Analysis of Symmetry of 2-point Correlation Functions in Finite Temperature QCD

Using Möbius Domain-Wall Fermions

Junxiong Nie, U. Osaka
for JLQCD Collaboration

Outline

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- Theoretical Background
- Lattice Simulation Setup
- Numerical Results
- Summary and Outlook

Research Background

- According to [Pisarski & Wilczek (1983)], the restoration of $U(1)_A$ axial symmetry is important to determine the order of the chiral phase transition (and universality if it is 2nd-order)
- Our previous work in the 2-flavor case by JLQCD [Phys. Rev. D **111**, 114506 (2025)] using the Möbius Domain-Wall fermions is consistent with the scenario that both $SU(2)_L \times SU(2)_R$ chiral symmetry and $U(1)_A$ symmetry are effectively restored near $T_c \approx 165(3)$ MeV.
- We extend this investigation to the more realistic $N_f = 2 + 1$ QCD adding strange quark, to determine if $U(1)_A$ restoration persists near the physical point.
- We examine the symmetry breaking by mesons screening masses

Symmetry of QCD

QCD: $\mathcal{Z} = \int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}qe^{-S}$, Gauge Boson+Fermion

$S = \int d^4x \left[-\frac{1}{4}\mathrm{Tr}F \wedge \star F + \bar{q}_a(i\not{D} - m_a)q_a\right]$, a in $[1, N_f]$

Specifically,in the up-down quark mass zero limit:
Flavored chiral symmetry $SU(2)_L \times SU(2)_R$:

$$q \to \exp(i\alpha_a T^a + i\beta_a T^a\gamma_5)q, \quad \bar{q} \to \bar{q}\exp(-i\alpha_a T^a + i\beta_a T^a\gamma_5),$$

$U(1)_A$ Symmetry :

$$q \to \exp(i\epsilon\gamma_5)q, \quad \bar{q} \to \bar{q}\exp(i\epsilon\gamma_5),$$

We use the mass difference of the screening masses of different channels to estimate the degree of the symmetry breaking.

Meson Correlation Functions

We consider the following set of meson operators

$$O_\Gamma(x) = \bar{q}(x)\Gamma F^a q(x),$$

$$\Gamma \in \left\{ \mathbf{1}(S), \gamma_5(P), \gamma_\mu(V), \gamma_\mu\gamma_5(A), \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu](T), \sigma_{\mu\nu}\gamma_5(X) \right\}$$

And we measure the two-points correlation function of the meson operators of different channels. Accroding to the symmetry properties, the asymptotic behavior of the 2-point correlation function at large distances should be approximately be $a\mathrm{Cosh}(\mathrm{m}_\Gamma(\mathrm{x} - \mathrm{L}/2))$

Symmetry of Correlation Functions

- Flavored chiral symmetry $SU(2)_L \times SU(2)_R$: $\delta_C O_\Gamma^a = \frac{i}{2}\beta_b \bar{q}\left(\{\Gamma,\gamma_5\}\frac{1}{2}\delta^{ab}\mathbf{1} + [\Gamma,\gamma_5] i\epsilon^{bac}\frac{\tau^c}{2}\right)q$
- $U(1)_A$ Symmetry : $\delta_A O_\Gamma^a = i\epsilon\bar{q}\{\Gamma,\gamma_5\}\frac{\tau^a}{2}q$

Γ	$[\Gamma,\gamma_5]$	$\{\Gamma,\gamma_5\}$
$\mathbf{1}(S)$		γ_5
$\gamma_5(P)$		$\mathbf{1}$
$\gamma_\mu(V)$	$\gamma_\mu\gamma_5$	
$\gamma_\mu\gamma_5(A)$	γ_μ	
$\sigma_{\mu\nu}(T)$		$\sigma_{\mu\nu}\gamma_5$
$\sigma_{\mu\nu}\gamma_5(X)$		$\sigma_{\mu\nu}$

Γ	Reference Name	Abbr.	Symmetry Correspondences	
$\mathbb{I}$	Scalar	S	$U(1)_A$	
γ_5	Pseudo Scalar	PS		
γ_1, γ_2	Vector	V	$SU(2)_L \times SU(2)_R$	$SU(2)_{CS}$
$\gamma_1\gamma_5, \gamma_2\gamma_5$	Axial Vector	A		
$\gamma_4\gamma_3$	Tensor	T_t	$U(1)_A$	
$\gamma_4\gamma_3\gamma_5$	Axial Tensor	X_t		

Order of Chiral Phase Transition

- Perturbative $4-\epsilon$ expansion Renormalization Group Analysis of effective linear sigma model for $\langle \bar{q} P_L q \rangle$. They found for $N_f > \sqrt{3}$ flavor symmetry, there's no stable IR fixed point. [Pisarski-Wilczek (1983)]
- The anomaly effect term is a determinant term in the effective linear sigma model, which is a N_f-th order term. For $N_f \geq 3$ flavor case, it doesn't change the order of the transition.
- But for $N_f = 2$ flavor case, it becomes a quadratic term, which can smooth out the singularity and change the transition to second order. If the anomaly effect still exists, the transition can be O(4) class. If the anomaly effect is suppressed and $\mathrm{U}(1)_{\mathrm{A}}$ symmetry is restored before or at the critical temperature T_c, then transition should be still first order
- But recently higher-loop computations and FRG computations suggested different possibilities [E.Vicari ,F.Basile, A.Pelissetto 2005 , G. Fejos 2022], and Conformal Boostrap results indicate some interesting points too[Nakayama Ohtsuki 2016

Possible Emergent Larger Symmetry

- Perturbative analysis of screening masses from NRQCD$_3$ [Laine, M., Vepsäläinen, M. (2004)]: $\left[2M - \frac{\nabla_r^2}{\pi T} + V(\boldsymbol{r})\right]\Psi_0 = m_{\rm scr}\Psi_0$, where M is the mass parameter in model
- $M = \pi T + g^2 T \frac{C_F}{8\pi} + \mathcal{O}(g^4 T)$
- We write $(m_{src} - 2M)$ as $g_{\rm E}^2 \frac{C_F}{2\pi}\hat{E}_0$, where $\hat{E}_0$(around 0.38237 for 2-flavor) is computed numerically
- For two falvor case, we got $m_{\rm scr} = 2\pi T + g_{\rm E}^2 \frac{C_F}{2\pi}(1/2 + \hat{E}_0) = 2\pi T + 0.1872 g^2 T$
- It predicts the flavor non-singlet channel will tend to behave same screening mass. Thus,a larger symmetry, Chiral-Spin Symmetry $SU(2)_{CS}$, may emerge

$$
\begin{aligned}
q &\to \exp(i\Sigma^i \epsilon_i) q, \\
\bar{q} &\to \bar{q}\gamma_0 \exp(-i\Sigma^i \epsilon_i)\gamma_0,
\end{aligned}
$$

where Σ^i has three components,

$$\Sigma^i = (\gamma_k, -i\gamma_5\gamma_k, \gamma_5), \quad k = 1, 2.$$

Lattice QCD Numerical Setup

- Möbius Domain-Wall Fermions: The effective 4D operator is[Brower, R. C., Neff, H., & Orginos, K. (2012)]:

$$D^{\rm 4D}_{\rm DW}(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \tanh(L_s \tanh^{-1}(H_M)),$$

where m is the bare quark mass, and the Möbius kernel H_M is $H_M = \gamma_5 \frac{2D_W}{2+D_W}$

The precision of chiral symmetry could be estimated by Residual Mass m_{res}, and at low beta case is not so good:

T (MeV)	138	145	153	157	164.5	176	202	241.6
m_{res}/m_l	0.9087	0.5360	0.3394	0.2744	0.1836	0.1041	0.0299	0.0049
m_{res}	7.31e-4	4.98e-4	3.40e-4	2.80e-4	1.91e-4	1.08e-4	2.81e-5	3.75e-6

Lattice QCD Numerical Setup

The simulation is run with lattice size $48^3 \times 16$ (near the physical point):

β	T (MeV)	am_s	am
4.13	138	0.043547	0.000805
4.15	145	0.040843	0.000930
4.17	153	0.038400	0.001001
4.18	157	0.037265	0.001022
4.20	164.5	0.035150	0.001041
4.23	176	0.032315	0.001033
4.30	202	0.026930	0.000939

Lattice Measurements & Configurations

- We performed measurements for each beta parameters, each channels, and each directions
- To improve the quality of data, we performed **16-source** measurements
- The total number of configurations (N_{conf}) analyzed for each temperature/β are counted below :

β	4.13	4.15	4.17	4.18	4.20	4.23	4.30
T (MeV)	138	145	153	157	164.5	176	202
No. of Confs (N_{conf})	395	398	396	402	407	395	404
Total Meas. ($16 \times N_{\text{conf}}$)	6320	6368	6336	6432	6512	6320	6464

Meson Channels & Directional Averaging

- To improve the data quality, the correlation functions of each meson channel are averaged over all equivalent spatial directions and components.
- The directional averaging components is summarized below:

Channel	Components	Averaged Directions (configured in `config.py`)
AV (Axial Vector)	6	X (`AVector2`, `AVector3`), Y (`AVector1`, `AVector3`), Z (`AVector1`, `AVector2`)
Vec (Vector)	6	X (`Vector2`, `Vector3`), Y (`Vector1`, `Vector3`), Z (`Vector1`, `Vector2`)
S (Scalar)	3	X, Y, Z (`TVector4`)
PS (Pseudo-Scalar)	3	X, Y, Z (`TAVector4`)
Tt (Tensor)	3	X (`TVector1`), Y (`TVector2`), Z (`TVector3`)
Xt (Axial Tensor)	3	X (`TAVector1`), Y (`TAVector2`), Z (`TAVector3`)

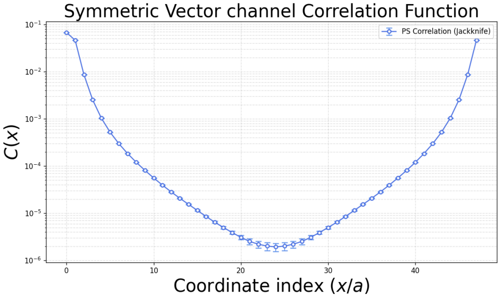
Extracting Screening Masses: Methodology

- We fit the ratio of averaged correlation functions to the analytic form:

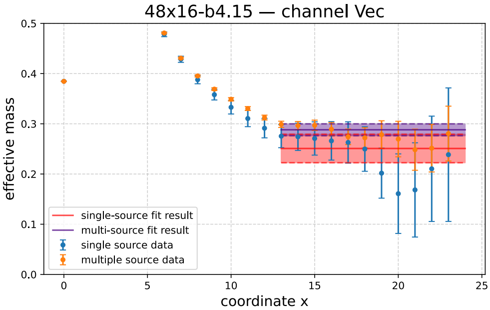
$$\frac{C(x)}{C(x+1)} = \frac{\cosh(m(x - L/2))}{\cosh(m(x + 1 - L/2))}$$

  to extract the effective mass $m(x)$ at each spatial coordinate x.
- **Jackknife Method**: Used to calculate unbiased statistical errors of the fitted masses and eliminate autocorrelation bias in HMC trajectories.
- **Binning Analysis**: Performed to ensure the Jackknife error estimate reaches saturation and accurately reflects the statistical uncertainty.
- **Screening Mass Extraction**: The physical screening mass m_{scr} is obtained by identifying the long-distance plateau of the effective mass $m(x)$.

Extracting Screening Masses: Fit Examples



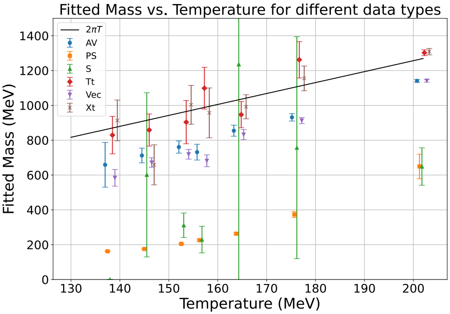
correlation data of Vec channel for
$\beta = 4.15$(T=153MeV)



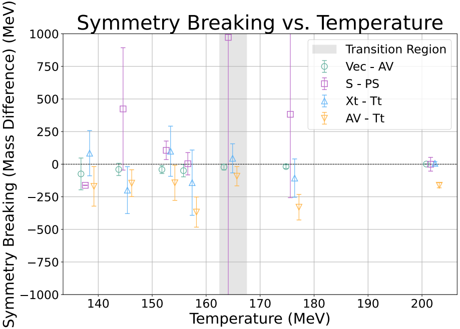
effective mass of Vec channel for
$\beta = 4.15$(T=153MeV)

Numerical Results

Multi-source analysis results(I draw also T_c =165(3) MeV range as the grey band)[Y. Aoki, D. Ward et al., Phys. Rev. D **111**, 114506 (2025)]:



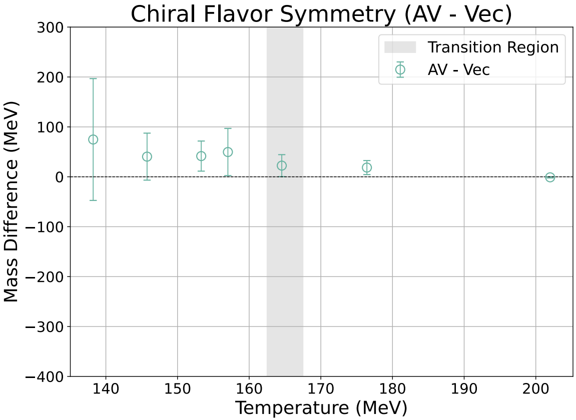
simulated effective masses of different channels



Symmetry breaking of different channels

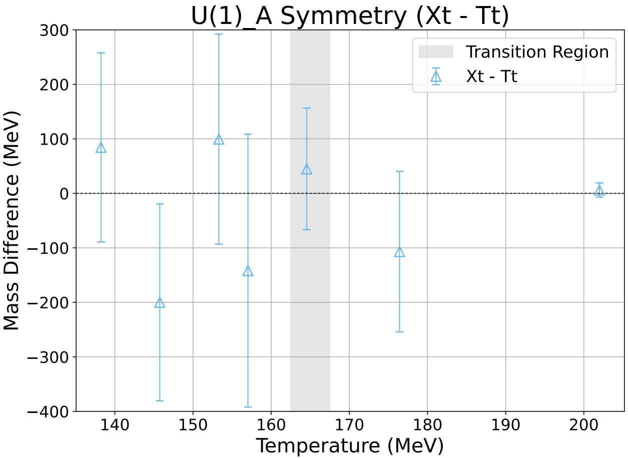
$SU(2)_L \times SU(2)_R$ Chiral Symmetry

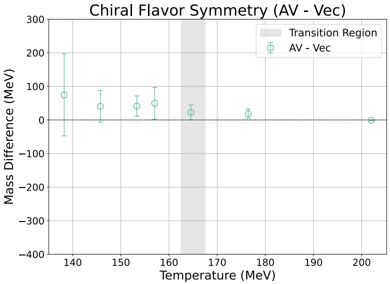
- This indicates the restoration of the $SU(2)_L \times SU(2)_R$ chiral symmetry (e.g., AV vs Vec channels).
- At $T = 0$, the difference between a_1 (1230 MeV) and ρ meson (775 MeV) is around 455 MeV.
- But the pseudo critical temperature is not very clear (due to finite quark mass)



$U(1)_A$ Symmetry

The difference tends to decrease at high temperatures, suggesting approximate restoration of $U(1)_A$ symmetry (anomaly suppression), compared to the flavour symmetry :



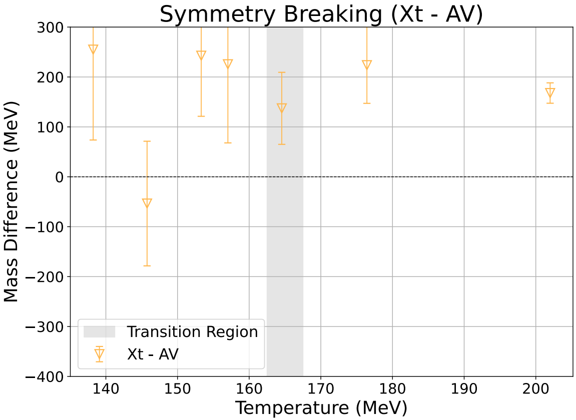


Scalar vs Pseudo-Scalar

$SU(2)_{CS}$ Symmetry

Observation: Chiral-Spin (CS) symmetry is not yet completely visible at the temperatures analyzed near the phase transition.

Requires exploration of higher temperature domains.



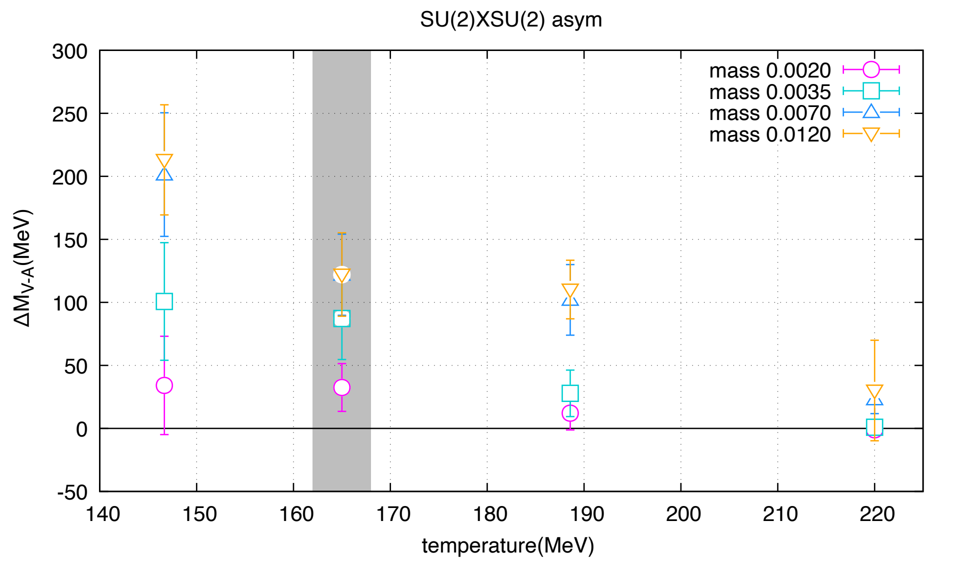
Summary and Outlook

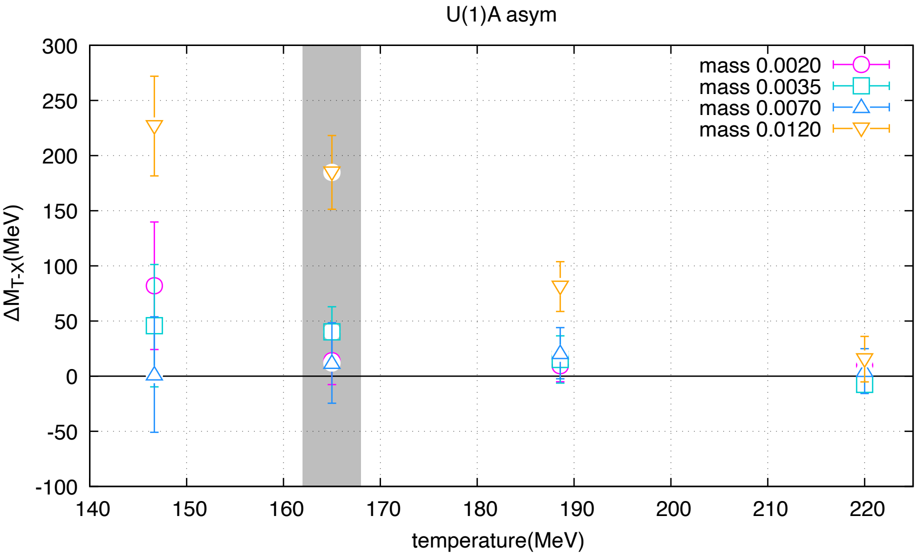
Summary of the preliminary results:

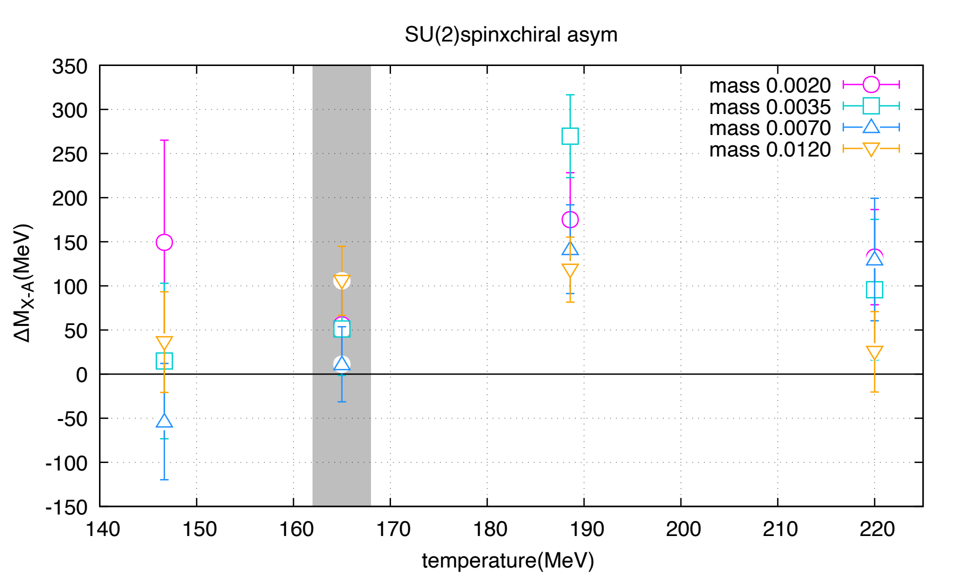
- Focusing on these data, the physical point $N_f = 2+1$ QCD does not show clear Chiral Phase Transition behavior as in the chiral limit (which is reasonable).
- For $T = 164$ MeV, the chiral flavor symmetry and $U(1)_{\mathrm{A}}$ symmetry breaking are consistent with zero (but with large errors).
- Chiral-Spin symmetry is not visible under T=202MeV.

Possible Improvement:

- The data still fluctuated very much, and we want to try to implement more sophisticated statstical method to improve the analysis
- We need to determine the critical temperature T_c by chiral susceptibility (also need to improve data quality).


SU(2)XSU(2) asym
mass 0.0020
mass 0.0035
mass 0.0070
mass 0.0120
$\Delta M_{V\text{-}A}$(MeV)
temperature(MeV)
300
250
200
150
100
50
0
-50
140
150
160
170
180
190
200
210
220


U(1)A asym
mass 0.0020
mass 0.0035
mass 0.0070
mass 0.0120
$\Delta M_{T\text{-}X}$(MeV)
temperature(MeV)
300
250
200
150
100
50
0
-50
-100
140
150
160
170
180
190
200
210
220



SU(2)spinxchiral asym
mass 0.0020
mass 0.0035
mass 0.0070
mass 0.0120
ΔM_{X-A}(MeV)
temperature(MeV)
350
300
250
200
150
100
50
0
-50
-100
-150
140
150
160
170
180
190
200
210
220