Baryon Momentum Shell from Holography

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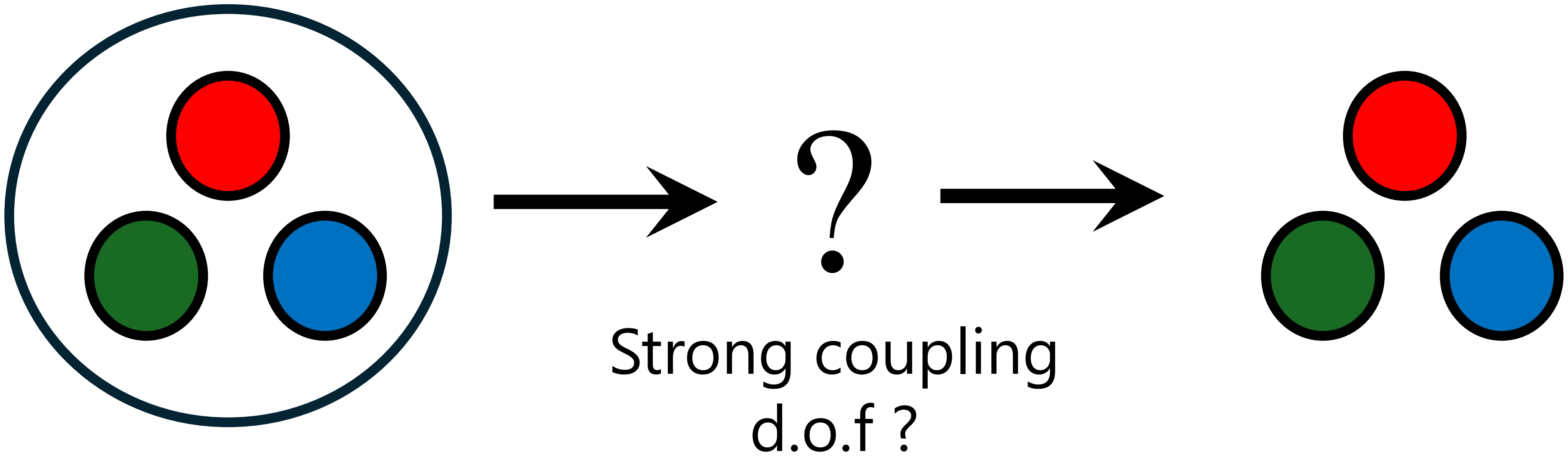
Buenas ideas, Kyoto 2026

Introduction

Motivation

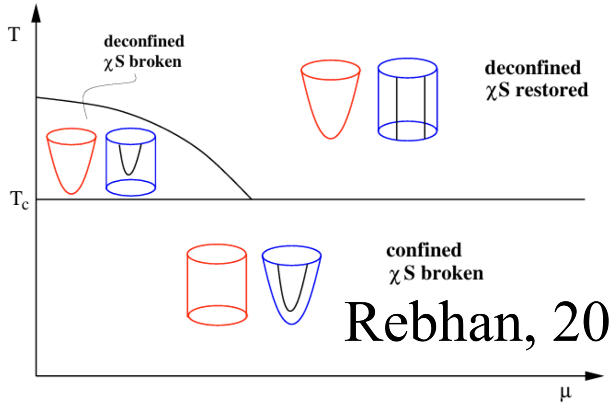


$n_B \sim n_0$

$n_B \sim 25 n_0$

Holography: Sakai-Sugimoto model

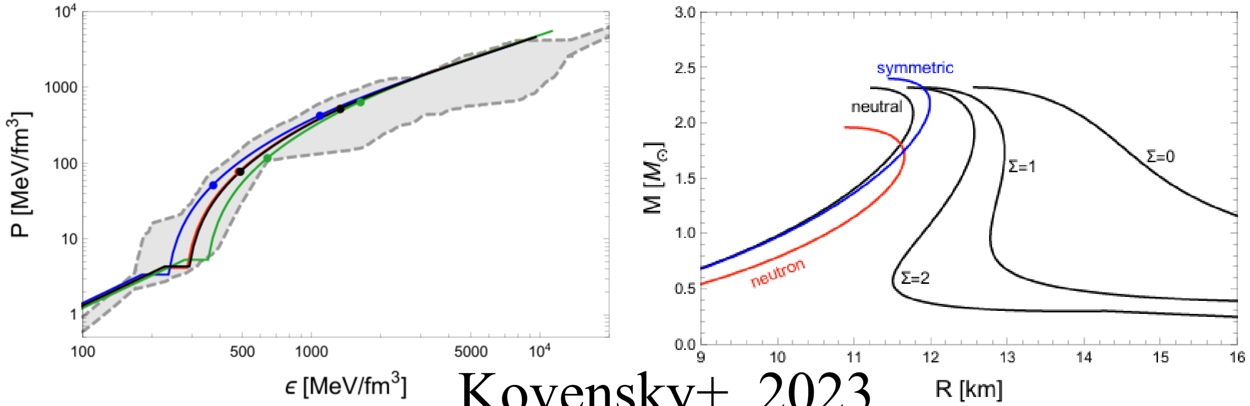
- top-down model for Large N_c QCD
- describe both confinement/deconfinement and chiral phase transitions



Rebhan, 2014

- useful description for strong coupling regime
- many applications including dense matter

  phase diagram, EoS, transport, etc.



Kovensky+, 2023

In-Medium Baryon Momentum Distribution

(μBMD = μ_B + BMD)

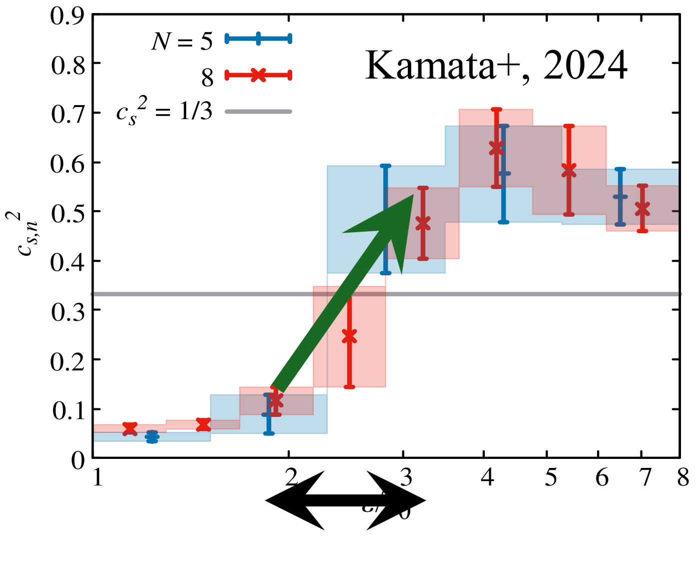
$$n(|\boldsymbol{p}|) = \int d\omega \, \mathrm{Tr_D}\left[\rho_{sp}(\omega, \boldsymbol{p})\right]$$

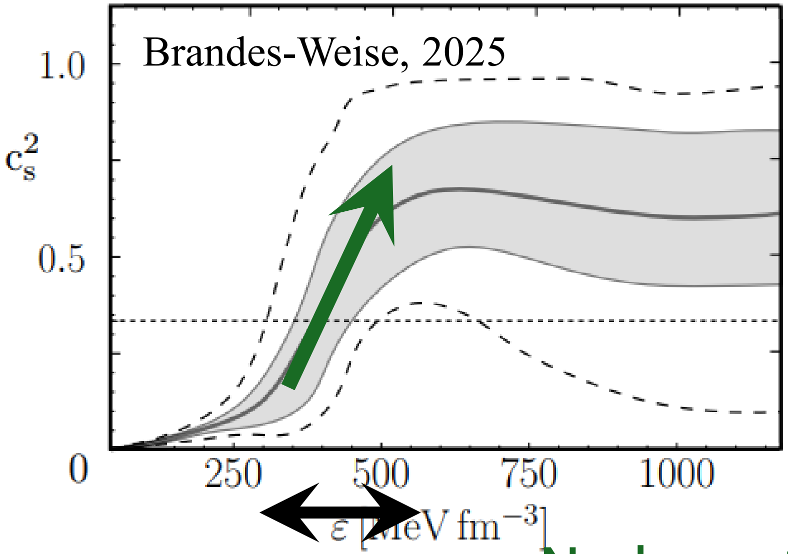
$$\rho_{sp}(\omega, \boldsymbol{p}) \sim \mathrm{Im}\langle BB^{\dagger}\rangle$$

baryon Field

thermal expectation value

Neutron star observation



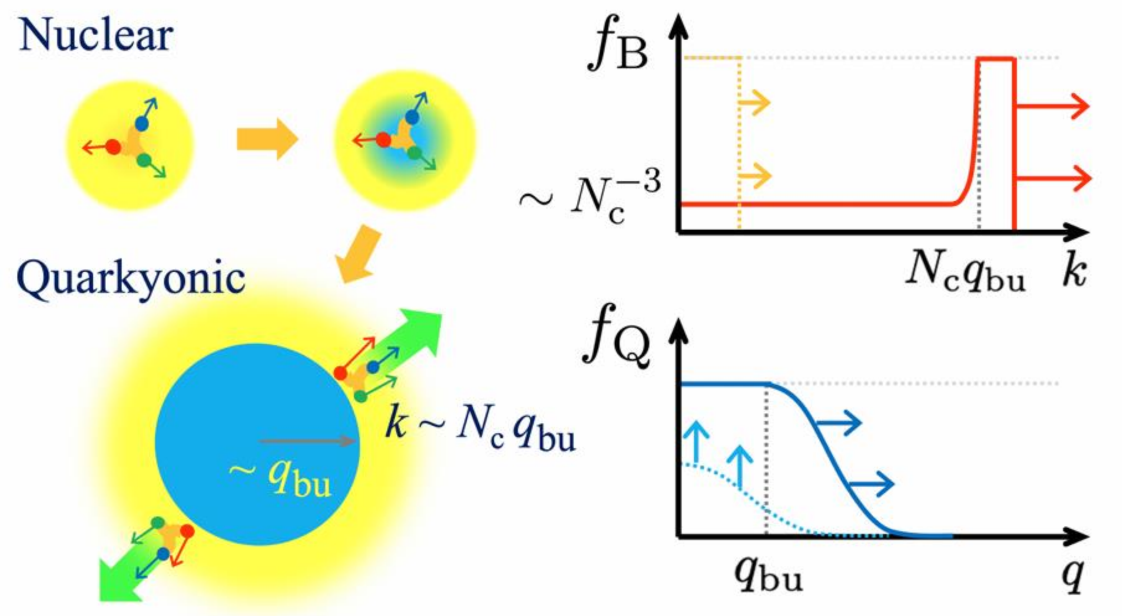


2-3 n_0 2-3 n_0

Nuclear saturation density
= 0.16 fm^-3

(Speed of Sound)^2 rapidly increases at 2-3 n_0

quarkyonic exact duality — IdelliQ (Fujimoto-Kojo-Mclerran, 2024)



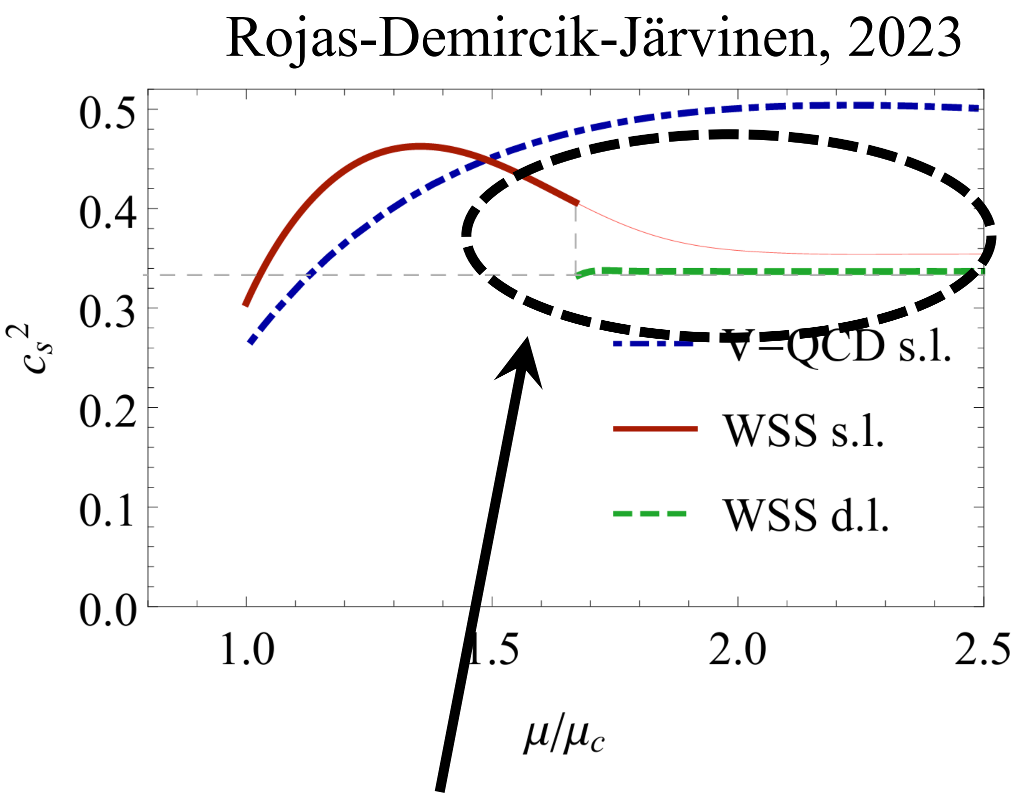
$$n_B = \int_k f_B(k) = \int_q f_q(k)$$

$$f_Q(q) = \int_k \varphi\left(q - \frac{k}{N_c}\right) f_B(k)$$

After quark saturation, n_B changes only weakly, while Fermi energy $\mu_B \sim E_F$ rises. Hence, the speed of sound increases:

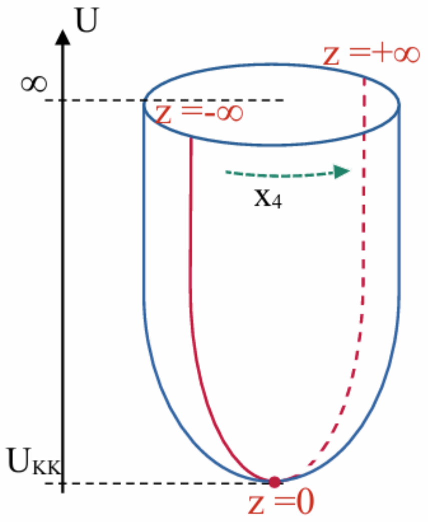
$$c_s^2 = \frac{n_B}{\mu_B} \frac{d\mu_B}{dn_B}$$

Single-layer solution of Sakai-Sugimoto model



Baryonic, yet nearly conformal

quarkyonic duality in SS model?



Setup:

- Two-flavor isospin symmetric matter

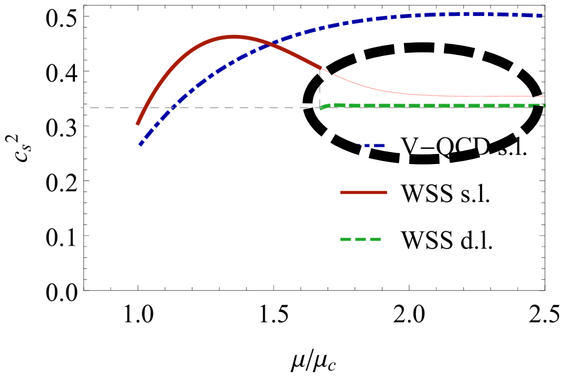
$$F_{\mu\nu} = \hat{F}_{\mu\nu} + F^a_{\mu\nu}\sigma^a$$

- DBI action + Chern-Simons term
- Homogeneous ansatz
- chemical potential as a boundary cond.

$$\hat{A}_0(z = \pm\infty) = \mu_q$$

Introduction

μBMD and Baryon Number Density



Assuming

$$n_B = \langle BB^\dagger \rangle_{\mu_B}$$

μBMD gives a momentum space decomposition of baryon density

$$n_B = \int_p n(|\mathbf{p}|) \qquad \left(n(|\boldsymbol{p}|) = \int d\omega \, \mathrm{Tr_D} \left[\rho_{sp}(\omega, \boldsymbol{p}) \right] \right)$$

$$\left(\rho_{sp}(\omega, \boldsymbol{p}) \sim \mathrm{Im}\langle BB^\dagger \rangle \right)$$

Good diagnosis for IdelliQ scenario?

This work:

Determine μBMD of **probe baryons** under the single-layer background

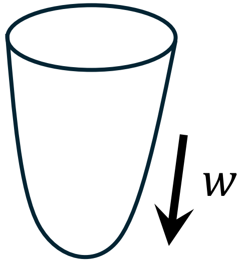
(Therefore $n_B \neq \langle BB^\dagger \rangle_{\mu_B}$)

In-Medium Baryon Momentum Distribution (μBMD) from holography

- Model

Pointlike approximation for baryons + single-layer background

(Low energy effective action for 5d instanton):



$$S_B = \int d^4x \int dw \; [i\bar{B}\gamma^M(\partial_M - iA_M)B - m_5(w)\bar{B}B]$$

Rojas-Demircik-Järvinen, 2023 Hong-Rho-Yee-Yi, 2007

5d Parity invariance

$$A_k(w) = h(z(w))\sigma_k, \quad h(-w) = -h(w), \quad A_0(-w) = A_0(w), \quad m_5(w) = m_5(-w)$$

Formalism

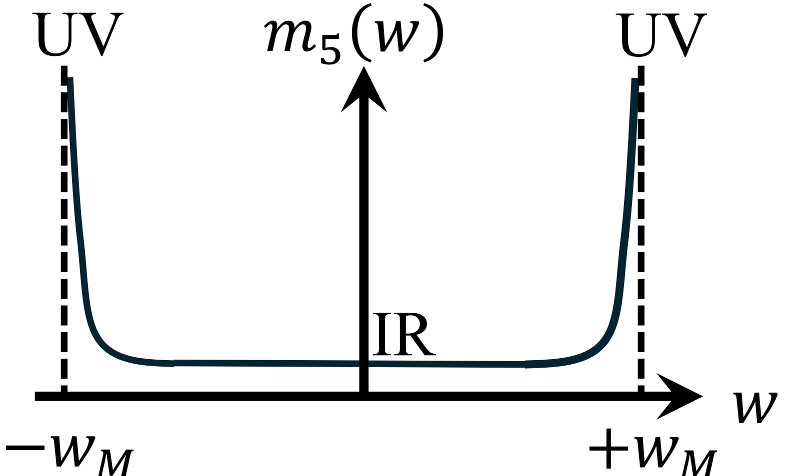
In-Medium Baryon Momentum Distribution (μBMD) from holography

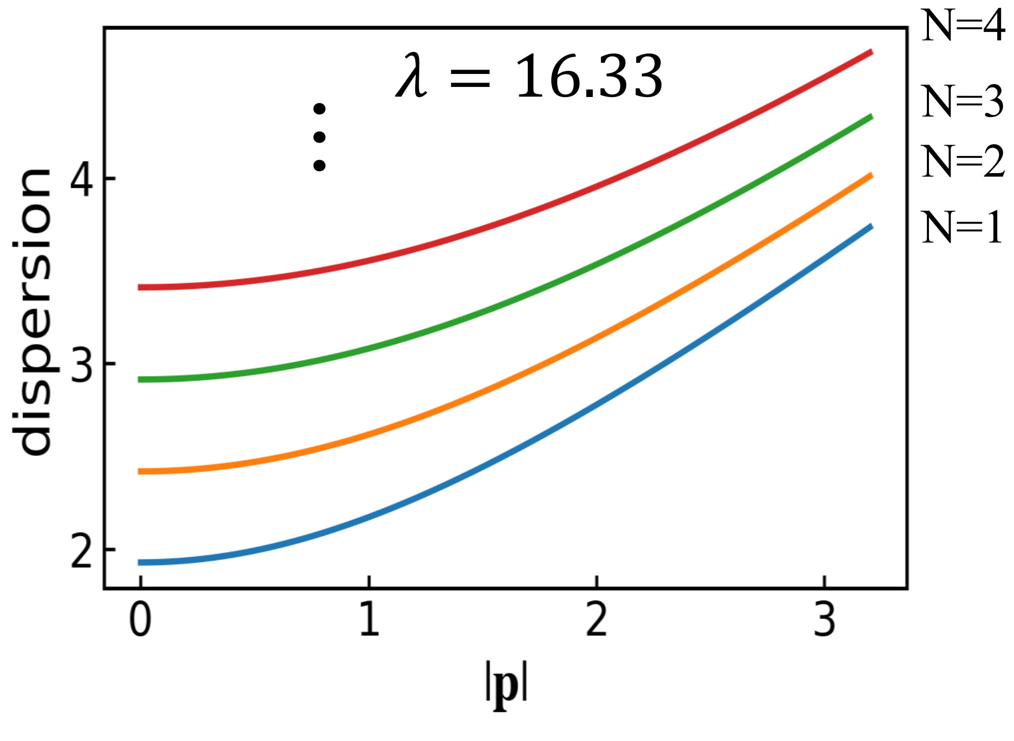
- Model

In vacuum, m_5 determines all spectra of isospin-1/2 baryons

Rest energy of 5d instanton (=baryon)

$$E_{inst} \sim 8\pi^2/g_{5d}(w)^2 \rightarrow +\infty \text{ (in the UV)}$$





Formalism

In-Medium Baryon Momentum Distribution (μBMD) from holography

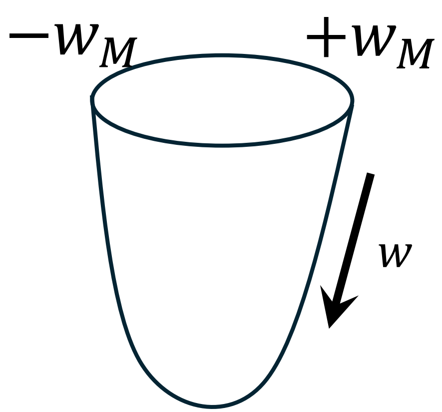
- Holographic prescription for retarded propagator

Source ~ Coefficirnt of non-normalizable mode at boundary

Response ~ Coefficient of normalizable mode at boundary

$$G_R = -\text{ Response / Source}$$

↖ For Dirac fermion (Iqbal-Liu, 2009)



- Asymptotic solution of Dirac equation

$$B(w) \sim B_{\text{div}}(w)\eta_R + B_{\text{reg}}(w)\psi_R$$

$B_{\text{div}}(w \to w_M) = e^{m_0(w_M - w)^{-1}}(1_2, 0)^T$, $\quad B_{\text{reg}}(w \to w_M) = e^{-m_0(w_M - w)^{-1}}(0, 1_2)^{\text{T}}$

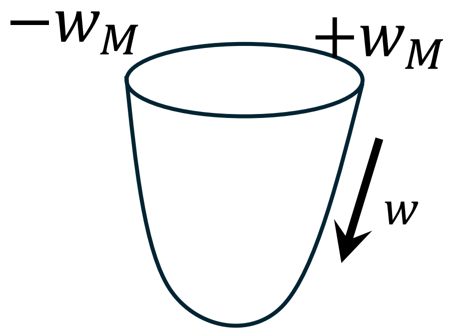
Formalism

In-Medium Baryon Momentum Distribution (μBMD) from holography

- IR boundary condition for Dirac equation

5d parity allows us to restrict w>0, and gives IR B.C.

$$B(w=0;p^0,\boldsymbol{p}) = \pm\gamma^0 B(w=0;p^0,-\boldsymbol{p})$$

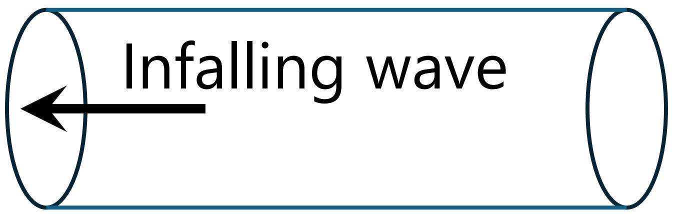


No horizon

↕

For geometries with horizon, this is replaced by the *infalling condition*

horizon



In-Medium Baryon Momentum Distribution (μBMD) from holography

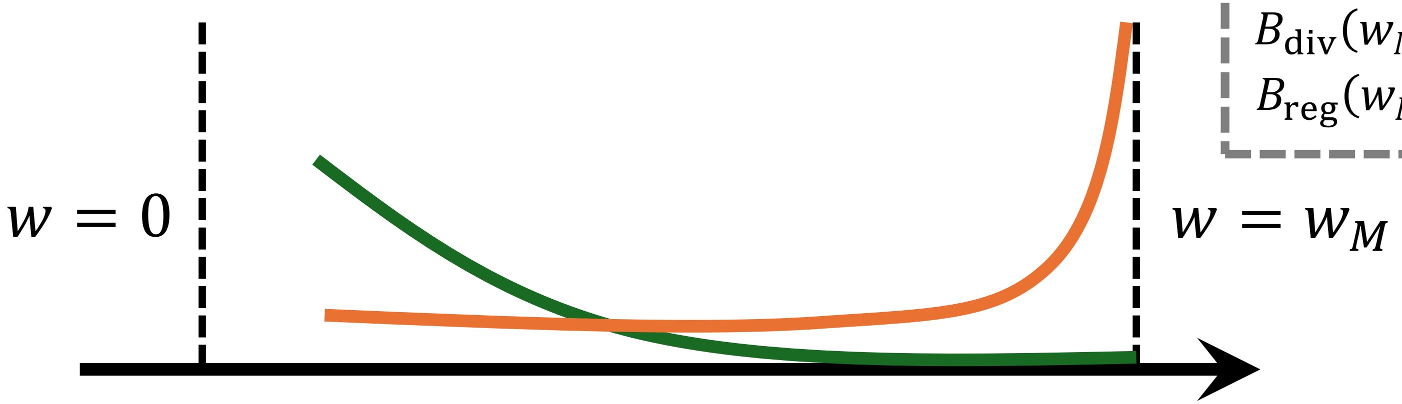
Numerical procedure

- Solve Dirac equation from UV to IR for two different four-momenta

$$B(w=0;p^0,\boldsymbol{p}) = \pm\gamma^0 B(w=0;p^0,-\boldsymbol{p})$$

Initial cond. at UV:

$$B_{\mathrm{div}}(w_M) = e^{m_0(w_M-w)^{-1}}(1_2,0)^T,$$
$$B_{\mathrm{reg}}(w_M) = e^{-m_0(w_M-w)^{-1}}(0,1_2)^{\mathrm{T}}$$



Singular solution (~Source) under $p=(p^0,\boldsymbol{p})$

Regular solution (~Response) under $p=(p^0,\boldsymbol{p})$

Formalism

In-Medium Baryon Momentum Distribution (μBMD) from holography

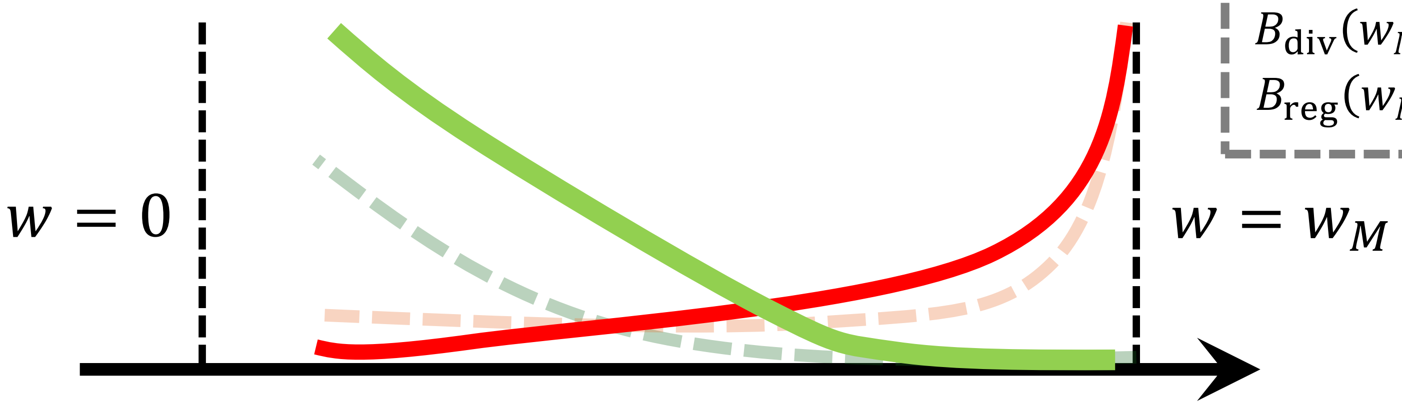
Numerical procedure

- Solve Dirac equation from UV to IR for two different four-momentums

$$B(w=0;p^0,\boldsymbol{p}) = \pm\gamma^0 B(w=0;p^0,-\boldsymbol{p})$$

Boundary cond. :

$$B(w=0;p^0,\boldsymbol{p}) = \pm\gamma^0 B(w=0;p^0,-\boldsymbol{p})$$



Initial cond. at UV:

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$$B_{\mathrm{reg}}(w_M) = e^{-m_0(w_M-w)^{-1}}(0,1_2)^{\mathrm{T}}$$

Singular solution (~Source) under $p=(p^0,\boldsymbol{p})$

Regular solution (~Response) under $p=(p^0,\boldsymbol{p})$

Singular solution (~Source) under $p=(p^0,-\boldsymbol{p})$

Regular solution (~Response) under $p=(p^0,-\boldsymbol{p})$

Formalism

In-Medium Baryon Momentum Distribution (μBMD) from holography

IR B.C. determines the response/response ratio

$$B_{\text{div}}(w;p^0,\boldsymbol{p})\eta_R(p^0,\boldsymbol{p}) + B_{\text{reg}}(w;p^0,\boldsymbol{p})\psi_R(p^0,\boldsymbol{p})$$
$$= \pm[B_{\text{div}}(w;p^0,-\boldsymbol{p})\eta_L(p^0,-\boldsymbol{p})B_{\text{reg}}(w;p^0,-\boldsymbol{p})\psi_L(p^0,-\boldsymbol{p})]$$

$$\longrightarrow \quad \psi = -R^{-1}S\,\eta$$

$(G_R = -\text{ Response / Source })$

$$\psi = (\psi_R, \pm\psi_L)^T,$$
$$S = \left(B_{div}(\boldsymbol{p})\ B_{div}(-\boldsymbol{p})\right),$$
$$R = (B_{reg}(\boldsymbol{p})\ B_{reg}(-\boldsymbol{p}))$$

Euclidean propagator in the parity basis

$$G_E = R^{-1}S \quad \longleftarrow \quad \langle BB^\dagger \rangle$$

In-Medium Baryon Momentum Distribution (μBMD) from holography

Analytical Continuation

$$G_E = R^{-1}S \xrightarrow{p^0 \to \omega + i0^+} G_R = R^{-1}S$$

- Pole condition

$\det R = 0$ has a single zero at the quasiparticle onshell

$$\det R \sim \omega - E_N(\boldsymbol{p}) + i0^+$$

- Spectral function

$$\rho_{sp}(\omega, \boldsymbol{p}) = \sum_N Z_N(\boldsymbol{p})\,\delta(\omega - E_N(\boldsymbol{p}))$$

$$\rho_{\rm sp}(p) = -\frac{1}{\pi}\,{\rm Im}\,G_{\rm R}(p)\,,$$

We confirmed $Z_N = Z_N^\dagger$ is satisfied

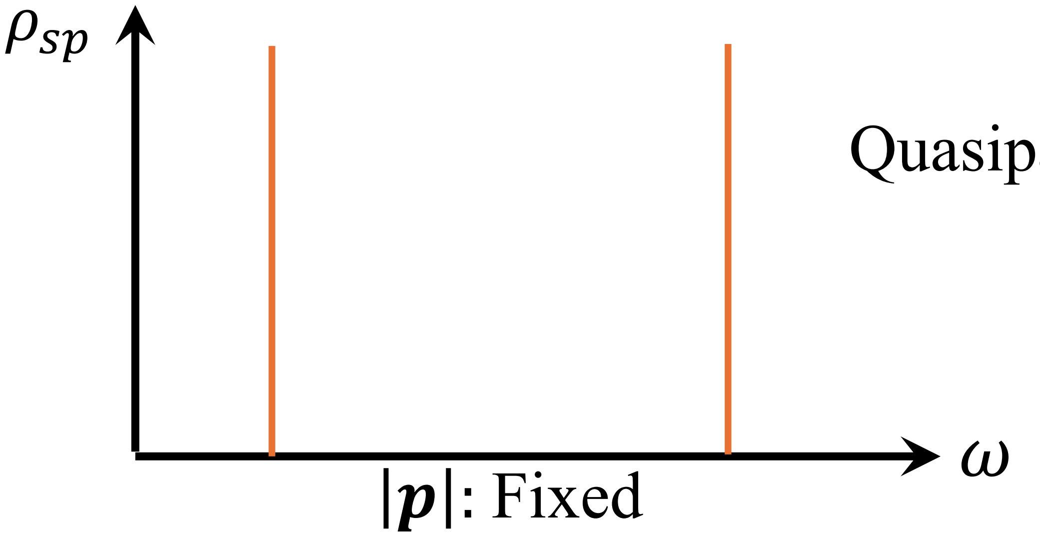
Formalism

In-Medium Baryon Momentum Distribution (μBMD) from holography

- μBMD

$$n_{\mathrm{B}}(|\boldsymbol{p}|) = 2\int_{-\infty}^{+\infty} \mathrm{d}\omega\, \theta(\mu_{\mathrm{B}} - \omega)\, \mathrm{Tr}_{\mathrm{D}}\big[\rho_{\mathrm{sp}}(\omega, \boldsymbol{p})\big]$$

$$= 2\int_{-\infty}^{+\infty} d\omega\, \theta\big(\mu_B - E_N(|\boldsymbol{p}|)\big)\, \mathrm{Tr}_{\mathrm{D}}\big(Z_N(|\boldsymbol{p}|)\big)$$



Quasiparticle poles w/ inifinite lifetime

(since no interaction)

In-Medium Baryon Momentum Distribution (μBMD) from holography

- μBMD

$$n_{\mathrm{B}}(|\boldsymbol{p}|) = 2\int_{-\infty}^{+\infty} \mathrm{d}\omega\, \theta(\mu_{\mathrm{B}} - \omega)\, \mathrm{Tr}_{\mathrm{D}}\big[\rho_{\mathrm{sp}}(\omega, \boldsymbol{p})\big]$$

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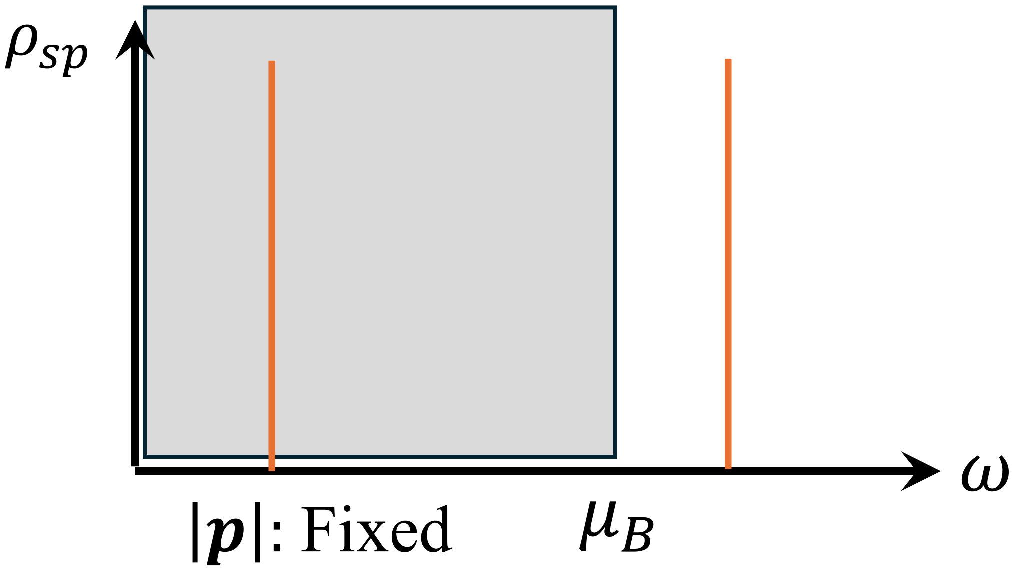

Only quasiparticles satisfying $E_N \leq \mu_B$ are excited

In-Medium Baryon Momentum Distribution (μBMD) from holography

- μBMD

$$n_{\mathrm{B}}(|\boldsymbol{p}|) = 2\int_{-\infty}^{+\infty} \mathrm{d}\omega\, \theta(\mu_{\mathrm{B}} - \omega)\, \mathrm{Tr}_{\mathrm{D}}[\rho_{\mathrm{sp}}(\omega, \boldsymbol{p})]$$
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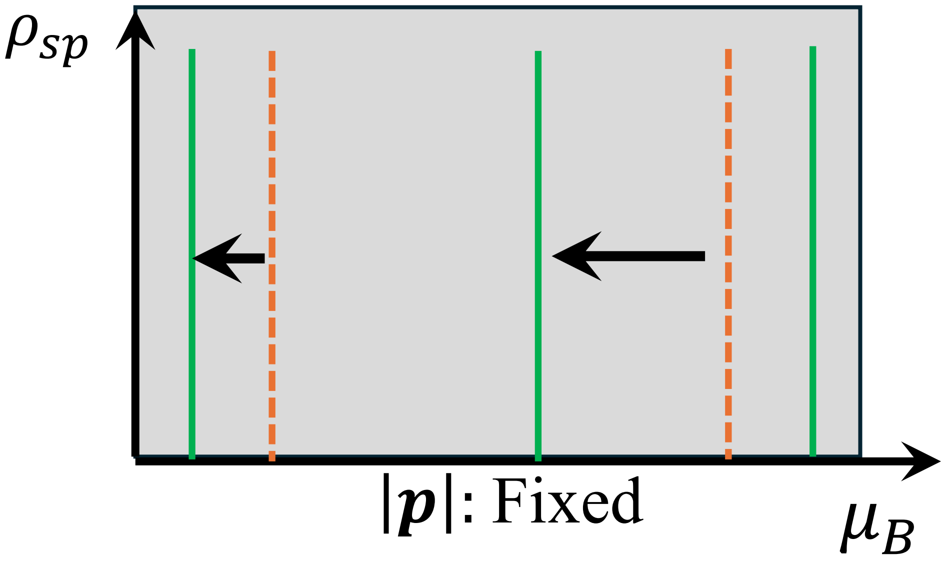
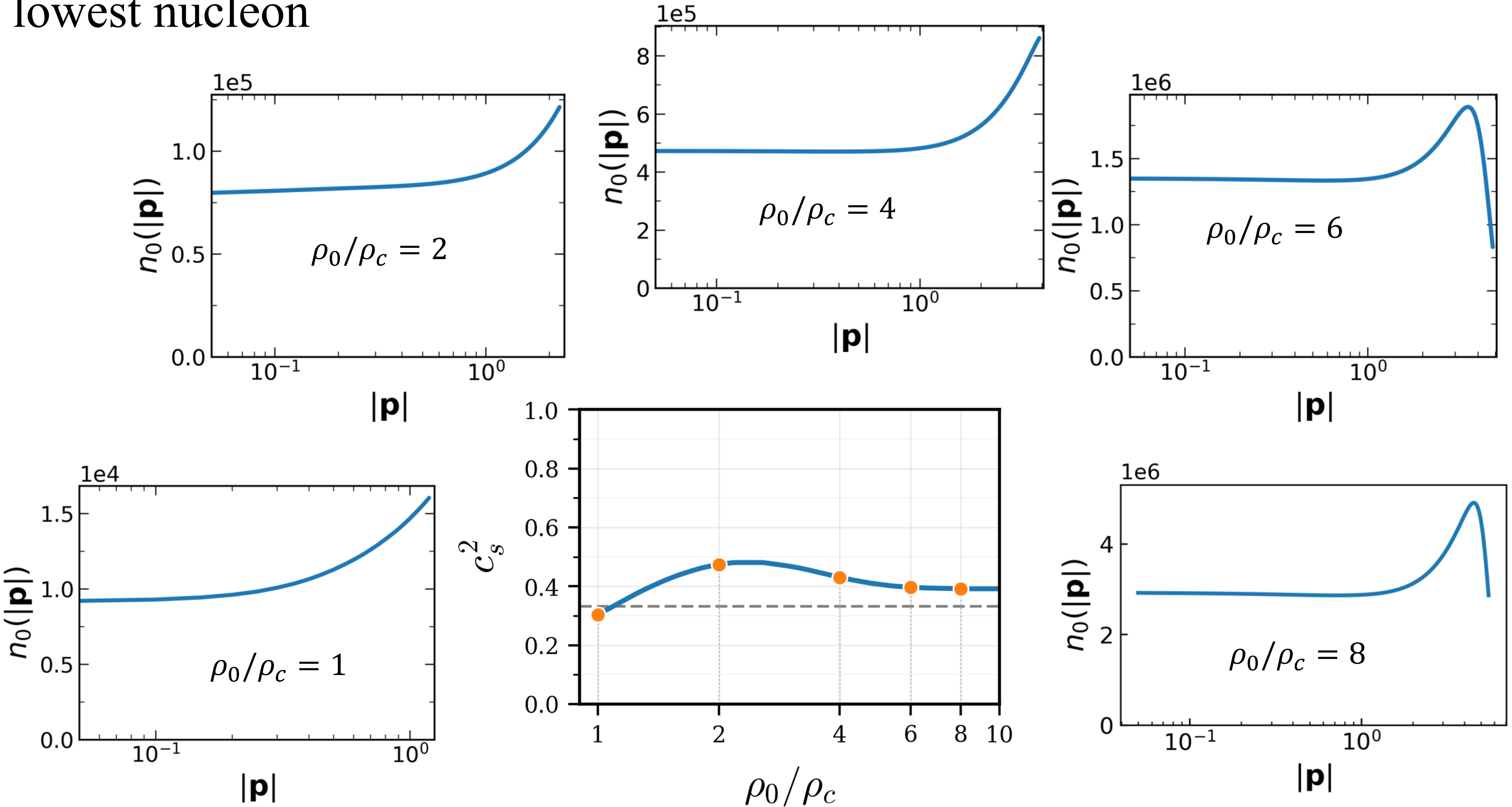


As the density increases,
not only the threshold but also the locations of each pole move
(since Dirac equation is solved in an in-medium background)

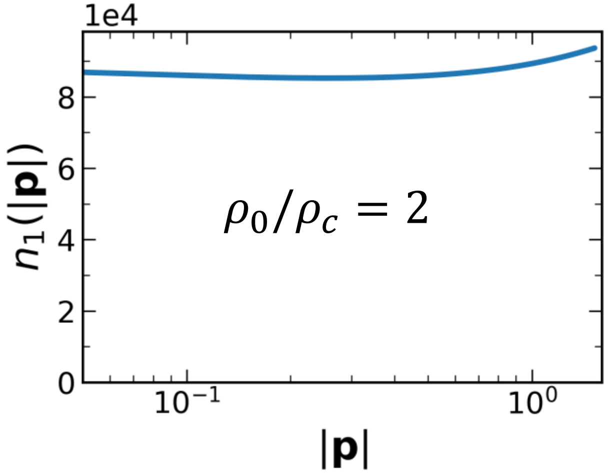
Results and Discussions

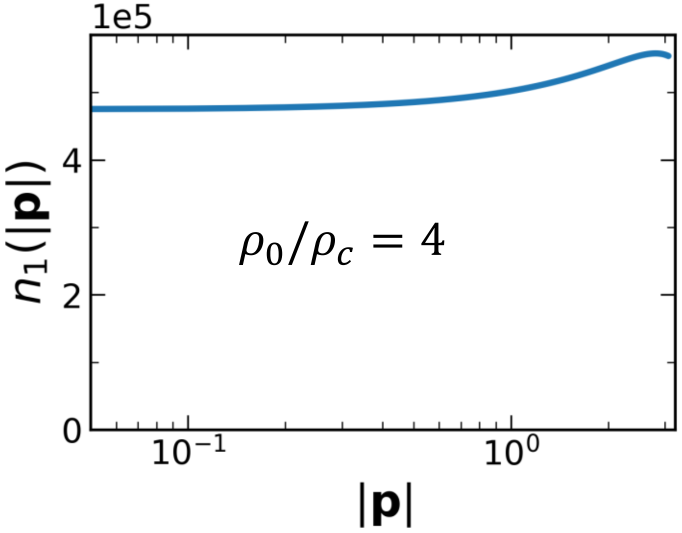
lowest nucleon

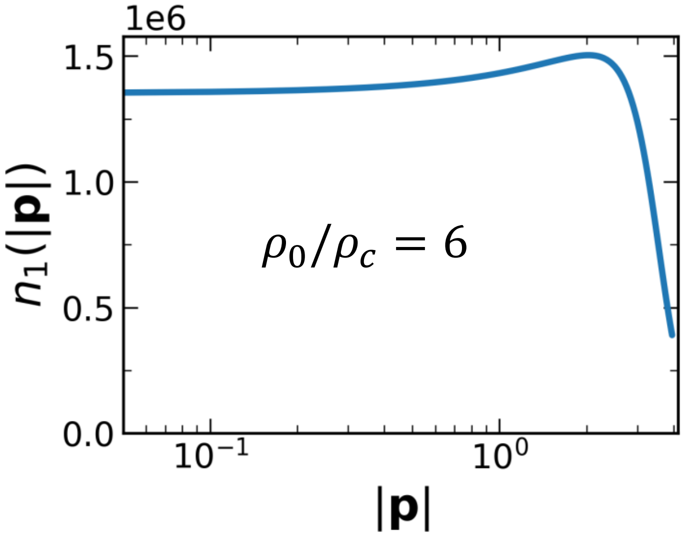


Results and Discussions

first excited state

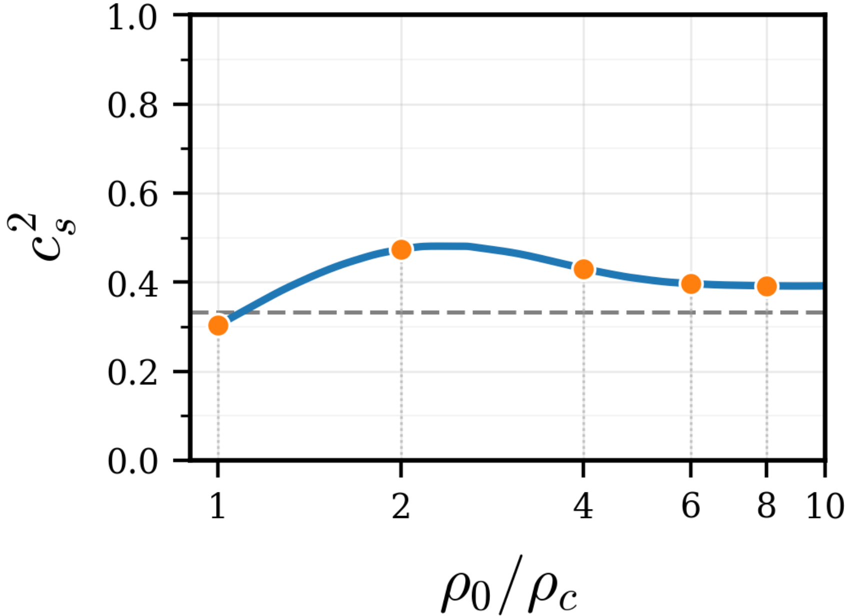


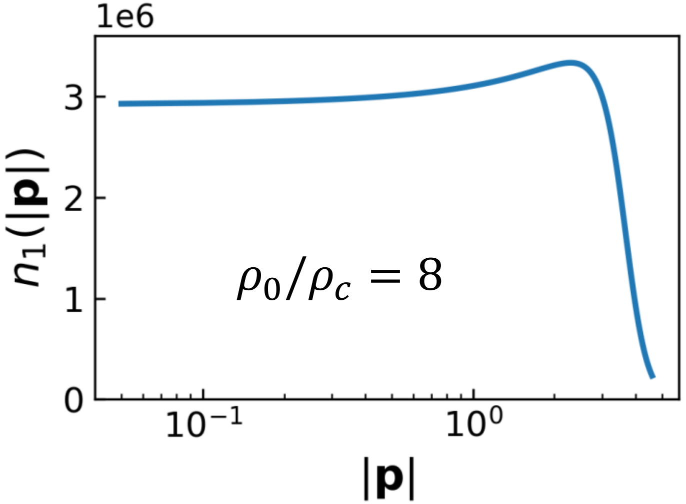




$\rho_0/\rho_c = 1$

not excited





Results and Discussions

Why peak?

$$\left[\partial_w - m_5(w)\right] b_r(w;s) + K_l(w;s)\, b_l(w;s) = 0\,,$$
$$\left[\partial_w + m_5(w)\right] b_l(w;s) - K_r(w;s)\, b_r(w;s) = 0$$

$$K_{r/l}(w;s) = p^0 - A^0(w) \mp s|\boldsymbol{p}| \pm 3h(w)$$

At $|\boldsymbol{p}| = 0$: the wavefunction (and μBMD) is suppressed by potential terms

Results and Discussions

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At $|\boldsymbol{p}| = 0$: the wavefunction (and μBMD) is suppressed by potential terms

At medium $|\boldsymbol{p}|$: A part of potential is canceled. μBMD enhanced.

Results and Discussions

Why peak?

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At $|\boldsymbol{p}| = 0$: the wavefunction (and μBMD) is suppressed by potential terms

At medium $|\boldsymbol{p}|$: A part of potential is canceled → μBMD enhanced.

$|\boldsymbol{p}| \to \infty$: potential terms are irrelevant. Exponential decaying $n_N \sim e^{-\#|\boldsymbol{p}|}$

Conclusions

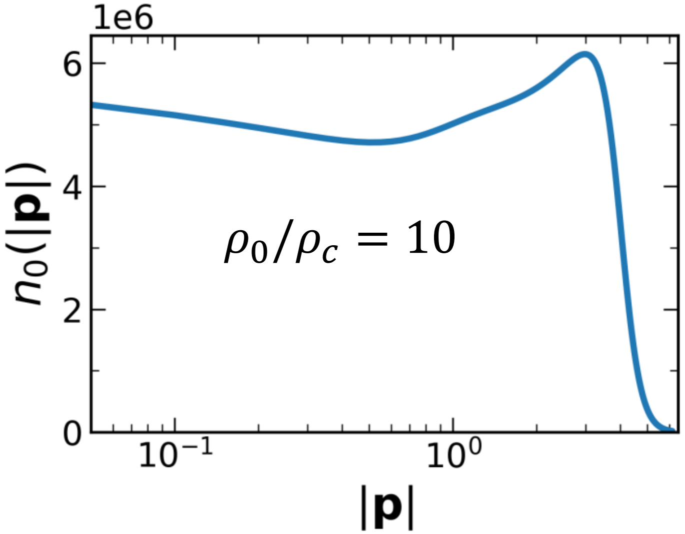
Goal: Diagnose IdelliQ behavior through μBMD in a holographic model with a peaked speed of sound

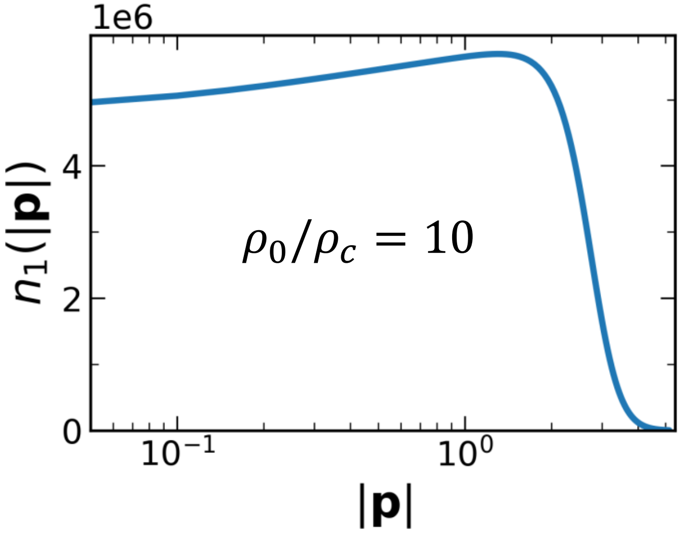
Try: Compute μBMD of probe baryons in the single-layer background

Result: (surprisingly) A shell-like peak appears, but it reflects the mobility of probe baryons under the mean-field backgrounds

Future: Compute μBMD of baryons that contribute the thermodynamics

Backup


1e6
6
4
2
0
$n_0(|\mathbf{p}|)$
$\rho_0/\rho_c = 10$
10^{-1}
10^{0}
$|\mathbf{p}|$


1e6
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