New Phases of Matter in QCD

Buenas Ideas on the QCD Phase Diagram
Yukawa Institute
Kyoto, May 2026

L. McLerran, Institute for Nuclear Theory, .
University of Washington (Emeritus)

First half:

The region between T_chiral and T_Hagedorn at finite T and zero baryon density

Collaborators: Fujimoto, Fukushima, Gyozo, Hidaka, Marczenko ,
Pawlovski,Redlich, Mikhaylova, Sasaki

Second Half:
Quarkyonic Matter at Finite Isospin Density
Collaborators: Fujimoto and Kojo, Hidaka and Fujikara

Everyone is entitled to his own opinion, but not his own facts.
Danial Patick Moynihan

As we heard in the lecture by Michal Marczenko:

The 3+1 dimensional string of large N_C QCD provides an excellent description of lattice thermodynamics of QCD for temperatures below T_c, a good description of the chiral phase transition, and spectra of hadrons below and near the chiral crossover; I believe we now have first principles understanding of this region.

For temperatures greater than the Hagedorn temperature, we have a viable description in terms of deconfined quarks and gluons. Hard thermal loops, resumed perturbation theory, etc, provide a good description in this region

For T_chiral < T < T_Hagedorn, there is little consensus of opinion, but there are also many facts arising from lattice gauge theory. The first part of this talk will be to try to make a minimalist simple model in an attempt to find consistency with these facts.

Much of the dispute revolves around whether the matter is confined or deconfined. But this question is not possibly to rigorously answer because at finite temperature, a deconfined particle would find its color force cutoff by pair production. There is no order parameter for deconfinement, so considerations are qualitative and semi-quantitative. As such, the answer one gets to a question with no crisp answer will depend somewhat upon the way the question is asked, and who answers the question

In large N_c limit, one can think of the three regions as (with confinement-deconfinement loosely defined)

$s/T^3 \sim 1 \; mesons$	$s/T^3 \sim N_c \; quarks$	$s/T^3 \sim N_c^2 \; quarks \; and \; gluons$
Chiral symmetry broken, confined	Chiral symmetry restored, confined?	Chiral symmetry restored, deconfined

The limits of description of this intermediate region using a quasi-particle model:

A quasi particle model has a gluon effective propagator, G, and and fermion propagator S. Interactions are assumed to be not so important.

Chiral symmetry constraint is that

tr S = 0

For massless quarks

$$T^\mu_\mu = -\frac{\beta(g)}{g}(E^2 - B^2) + m_q(1 + \gamma_q)\overline{\psi}\psi$$

The QCD trace anomaly is

$$e - 3P = T^\mu_\mu(T) - T^\mu_\mu(0)$$

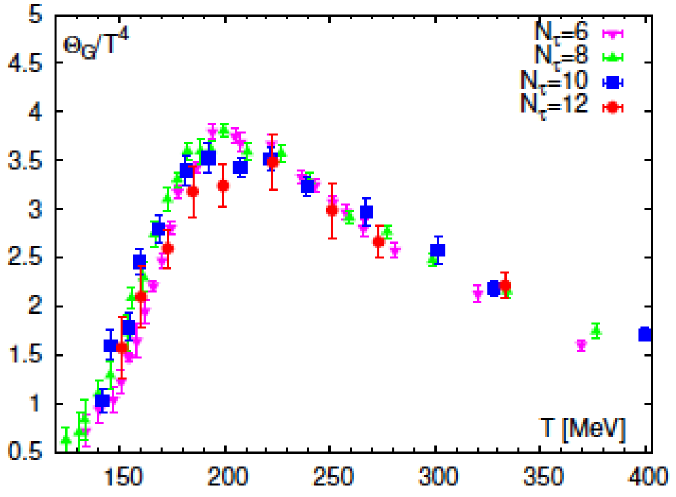
$$T\frac{d}{dT}P/T^4 = (e - 3p)/T^4$$

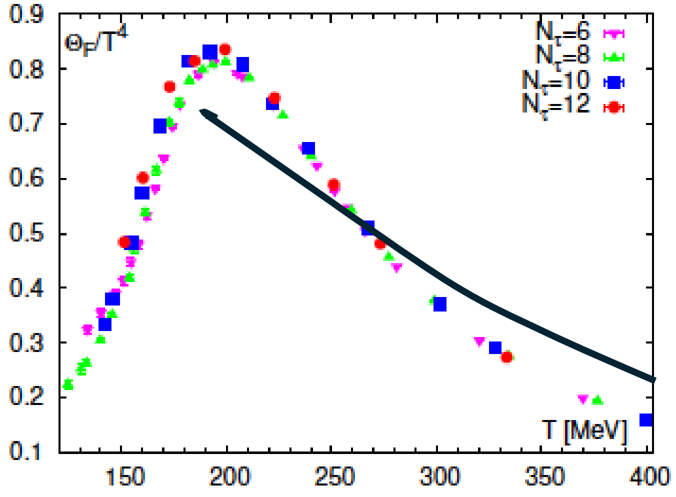
$$P = P_{glue} + P_{quark} \qquad T\frac{d}{dT}\frac{P_{quark}}{T^4} = \sum_q m_q(1+\gamma_q)\overline{\psi}\psi$$

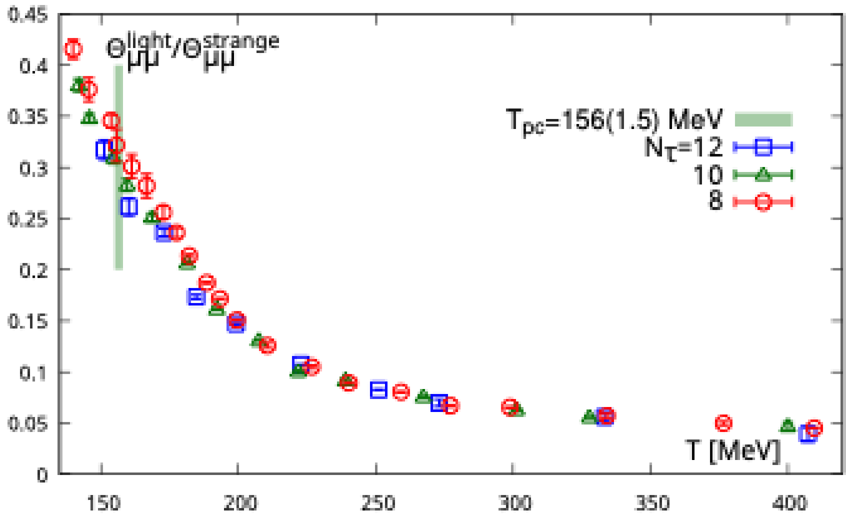
If trace anomaly is small, P/T^4 -> constant, ideal massless gas limit

Because the up and down quark masses are small, this is dominated by the strange quark.
If it becomes substantial then the quasi-particle approximation breaks down

Hot QCD







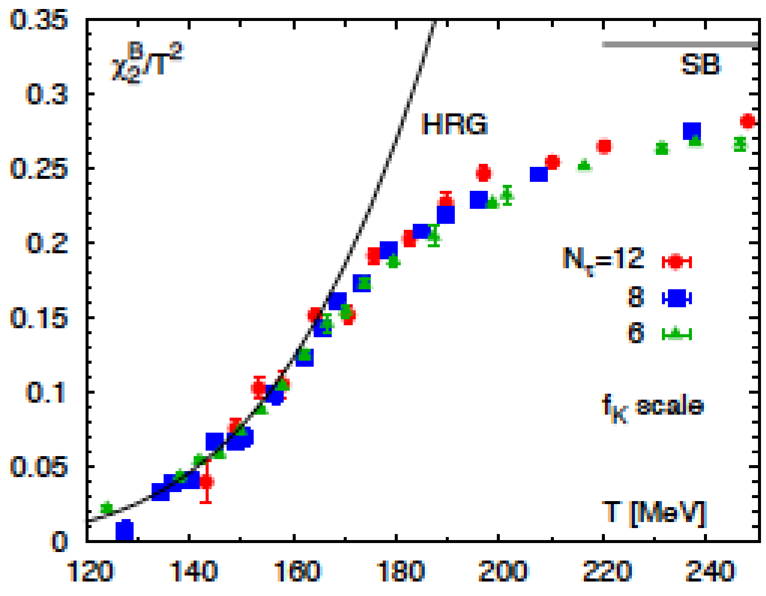
Anomalous dimension 1strange mass .220 GeV
Massless up and down quarks
In fact anomalous dimension should decrease making better fit

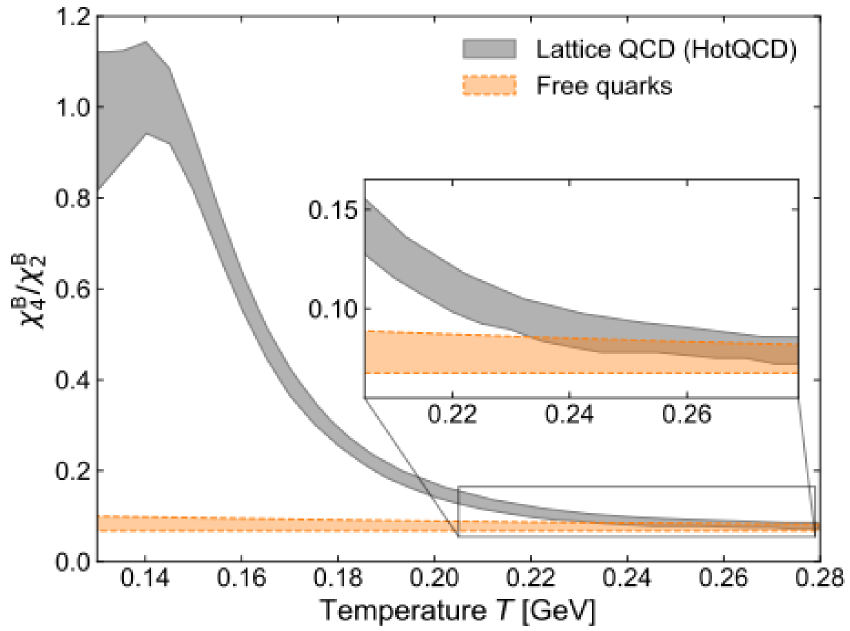
Baryon Susceptibilities are Sensitive to Quark Densities.

If we use a Boltzman approximation on the finite T part of quark propagator
(~ 10% accuracy for the pressure)

$$P_{quark}(T, \mu_B) \sim cosh(\mu_B/N_c T) P_{quarks}(T, \mu_B) = 0$$

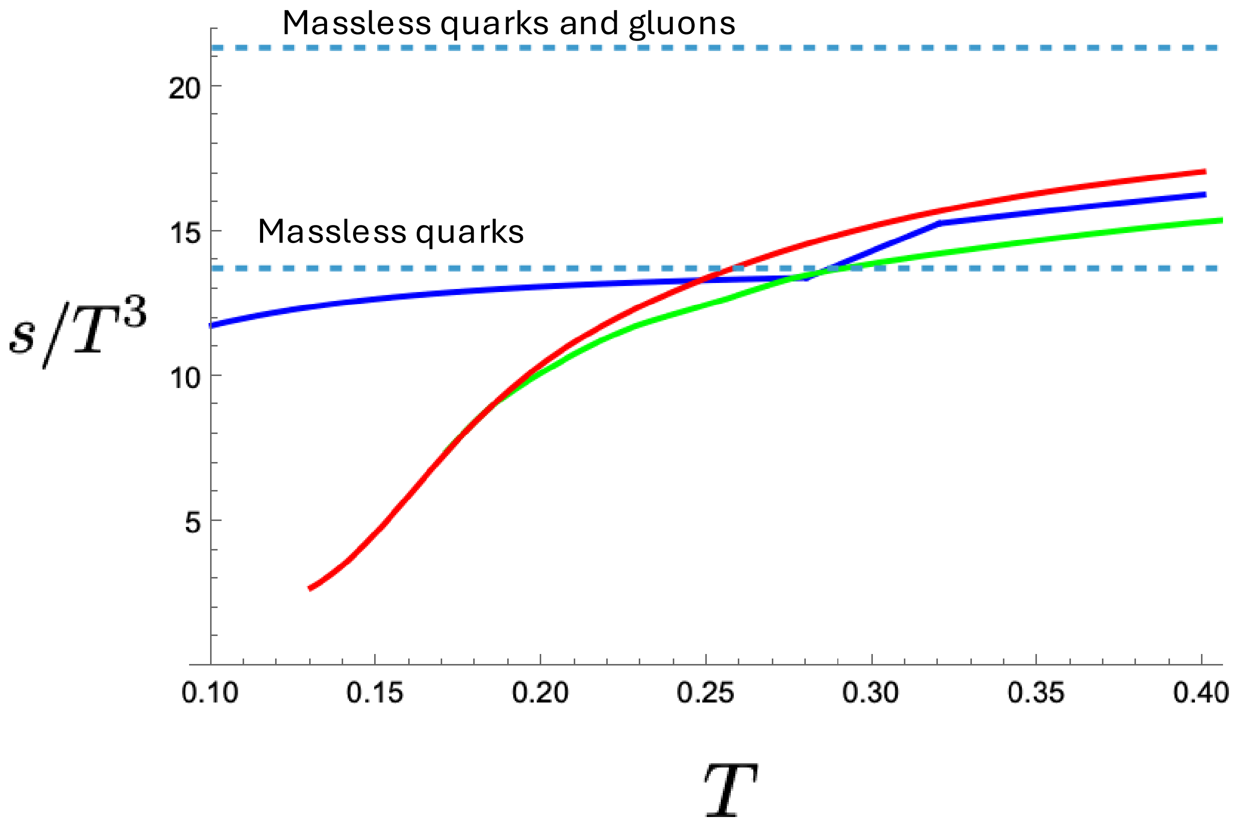
$$C_n = T^n (d/d\mu)^n P$$





C_4/C_2 = 1/9 in Boltzman approximation, about 10% change for ideal Fermi gas

Simple computation: Take quark gas with zero mass up and down, finite mass strange. assume a smooth interpolation exists to a quark gas with gluons. Take effective gluon mass to be 1.2 GeV. Interpolation region centers at 300 MeV with width of 20 MeV



Blue: Computation with massless quarks
Red: HOT QCD
Green WB

The total trace anomaly for this system will be the fermion trace anomaly below the interpolation region. This is about an order of magnitude too small!

Note that the ideal gas for a pure quark gas works for T down to about 220 MeV and by 160 MeV, the data is three times too small.

Need an interaction contribution to the trace anomaly

For T > 270 MeV, we assume trace anomaly comes from perturbative value.

$$T_A(T) = T_{NP}(T),\ T < T_o = 270\ MeV$$

$$T_A(T) = s(T) - 4P(T) = T_A(T_0) + s(T) - s(T_0) - 4\int_{T_0}^{T} dT' s(T'),\ T > T_0$$

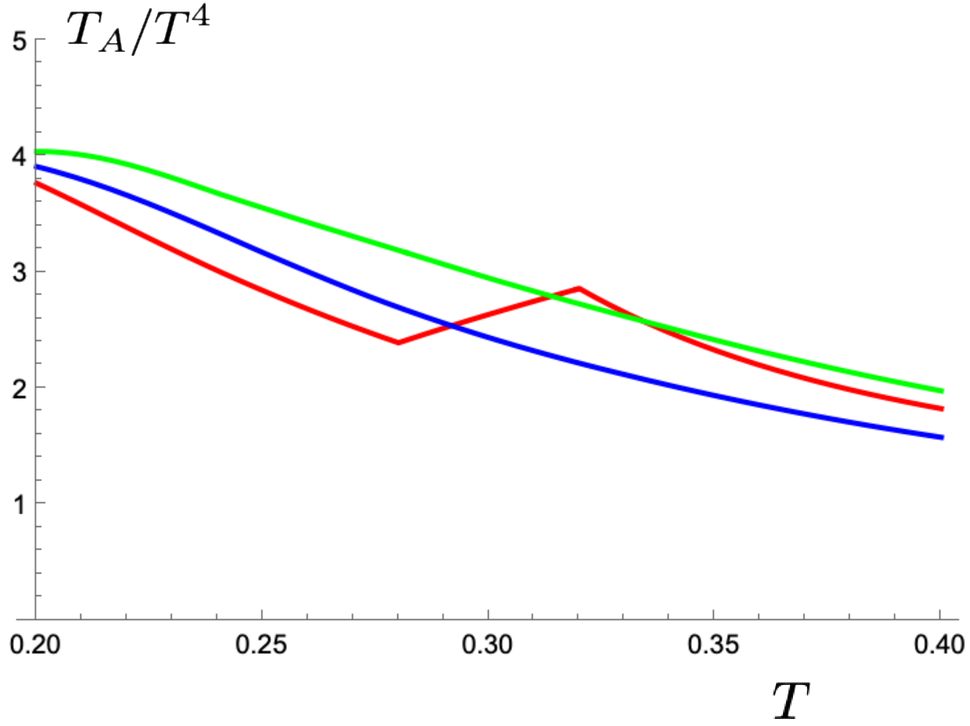
For the nonperturbative value, we assume strings connect quarks so that

$$T_{NP}(T) = \sigma < R > \rho_{quarks} \sim \sigma T^2 (s/T^3)^{2/3}$$

Where I use

$$\rho_{quarks} \sim s_{quarks}$$

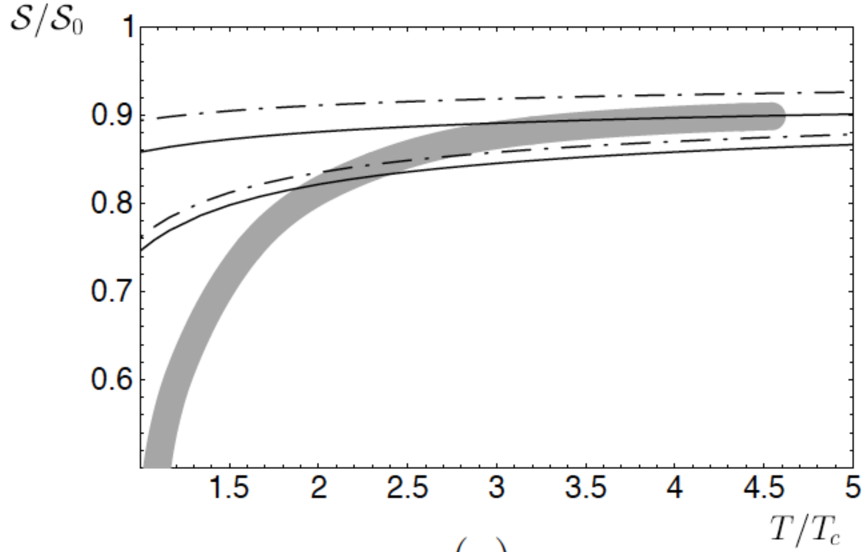
$$R \sim 1/\rho_{quarks}^{1/3}$$



T_vs T for simple interpolation model.

Need a little better model and better data.

Semiquantitative and qualitative features of data consistent with quasi particle picture of quarks above about 220 -240 MeV. Above 300 MeV, consistent with quasiparticle picture including gluons in lower T region



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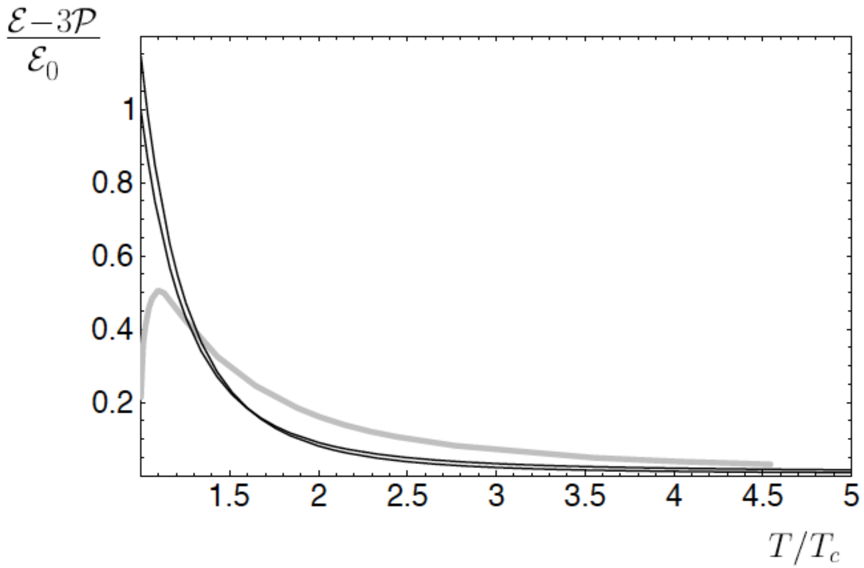
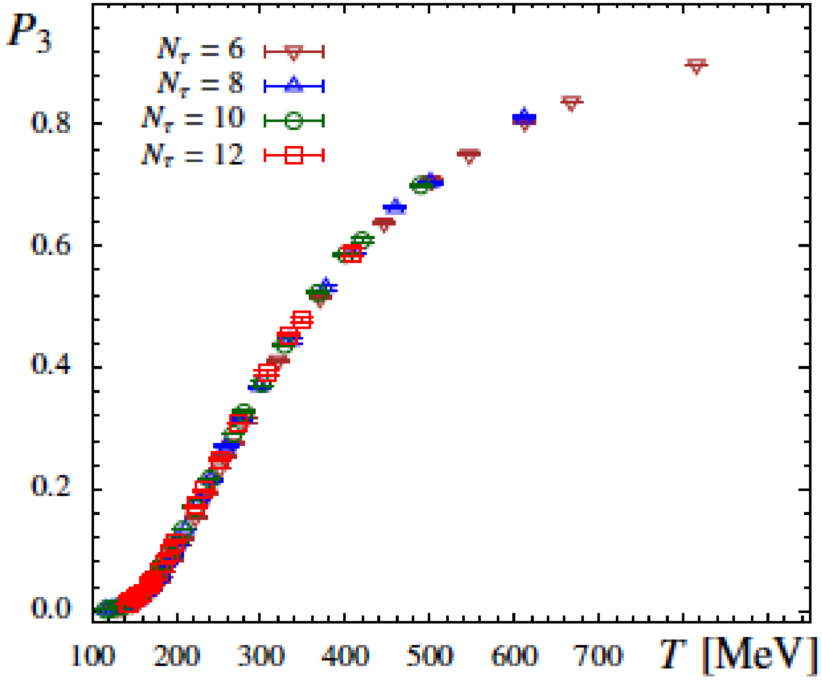


Grey band is lattice and lines are computation

Results from Blaizot, Iancu and Rebhan with HTL quasi particle model. Includes glue at all T but effective gluon mass is large. T/T_c ~ 1.5, is T about 240 MeV. Overshooting data between 200-300 is because too many gluons.? Between 200-300 MeV, undershoot because no string contribution

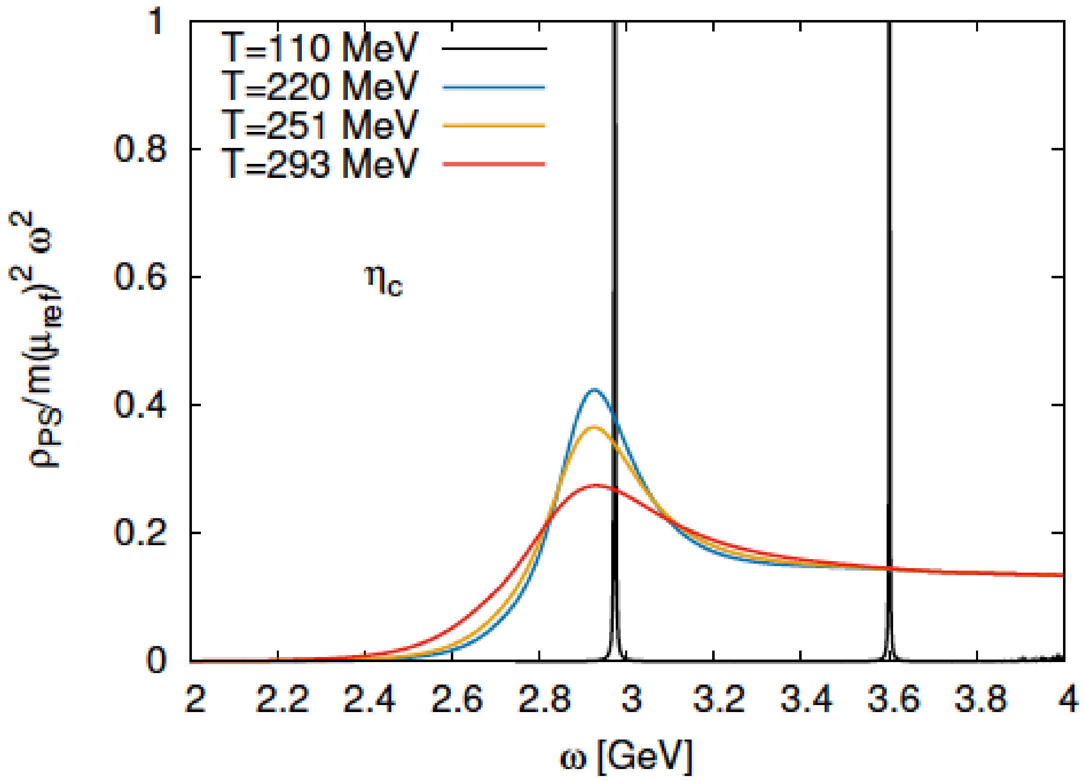
Note that the typical energy of excitations is about 3T, so already by T ~ 300 MeV, the typical particle energy is about 1 GeV

Story is more complicated for correlations which probe confinement scale

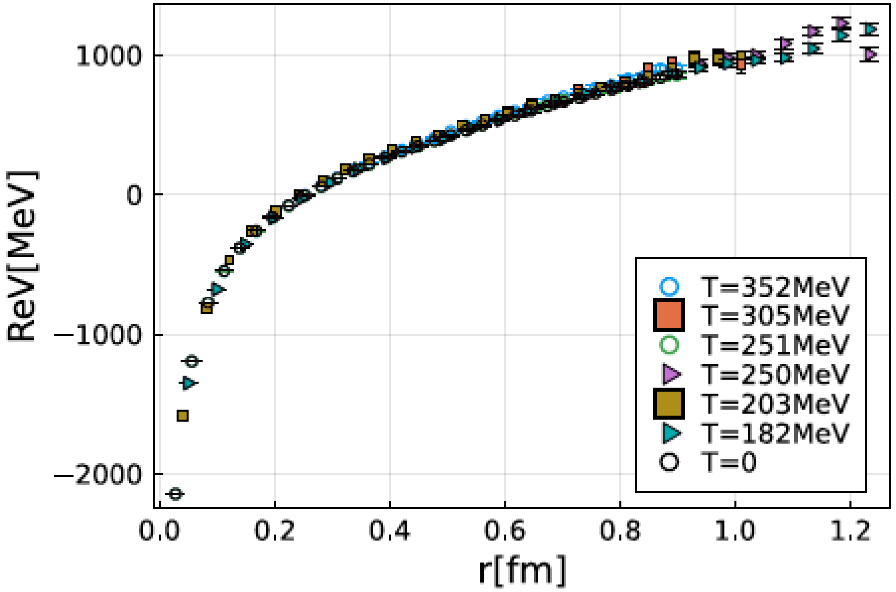


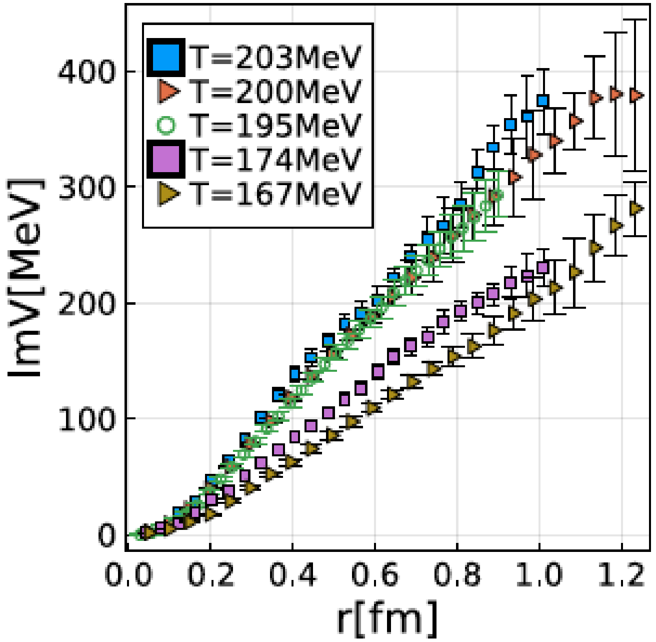
Hot QCD

Renormalized Polyakov loop is very small at the chiral restoration temperature and is about ½ of it's "deconfined" value by about 300~MeV



Charmonium not so strongly broadened even at T ~ 300 MeV
HOT QCD





Real part of string tension not much changed, but a large imaginary part

Similar to vacuum where string break into meson pairs.

Quark not confined into heavy bound state, but are confined inside meson pair

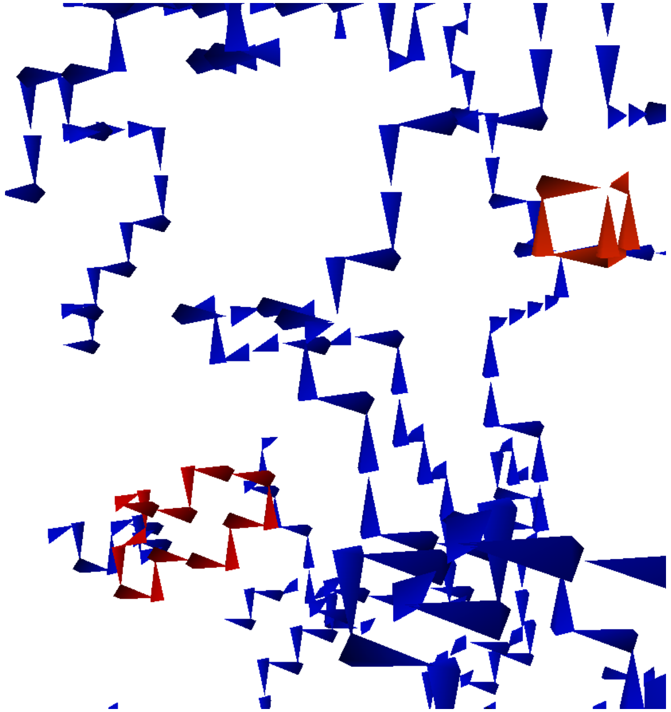
$$\rho + J/\psi \to D\overline{D}$$

Standard argument for confinement at zero T is if you pull quarks apart a string breaks and you make mesons

At high energy, the ratio of total cross section of electro-positron to hadrons vs muons

$$R \sim N_c \sum_i q_i^2$$

is well verified for E > 4 GeV. (experimental evidence for color), but this is not evidence the vacuum is deconfined



String percolation studies shows a percolation transition at T ~ 300 MeV

Kamleh, Leinweber, Mickley

Can we live with a picture where the physics is confinement like at long distances, but for bulk thermodynamic quantities behaves like a quark gas?

Part Two:

Quarkyonic Matter at Zero Baryon Density and Finite Isospin Density

Finite isospin density, zero baryon density

Quarkyonic, because

$$M^2_{Debye} \sim \alpha_{'tHooft} \frac{\mu_I^2}{N_c}$$

Low temperatures: pion condensate

$$\mu \leq \Lambda_{QCD}$$

Very High Temperatures: Fermi gas with cooper pairing

$$\mu \gg \sqrt{N_c}\Lambda_{QCD}$$

Quarkyonic Region

$$\Lambda_{QCD} \ll \mu \ll \sqrt{N_c}\Lambda_{QCD}$$

In quarkyonic region should be a Fermi sea with a Fermi shell composed of either bosons or Cooper pairs. Scale for cooper pairing would be the QCD scale and is not computable by weak coupling methods

Duality relation between quark up quark and charged meson for phase space densities

$$\rho^Q_{PS}(k) = \frac{1}{2N_c} \int \frac{d^3p}{(2\pi)^3} K(k - p/2) \rho^M_{PS}(p)$$

The quark fermi sea corresponds to a flat phase space density of mesons of height N_c/4

$$p < p^M_F = 2k^Q_F$$

For an initial phase space distribution at low density (corresponding to a constant scalar field) which is,

$$\rho^0_{PS} = \rho^0 (2\pi)^3 \delta^{(3)}(\vec{p})$$

There is a limiting density

$$\rho^0 < 2N_c/K(0)$$

At low densities there is a condensate, but at some density the phase space density for bosons occupies momentum states up to some maximum momentum, corresponding to a Fermi surface for quarks

On the "Fermi" surface there is a condensate of bosons and possibly Cooper pairs

Why does this happen?

If there is some limiting density, then in a scalar field theory the repulsive quartic scalar field interactions cease to keep up with the negative mass squared term

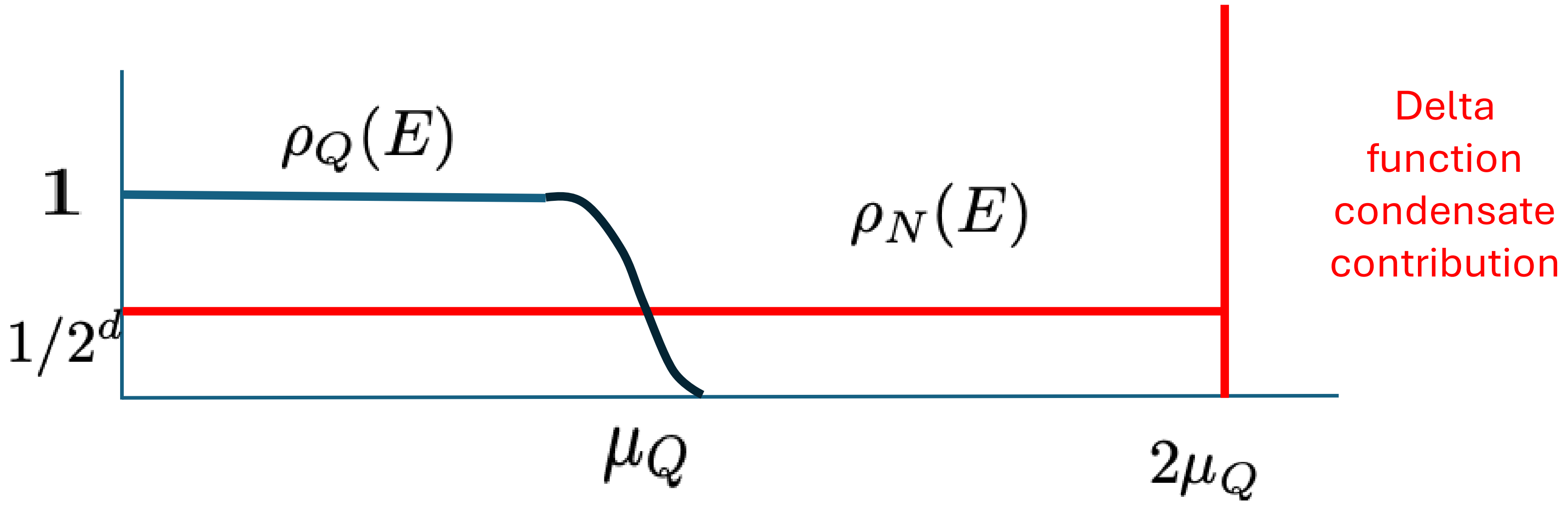
$$\frac{1}{2}\phi \left\{k^2 - \mu_I^2\right\} \phi + \frac{\Lambda}{4N_c}\phi^4 \sim \frac{1}{2}\phi \left\{k^2 - \mu_I^2\right\} \phi$$

For small k, this term is unstable and states occupy until the Fermi exclusion principle of the underlying quark degrees of freedom becomes important

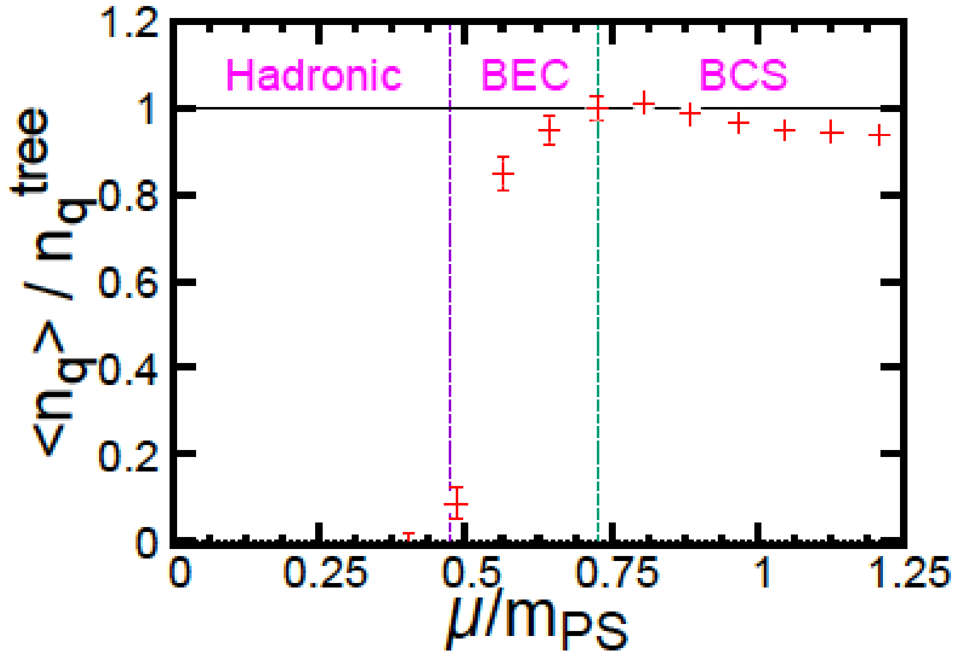
Can this be seen using Monte Carlo methods and composite operators?

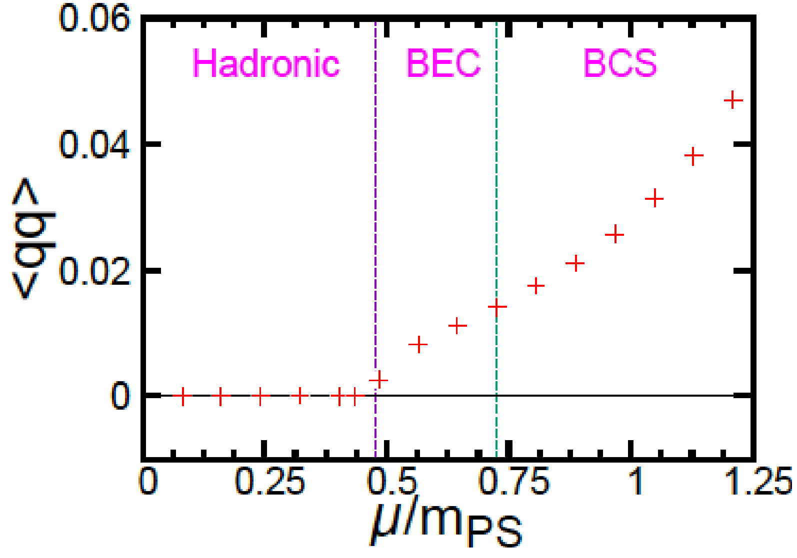
For N_c = 2 QCD: Situations should be similar

When quarks are fully occupied, the bosonic baryons look like a Fermi gas with occupation number 1/N_c^d



Quark Fermi surface broadened from the surface structure of the baryon number distribution:
It is a small correction to the total density at large density, and in 1+1 d generates a constant contribution to the trace
Surface can arise for either Cooper pair or chiral spiral like effects





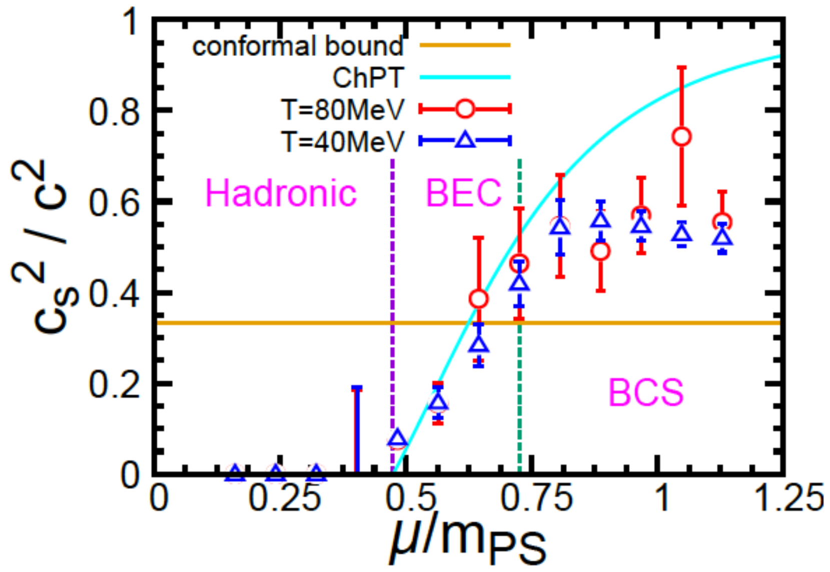
Iida, Itou, Murakami, Suenega

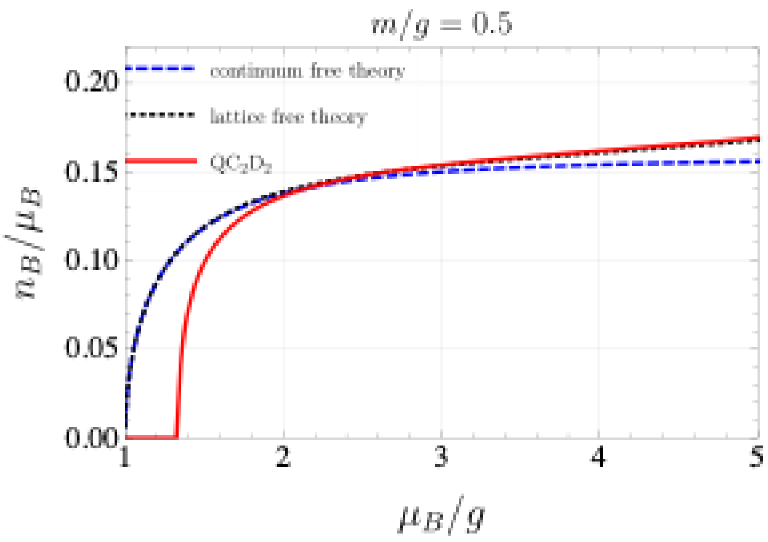
This occurs at densities less than 1/fm^3 assuming nucleon mass of .6-1 GeV (two colors probably closer to .6). Needs a closer look.

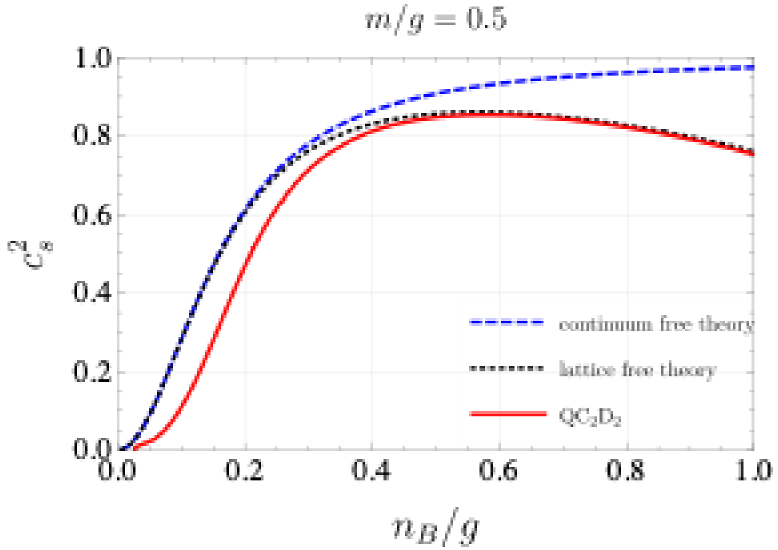
I also would think baryon number density is linear in \mu, but on the other hand a Fermi surface effect on a shell goes like surface area which is \mu^2

Correlation functions show confining behaviour at large distances;

Potential between quarks is linear

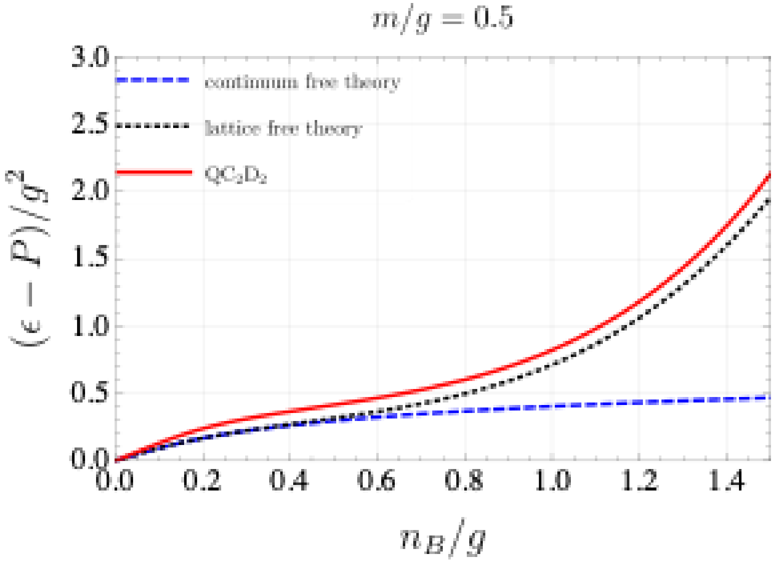


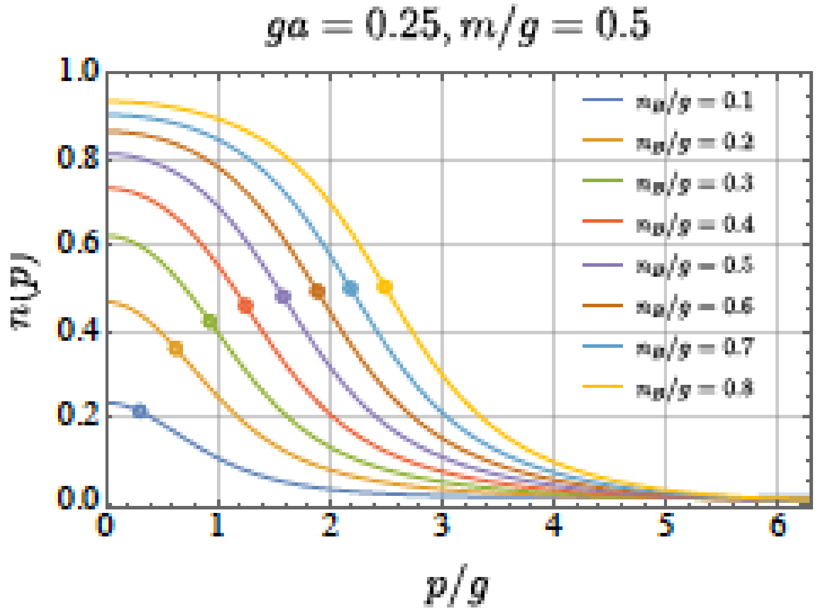




Fujikura and Hidaka

Trace anomaly is probably a Fermis surface effect. Particle hole pairs or cooper pairs which are diminished by fluctuations in 1+1 d?





Density is low where transition occurs,

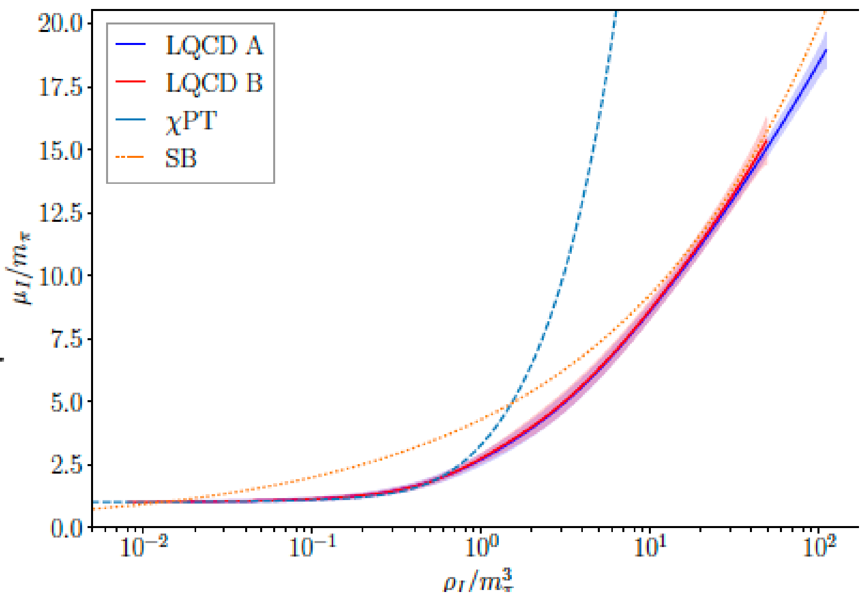
About .2 fm^-1?

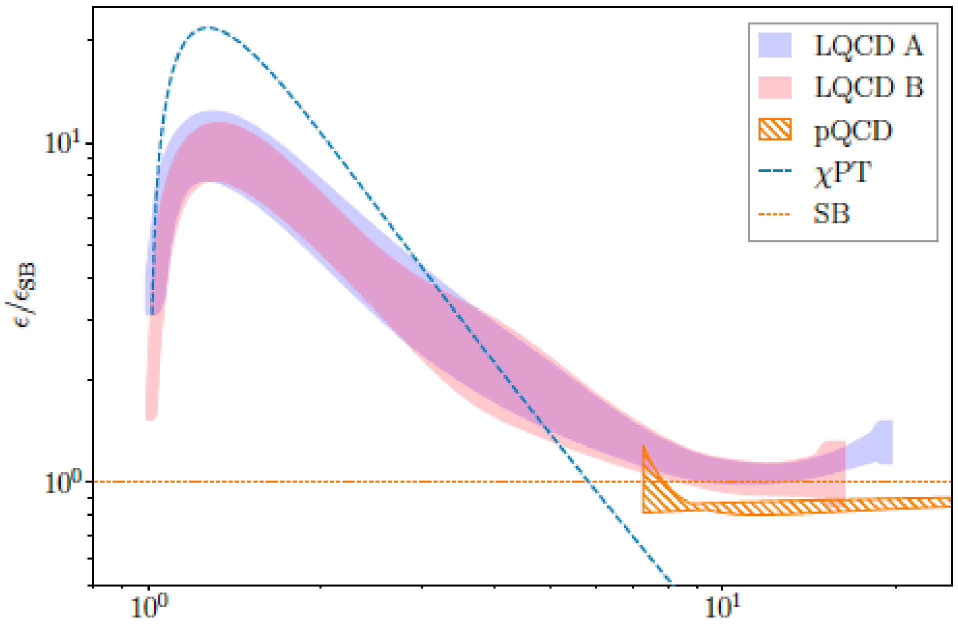
N_c = 3 at Finite Isospin: NPLQCD

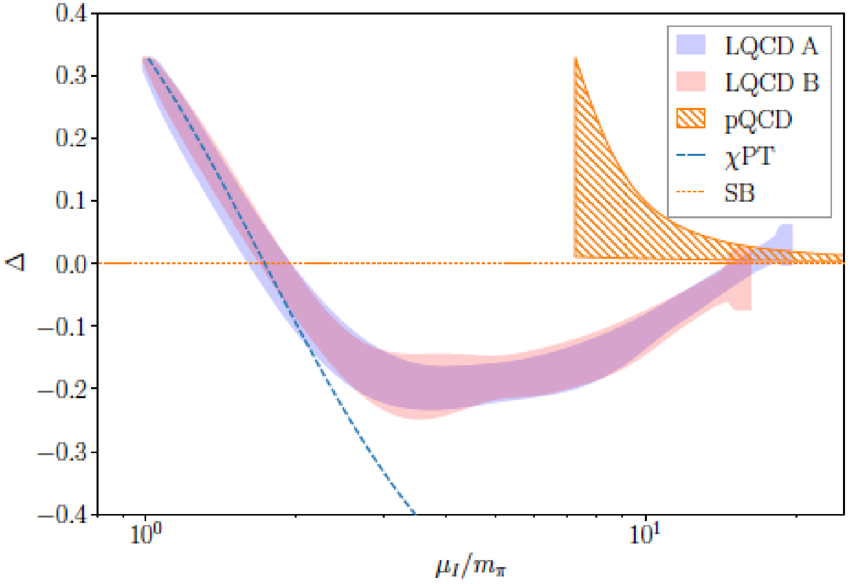
$\mu_I \sim 700\ MeV \sim 5/m_\pi$

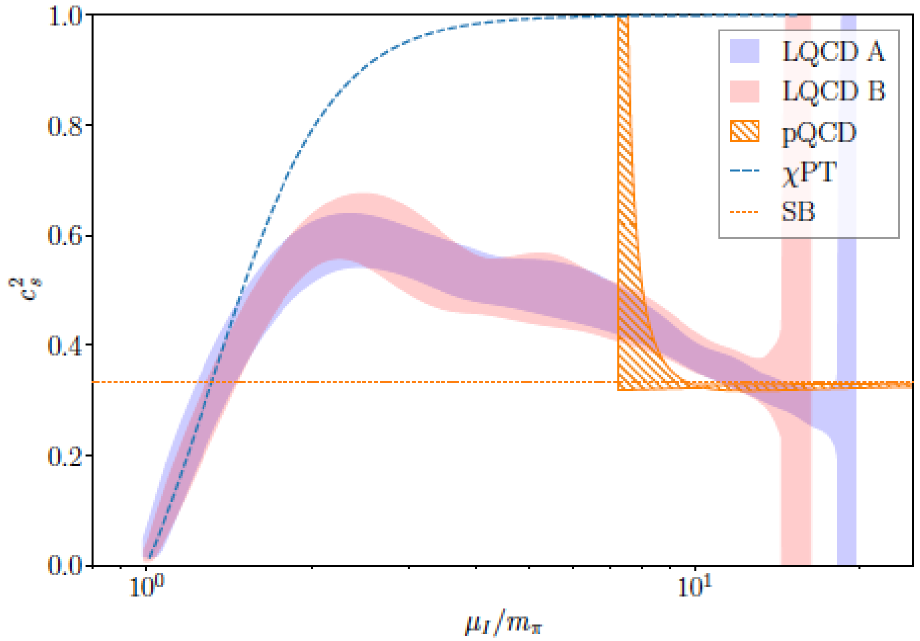
$\mu_Q \sim 350 MeV$

Quarkyonic behaviour sets in at low density









My Opinions:

Finite Temperature

Finite temperature data consistent with a region $T_{chiral} < T < T_{Hagedorn}$

We now understand from first principles computations the thermodynamics below and near T_{chiral} and above $T_{Hagedorn}$

Lattice data shows onset of quark like fluid behaviour for a temperature about midway between the upper and lower limit

In the intermediate region, long range correlations look confined

Not yet consensus on physics of this region

Finite Density

Data for $N_c = 2$ in 2 and 4 d at finite baryon number density, and for $N_c = 3$ in 4 d at finite isospin

VERY EXCITING

Results show transition at low density to behaviour close to a weakly interacting gas of quarks

For $N_c = 2$, correlations at large distance are confined

Quarkyonic Matter