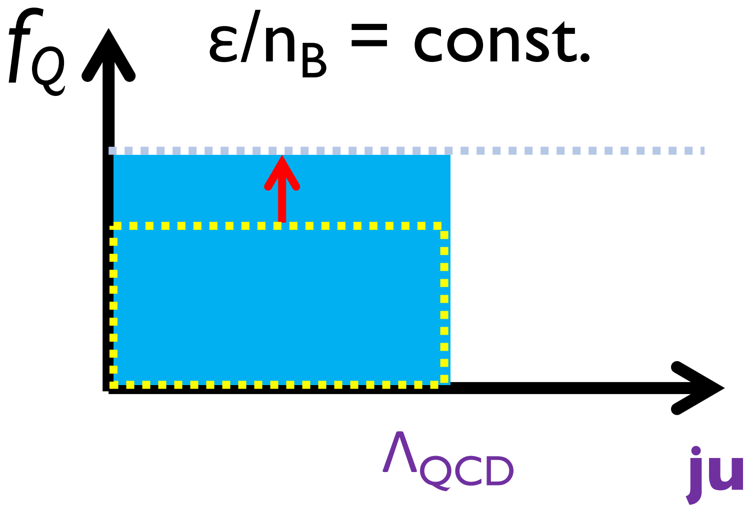
$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B} \left(\frac{\varepsilon}{n_B} \right)$$

energy per particle



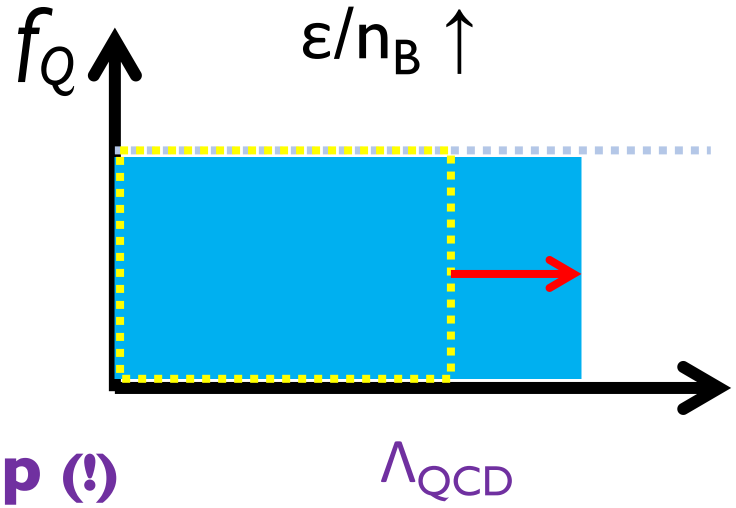


P = **0**



P = **0**

jump (!)



P = **finite**