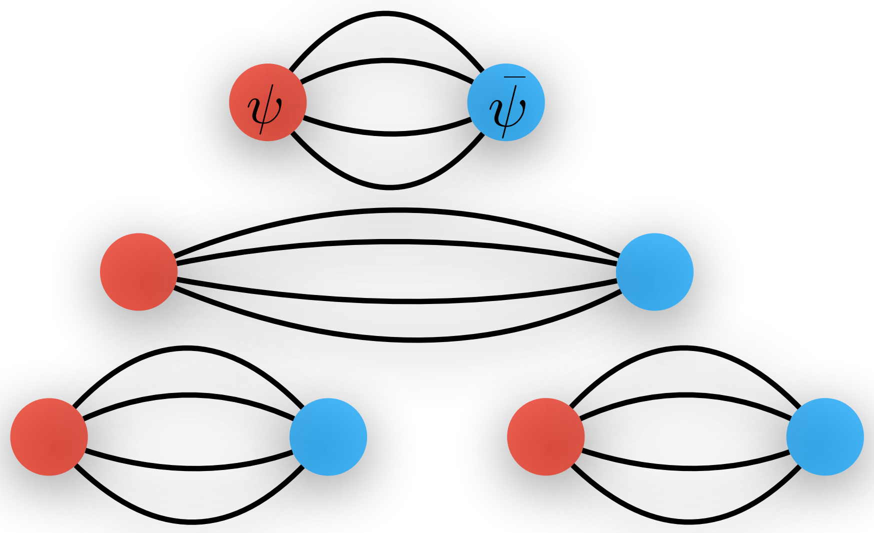
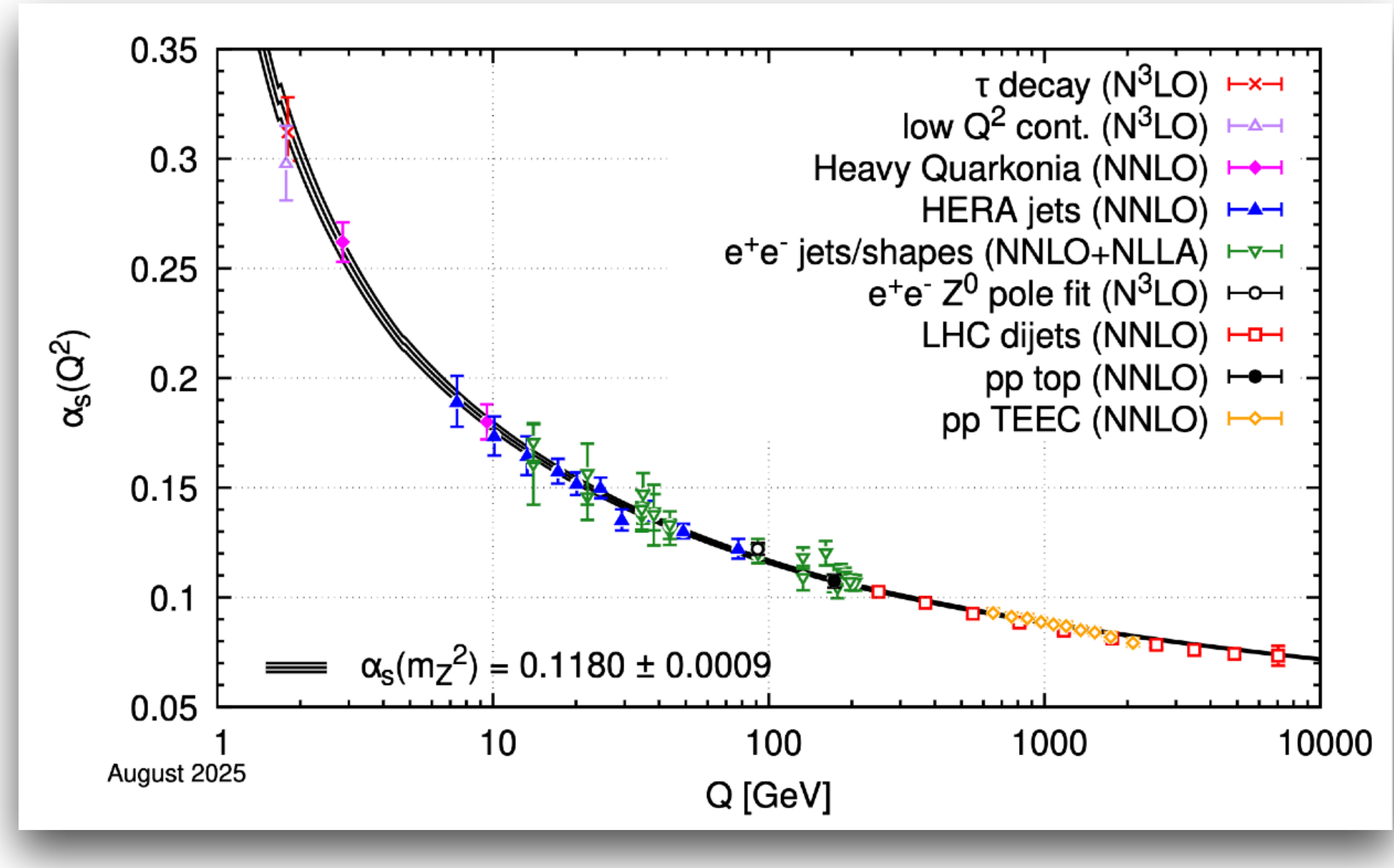
Ryōan-ji, Kyoto

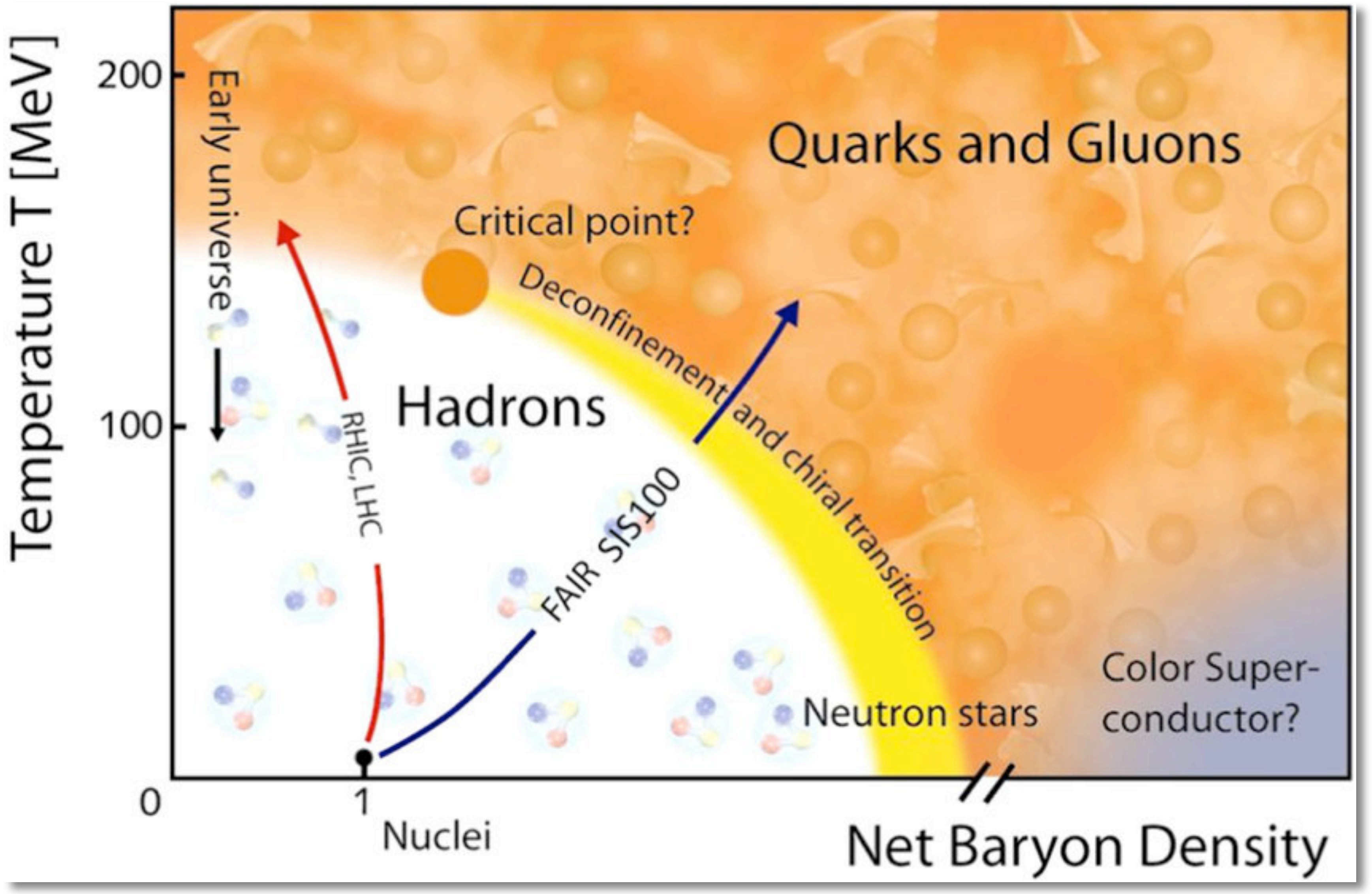
Strong dynamics = rich dynamics

- Example: physical QCD
  - Well-defined fundamental limit: asymptotic freedom
  - Colour confinement
    - Absence of coloured asymptotic states
    - Massive spectrum of gauge resonances
  - Dynamical chiral symmetry breaking
    - Chiral condensate and massive spectrum of hadrons
    - Nuclear forces
  - Axial anomaly
  - Exotic phases at finite temperature and density
  - …



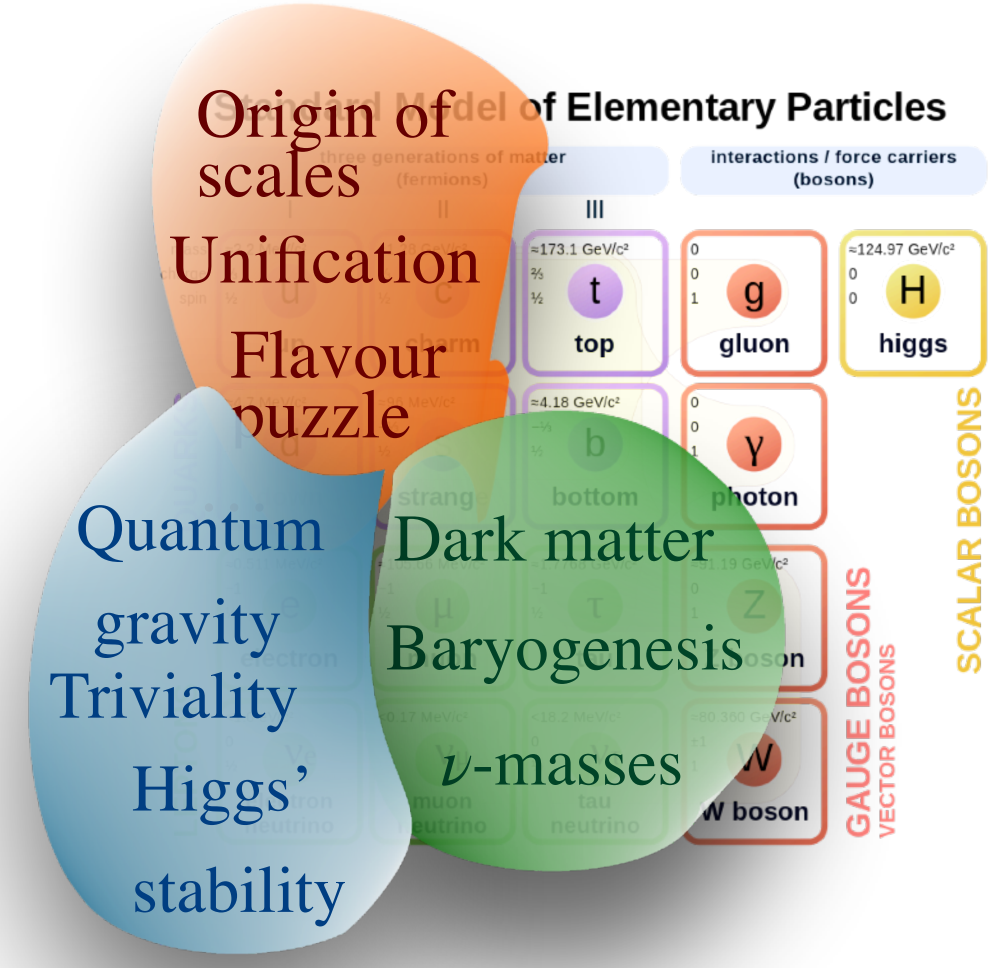


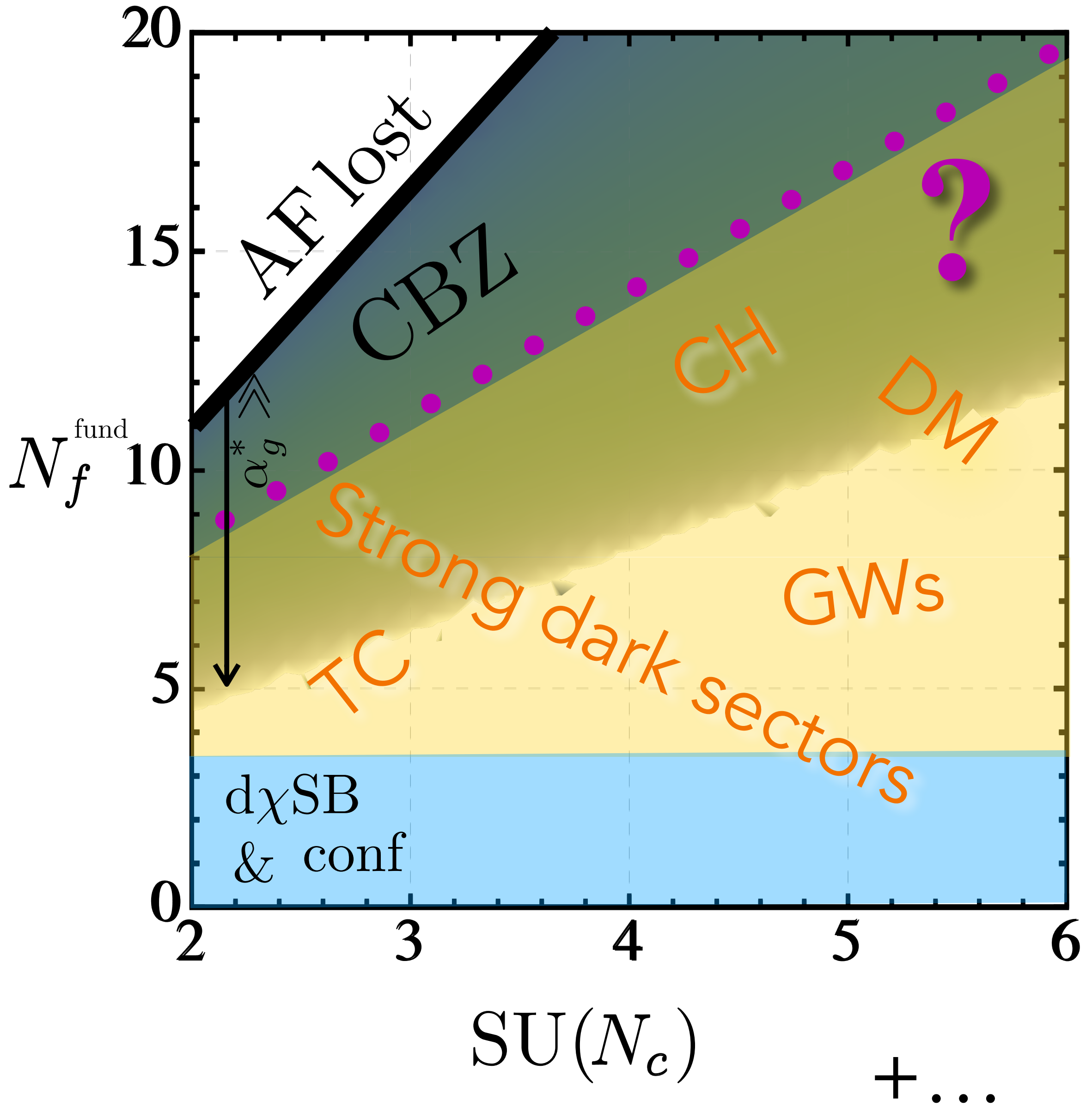
$$
\begin{aligned}
m_{\text{proton}} &\sim 1\text{GeV} \sim 3\, m_{\text{up}} \\
m_{\text{up}} &\sim h\langle\bar\psi\psi\rangle^{1/3} + m_{\text{up}}^{\text{Higgs}} \\
&\sim 350\,\text{MeV} + 2\,\text{MeV}
\end{aligned}
$$



Strong dynamics = rich dynamics

- QFTs beyond QCD:
  - New dynamics?
  - New physics?





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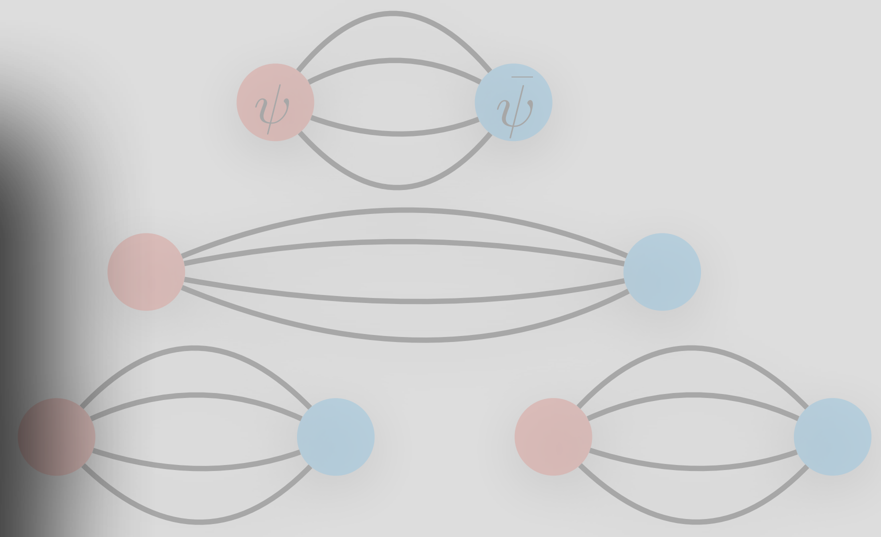
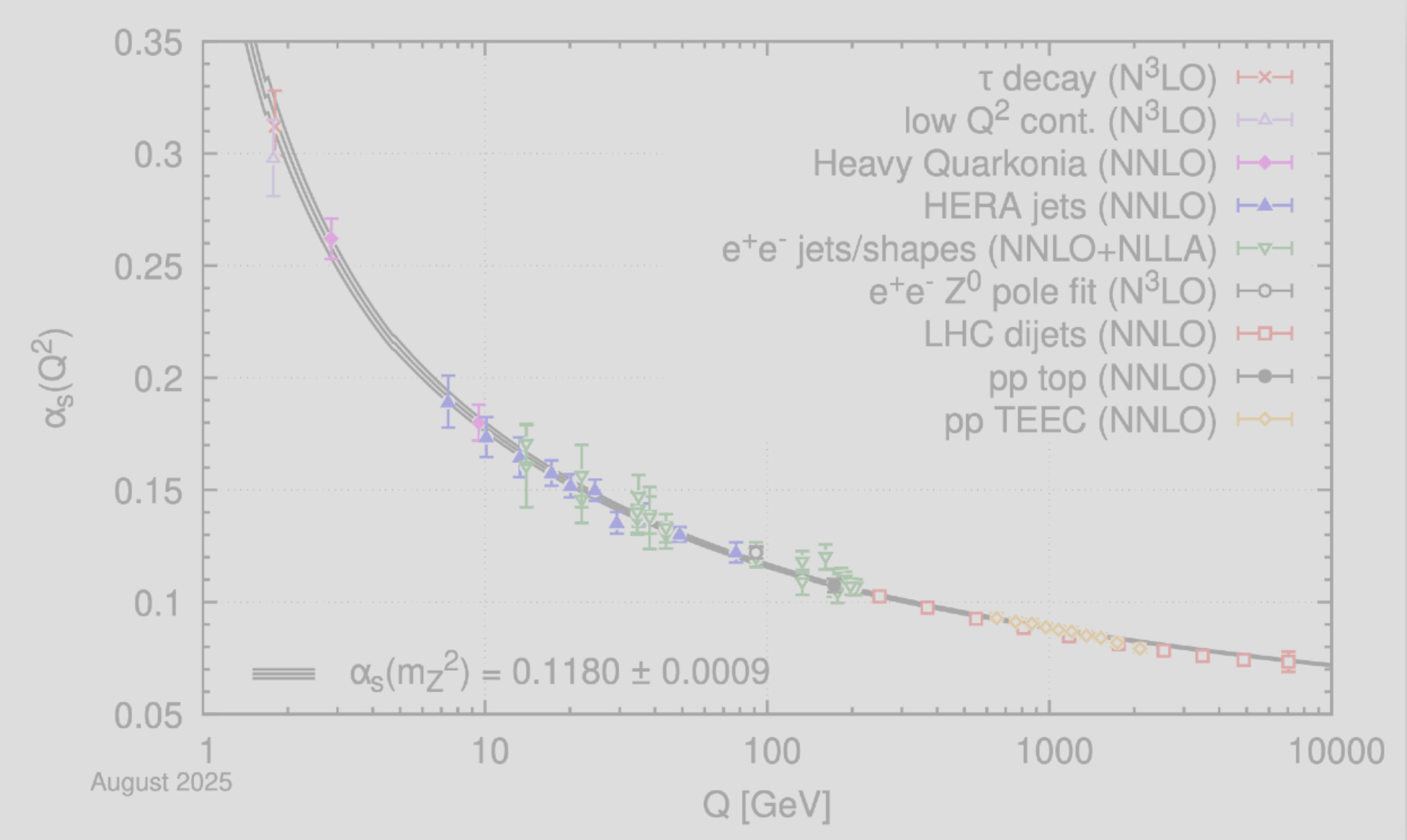
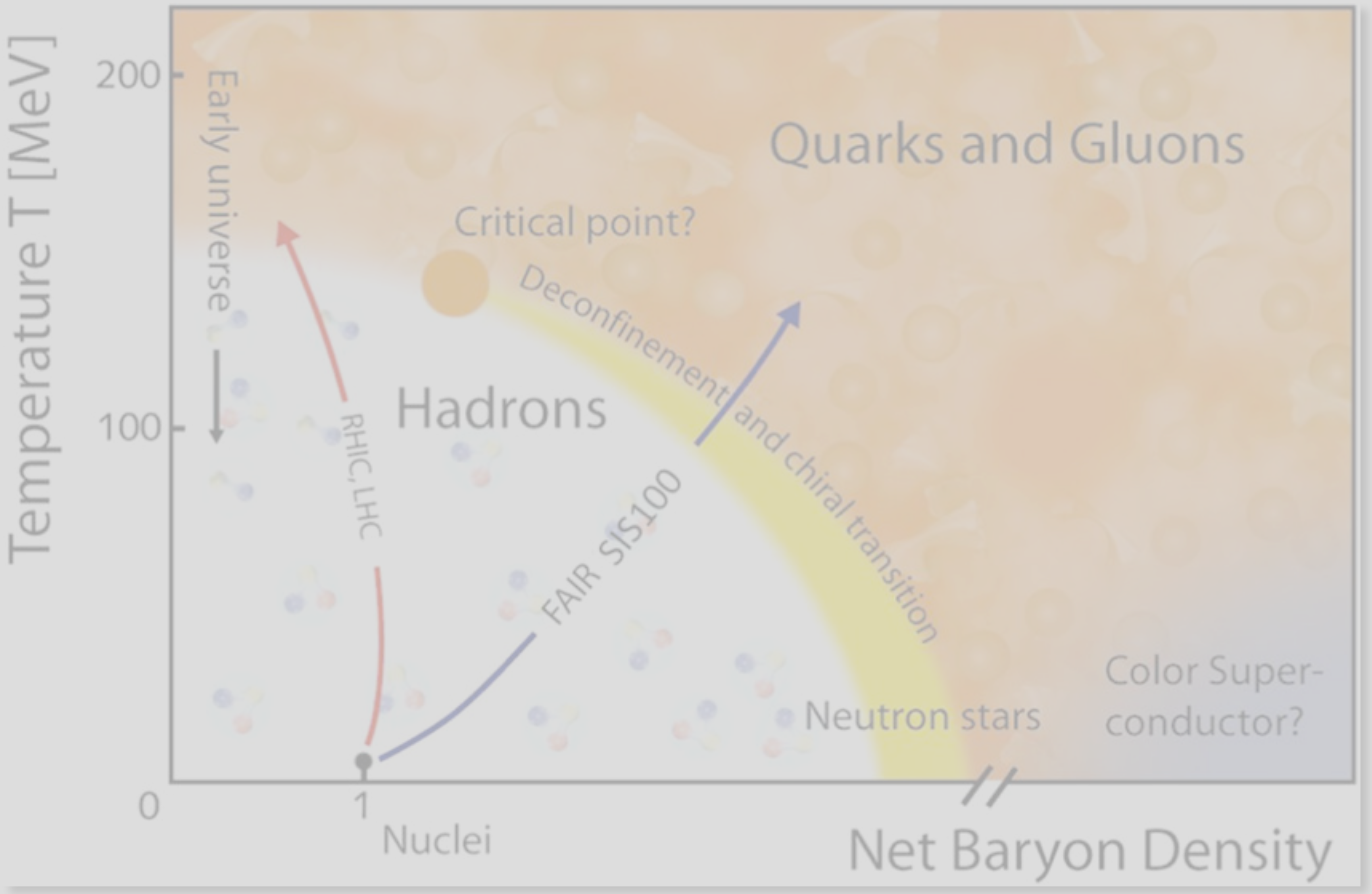




GeV $\sim 3\,m_{\rm up}$

$\langle\bar{\psi}\psi\rangle^{1/3} + m_{\rm up}^{\rm Higgs}$

50 MeV + 2 MeV



- …

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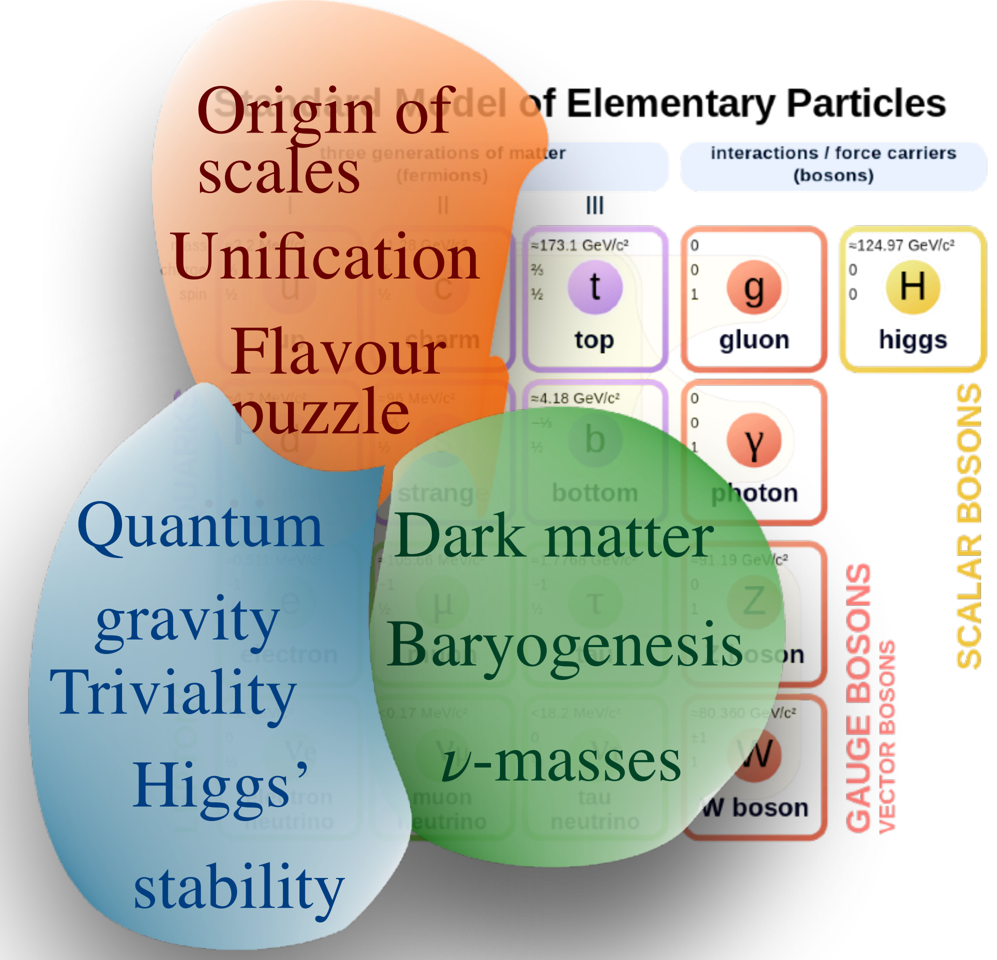
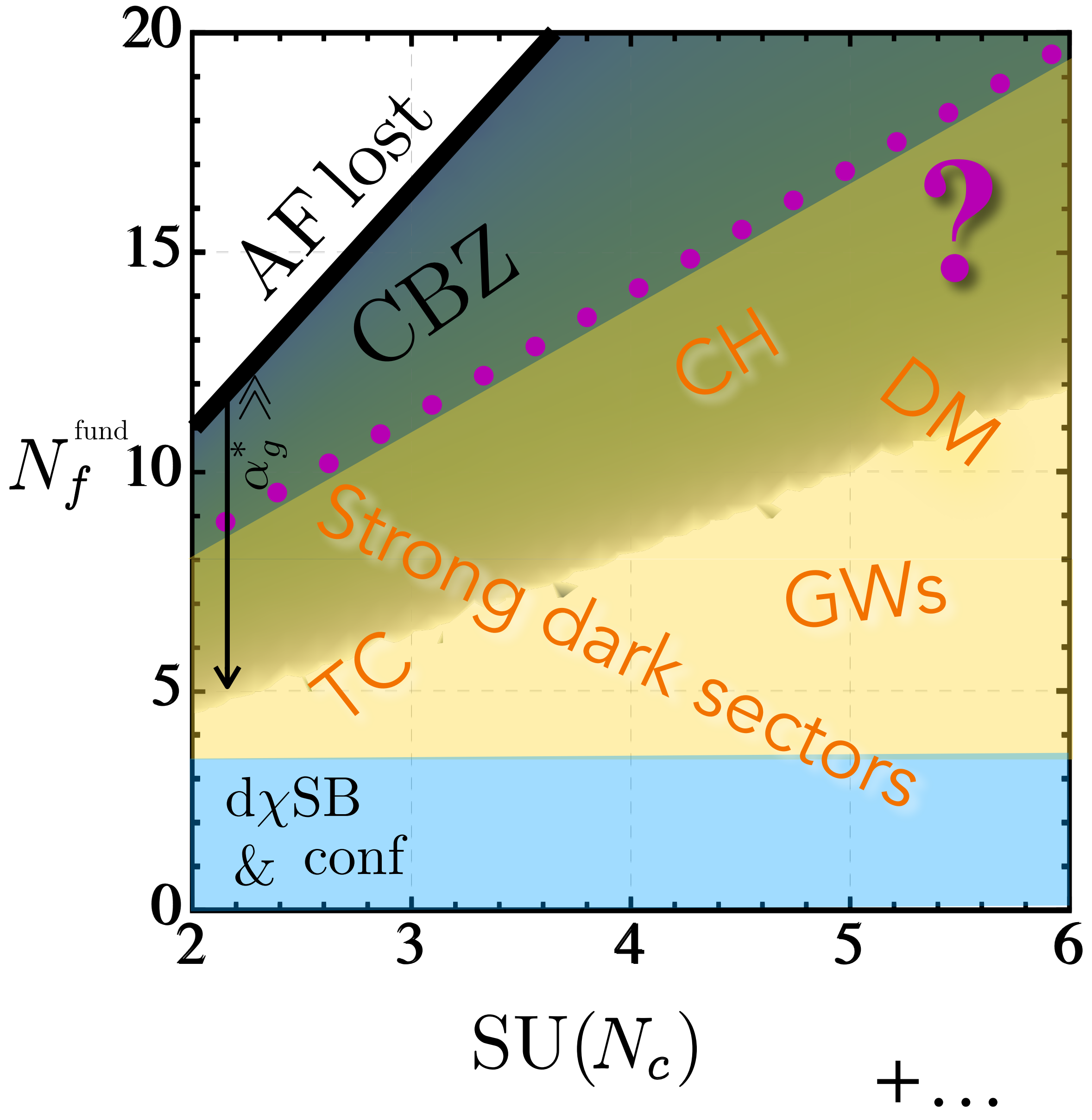


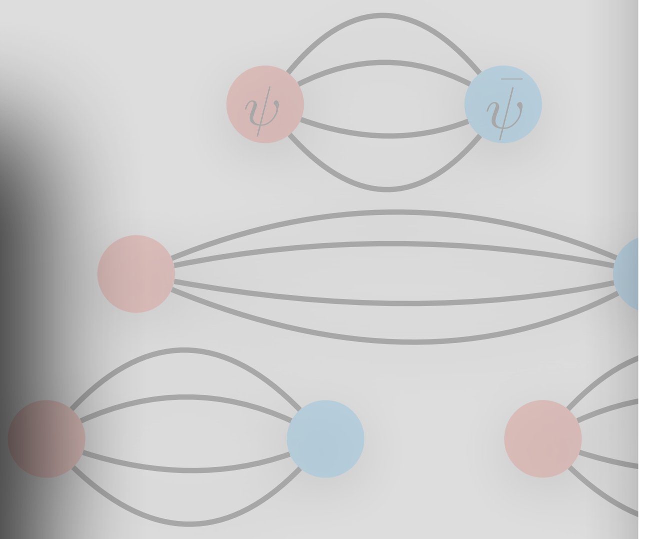


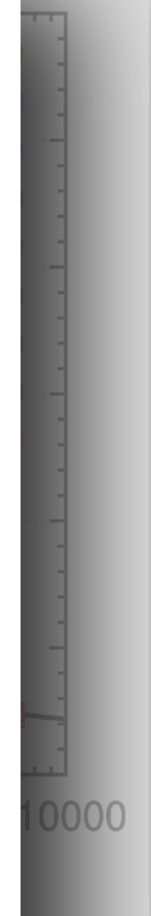
+...

- ...

Challenges:

- Non-perturbative
- Expensive
- Tricky limits (eg. chiral, near-conformal,...)
- Versatility
- ...





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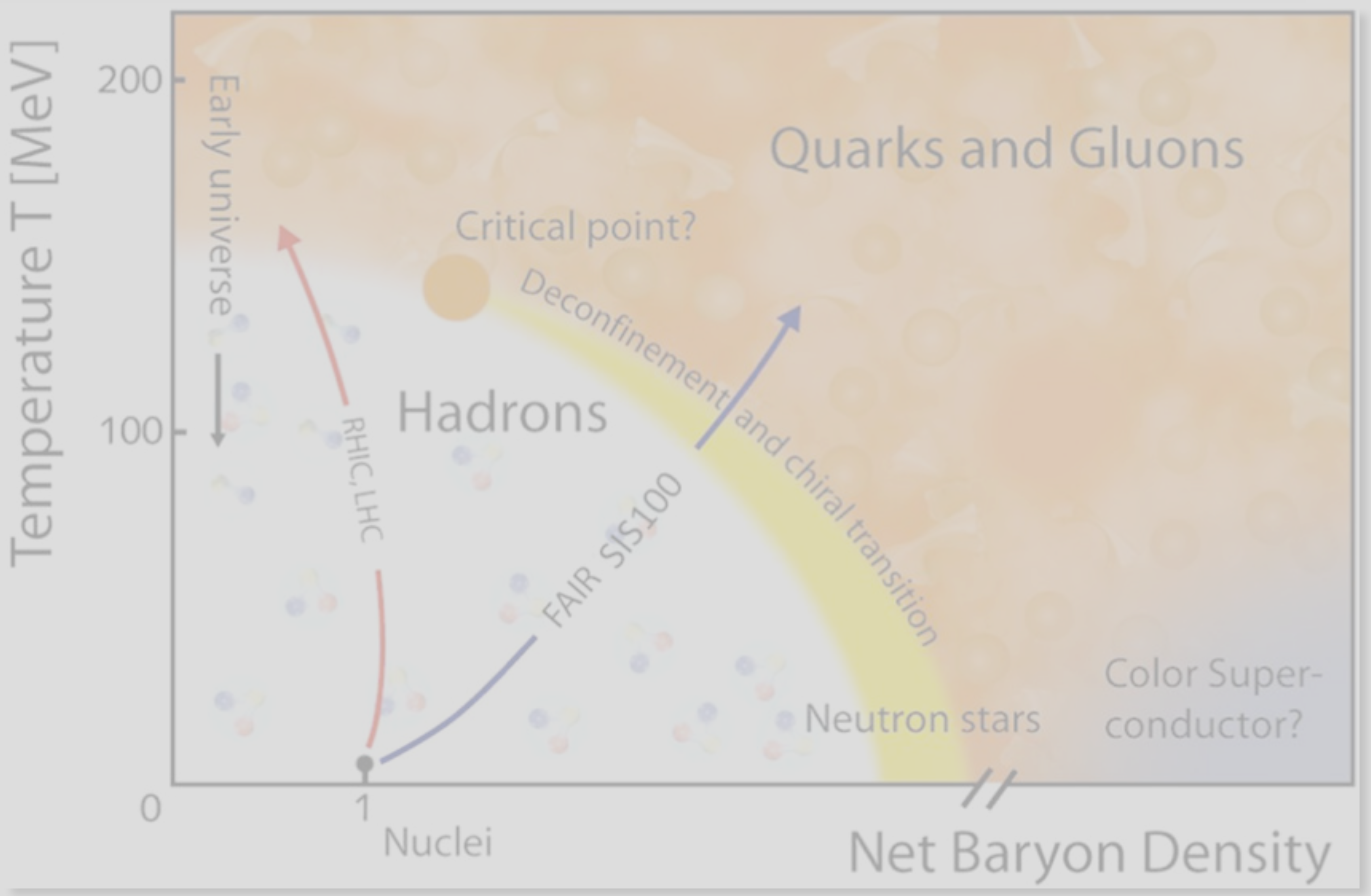

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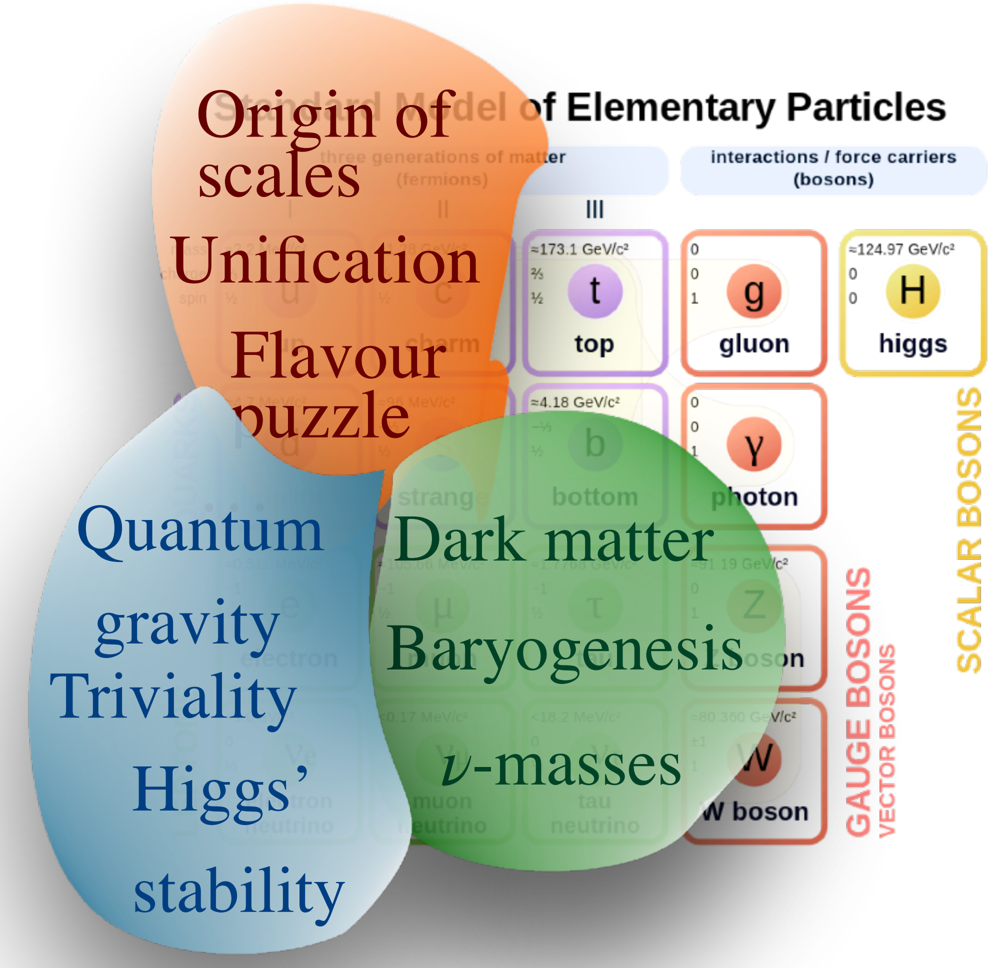

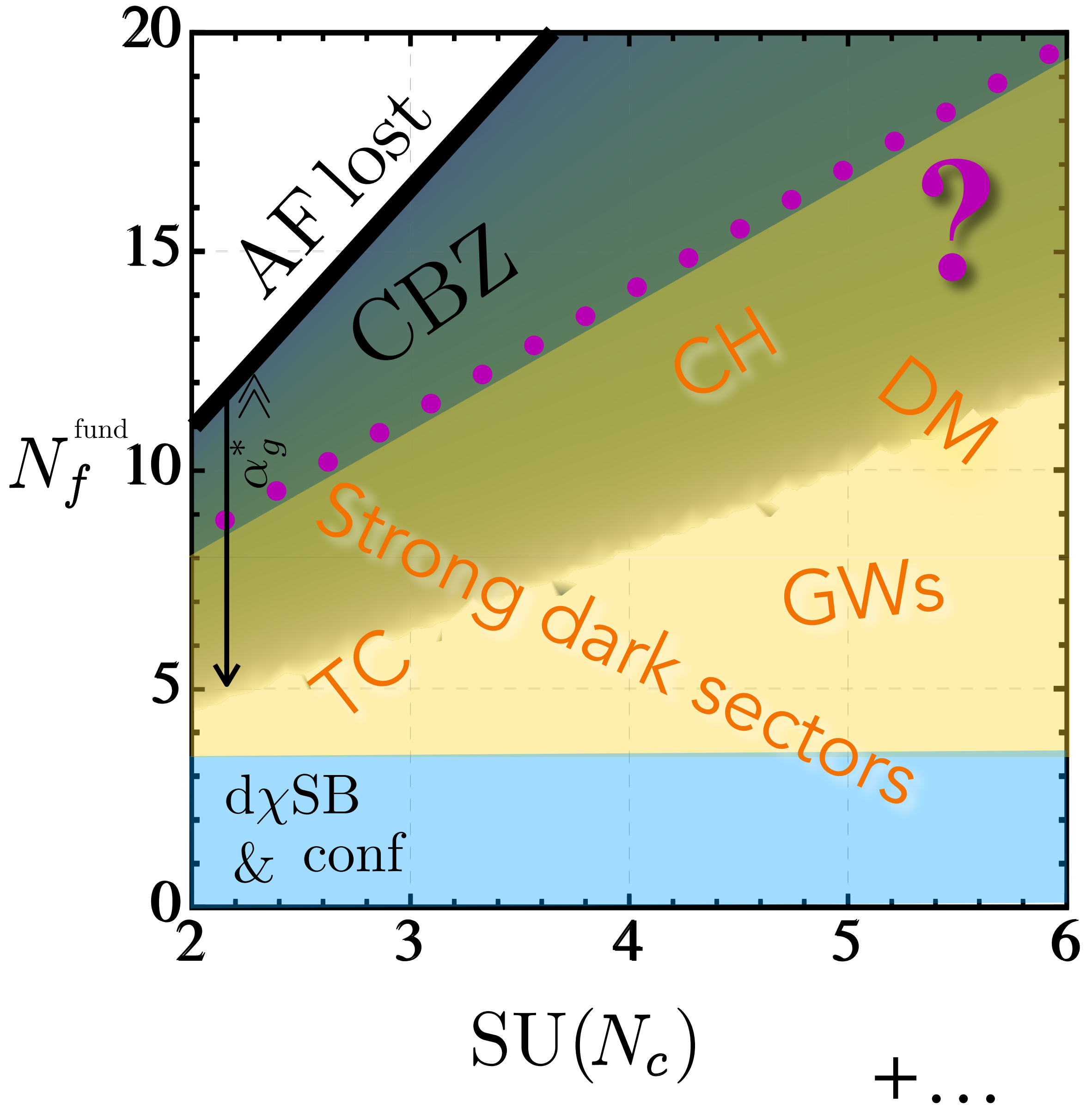

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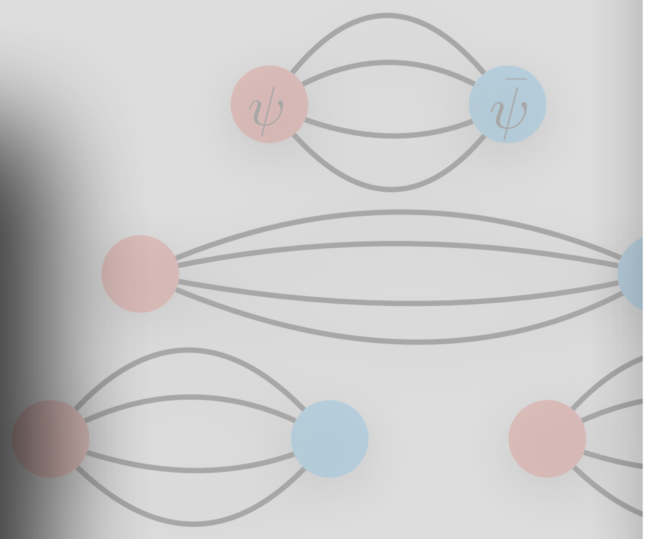

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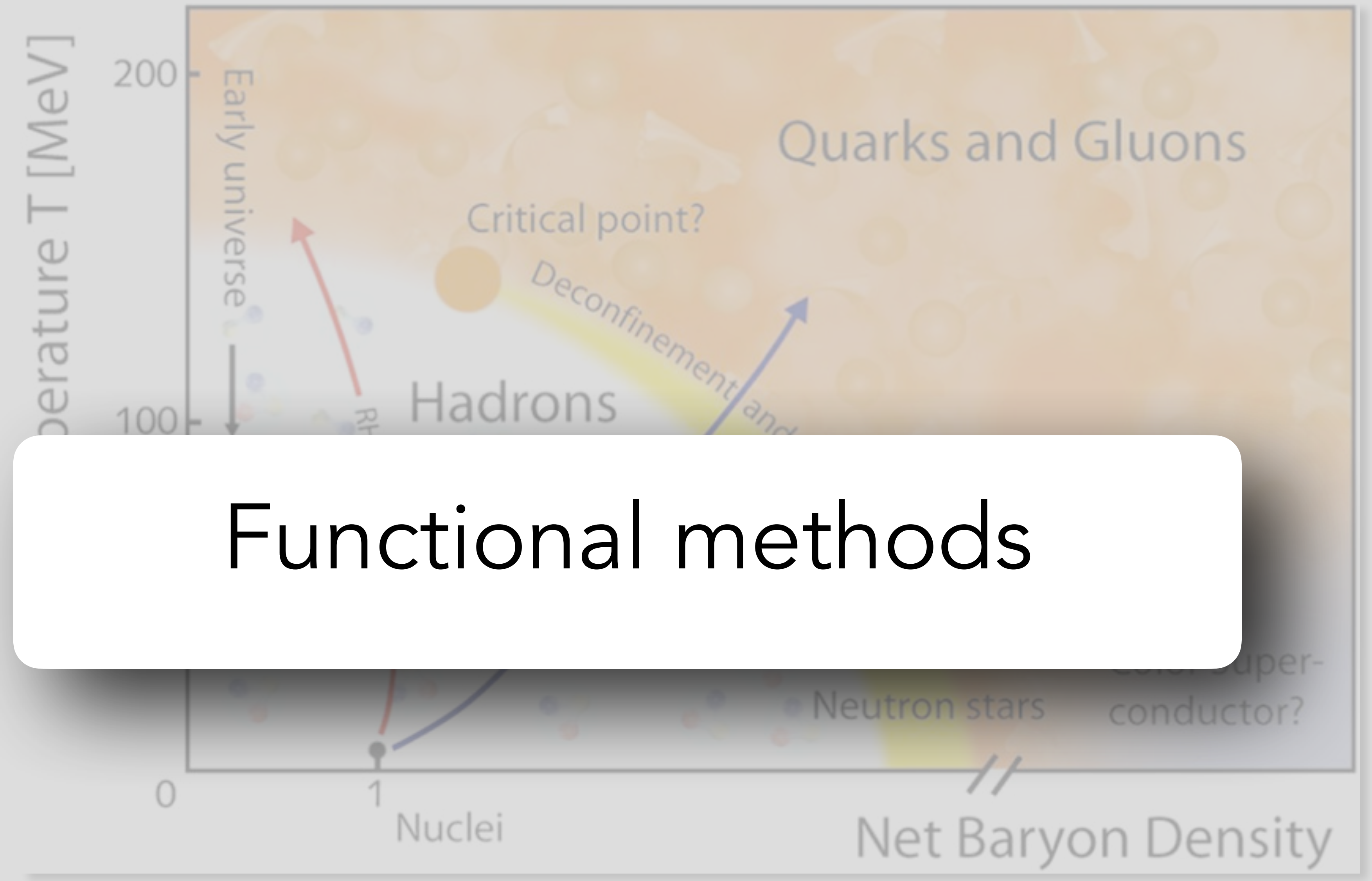


Functional methods

- …

Phases of QCD-like and chiral gauge theories

1. The functional RG and the approach gauge-fermion theories
    i. **Colour confinement** in correlation functions
    ii. Dynamical symmetry breaking
2. Phases of QCD-like theories (**many colours and flavours**)
3. Phases of **chiral gauge theories**
4. Conclusions

Goertz,APG,Pawlowski[2412.12254]

$$S = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}\gamma_\mu D_\mu \psi$$

$$S = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \psi^\dagger \bar{\sigma}^\mu D_\mu \psi + \chi^\dagger \bar{\sigma}^\mu D_\mu \chi$$

Li,APG,Vatani,Xu[2507.21208]
Li,APG,Vatani[2603.19355]

fRG for gauge-fermion QFTs

Kenninji Temple, Kyoto

Functional Renormalisation Group

- Progressive integration of momentum shells:

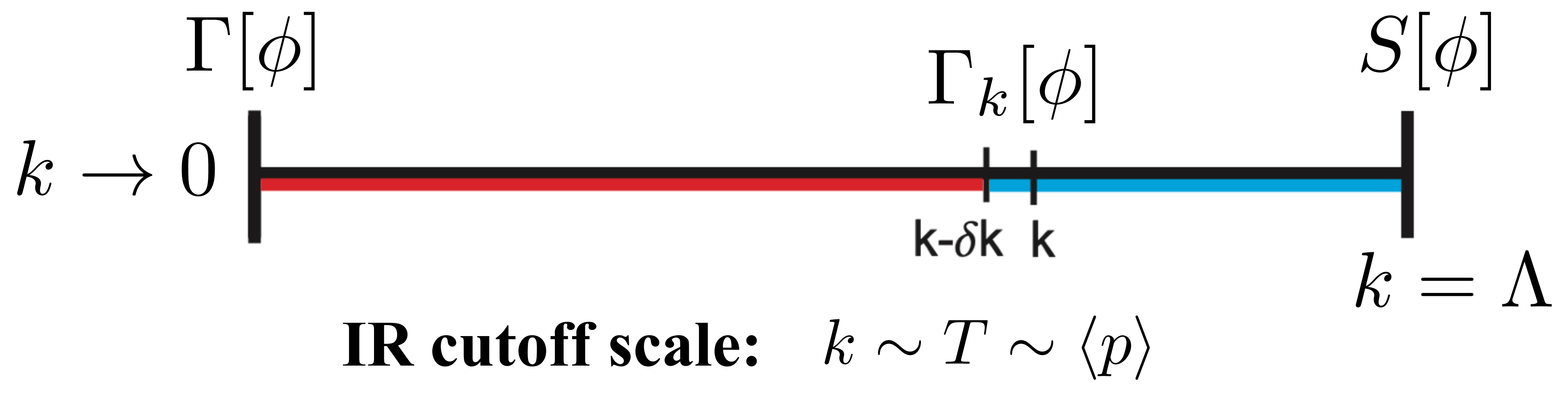
$$\int [\mathcal{D}\phi]_{p>k} = \int \mathcal{D}\phi \, \exp(-\Delta S_k[\phi]) \qquad \Delta S_k[\phi] = \int_p \phi(p)\, R_k(p)\, \phi(-p)$$

- **Effective average action**:

$$\Gamma_k[\phi] = \int_x J(x)\phi(x) - \mathcal{W}_k[J] - \Delta S_k[\phi]$$

Wetterich '89

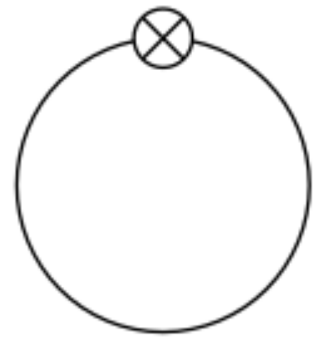
  - Average action of fields over a k^{-d} space-time volume
  - Kadanoff's block-spinning idea in the continuum limit



IR cutoff scale: $k \sim T \sim \langle p \rangle$

Flow equation:

$$\boxed{\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k\right] = \frac{1}{2}}$$



$\partial_t \equiv k\, \partial_k$

Wetterich '93

- Exact
- Non-perturbative
- One loop
- Diagrammatic
- Mass-dependent
- Analytic regulators
- Versatile
- Real-time formulation
- UV-IR finite
- Systematic expansion schemes

⋮

➡ Pedagogical introduction: Gies [0611146] ➡ Review: Dupuis,et.al [2006.04853]

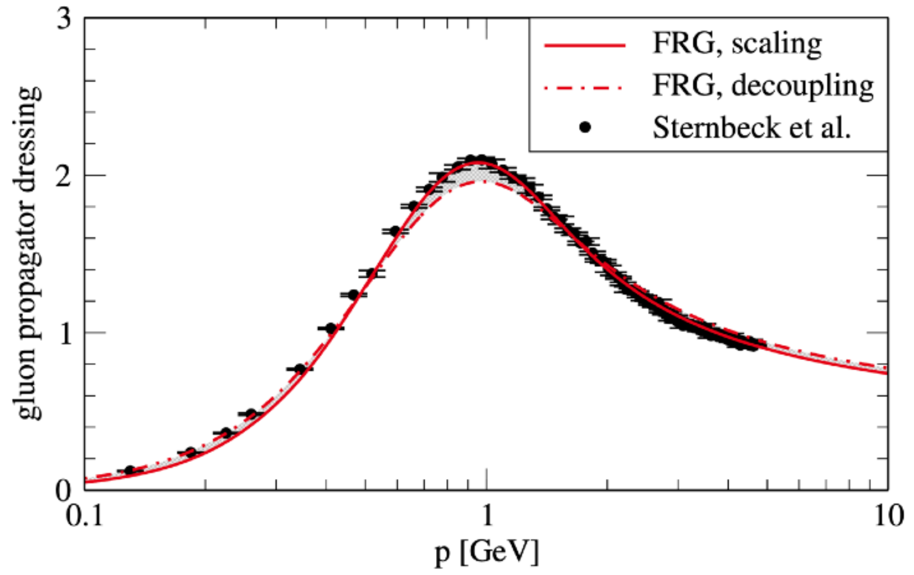
(a.k.a. QCD, see Jan's and Christian's talks)

Some results in gauge-fermion theories

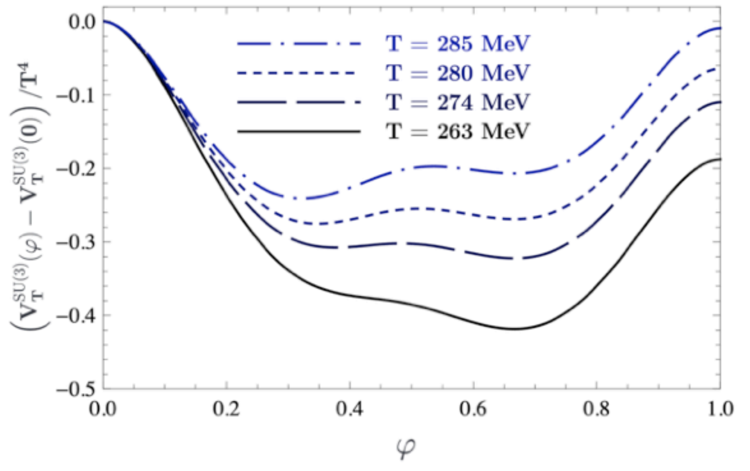
Today

Colour confinement

Cyrol,Fister,Mitter,Pawlowski [1605.01856]

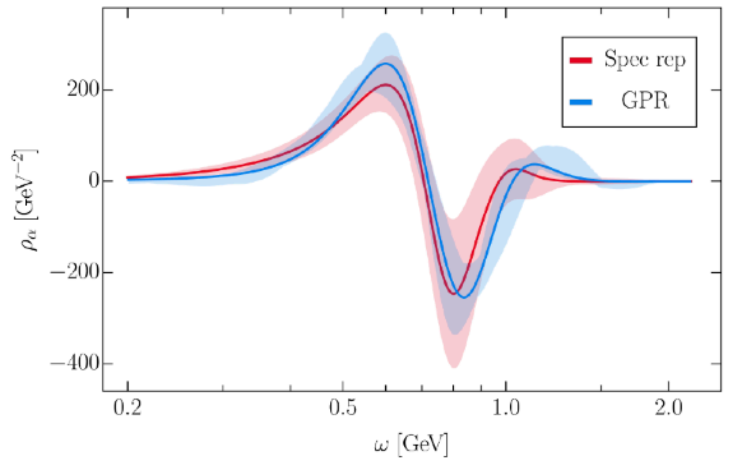


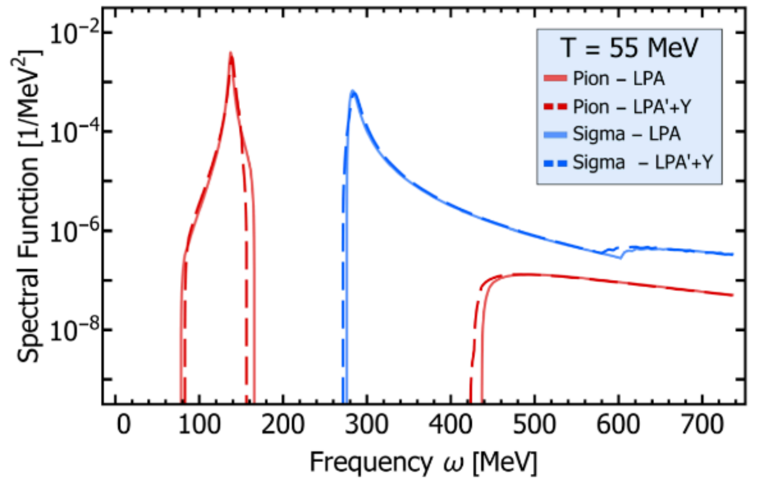
Fister,Pawlowski [1301.4163]



Real time properties

Horak,Pawlowski,Turnwald,Urban,Wink,
Zafeiropoulos[1711.07444]





Pawlowski,Wink,Strodthoff[1711.07444]

Many flavour and colours

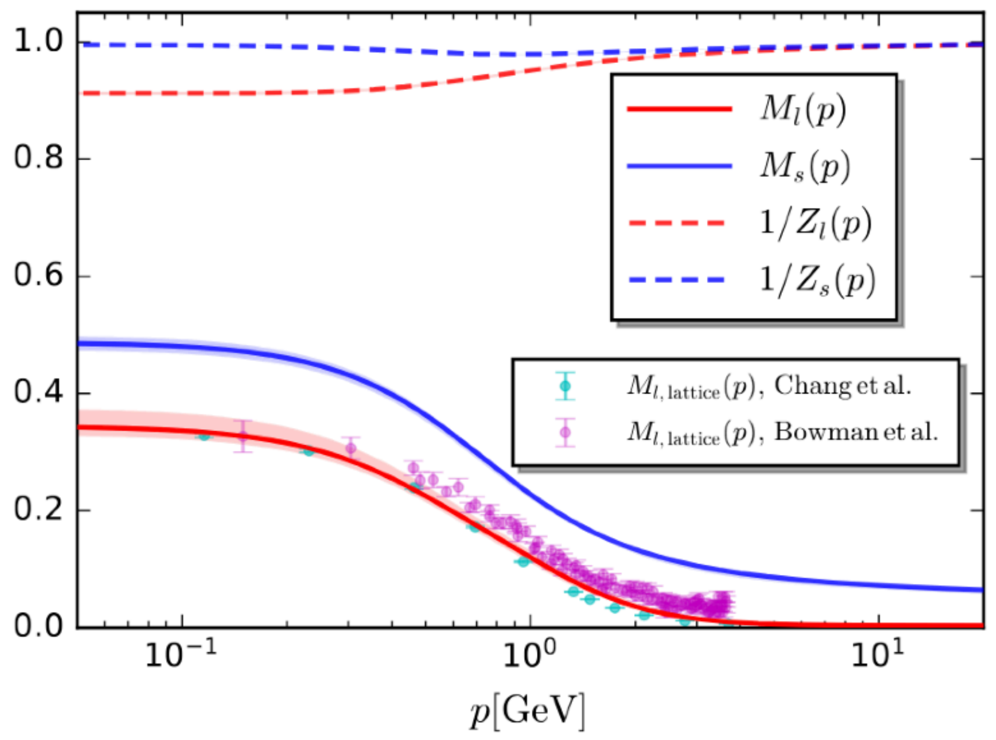
Gies,Jaeckel[hep-ph/0507171]
Braun,Gies[0912.4168]
Braun,Fischer,Gies[1012.4279]
Goertz,APG,Pawlowski[2412.12254]

Chiral gauge theories

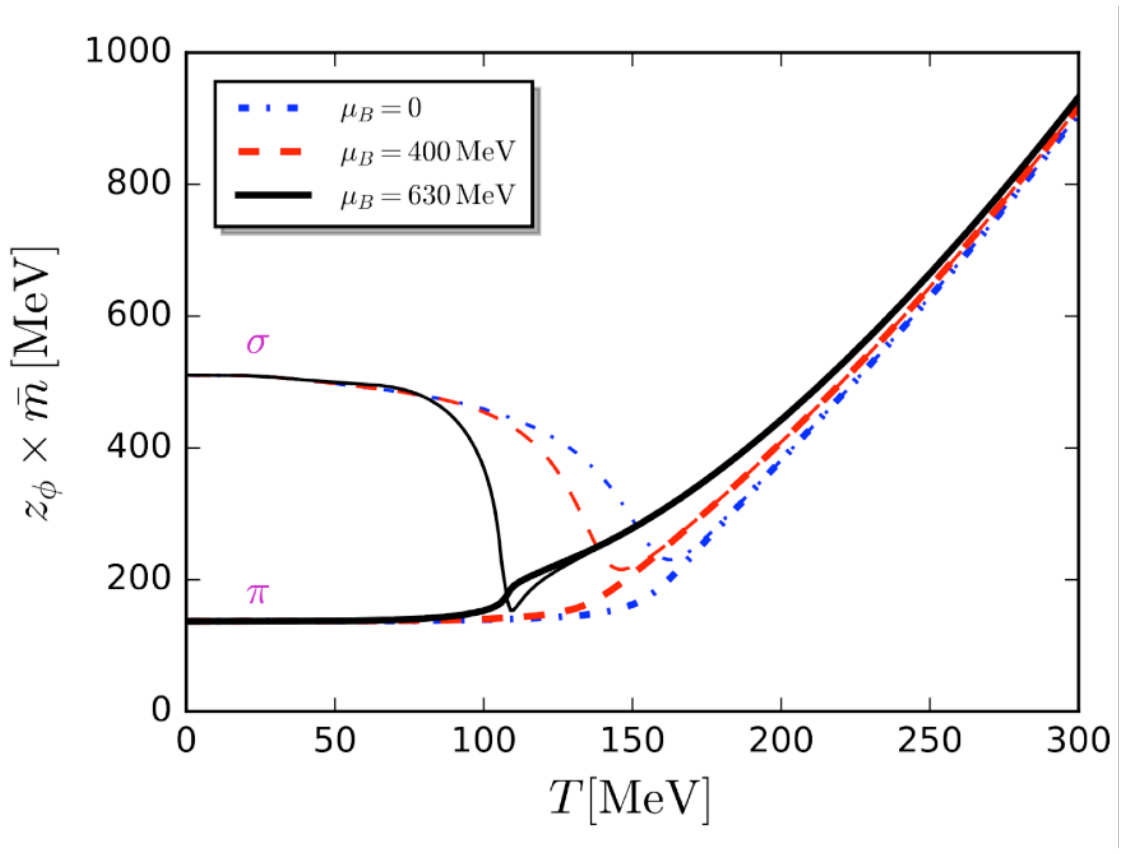
Li,APG,Vatani,Xu[2507.21208]
Li,APG,Vatani[2603.19355]

Dynamical chiral symmetry and bound states

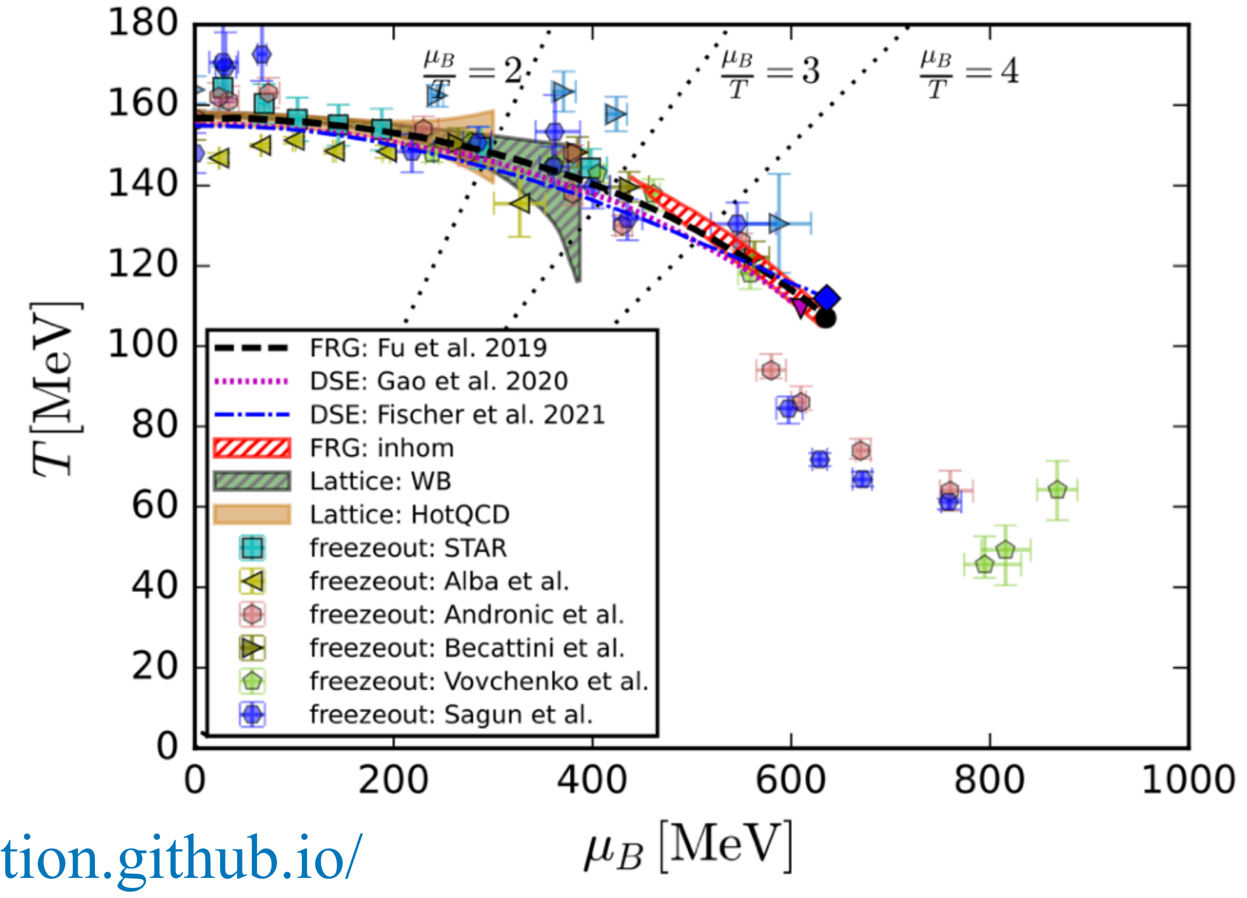
Fu,Huang,Pawlowski,Tan,Zhou[2502.14388]



Fu,Pawlowski,Rennecke [1909.02991]



Finite temperature and chemical potential



https://fqcd-collaboration.github.io/

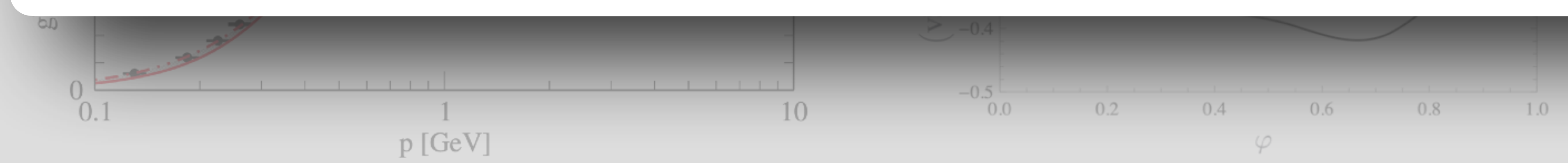
(a.k.a. QCD, see Jan's and Christian's talks)

Some results in gauge-fermion theories

Physics in correlation functions:

Colour confinement, dynamical symmetry breaking and/or anything else

Colour confinement



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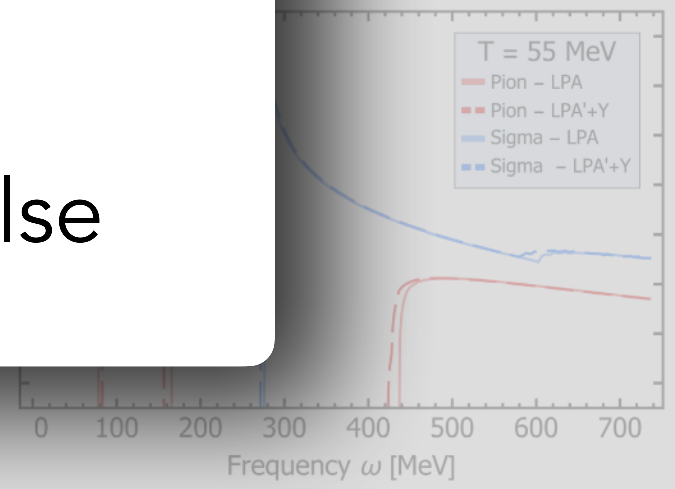

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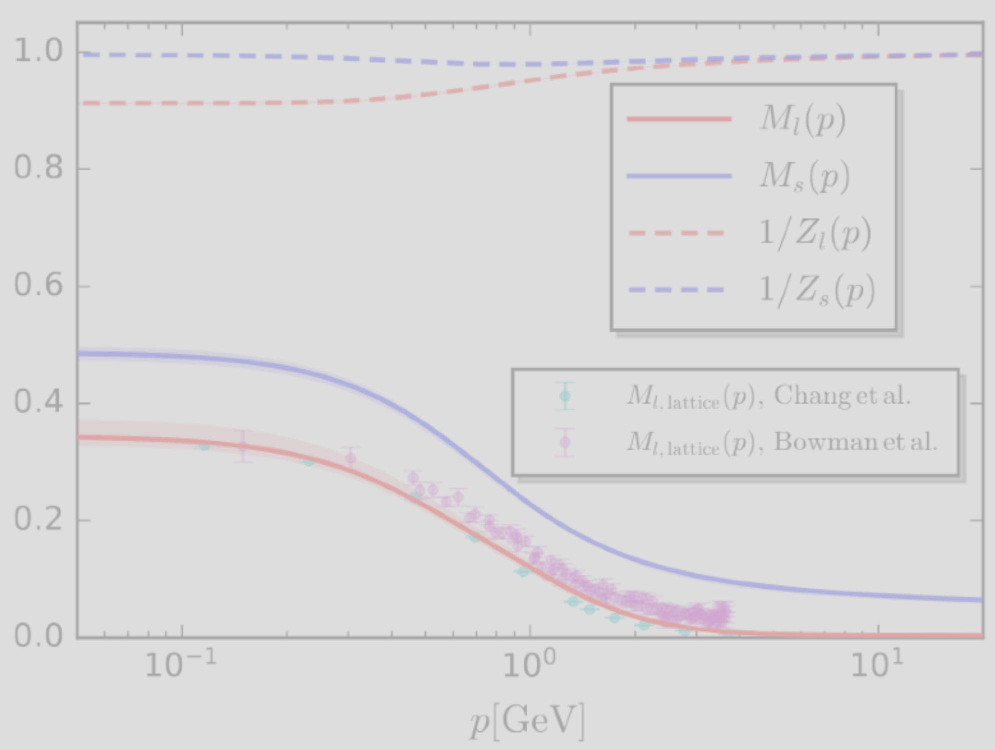

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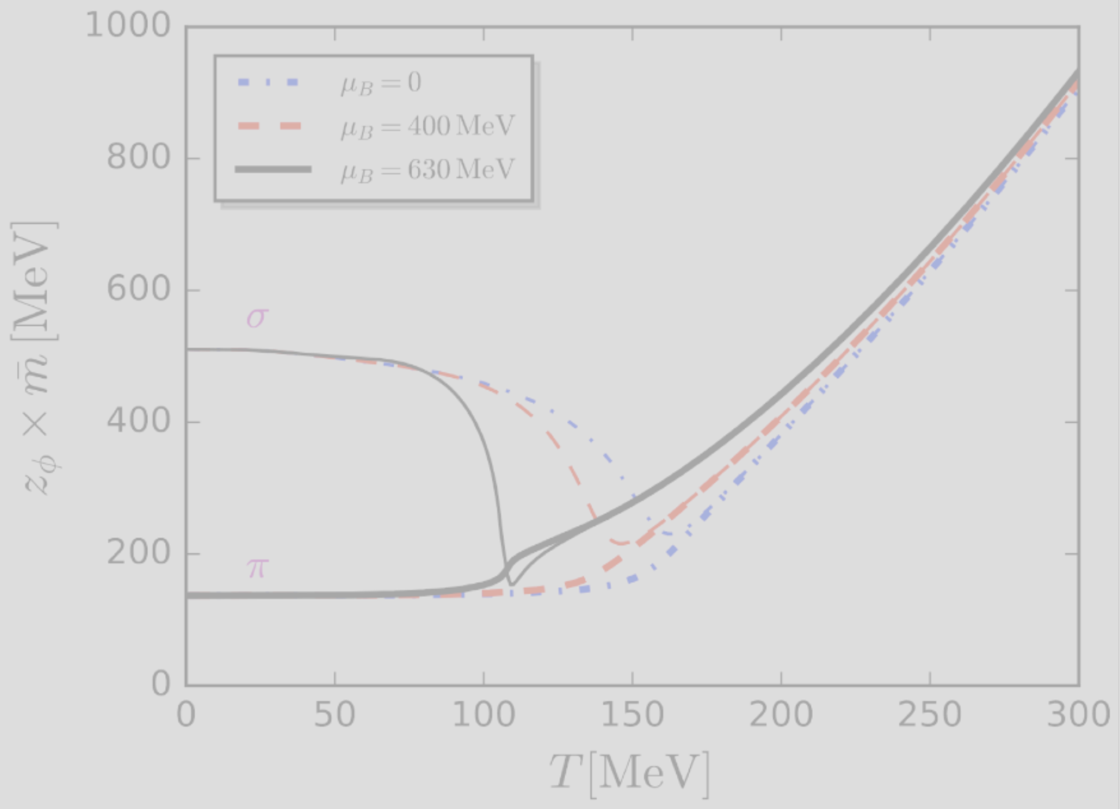

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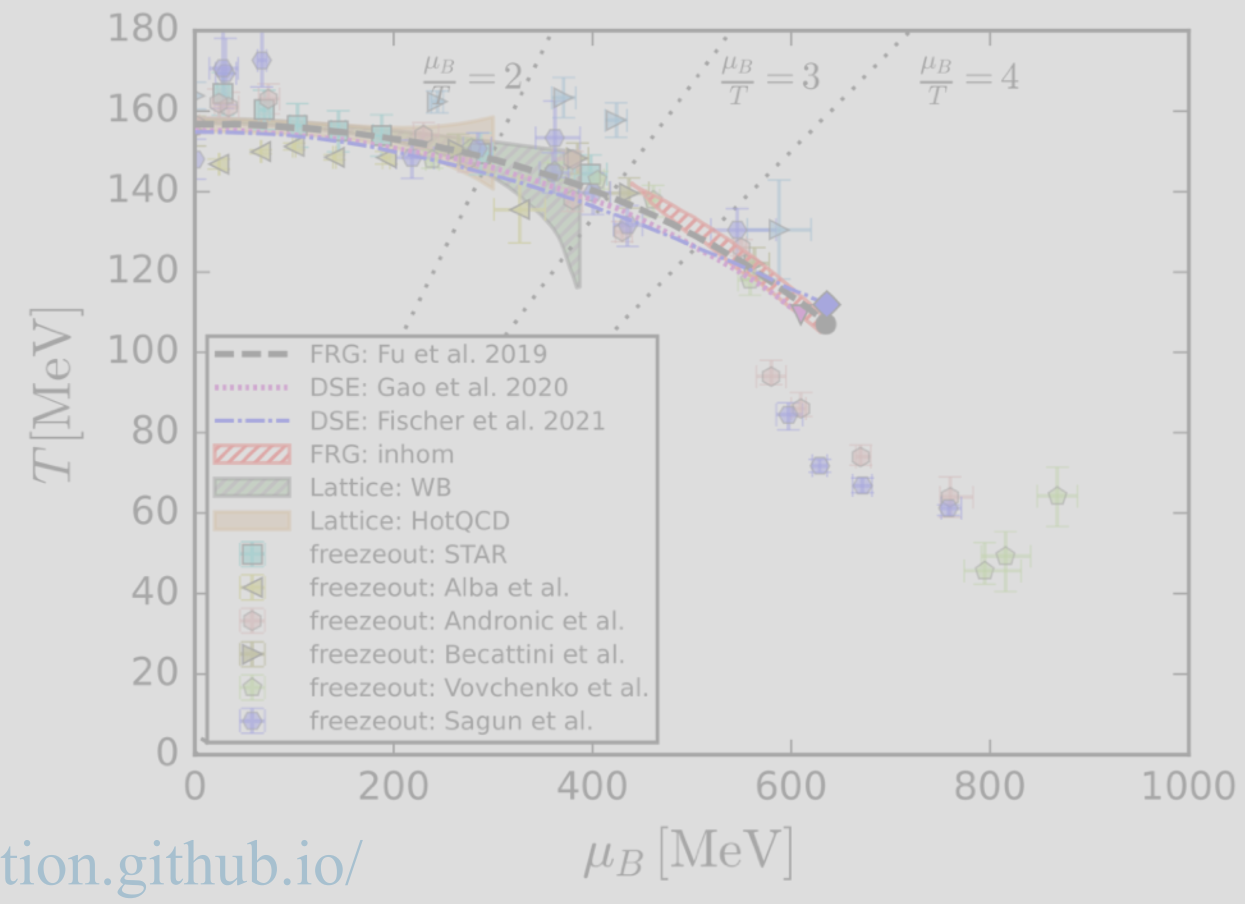

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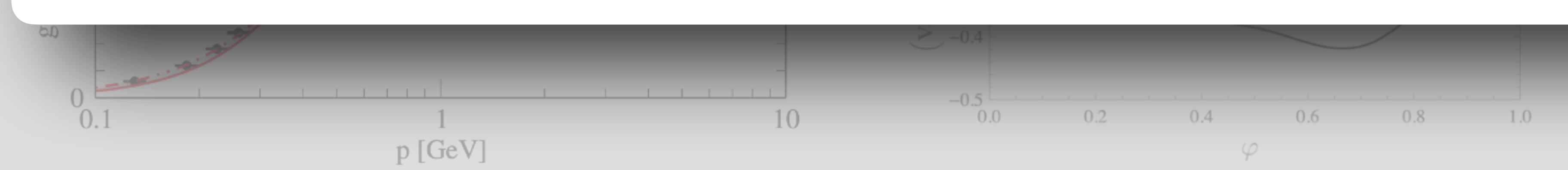

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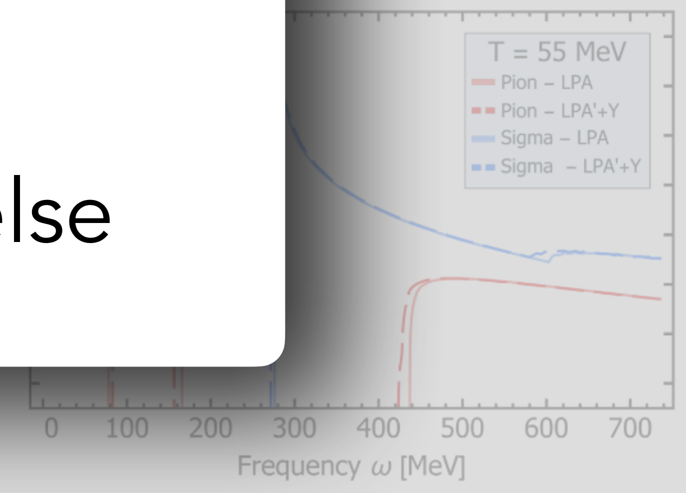

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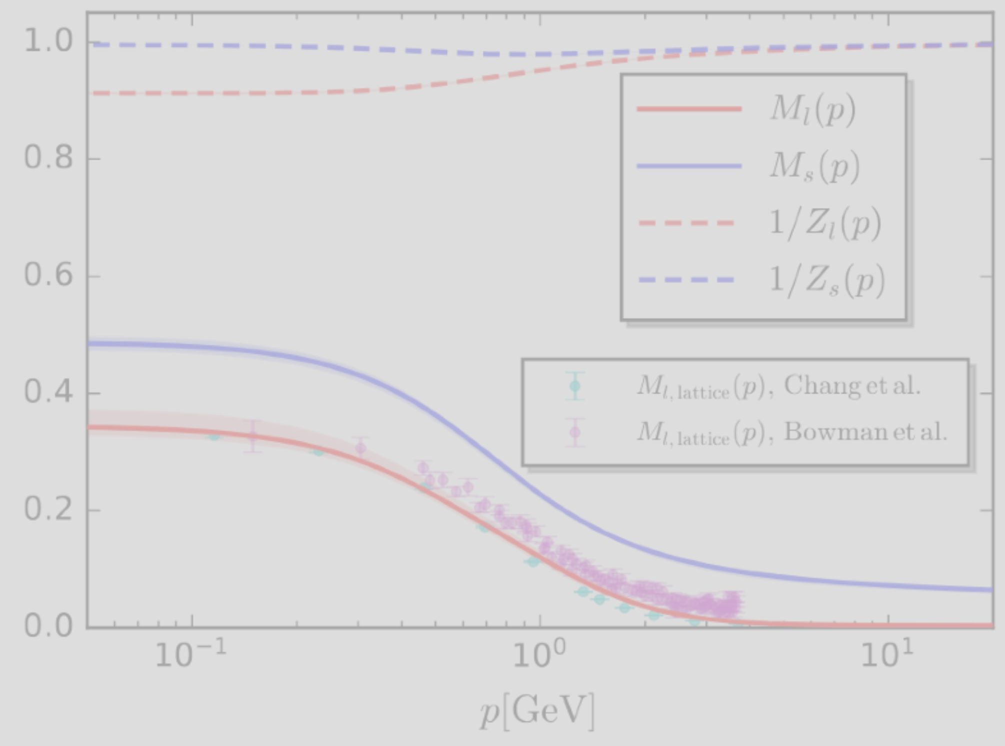

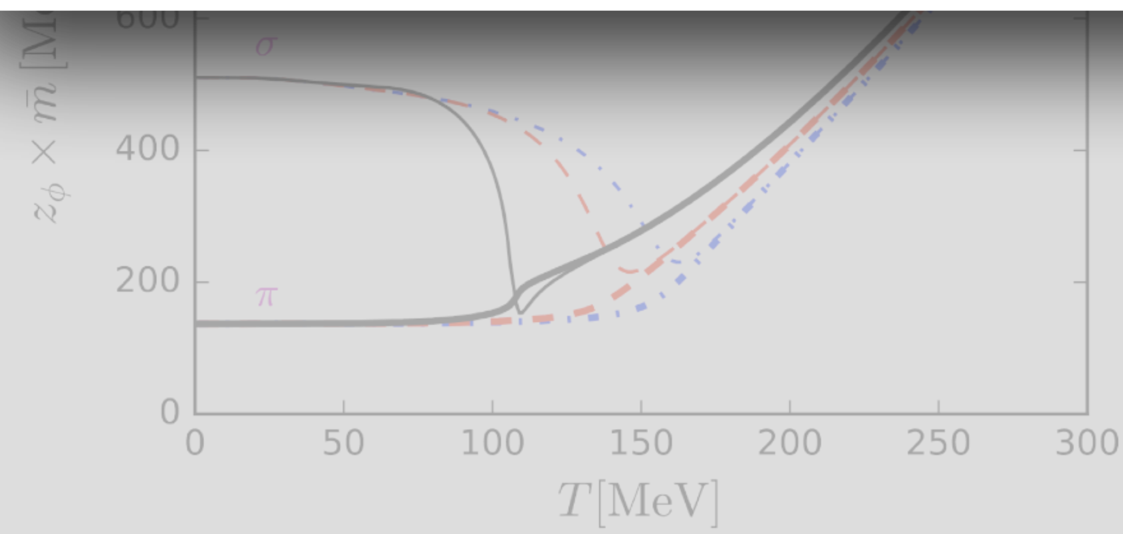

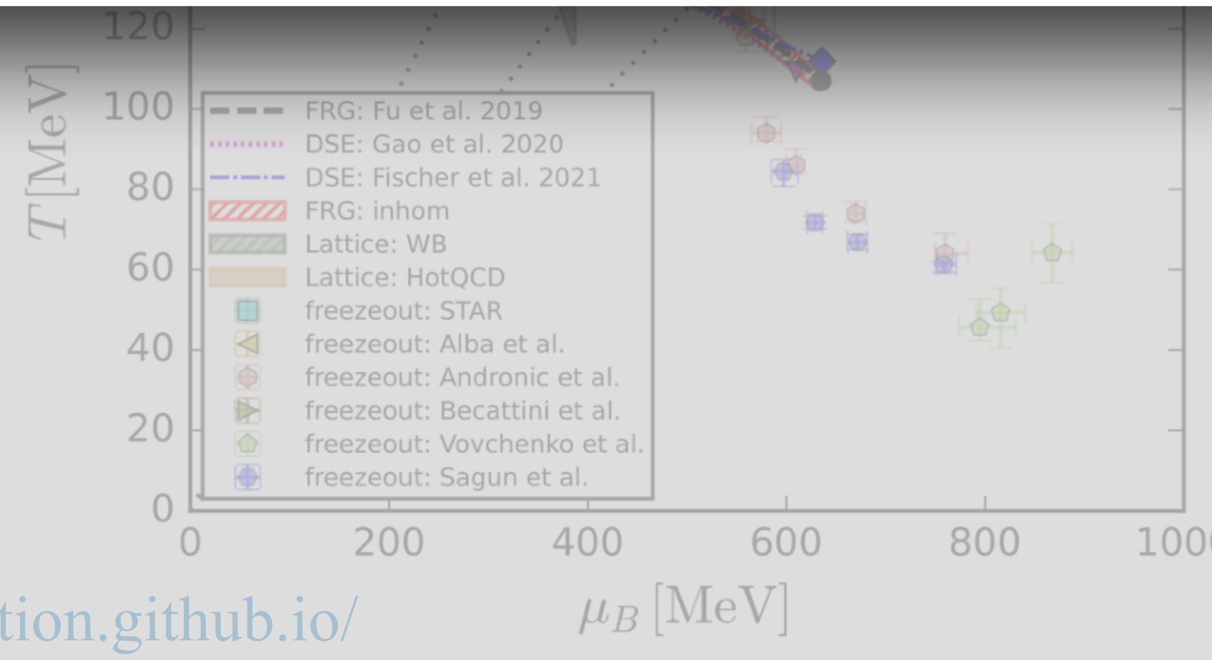

https://fqcd-collaboration.github.io/

Fundamental, first principles. Necessary ingredients: symmetries and relevant deformation ☛ predictions for LEFTs

Colour confinement

Otaru, Hokkaido

Colour confinement and the gluon mass gap

- Observables:
  - **Absence** of **coloured asymptotic states**
  - **Massive spectrum** of bound states (glueballs)
- Hand in hand with existence of a **gluon mass gap:**
  - Area law Wilson'74
  - Confinement-deconfinement phase transition Polyakov'75

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Alkofer,von Smekal'96'00 Pawlowski,Litim,Nedelko,von Smekal[hep-th/0312324]
Fischer, Maas, Pawlowski[0810.1987] Cyrol,Fister,Mitter,Pawlowski [1605.01856]

- Functional *bootstrap* approach:
  - **Gluon mass gap** generated by **quantum fluctuations**

$$\Gamma_k^{(AA)}(p^2) = Z_{A,k}(p)(p^2 + m^2_{\text{gap},k}) = \hat{Z}_{A,k}(p)\, p^2$$

  - Agnostic to precise mechanism (eg. Schwinger, quartet mechanisms) but accounted by non-perturbative approaches
  - How we know its confinement?
    - ◉ **Kugo-Ojima criterion**

      Kugo, Ojima'79
      Nakanishi,Ojima'90
      - **Sufficient conditions:**
        - **Massive spectrum** of **physical states** in the **Hilbert space** and a **global BRST** transformation
      - **Unique** IR scaling of correlation functions

Kugo[hep-th/9511033]

$$\lim_{p\to 0} Z_c(p^2) \propto (p^2)^{\kappa} \qquad \lim_{p\to 0} Z_A(p^2) \propto (p^2)^{-2\kappa}$$

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New*

"*easy*" confinement: Goertz,APG,Pawlowski[2412.12254]

- *k* dependences suffice
- Semi-analytical (no heavy numerics)
- Facilitate study beyond QCD-limit

Alkofer,von Smekal'96'00 Pawlowski,Litim,Nedelko,von Smekal[hep-th/0312324]
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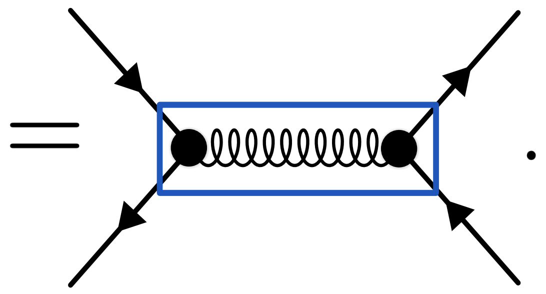
Confinement in correlation functions

- Flows computed: $\{g_{A\bar\psi\psi}, g_{A\bar c c}, g_{A^3}, g_{A^4}, \bar m^2_{\text{gap}}, Z_A, Z_c\}$
- **Exchange couplings**:

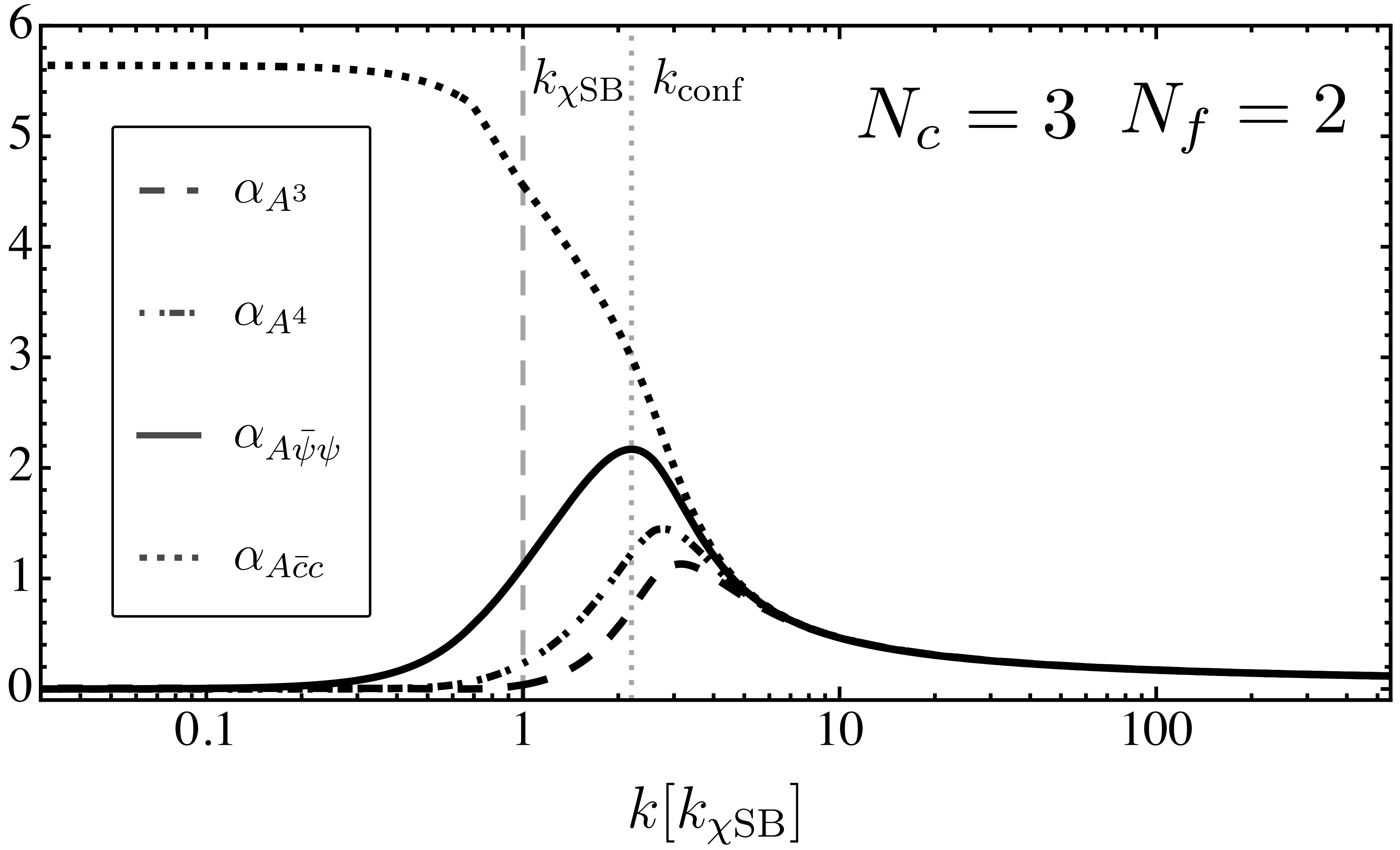
$\alpha_{A\psi\bar\psi} =$  $\cdot p^2 = \dfrac{g^2_{A\psi\bar\psi}}{4\pi\,(1+\bar m^2_{\text{gap}})}$

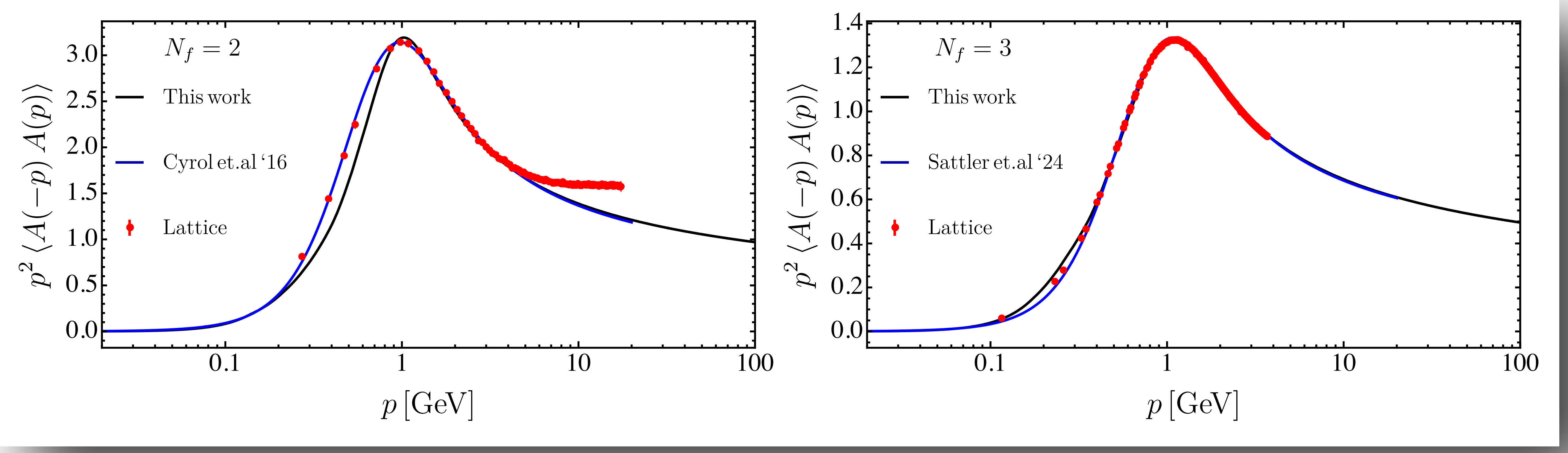
- **Decay** of correlation functions below the **mass gap scale**

$$k_{\text{conf}} \sim m_{\text{gap}} \sim T_{\text{conf}} \sim \Lambda_{\text{QCD}}$$

- The **interplay** of **gapped gauge** and **ghost** contributions
- **Signatures** of Kugo-Ojima confinement in Landau gauge
  - IR constant $\alpha_{A\bar c c}$ and α_{A^3}
  - Zero crossing g_{A^3}

Goertz,APG,Pawlowski[2412.12254]





Dynamical symmetry breaking

Kenroku-en, Kanazawa

Dynamical symmetry breaking in the effective action

a chiral QCD example

$$S = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm gh} + \bar{\psi}\gamma_\mu D_\mu \psi$$

Dynamical symmetry breaking in the effective action

a chiral QCD example

$$\Gamma = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \lambda_i \left(\bar{\psi}\mathcal{T}_i\psi\right)^2 + \kappa_i \left(\bar{\psi}\mathcal{T}_i\psi\right)^3 + \ldots$$

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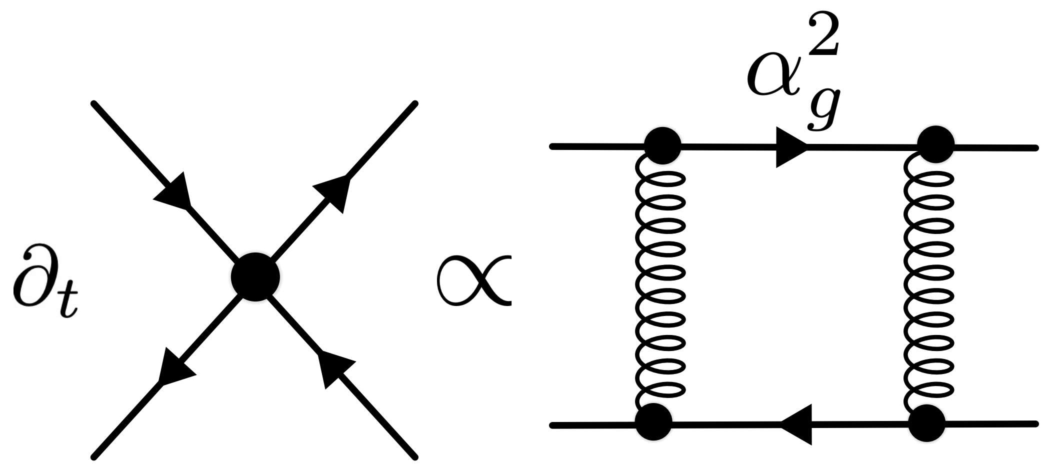

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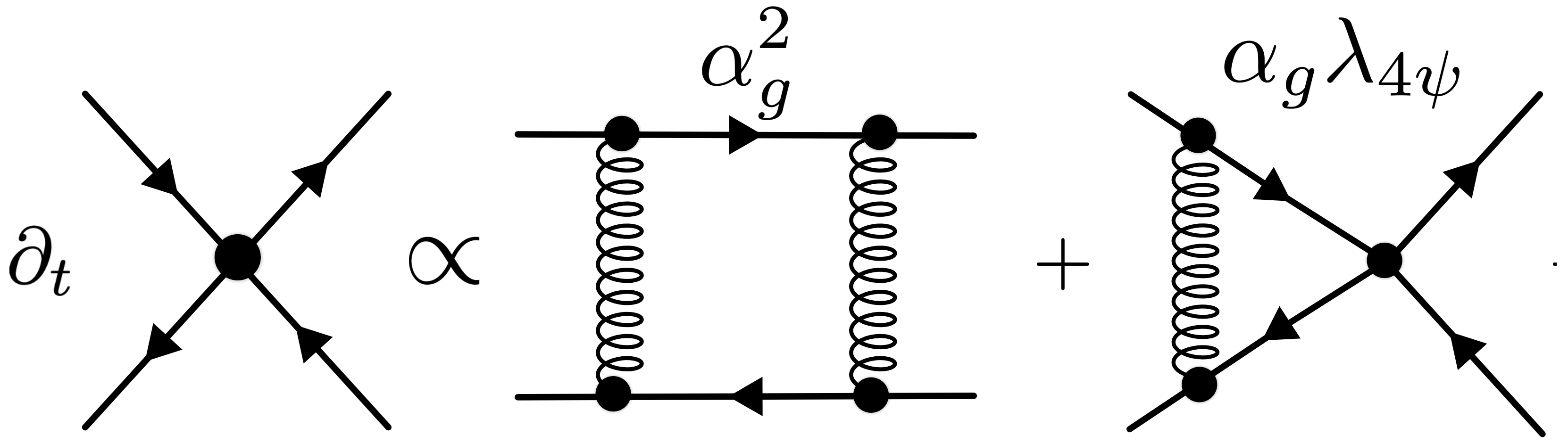
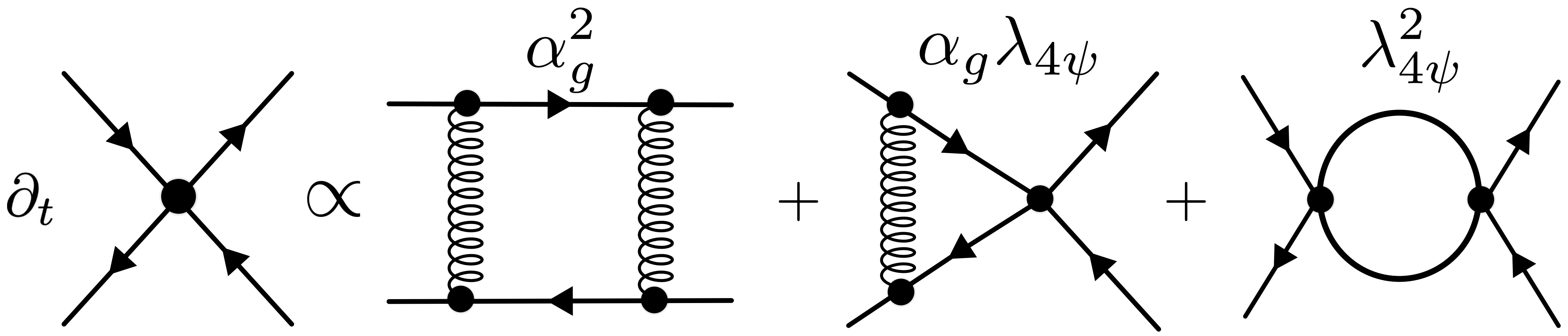


Dynamical symmetry breaking in the effective action

a chiral QCD example

$$\Gamma = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \lambda_i \left(\bar{\psi}\mathcal{T}_i\psi\right)^2 + \kappa_i \left(\bar{\psi}\mathcal{T}_i\psi\right)^3 + \ldots$$



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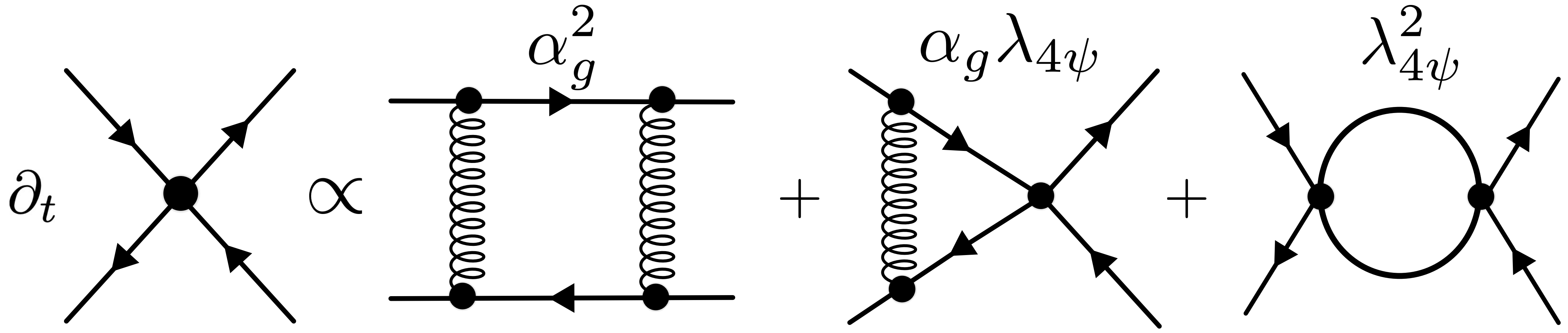

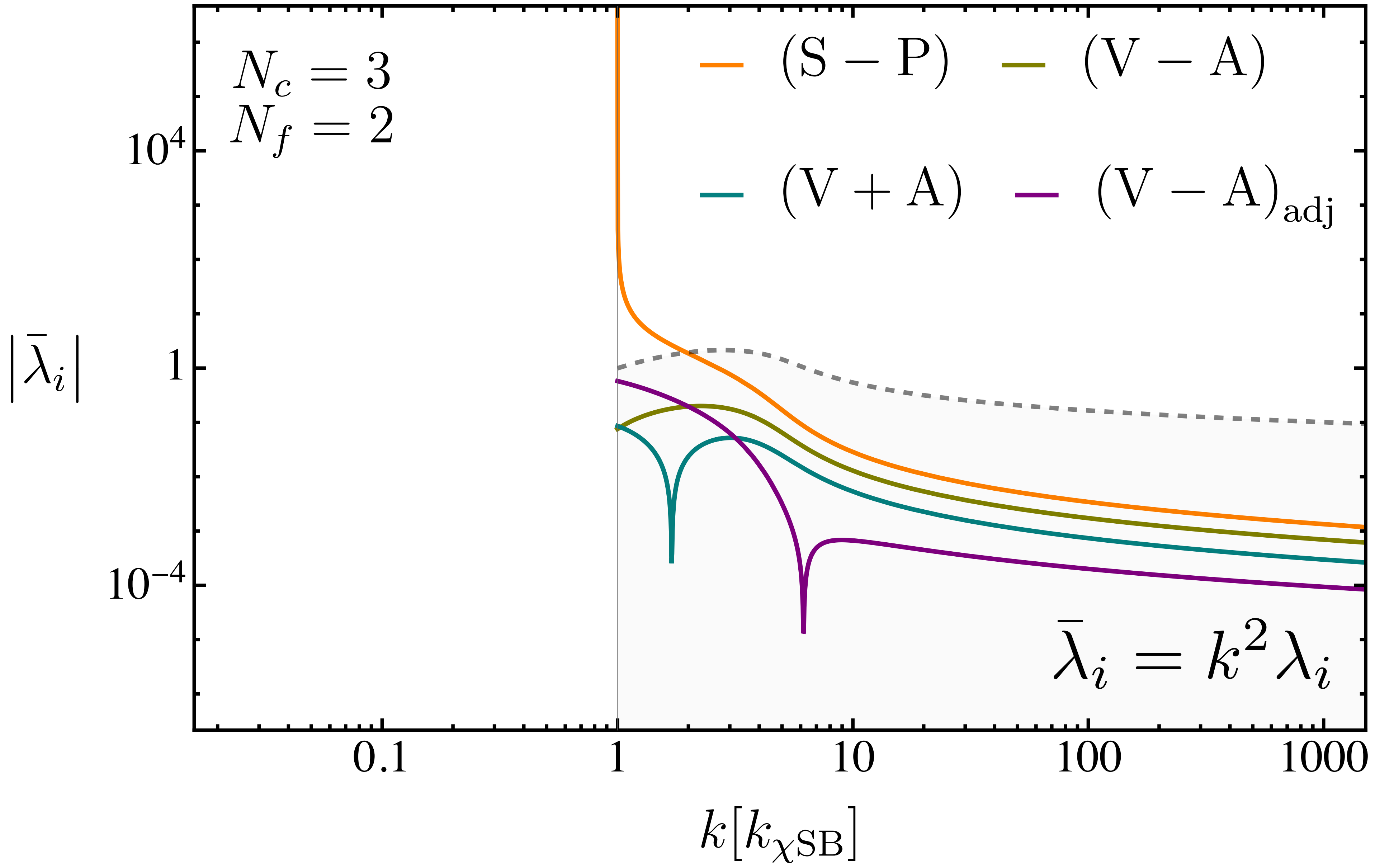

Dynamical symmetry breaking in the effective action

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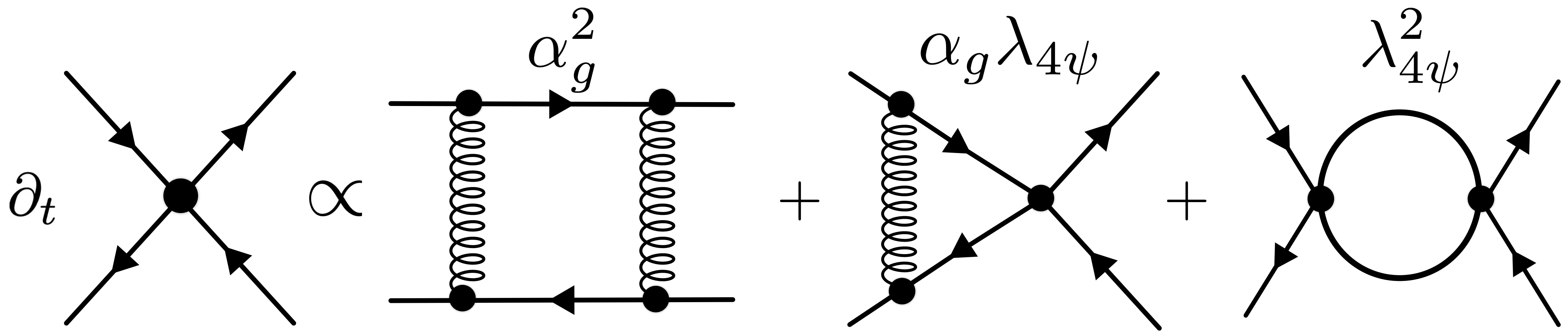
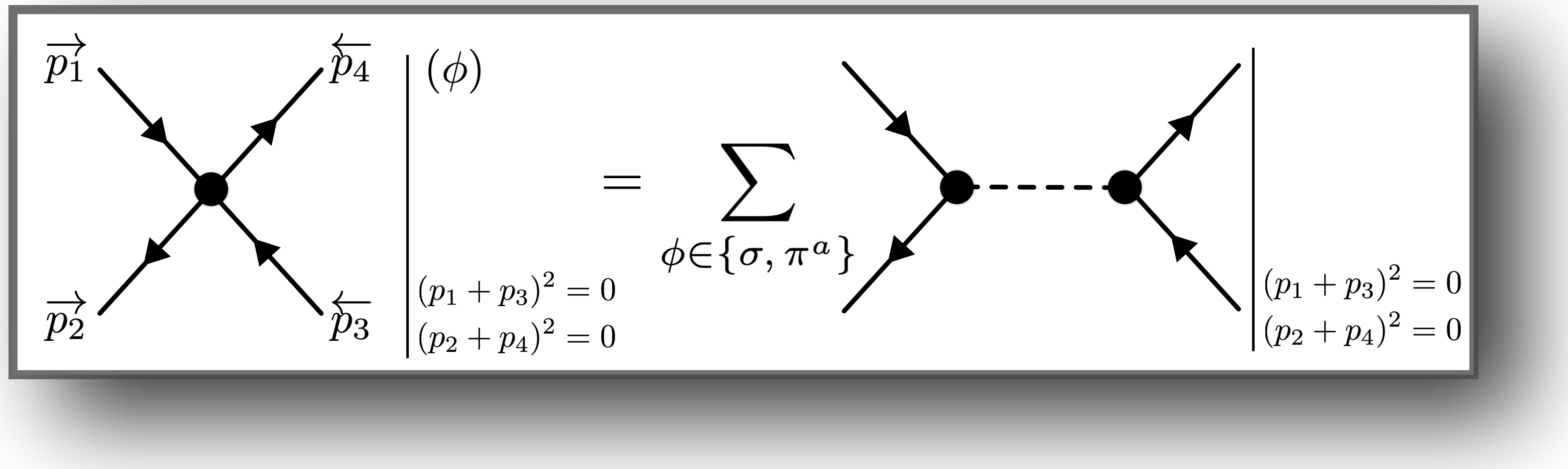
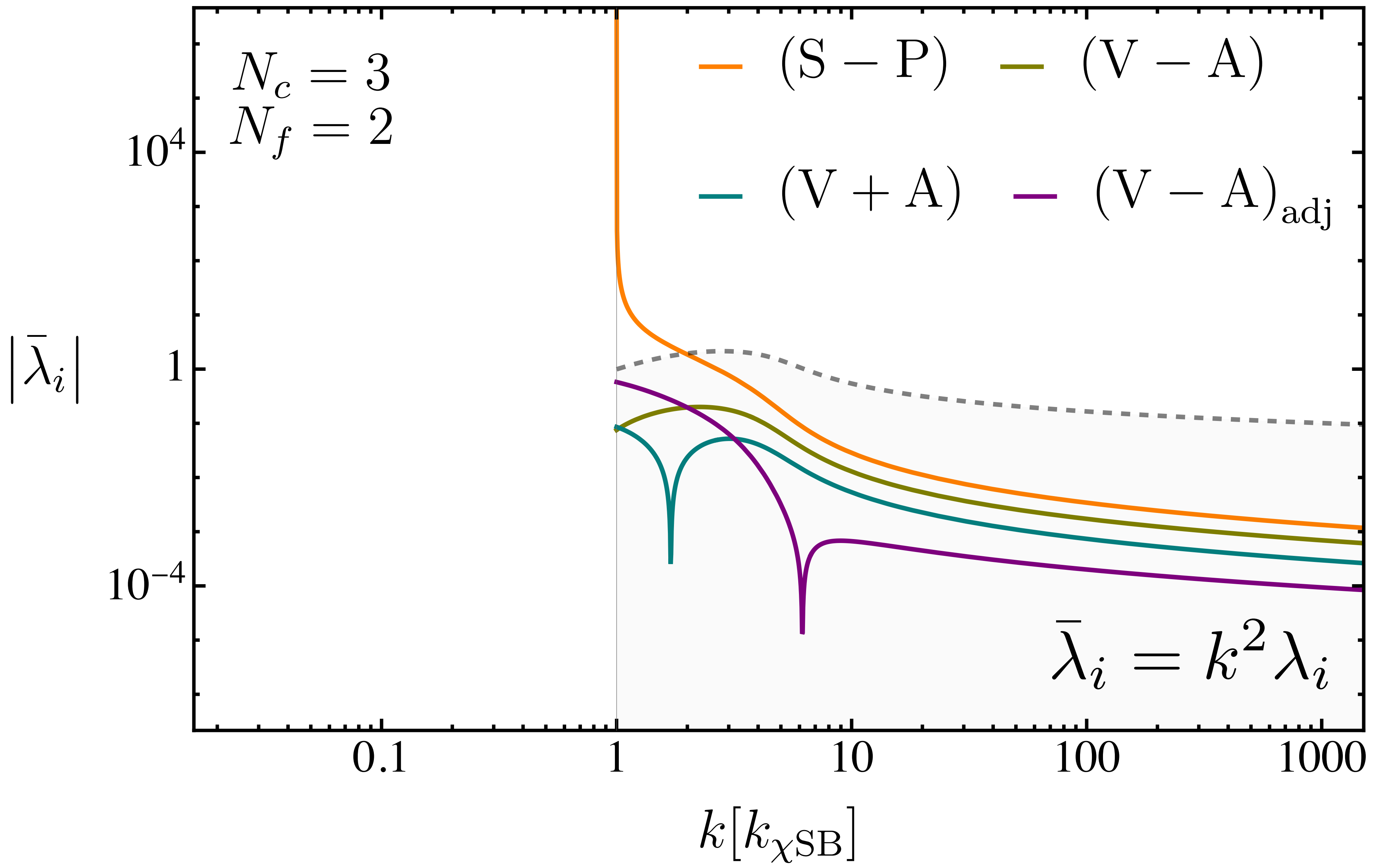
$$\Gamma = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm gh} + \lambda_i \left(\bar\psi \mathcal{T}_i \psi\right)^2 + \kappa_i \left(\bar\psi \mathcal{T}_i \psi\right)^3 + \dots$$



Stratonovich’57 Hubbard’59





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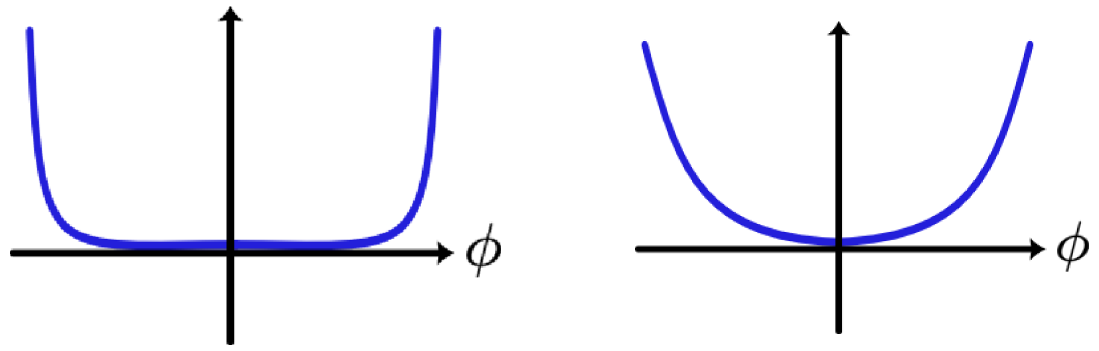
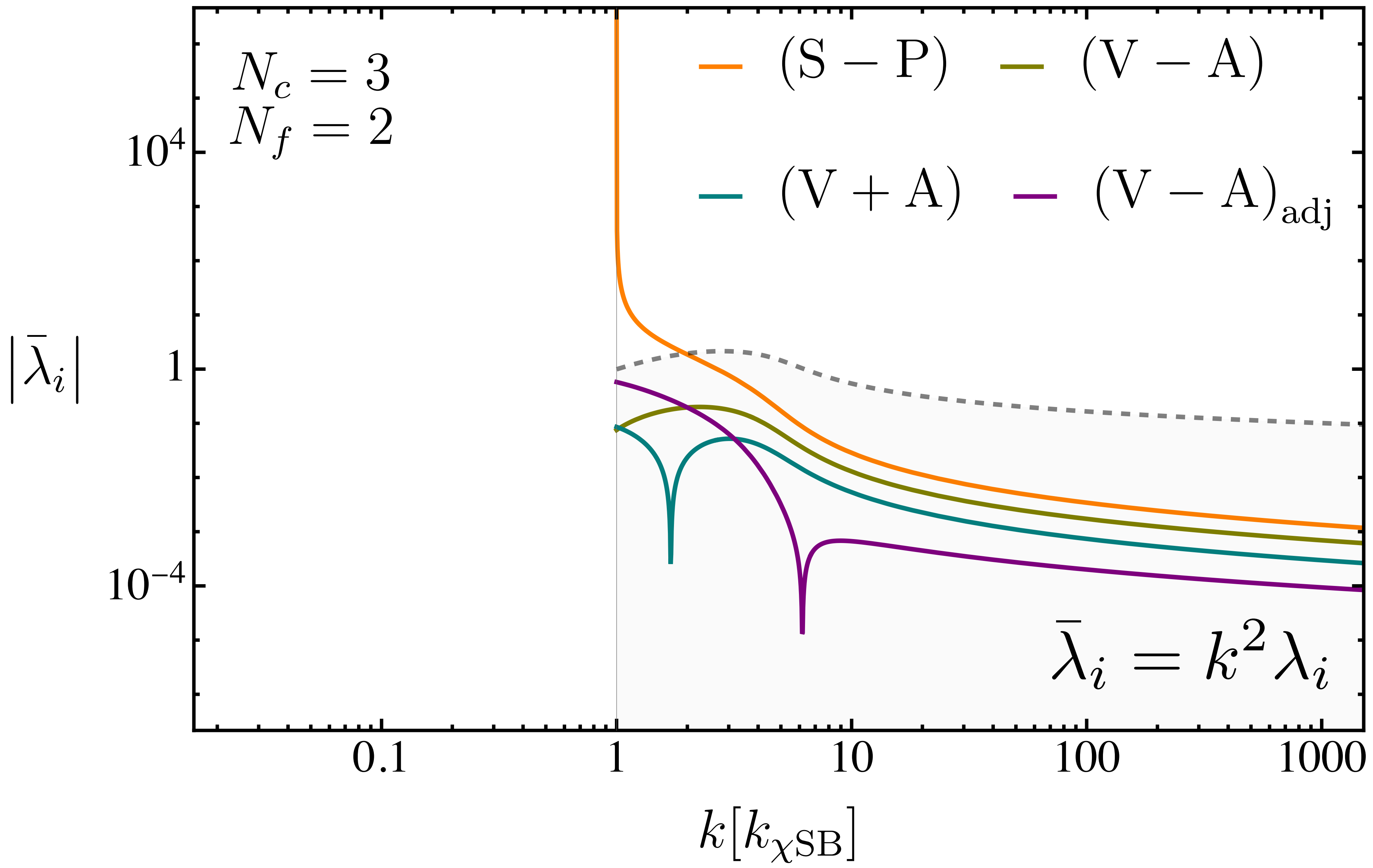
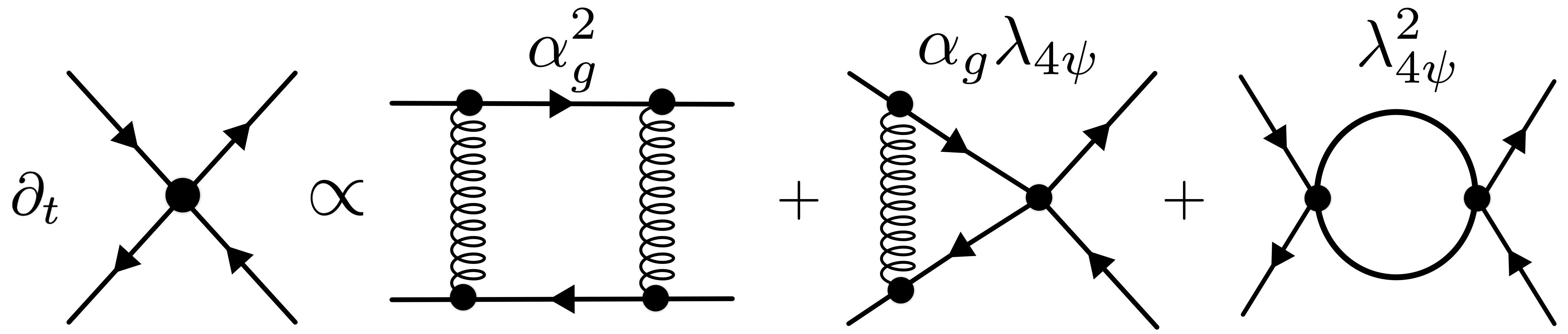
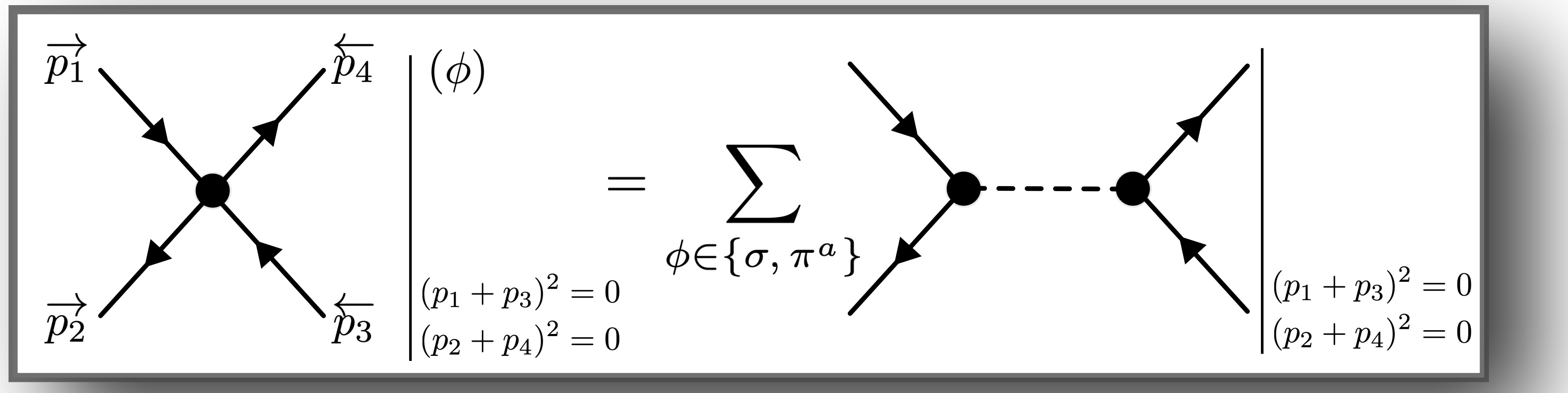






Stratonovich'57 Hubbard'59



$\lambda_i \propto 1/m^2_{\phi_i}$

$m^2_{\phi_i} = \partial_{\phi_i^2} V(\phi_i)$

$\lambda_i \to \infty \qquad m^2_{\phi_i} = 0$

Infinite correlation length:

$\xi \to \infty$

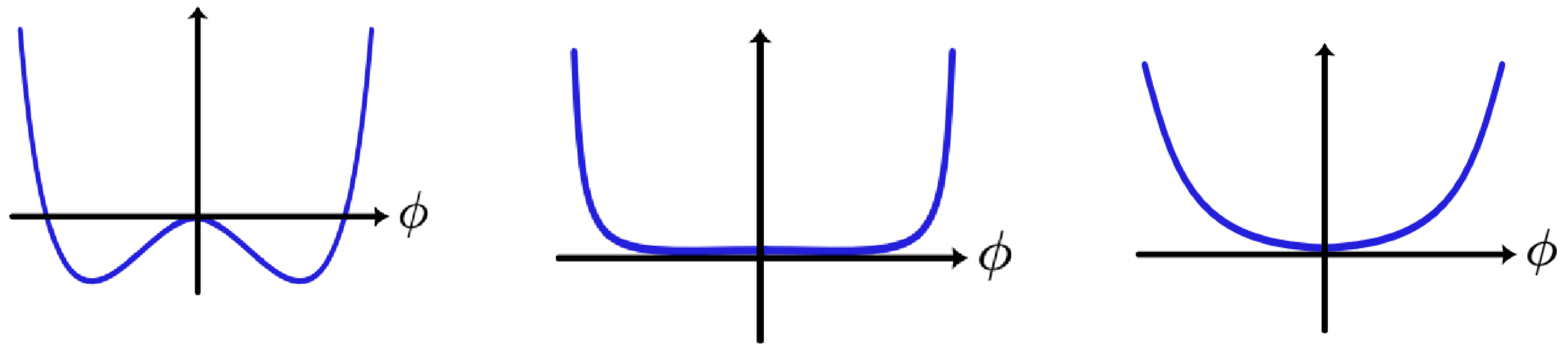
Phase transition

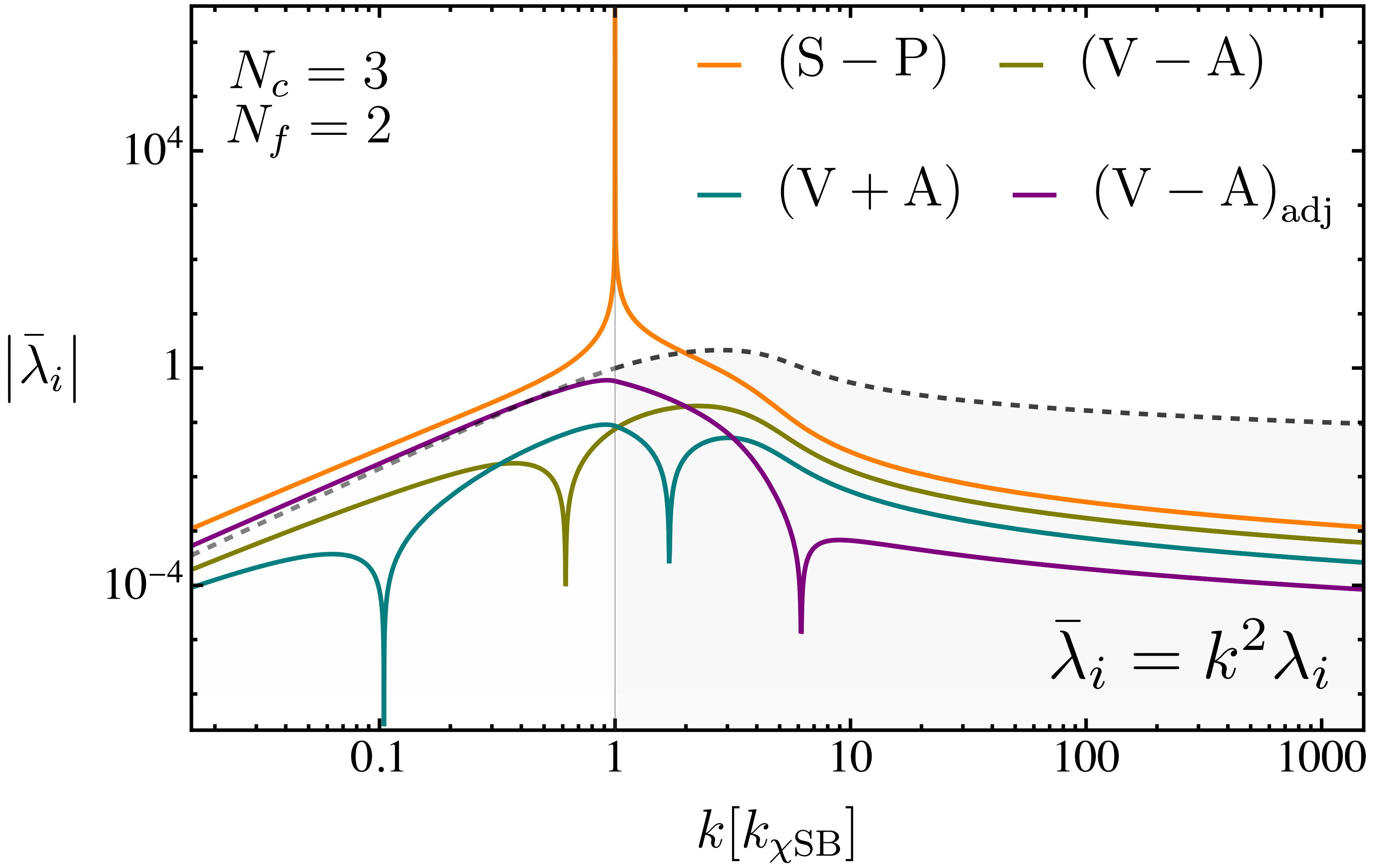
Nambu,Jona-Lasinio'60'61

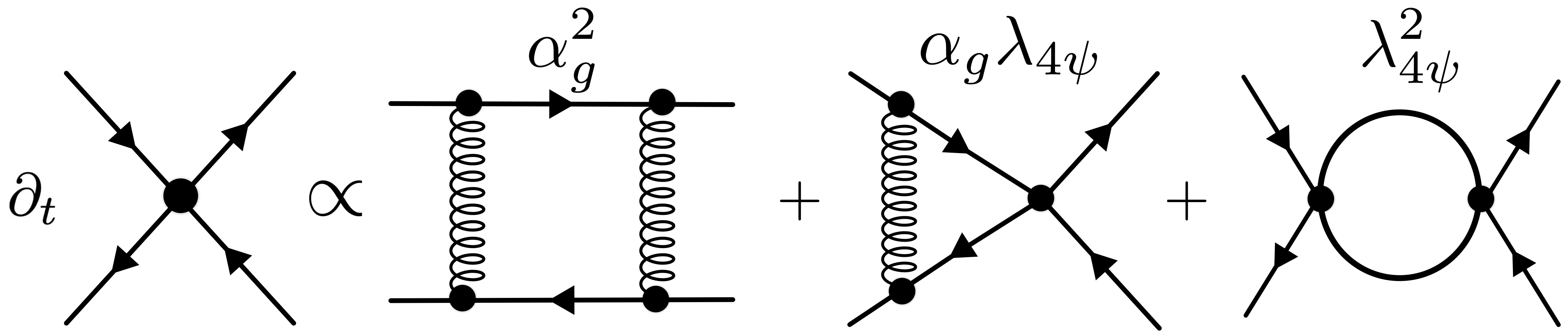
Dynamical symmetry breaking in the effective action

a chiral QCD example

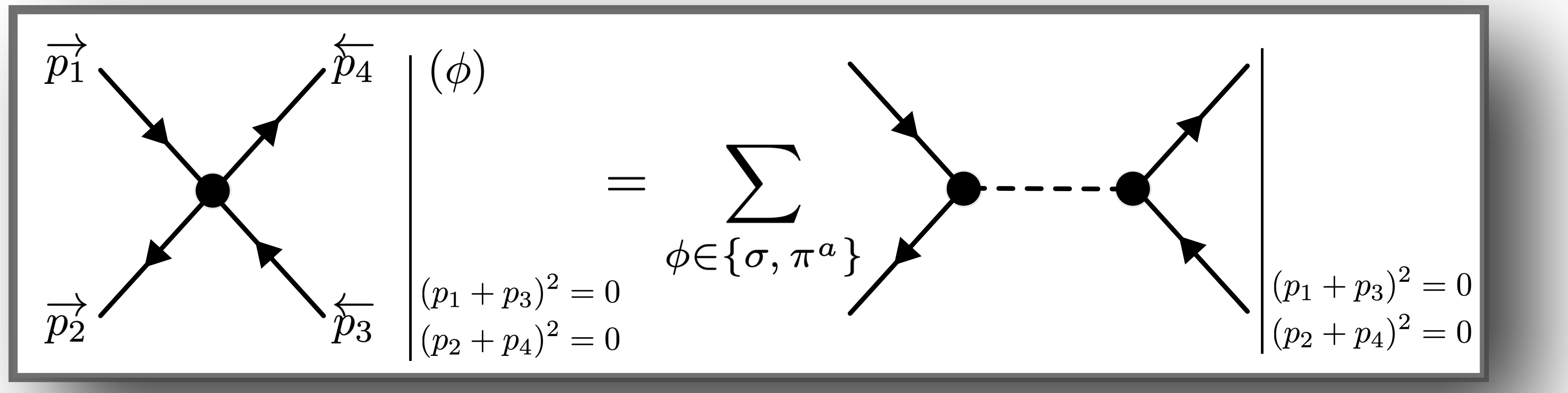
$$\Gamma = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm gh} + \lambda_i \left(\bar{\psi}\mathcal{T}_i\psi\right)^2 + \kappa_i \left(\bar{\psi}\mathcal{T}_i\psi\right)^3 + \ldots$$







Stratonovich’57 Hubbard’59



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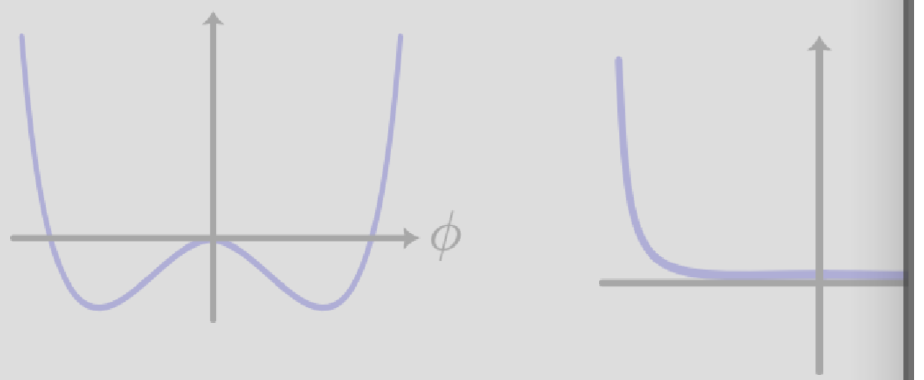
Phase transition

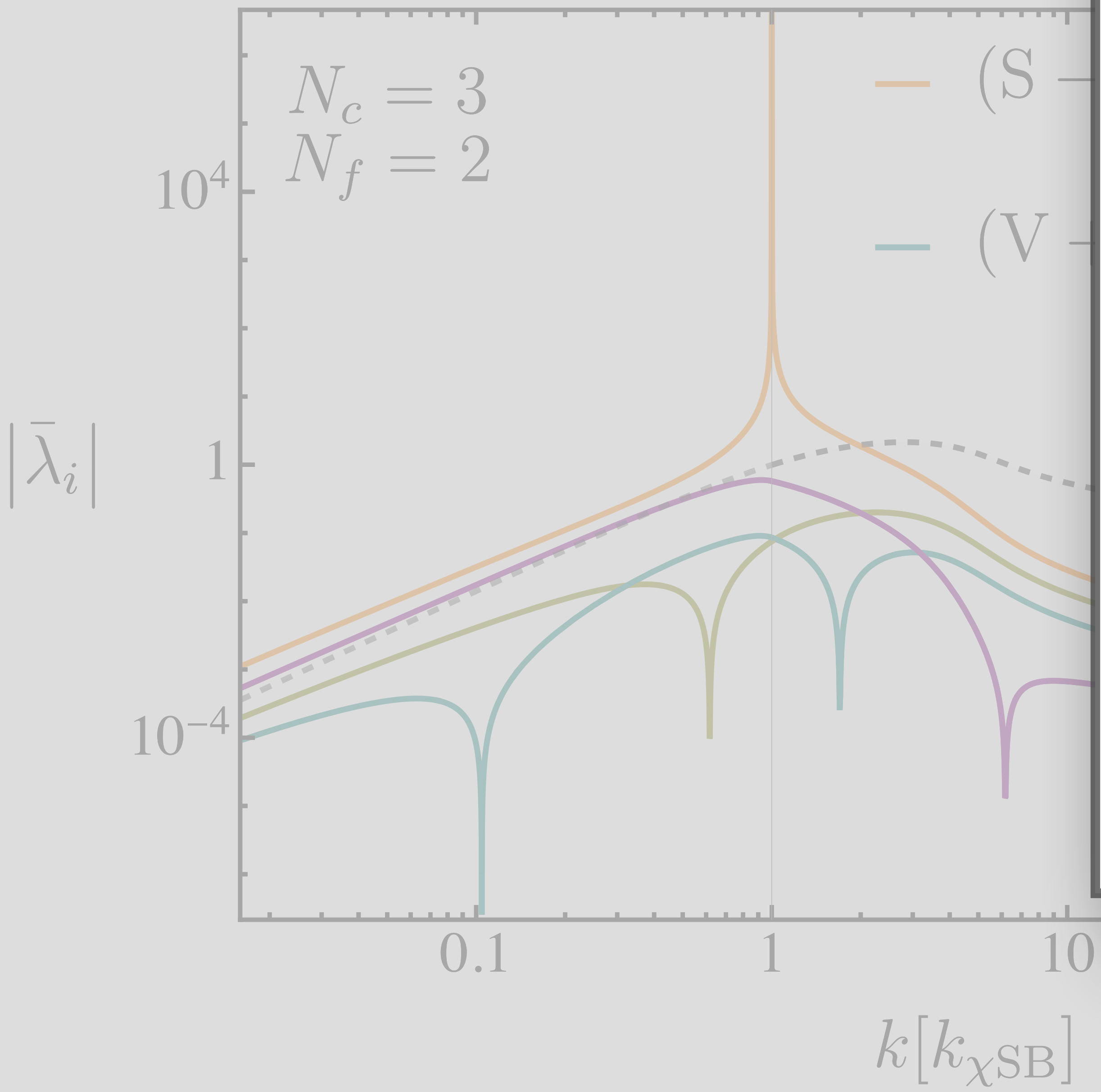
Nambu,Jona-Lasinio’60’61

Dynamical s effective action

$$\Gamma = \int_x \frac{1}{4}$$

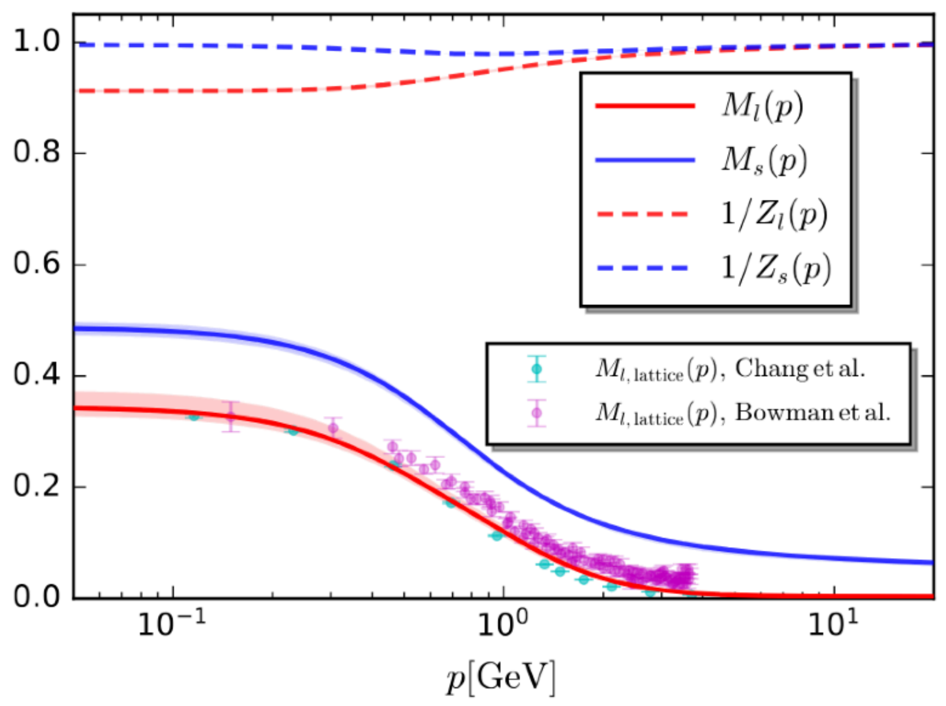
ral QCD example



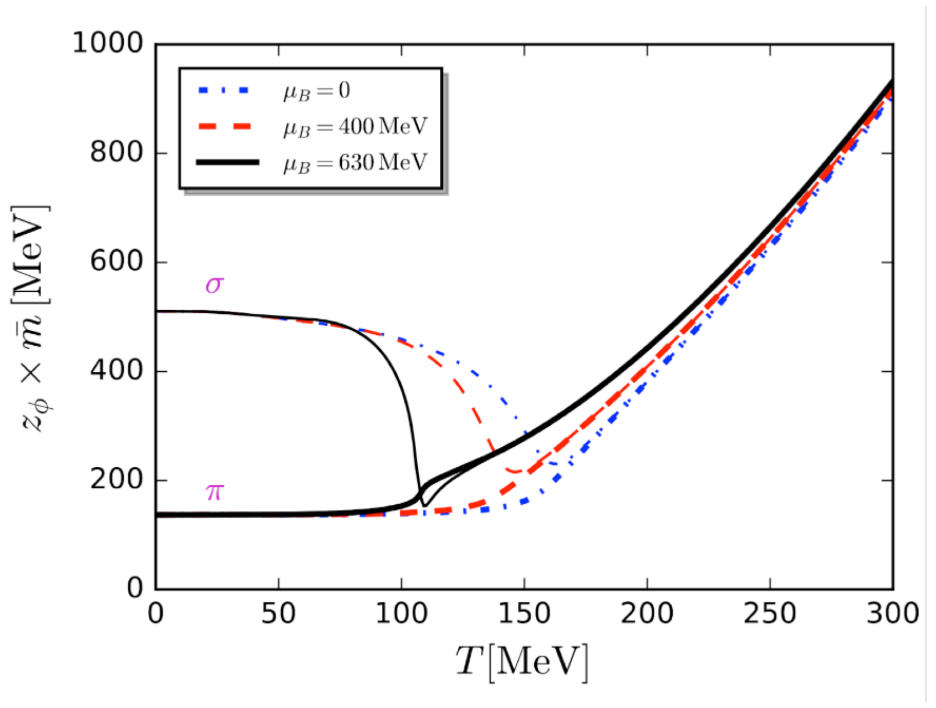


Dynamical bosonisation and emergent hadrons

Fu,Huang,Pawlowski,Tan,Zhou[2502.14388]

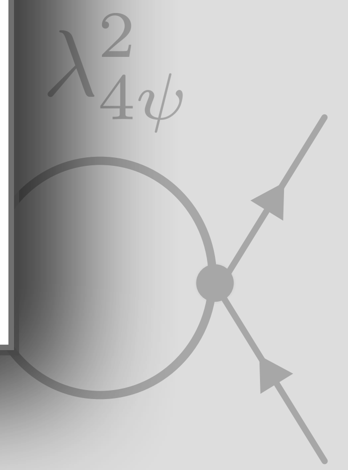


Fu,Pawlowski,Rennecke [1909.02991]

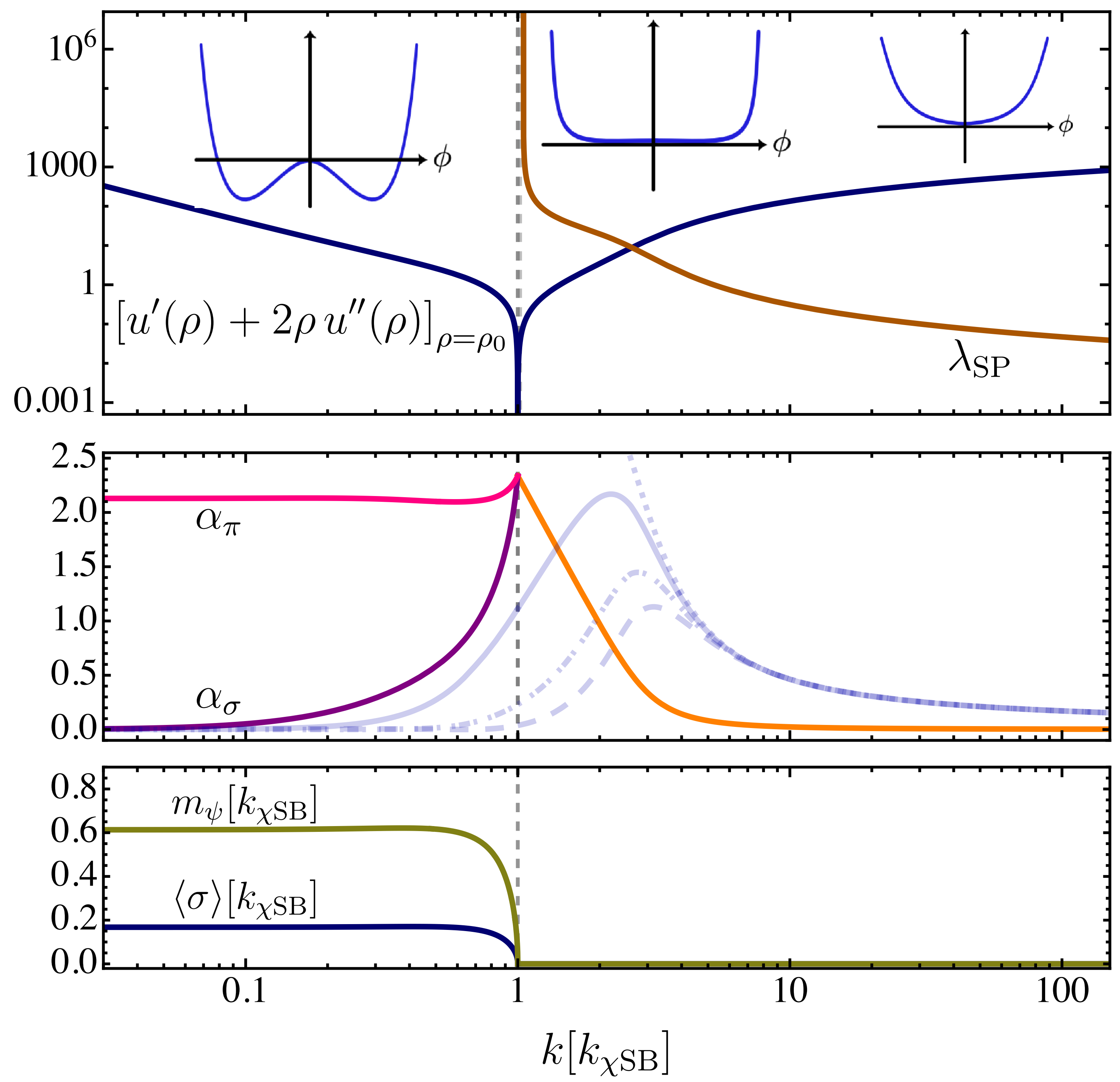


$$\left(\partial_t + \int \dot\phi_c \frac{\delta}{\delta\phi_c}\right)\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{1}{\Gamma_k^{(2)}+R_k}\right)_{ij}\left(\partial_t\delta^{jn} + 2\frac{\delta\dot\phi_n}{\delta\phi_j}\right)R_k^{mi}\right]$$

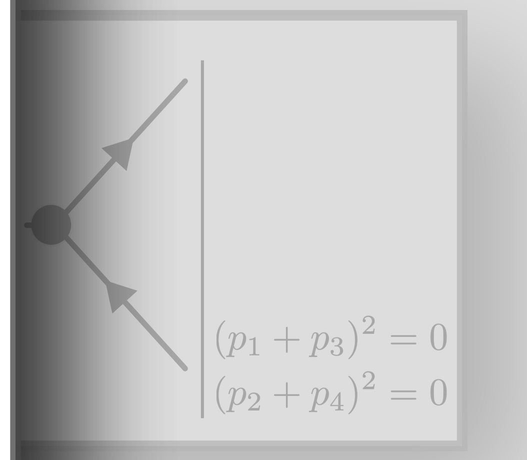
$\lambda_{4\psi}^2$



Goertz,APG,Pawlowski[2412.12254]



h'57 Hubbard'59



$\partial_{\phi_i^2} V(\phi_i)$

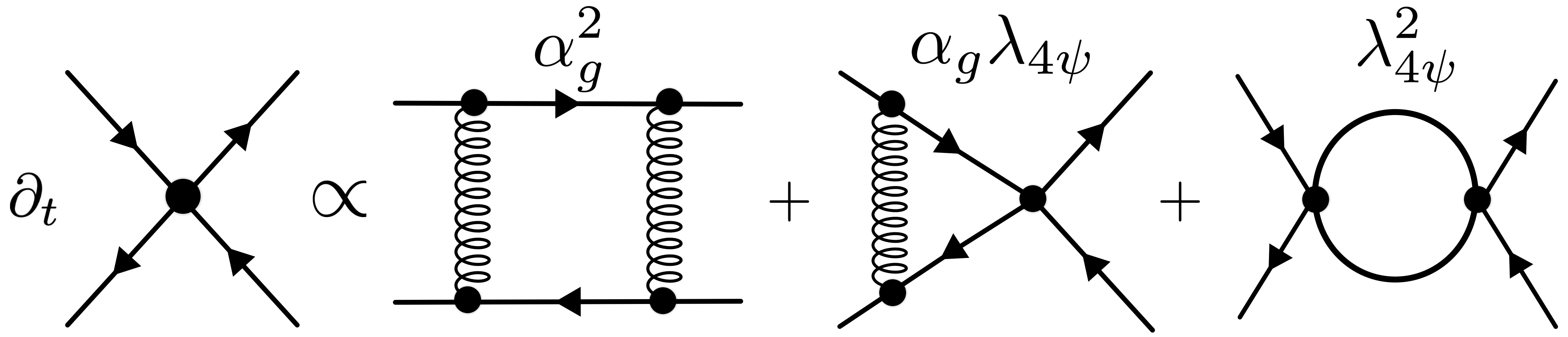
rrelation length:

$\xi \to \infty$

Jona-Lasinio'60'61

Mechanism underlying dSB

Nambu,Jona-Lasinio’60’61 Kosterlitz’74 Miransky’85 Jungnickel,Wetterich[hep-ph/9505267] Gies,Jaeckel[hep-ph/0507171] B.Kaplan,Lee,T.Son,Stephanov[0905.4752] Braun[1108.4449]



$$\partial_t\bar{\lambda}_i \propto 2\,\bar{\lambda}_i + \boldsymbol{c}_{\mathrm{A},i}\cdot\alpha_g^2 + \boldsymbol{c}_{\mathrm{B},ij}\cdot\alpha_g\,\bar{\lambda}_j + \boldsymbol{c}_{\mathrm{C},ijk}\cdot\bar{\lambda}_j\bar{\lambda}_k + \ldots$$

- Necessary conditions for dSB (FP merger):

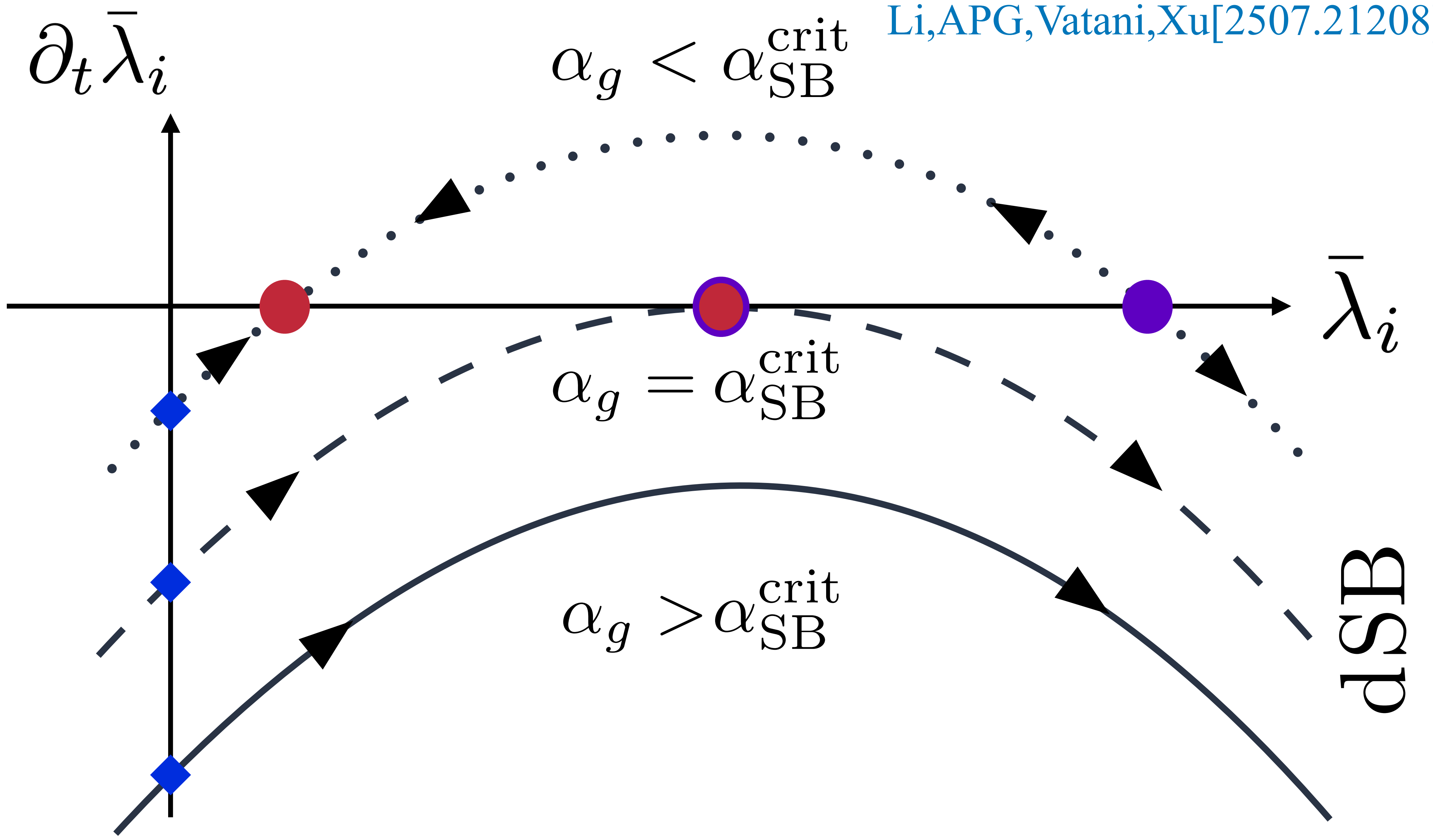
  1. Resonant structure:

$$\frac{\boldsymbol{c}_{\mathrm{A},i}}{\boldsymbol{c}_{\mathrm{C},iii}} > 0$$

  2. Critical strength of gauge dynamics is reached:

$$\alpha_g > \alpha_{\mathrm{SB}}^{\mathrm{crit}}$$

Goertz,APG,Pawlowski[2412.12254]
Li,APG,Vatani,Xu[2507.21208]



$$\alpha_{\mathrm{SB}}^{\mathrm{crit}} = \inf\left\{\alpha_g \,\middle|\, \exists\, i : \left(\partial_t\bar{\lambda}_i \leq 0 \quad \forall\,\bar{\lambda}_j\right)\right\}$$

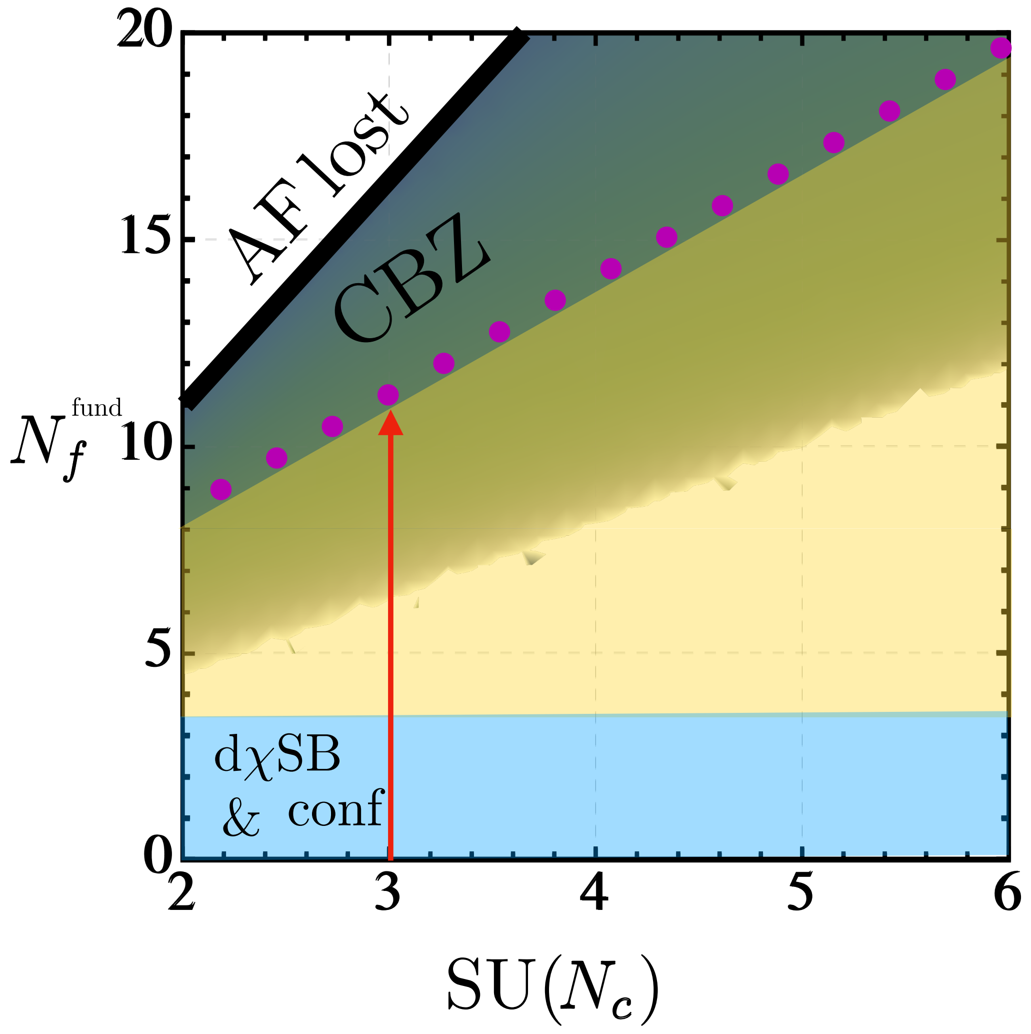
Phases of QCD-like theories

Osaka castle

Many flavour dynamics

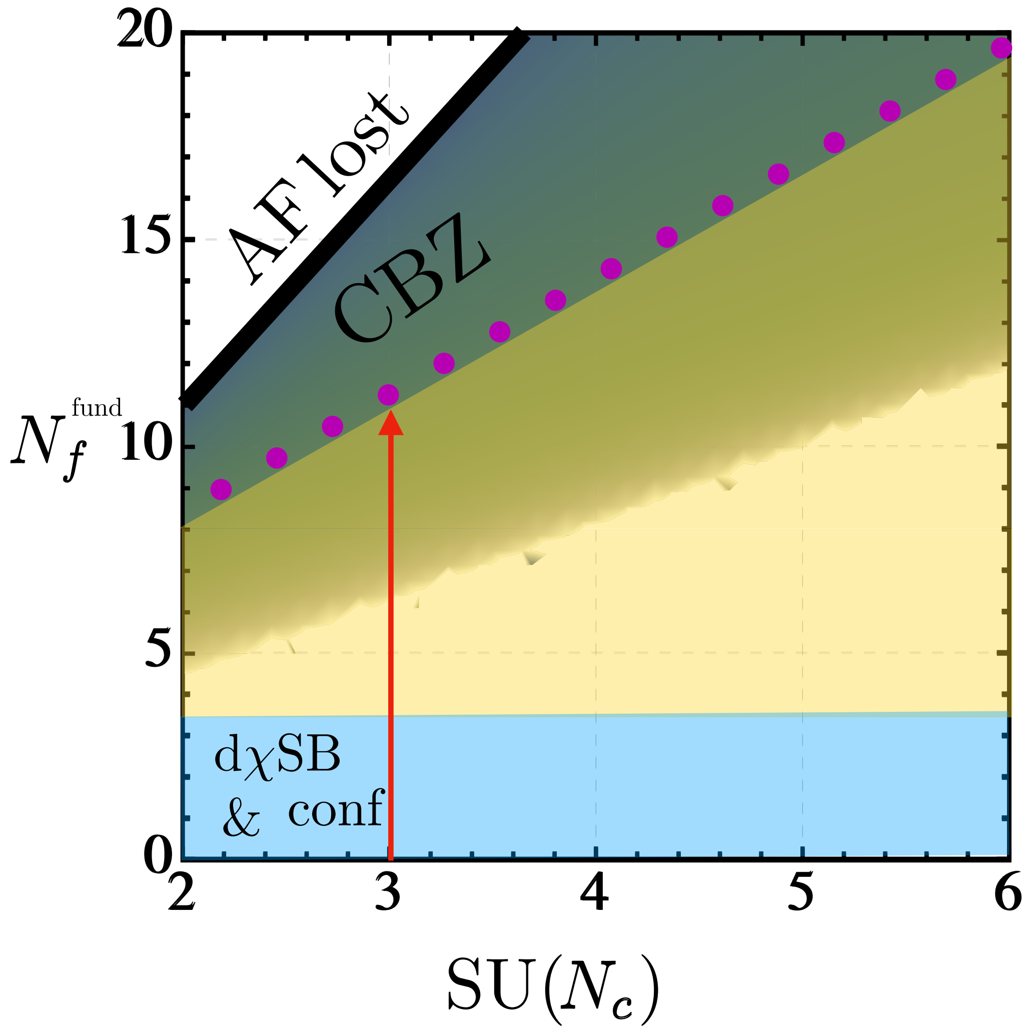
$$\qquad\qquad\qquad\mathrm{SU}(N_c) \qquad\qquad \mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$$

$$S = \int_x \boxed{\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{gh}}} + \boxed{\bar{\psi} \not{D}\, \psi}$$

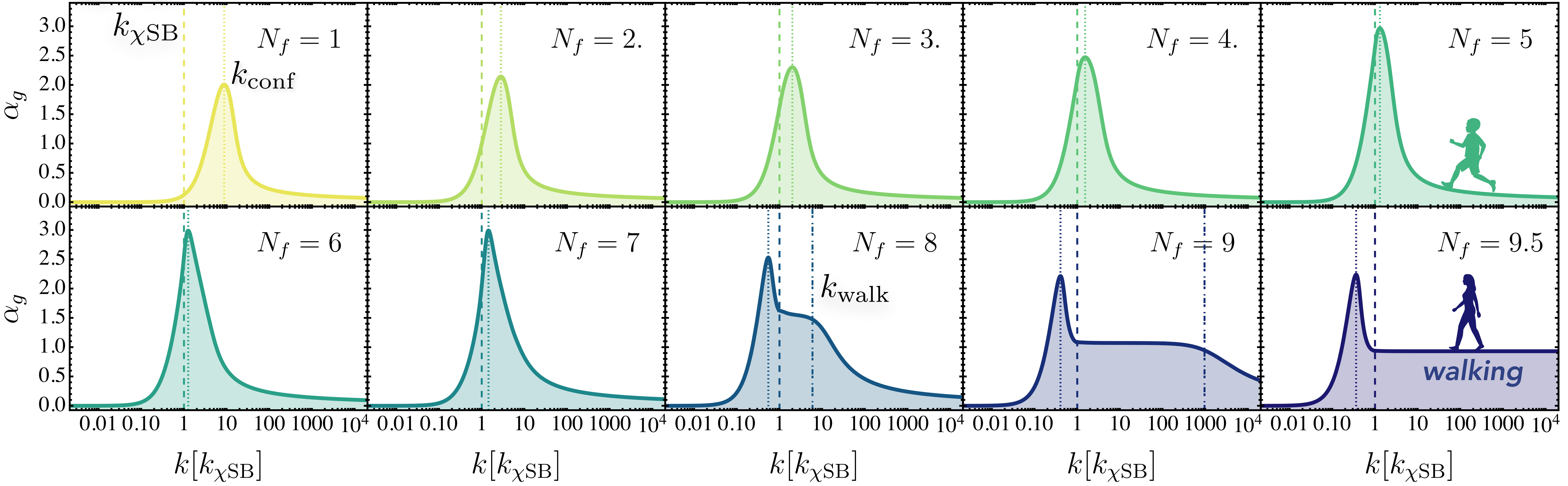


Many flavour dynamics

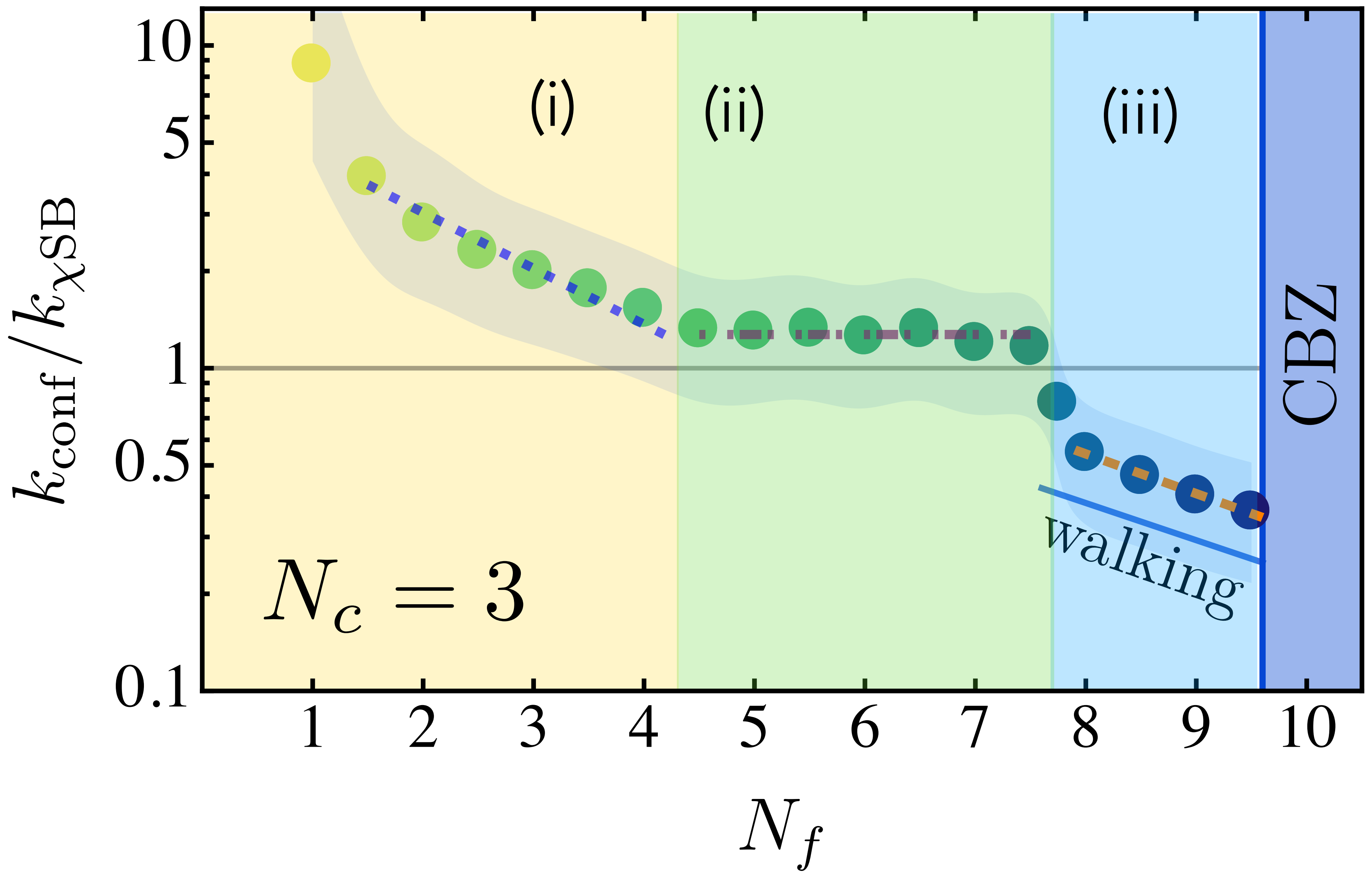
$$S = \int_x \overset{\mathrm{SU}(N_c)}{\boxed{\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{gh}}}} + \overset{\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R}{\boxed{\bar{\psi} \not{D}\, \psi}}$$



$N_c = 3$



Phases of gauge-fermion QFTs



(i) $N_f \lesssim 4$: QCD-like

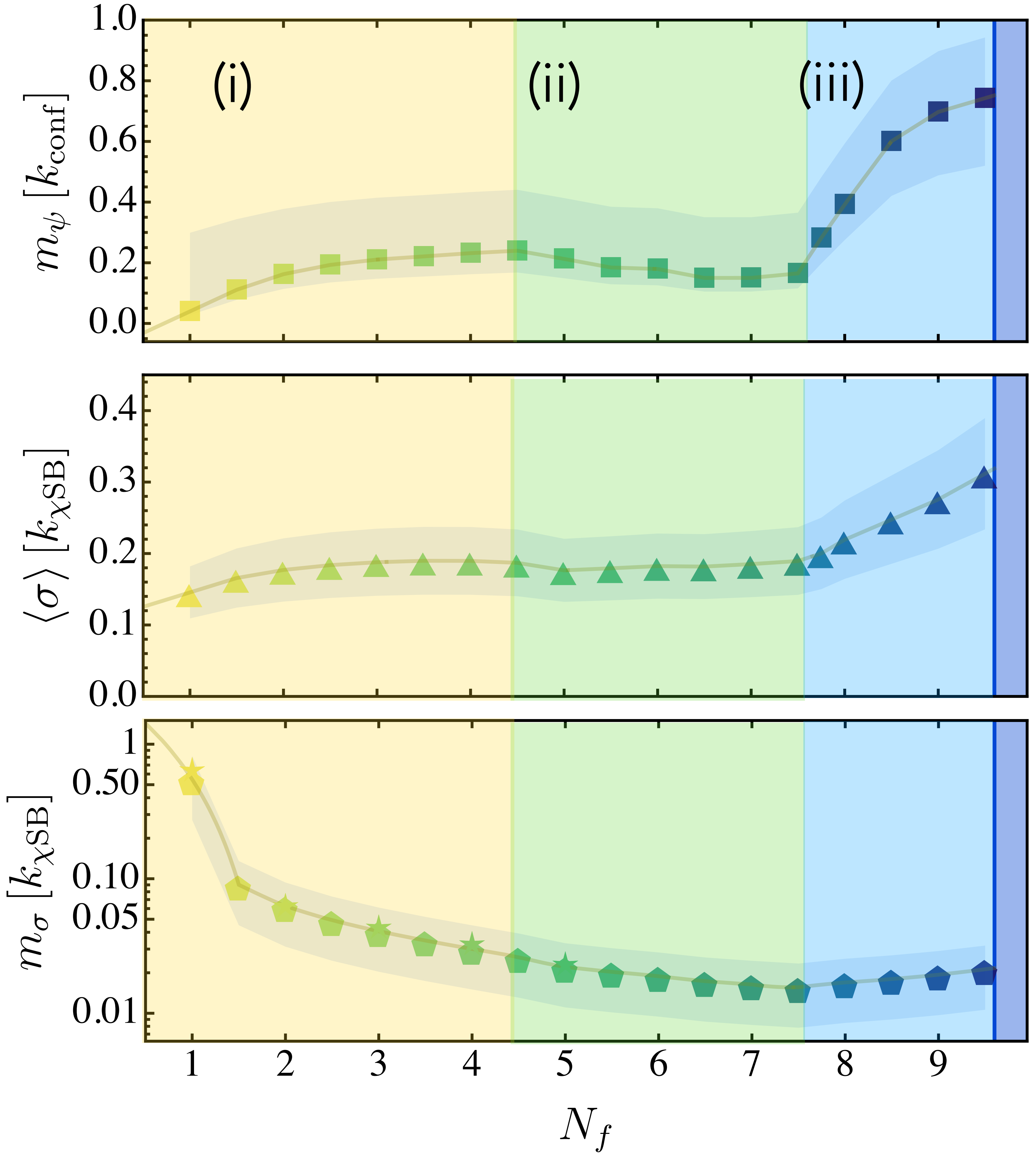
- Criticality for $N_f \lesssim 2$: potential *confinement without dχSB*

(ii) $4 \lesssim N_f \lesssim 7.5$: *Locking*

- Very entangled (qualitative)
- ***No** confinement without dχSB*

(iii) $7.5 \lesssim N_f < N_f^{\rm crit}$: Walking

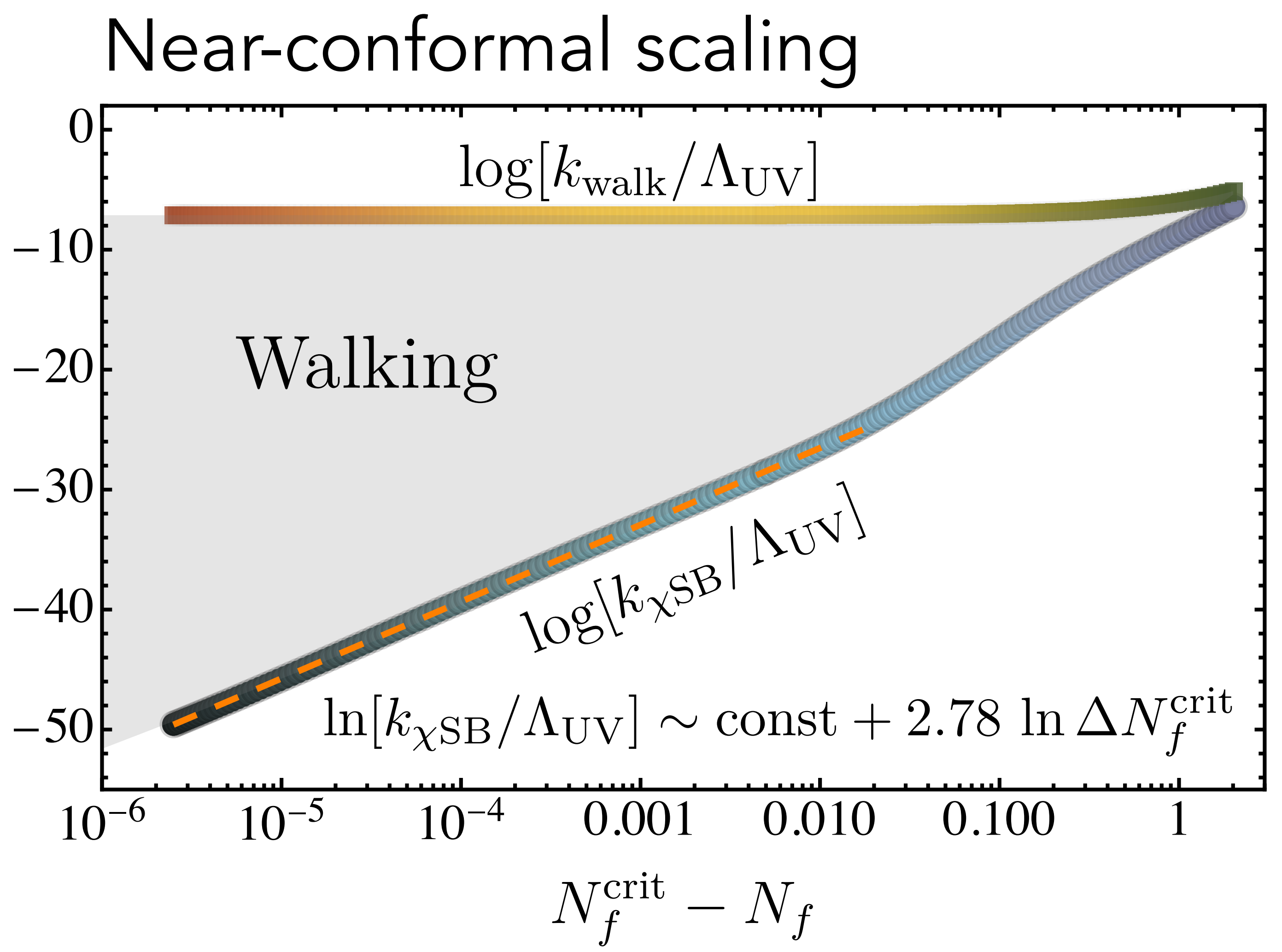
- Non-NGB bound states heavier than glueballs
- **Swicthed order parameters**
- Size of walking regime from first-principles

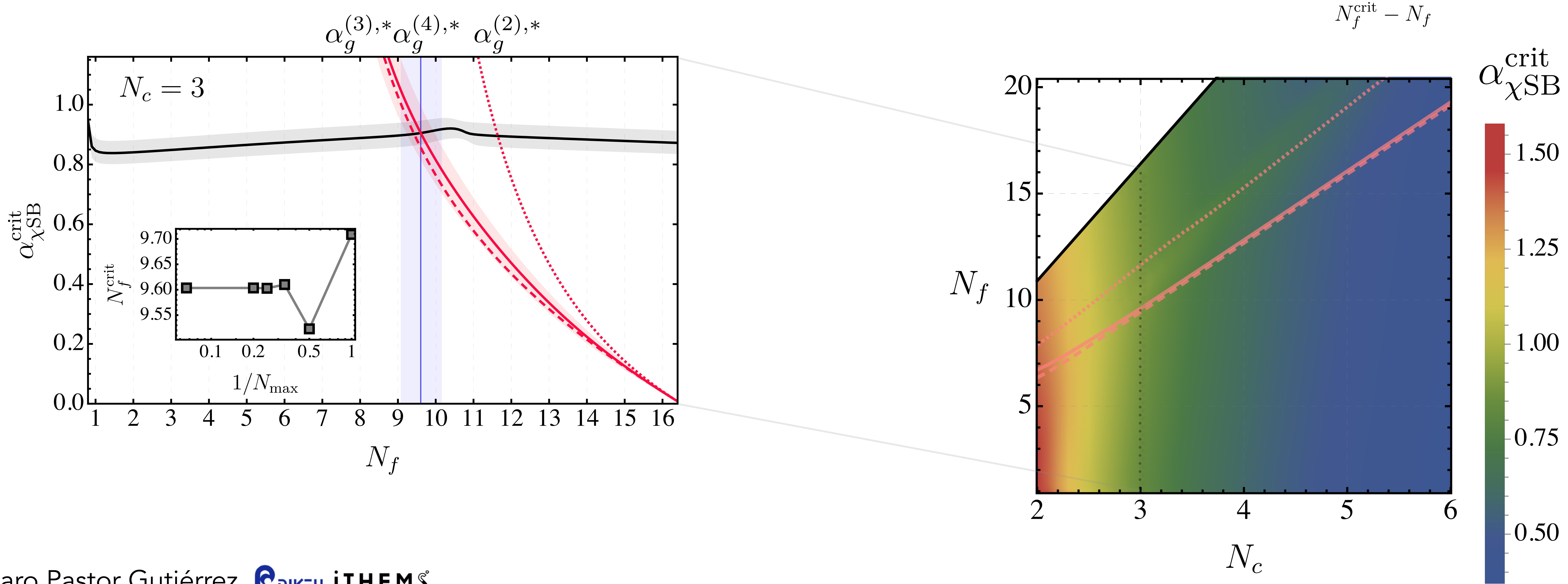


Conformal window and scaling

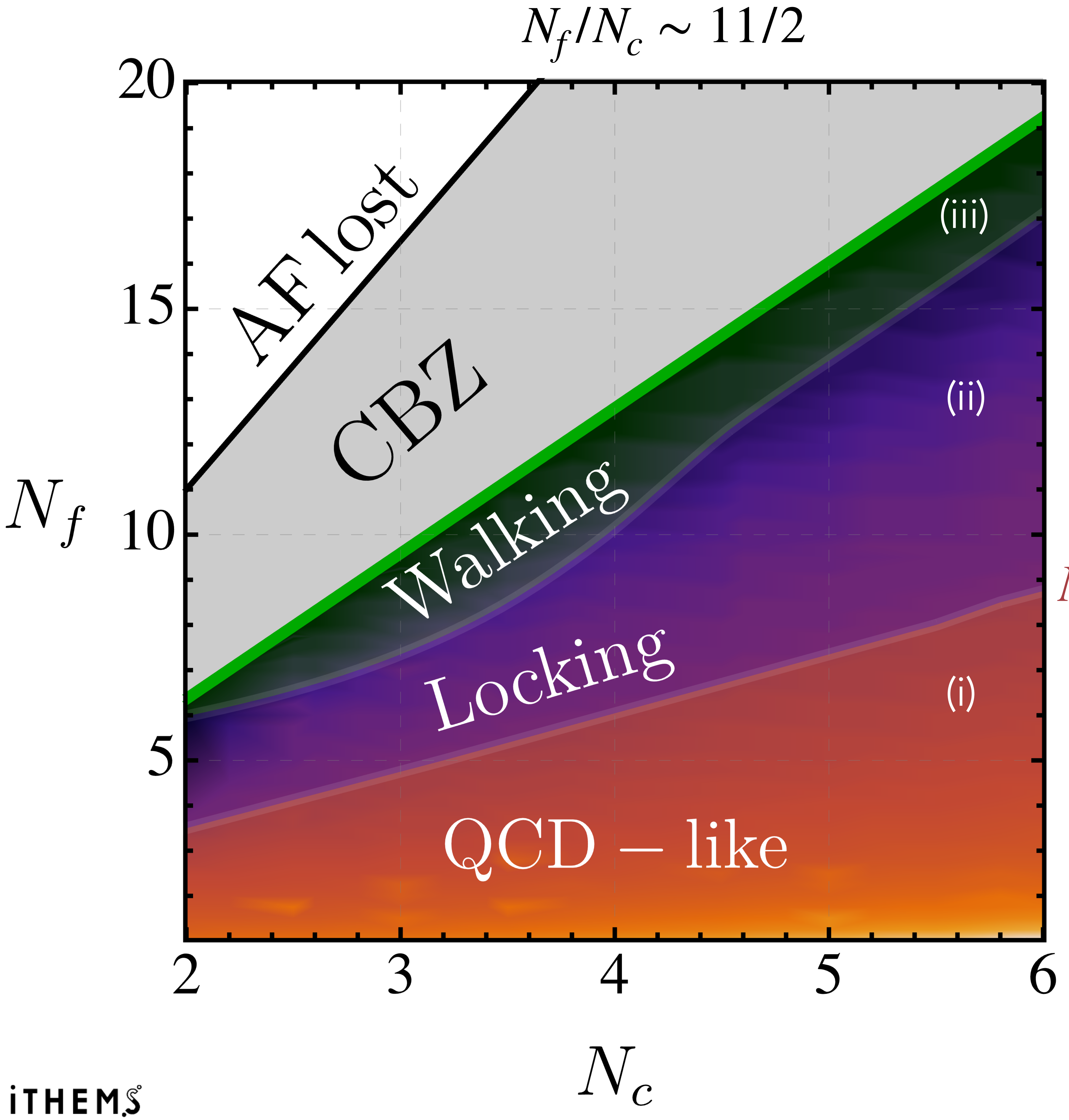
Gies, Jaeckel ’05 Braun, Gies’05’06 Braun,Gies[0912.4168] Braun,Fischer,Gies[1012.4279]

$$\boxed{N_f^{\text{crit}}(N_c=3) = 9.60^{+0.55}_{-0.53}}$$


Near-conformal scaling



Cartography



IR phases of chiral gauge theories

Kasugataisha Shrine, Nara

Chiral gauge theories

Li,APG,Vatani,Xu[2507.21208]

Li,APG,Vatani[2603.19355]

$$S = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \psi^\dagger \bar{\sigma}^\mu D_\mu \psi + \chi^\dagger \bar{\sigma}^\mu D_\mu \chi$$

Chiral: Weyl fermions, only left (right)-handed

- Phenomenology:
  - SM and BSM physics (GUTs, preon models, …)

    Georgi,Glashow'74 Raby,Dimopoulos,Susskind '79 Bars,Yankielowitz'81 …

    Bolognesi,Konishi,Orso[2605.10416] Cacciapaglia,Sannino,Wagner [2605.08294]
- Theory:
  - understand QFT
  - conjectured **rich** and **unexplored** dynamics, a challenge
    - Exotic condensates
    - Tumbling: several patterns of SB
    - Symmetric Mass Generation Wang,You[2204.14271] Karasik,Önder,Tong[2208.07842] Tong[2104.03997]

	$SU(N_c)$	$SU(N_c-4)$	$U(1)$
ψ	$\overline{\square}$	$\square$	$-(N_c-2)$
χ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	N_c-4

Bars-Yankielowitz (BY)

	$SU(N_c)$	$SU(N_c+4)$	$U(1)$
ψ	$\overline{\square}$	$\square$	$-(N_c-2)$
χ	$\square\square$	1	N_c+4

What do we know?

Method	**Major challenges**	**Results**
Monte Carlo Lattice simulations Nielsen,Ninomiya’80’81Eichten,Preskill’86	Doubler and sign problems	
Most Attractive Channel Raby,Dimopoulos,Susskind ’79 Dimopoulos,Raby,Susskind ’80 Eichten,Feinberg ‘82	Too naive	*Color-flavor-locked* IR with $\langle\chi\psi\rangle$
Large N Eichten,Peccei,Preskill,Zeppenfeld’86 Karasik,Önder,Tong[2208.07842]	Finite N	If confinement, no dSB, massless boundstates
Anomaly mediated SUSY breaking Csáki,Murayama,Telem[2105.03444]	Non-SUSY limit	Conformal theories for few N<13 and for larger, $\langle\chi\psi\rangle$ and $\langle\chi\chi\rangle$
Anomaly matching, discrete symmetries and higher forms ’t Hooft ’80 Bolognesi,Konishi,Luzio[2101.02601] Bars,Yankielowitz’81	Not sufficiently constraining	Massless $\mathcal{B}\sim\langle\chi\psi\psi\rangle$ baryons with no dSB, or *color-flavor-locked* phase

What do we know?

Method **Major challenges** **Results**

In conclusion:

no good understanding of the IR

Monte

Nielsen,Ninom

Most A

Raby,Dimopo

Dimopoulos,R

Large N

Eichten,Peccei,Preskill,Zeppenfeld'86

Karasik,Önder,Tong[2208.07842]

Finite N

boundstates

Anomaly mediated SUSY breaking

Csáki,Murayama,Telem[2105.03444]

Non-SUSY limit

Conformal theories for few N<13 and for larger, $\langle\chi\psi\rangle$ and $\langle\chi\chi\rangle$

Let's do fRG

Anomaly matching, discrete symmetries and higher forms

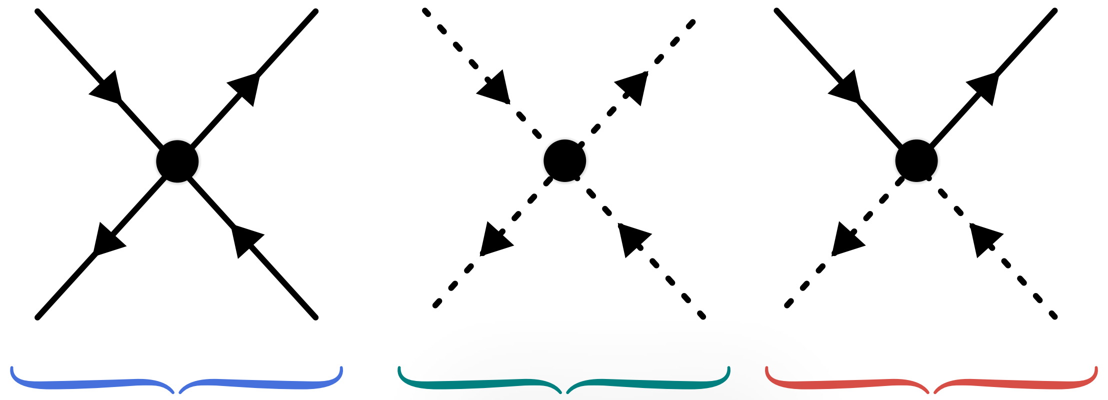
't Hooft '80 Bolognesi,Konishi,Luzio[2101.02601] Bars,Yankielowitz'81

Not sufficiently constraining

Massless $\mathcal{B}\sim\langle\chi\psi\psi\rangle$ baryons with no dSB, or *color-flavor-locked* phase

The Fierz-complete four-fermion basis

$$\Gamma_{\mathrm{fermion},k}[\chi^\dagger,\chi,\psi^\dagger,\psi] = -\int_x \underbrace{Z_\psi^2 \sum_{i=1}^{2} \lambda_i \mathcal{O}_i^{\psi}} + \underbrace{Z_\chi^2 \sum_{i=4}^{5} \lambda_i \mathcal{O}_i^{\chi}} + \underbrace{Z_\psi Z_\chi \sum_{i=6}^{7} \lambda_i \mathcal{O}_i^{\chi\psi}}$$



$$\left\{\begin{aligned} \mathcal{O}_1^{\psi} &= (\psi^\dagger \bar{\sigma}^\mu \psi)(\psi^\dagger \bar{\sigma}^\mu \psi) \\ \mathcal{O}_2^{\psi} &= (\psi^{\dagger\, f_1} \bar{\sigma}^\mu \psi_{f_2})(\psi^{\dagger\, f_2} \bar{\sigma}^\mu \psi_{f_1}) \end{aligned}\right.$$

$$\left\{\begin{aligned} \mathcal{O}_4^{\chi} &= (\chi^\dagger \bar{\sigma}^\mu \chi)(\chi^\dagger \bar{\sigma}^\mu \chi) \\ \mathcal{O}_5^{\chi} &= (\chi^\dagger \bar{\sigma}^\mu T_{\mathrm{sym}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\mathrm{sym}} \chi) \end{aligned}\right.$$

$$\left\{\begin{aligned} \mathcal{O}_6^{\chi\psi} &= (\psi^\dagger \bar{\sigma}^\mu \psi)(\chi^\dagger \bar{\sigma}^\mu \chi) \\ \mathcal{O}_7^{\chi\psi} &= (\psi^\dagger \bar{\sigma}^\mu T_{\mathrm{anti}} \psi)(\chi^\dagger \bar{\sigma}^\mu T_{\mathrm{sym}} \chi) \end{aligned}\right.$$

The Fierz-complete four-fermion basis

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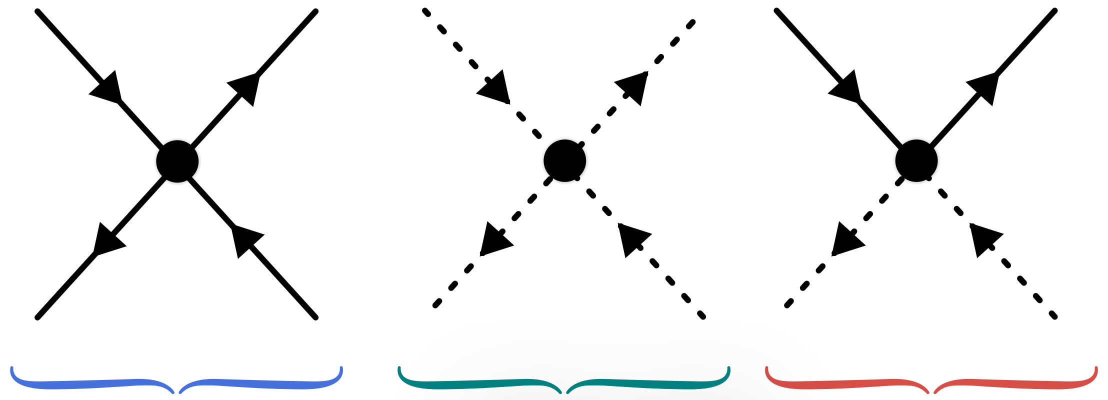

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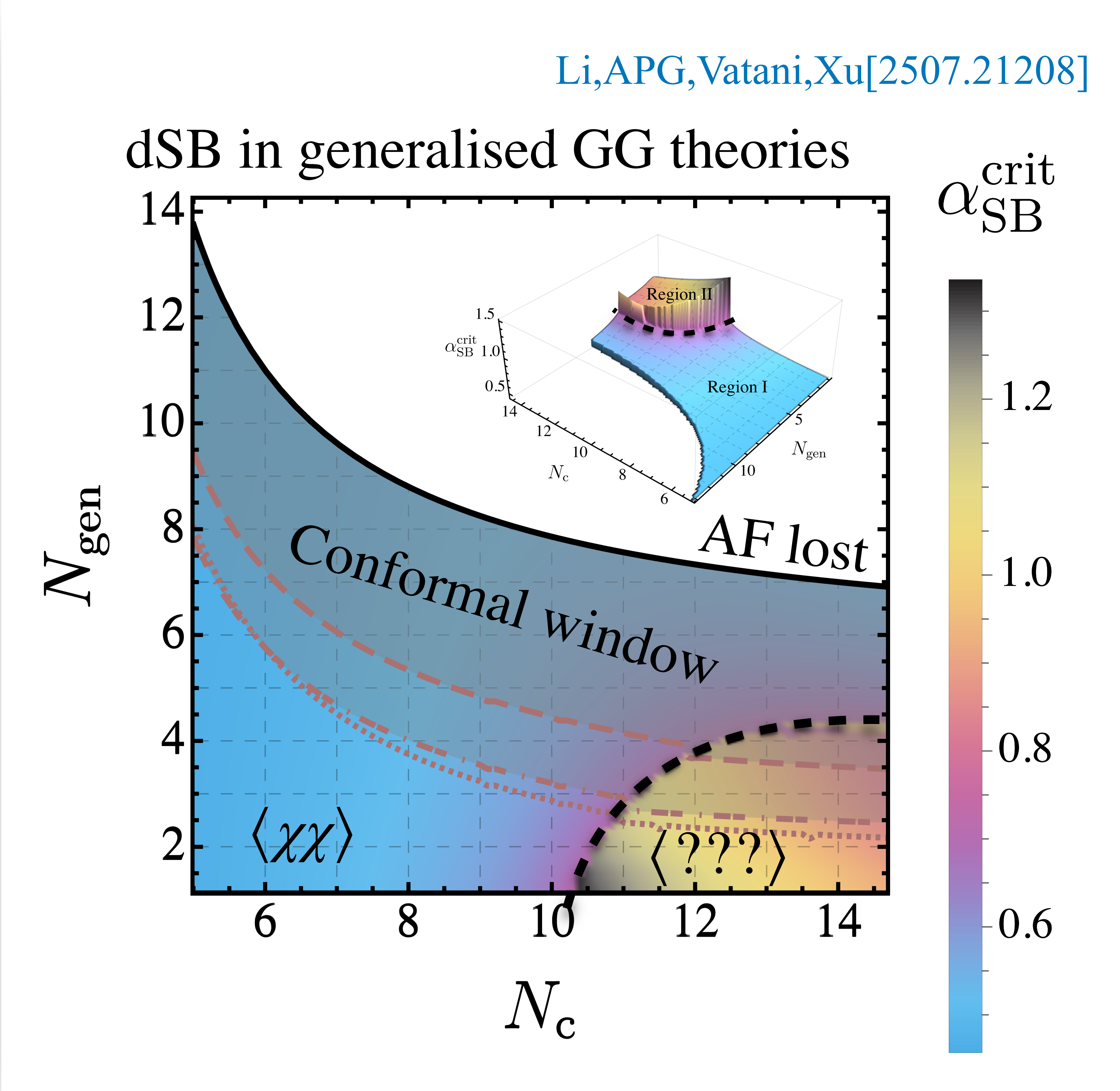
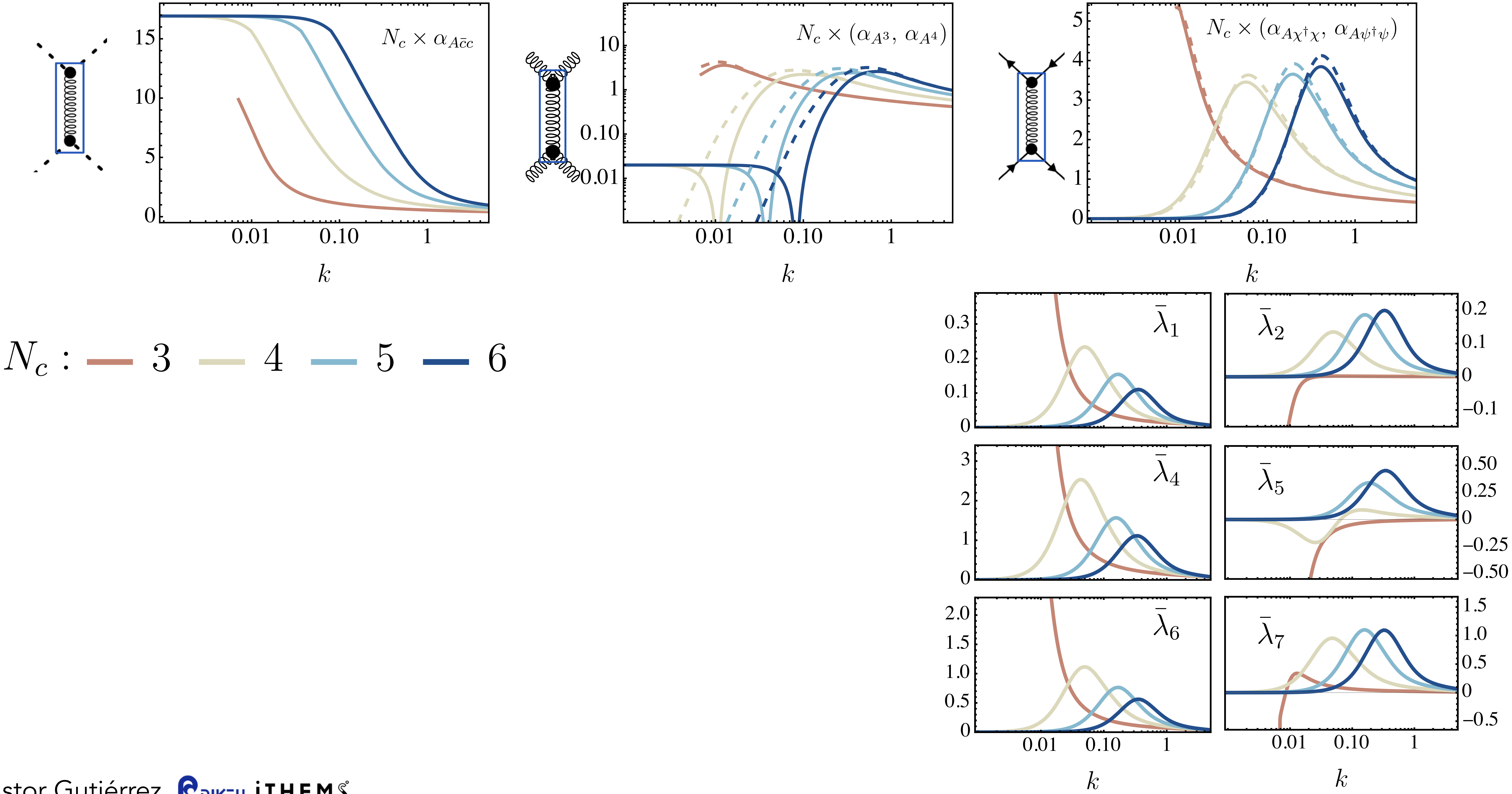


Li,APG,Vatani,Xu[2507.21208]

dSB in generalised GG theories

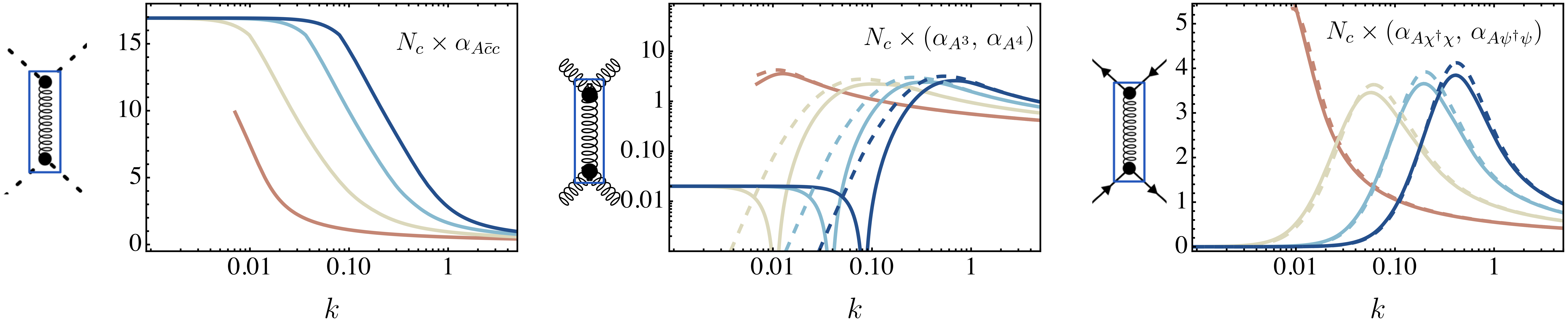
Colour-confinement and dSB in BY

Li,APG,Vatani[2603.19355]

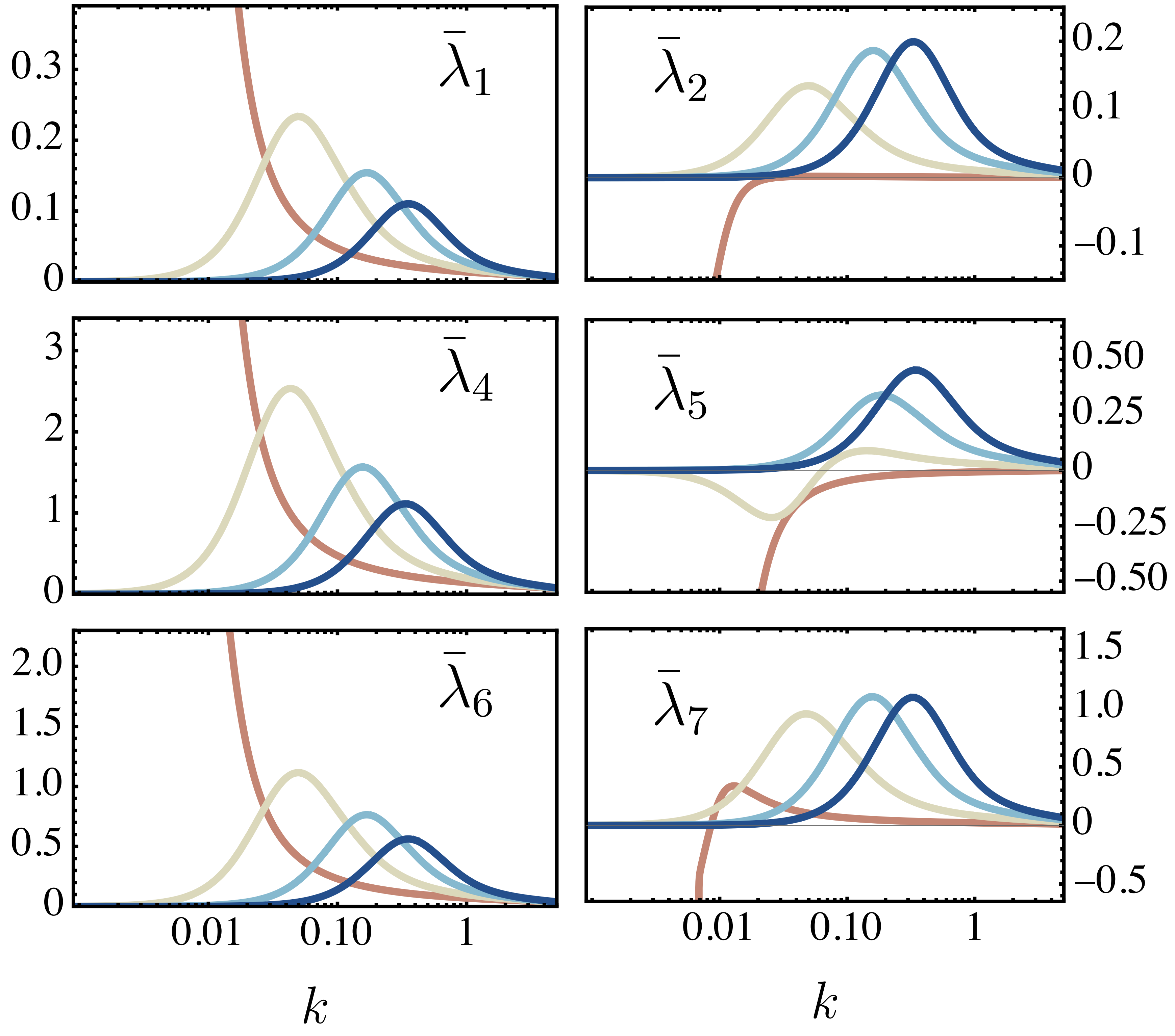


Colour-confinement and dSB in BY

Li,APG,Vatani[2603.19355]



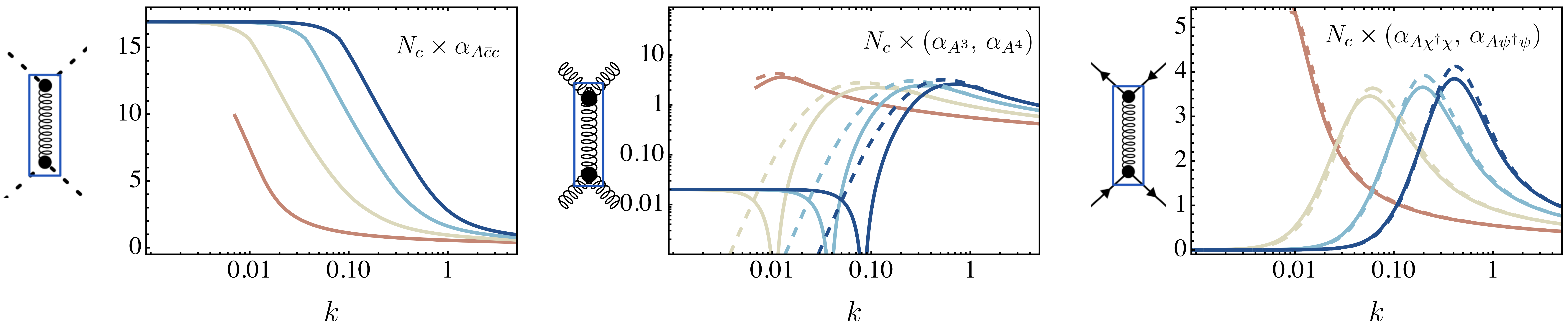
N_c : — 3 — 4 — 5 — 6



Signatures of confinement in the Landau gauge

Colour-confinement and dSB in BY

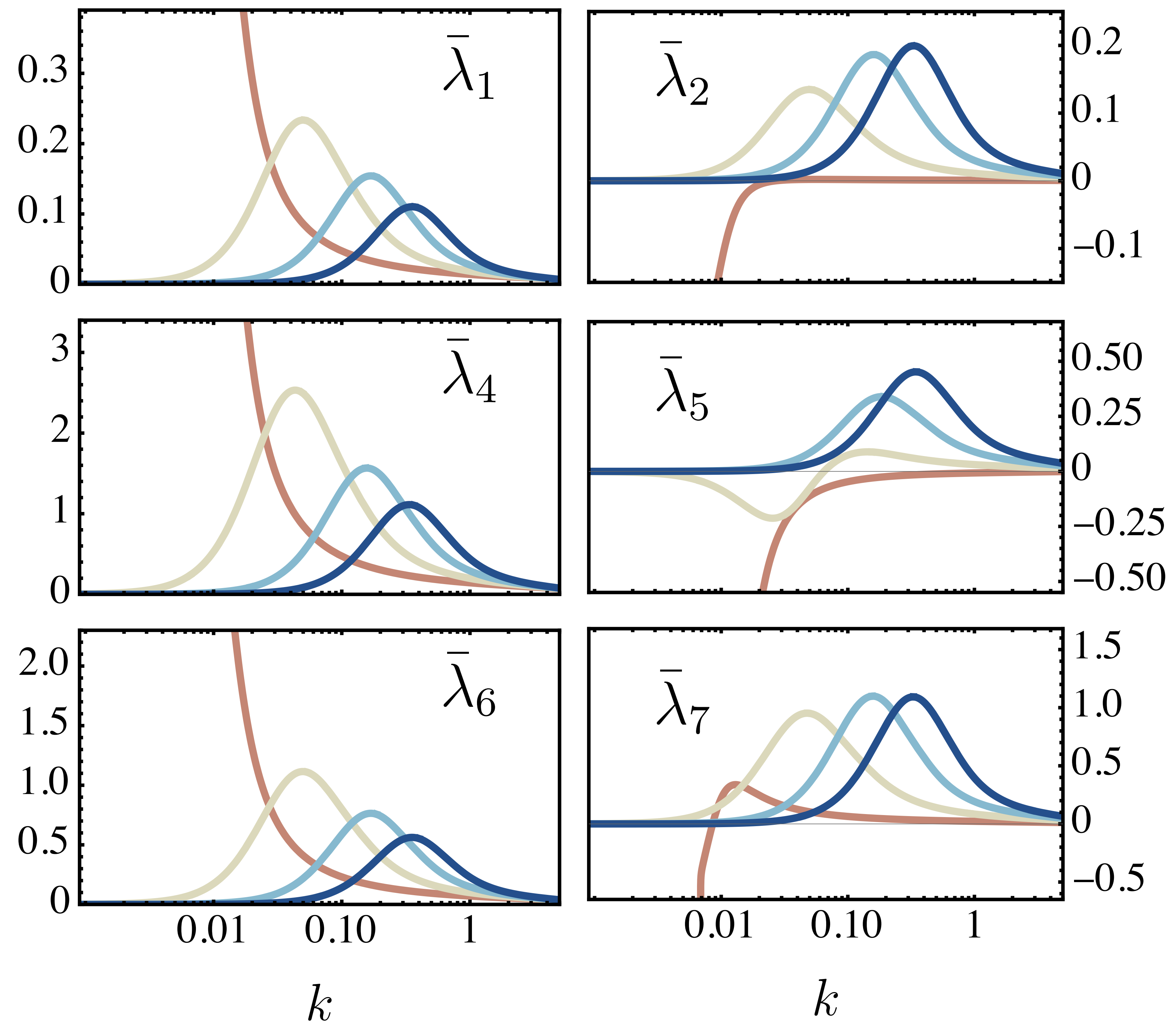
Li,APG,Vatani[2603.19355]



N_c : — 3 — 4 — 5 — 6

Divergence → dSB

No divergence → no dSB



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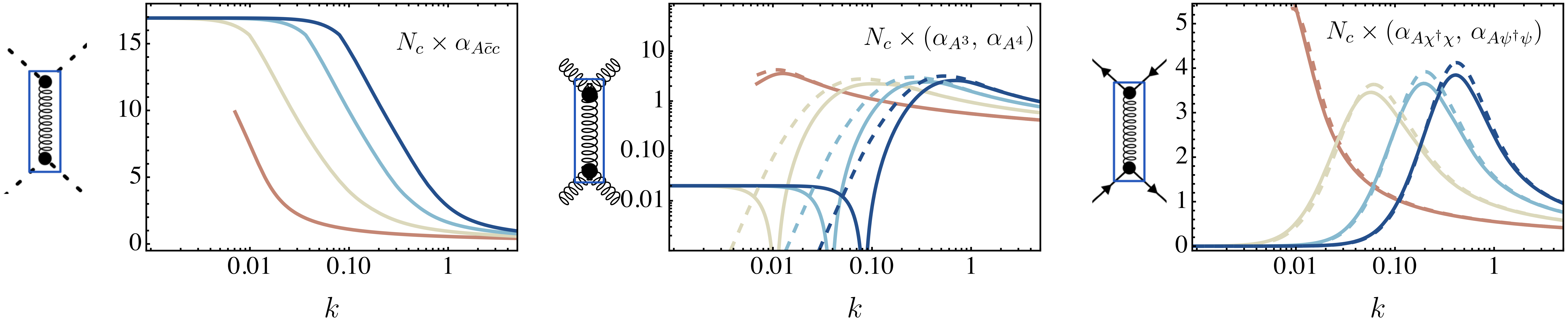


N_c : — 3 | — 4 — 5 — 6

Divergence → dSB

No divergence → no dSB

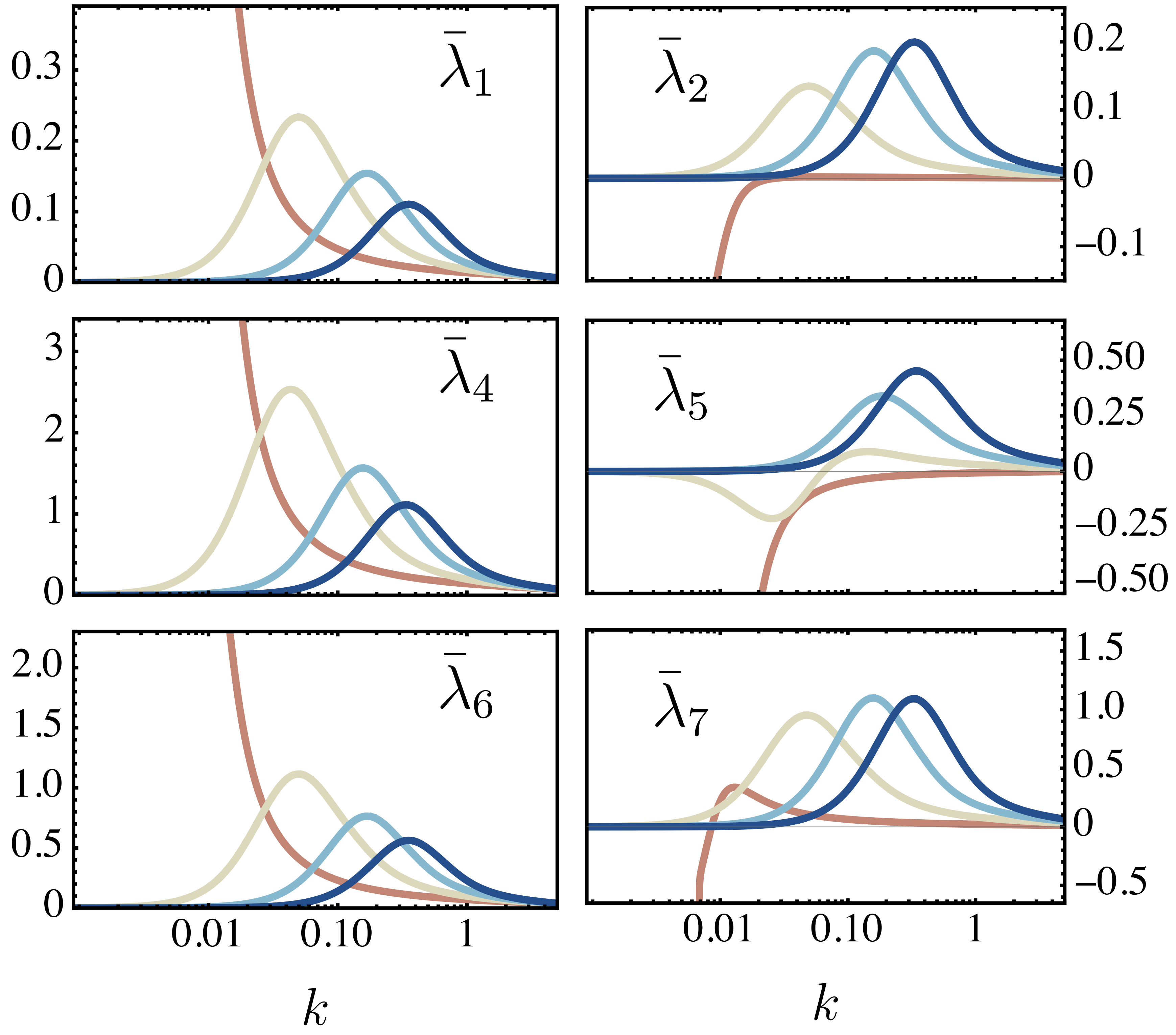
Signatures of confinement in the Landau gauge

Dominant four-fermion operators:

$$\mathcal{O}_4^\chi = (\chi^\dagger \bar{\sigma}^\mu \chi)(\chi^\dagger \bar{\sigma}^\mu \chi)$$

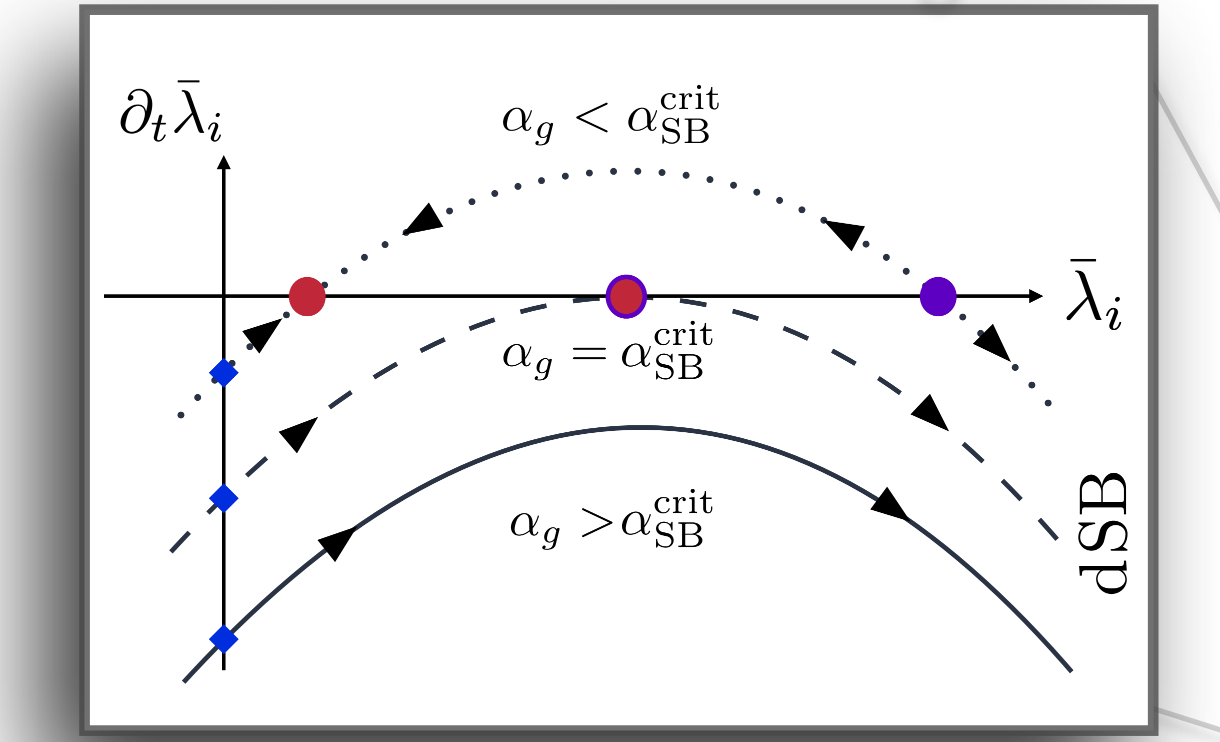
$$\mathcal{O}_5^\chi = (\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)$$

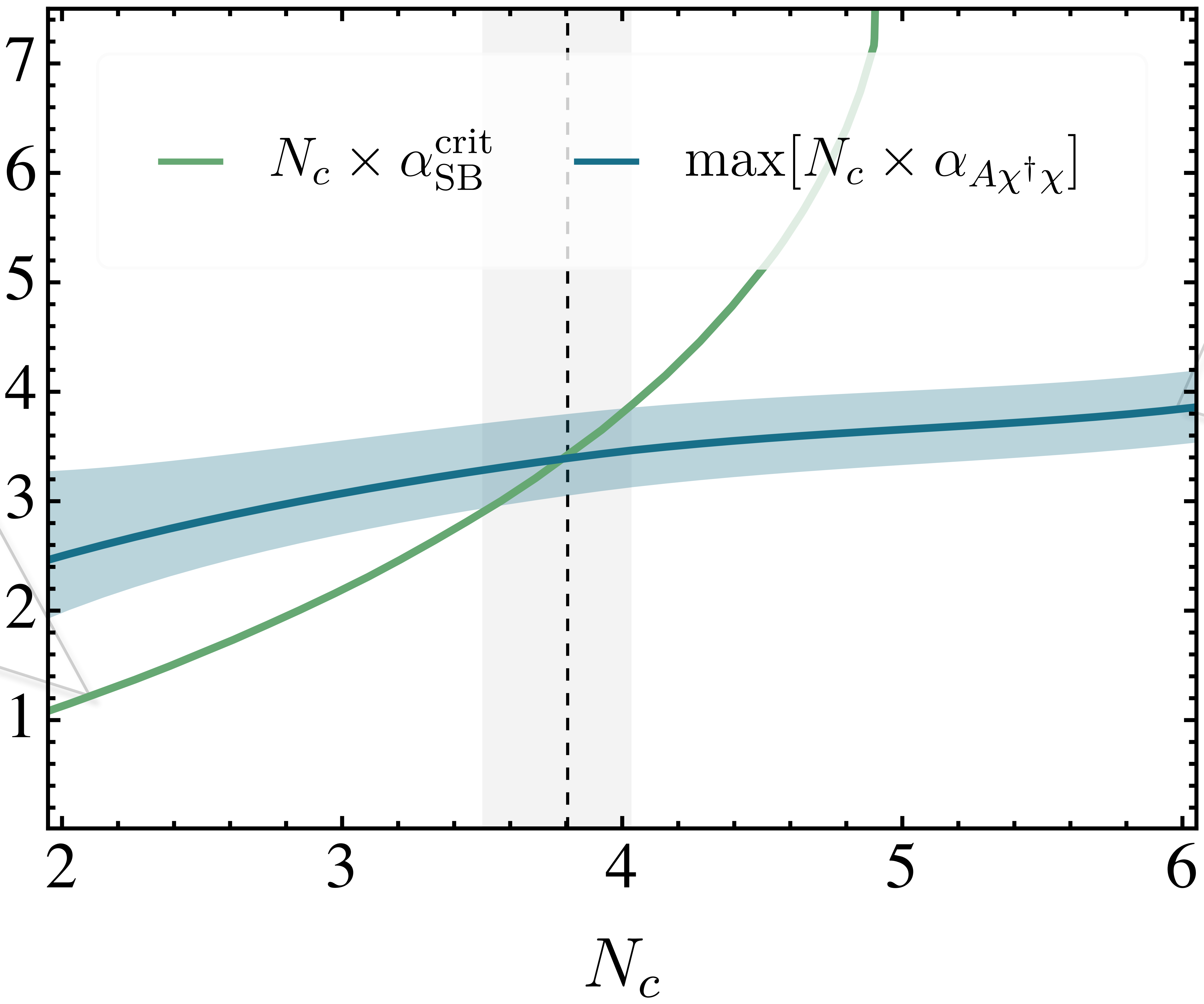
Diquark-like condensate: $\sim \langle \chi\chi \rangle$



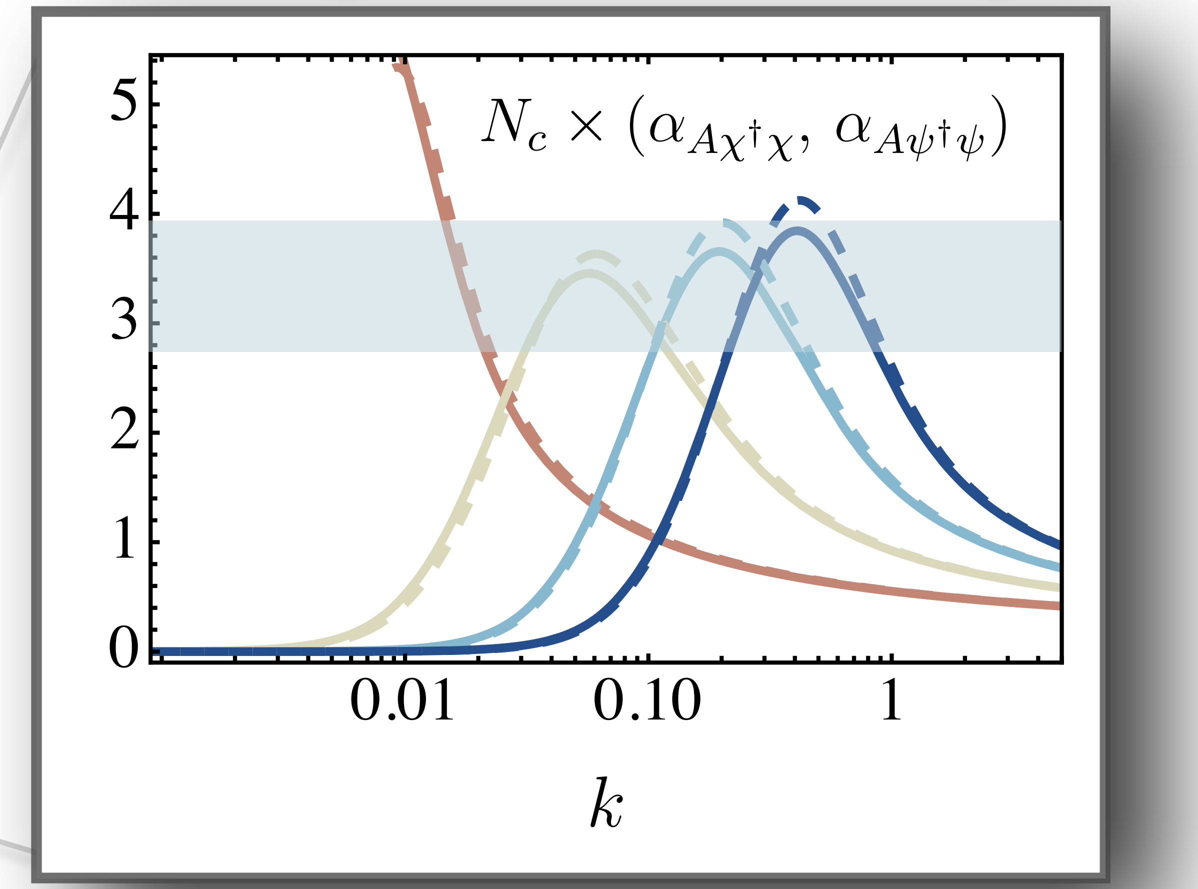
Phase diagram

Critical dSB strength





Gauge-fermion strength



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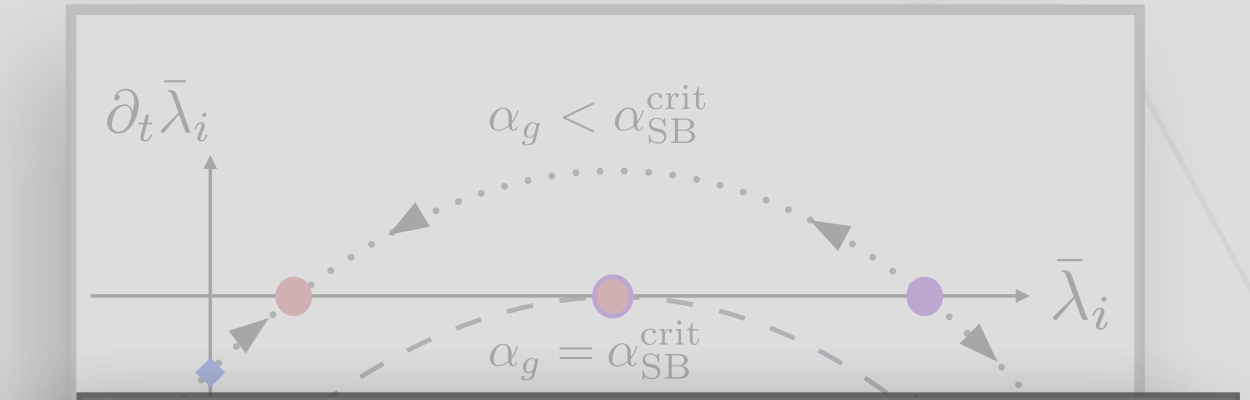
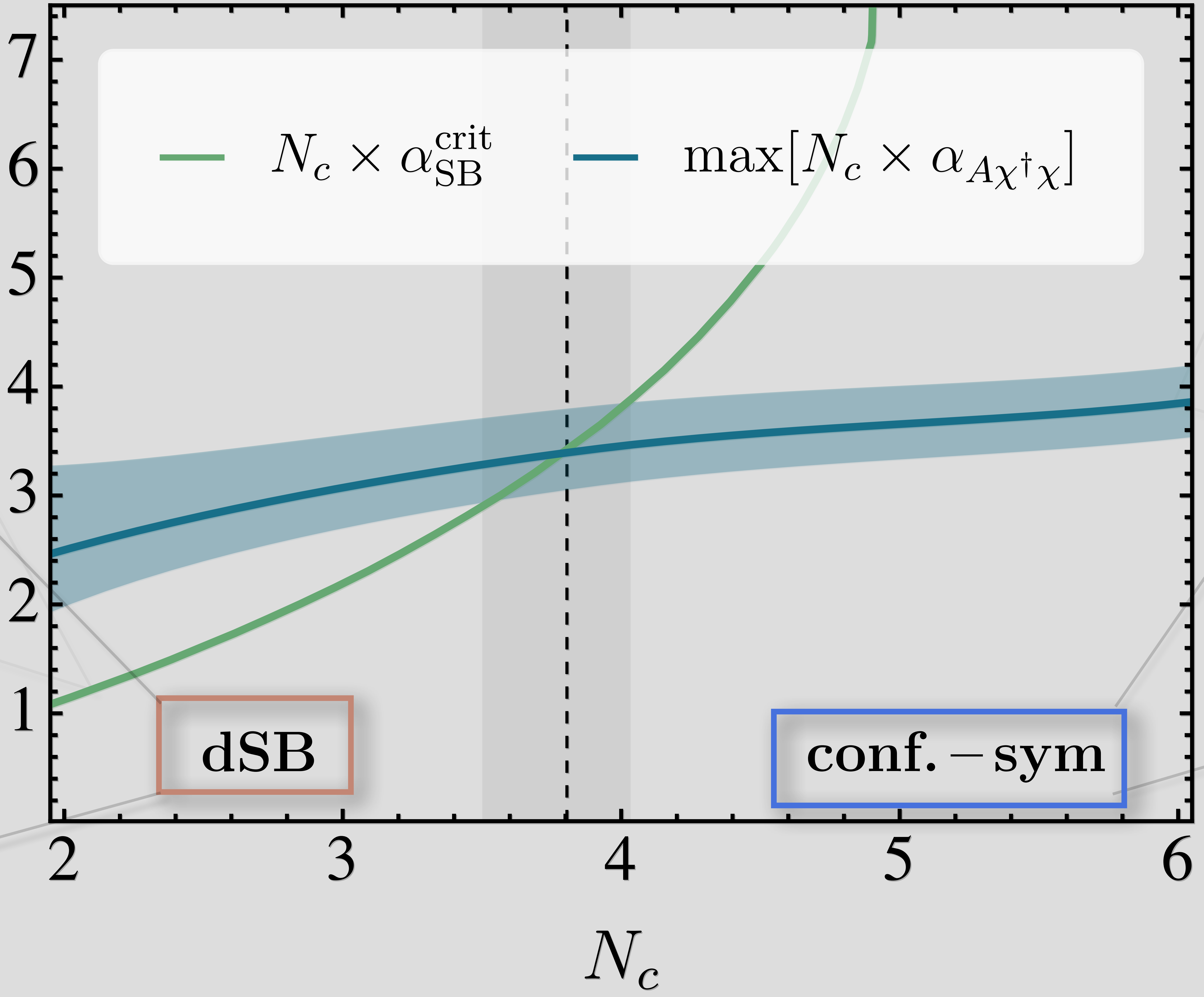


Highest condensate

Di-quark like $\sim \langle\chi\chi\rangle$

Higgsing the gluons, symmetry tumbling, … (tbd)



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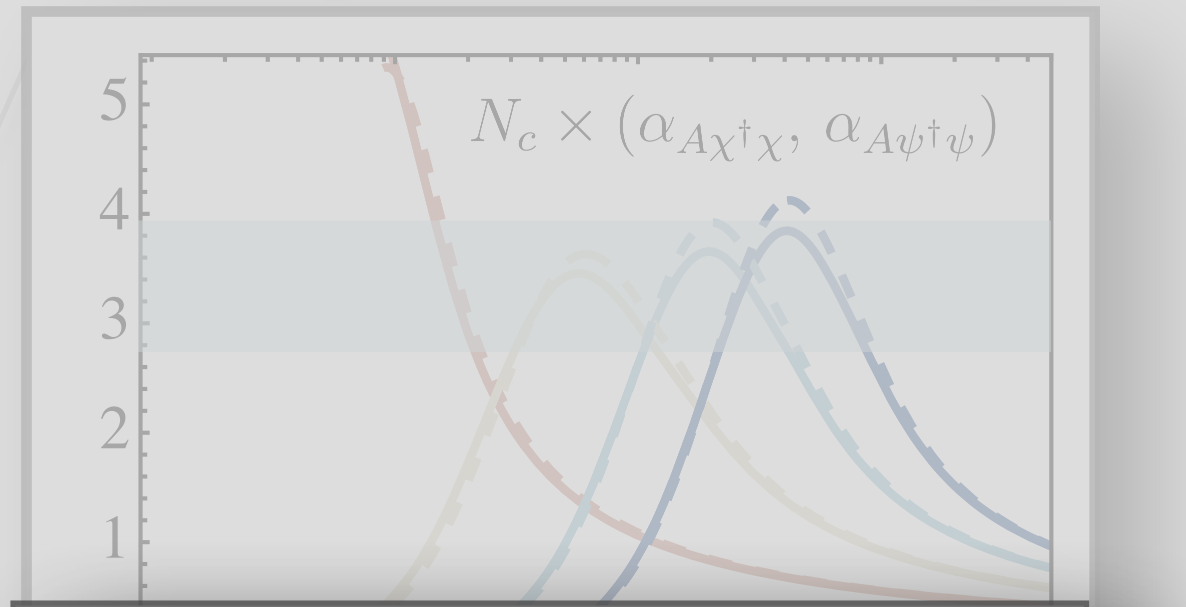


Confinement without dSB

Bars, Yankielowitz'81

Massless baryons saturate the anomaly

Raby, Dimopoulos, Susskind '79
Eichten, Peccei, Preskill, Zeppenfeld'86

What is this dynamics? SMG?,…

Wang, You[2204.14271]
Karasik, Önder, Tong[2208.07842]

Conclusions

Kenninji Temple, Kyoto

Conclusions

Very **rich dynamics** beyond the QCD limit!

- chiral limit, many flavours, colours and chiral-gauge theories

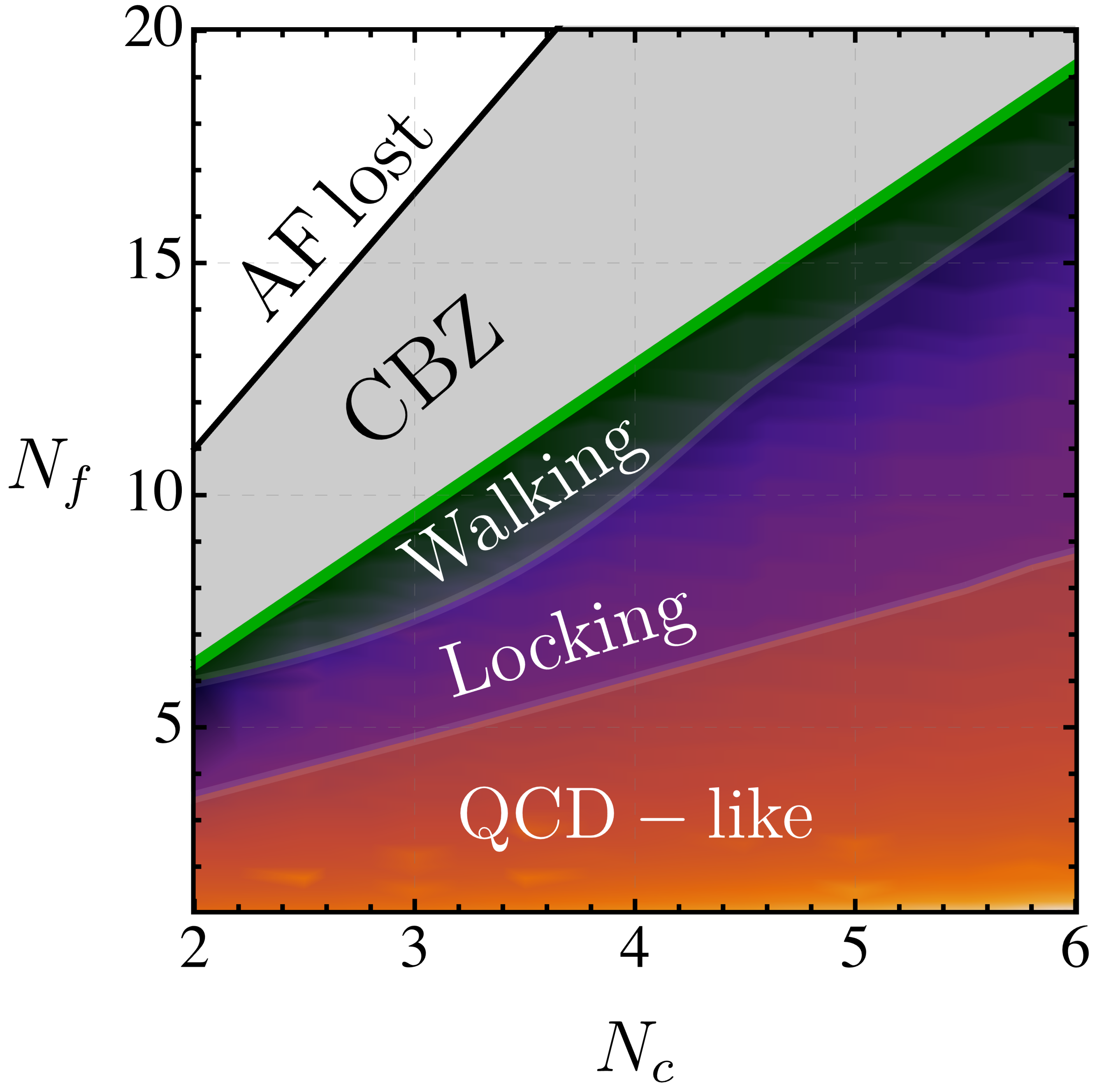
fRG: non-perturbative, versatile

- well-defined criteria for phenomena
- dynamics: encoded in full correlation functions
- systematically improvable, …

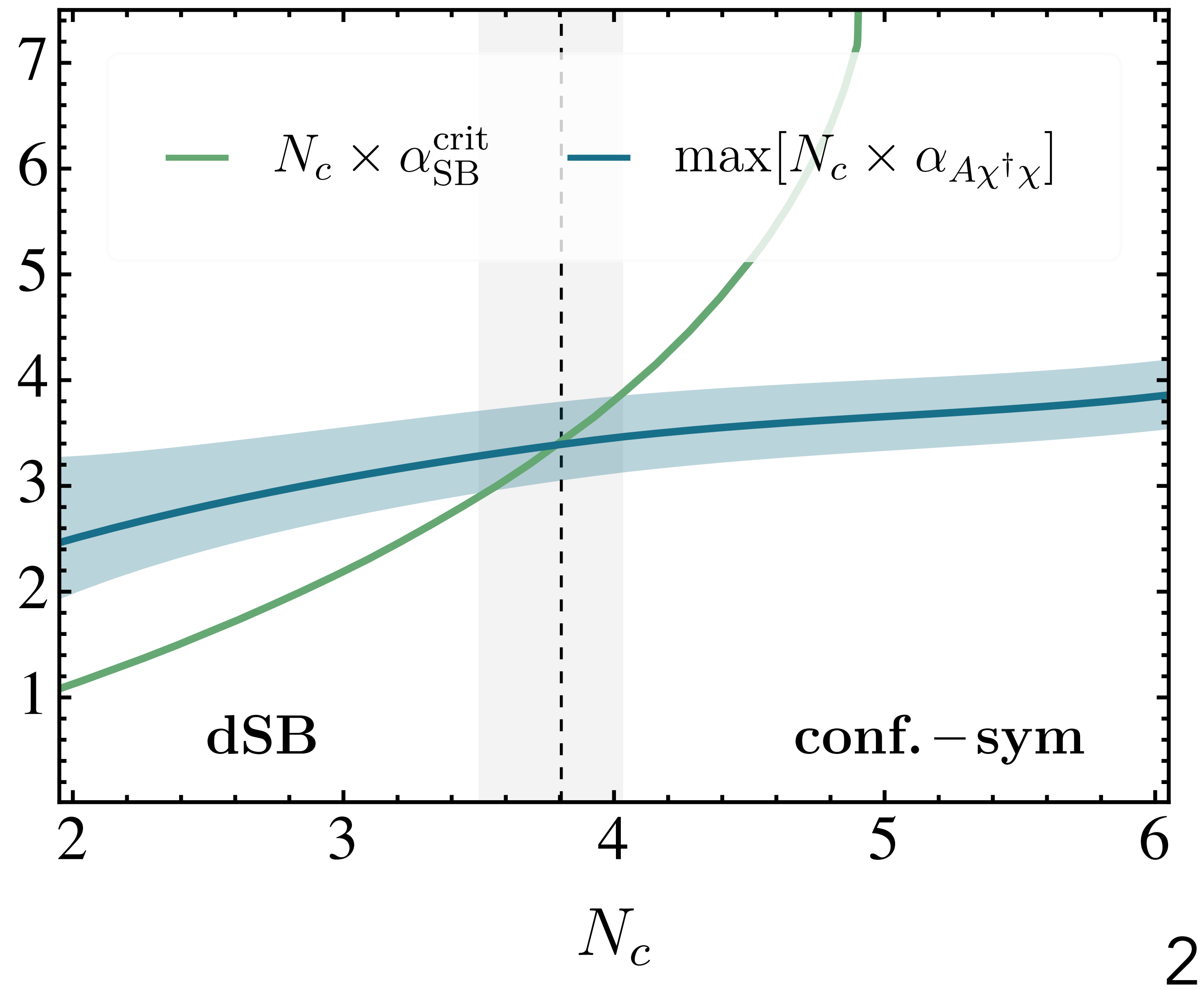
Charting unexplored and unknown landscape (also at finite T)

- More to come!

Goertz,APG,Pawlowski[2412.12254]



Li,APG,Vatani,Xu[2507.21208] Li,APG,Vatani[2603.19355]



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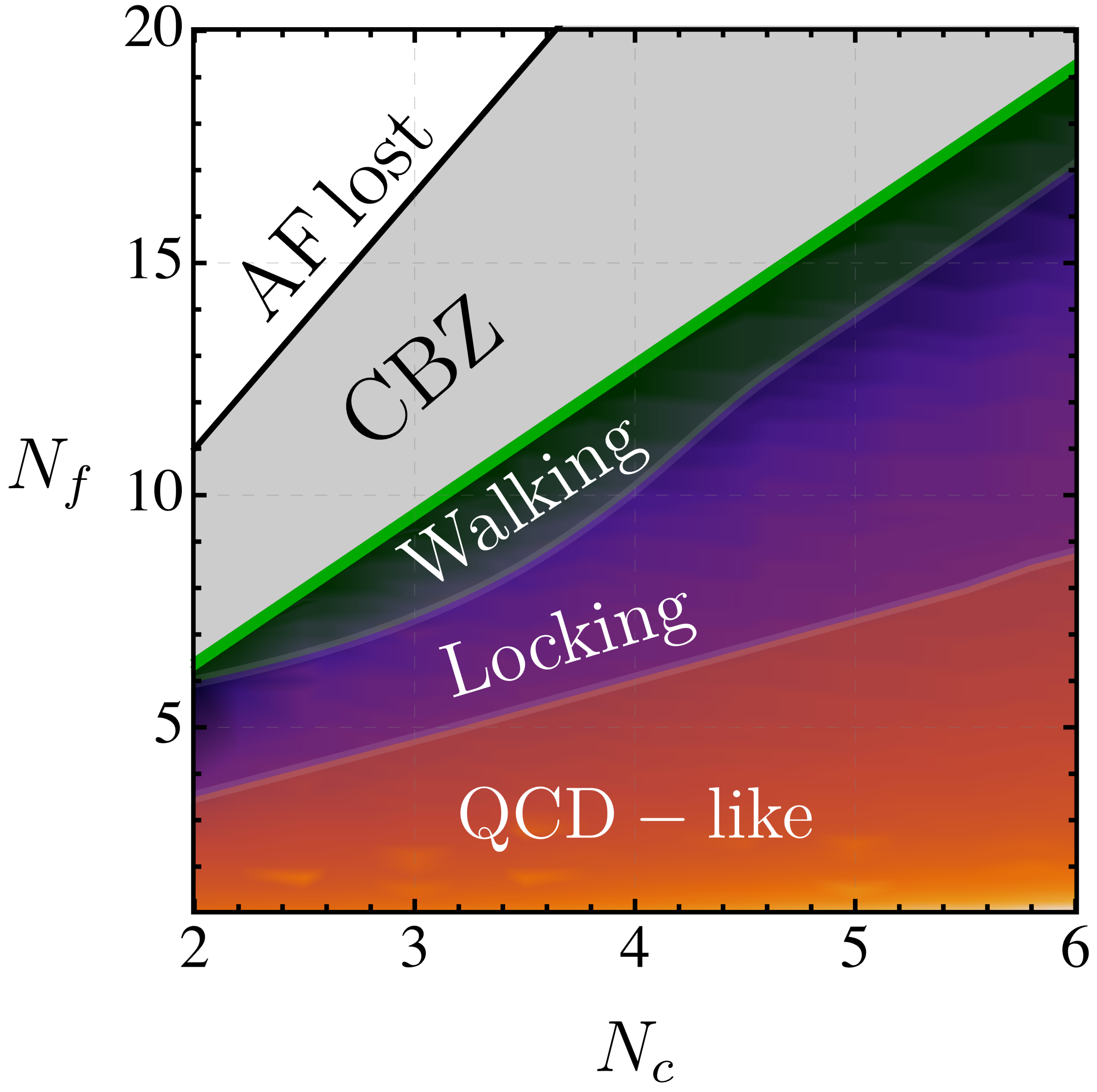
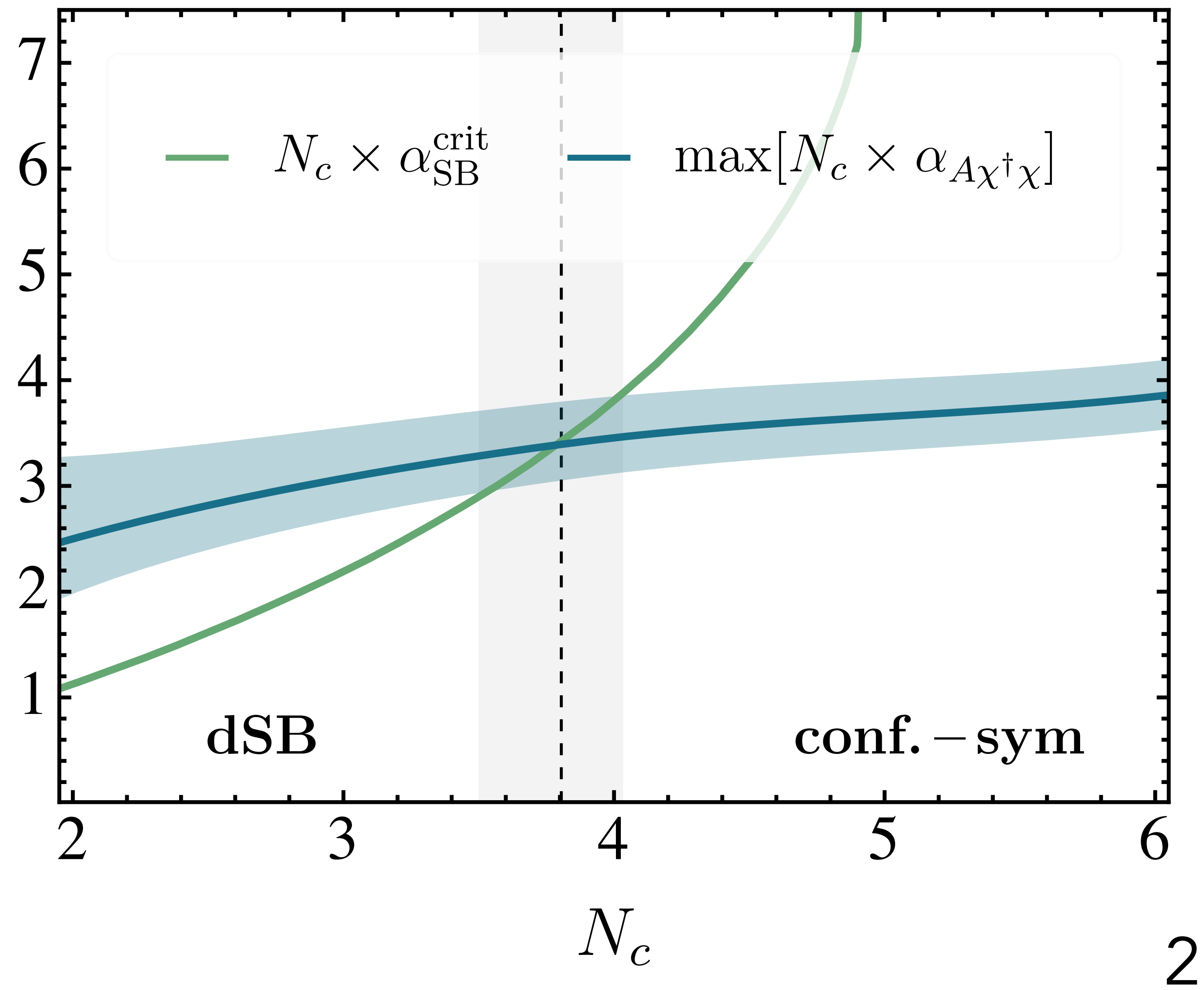
Thank you for your attention!



Goertz,APG,Pawlowski[2412.12254]



Li,APG,Vatani,Xu[2507.21208] Li,APG,Vatani[2603.19355]

Additional slides

Effective action 1

$$\boxed{\Gamma_k[A_\mu,\bar{c},c,\chi^\dagger,\chi,\psi^\dagger,\psi] = \Gamma_{\text{gauge},\,k}[A_\mu,\bar{c},c] \;+\; \Gamma_{\text{gauge}-\text{fermion},k}[A_\mu,\chi^\dagger,\chi,\psi^\dagger,\psi] + \Gamma_{\text{fermion},k}[\chi^\dagger,\chi,\psi^\dagger,\psi]}$$

$$\Gamma_{\text{gauge},k}[A,c,\bar{c}] = \frac{1}{2}\int_p A^a_\mu(p)\left[\tilde{Z}_{A,k}\;\left(p^2+m^2_{\text{gap},k}\right)\Pi^\perp_{\mu\nu}(p) + \frac{1}{\xi}Z^\parallel_{A,k}\;\left(p^2+m^2_{\text{mSTI},k}\right)\frac{p_\mu p_\nu}{p^2}\right]A^a_\nu(-p)$$

$$+\frac{1}{3!}\int_{p_1,p_2}\tilde{Z}^{3/2}_{A,k}\;g_{A^3,k}\left[\mathcal{T}^{(1)}_{A^3}(p_1,p_2)\right]^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}\prod_{i=1}^{3}A^{a_i}_{\mu_i}(p_i) + \frac{1}{4!}\int_{p_1,p_2,p_3}\tilde{Z}^2_{A,k}\;g_{A^4,k}\left[\mathcal{T}^{(1)}_{A^4}(p_1,p_2,p_3)\right]^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}\prod_{i=1}^{4}A$$

$$+\int_p Z_{c,k}\,\bar{c}^{\,a}(p)\;p^2\delta^{ab}c^b(-p) + \int_{p_1,p_2}Z_{c,k}\tilde{Z}^{1/2}_{A,k}\;g_{c\bar{c}A,k}\left[\mathcal{T}^{(1)}_{A\bar{c}c}(p_1,p_2)\right]^{a_1a_2a_3}_{\mu}\bar{c}^{a_2}(p_2)c^{a_1}(p_1)A^{a_3}_\mu(-p_1-p_2)$$

$$\Gamma^{(\Phi_{i_1}\cdots\Phi_{i_n})}(p_1,...,p_n) = \frac{\delta}{\delta\Phi_{i_1}(p_1)}\cdots\frac{\delta}{\delta\Phi_{i_n}(p_n)}\Gamma[\Phi] = \tilde{\Gamma}^{(\Phi_{i_1}\cdots\Phi_{i_n})}(p_1,...,p_n)\;\mathcal{T}^{(1)}_{\Phi_{i_1}\cdots\Phi_{i_n}}(p_1,...,p_n)$$

Effective action 2

$$\Gamma_{\mathrm{gauge-fermion},k}\left[A_\mu,\psi^\dagger,\psi,\chi^\dagger,\chi\right] =$$

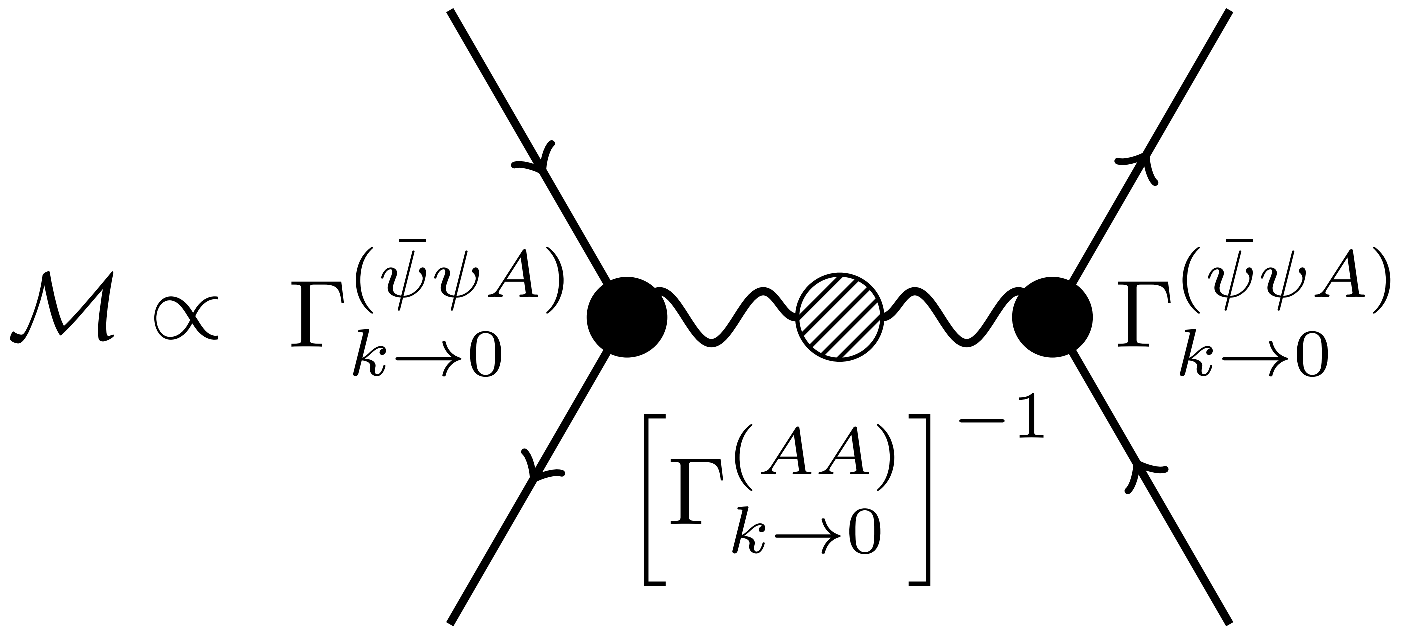
$$\int_p Z_{\psi,k}\,\psi^{\dagger\,,i,\,f}(p)\,\bar{\sigma}_\mu\partial_\mu\,\psi^{i,\,f}(-p) - \mathrm{i}\int_{p_1,\,p_2}\tilde{Z}_{\psi,k}\tilde{Z}^{1/2}_{A,k}\,g_{A\psi^\dagger\psi,k}\;\psi^{\dagger\,,i_1 f_1}(p_2)\,\left[\mathcal{T}^{(1)}_{A\psi^\dagger\psi}(p_1,p_2)\right]^{a\,i_1 i_2 f_1 f_2}_{\mu}\,A^a_\mu(-p_1-p_2)\psi$$

$$+\int_p Z_{\chi,k}\,\chi^{\dagger\,,\alpha}(p)\,\bar{\sigma}_\mu\partial_\mu\chi^{\alpha}(-p) - \mathrm{i}\int_{p_1,\,p_2} Z_{\chi,k}\tilde{Z}^{1/2}_{A,k}\,g_{A\chi^\dagger\chi,k}\,\chi^{\dagger\,,\alpha_1}(p_2)\,\left[\mathcal{T}^{(1)}_{A\chi^\dagger\chi}(p_1,p_2)\right]^{a\,\alpha_1\alpha_2}_{\mu}\,A^a_\mu(-p_1-p_2)\chi^{\alpha_2}(p_1)$$

$$\left[\mathcal{T}^{(1)}_{A\psi^\dagger\psi}(p_1,p_2)\right]^{a\,i_1 i_2 f_1 f_2}_{\mu} = \bar{\sigma}_\mu(T^a_{\mathrm{F}})^{i_1 i_2}\delta^{f_1 f_2} \quad \text{and} \quad \left[\mathcal{T}^{(1)}_{A\chi^\dagger\chi}(p_1,p_2)\right]^{a\,\alpha_1\alpha_2}_{\mu} = \bar{\sigma}_\mu(T^a_{\mathrm{S}})^{\alpha_1\alpha_2}$$

$$\Gamma_{\mathrm{fermion},k}[\chi^\dagger,\chi,\psi^\dagger,\psi] = -\int_x Z^2_\psi\sum_{i=1}^{2}\lambda_i\mathcal{O}^{\psi}_i + Z^2_\chi\sum_{i=4}^{5}\lambda_i\mathcal{O}^{\chi}_i + Z_\psi Z_\chi\sum_{i=6}^{7}\lambda_i\mathcal{O}^{\chi\psi}_i,$$

Deriving full correlation functions



Vertex expansion of the effective average action

$$\Gamma_k[\Phi] = \sum_n^{N_{\rm tr}} \int_{\boldsymbol{p}} \Gamma_k^{(\Phi_{i_1}\cdots\Phi_{i_n})}(\boldsymbol{p})\, \Phi_{i_n}(p_n)\cdots\Phi_{i_1}(p_1)$$

with

$$\Gamma_k^{(\Phi_{i_1}\cdots\Phi_{i_n})}(p_1,\ldots,p_n) = \frac{\delta}{\delta\Phi_{i_1}(p_1)}\cdots\frac{\delta}{\delta\Phi_{i_n}(p_n)}\Gamma_k[\Phi]$$

<u>Example:</u> fermion-gauge vertex and flow of the coupling

$$\partial_t\Gamma_k^{(\bar\psi\psi A)} = \partial_t\left(Z_A^{1/2} Z_\psi\ g_{\bar\psi\psi A}\cdot\mathcal{T}_\mu\right)$$

☞

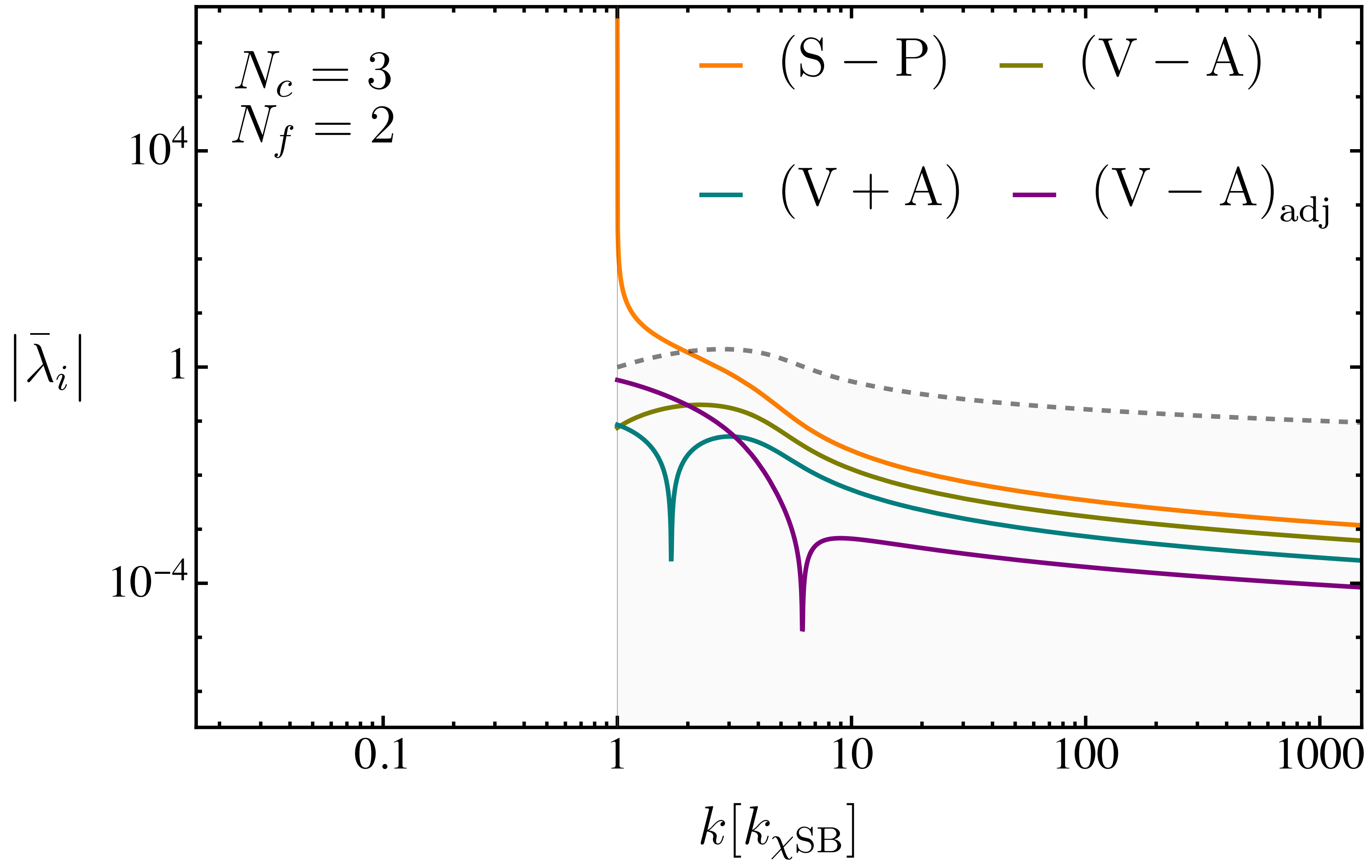
$$\partial_t g_{\bar\psi\psi A} = \frac{\mathrm{Tr}\left[\mathcal{T}_\mu\,\partial_t\Gamma_k^{(\bar\psi\psi A)}\right]}{\mathrm{Tr}\left[\mathcal{T}_\mu^2\right] Z_A^{1/2} Z_\psi} + \left(\frac{1}{2}\eta_A+\eta_\psi\right) g_{\bar\psi\psi A}$$

with anomalous dimensions$\eta_{\phi_i} = -\frac{\partial_t Z_{\phi_i}}{Z_{\phi_i}} = -\frac{\partial_{p^2}\,\partial_t\Gamma_k^{(\phi_i\phi_i)}}{Z_{\phi_i}}$

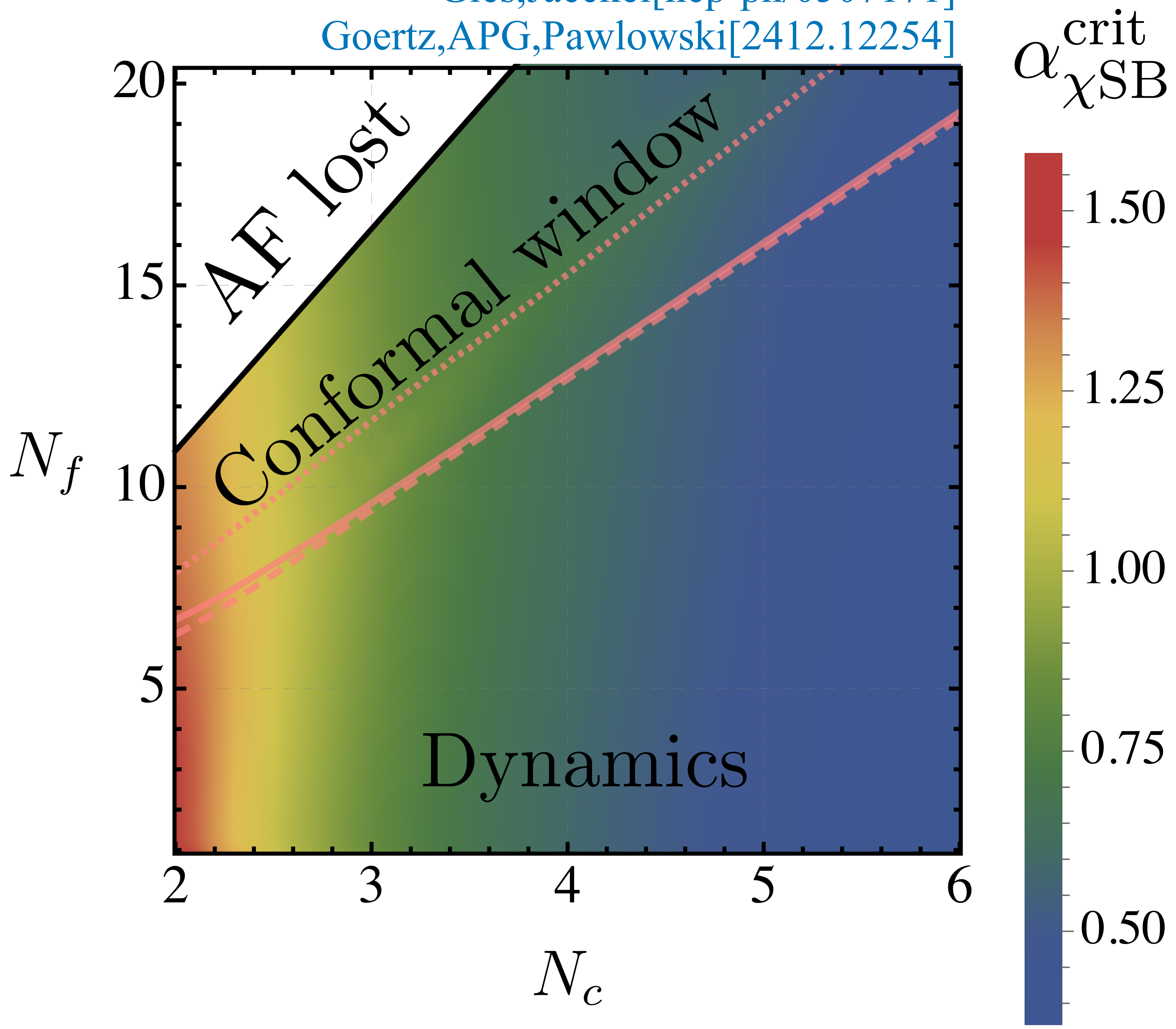
Diagramatic flows to be computed

Dynamics from $(\bar{\psi}\,\mathcal{T}\psi)^2$

- Wrap up: what do we learn from this simple picture?
  - First dSB mechanism
    - ★ **Global symmetry breaking**:
      - Quantum numbers of the highest **condensate**
    - ★ Breaking of **quantum scale invariance**
  - $\alpha_{\mathrm{SB}}^{\mathrm{crit}}$
  - dSB **scale**: $k_{\mathrm{SB}} \sim T_{\mathrm{c}} \sim \mathcal{O}(\langle\phi\rangle)$



Gies,Jaeckel[hep-ph/0507171]
Goertz,APG,Pawlowski[2412.12254]



Gauge symmetry in the RG flow

$$\Delta S_k[\phi] = \int_p \phi(p) R_k \phi(-p)$$

- Gauge symmetry at the quantum level ☞ Slavnov-Taylor identities (STIs) ☞ In the presence of a cutoff derive the modified STIs (mSTIs).

Becchi[9607188],Bonini,D'Attanasio,Marchesini'95 Ellwanger[9402077]

$$\int_x \frac{\delta\Gamma}{\delta\mathcal{Q}_i(x)}\frac{\delta\Gamma}{\delta\Phi_i(x)} = 0 \qquad \int_x \frac{\delta\Gamma_k}{\delta\mathcal{Q}_i(x)}\frac{\delta\Gamma_k}{\delta\Phi_i(x)} = \mathrm{Tr} R^{ij} G_{jl} \frac{\delta^2\Gamma_k[\Phi,\mathcal{Q}]}{\delta\Phi_l\delta\mathcal{Q}^i}$$

$\Phi_i = \{A_\mu, c, \bar{c}, \ldots\}$

$\mathcal{Q}^i$: BRST sources of the Φ_i fields

  - In the physical limit $k \to 0$ and $R_k \to 0$ and mSTI ☞ STI and no gauge symmetry breaking is present. (*)

- **Under control:** concentually and quantitatively

Pawlowski[0512261] Gies[0611146]

  - Deviation from the STI can be checked along the RG-flow

Pawlowski,Schneider,Wink[2202.11123]

  - Gauge invariant flow equations and field transformations

Ihssen,Pawlowski[2503.22638]

Morris,Rosten[0606189]
Wetterich[1607.02989]

STIs and mSTIs and confinement

$$\Delta S_k[\phi] = \int_p \phi(p) R_k \phi(-p)$$

- Insertion of mass-like regulator: Slavnov-Taylor identities (STIs) for gauge invariance → modified STIs (mSTIs)

$$\int_x \frac{\delta\Gamma}{\delta\mathcal{Q}_i(x)}\frac{\delta\Gamma}{\delta\Phi_i(x)} = 0 \qquad \longrightarrow \qquad \int_x \frac{\delta\Gamma_k}{\delta\mathcal{Q}_i(x)}\frac{\delta\Gamma_k}{\delta\Phi_i(x)} = \mathrm{Tr} R^{ij} G_{jl} \frac{\delta^2\Gamma_k[\Phi,\mathcal{Q}]}{\delta\Phi_l \delta\mathcal{Q}^i}$$

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- “easy” approach: physics at k-dependent correlation functions
  - Minimise mSTI and STI deviations for $k \neq 0$
  - Hands-on approach: include problematic power-law corrections to the flow of the gauge mass gap only in the confinement regime

$$\partial_t \bar{m}^2_{\mathrm{gap}} = (-2 + \eta_A)\, \bar{m}^2_{\mathrm{gap}} + \underbrace{\overline{\mathrm{Flow}}_{AA}(p^2 = 0)}_{}$$

  - Upgrade with full momentum dependencies

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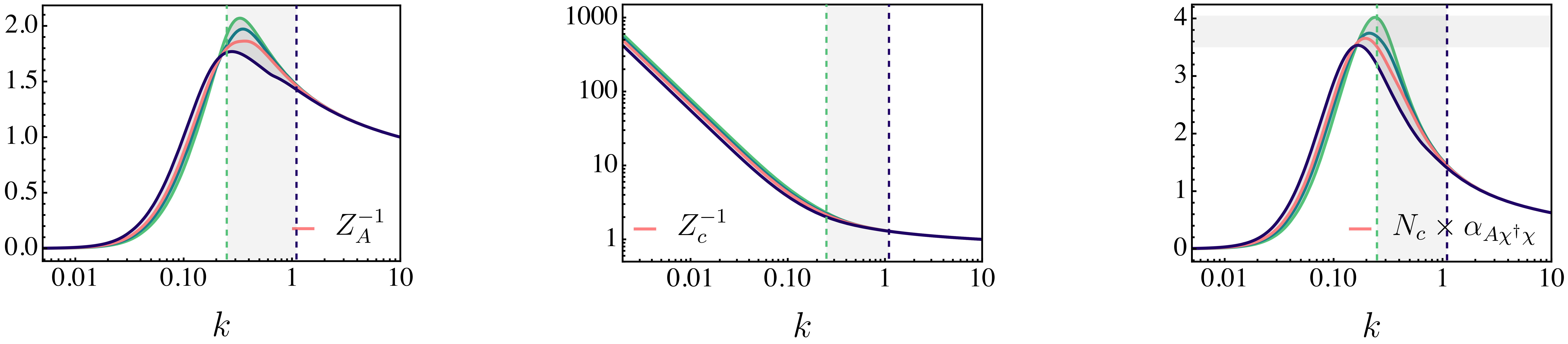
  - Upgrade with full momentum dependencies

Chiral symmetry intact:

$$R_{\psi/\chi,\,k}(p^2) = \mathrm{i}\, Z_{\psi/\chi}\, \bar{\sigma}_\mu p_\mu\, r_{\psi/\chi}(x)$$

$$r_{\psi/\chi}(x) = (1/\sqrt{x} - 1)\, \theta\,(1 - x)$$

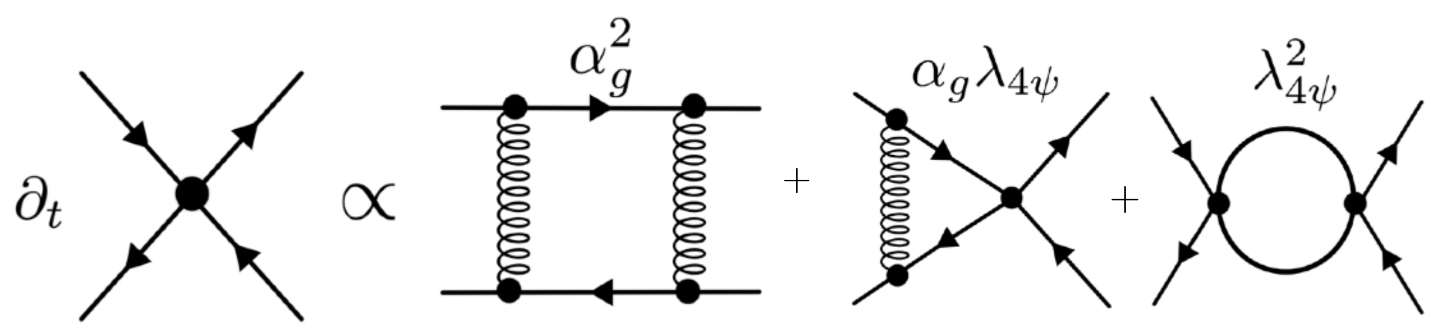
Systematics: minimasing the mSTI/STI deviation



- control onset of power-law divergences in the flow of the gauge mass gap.

$$\partial_t \bar{m}^2_{\text{gap}} = (-2 + \eta_A)\, \bar{m}^2_{\text{gap}} + \overline{\text{Flow}}_{AA}(p^2 = 0)$$

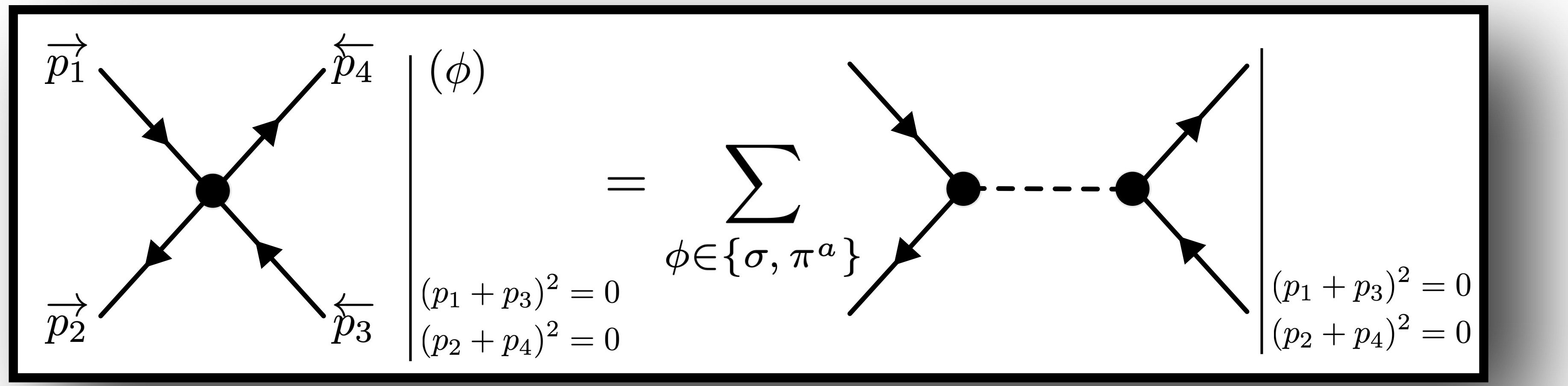
Dynamical chiral symmetry breaking



$$\Gamma_k[\Phi] \supset -\int_x \bar{\lambda}_\sigma \left(\bar{\psi}\,\mathcal{T}_{(\mathrm{S-P})}\psi\right)^2 + \dots$$

Stratonovich'57 Hubbard'59

Gies, Wetterich '01



Fukushima,Pawlowski,Strodthoff [2103.01129]

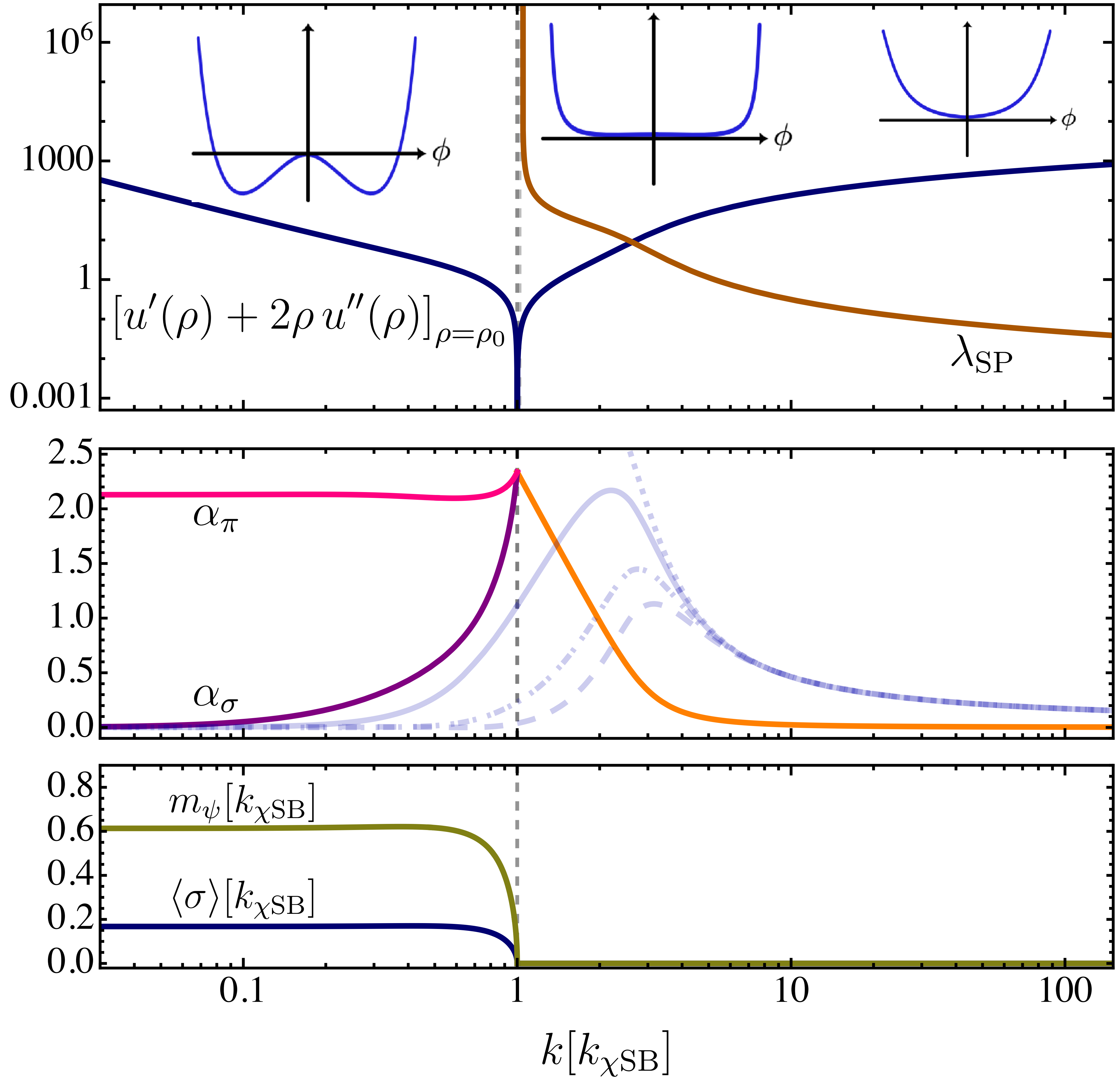
Dynamical bosonisation and generalised flow equation

Gies, Wetterich '01Pawlowski[hep-th/0512261]

$$\Gamma_k[\Phi] \supset \int_x \bar{h}\,\bar{\psi}\left(T_f^0\sigma + \mathrm{i}\gamma_5 T_f^a\pi^a\right)\psi + Z_\phi\left(\partial_\mu\phi\right)^2 + V(\phi^2) + \dots$$

Goertz,APG,Pawlowski[2412.12254]

$N_c = 3 \quad N_f = 2$



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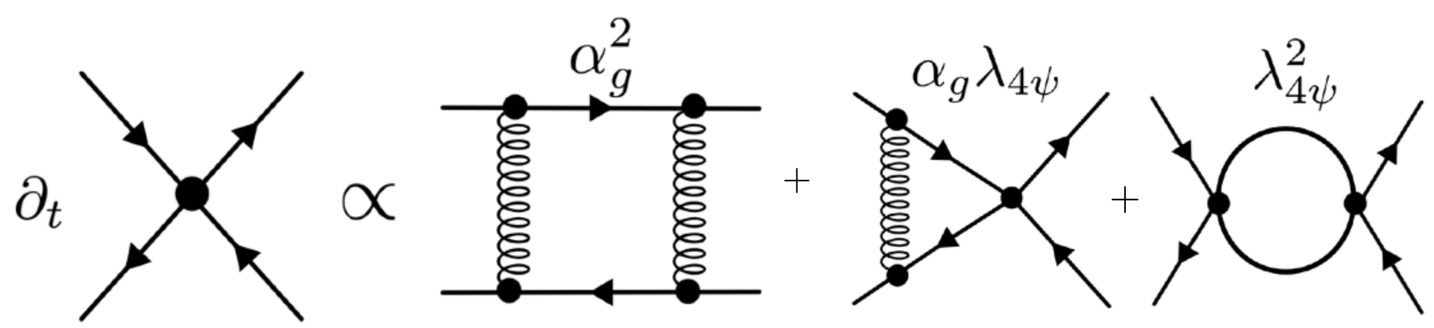

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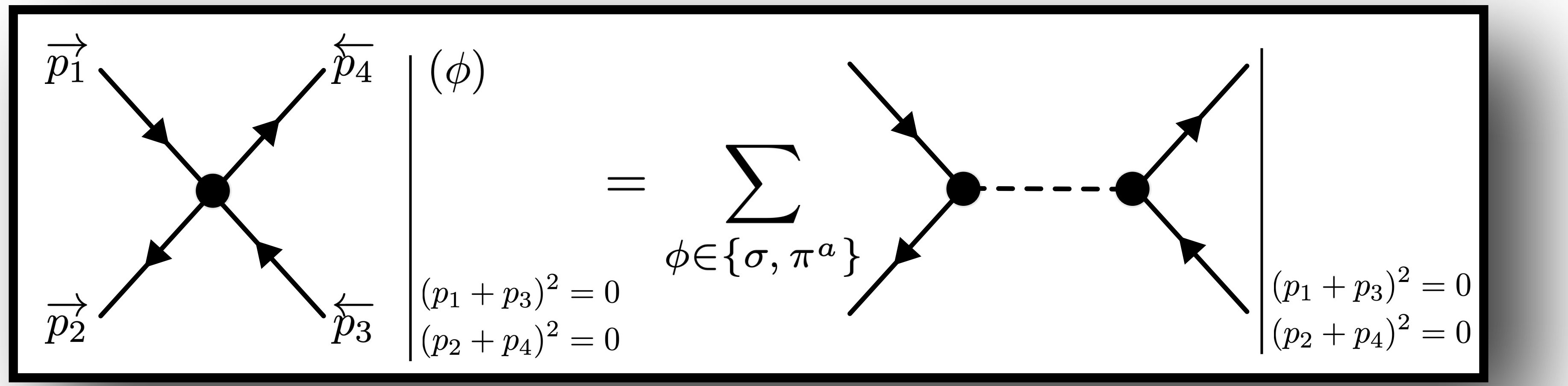

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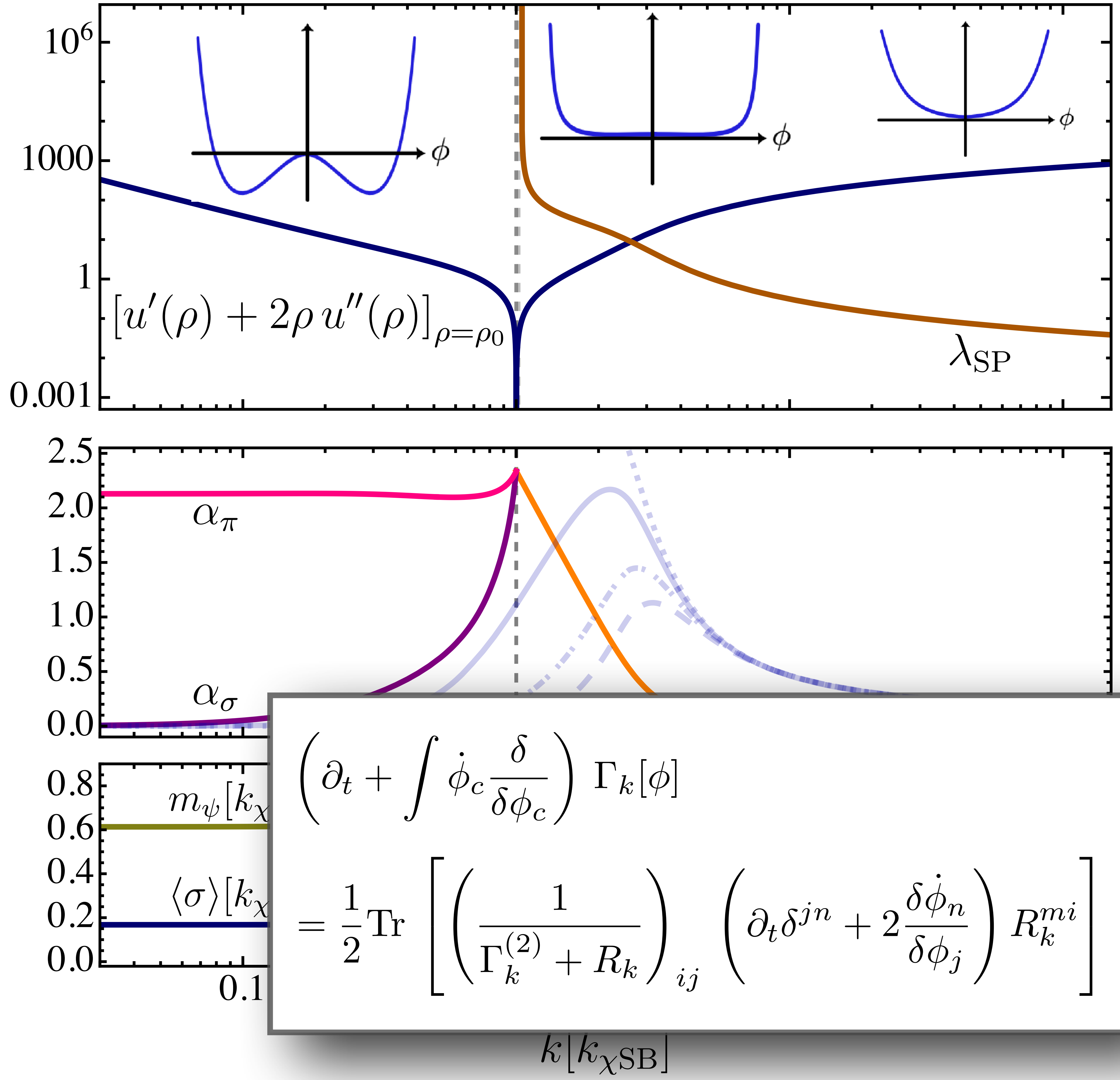
Goertz,APG,Pawlowski[2412.12254] $N_c = 3 \;\; N_f = 2$



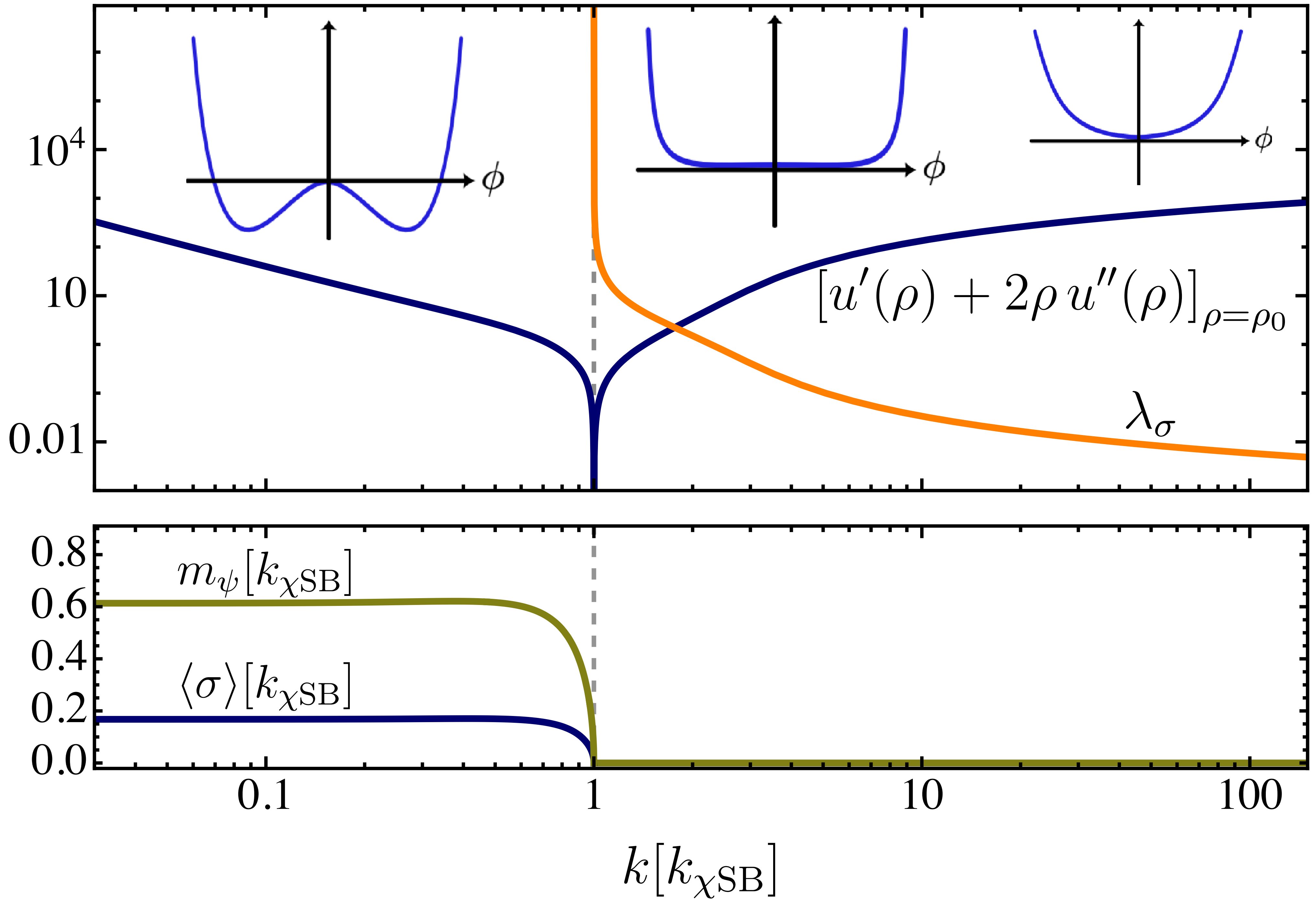
$$\left(\partial_t + \int \dot{\phi}_c \frac{\delta}{\delta \phi_c}\right) \Gamma_k[\phi]$$

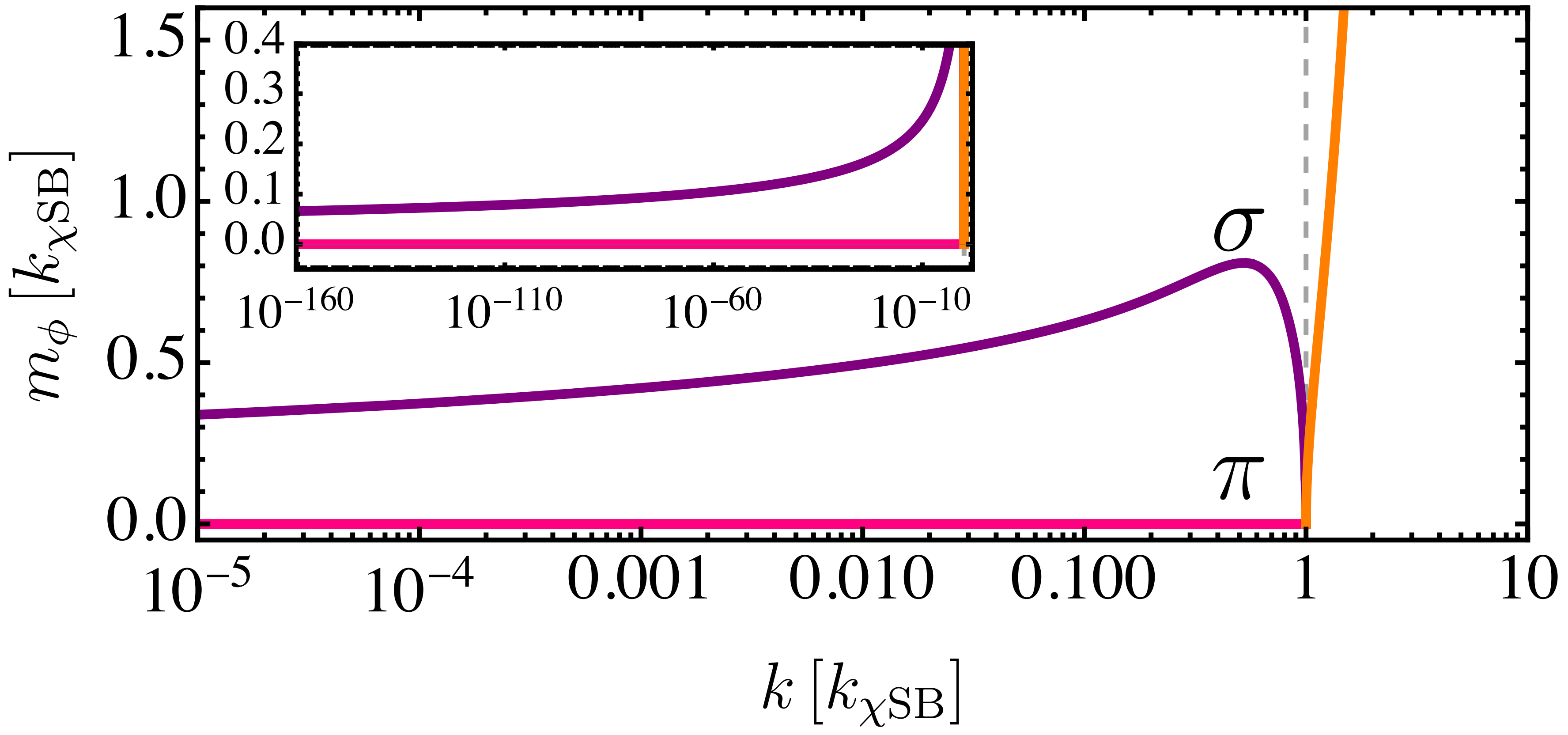
$$= \frac{1}{2}\mathrm{Tr}\left[\left(\frac{1}{\Gamma_k^{(2)} + R_k}\right)_{ij} \left(\partial_t \delta^{jn} + 2\frac{\delta \dot{\phi}_n}{\delta \phi_j}\right) R_k^{mi}\right]$$

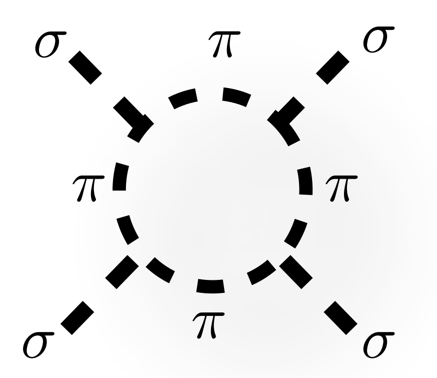
Dynamics in the chirally broken phase

$N_c = 3 \;\; N_f = 2$

- ✦ Flows computed: $\{h, V(\phi), Z_\psi, Z_\phi, \lambda_i\}$
- ✦ **Continuous** interpolation between chirally **symmetric** and **broken** regimes
- ✦ A **clear** and **precise** way to **diagnose** χ**SB**







- ✦ Obtaining **fundamental parameters**
  - Constituent fermion masses: m_ψ
  - Chiral condensate: $\langle\sigma\rangle \sim f_\pi$
  - Composite masses of bosonised channels: m_σ, m_π
- ✦ Account for **higher dimensional fermionic** operators via higher-order scalar potential:
  - Non-perturbative effects and higher precision

Fischer,Maas,Pawlowski[0810.1987]
Cyrol,Fister,Mitter,Pawlowski [1605.01856]

Yang-Mills phases and confinement

$$\Gamma_k^{(AA)}(p^2) = Z_{A,\,k}(p)\,\left(p^2 + m^2_{A,\,k}\right)$$

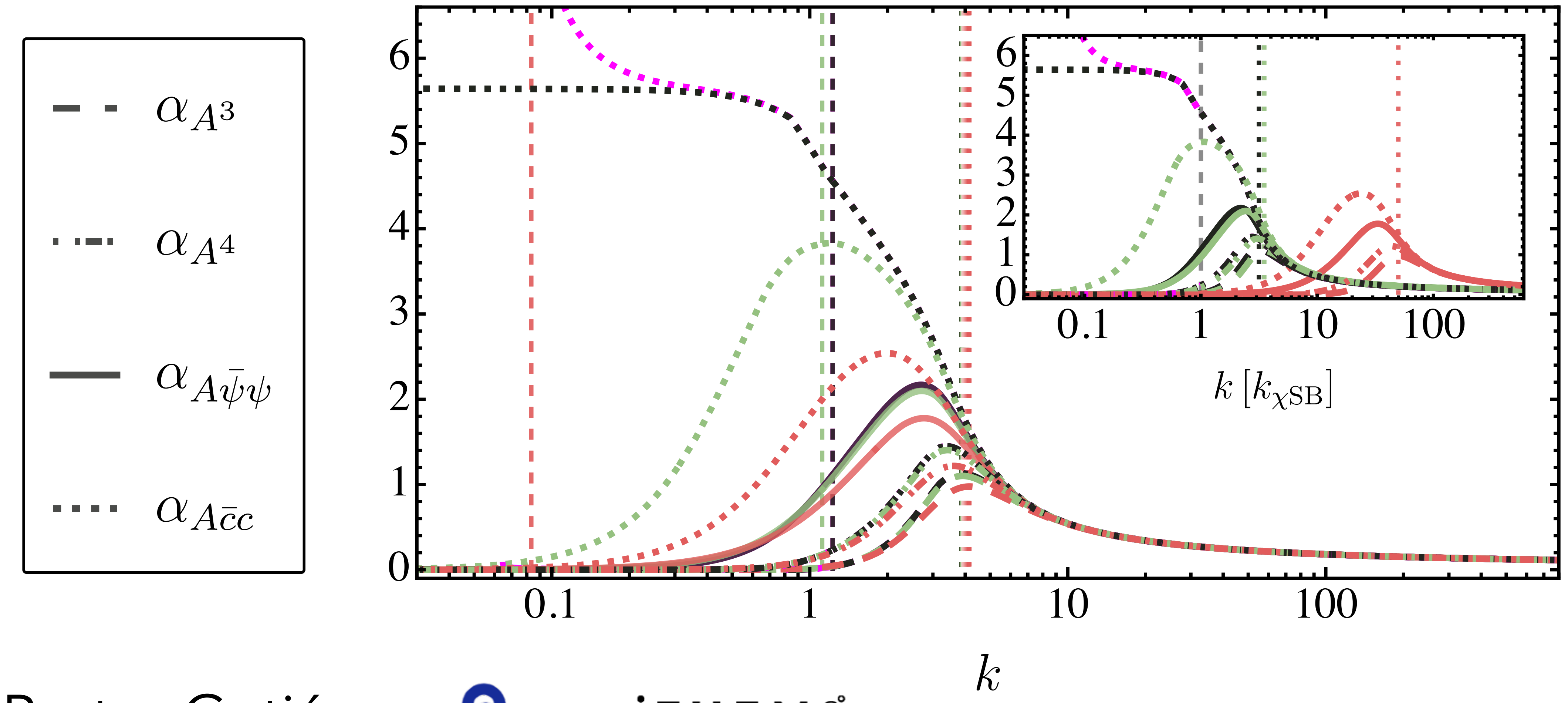
✦ Mass gap as a dialing parameter

✦ Confining solutions:

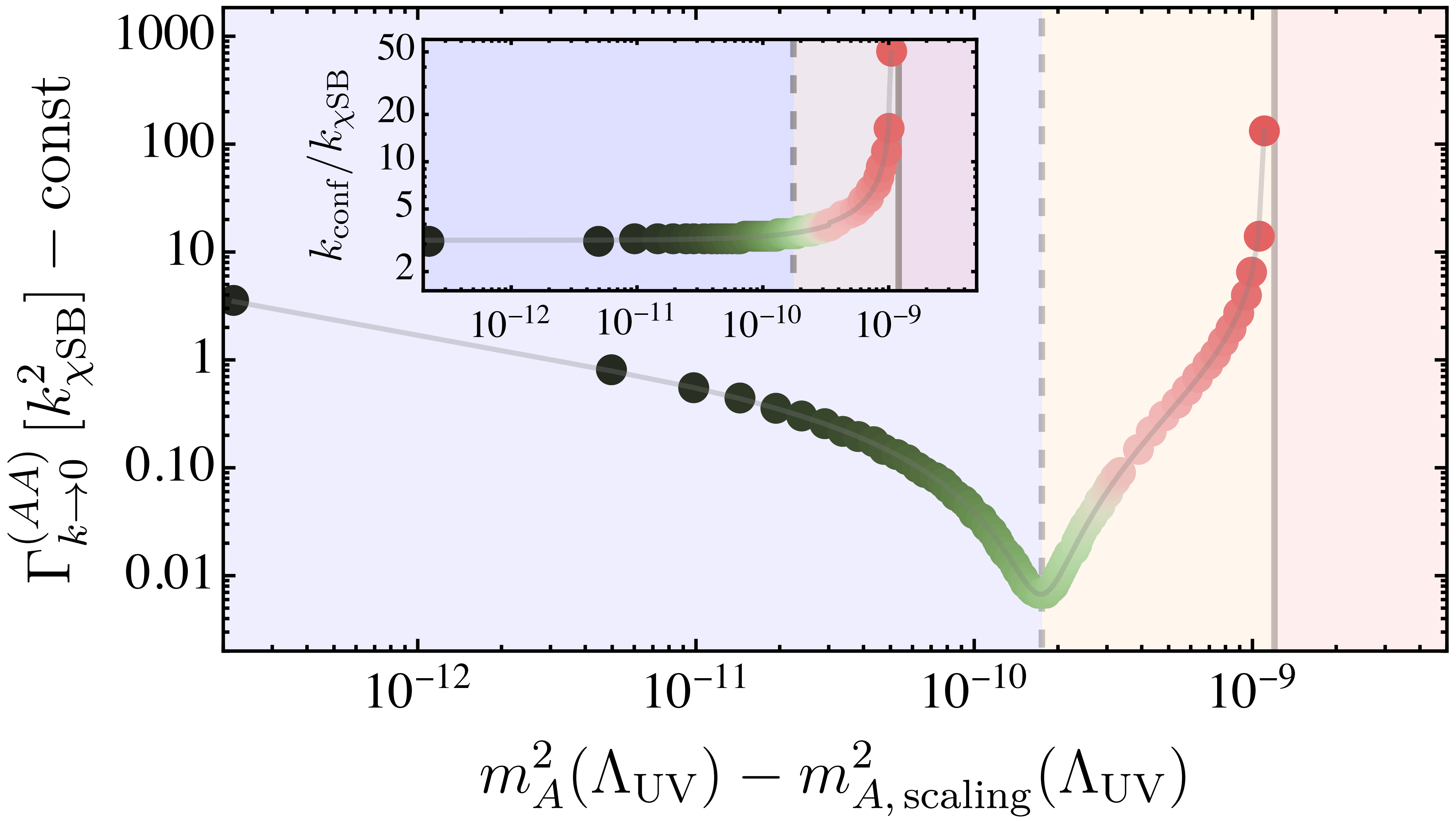
- Scaling
- Decoupling

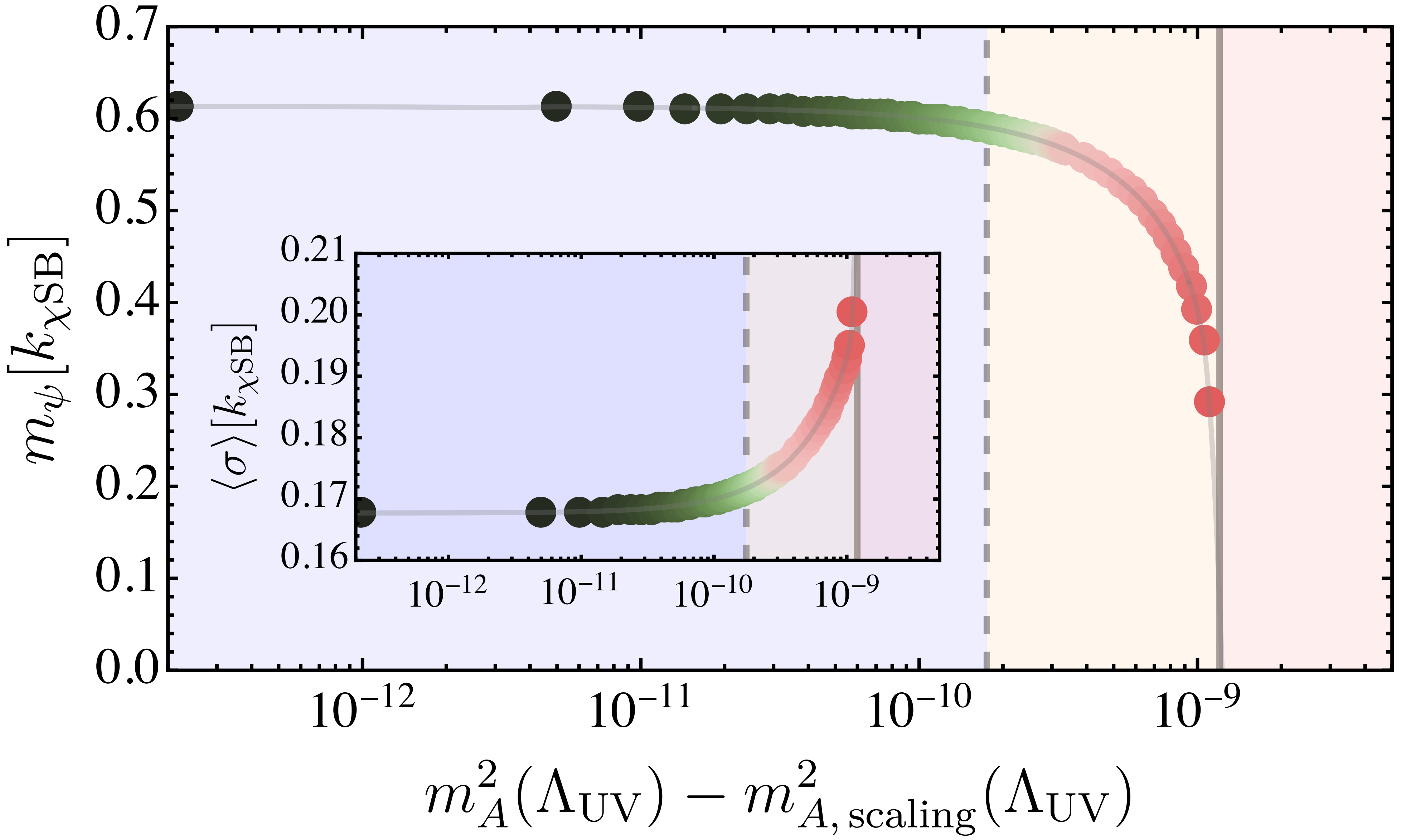
✦ Non-confining solutions:

- Coulomb phase
- Massive Yang-Mills



Goertz,APG,Pawlowski[2412.12254]





Thermal precondensation in chiral theories

What about finite temperature?

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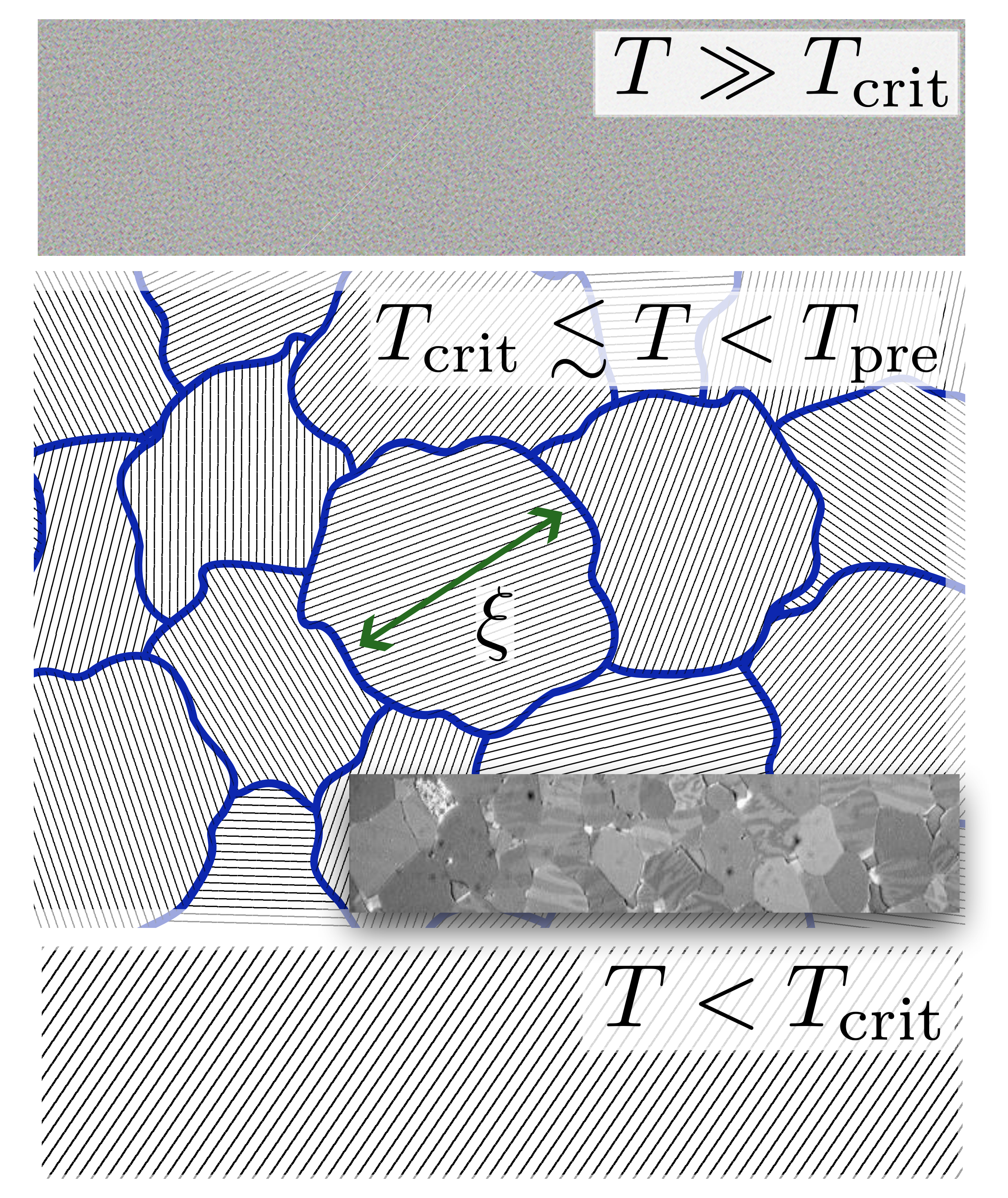
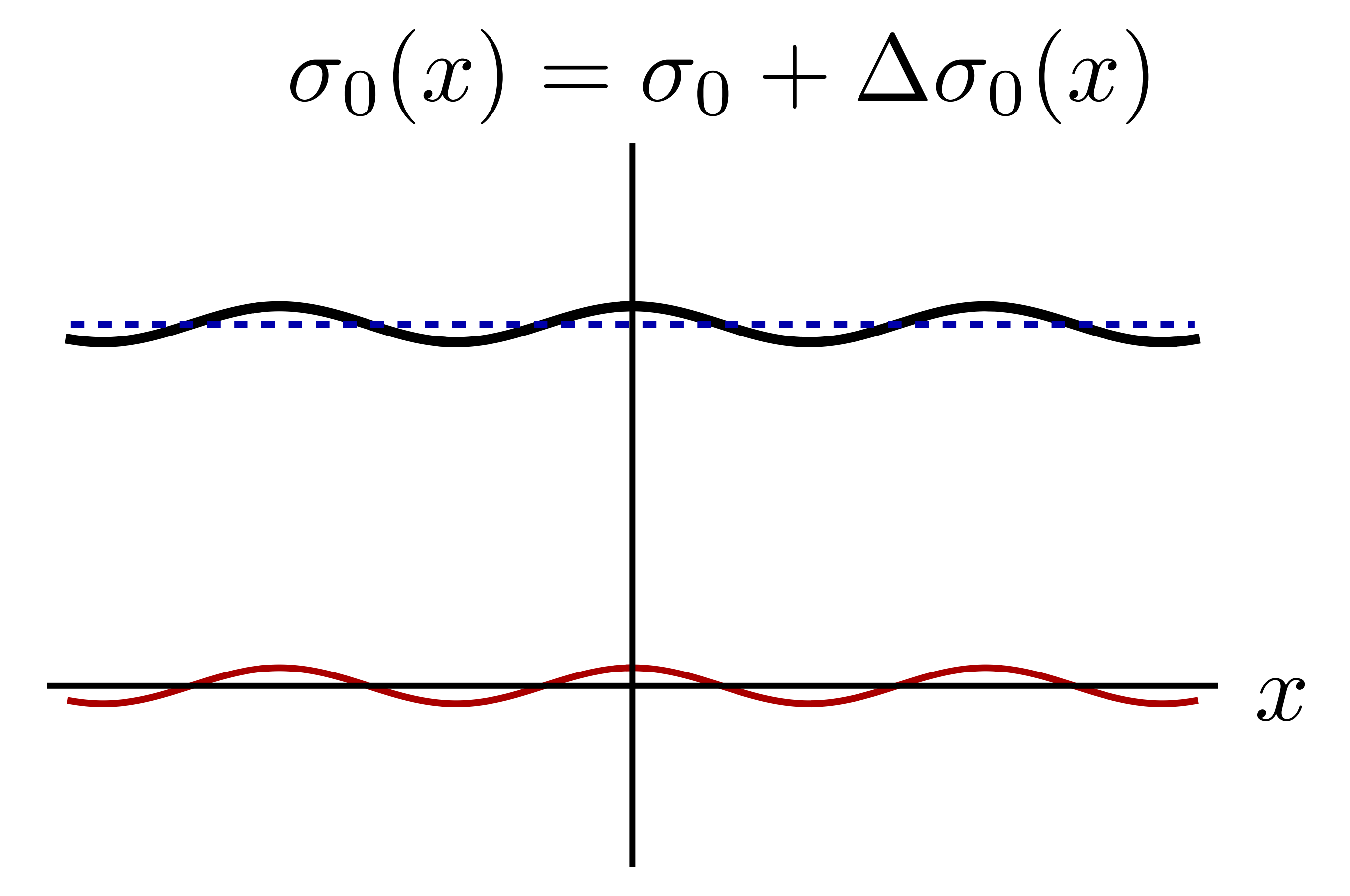
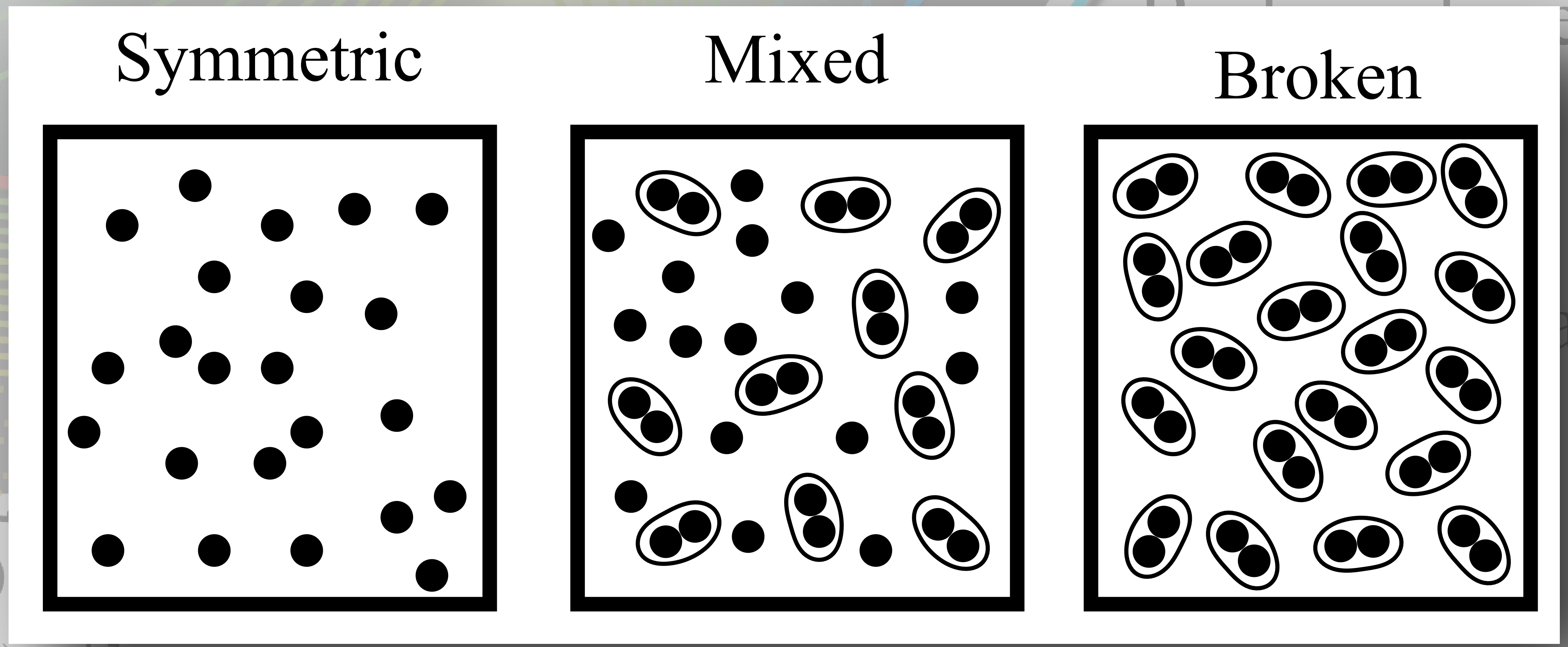
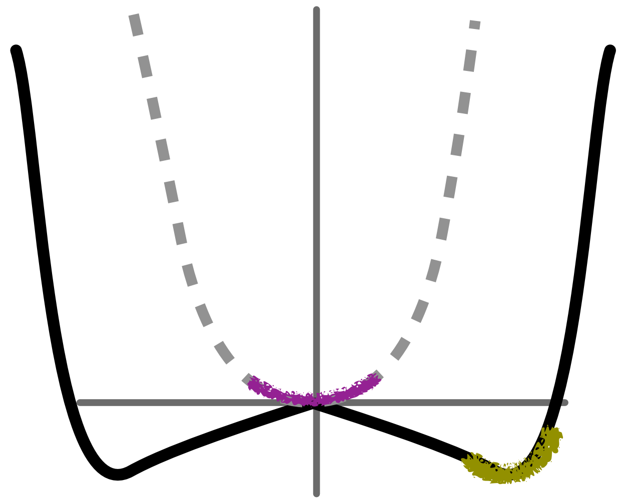






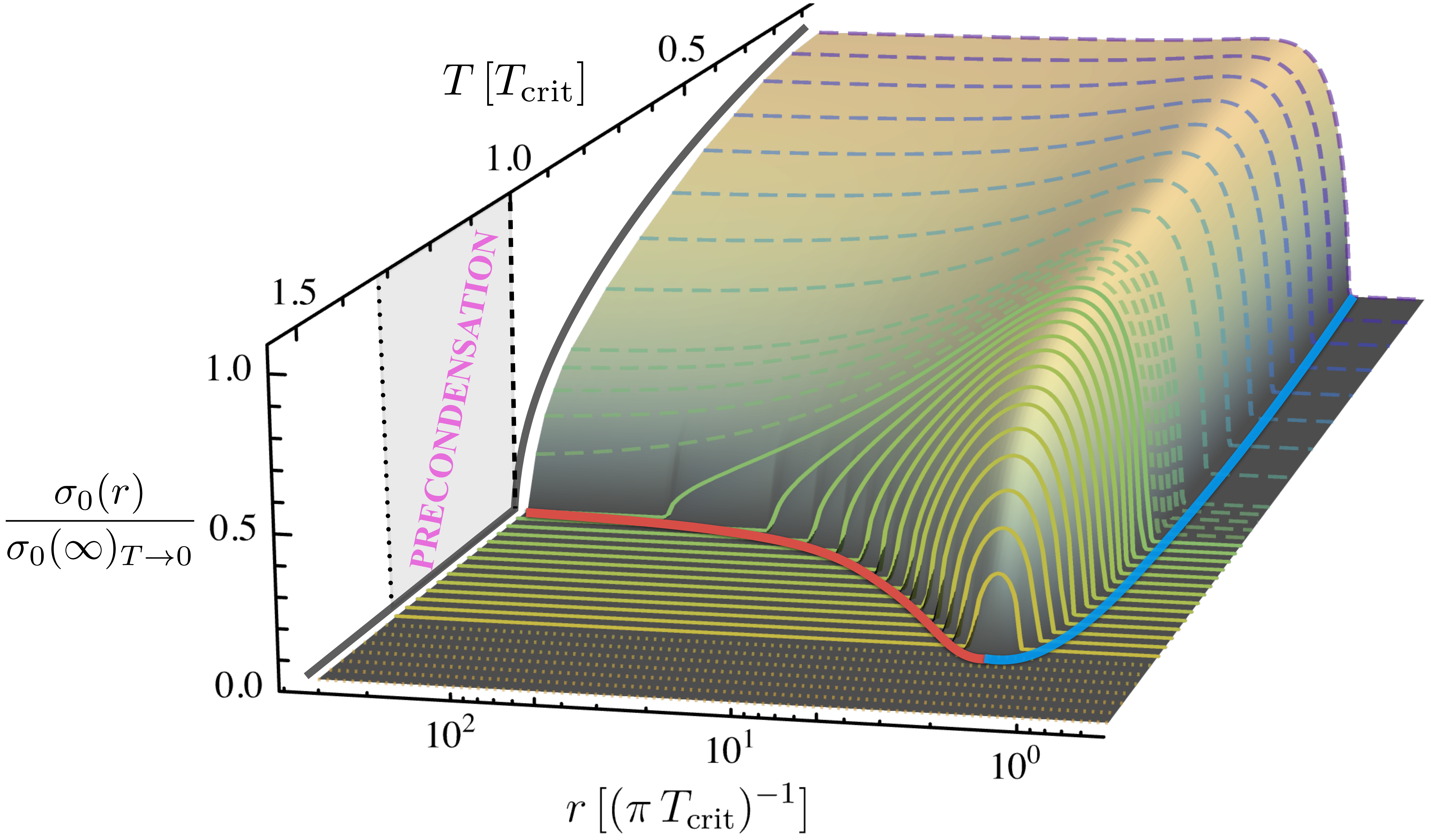
Microscopic origin

Flow of the effective chiral potential:



$\propto m_\sigma^2$

$N_c = 3 \quad N_f = 3$



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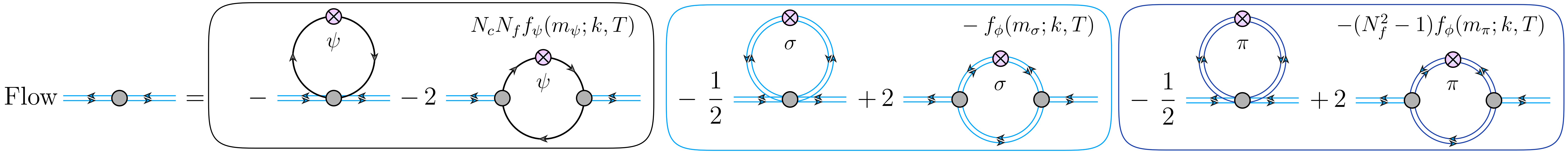

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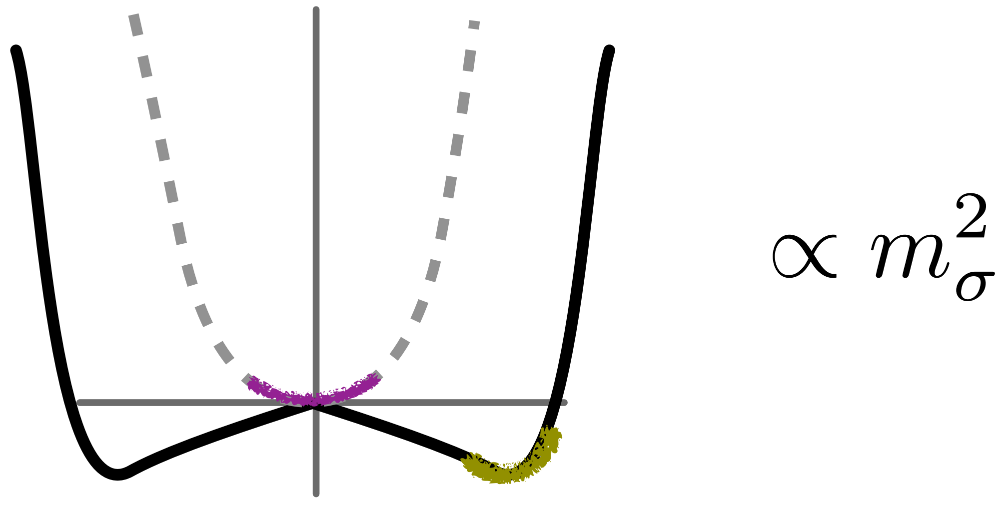

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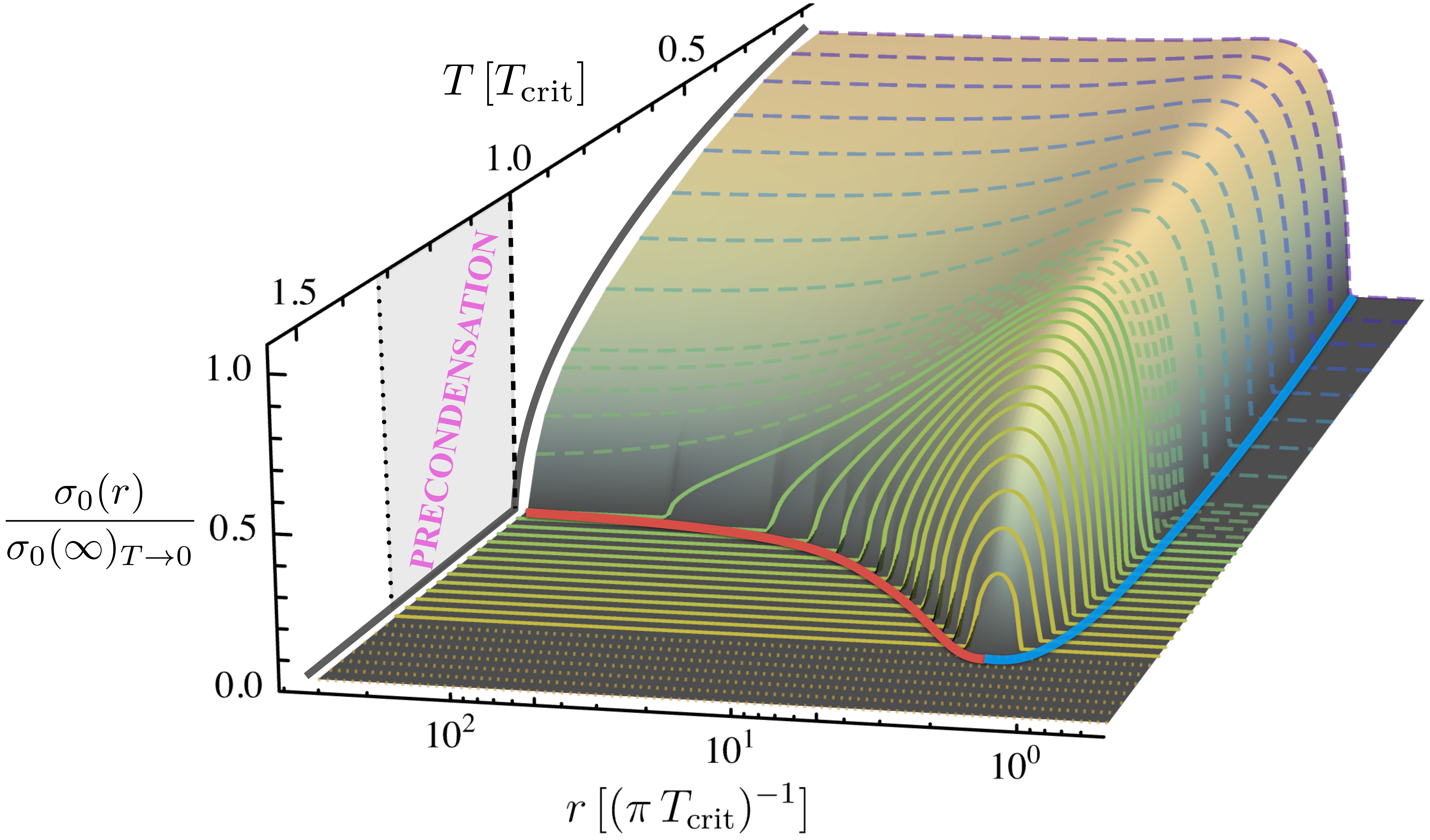

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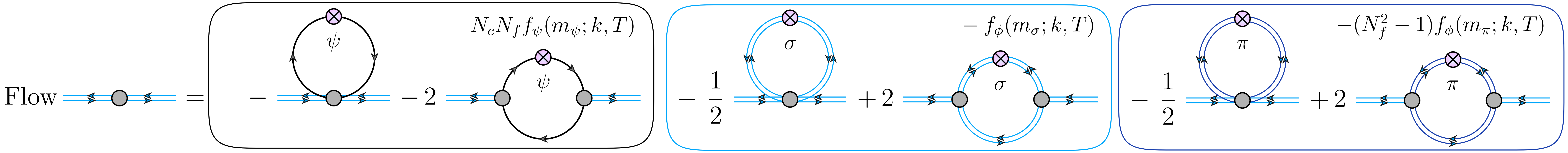

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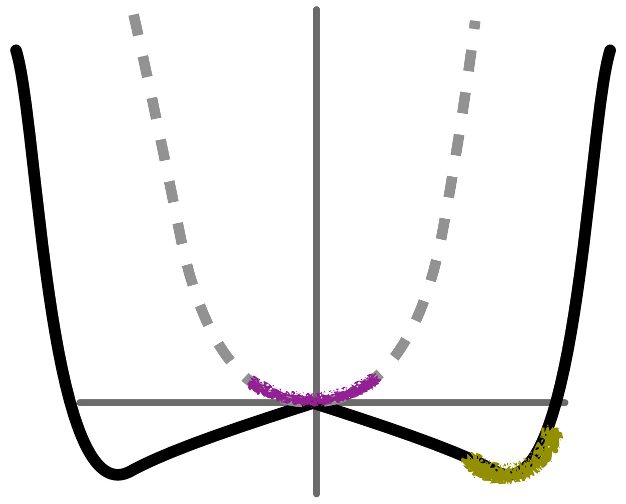

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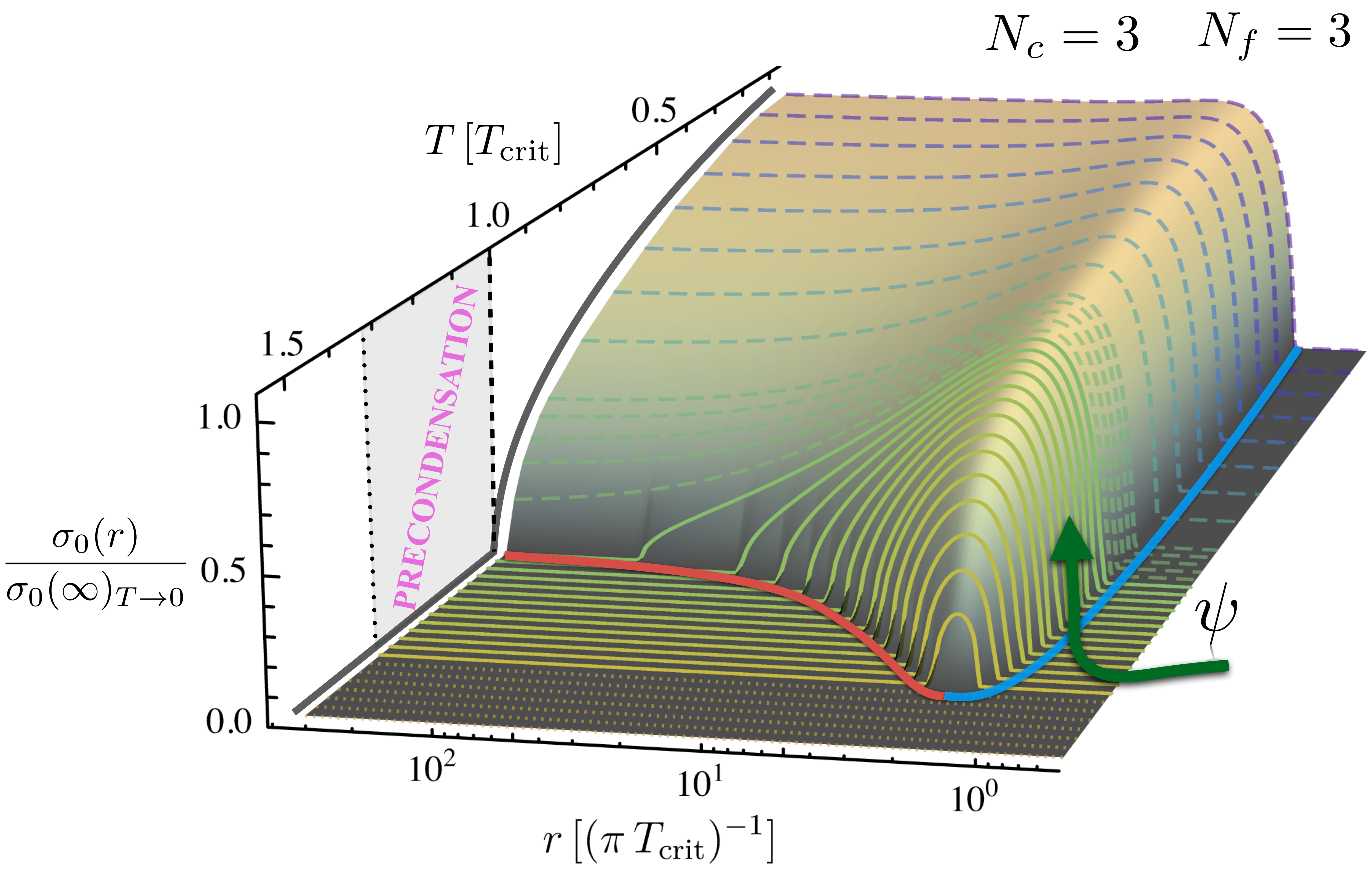

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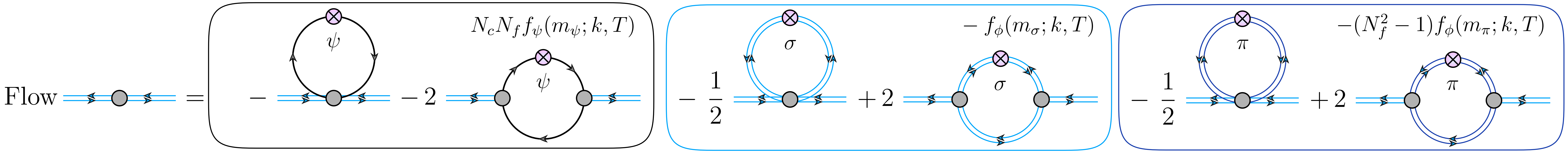

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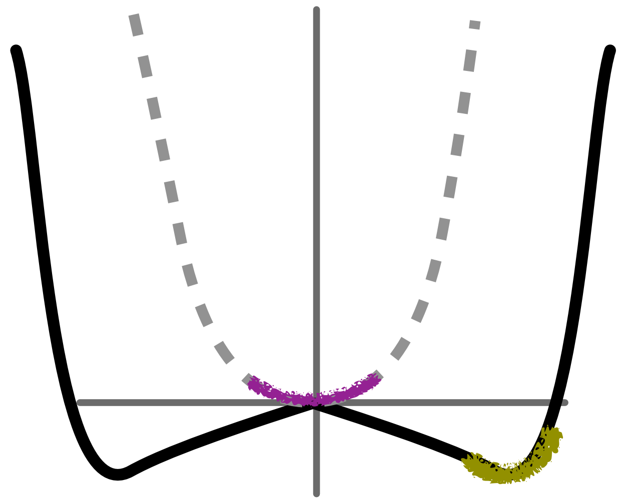

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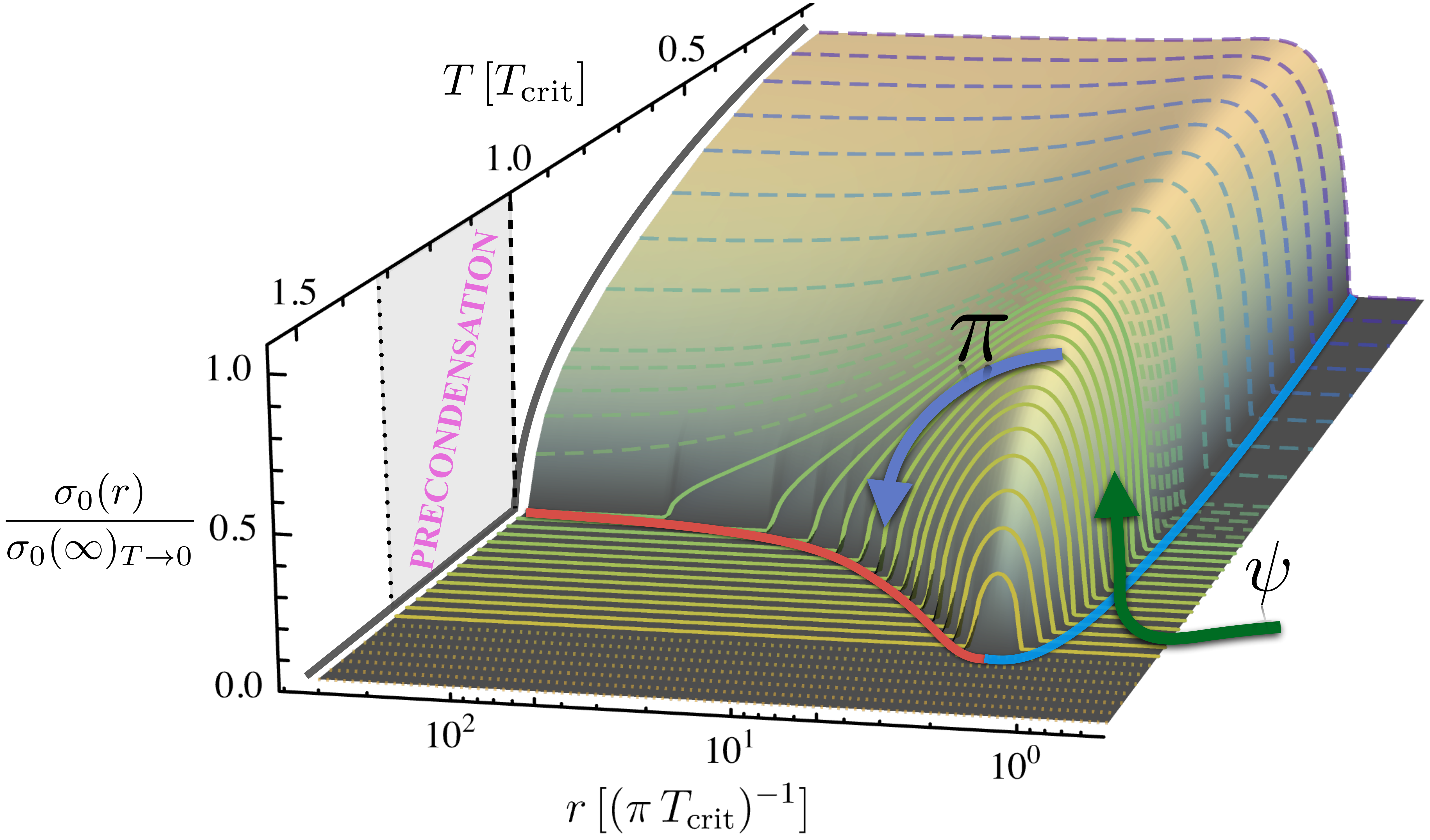

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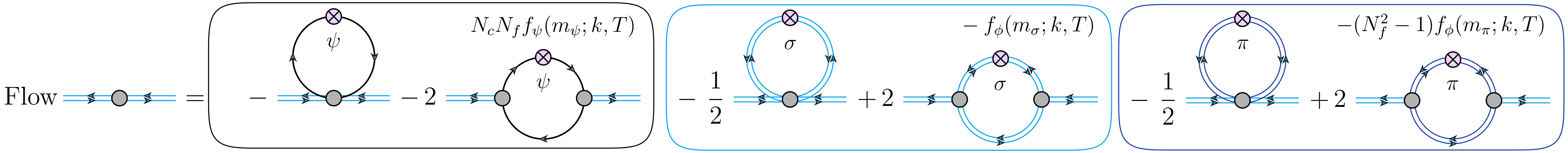

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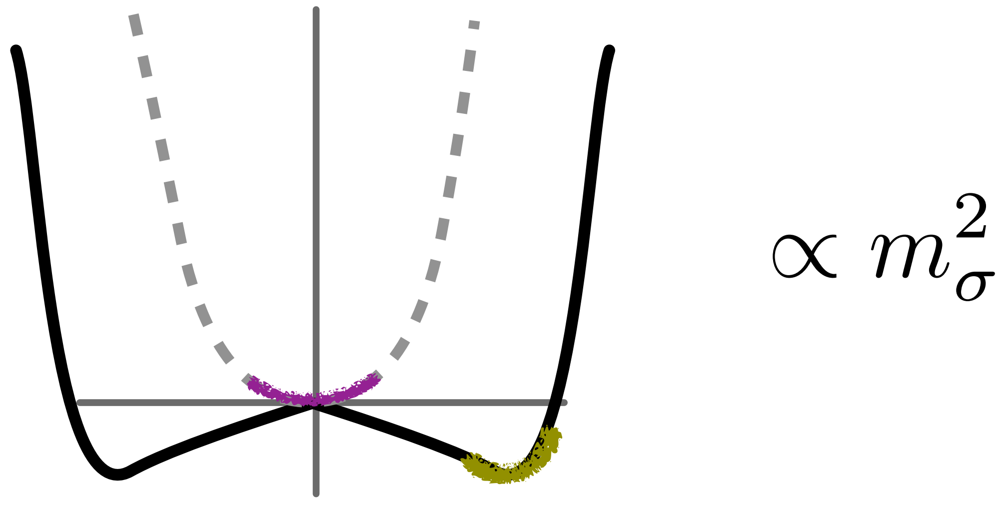
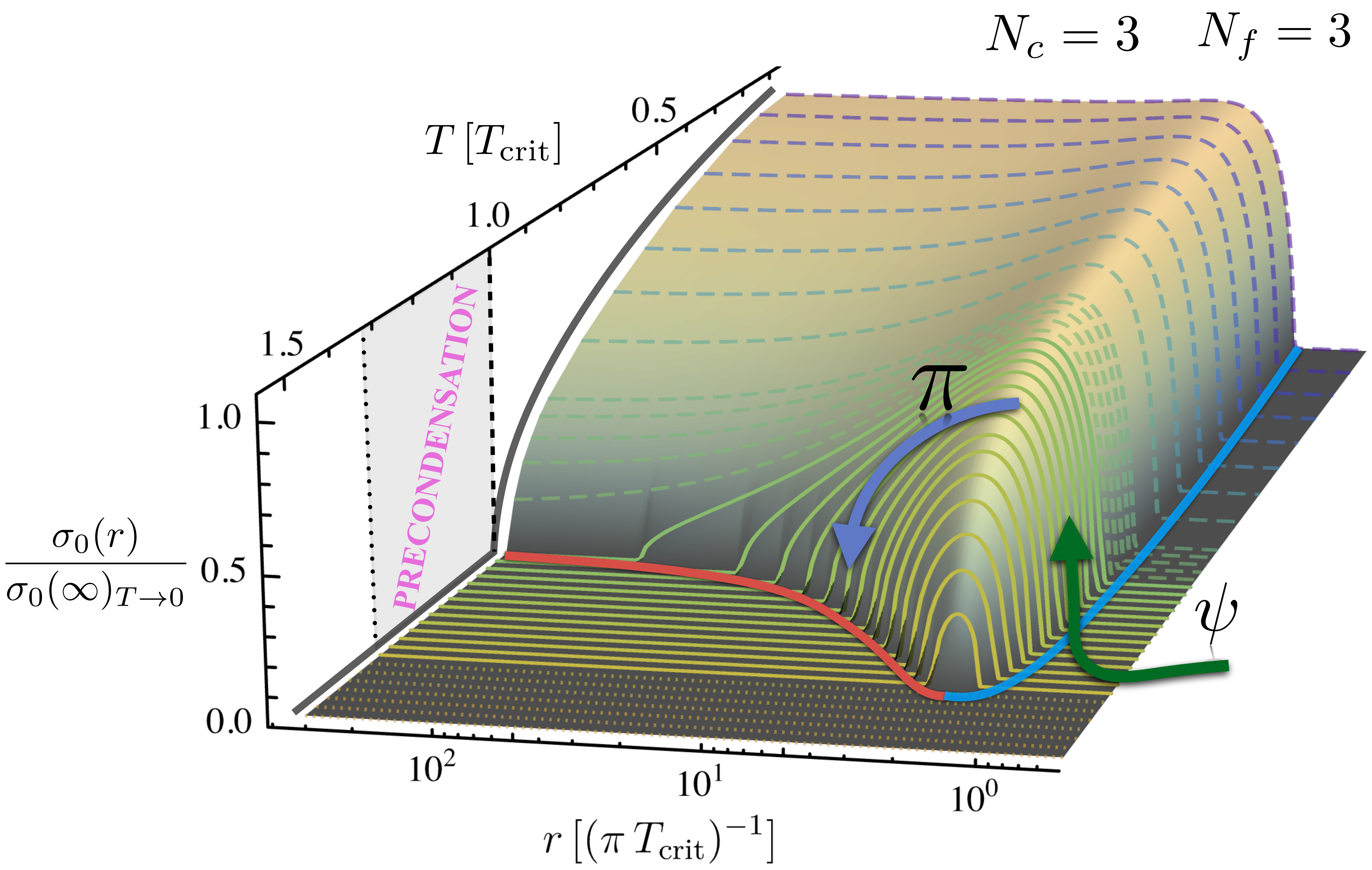
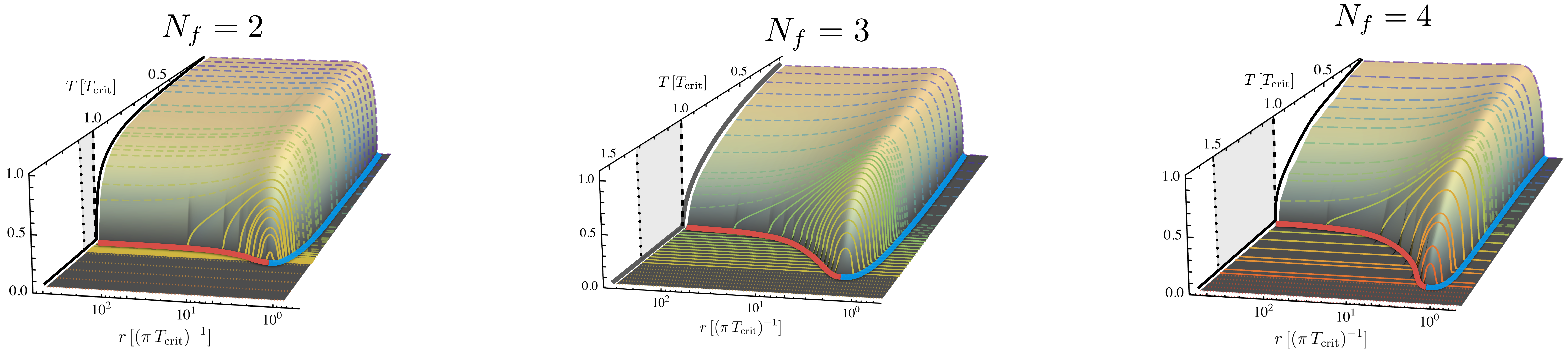


- Necessary **key ingredients**:
  - Competing counteracting effects: fermions and bosons
  - External parameter: $T, \mu, \ldots$
  - Massless modes: exact Goldstones



Many flavour scaling



- **Growing size** of precondensation regime in T
- **Growing** number of **Goldstones** $N_f^2 - 1$. Axial anomaly
- **Relevant role** in the **near-critical dynamics**