Hadron spectra and thermodynamics for all quark flavors from a universal Hagedorn temperature

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with Győző Kovács, Larry McLerran, Krzysztof Redlich

- MM, Kovács, McLerran, Redlich, PRD 112 (2025) 9, 096010
- MM, Redlich, 2601.22902 [hep-ph] (2026)
- MM, McLerran, Redlich, 2603.28668 [hep-ph] (2026)

Buenas Ideas on the QCD Phase Diagram, YITP, Kyoto University, Japan, 2026/05/26

The Hagedorn Hypothesis

$$\rho(m) \simeq m^{-a} \exp(m/T_H)$$

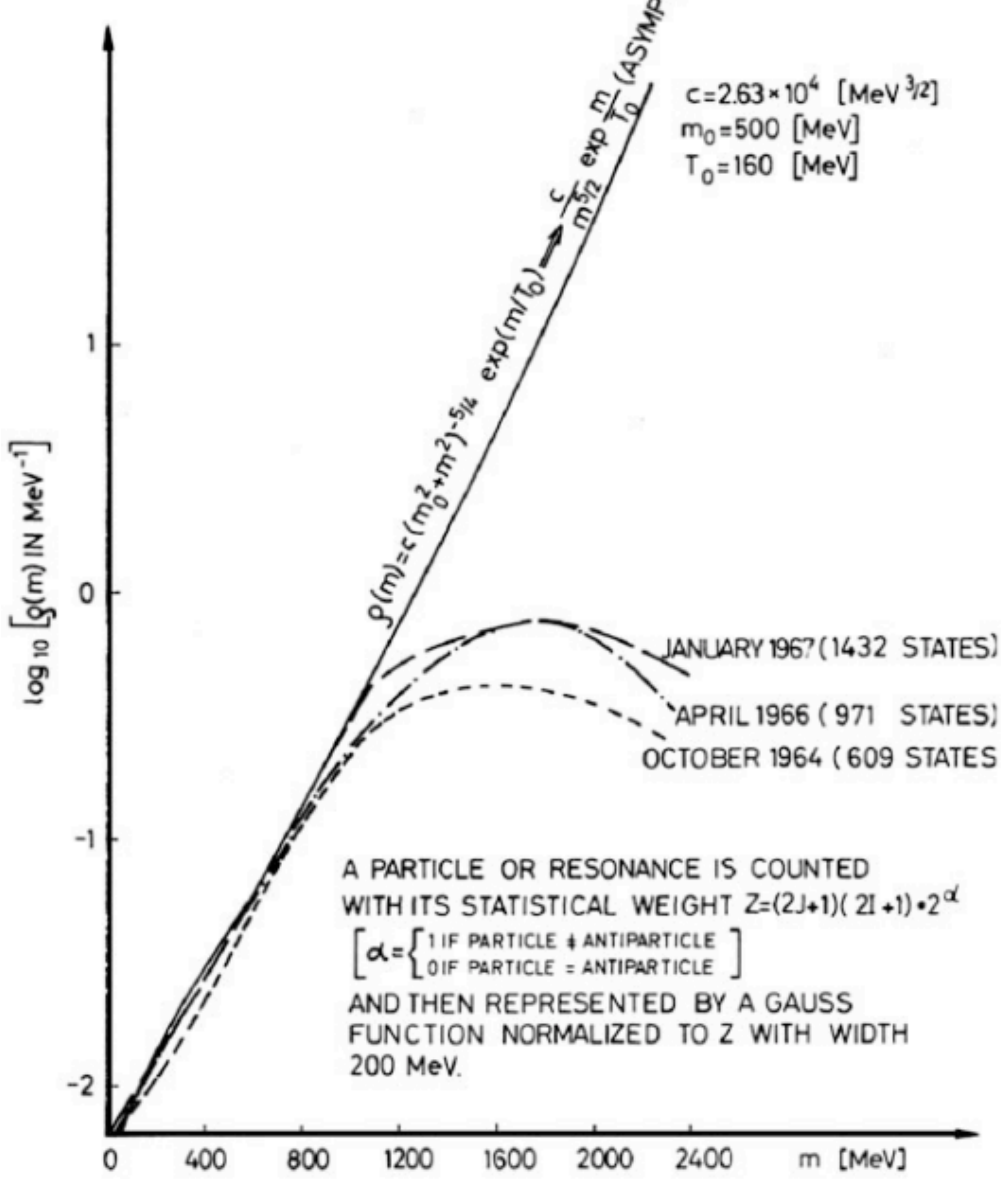
scale controlling the asymptotic growth of resonances

Singularity at T_H

$$Log\mathcal{Z} \sim \int \rho(m) e^{-m/T}$$

Hagedorn: Limiting T for strong systems

Hagedorn, 1965



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↓

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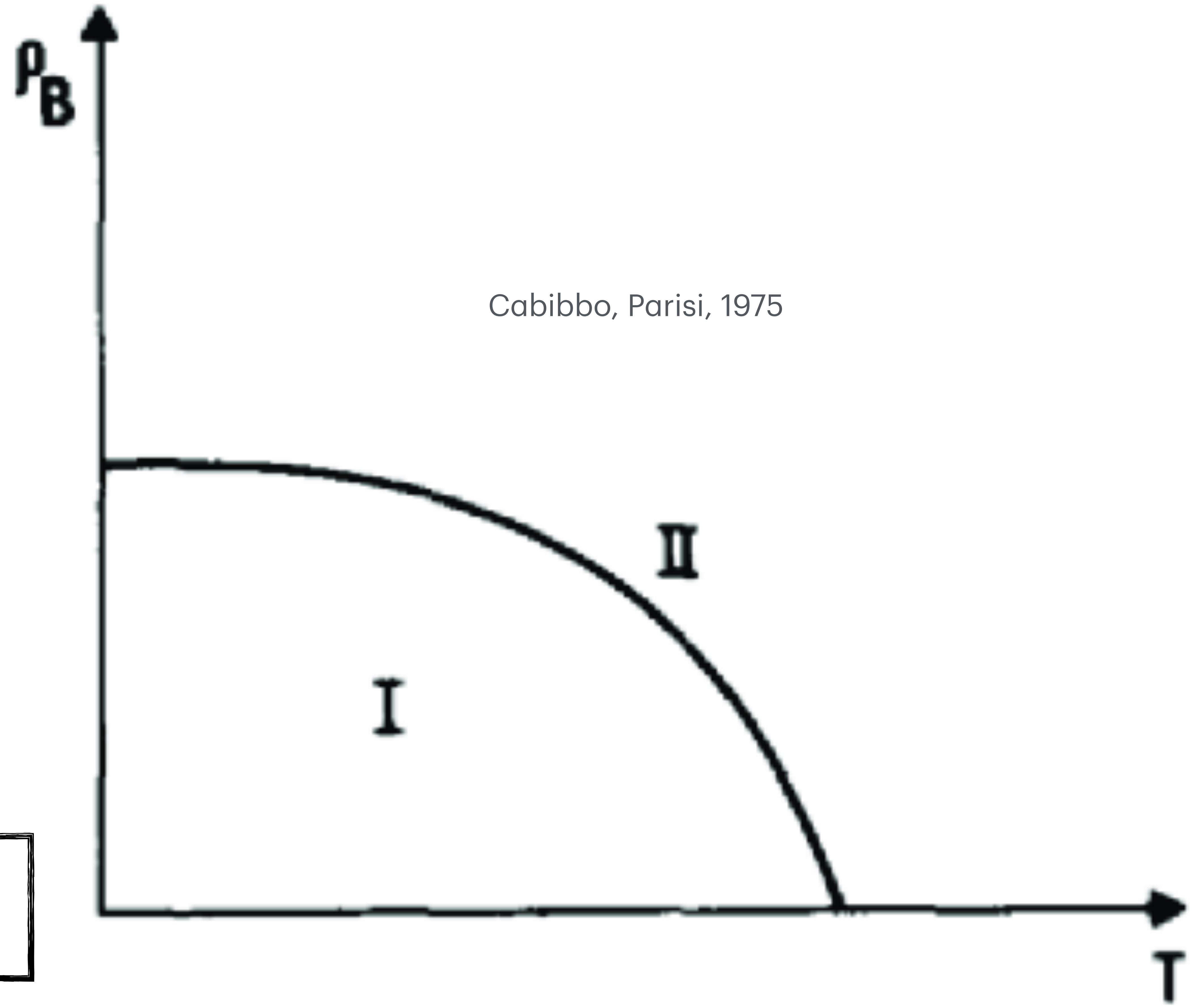
Cabibbo, Parisi: transition to QGP



Cabibbo, Parisi, 1975

The Hagedorn Hypothesis

$$\rho(m) \simeq f(m)\ \exp(m/T_H)$$

Typical Prefactor

$$f(m) = A/(m^2 + m_0^2)^{5/4} \sim m^{-5/2}$$

But depends on Bootstrap Condition

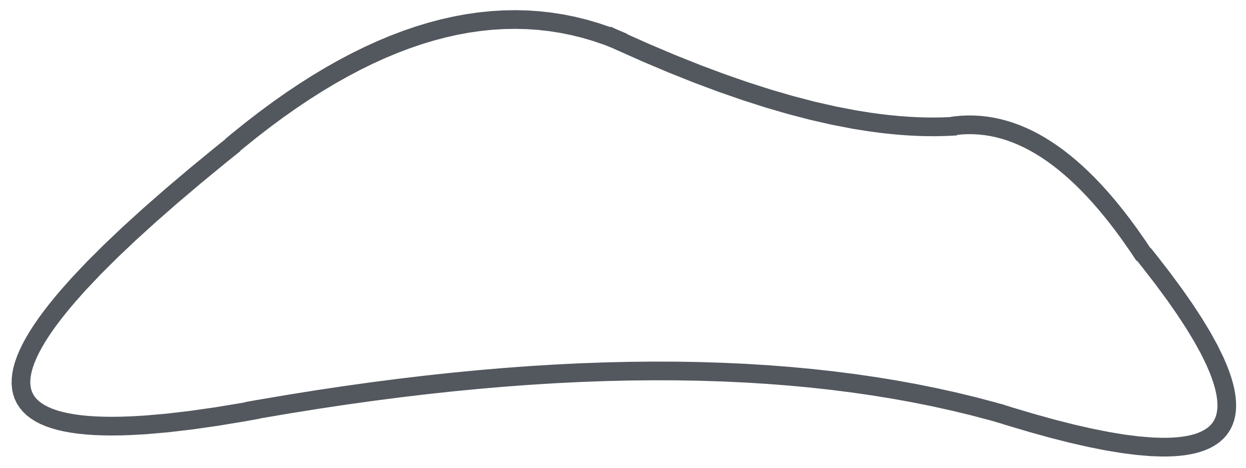
Typical Hagedorn Temperature

$$T_H \approx 150 - 180\,\mathrm{MeV}$$

Associated with Deconfinement

Hagedorn Spectrum from Relativistic String

Closed String



$$\rho_{\rm cl}(m) = \frac{1}{T_H}\left(\frac{2\pi}{3}\right)^3\left(\frac{m}{T_H}\right)^{-4} e^{m/T_H}$$

Open String

$$\rho_{\rm op}(m) = \frac{\sqrt{2\pi}}{6T_H}\left(\frac{m}{T_H}\right)^{-3/2} e^{m/T_H}$$

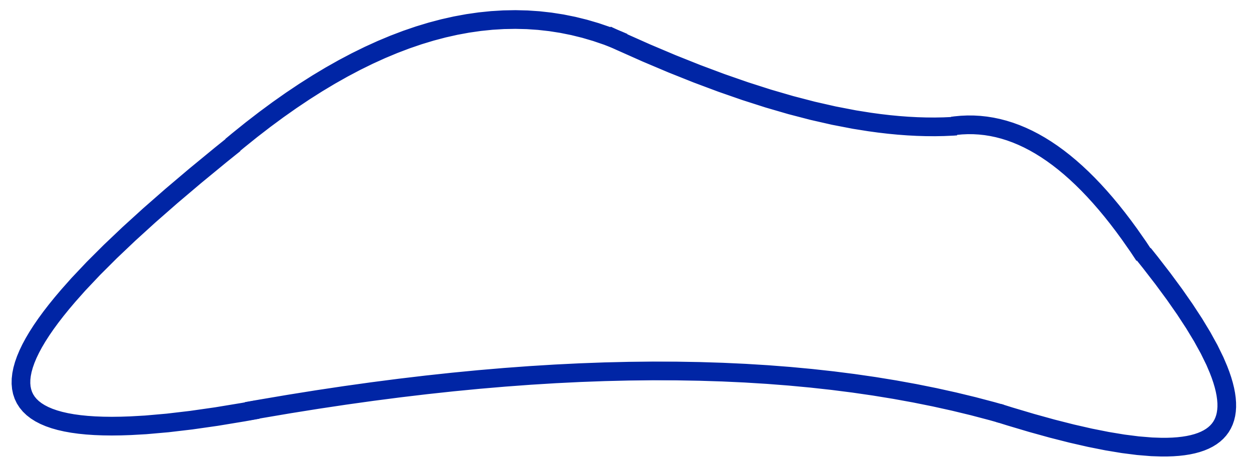
The same Hagedorn Temperature: $T_H = \sqrt{\frac{3\sigma}{2\pi}}$ → Effective scale set by string tension $\sqrt{\sigma}$

Recent estimates on $\sqrt{\sigma} \approx 480\,\mathrm{MeV}$ give $T_H \approx 330\,\mathrm{MeV}$ (Brambilla et al, 2023)

Link to deconfinement in Pure Gauge Theory: $T_H = 1.069(5)\,T_{\rm dec}$ (Lucini et al, 2004)

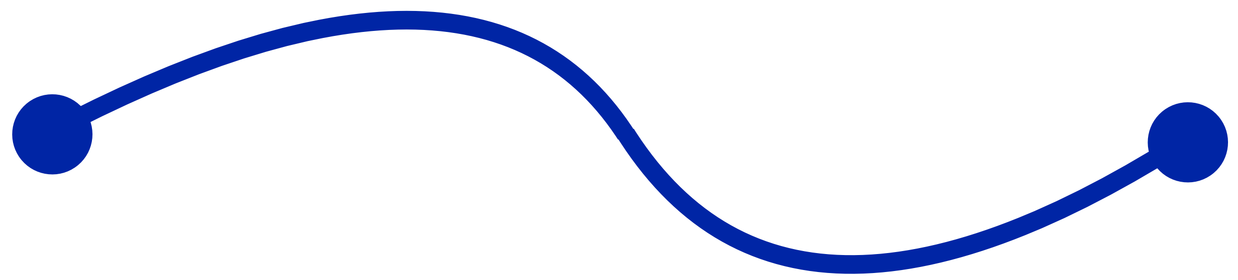
Hagedorn Spectrum from Relativistic String

GLUEBALLS AS CLOSED STRINGS



$$\rho_{\rm cl}(m) = \frac{1}{T_H}\left(\frac{2\pi}{3}\right)^3\left(\frac{m}{T_H}\right)^{-4} e^{m/T_H}$$

MESONS AS OPEN STRINGS



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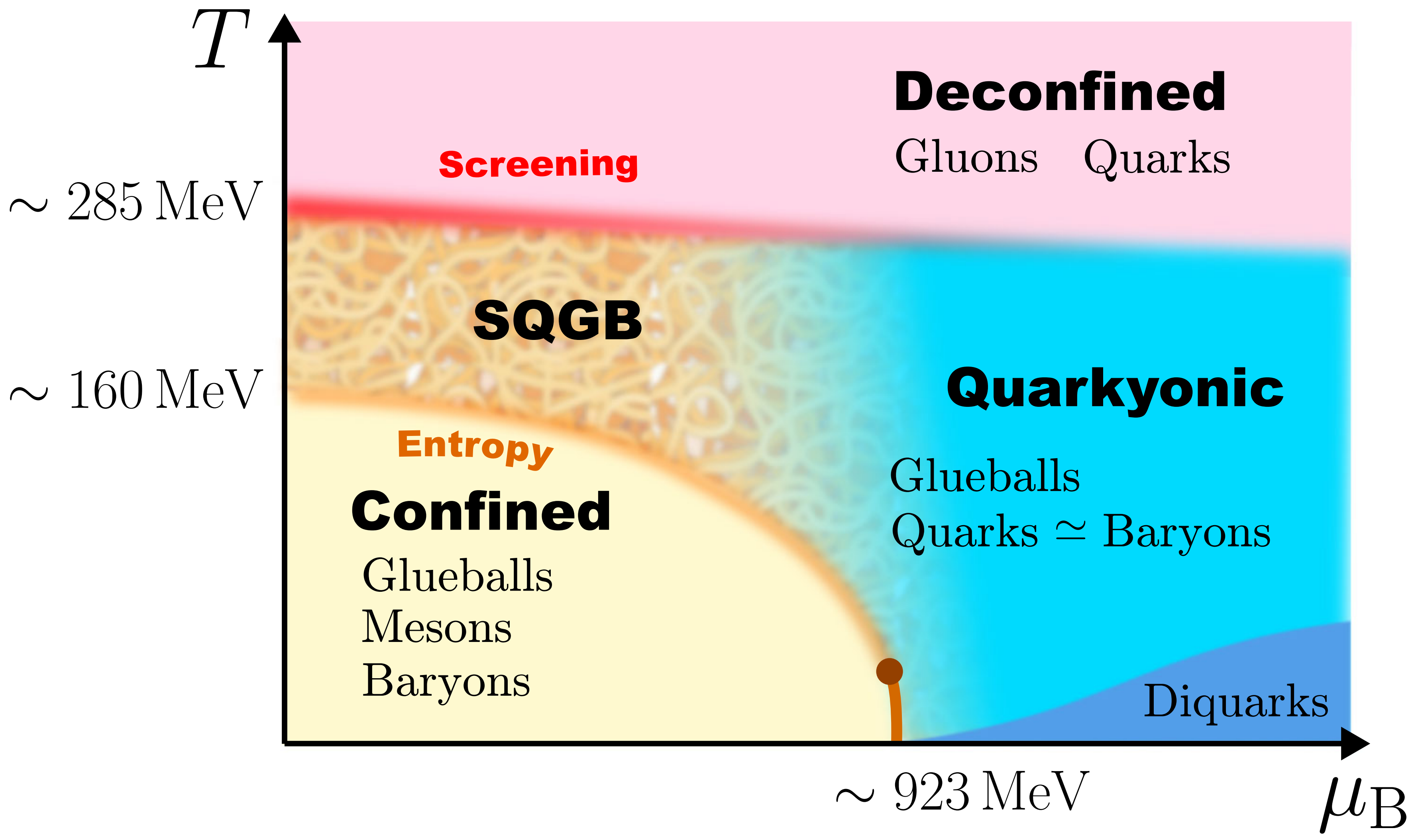
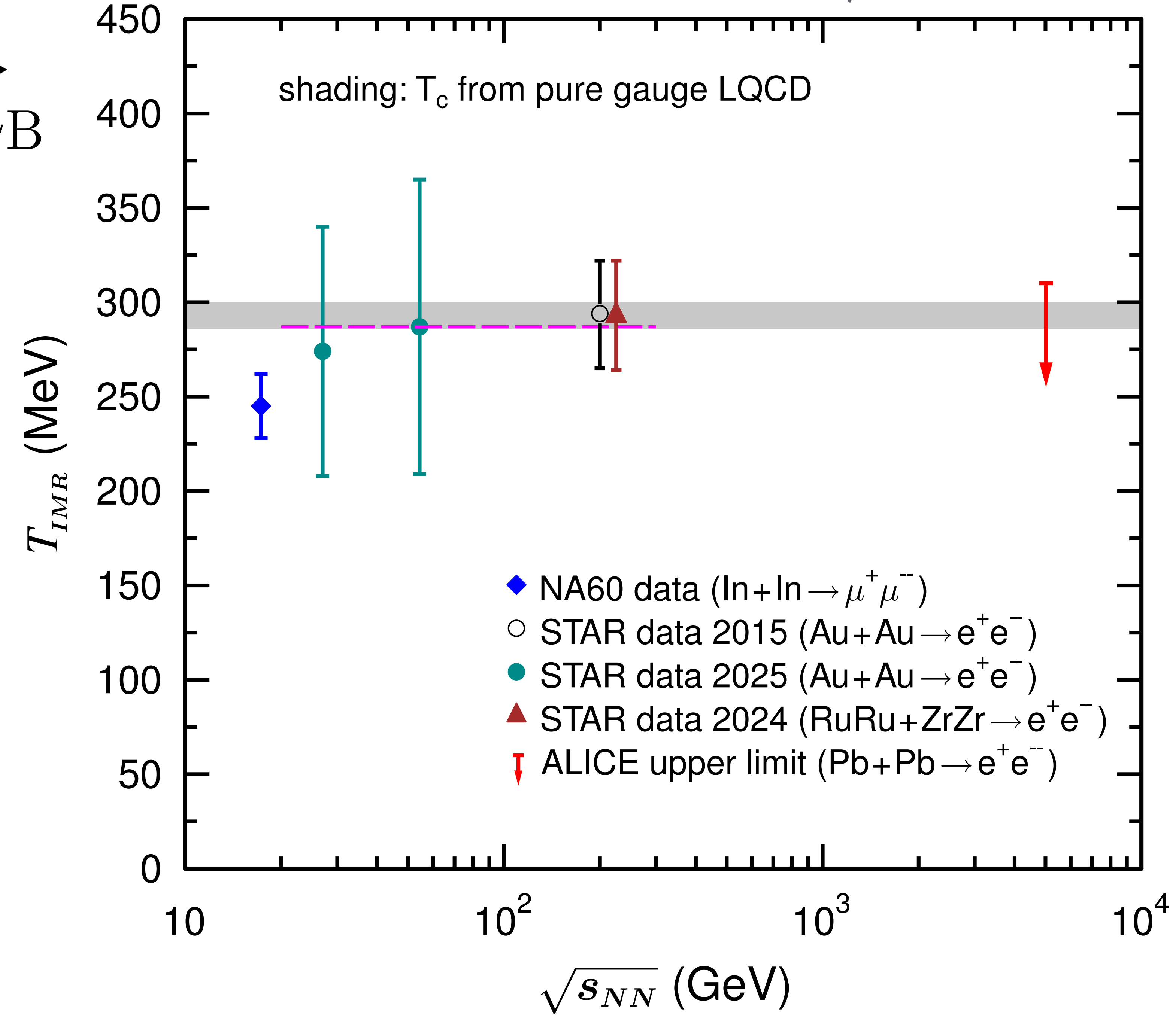
The $\sqrt{\sigma}$ scale Reappears

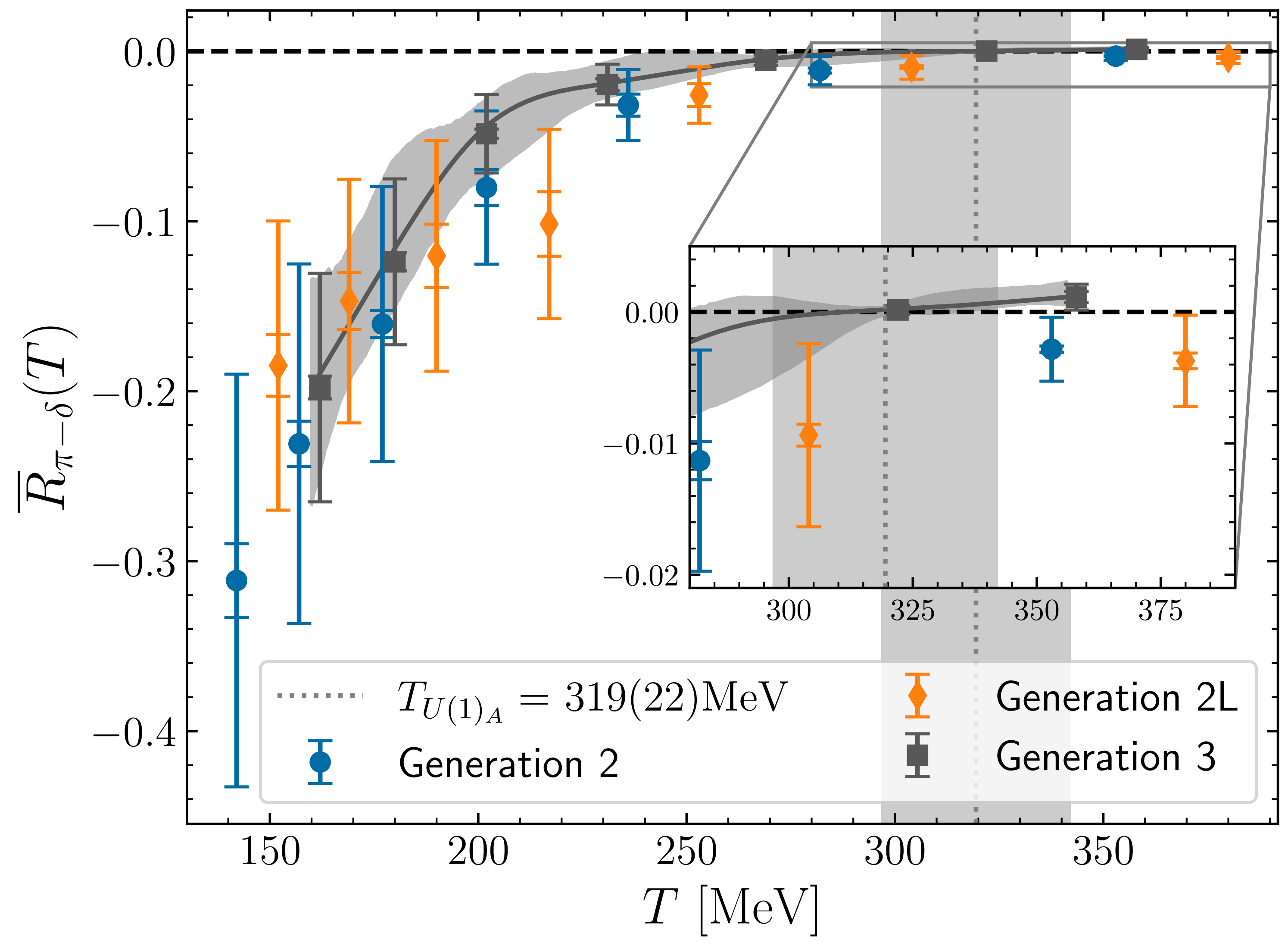
Fujimoto et al, 2025



Vovchenko et al, 2026



Aarts et al, 2025



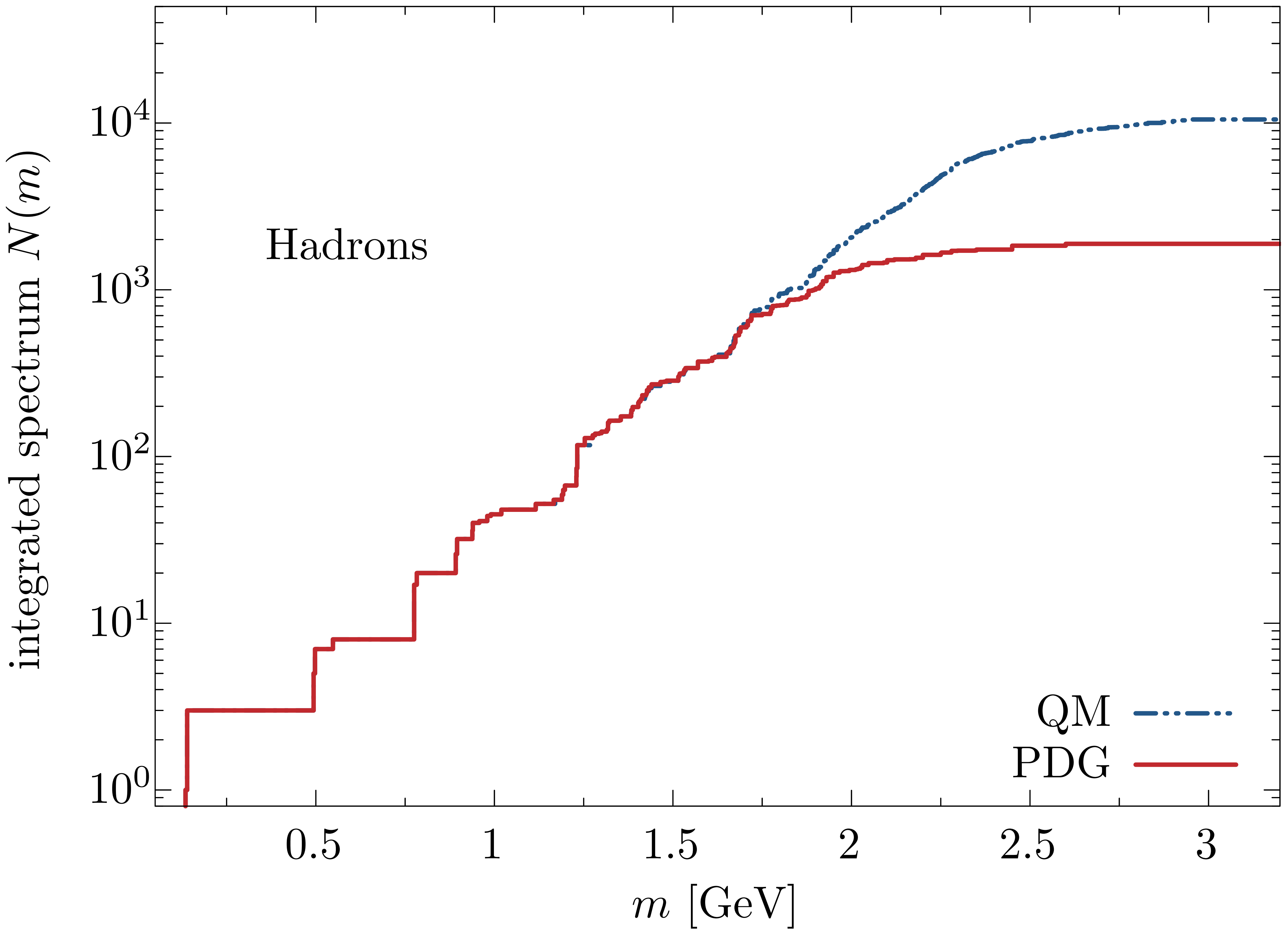
Counting the States

It is customary to consider cumulative spectrum

$$N(m) = \int_0^m dm' \rho(m') \quad \text{such that} \quad \rho(m) = \left.\frac{dN(m')}{dm'}\right|_m$$

Discrete spectrum

$$\rho(m) = \sum_i g_i \delta(m - m_i) \qquad N(m) = \sum_i g_i \theta(m - m_i)$$



Experimental spectrum → Particle Data Group

Theoretical spectrum → Quark Model

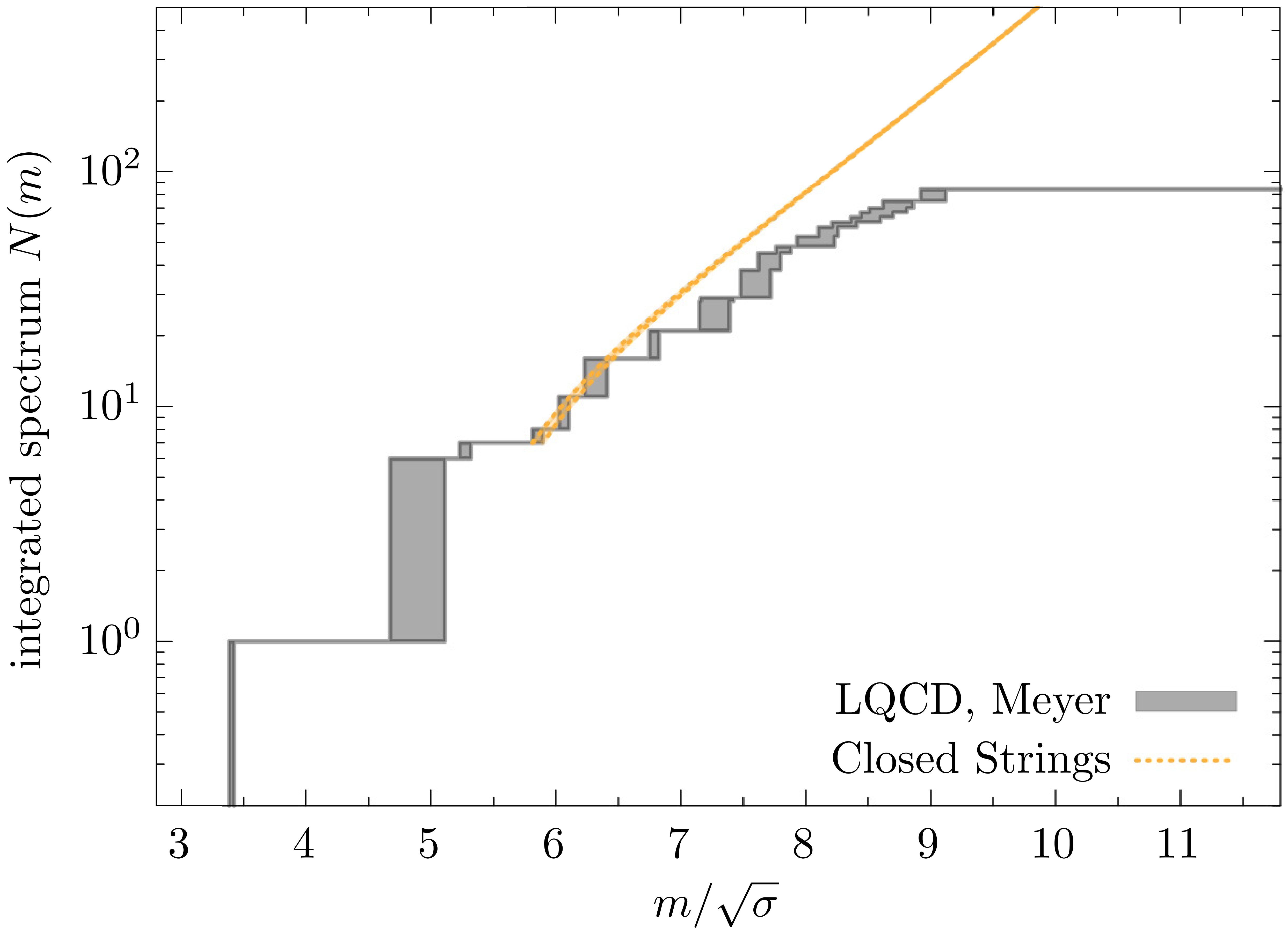
Closed Strings Describe LQCD Glueballs

In Pure Gauge Theory scale is set by $\sqrt{\sigma}$

Hagedorn Temperature: $\frac{T_H}{\sqrt{\sigma}} = \sqrt{\frac{3}{2\pi}}$

Spectrum is practically parameter-free

$$\rho_{\mathrm{gb}}(m) = \theta(m - m_{\mathrm{thr}})\rho_{\mathrm{cl}}$$



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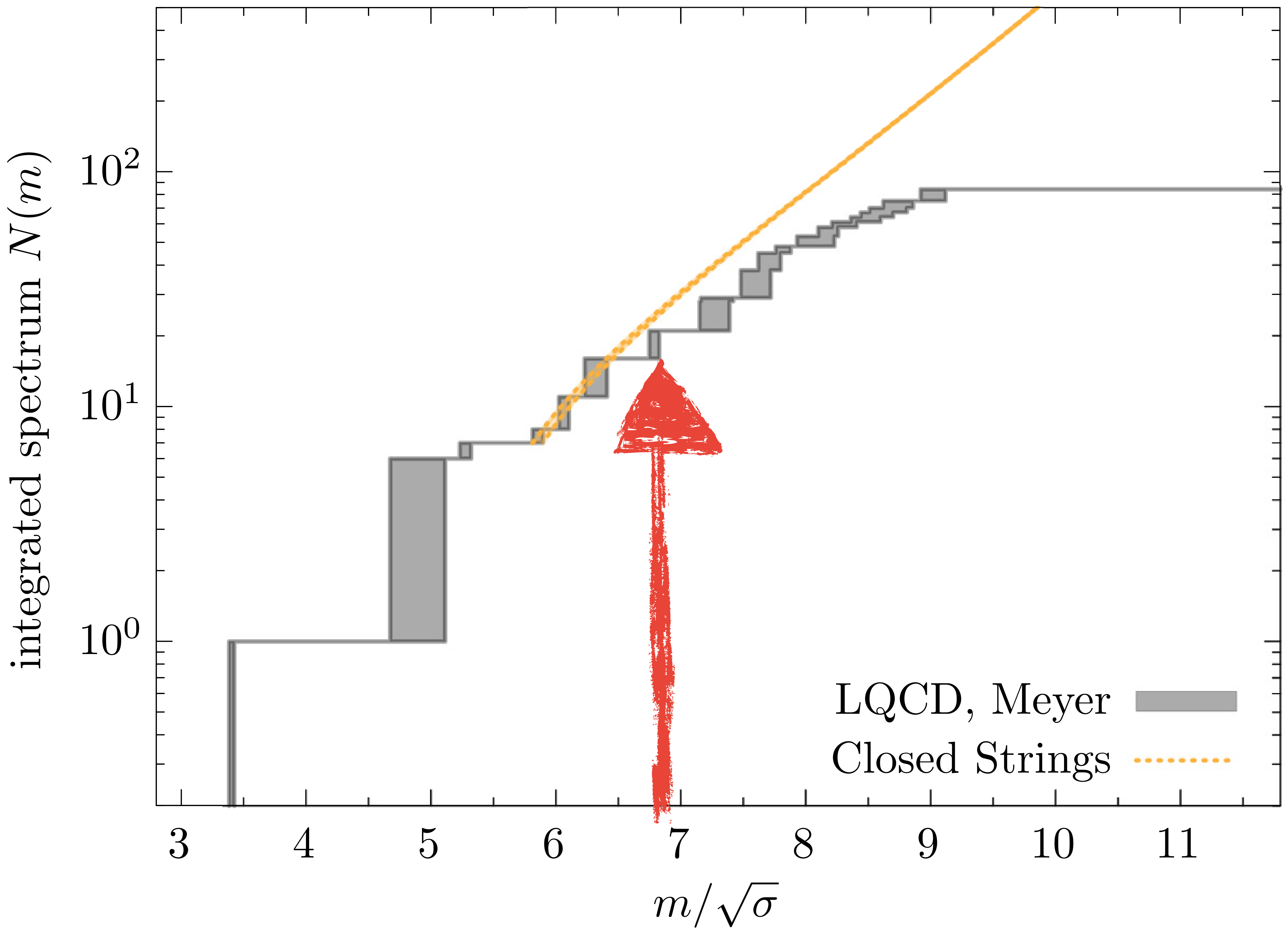


Above $2\,M_0$, LQCD spectrum saturates due to difficulties in extraction of the states

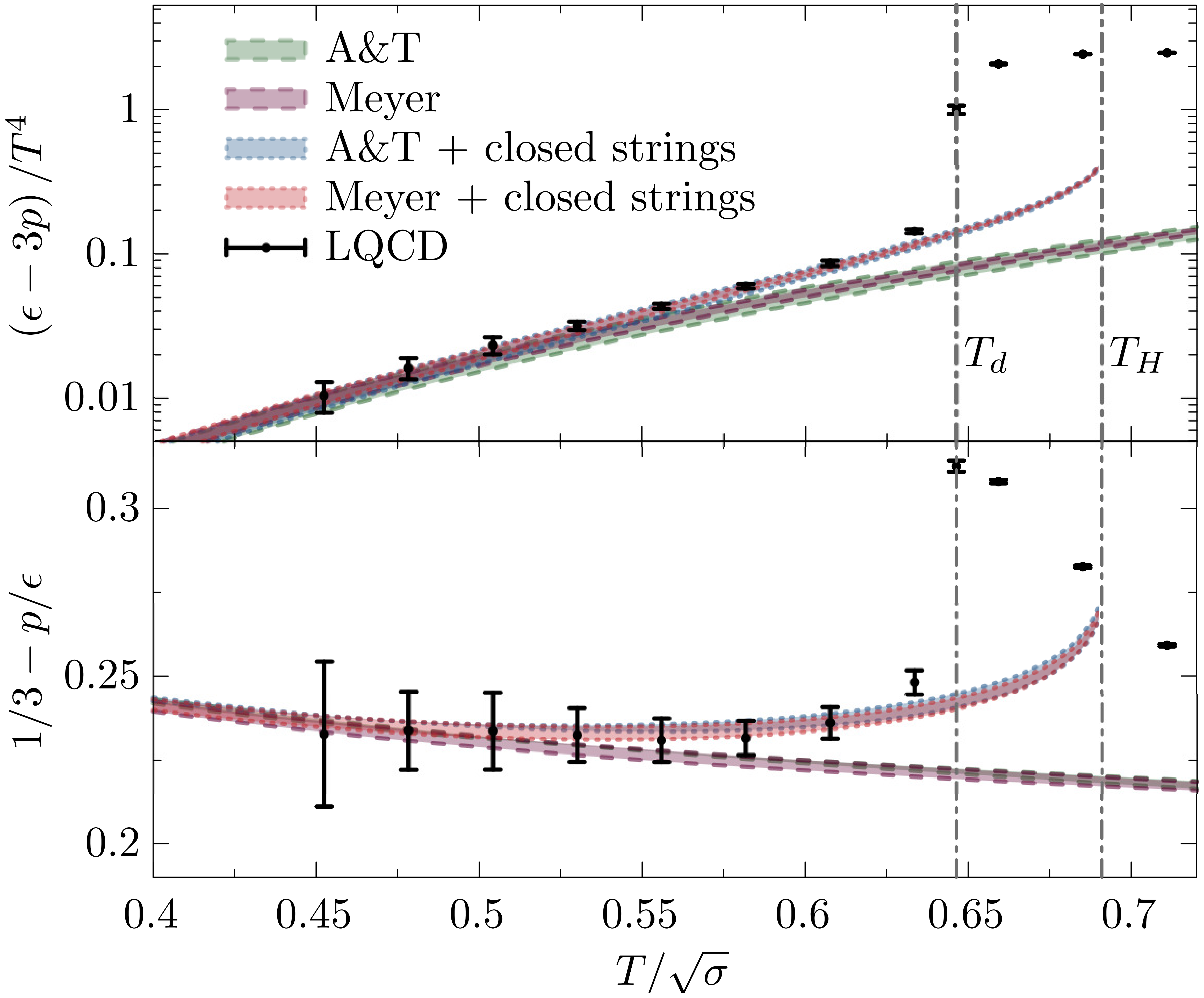
Thermodynamics of Glueballs

Boltzmann Approximation

$$\hat{P}(m) = \frac{1}{2\pi^2}\left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right) e^{\mu/T}$$

$$\hat{P} = \int_0^\infty dm\, \rho(m)\, \hat{P}(m)$$

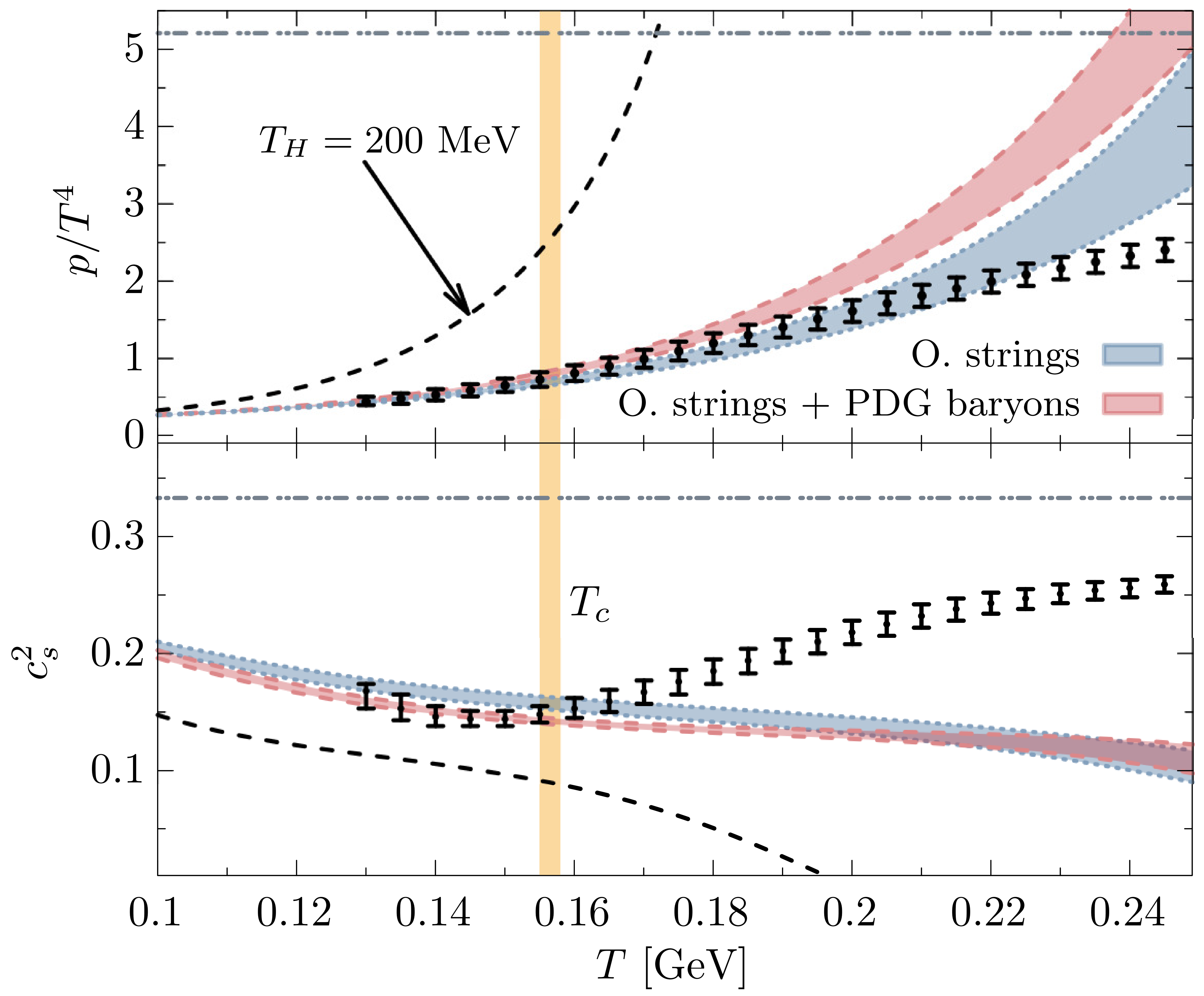
Trace Anomaly

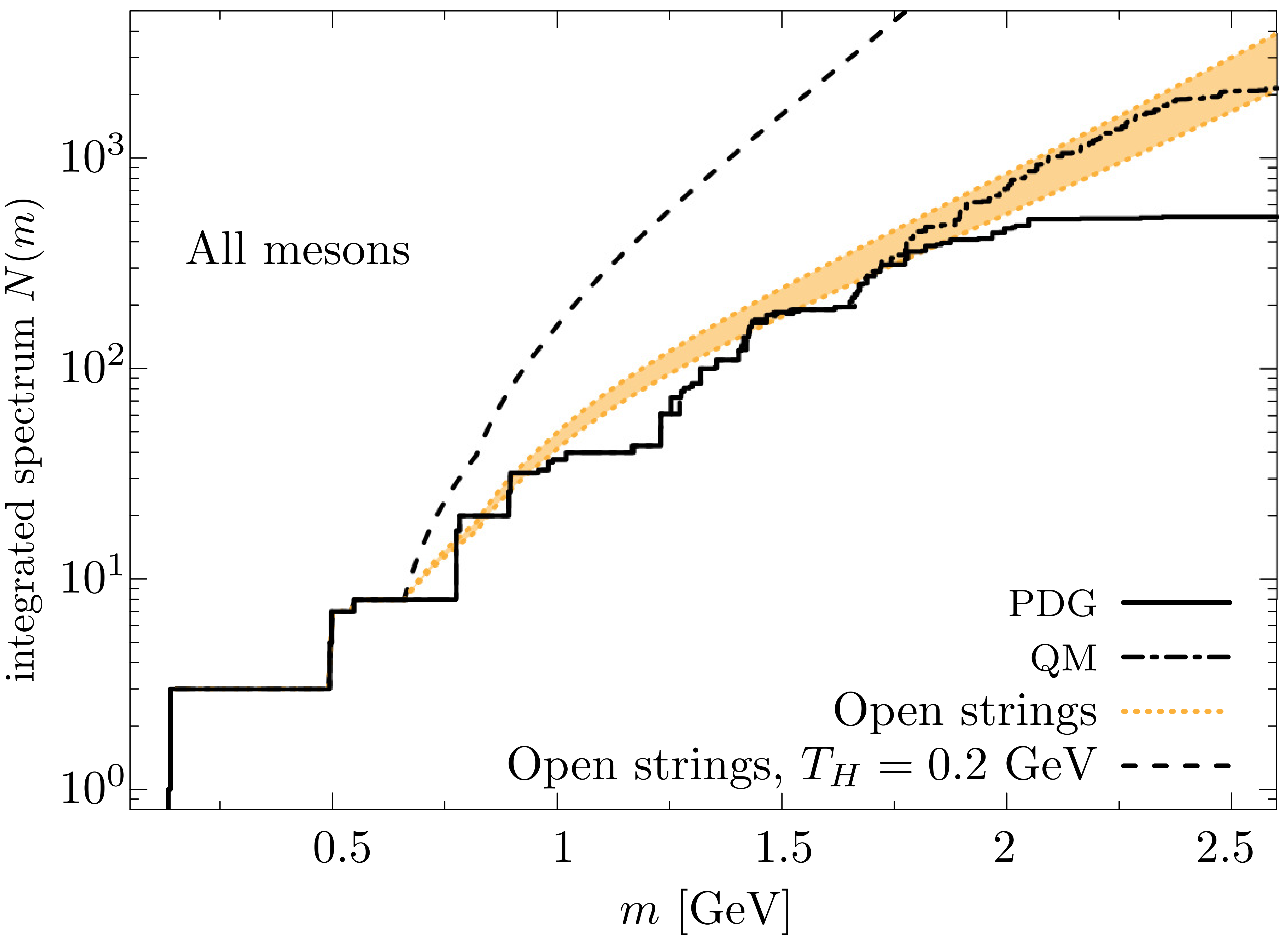


Thermodynamics described well with asymptotic Hagedorn tail (Meyer, 2004)

Open-String Spectrum and Thermodynamics

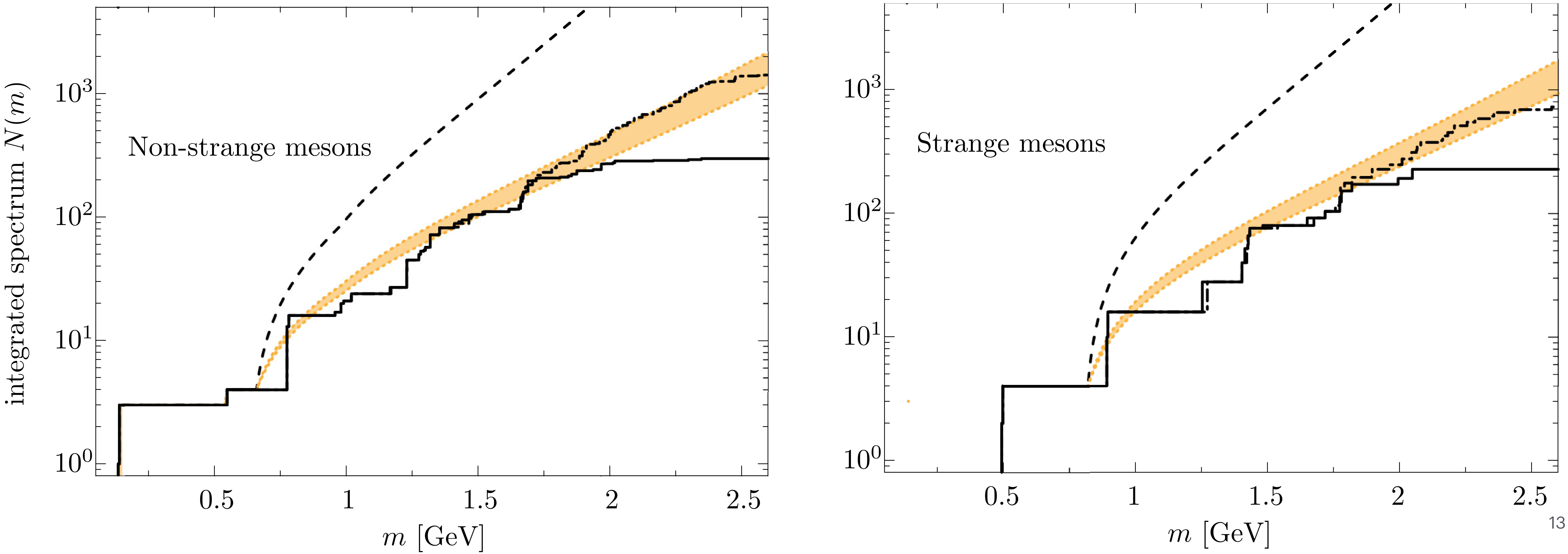
The same set of parameters describes mesonic sector too





Definite quantum-number sectors

Density of state is additive: $\rho(m) = \rho^{S=0}(m) + \rho^{|S|=1}(m)$



Baryons as quark-diquark strings

Sharov, 2000

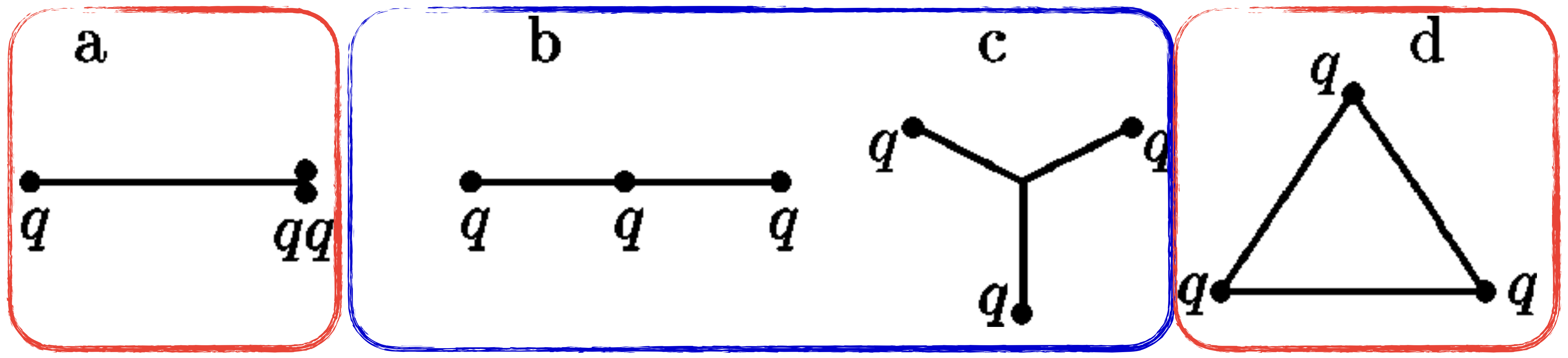
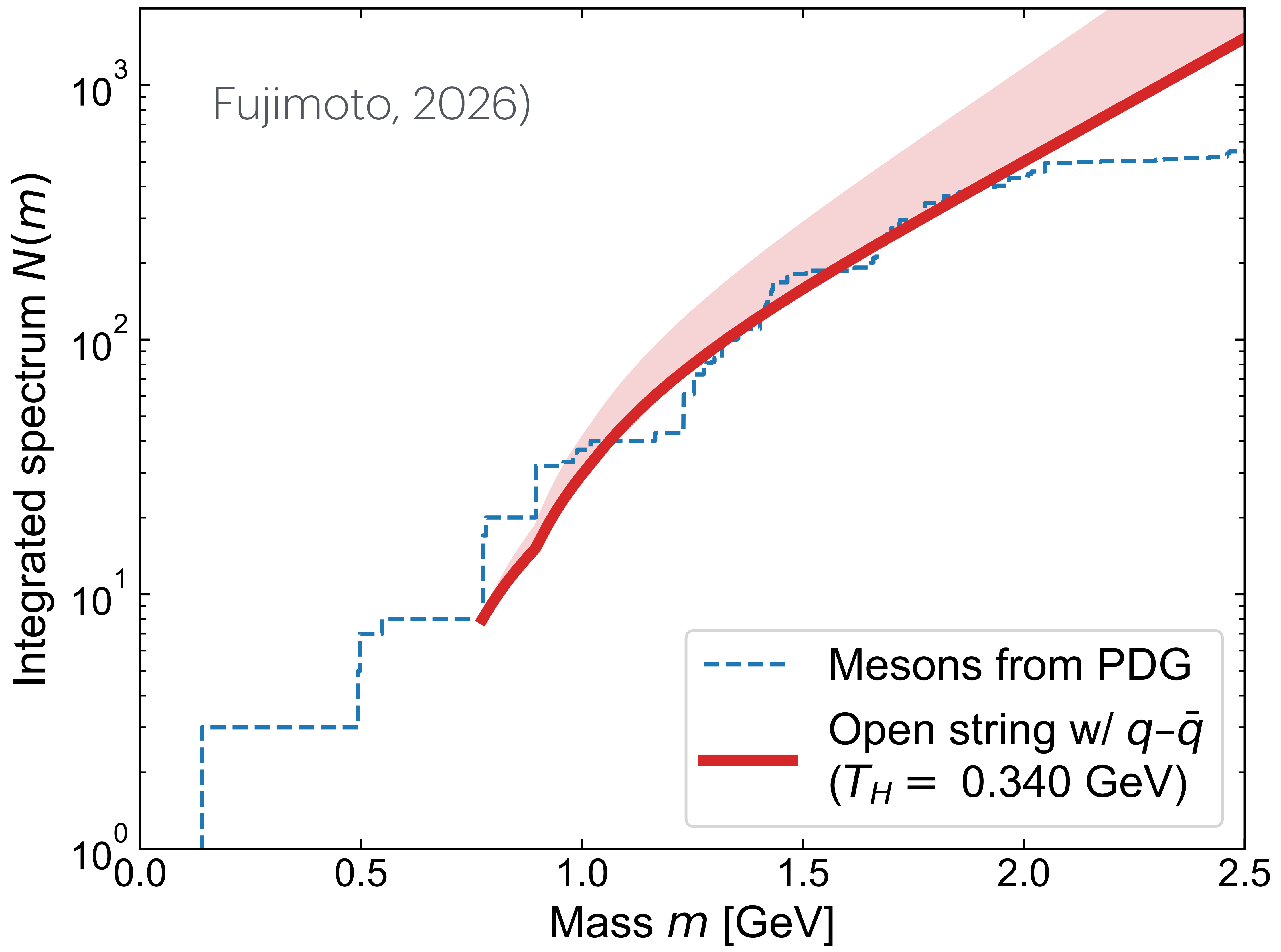


FIG. 1. String baryon models.

Classical rotational motion

- qqq and $\mathbf{Y}$ are unstable
- $q(qq)$ and $\boldsymbol{\Delta}$ are stable

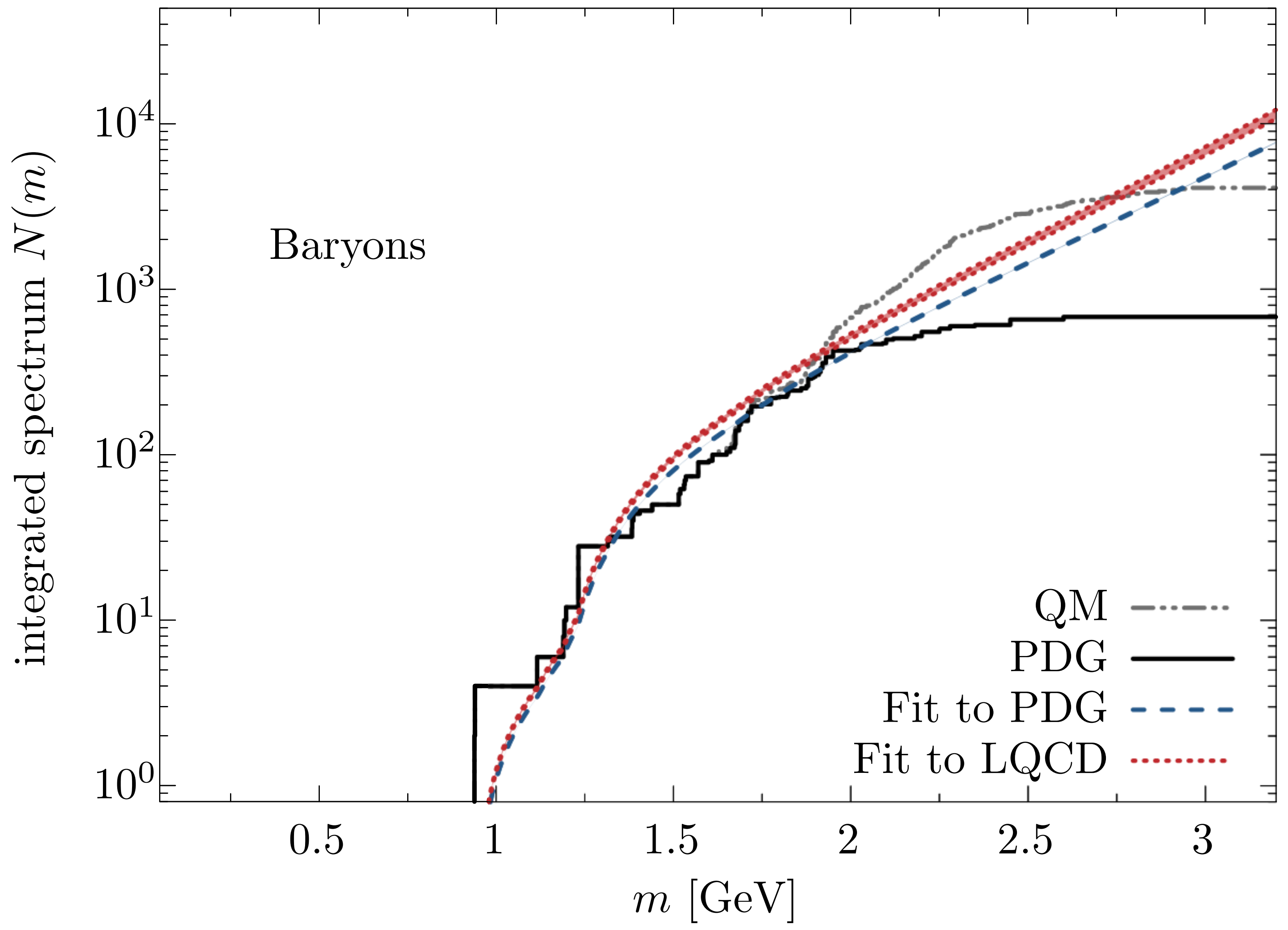


- Closed strings thermally suppressed: $n_{q(qq)} \gg n_{\Delta}$
- Quark-diquark picture supported by Regge trajectories (Selem, Wilczek 2006)

Baryonic spectrum described with the same T_H as mesons (Fujimoto, 2026)

Extracting T_H from data

Experimental spectrum underestimates LQCD (Bazavov, et al., 2014)



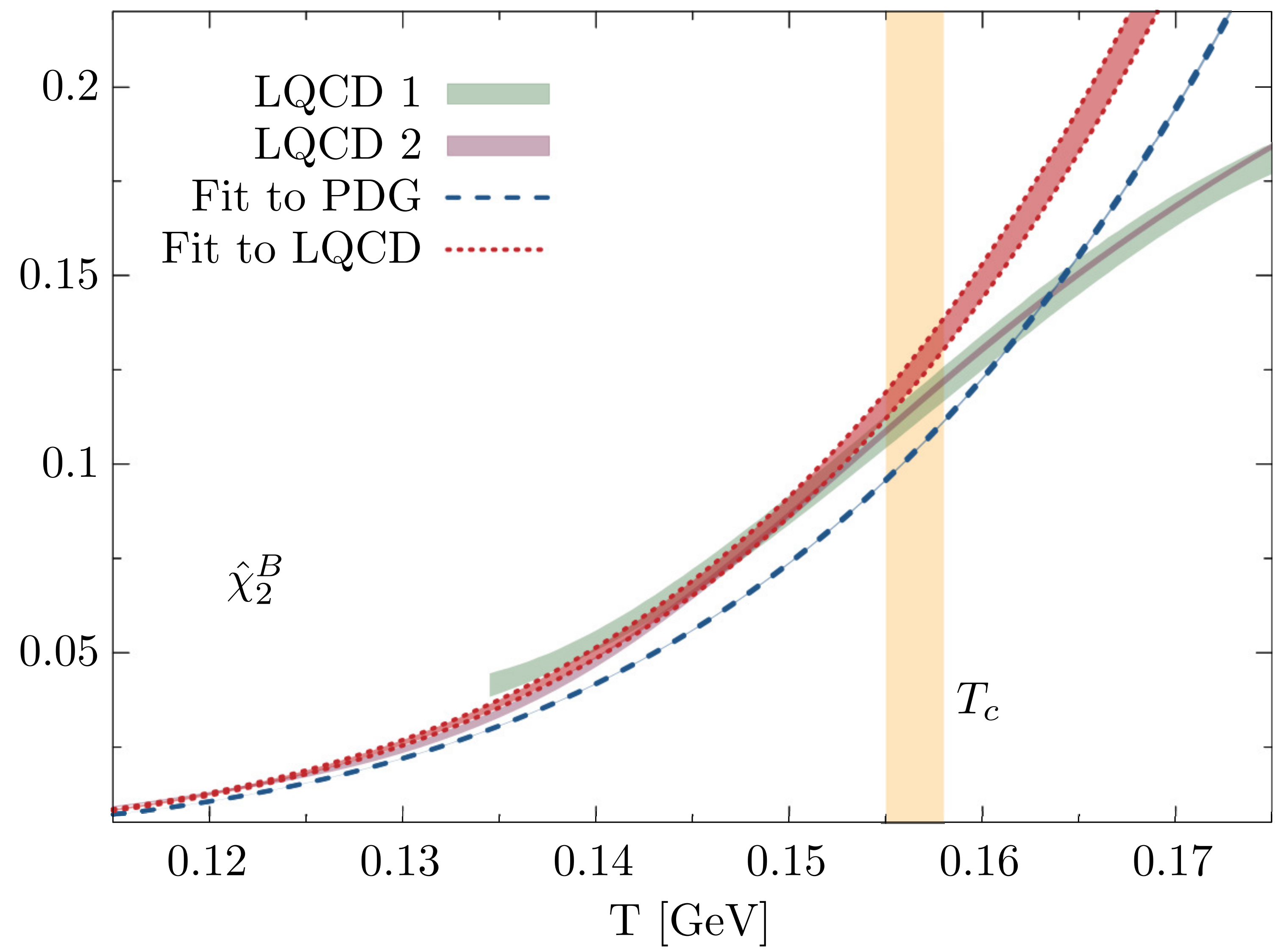
Extend the spectrum with the Hagedorn tail

- Accounts for asymptotic states
- Fit to spectrum gives: $T_H \simeq 340\,\mathrm{GeV}$
- Underestimates LQCD thermodynamics

$$\hat{\chi}^{BQS}_{jkl}(T,\mu) = \left.\frac{\partial^n \hat{P}(T,\mu)}{\partial\hat{\mu}^j_B \partial\hat{\mu}^k_Q \partial\hat{\mu}^l_S}\right|_T$$

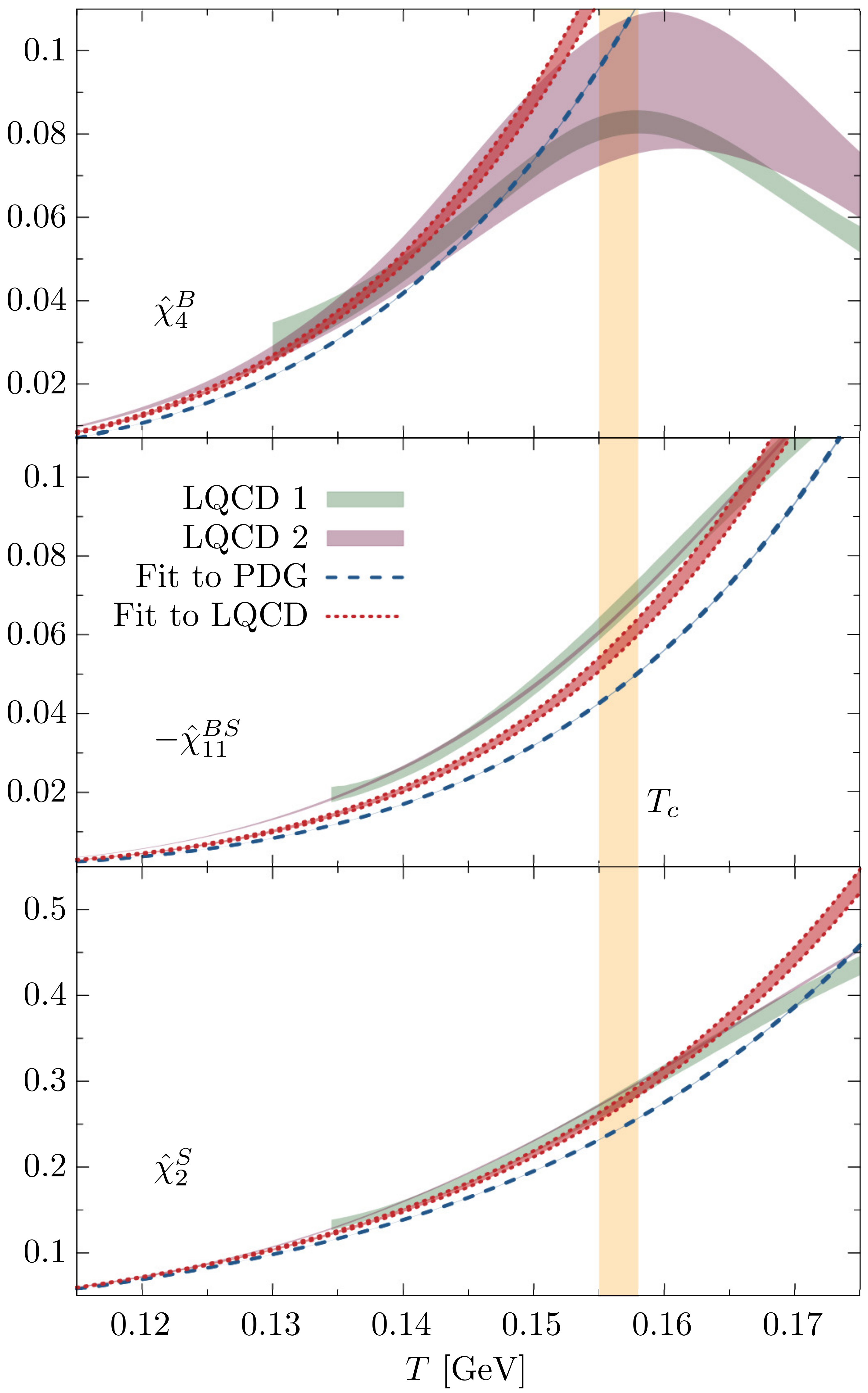
Extract spectrum from LQCD fluctuations

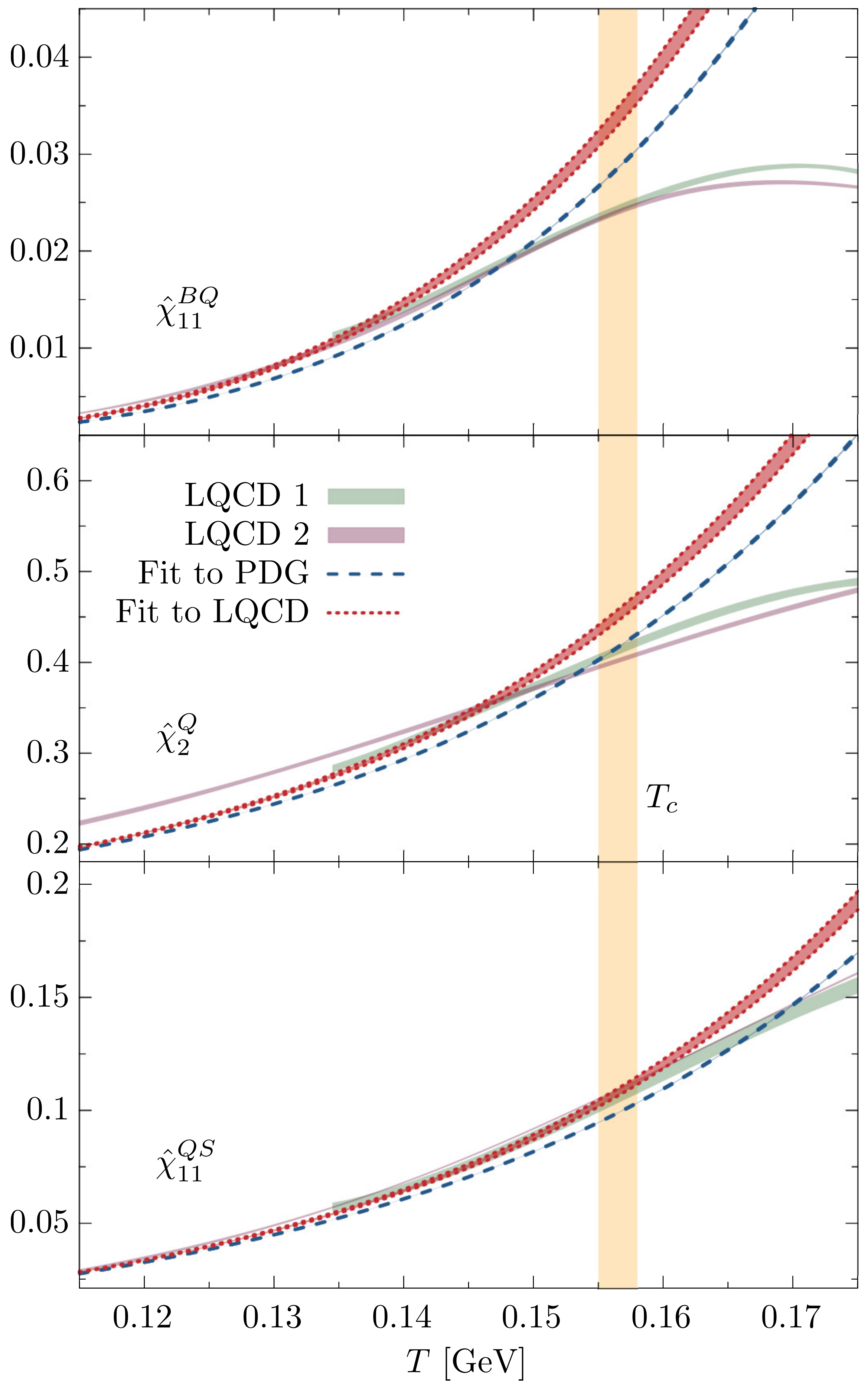
- Fit to $\hat{\chi}^B_2$ gives: $T_H \simeq 323\,\mathrm{GeV}$
- Spectrum consistent with PDG



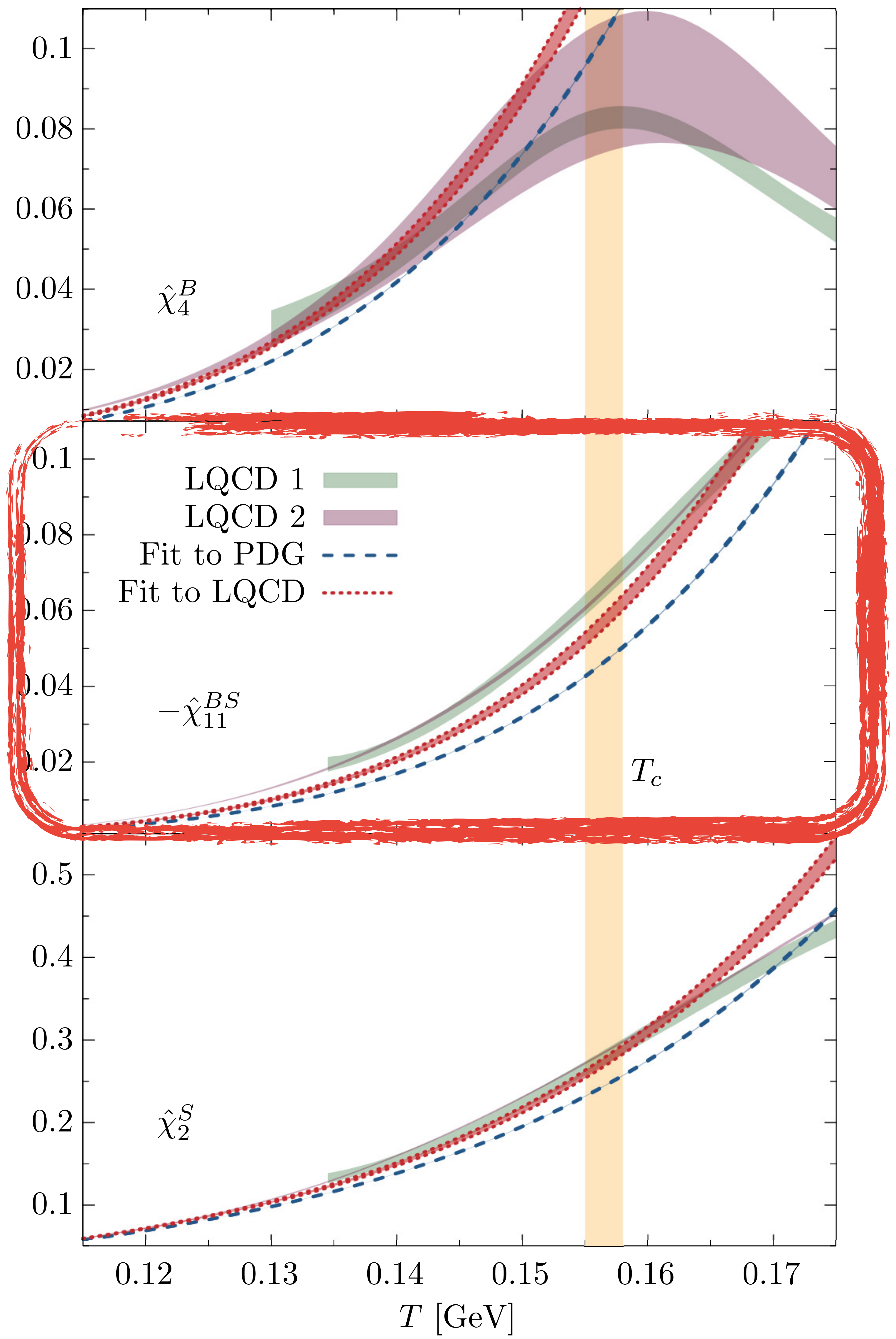
Thermodynamics exposes the full spectrum

Correlations in the Baryonic Sector





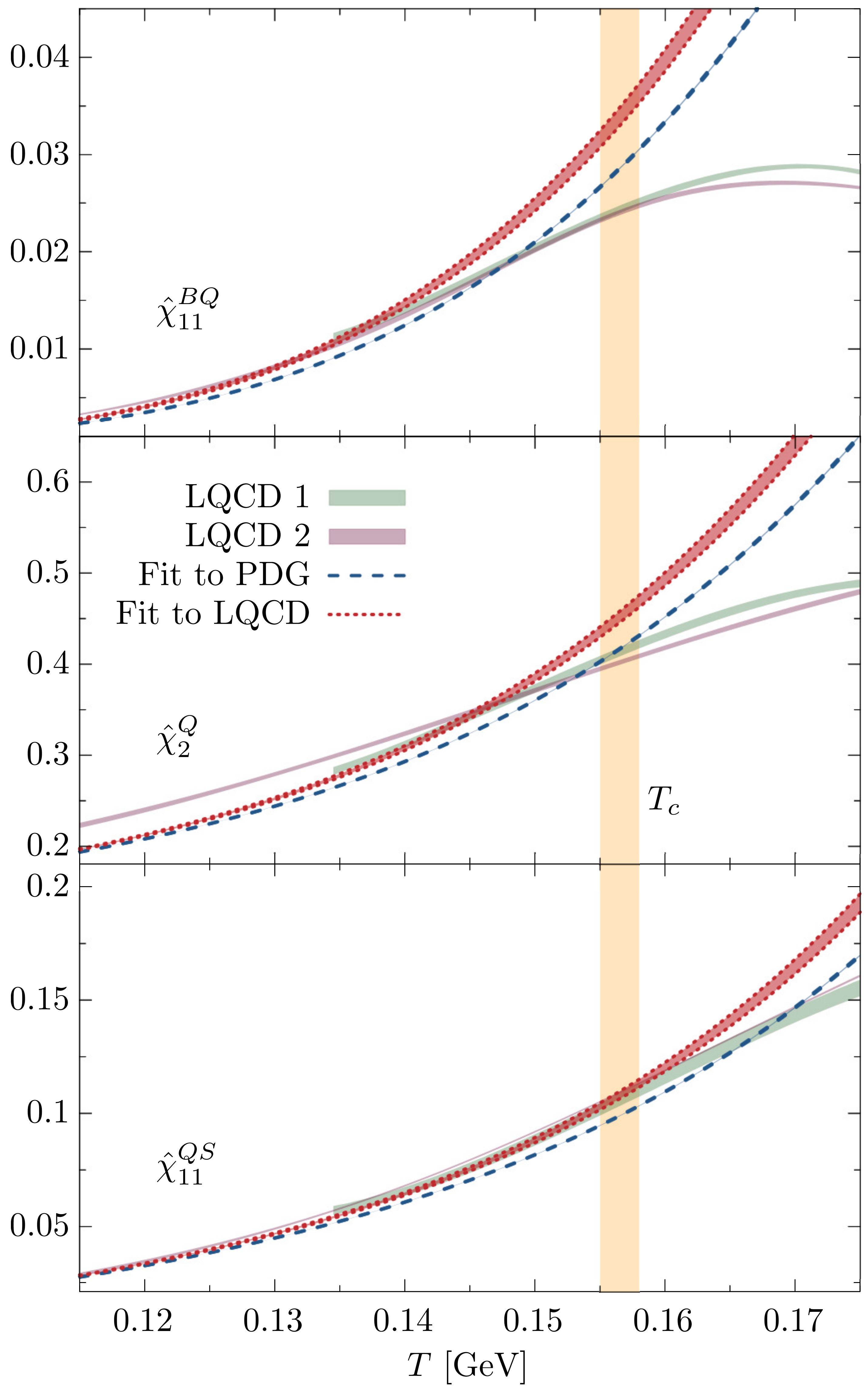
Incomplete Meson-Baryon Interactions



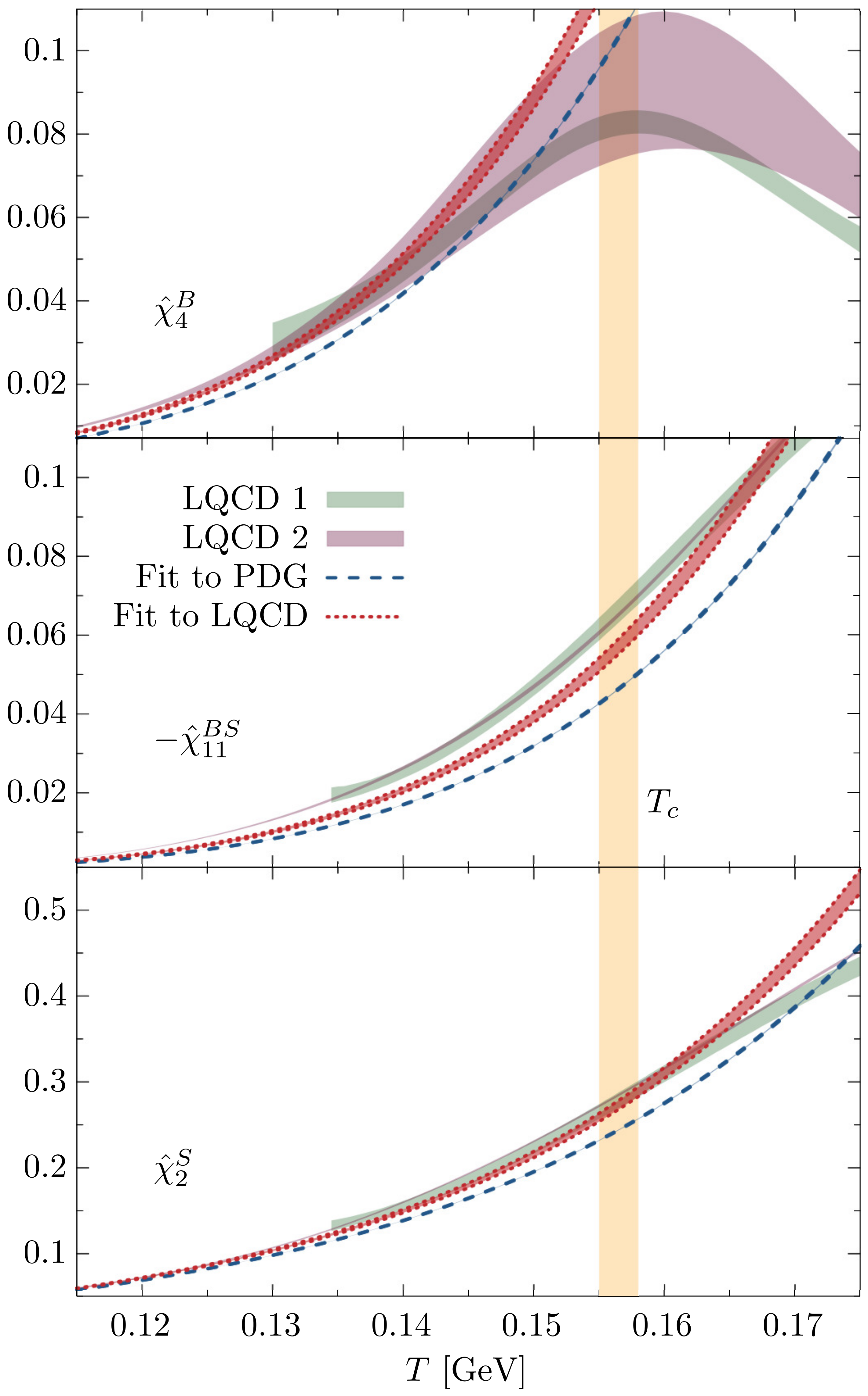
Small undershoot of $-\hat{\chi}_{11}^{BS}$

Fit based on $\hat{\chi}_2^B$

Insensitive to Strangeness



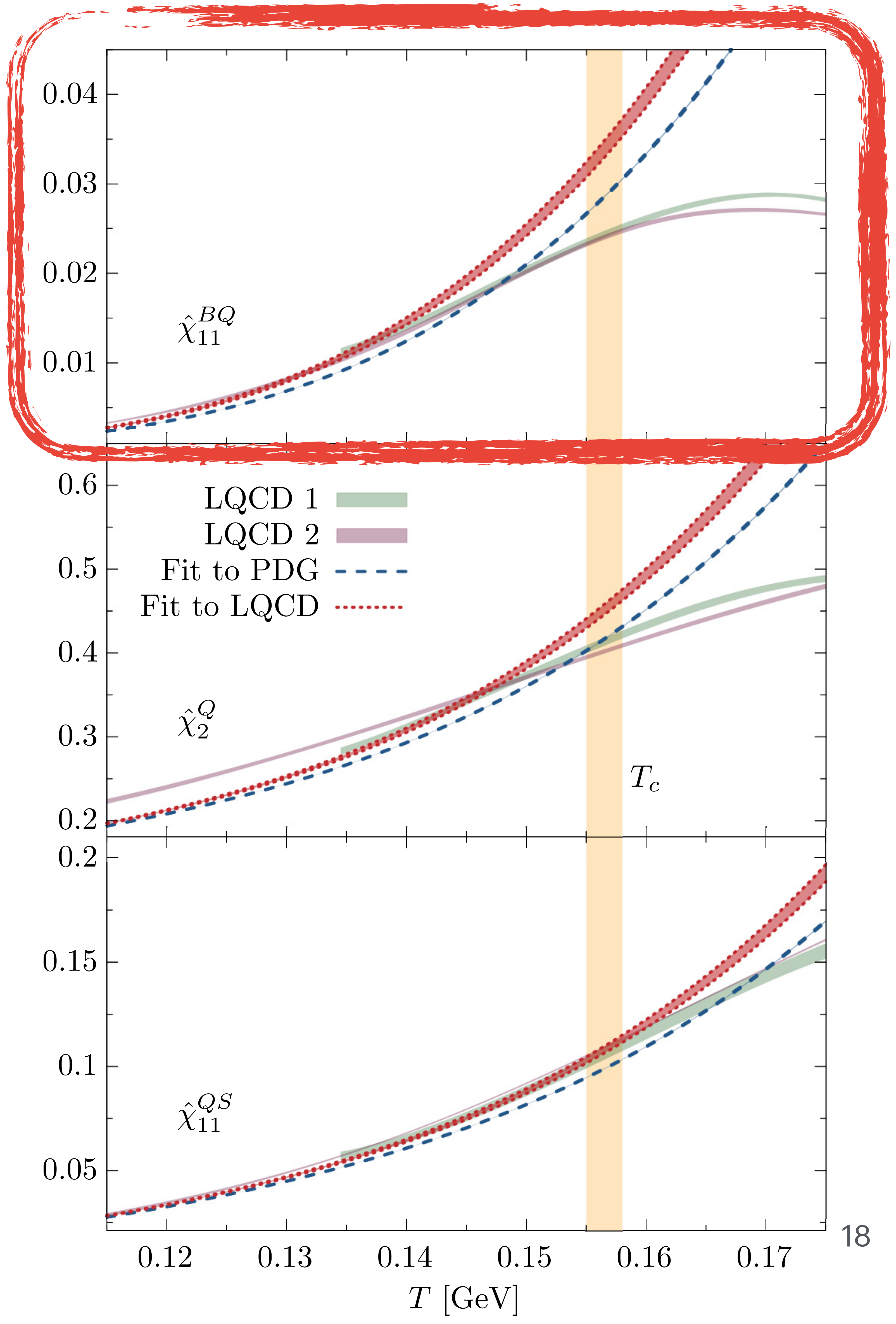
Incomplete Meson-Baryon Interactions



Overshoot of $\hat{\chi}_{11}^{BQ}$

Ideal treatment neglects non-resonant interaction

S-Matrix phase-shift analysis (Andronic et al, 2018)



Incomplete Meson-Baryon Interactions

Thermodynamics of Charm

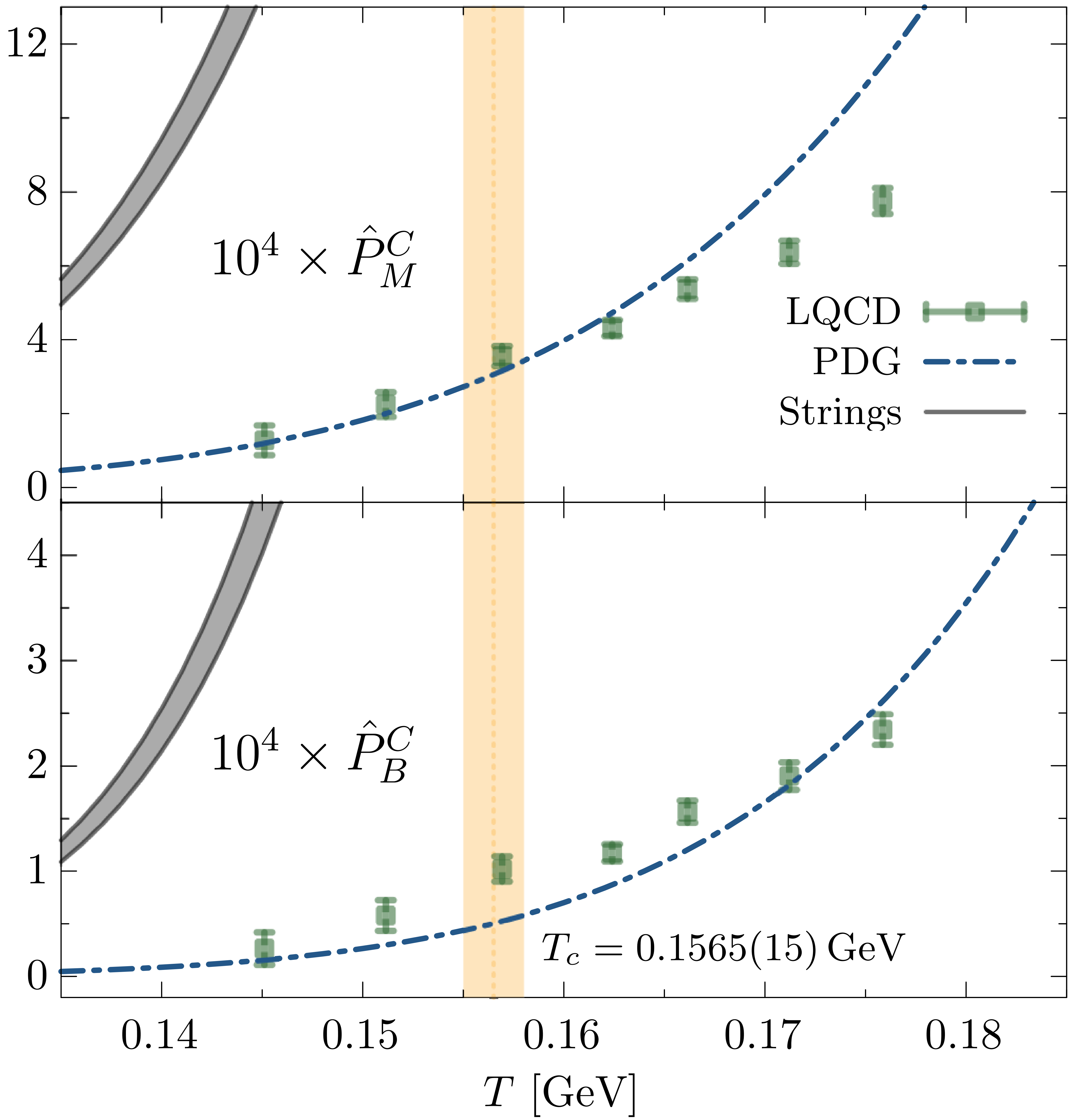
Hagedorn Spectrum overpredicts thermodynamics

- String → intrinsic flux-tube excitation
- Additional heavy quarks

Separate string excitation from the endpoint masses

$$E = m - \sum n_q m_q$$

$$\rho(m) \longrightarrow \rho(E)$$



Thermodynamics of Charm

No (re)fitting

T_H fixed from light sector

+

Quark masses from PDG

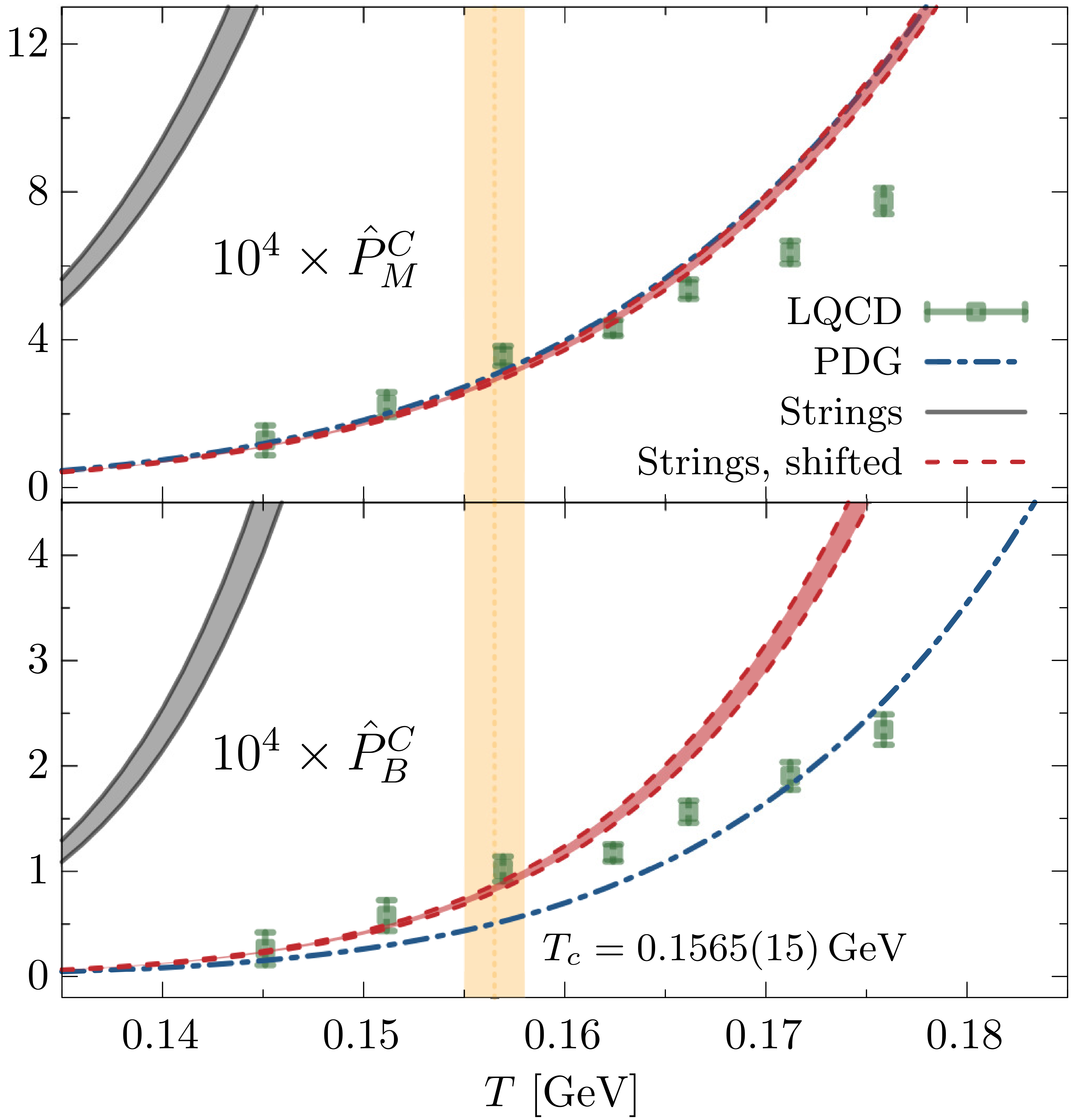
$m_s = 0.093\,\mathrm{GeV},\ m_c = 1.273\,\mathrm{GeV}$

+

$\rho(m) \to \rho(E)$

Parameter-free agreement with LQCD open-charm meson and baryon pressures

Hadronic spectra are controlled by excitation energy above the current quark mass

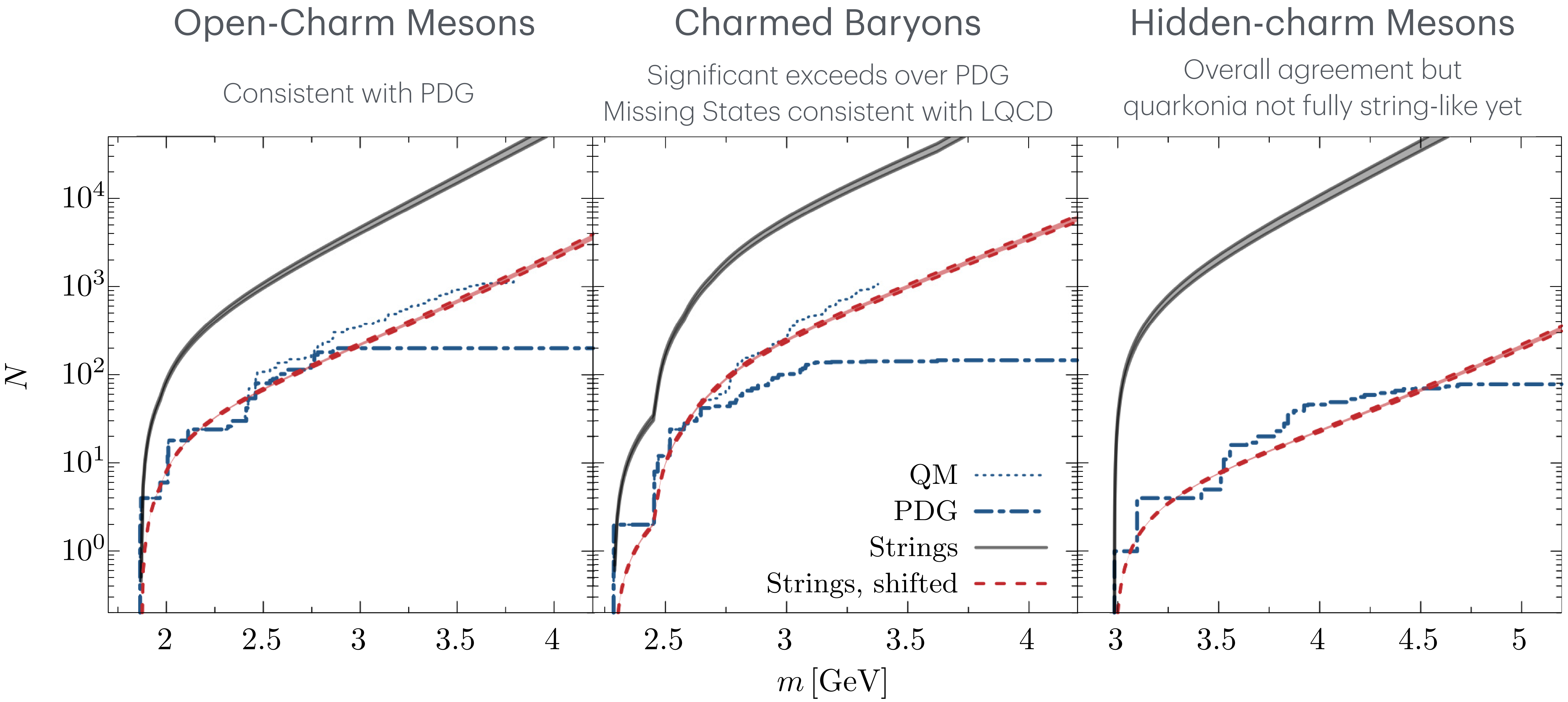


Charm Susceptibilities up to 4th Order

$$\hat{\chi}^{BQSC}_{jklm}(T,\mu) = \frac{\partial^n \hat{P}(T,\mu)}{\partial \hat{\mu}_B^j \partial \hat{\mu}_Q^k \partial \hat{\mu}_S^l \partial \hat{\mu}_C^m}\Bigg|_T$$

Overall description of LQCD thermodynamics quantitatively consistent across channels

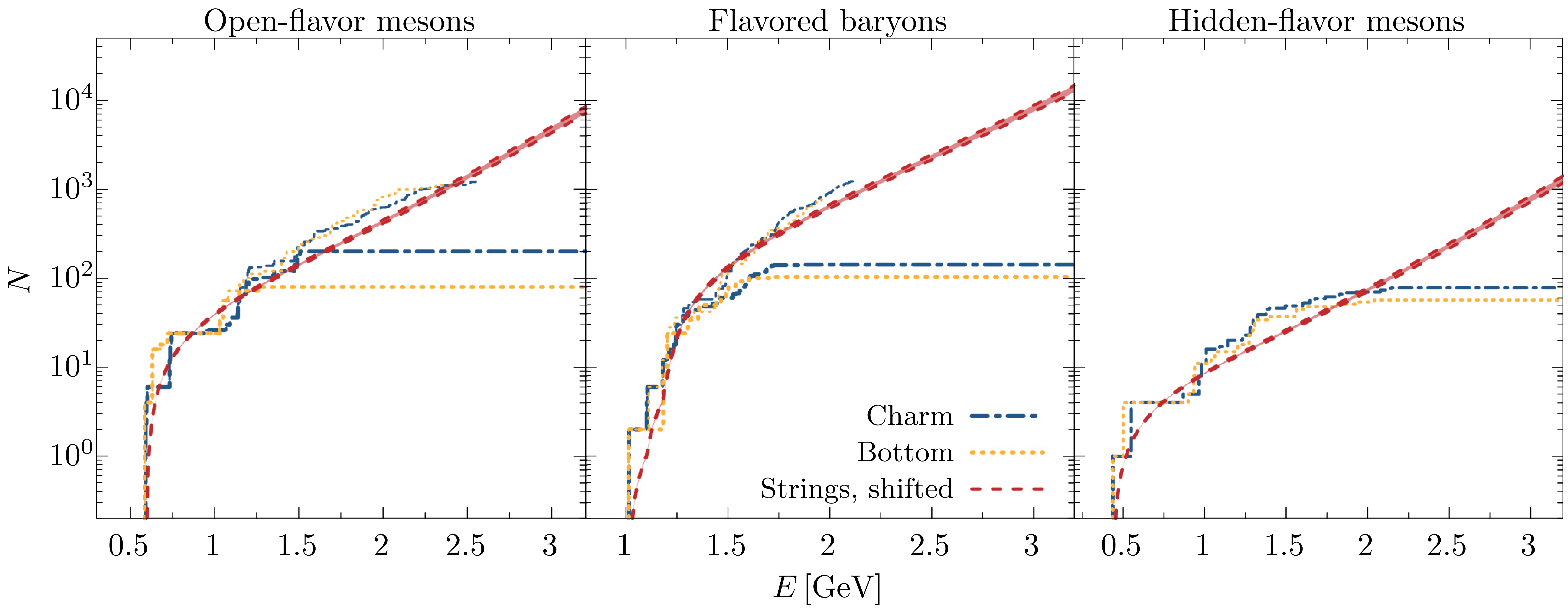
Charm Spectral Abundances



Overall excitation-energy string spectrum captures global charm-hadron systematics

Universality across quark flavors

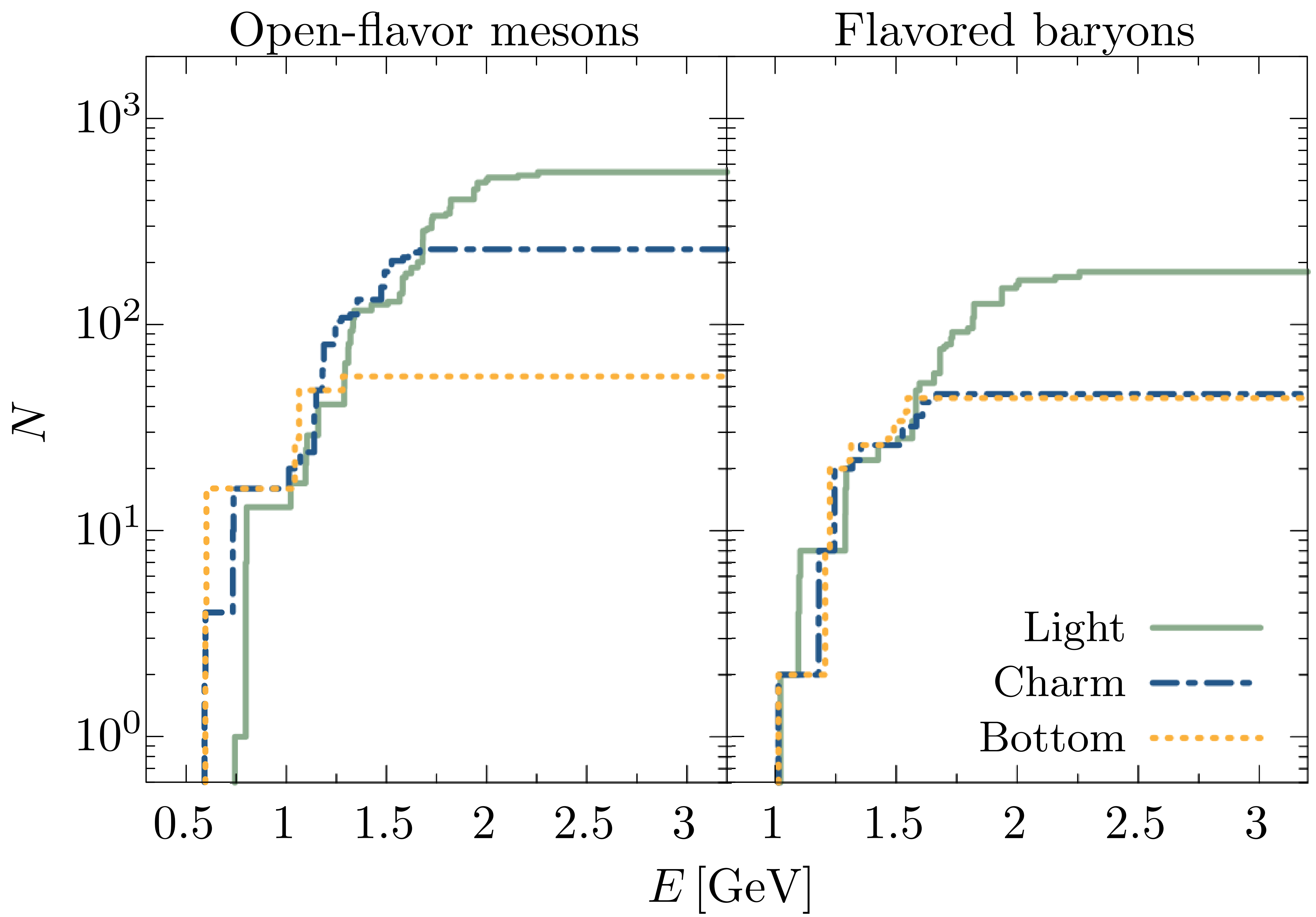
Universal Excitation Energy: $E_{\text{thr}} = m_{\text{thr}} - \sum m_q$ $\qquad E_{\text{thr}}^{\text{charm}} = E_{\text{thr}}^{\text{bottom}} \Rightarrow m_b \simeq 4.5 - 4.6\,\text{GeV}$



$\rho^{\text{charm}}(E) \simeq \rho^{\text{bottom}}(E)$ within uncertainties → Charm and bottom have the same excitation pattern

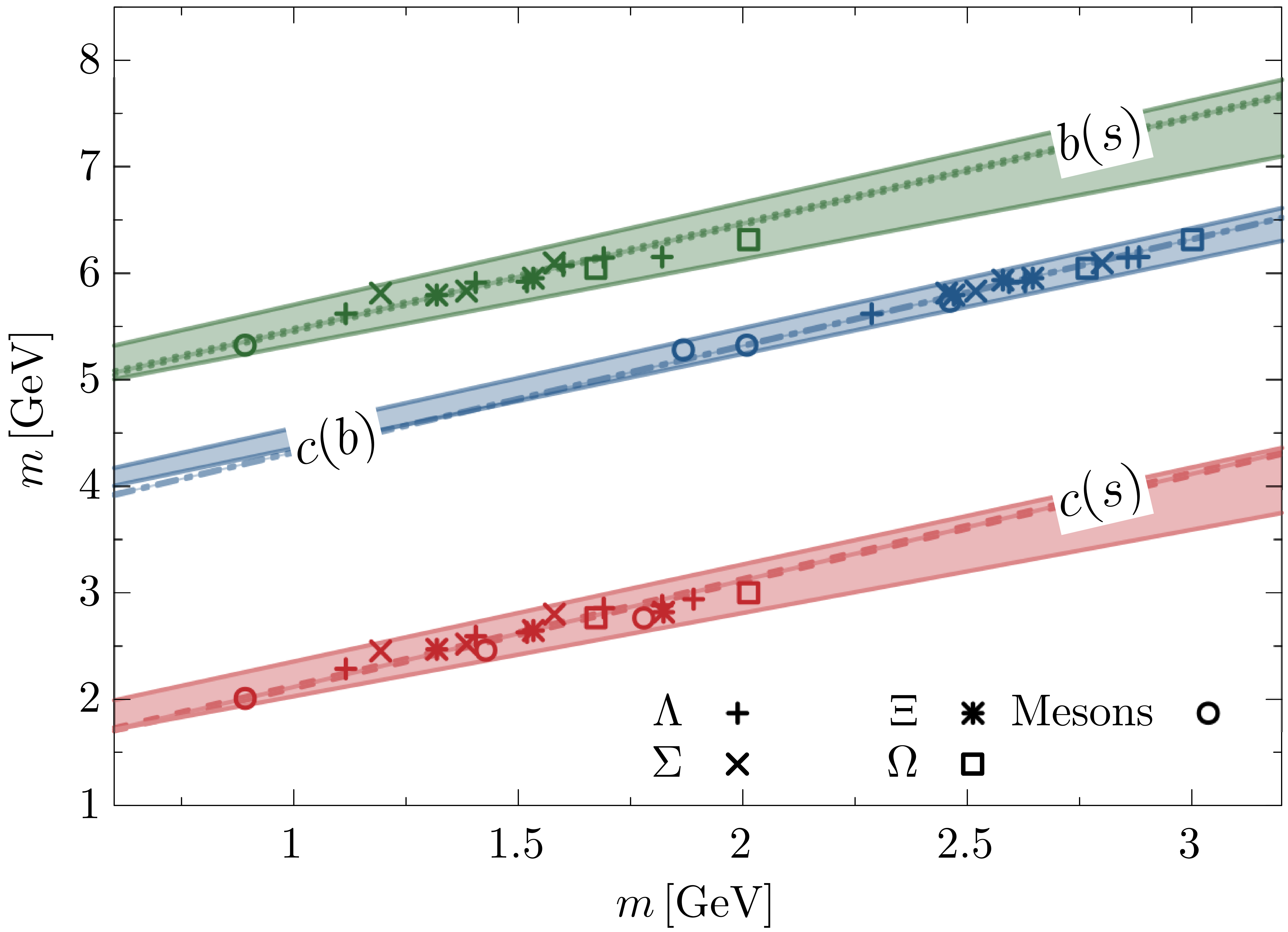
Universality across quark flavors

Collapse of experimental spectra across flavors



T_H is a fundamental and flavor-independent property of QCD

Universality across quark flavors



Regge Phenomenology and Flavor Dependence

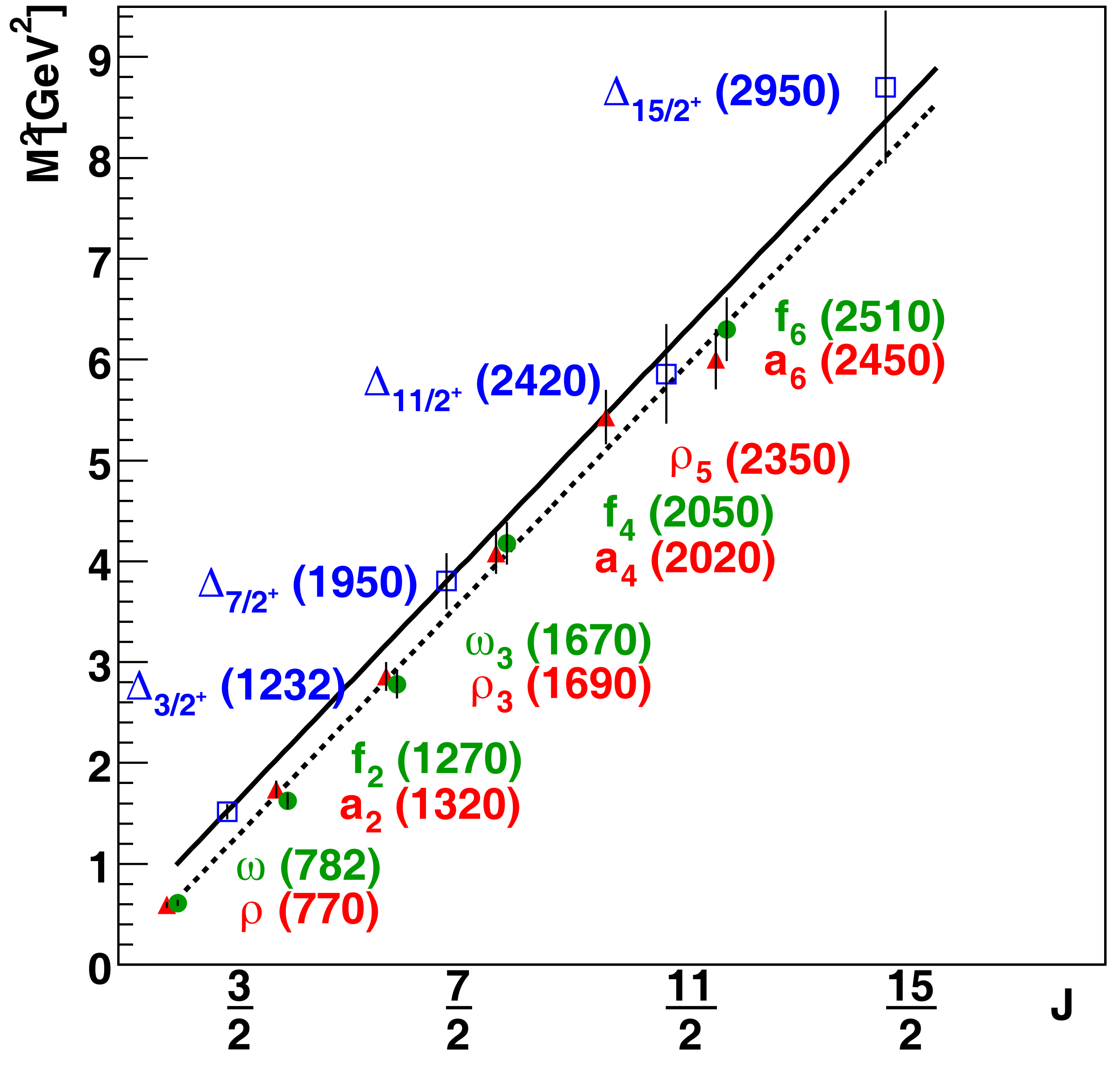
Experimental observation

Light hadrons follow approximately linear trajectories

$$J \simeq \alpha M^2 + \beta \quad \text{where} \quad \alpha \equiv \frac{dJ}{dM^2} = (2\pi\sigma)^{-1}$$

see Chew & Frautschi (1962), Collins (1977)

Klempt, Mensch (2012)

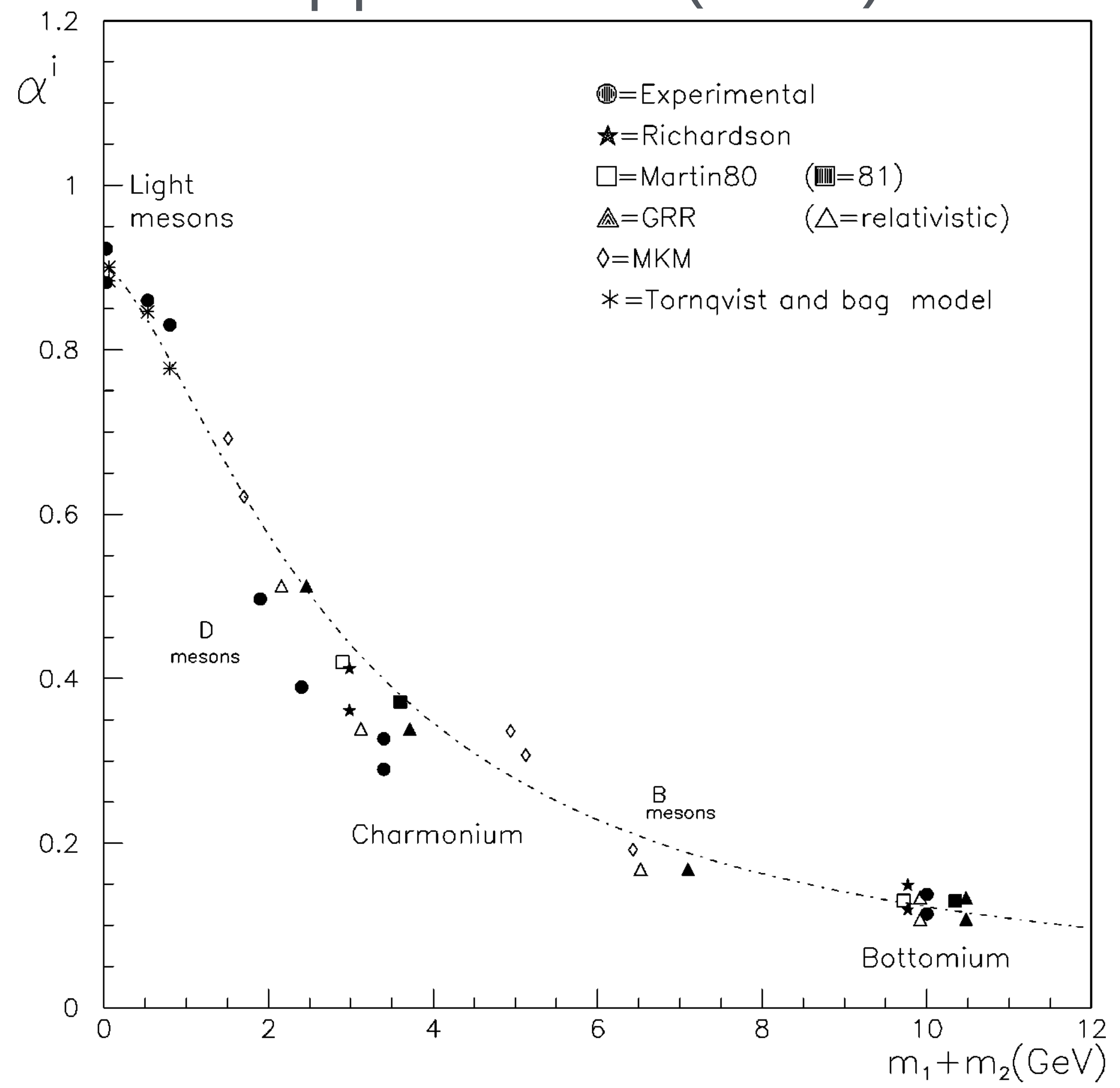


Heavy Flavor suppresses the slope

reduced effective slopes and apparent flavor dependence

see Filipponi & Srivastava (1998), Afonin (2007), Sonnenschein & Weissman (2014)

Filipponi et al (1998)



Is the flavor dependence dynamical or kinematic?

Excitation Energy Reveals Linearity

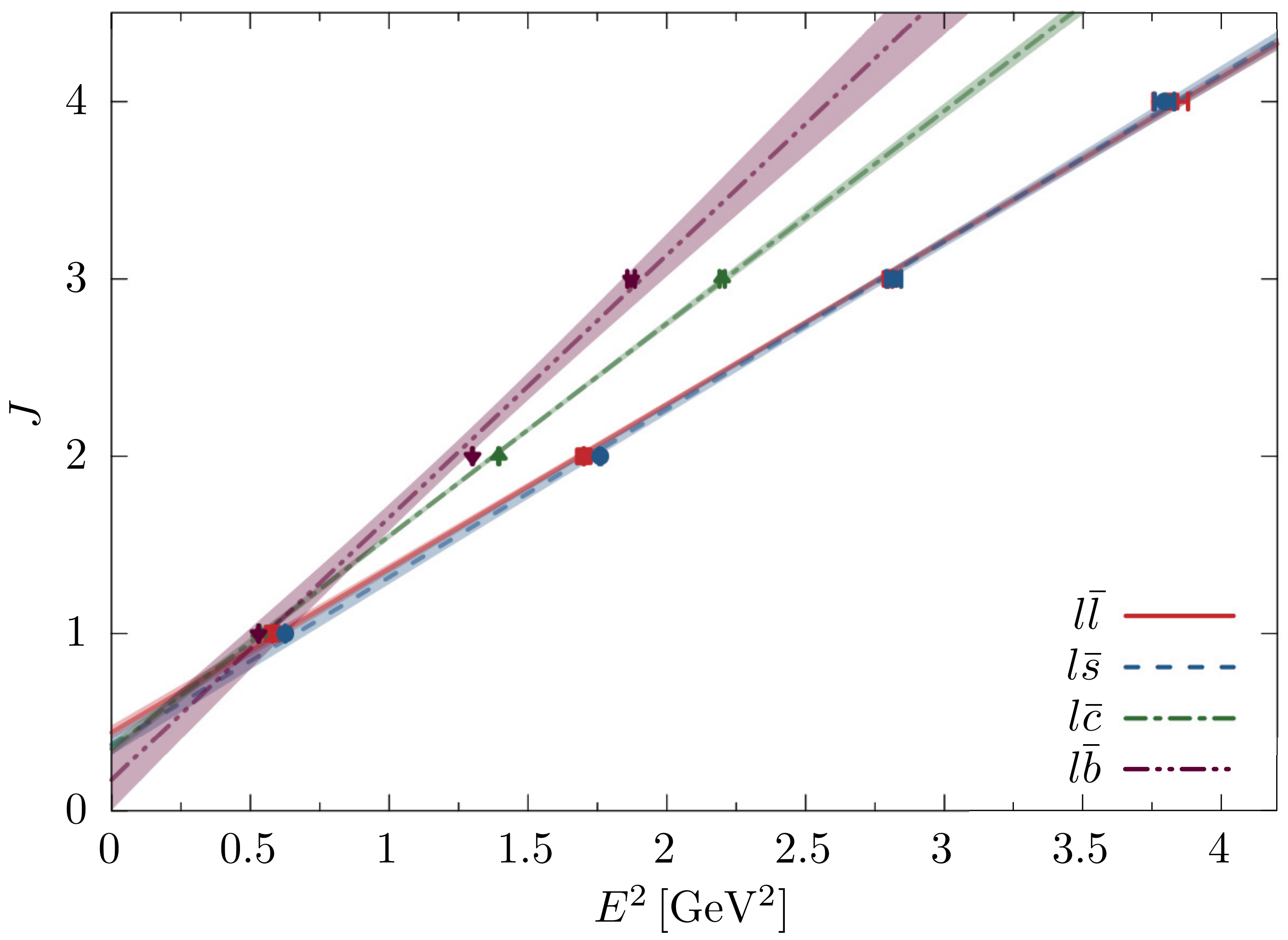
Excitation Energy Representation

- $J(E^2) = \alpha_E E^2 + \beta_E$
- $\alpha_E = \dfrac{dJ}{dE^2} = const$

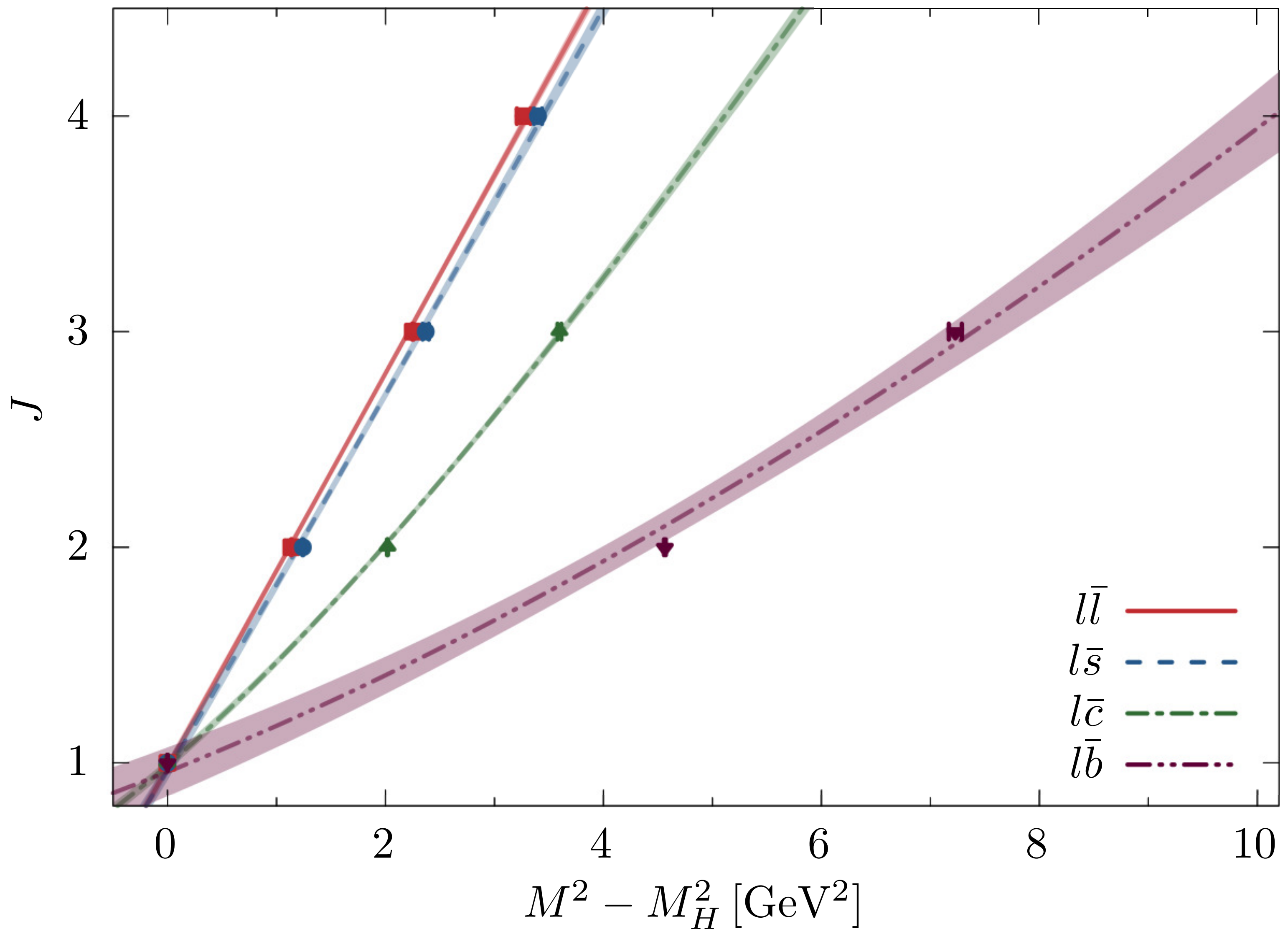
Physical-Mass Representation

- $J(M^2) = \alpha_M M^2 + \beta_M$
- $\alpha_M = \dfrac{dJ}{dM^2} = \alpha_E \left(1 - \sum m_q/M\right)$

Trajectories for ρ, K^*, D^*, B^*



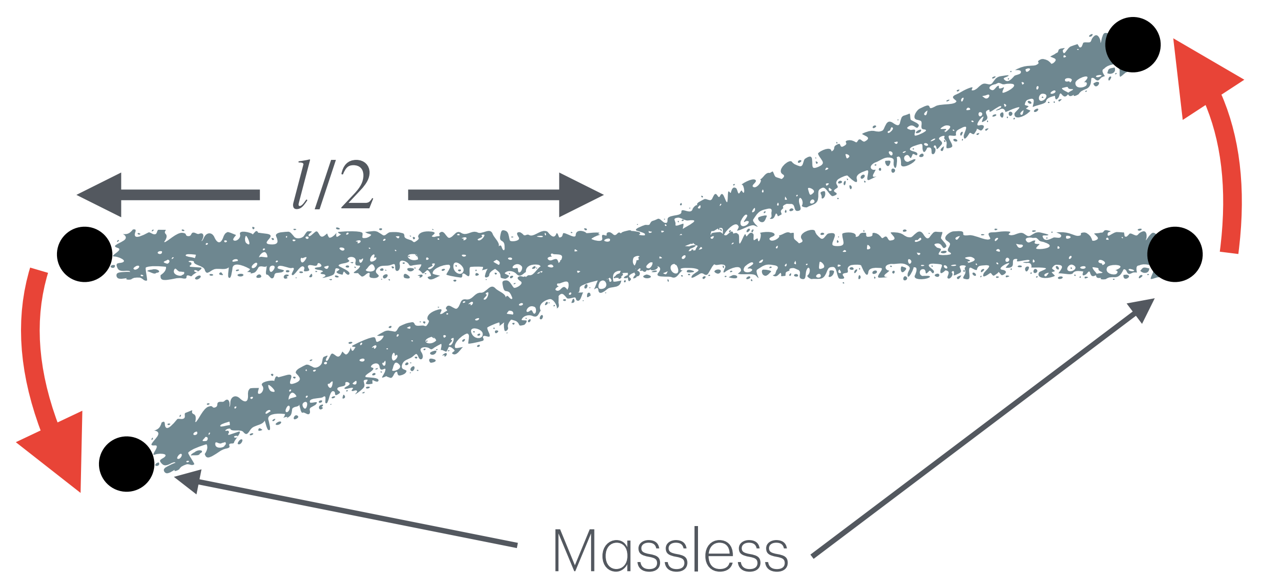
Slope hierarchy: $\alpha_E^{l\bar{l}} < \alpha_E^{l\bar{s}} < \alpha_E^{l\bar{c}} < \alpha_E^{l\bar{b}}$



Slope hierarchy: $\alpha_M^{l\bar{l}} > \alpha_M^{l\bar{s}} > \alpha_M^{l\bar{c}} > \alpha_M^{l\bar{b}}$

Geometry of Rotating String

Light-Light String: $m_1 = m_2 = 0$

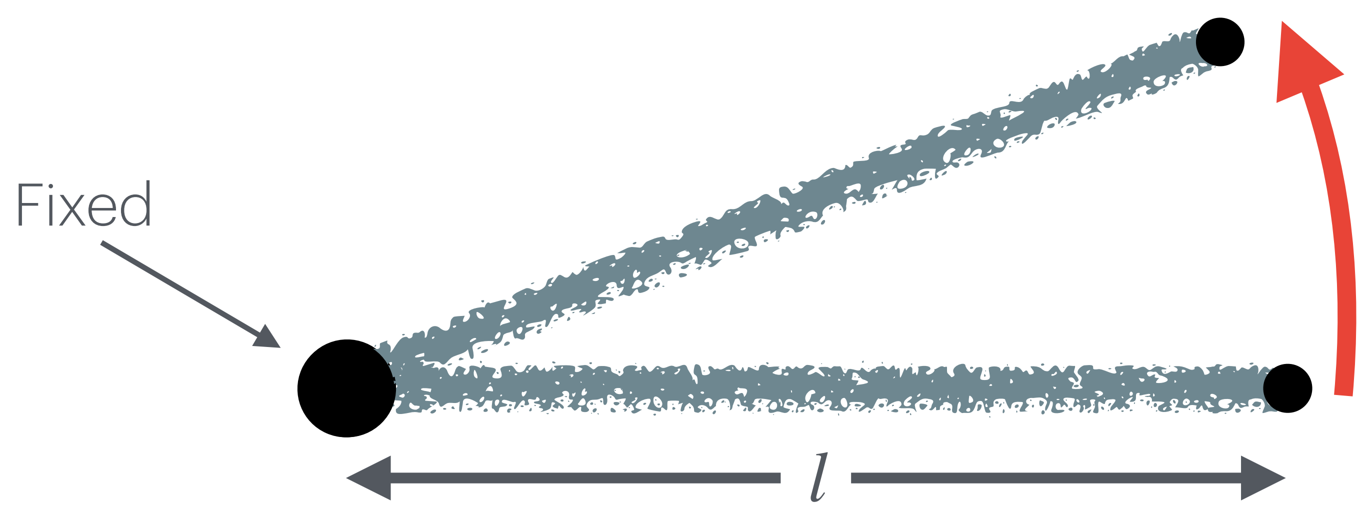


$$M = 2\int_0^{l/2} dr \frac{\sigma}{\sqrt{1 - v^2(r)}} = \frac{1}{2}\pi\sigma l$$

$$L = 2\int_0^{l/2} dr \frac{\sigma r v(r)}{\sqrt{1 - v^2(r)}} = \frac{1}{8}\pi\sigma l^2$$

$$\longrightarrow \alpha_{LL} = \frac{1}{2\pi\sigma}$$

Heavy-Light String: $m_1 = 0,\ m_2 = \infty$



$$M = 2\int_0^{l} dr \frac{\sigma}{\sqrt{1 - v^2(r)}} = \frac{1}{2}\pi\sigma l$$

$$L = 2\int_0^{l} dr \frac{\sigma r v(r)}{\sqrt{1 - v^2(r)}} = \frac{1}{4}\pi\sigma l^2$$

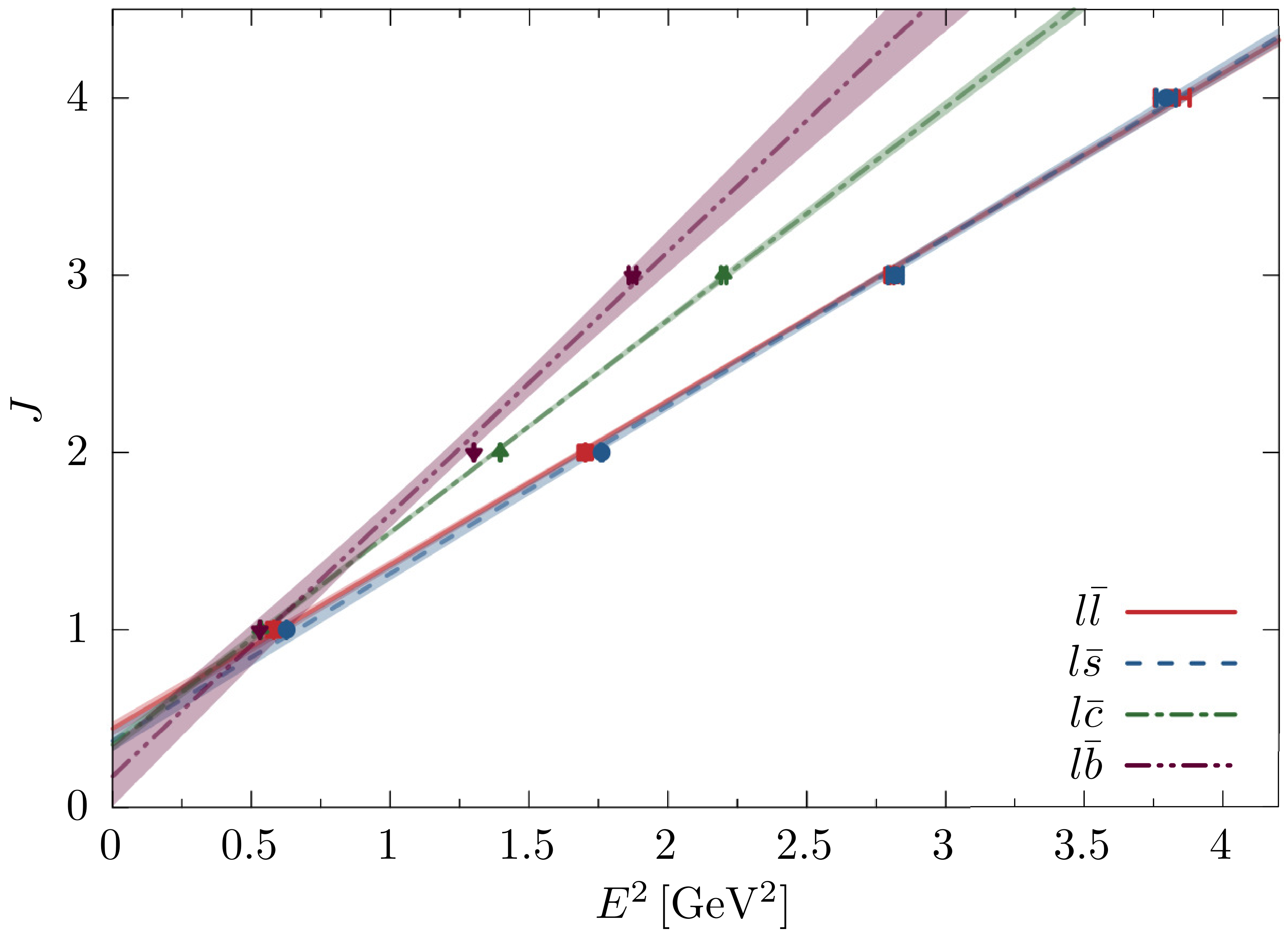
$$\longrightarrow \alpha_{HL} = \frac{1}{\pi\sigma}$$

$$\alpha_{LL} = \frac{1}{2}\alpha_{HL}$$

Geometry changes the slope, but string tension is universal

Geometrical Correction

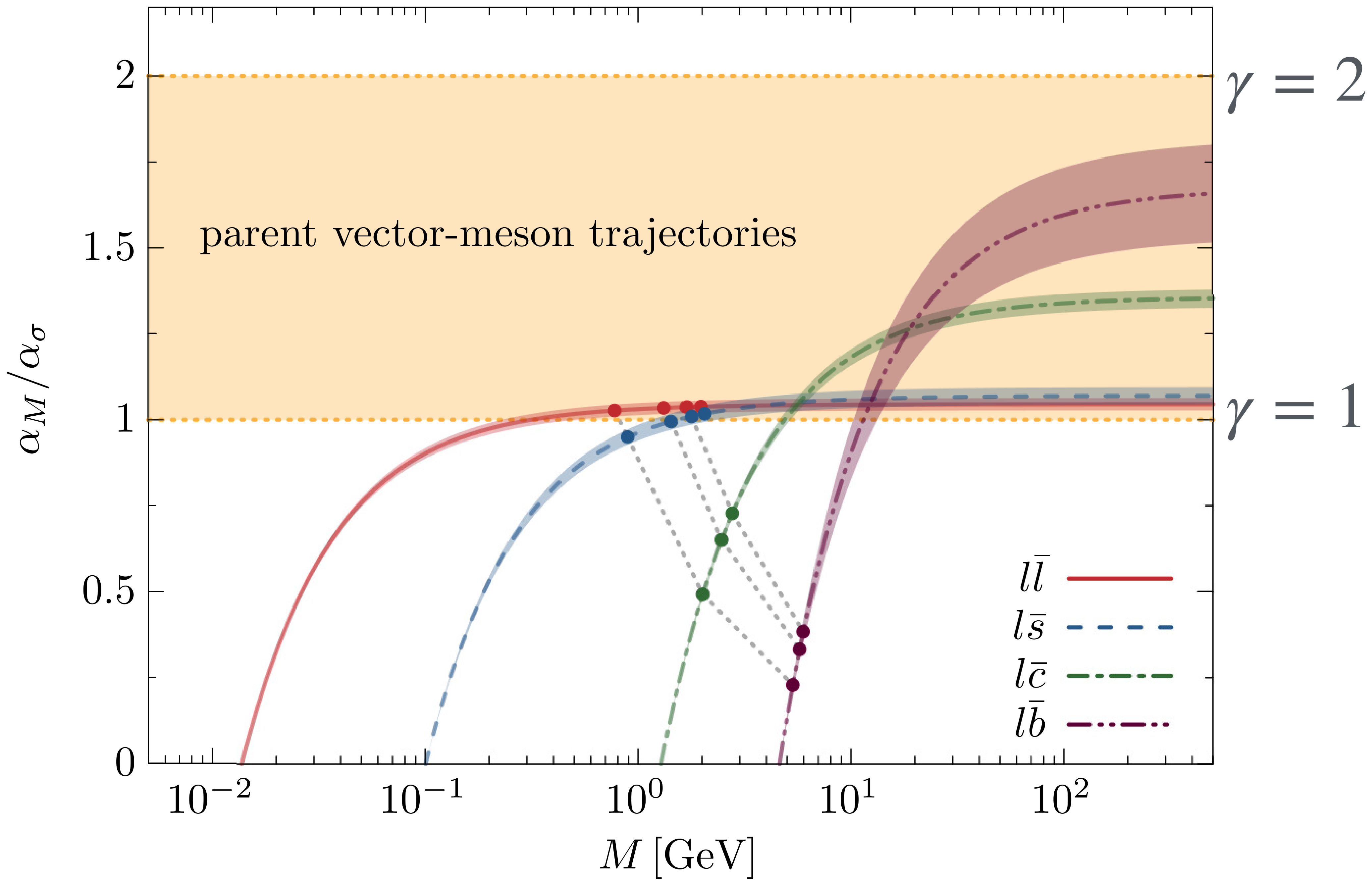
Introduce correction: $\alpha_E \longrightarrow \gamma \cdot \alpha_E = \gamma \cdot (2\pi\sigma)^{-1}$



For $\sigma = 0.18\,\mathrm{GeV}^2$

- $\gamma_l = 1.046(18)$
- $\gamma_s = 1.070(25)$
- $\gamma_c = 1.356(27)$
- $\gamma_b = 1.673(145)$

As $M \to \infty$ the ratio $\alpha_M/\alpha_\sigma \to \gamma$



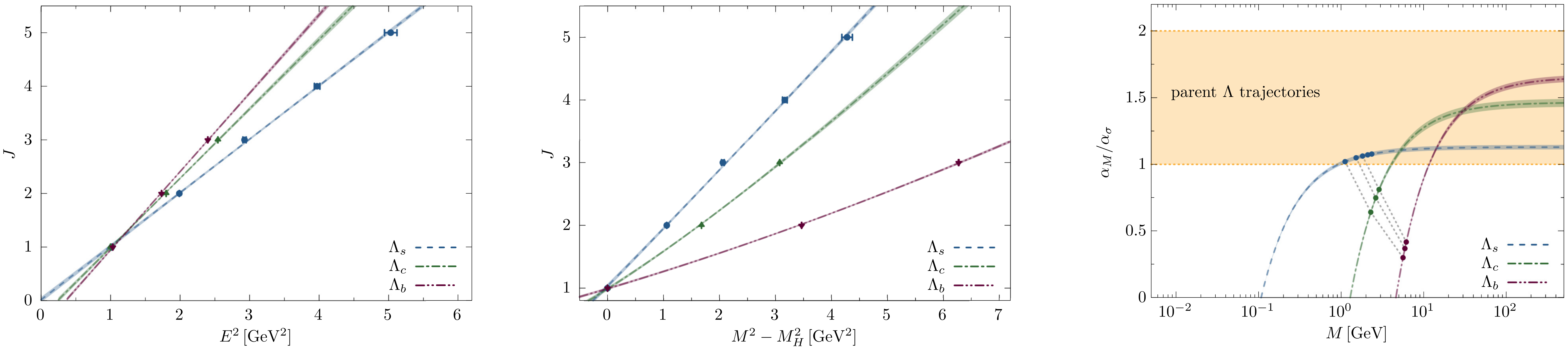
Low M:

- Hierarchy dominated by kinematic mass effects
- $J(M^2)$ trajectories non-linear

High M:

- Hierarchy from asymptotic string geometry
- $J(M^2)$ trajectories linearize

Λ Baryons as quark-diquark strings



Vector Mesons	Lambda Baryons
$\gamma_s = 1.070(25)$	$\gamma_s = 1.129(17)$
$\gamma_c = 1.356(27)$	$\gamma_c = 1.464(30)$
$\gamma_b = 1.673(145)$	$\gamma_b = 1.655(24)$

Similar enhancement factors for mesons and baryons support Hagedorn universality and the quark–diquark picture of baryons.

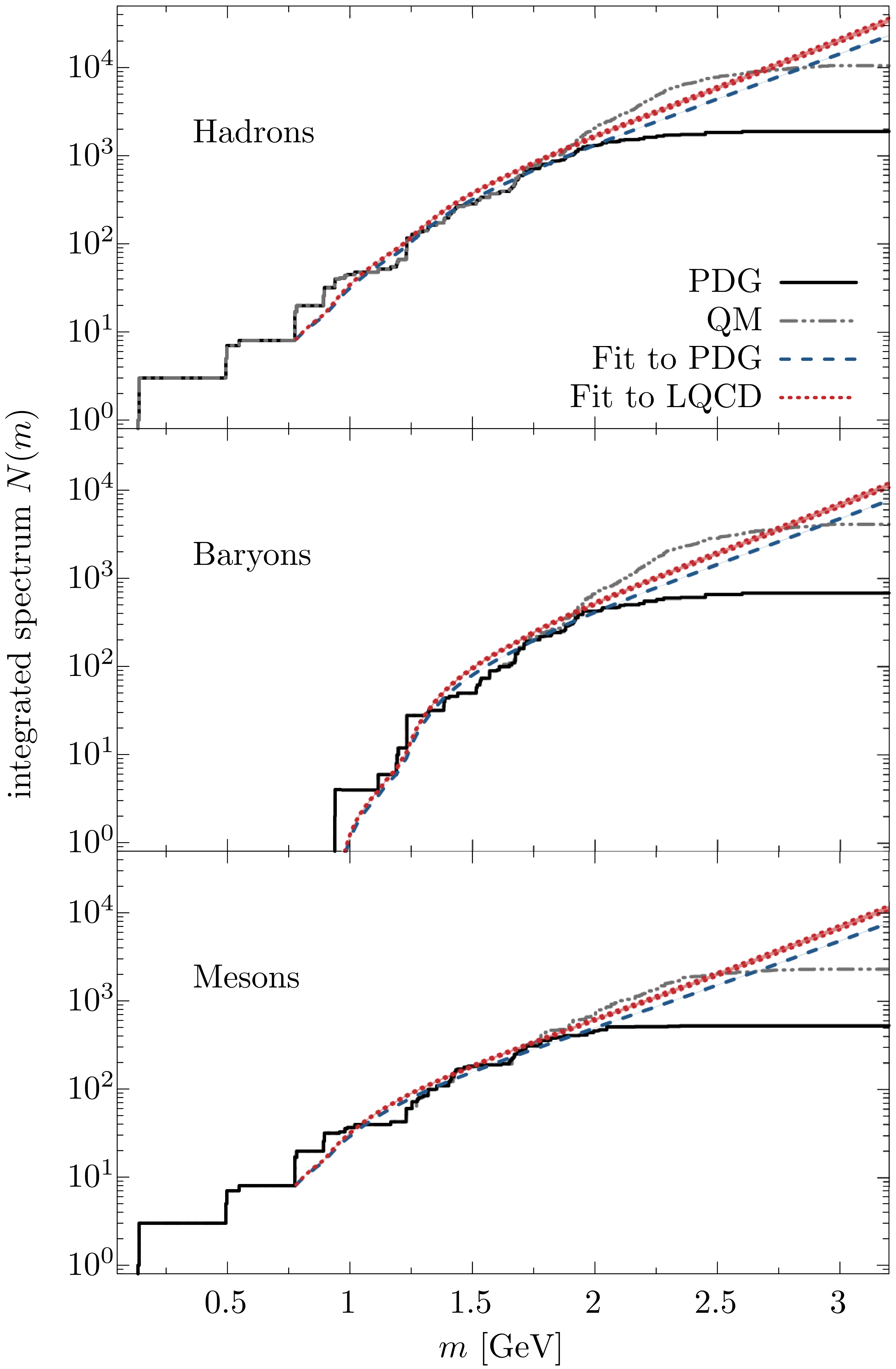
QCD spectroscopy governed by a universal excitation-energy string structure

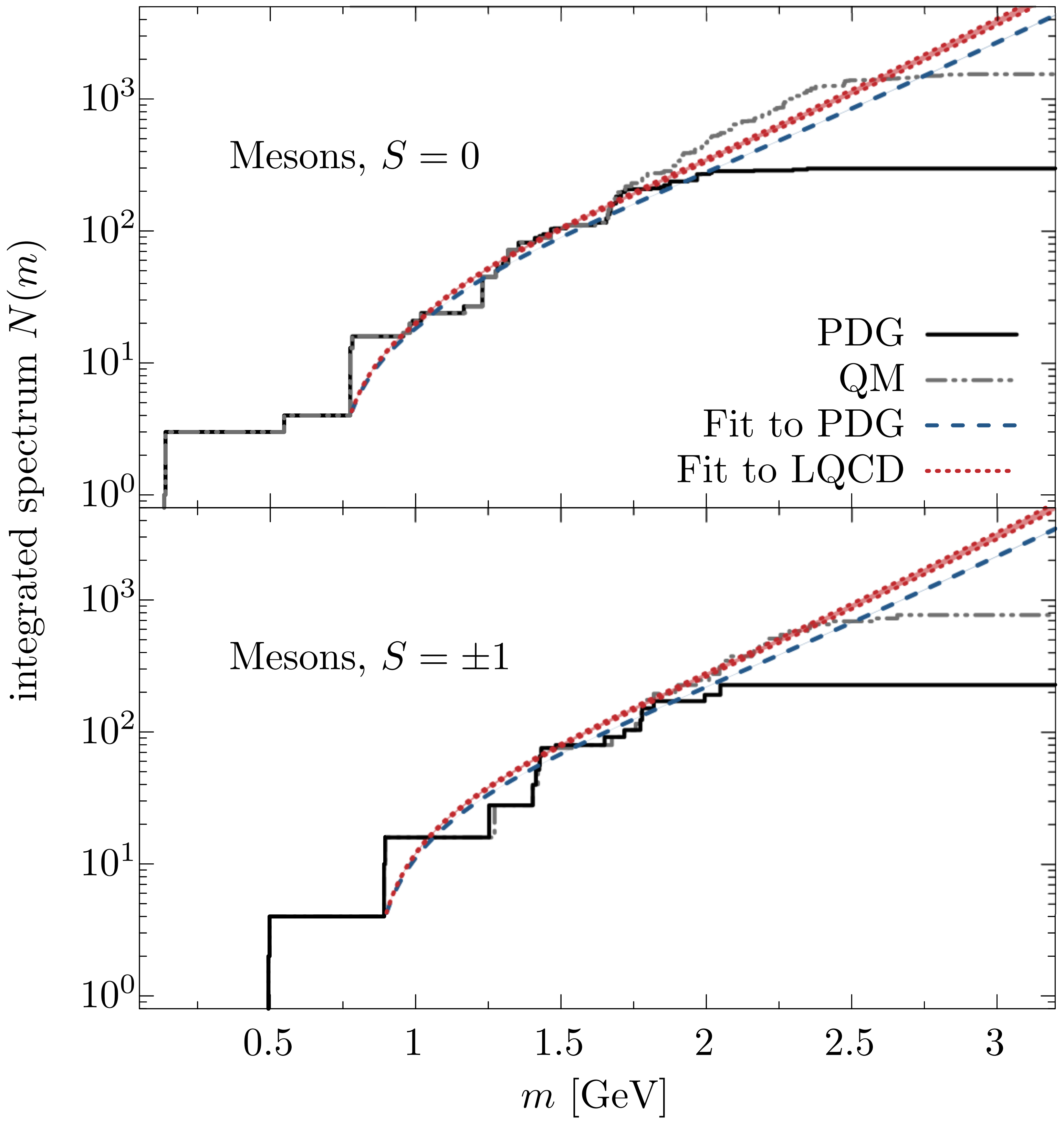
Hadrons emerge naturally as strings

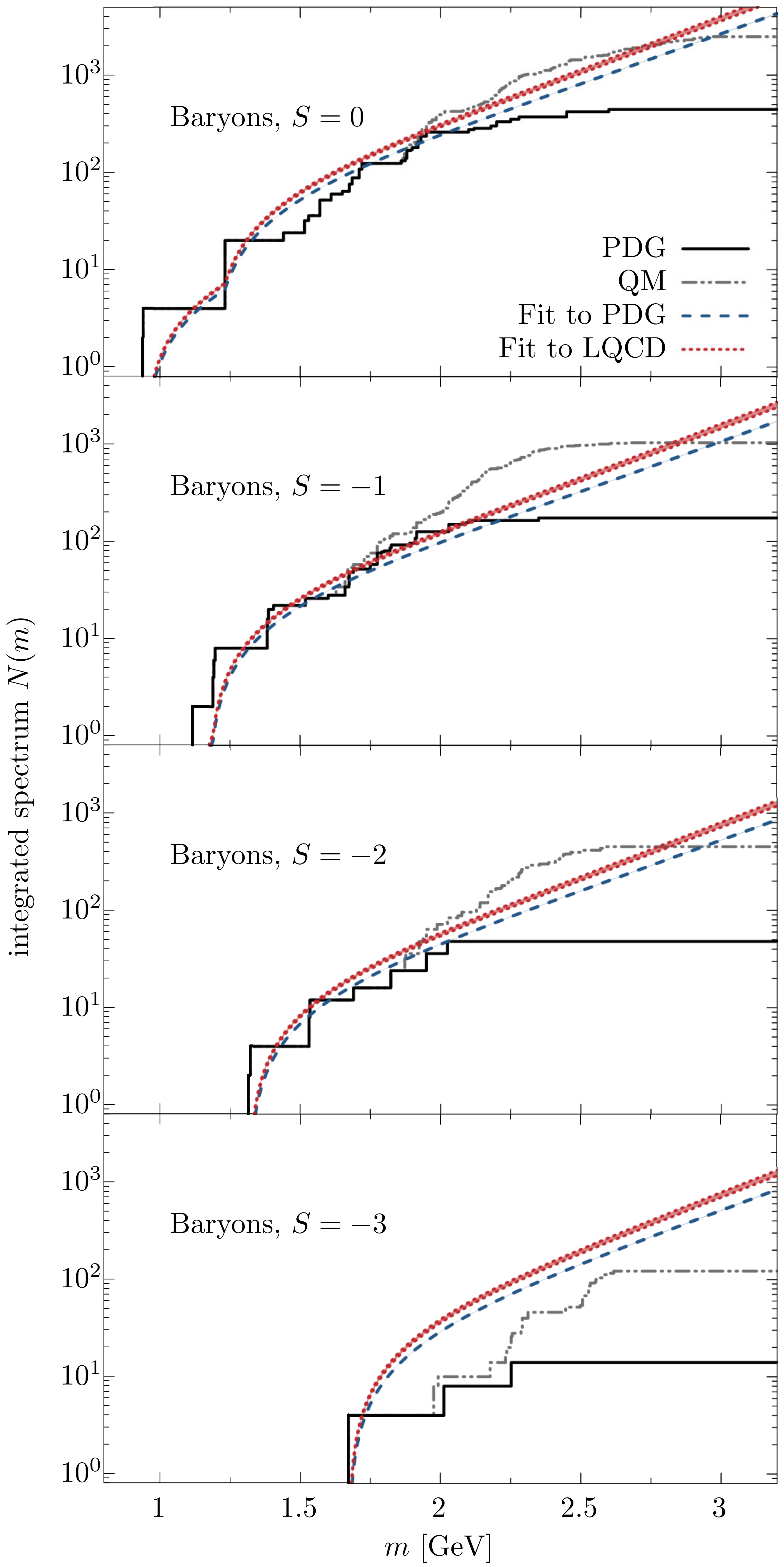
Exponentially growing spectra are governed by a universal T_H

Scale set by the string tension

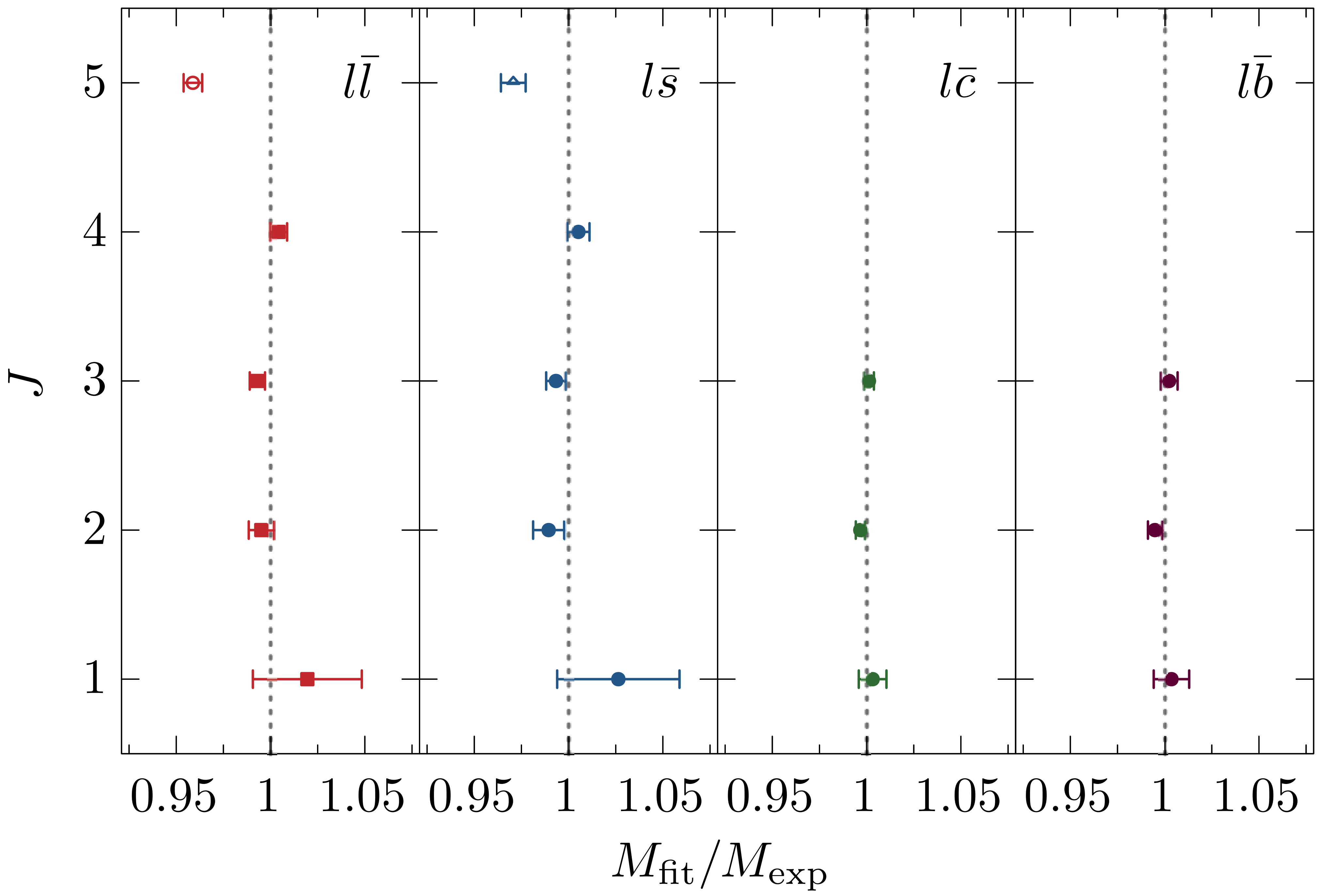
Thank You


Hadrons
Baryons
Mesons
PDG
QM
Fit to PDG
Fit to LQCD
integrated spectrum $N(m)$
m [GeV]


Mesons, $S = 0$
Mesons, $S = \pm 1$
PDG
QM
Fit to PDG
Fit to LQCD
integrated spectrum $N(m)$
m [GeV]


Baryons, $S = 0$
Baryons, $S = -1$
Baryons, $S = -2$
Baryons, $S = -3$
PDG
QM
Fit to PDG
Fit to LQCD
integrated spectrum $N(m)$
m [GeV]

Regge trajectories Fit



Mesons

sector	α_E [GeV^{-2}]	β_E	$\gamma = \alpha_E/\alpha_\sigma$
$l\bar{l}$	0.924(16)	0.442(40)	1.046(18)
$l\bar{s}$	0.946(23)	0.372(57)	1.070(25)
$l\bar{c}$	1.199(25)	0.350(37)	1.356(27)
$l\bar{b}$	1.479(127)	0.174(172)	1.673(145)

Baryons

sector	α_E [GeV^{-2}]	β_E	$\gamma = \alpha_E/\alpha_\sigma$
Λ ([ud]s)	0.998(15)	0.018(49)	1.129(17)
Λ_c ([ud]b)	1.295(27)	-0.305(50)	1.464(30)
Λ_b ([ud]b)	1.463(26)	-0.518(39)	1.655(24)

Channel	mass [GeV]	I	Q	S	deg
$\rho\ (l\bar{l})$	0.770	1	$0, \pm 1$	0	16
$K^*\ (l\bar{s},\ s\bar{l})$	0.896	1/2	$0, \pm 1$	± 1	16
$\phi\ (s\bar{s})$	1.019	0	0	0	4

State	String	J	I	S	Mass [GeV] $Q = c$	Mass [GeV] $Q = b$	g
$D,\ B$	$Q\bar{l},\ \bar{Q}l$	0	1/2	0	1.867	5.279	16
$D_s,\ B_s$	$Q\bar{s},\ \bar{Q}s$	0	0	± 1	1.968	5.367	8
$\eta_c,\ \eta_b$	$Q\bar{Q}$	0	0	0	2.984	9.398	4

Channel	mass [GeV]	I	Q	S	deg
$N\ (l[ll])$	0.938	1/2	$(0,\ 1)$	0	4
$\Lambda\ (s[ll])$	1.116	0	(0)	-1	2
$\Sigma\ (l[ls])$	1.189	1	$(0,\ \pm 1)$	-1	6
$\Delta\ (l\{ll\})$	1.232	3/2	$(0,\ \pm 1,\ 2)$	0	16
$\Xi\ (s[ls])$	1.315	1/2	$(0,\ -1)$	-2	4
$\Omega\ (s\{ss\})$	1.672	0	(-1)	-3	4

State	String	J	I	S	Mass [GeV] $Q = c$	Mass [GeV] $Q = b$	g
Λ_Q	$Q[ll]$	1/2	0	0	2.286	5.620	2
Σ_Q	$Q\{ll\}$	1/2, 3/2	1	0	2.453	5.811	18
Ξ_Q	$Q[ls]$	1/2	1/2	-1	2.468	5.793	4
Ξ'_Q	$Q\{ls\}$	1/2, 3/2	1/2	-1	2.576	5.935	12
Ω_Q	$Q\{ss\}$	1/2, 3/2	0	-2	2.695	6.046	6
Ξ_{QQ}	$l\{QQ\}$	1/2, 3/2	1/2	0	3.519	–	12
Ω_{QQ}	$s\{QQ\}$	1/2, 3/2	0	-1	3.778	–	6
Ω_{QQQ}	$Q\{QQ\}$	3/2	0	0	4.800	–	4