Demystified Mysteries in the QCD Phase Diagram Remystified

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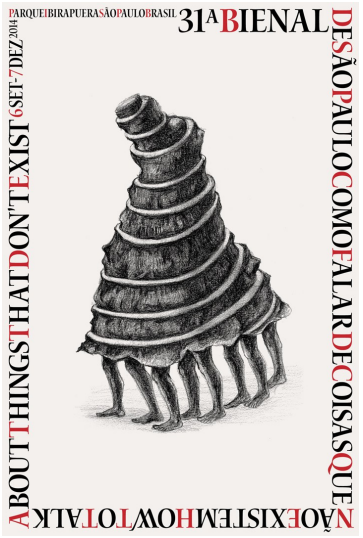
— Buenas Ideas on the QCD Phase Diagram —

Folklores

We can define a *quark* confinement-deconfinement phase transition only in a gluonic theory that has *no quarks* in the dynamics.

This is confinement of something that does not exist in the theory…



(Biennial Foundation)

Confinement is theoretically "defined" in a way which is not the same as experimentally realized.

Folklores

Truly Confining Supersymmetric Gauge Theories

2505.11354

Riku Ishikawa,[a] Hitoshi Murayama,[a,b,c,1] Shota Saito[a]

proposing the *t*-confinement not screening all center elements.

(A) Pure Yang–Mills theories

(B) $SO(k)$ gauge theories with $k-4$ flavors in the vector representation

(C) $SO(k)$ gauge theories with $k-3$ flavors in the vector representation

(D) $SU(6)$ gauge theory with one field in the rank-three anti-symmetric tensor representation

(E) $Sp(k)$ gauge theories with one field in the rank-two anti-symmetric tensor representation

(F) $SU(2k)$ gauge theories with one field in the rank-two anti-symmetric tensor representation and another in its conjugate representation

(G) $SO(12)$ gauge theories with two flavors in the spinor representation

Where is QCD?

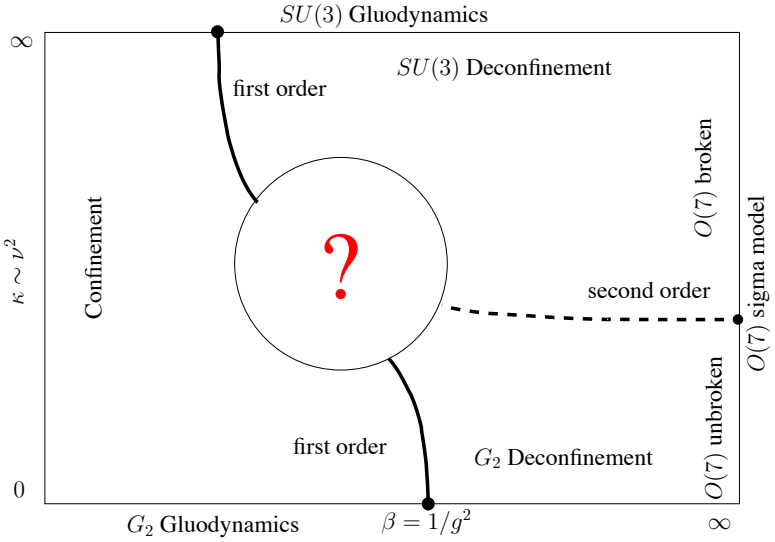
Is center so crucial for confinement?

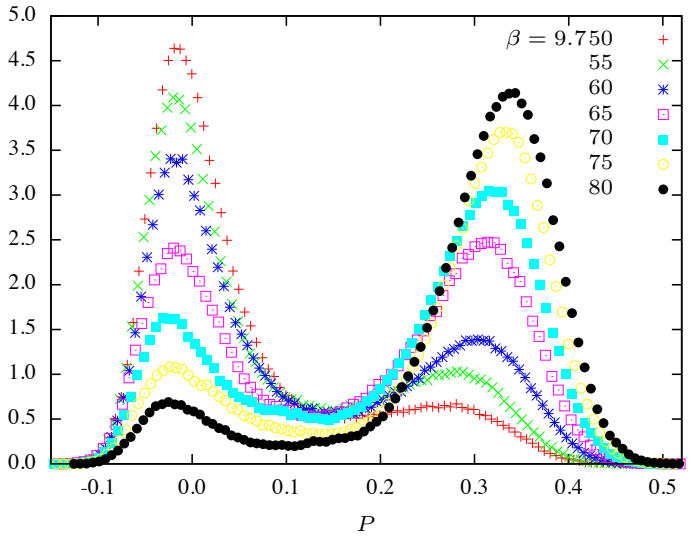
Folklores

Phase diagram of the lattice G_2 Higgs Model

Björn H. Wellegehausen, Andreas Wipf, and Christian Wozar*

1102.1900





Our deconfinement (with center) is a partial deconfinement of a larger theory (G_2 has only trivial center) ?

Folklores

Can we distinguish confinement-deconfinement with dynamical quarks?

Center symmetry explicitly broken… but it is always possible that symmetry is restored…!?

Canonical versus grand canonical ensemble in QCD #109

Michael Oleszczuk (MIT, LNS and Strasbourg, CRN), Janos Polonyi (MIT, LNS and Strasbourg, CRN) (Sep 25, 1992)

pdf cite claim reference search 1 citation

Theory can equivalently be reformulated with center symmetry restored.
The problem is… the idea fails in the thermodynamic limit.

Cited in my review article

Folklores

Some (conservative) people say:

Thus, QCD has only one phase transition (chiral), but deconfinement is smooth and irrelevant…

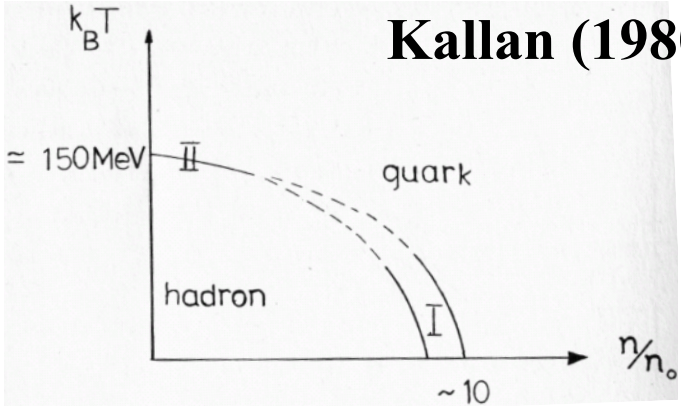
QCD has *only one* scale $\Lambda_{\rm QCD}$, so there should be *only one* T_c for whatever QCD phase transition.

Really???

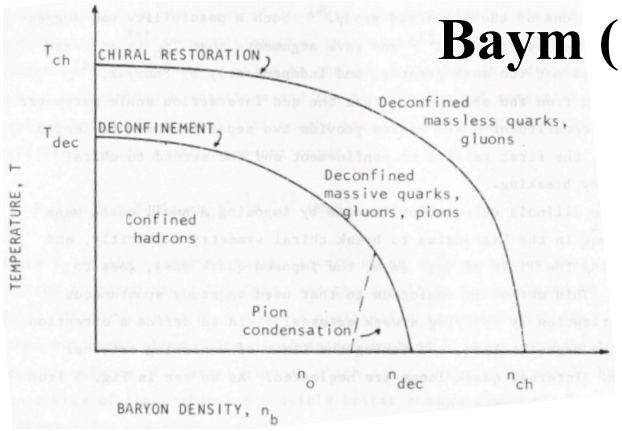
Folklores

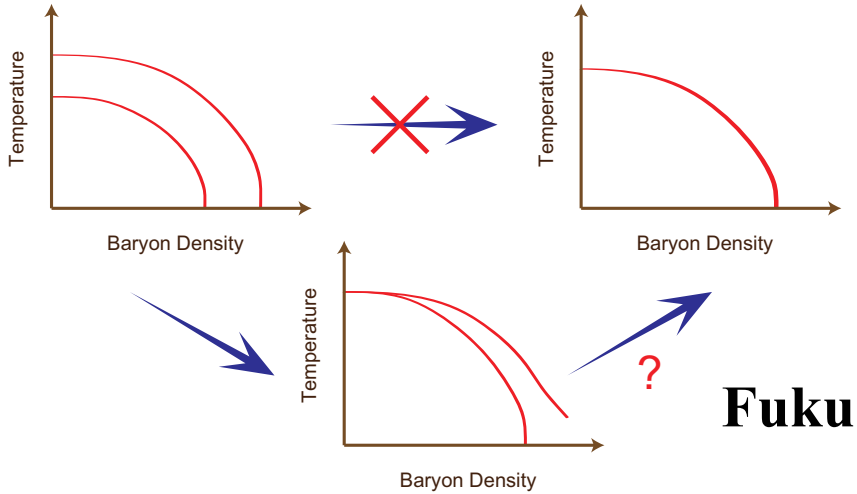
Quark Matter Proceedings

Kallan (1980)



Baym (1982)



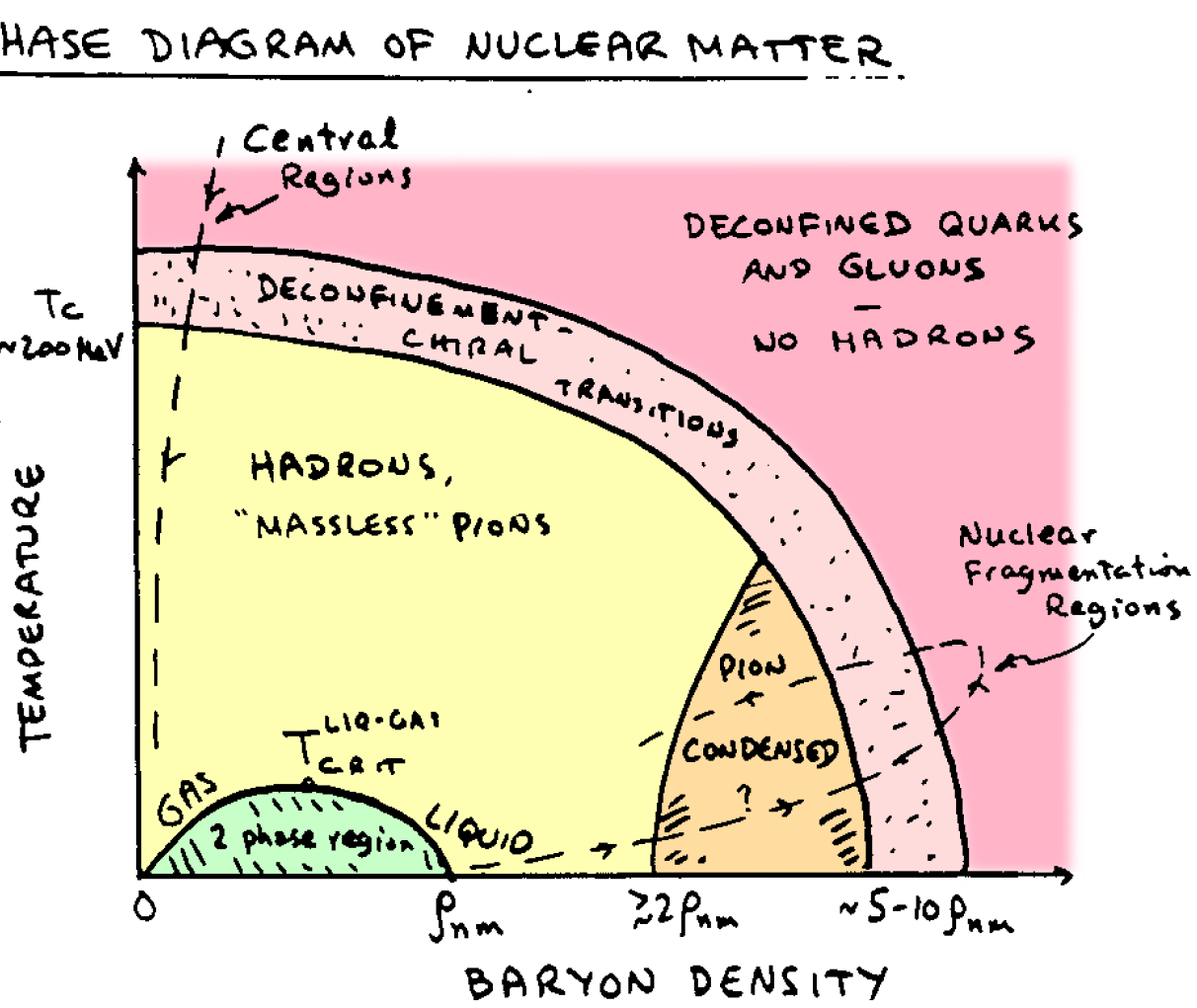


How to get *different* scales dynamically?

Fukushima (2008)

Folklores converging →

Baym (1986)



Refinements:

1st-order Phase Transition (QCD CP)

Color-super Conductors

Quarkyonic

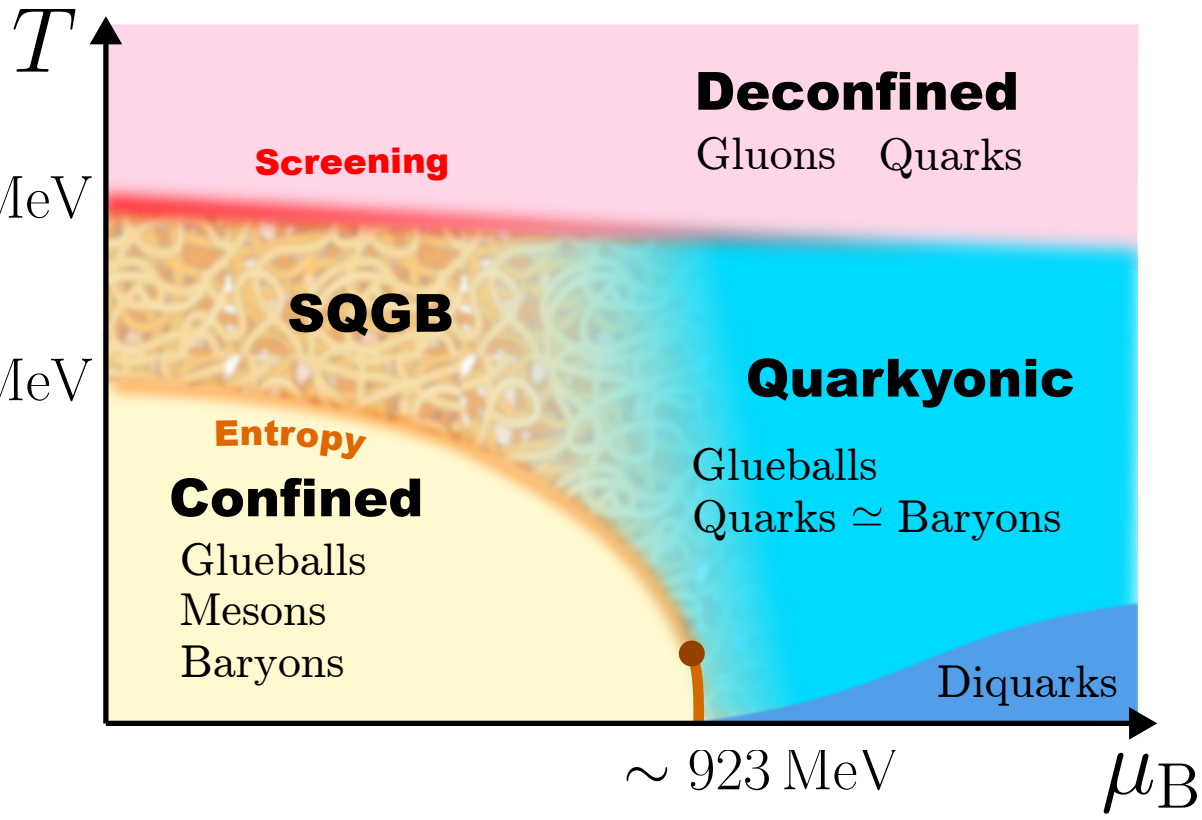
t-QCD Phase Diagram

Spaghetti and meatballs

sQGP = strongly-correlated QGP

SQGB = Spaghetti of Quarks with Glueballs

Fujimoto-Fukushima-Hidaka-McLerran (2025)



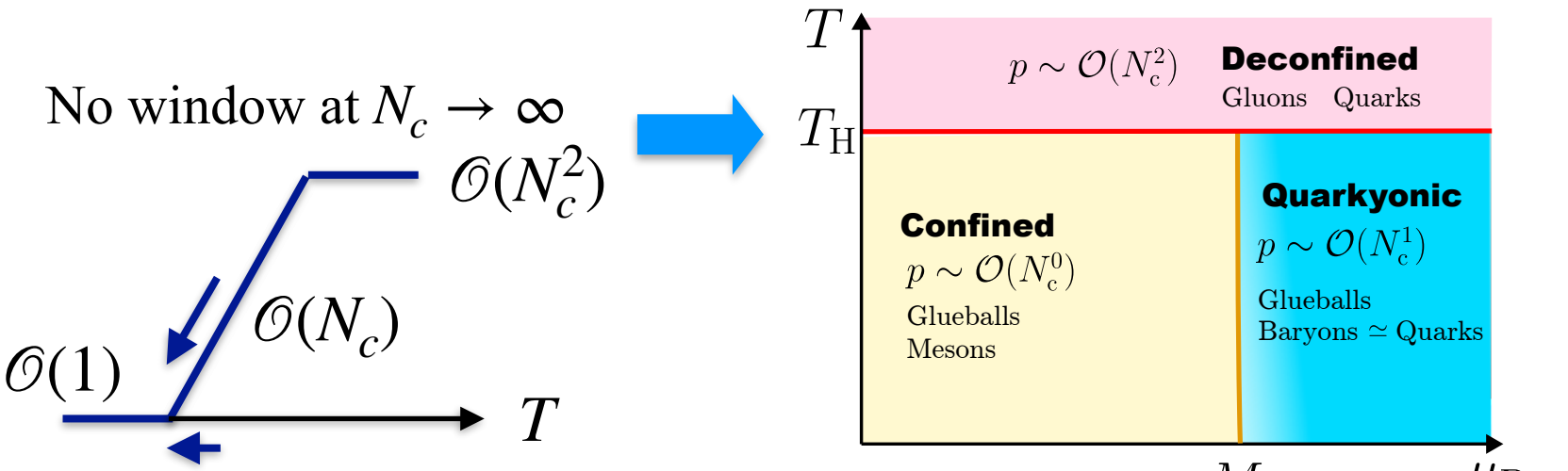
Inspiration came from

Large N_c QCD phase diagram at $\mu_B = 0$.

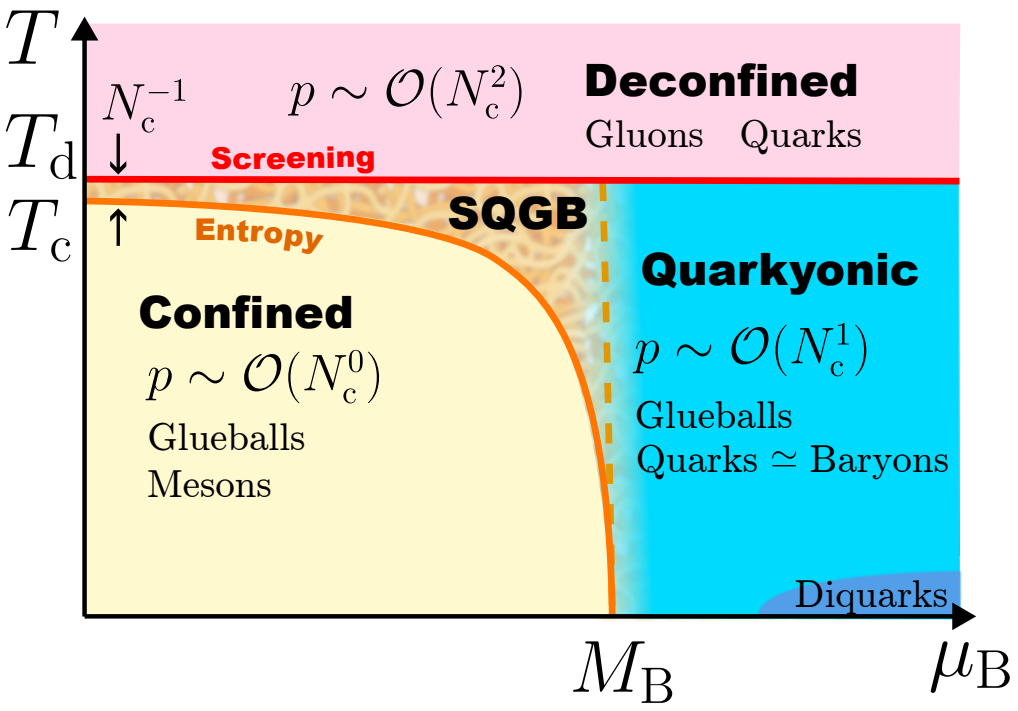
T. D. Cohen[1] and L. Ya. Glozman[2]

2311.07333

quark-gluon plasma which is often considered to reflect "deconfinement" in some way. This paper explores the behavior of these regimes of QCD as the number of colors in the theory, N_c, gets large. In the large N_c limit the theory is center-symmetric and notions of confinement and deconfinement are unambiguous. The energy density is $\mathcal{O}(N_c^0)$ in the meson gas, $\mathcal{O}(N_c^1)$ in the intermediate regime and $\mathcal{O}(N_c^2)$ in the quark-gluon plasma regime. In the large N_c limit these regimes may become distinct phases separated by first order phase transitions. The intermediate phase has the peculiar feature that glueballs should exist



Spaghetti Window



Gluons are physically deconfined when they are Debye screened.

The Debye mass has weak μ_B dep. because quarks are suppressed.

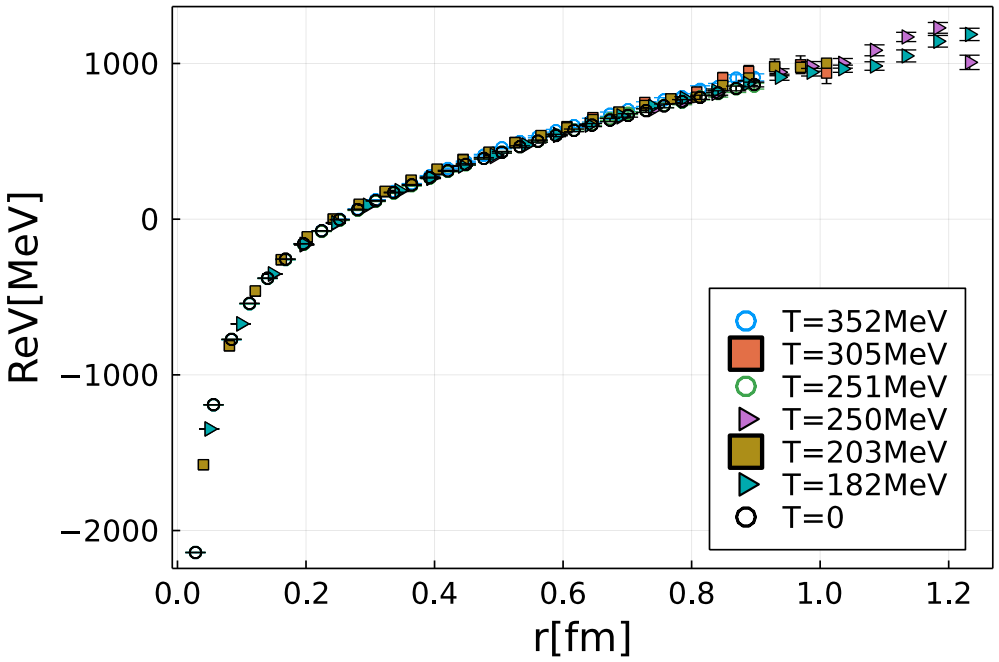
We can talk about the thermal d.o.f. by looking at the entropy behavior — hadron resonance gas!

Spaghetti Window

arXiv:2308.16587

Un-screened forces in Quark-Gluon Plasma ?

Alexei Bazavov,[1] Daniel Hoying,[2] Rasmus N. Larsen,[3] Swagato Mukherjee,[4]
Peter Petreczky,[4] Alexander Rothkopf,[3] and Johannes Heinrich Weber[5]



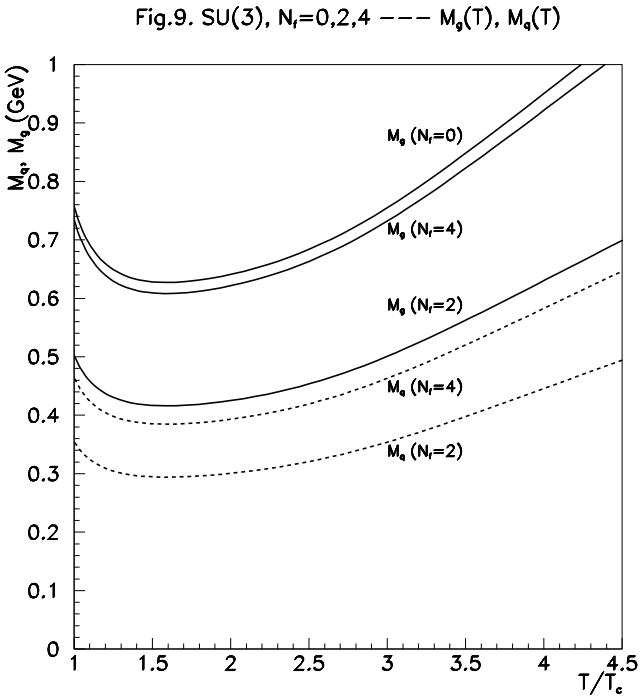
Quarks are only diffused but still confined even above T_c, while quark thermal d.o.f. can be liberated there…

Spaghetti Window

Gluons are really heavy and suppressed above T_c ?
… Yes, they are!

hep-ph/9710463

Massive gluons and quarks and the equation of state obtained from SU(3) lattice QCD

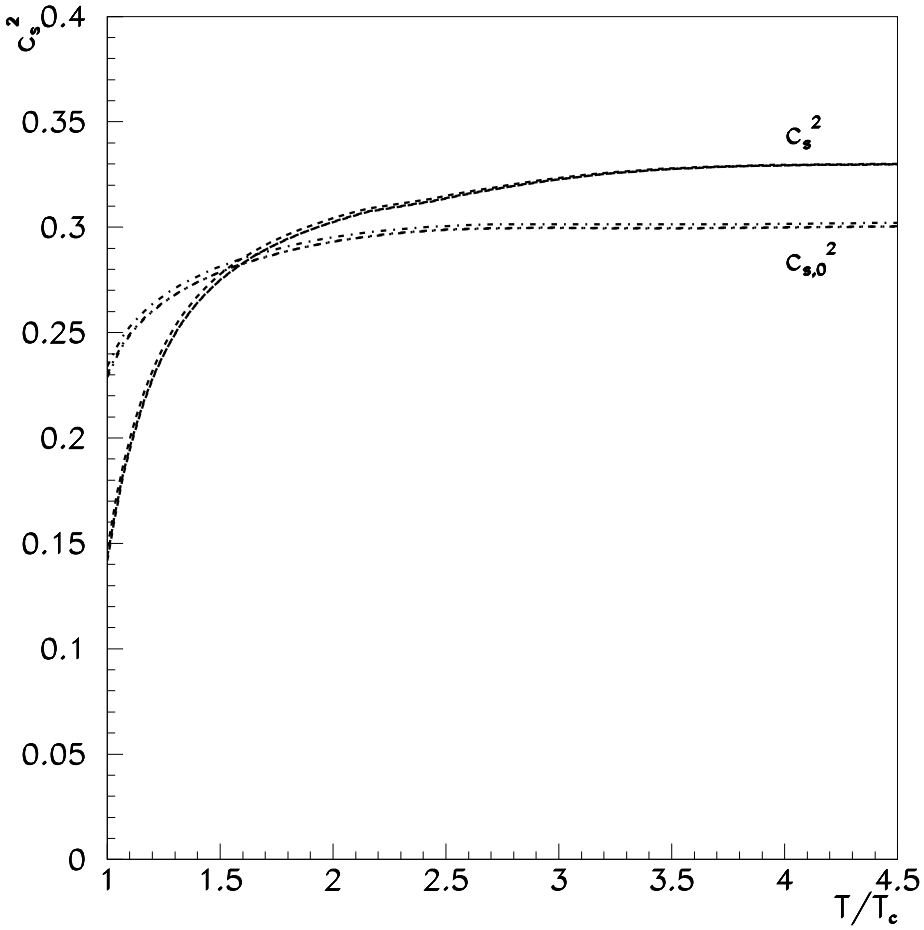


Péter Lévai[1,2] and Ulrich Heinz[2]

Gluons are massive (700 ~ 800 MeV) heavier than **quarks** (300 ~ 400 MeV) even above $\sim 3T_c$.

Spaghetti Window

Fig.8. SU(3), N_f=0,2,4 −−− c_s^2, $c_{s,0}^2$



Speed of sound approaches the conformal value $1/\sqrt{3}$.

$$c_s^2 = \frac{dp}{d\varepsilon} \qquad c_{s,0}^2 = \frac{dp_0}{d\varepsilon_0}$$

Dotted line represents the results without interaction (gluon condensate).

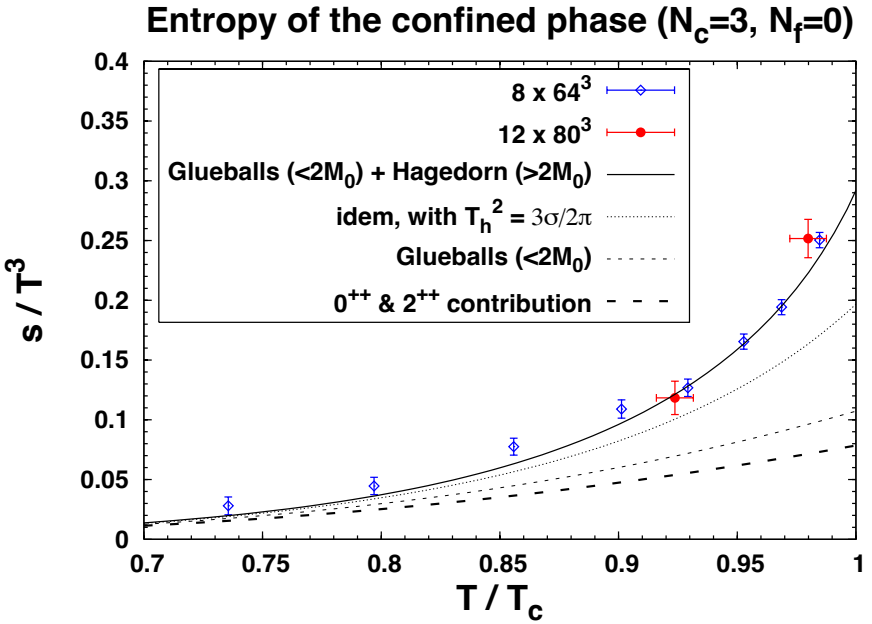
Emergent "conformality" realized by gluon condensate

How Different Scales Appear

arXiv:0905.4229

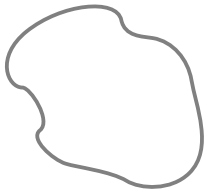
High-Precision Thermodynamics and Hagedorn Density of States

Harvey B. Meyer



$$\rho(M) = \frac{(2\pi)^3}{27\,T_h}\left(\frac{T_h}{M}\right)^4 e^{M/T_h}$$

Hagedorn spectrum of glueballs identified as the spectrum of **closed strings**.

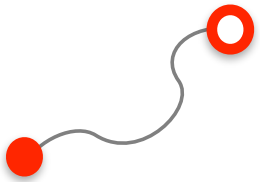


Zwiebach (2004) 558p…

Derivation given in our paper based on Green-Schwarz-Witten

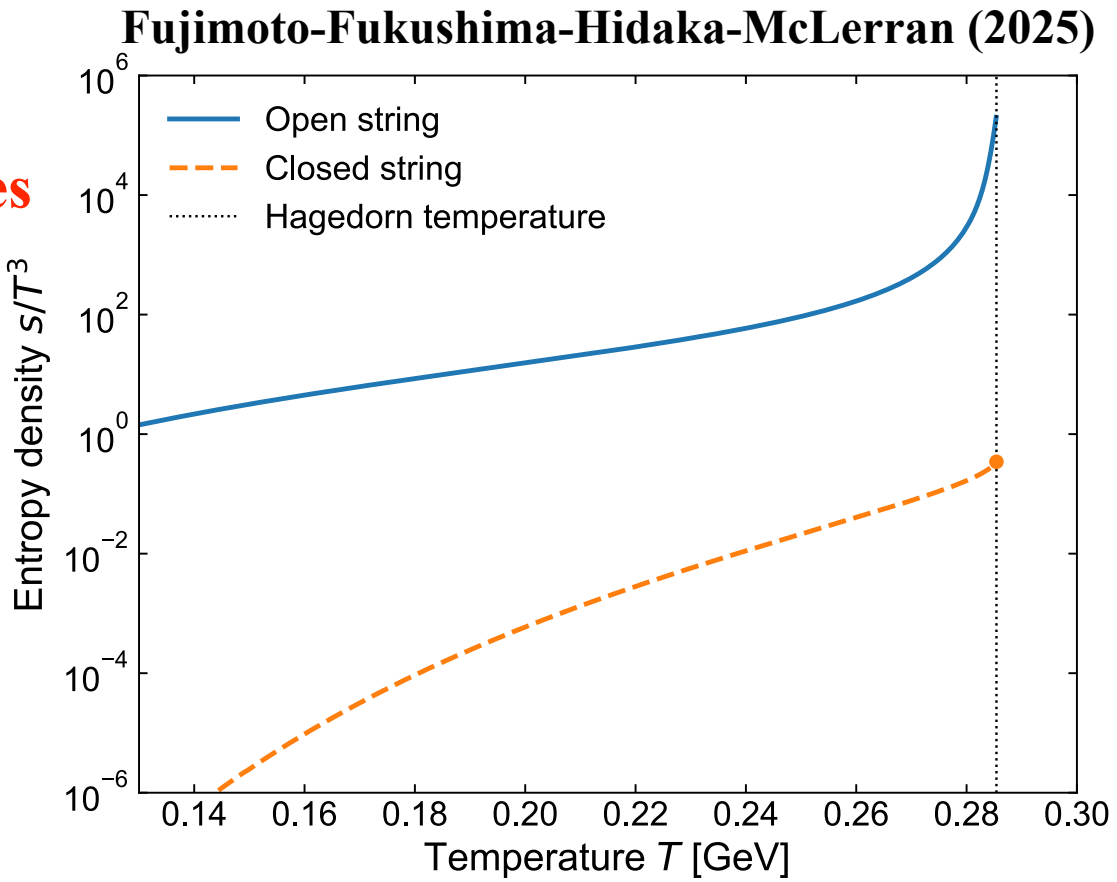
How Different Scales Appear

We can approximate the meson spectrum by **open strings**.



Hagedorn temperatures for open/closed strings are the same!

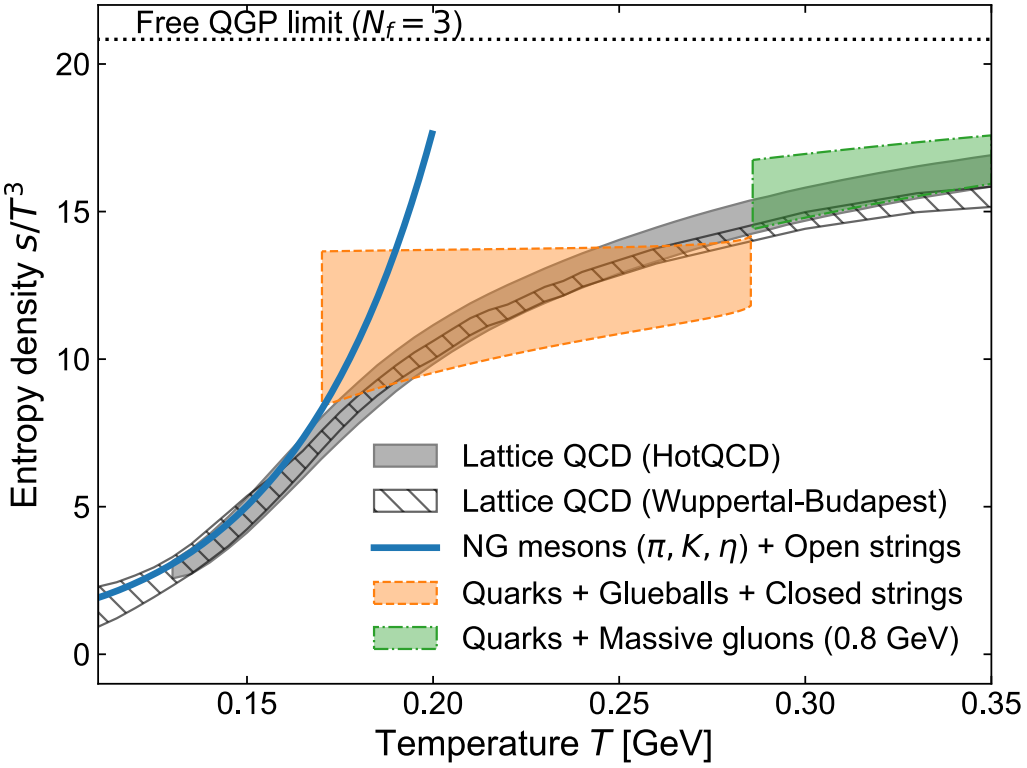
The entropy density is singular for mesons, but not glueballs…



How Different Scales Appear

Successive Crossovers with One Scale T_h

Fujimoto-Fukushima-Hidaka-McLerran (2025)

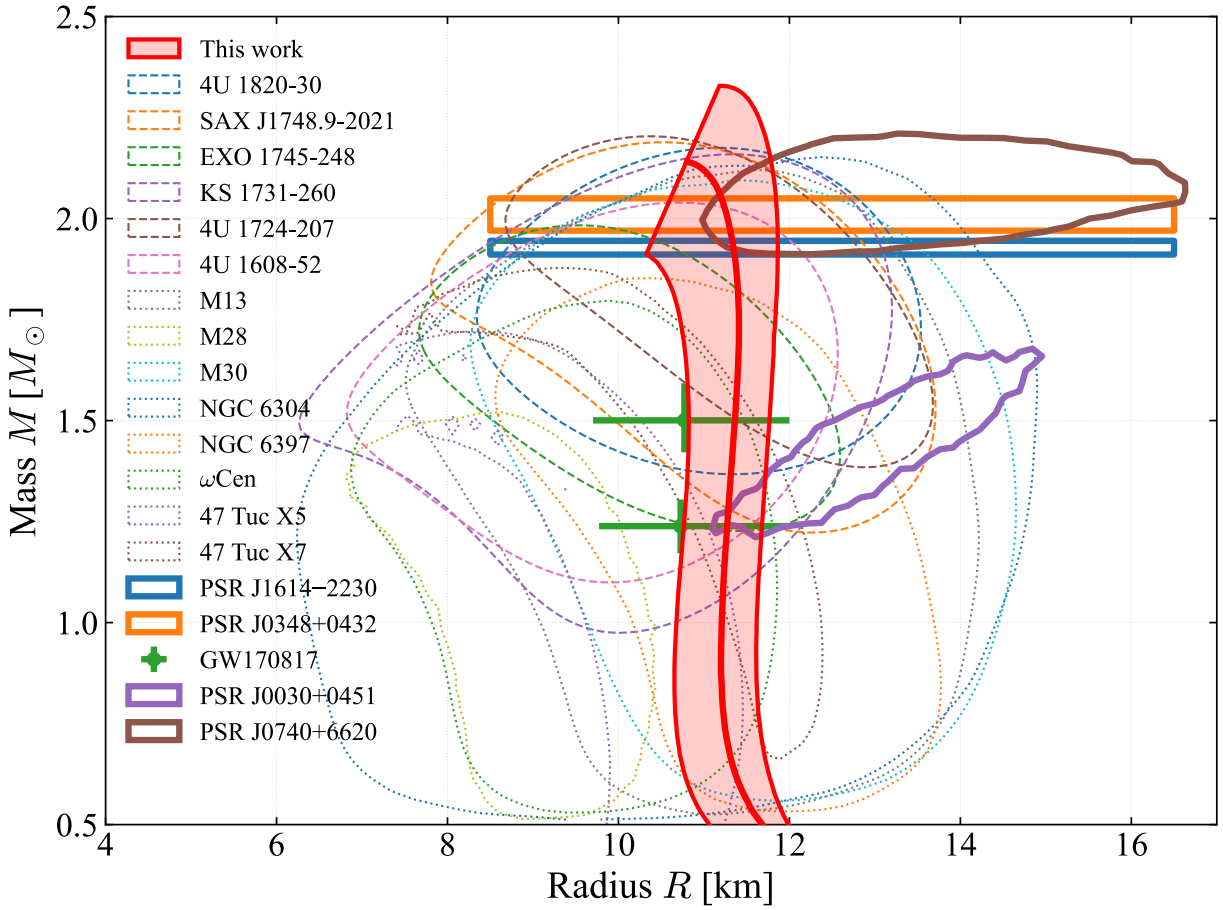


Quarks (mesons) blow up much earlier than T_h, but gluons never blow up.

Not the exponential rise but the power-law prefactor is important!

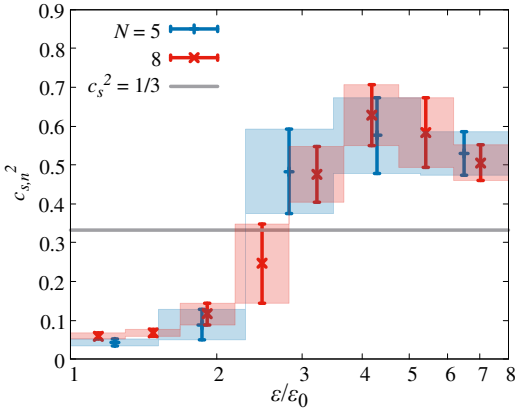
Messages from Another World

Fujimoto-Fukushima-Kamata-Murase (2018-2024)

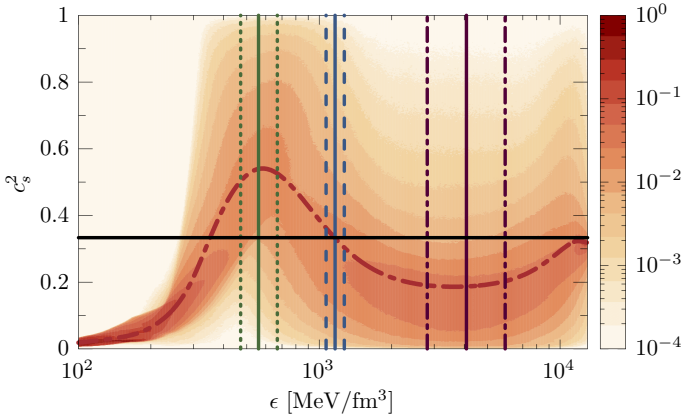


Messages from Another World

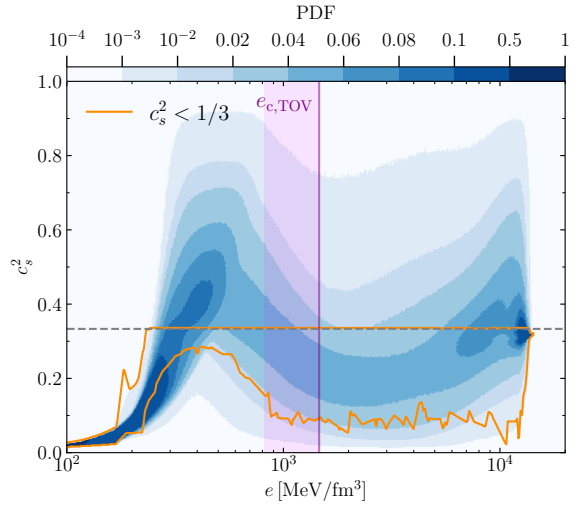
Fujimoto-Fukushima-Kamata-Murase (2018-2024)



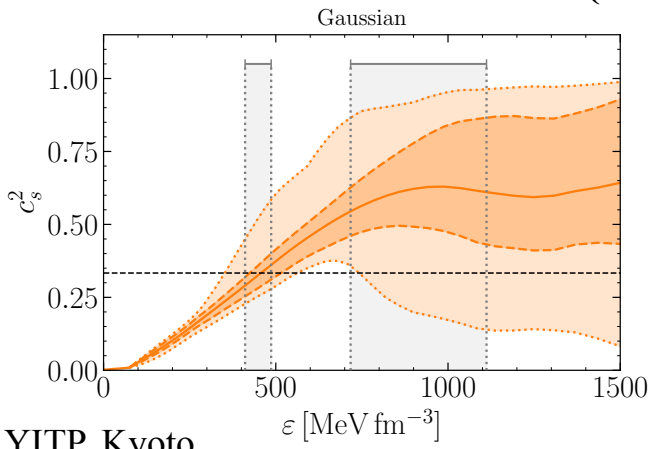
Marczenko-McLerran-Redlich-Sasaki (2022)



Altiparmak-Ecker-Rezzolla (2022)



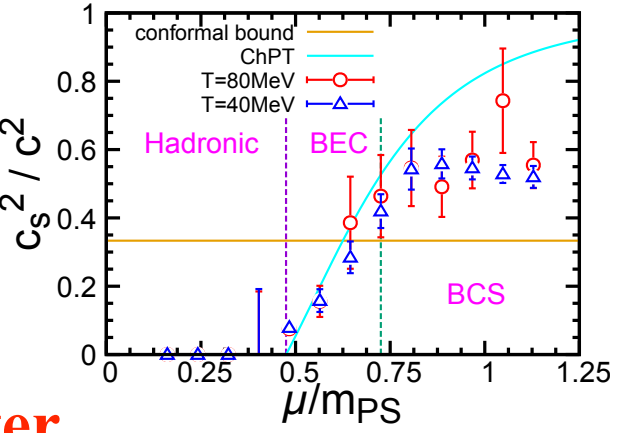
Brandes-Weise-Kaiser (2022)



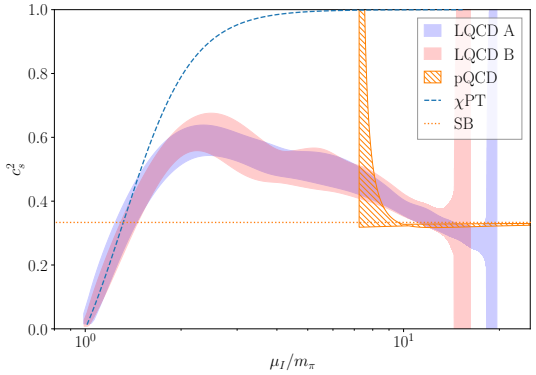
Messages from Another World

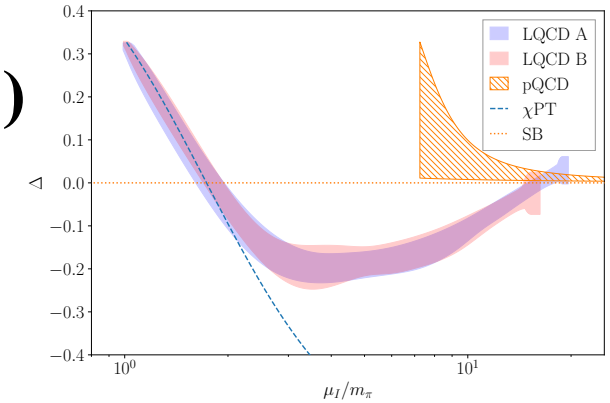
Lattice results for QCD-like theories

* **Diquark superfluid in QC_2D**
  To be compared with
  Lattice: Itou+ (2022–)



* **Pion-condensed high-isospin matter**
  To be compared with
  Lattice: Abbott+ (2023)





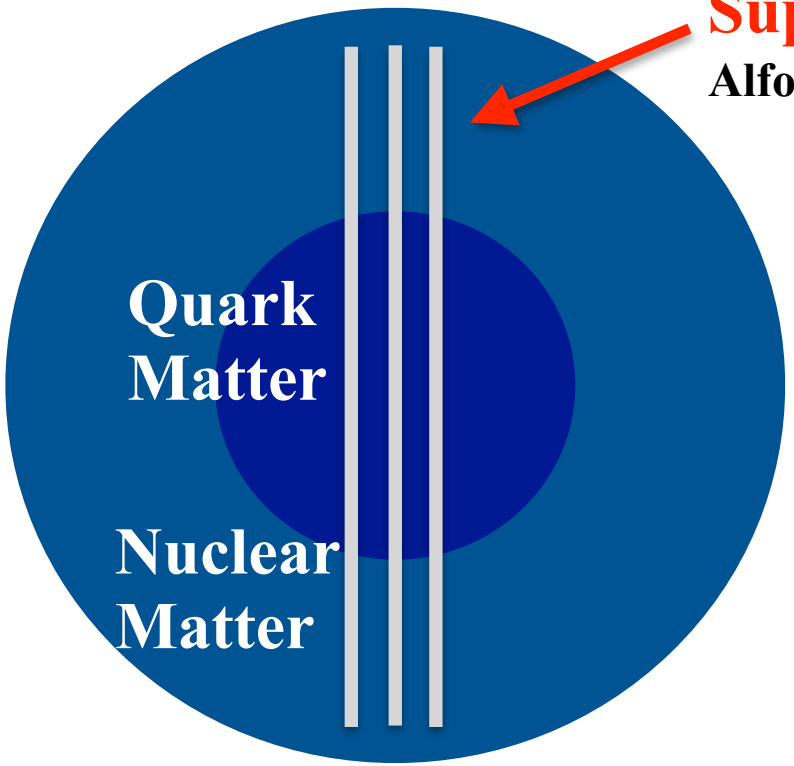
$$\Delta = \frac{1}{3} - \frac{p}{\varepsilon}$$

conformality measure first introduced by

Gavai-Gupta-Mukherjee (2004)
Fujimoto-Fukushima-McLerran-Praszalowicz (2022)

Messages from Another World

NS data imply no phase transition… however!



Superfluid Vortices

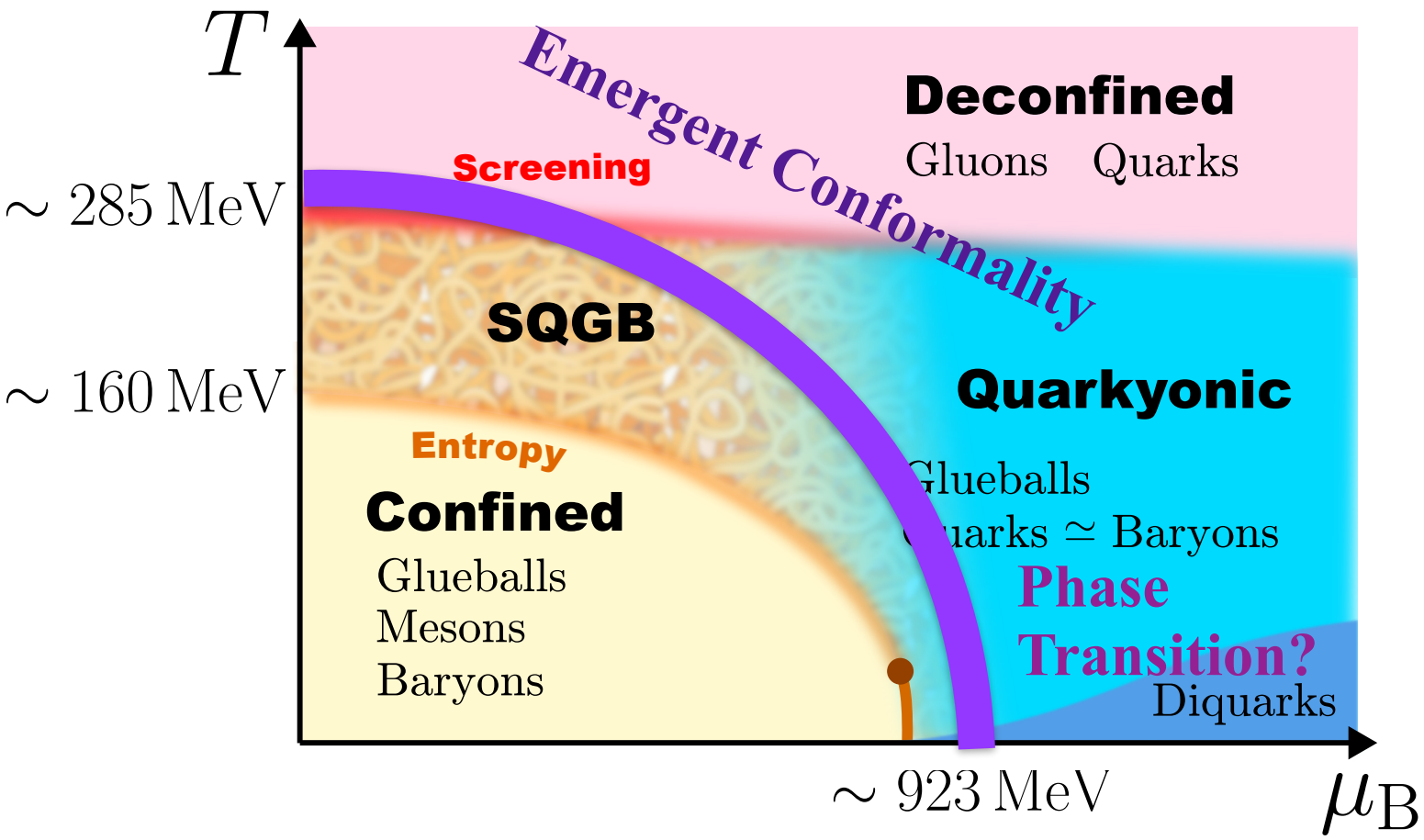
Alford-Baym-Fukushima-Hatsuda-Tachibana (2018)

In nuclear matter, no magnetic flux inside the vortices, while in quark matter, a partial magnetic flux due to electro-color mixing.

The magnetic flux (probed by Wilson loop) can distinguish confined and deconfined states of matter!

Chatterjee-Nitta-Yasui (2018) **Cherman-Sen-Yaffe (2018)**

Phase Diagram More Enriched?



Driven by the gluon condensate (bag pressure)? **cf. Levai-Heinz**

Summary

- **QCD Phase Diagram Research:** **Long and confusing history… so many interesting ideas, some faded out, and some still survive.**
- **Confinement Remystified:** **No way to diagnose gluon confinement (adjoint Polyakov loop may be an approximate order parameter).**
- **Emergent Conformality:** **Neutron stars have nearly conformal matter, possibly driven by gluon condensate.**
- **More Mysteries:** **To be discussed this week!**