

# Deconfinement-Higgs continuity in $SU(2)$ adjoint Higgs model at finite temperature

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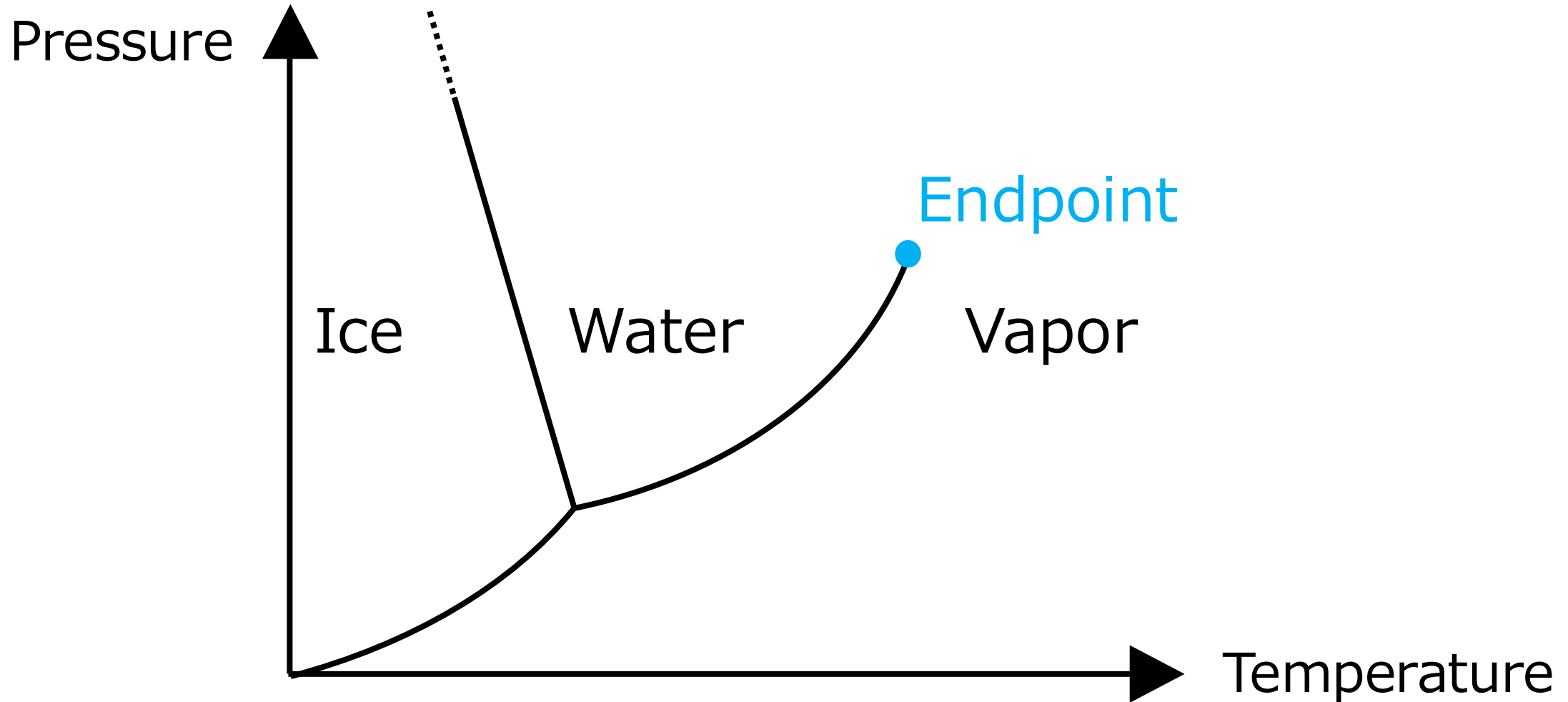
Based on arXiv:2510.15493[hep-th]

- Motivation
- Our setup and proposal
- Lattice model and its schematic phase diagram
- Center-destabilizing deformation
- Summary and Discussion

- **Motivation**
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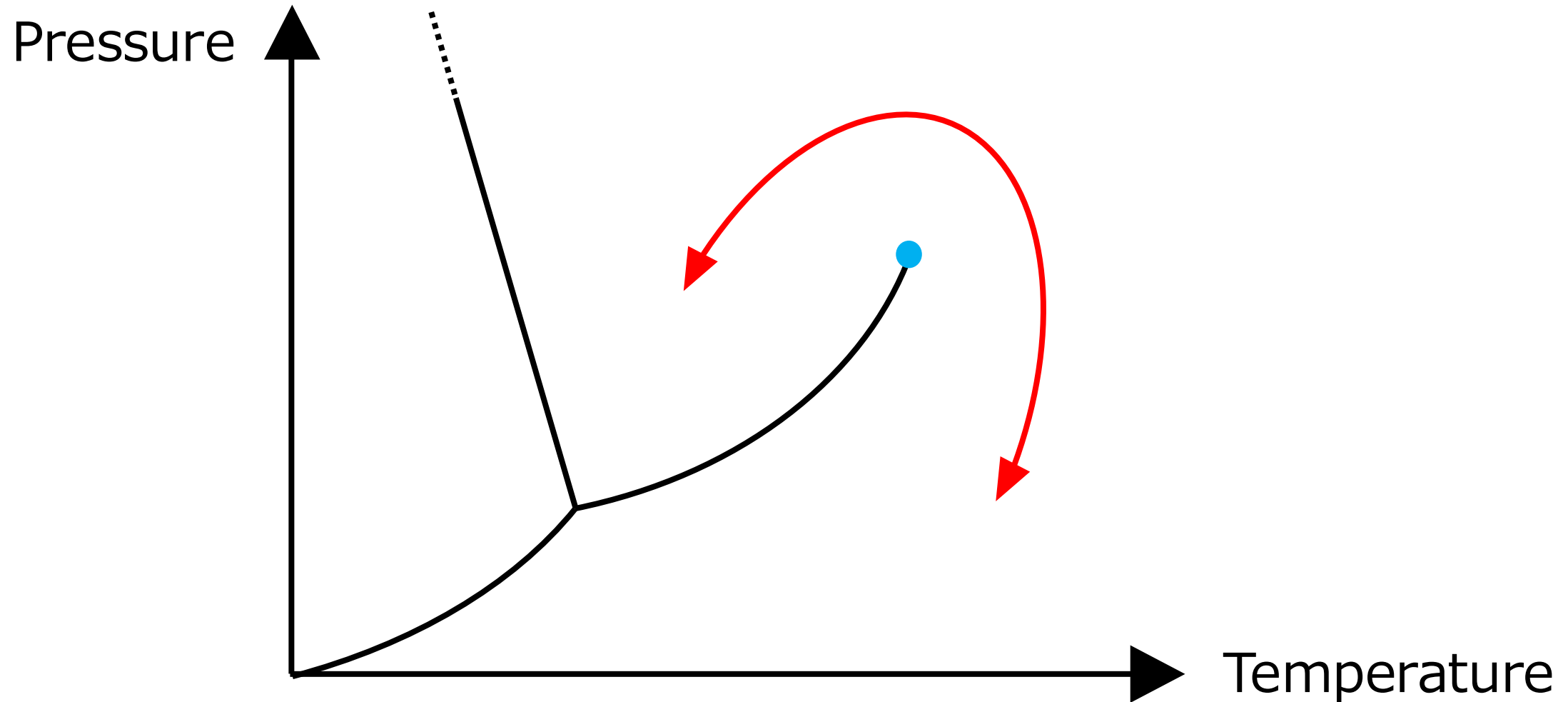
Continuity appears  
in a wide range of physical phenomena.

# Example 1: H<sub>2</sub>O



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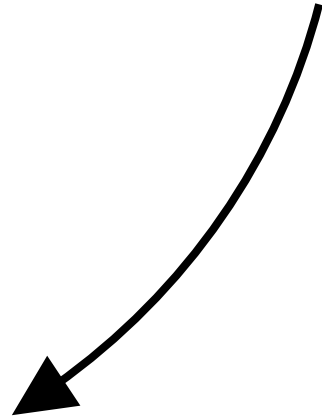
This is the continuity between water and vapor.



# Example 2: the Standard Model

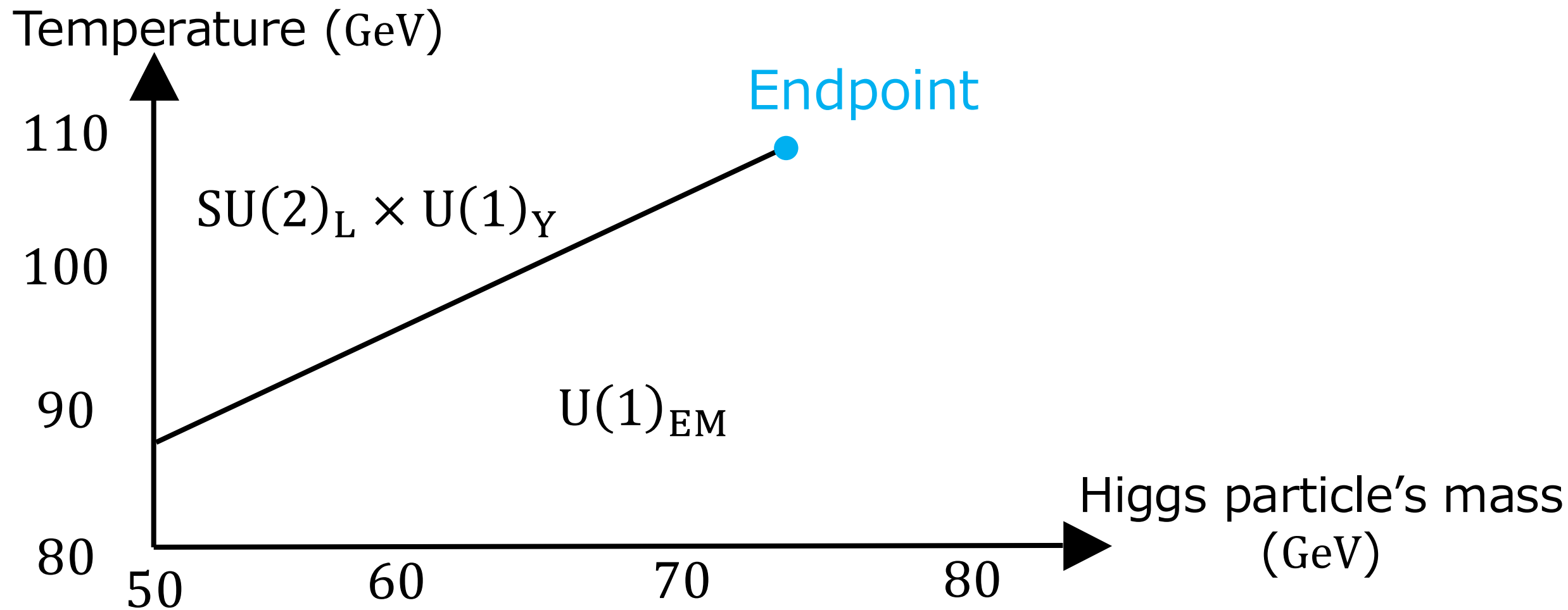
$U$   
Electroweak sector

$SU(2)_L \times U(1)_Y$  (at high temperature)



$U(1)_{EM}$  (at low temperature)

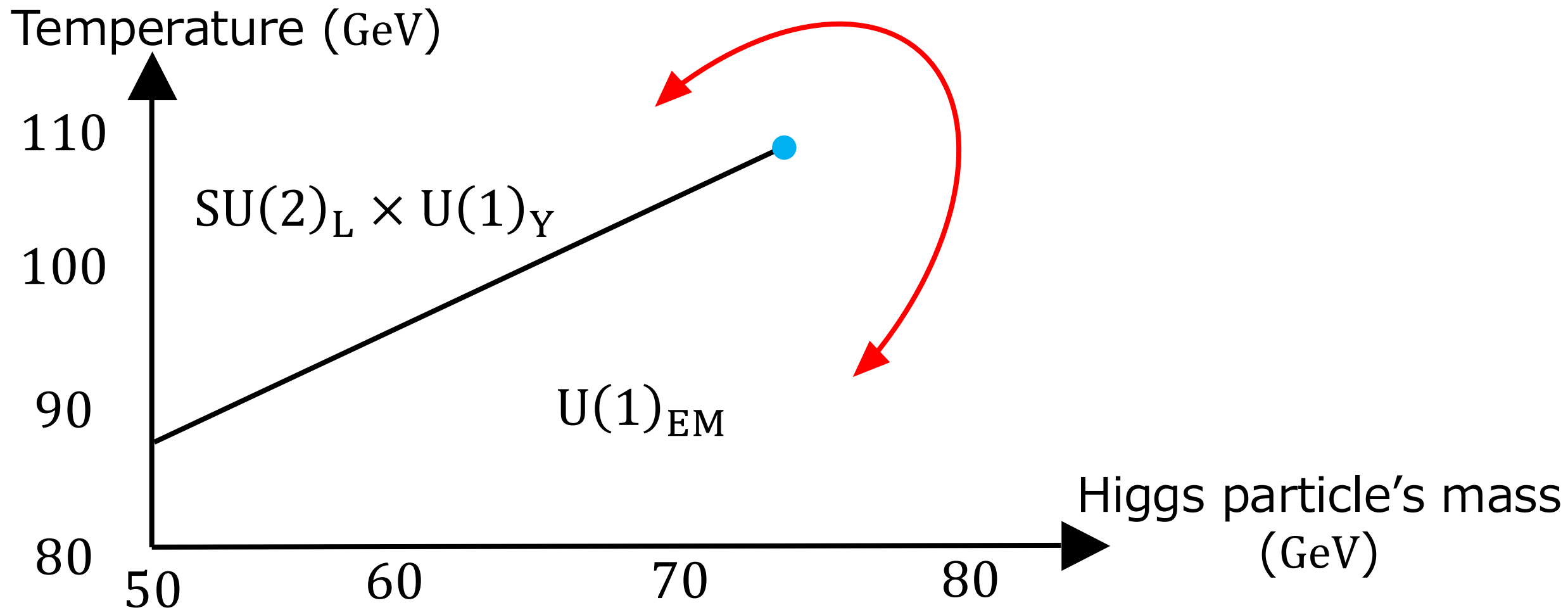
# Example 2: the Standard Model



(based on [Laine(2000),...])

# Example 2: the Standard Model

The continuity between symmetric and Higgs phases



(based on [Laine(2000),...])

# Our motivation

The phase diagram with a fundamental Higgs

- ▶ Many studies

e.g. Fradkin-Shenker's work, Electroweak Higgs, ...

The phase diagram with an **adjoint Higgs**

- ▶ Few studies

# Our motivation

The phase diagram with a fundamental Higgs

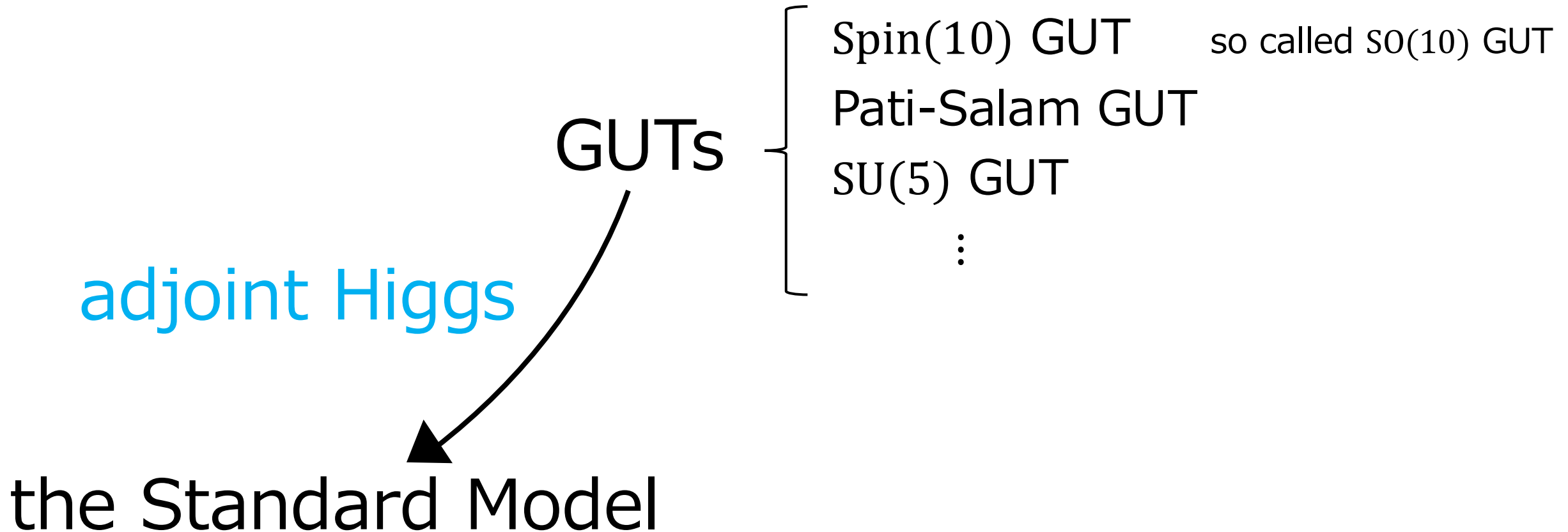
- ▶ Many studies

e.g. Fradkin-Shenker's work, Electroweak Higgs, ...

The phase diagram with an **adjoint Higgs**

- ▶ Few studies, but important for BSMs like GUTs

GUTs need adjoint Higgs fields.



Phenomenology and cosmology usually assume that the **adjoint Higgs field** induces the phase transition.

However, no one knows whether this assumption is valid at the nonperturbative level.

If there is continuity, some changes might be needed these areas.

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# Our setup

SU(2) gauge field + adjoint Higgs field

$$a_\mu = a_\mu^i \sigma^i$$

$$\phi = \phi^i \sigma^i$$

$$\blacktriangleright f_{\mu\nu} = f_{\mu\nu}^i \sigma^i$$

$$S = \int_0^\beta dt \int d^3 \mathbf{x} \left[ \frac{1}{4g^2} \text{tr}(f^{\mu\nu} f_{\mu\nu}) + \frac{1}{2} \text{tr}(D^\mu \phi D_\mu \phi) + V(\phi) \right]$$

$$D_\mu \phi = \partial_\mu \phi + i[a_\mu, \phi]$$

$$V(\phi) = m^2 \text{tr}(\phi^2) + \lambda \text{tr}(\phi^4)$$

# Phase diagram

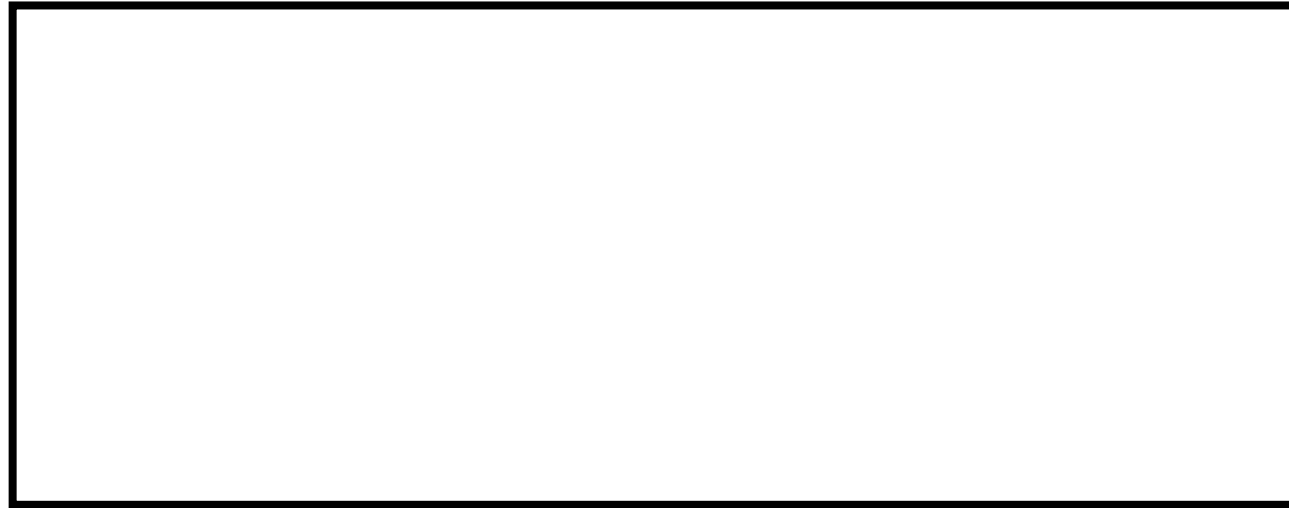
$$S = \int_0^\beta dt \int d^3 \mathbf{x} \left[ \frac{1}{4g^2} \text{tr}(f^{\mu\nu} f_{\mu\nu}) + \frac{1}{2} \text{tr}(D^\mu \phi D_\mu \phi) + m^2 \text{tr}(\phi^2) + \lambda \text{tr}(\phi^4) \right]$$

$T = \infty$

$T = 0$

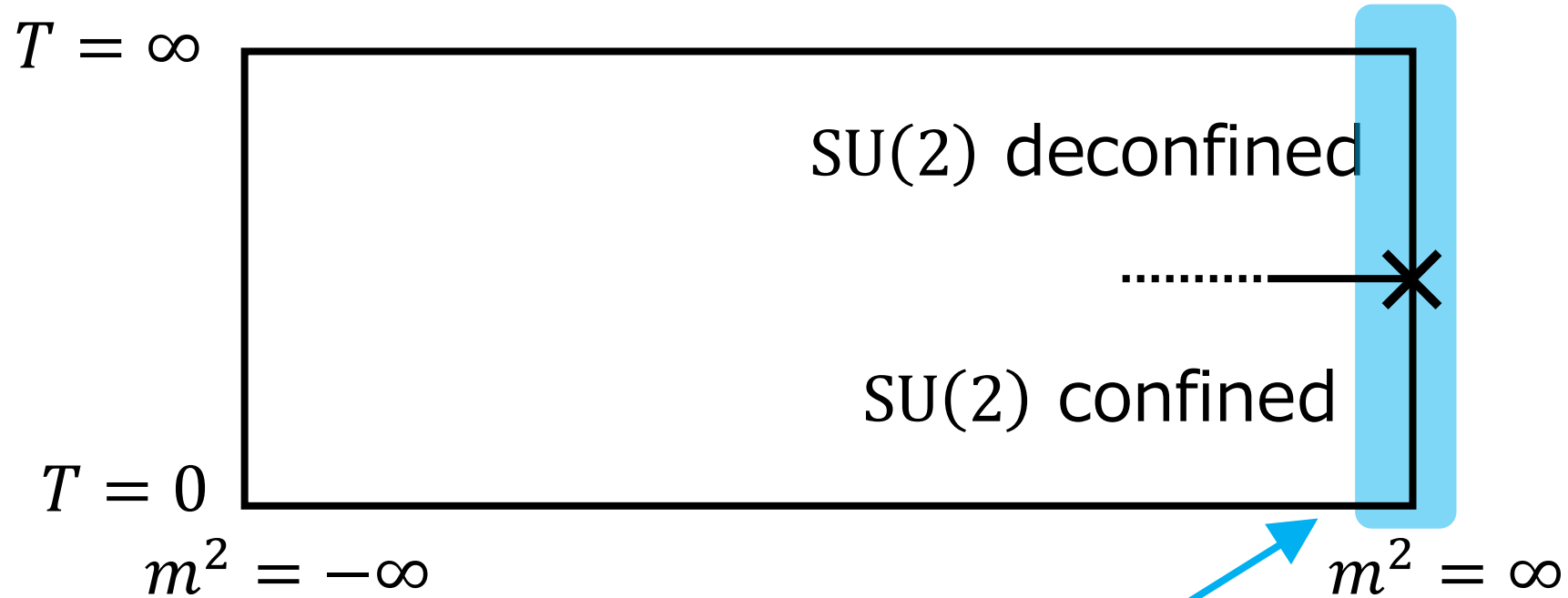
$m^2 = -\infty$

$m^2 = \infty$



# Phase diagram

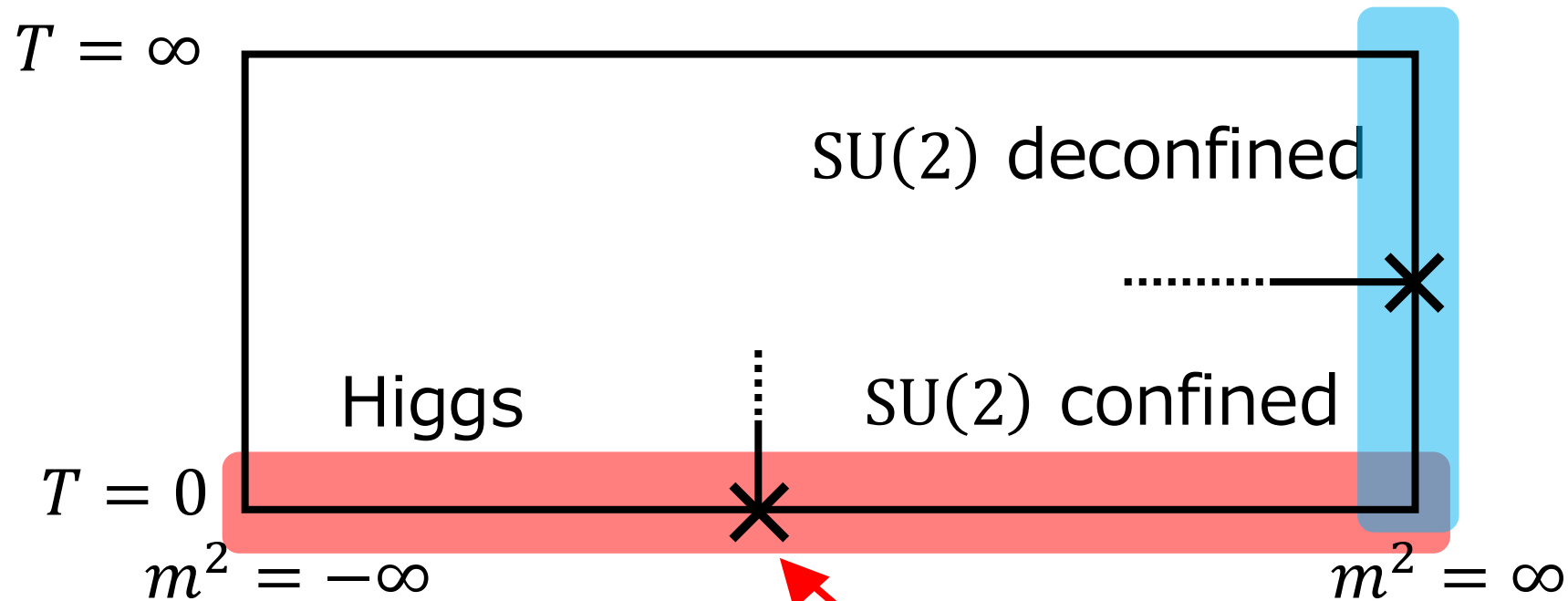
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Related to temporal center symmetry

# Phase diagram

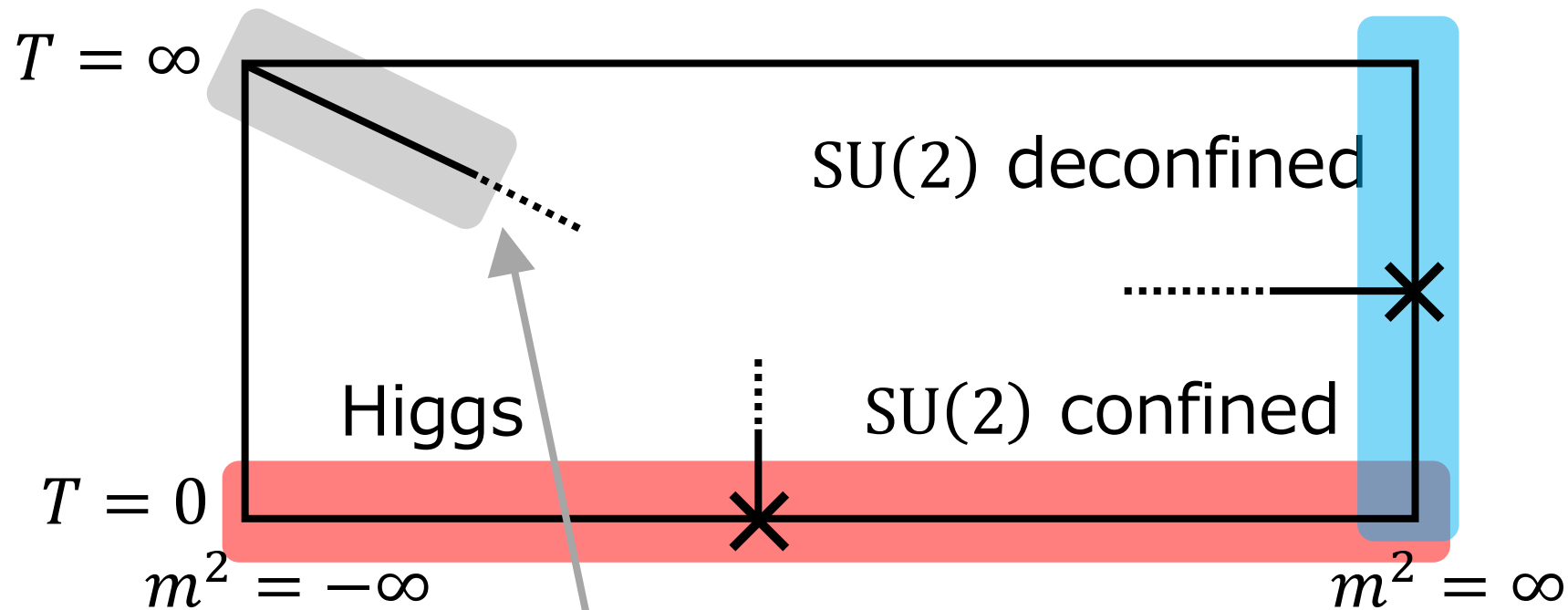
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Related to center symmetry

# Phase diagram

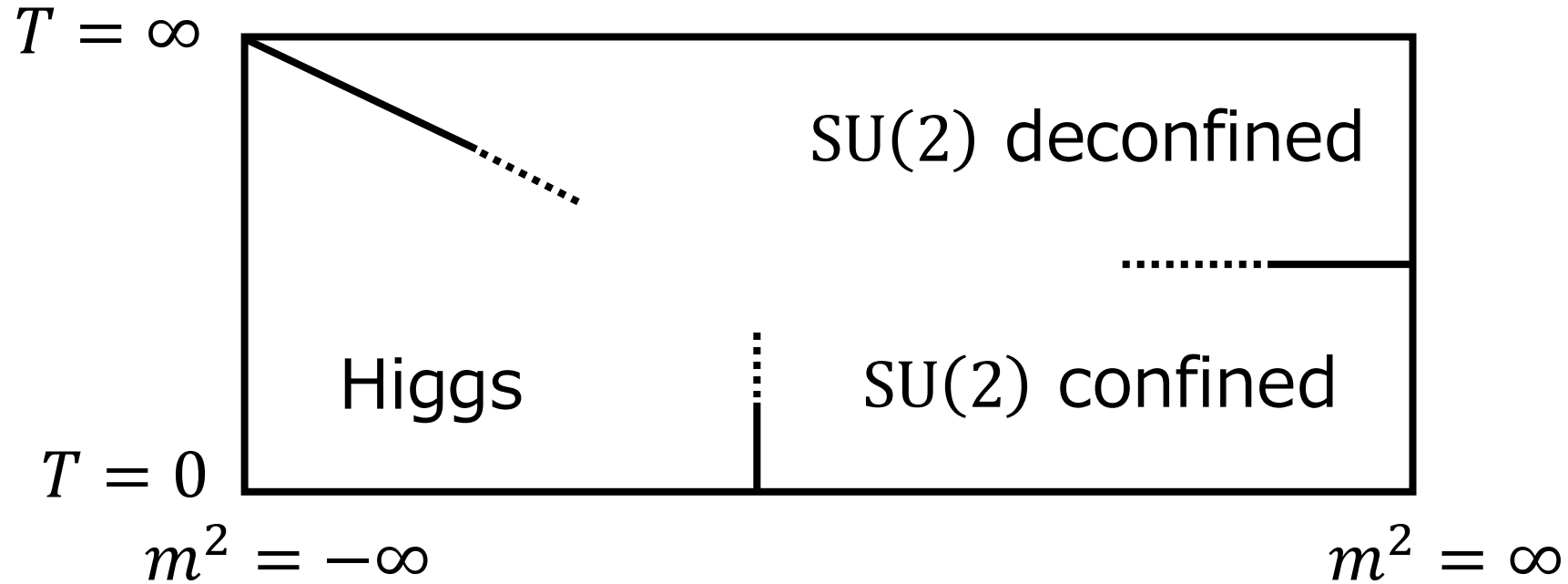
$$S = \int_0^\beta dt \int d^3 \mathbf{x} \left[ \frac{1}{4g^2} \text{tr}(f^{\mu\nu} f_{\mu\nu}) + \frac{1}{2} \text{tr}(D^\mu \phi D_\mu \phi) + m^2 \text{tr}(\phi^2) + \lambda \text{tr}(\phi^4) \right]$$



From perturbative calculation  
(not necessarily related to a global symmetry)

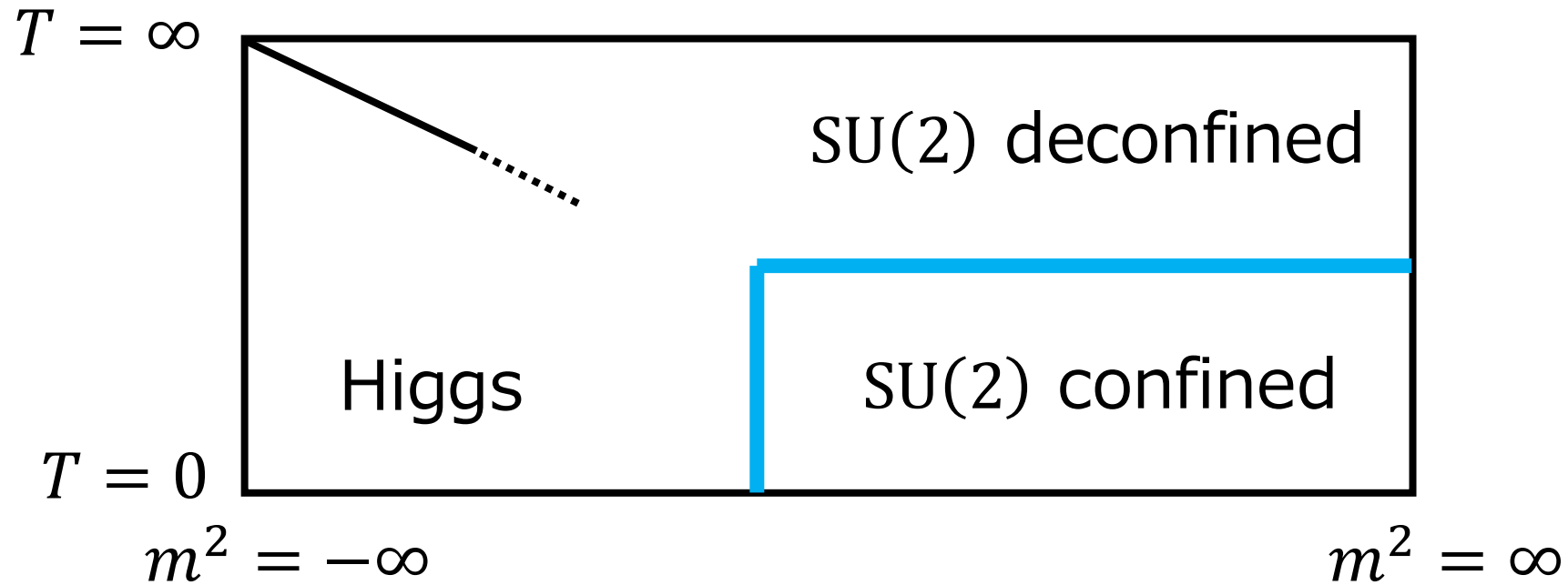
# Phase diagram

$$S = \int_0^\beta dt \int d^3 \mathbf{x} \left[ \frac{1}{4g^2} \text{tr}(f^{\mu\nu} f_{\mu\nu}) + \frac{1}{2} \text{tr}(D^\mu \phi D_\mu \phi) + m^2 \text{tr}(\phi^2) + \lambda \text{tr}(\phi^4) \right]$$



# Phase diagram

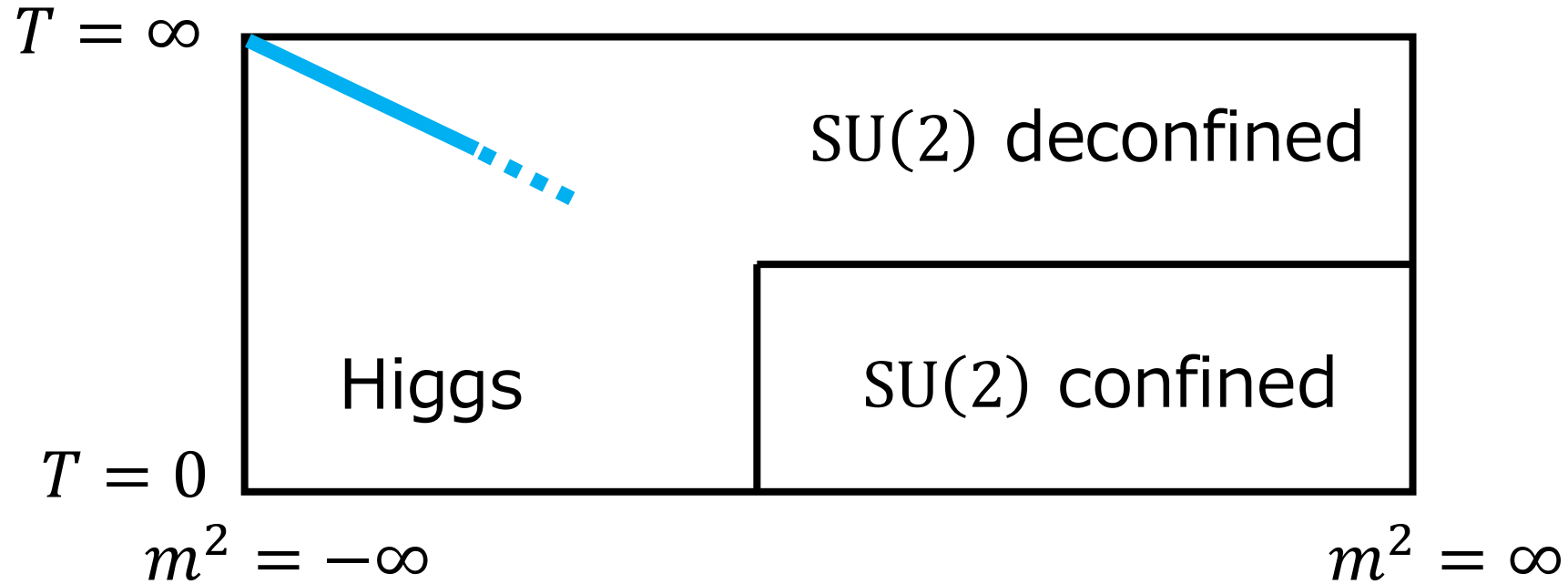
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SSB completely separates the phases.

# Phase diagram

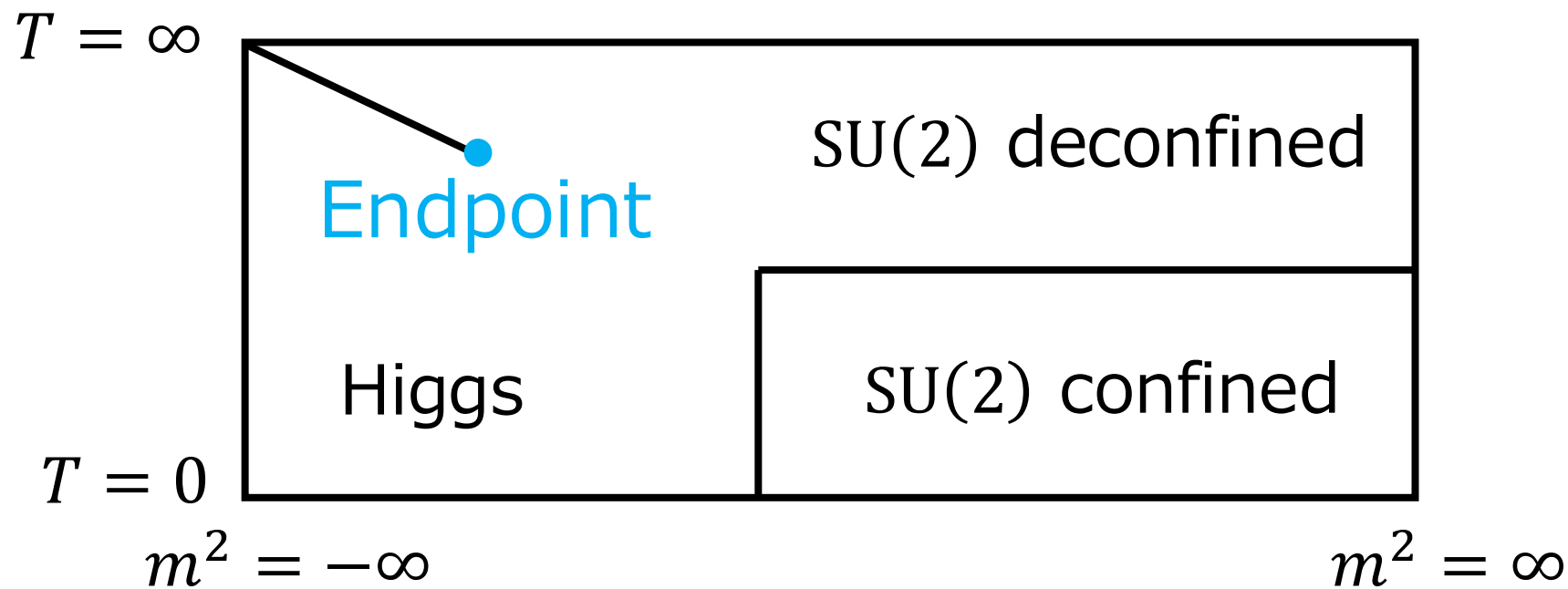
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It is unclear whether this line separates the phases.

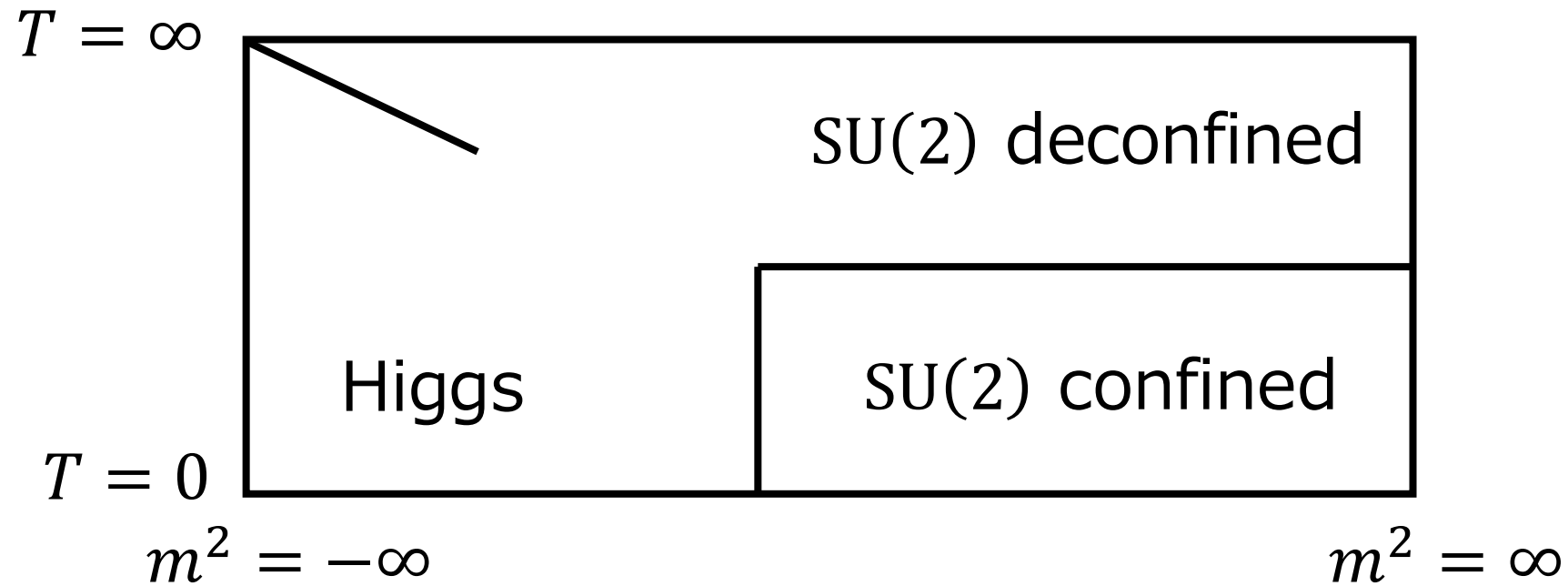
# Our proposal

$$S = \int_0^\beta dt \int d^3 \mathbf{x} \left[ \frac{1}{4g^2} \text{tr}(f^{\mu\nu} f_{\mu\nu}) + \frac{1}{2} \text{tr}(D^\mu \phi D_\mu \phi) + m^2 \text{tr}(\phi^2) + \lambda \text{tr}(\phi^4) \right]$$



Continuity between deconfined phase and Higgs phase.  
(Deconfinement-Higgs continuity)

Today, I will discuss this proposal in a lattice model.



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Lattice model

$$S[U, \phi] = \frac{\beta}{2} S_{\text{Plaquette}}[U]$$

$$- \frac{\beta_H}{2} \sum_{x, \mu} \phi^i(x) \phi^j(x + \hat{e}_\mu) \text{tr}[U_\mu(x) \sigma^i U_\mu^\dagger(x) \sigma^j]$$

where we imposed a condition :

$$\sum_{i=1}^3 (\phi^i)^2 = 1$$

Roughly, it corresponds to  $\lambda \rightarrow \infty$ .

$$S[U, \phi] = \frac{\beta}{2} S_{\text{Plaquette}}[U]$$

$$- \frac{\beta_{\text{H}}}{2} \sum_{x, \mu} \phi^i(x) \phi^j(x + \hat{e}_\mu) \text{tr}[U_\mu(x) \sigma^i U_\mu^\dagger(x) \sigma^j]$$

In addition, we take a unitary gauge

$$\phi^i(x) \sigma^i = \sigma^y$$

►  $\phi^i(x) \phi^j(x + \hat{e}_\mu) \text{tr}[U_\mu(x) \sigma^i U_\mu^\dagger(x) \sigma^j] \rightarrow \text{tr}[U_\mu(x) \sigma^y U_\mu^\dagger(x) \sigma^y]$

$$S[U, \phi] = \frac{\beta}{2} S_{\text{Plaquette}}[U] - \frac{\beta_{\text{H}}}{2} \sum_{x, \mu} \text{tr}[U_{\mu}(x) \sigma^y U_{\mu}^{\dagger}(x) \sigma^y]$$

Schematic phase diagram

$$S[U, \phi] = \frac{\beta}{2} S_{\text{Plaquette}}[U] - \frac{\beta_{\text{H}}}{2} \sum_{x, \mu} \text{tr}[U_{\mu}(x) \sigma^y U_{\mu}^{\dagger}(x) \sigma^y]$$

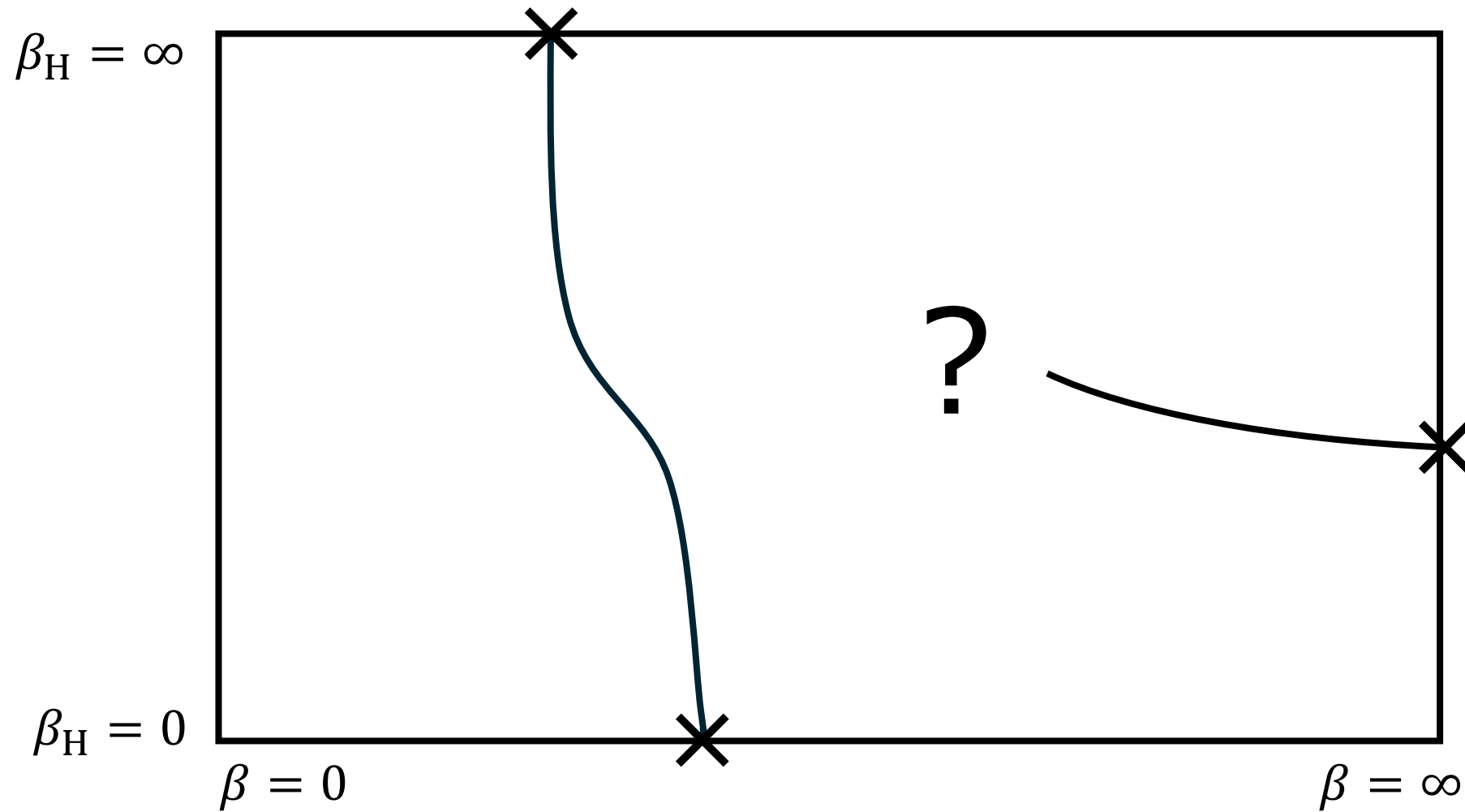
We use a lattice  $N_t \times N_s^3$  with finite  $N_t$  and large  $N_s$ .  
(Finite temperature)

$$S[U, \phi] = \frac{\beta}{2} S_{\text{Plaquette}}[U] - \frac{\beta_{\text{H}}}{2} \sum_{x, \mu} \text{tr}[U_{\mu}(x) \sigma^y U_{\mu}^{\dagger}(x) \sigma^y]$$

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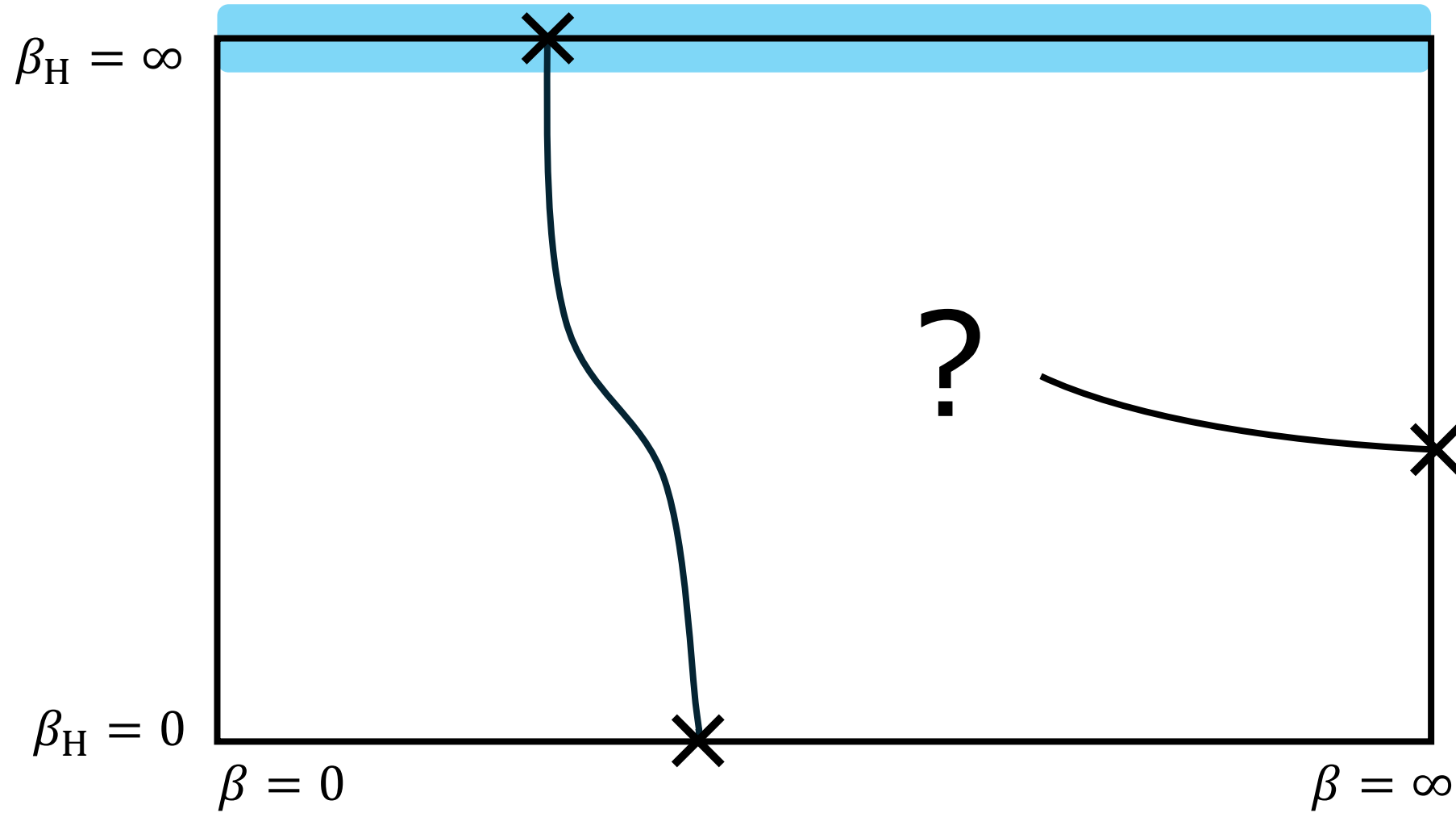
Instead of varying the physical temperature,  
we consider the  $\beta - \beta_{\text{H}}$  phase diagram.

$$S[U, \phi] = \frac{\beta}{2} S_{\text{Plaquette}}[U] - \frac{\beta_H}{2} \sum_{x, \mu} \text{tr}[U_\mu(x) \sigma^y U_\mu^\dagger(x) \sigma^y]$$



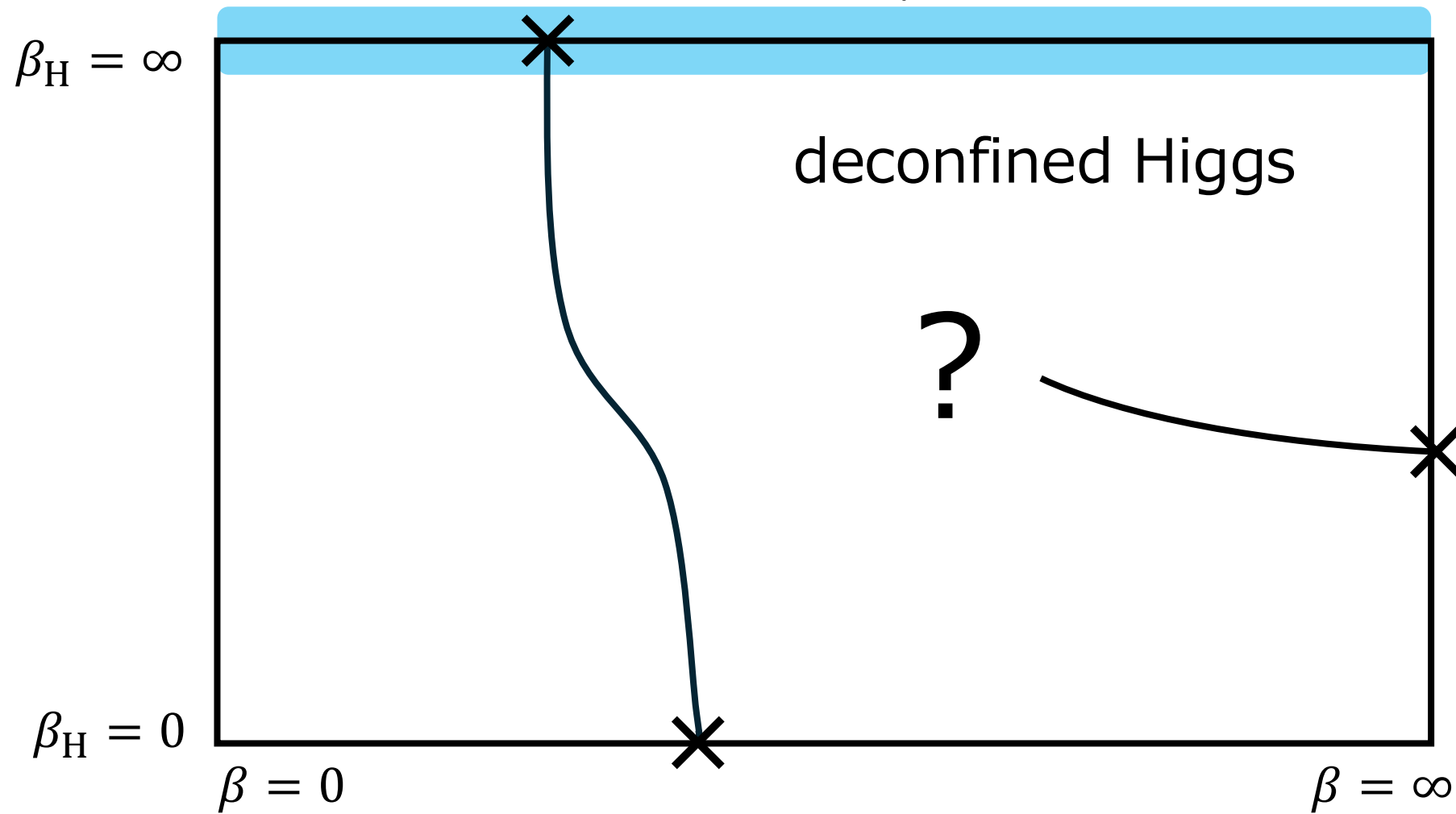
(schematic phase diagram based on [Karsch-Seiler-Stamatescu(1983)])

$$S[U, \phi] = \frac{\beta}{2} S_{\text{Plaquette}}[U] - \frac{\beta_H}{2} \sum_{x, \mu} \text{tr}[U_\mu(x) \sigma^y U_\mu^\dagger(x) \sigma^y]$$



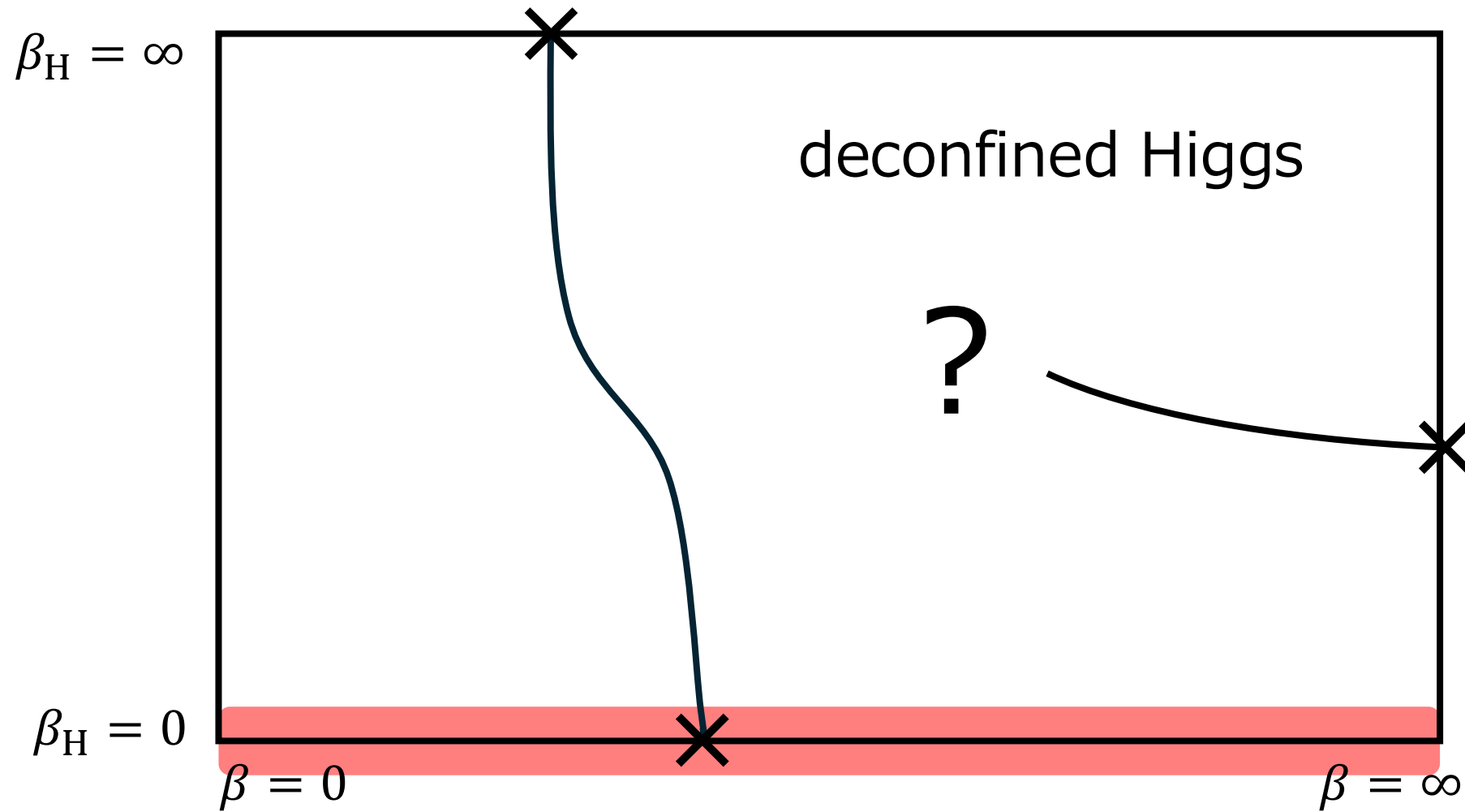
compact U(1) gauge theory

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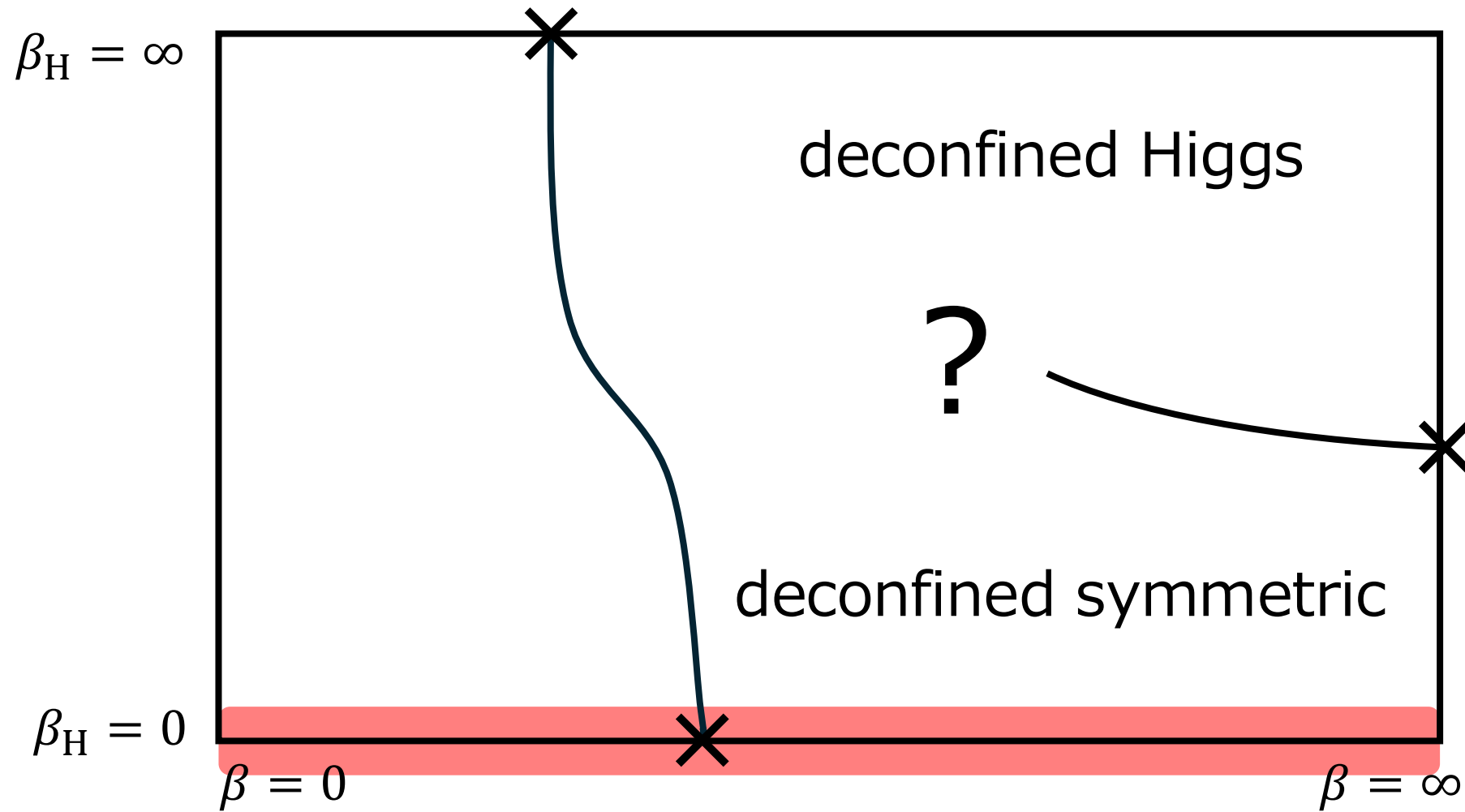
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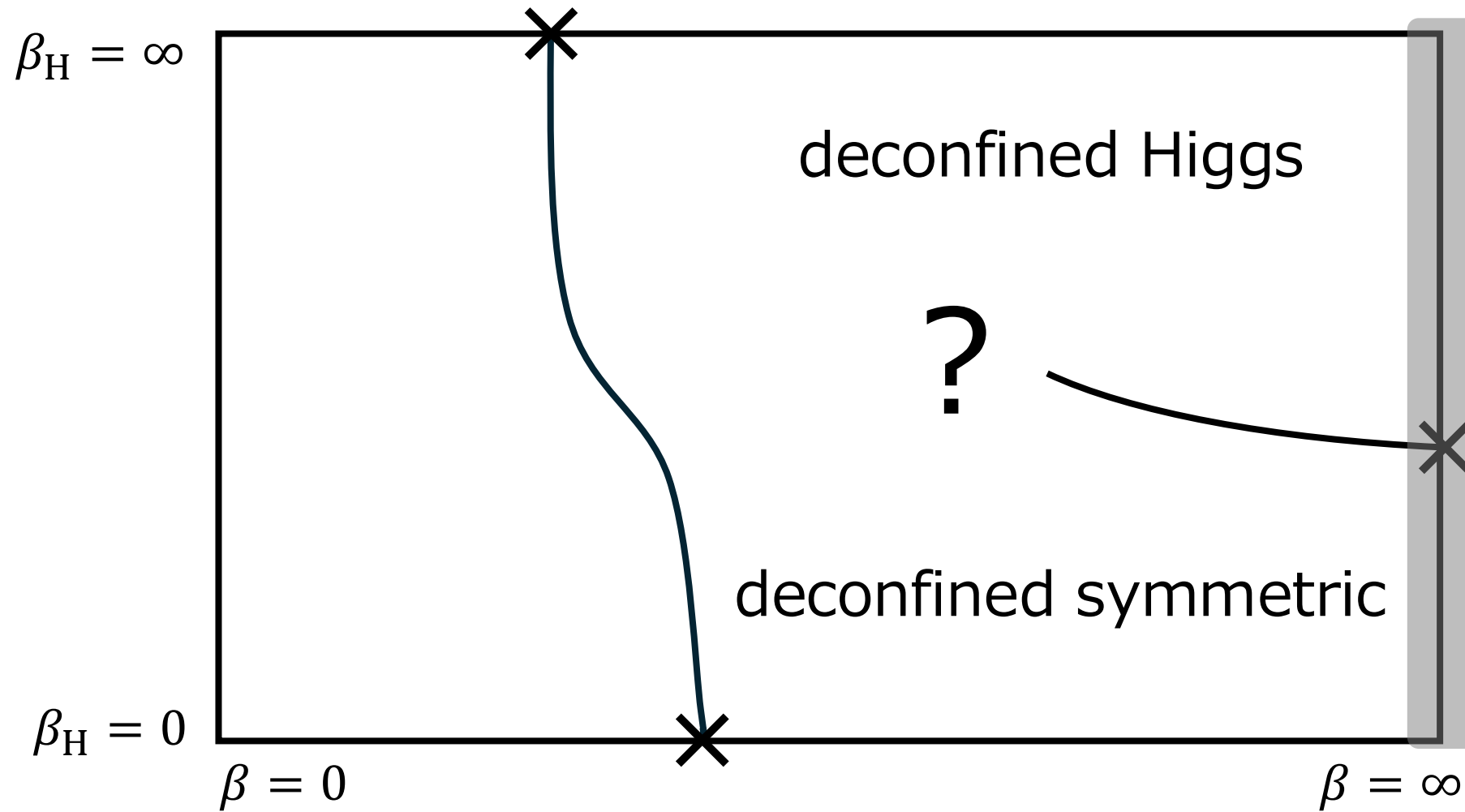
SU(2) gauge theory

$$S[U, \phi] = \frac{\beta}{2} S_{\text{Plaquette}}[U] - \frac{\beta_H}{2} \sum_{x, \mu} \text{tr}[U_\mu(x) \sigma^y U_\mu^\dagger(x) \sigma^y]$$



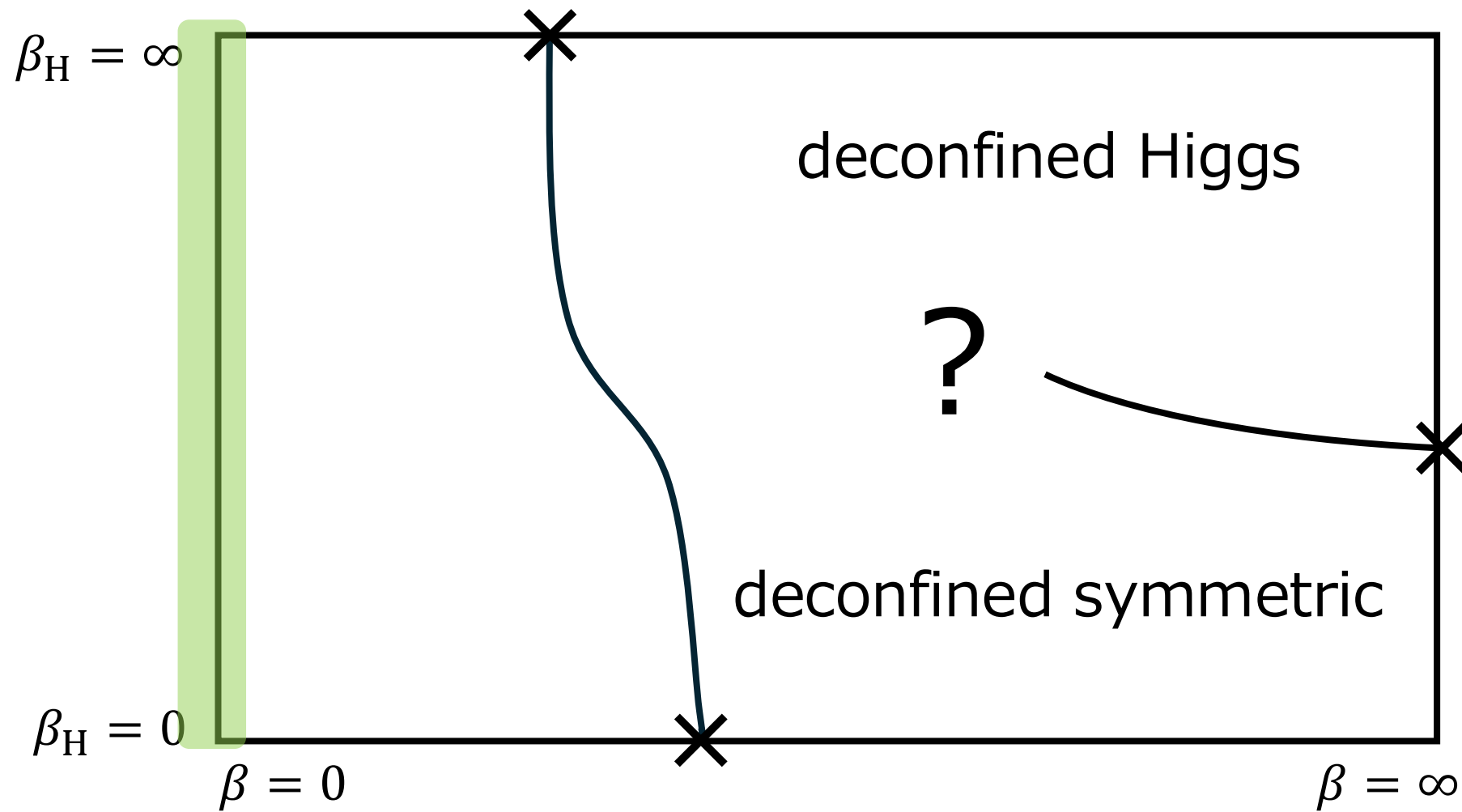
SU(2) gauge theory

$$S[U, \phi] = \frac{\beta}{2} S_{\text{Plaquette}}[U] - \frac{\beta_H}{2} \sum_{x, \mu} \phi^i(x) \phi^j(x + \hat{e}_\mu) \text{tr}[U_\mu(x) \sigma^i U_\mu^\dagger(x) \sigma^j]$$



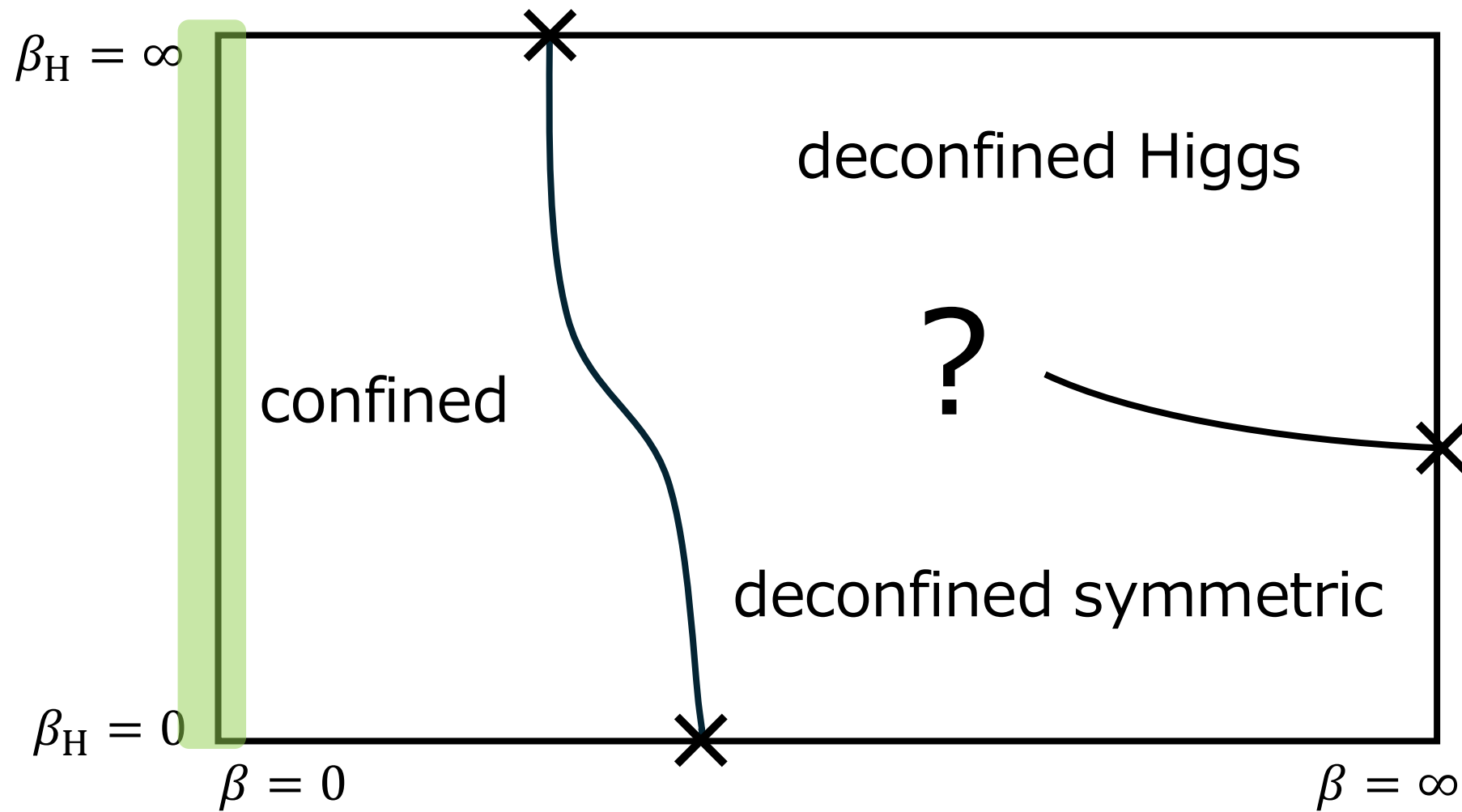
$O(3)$  Heisenberg model ( $\because U \rightarrow 1$  as  $\beta \rightarrow \infty$ )

$$S[U, \phi] = \frac{\beta}{2} S_{\text{Plaquette}}[U] - \frac{\beta_H}{2} \sum_{x, \mu} \text{tr}[U_\mu(x) \sigma^y U_\mu^\dagger(x) \sigma^y]$$



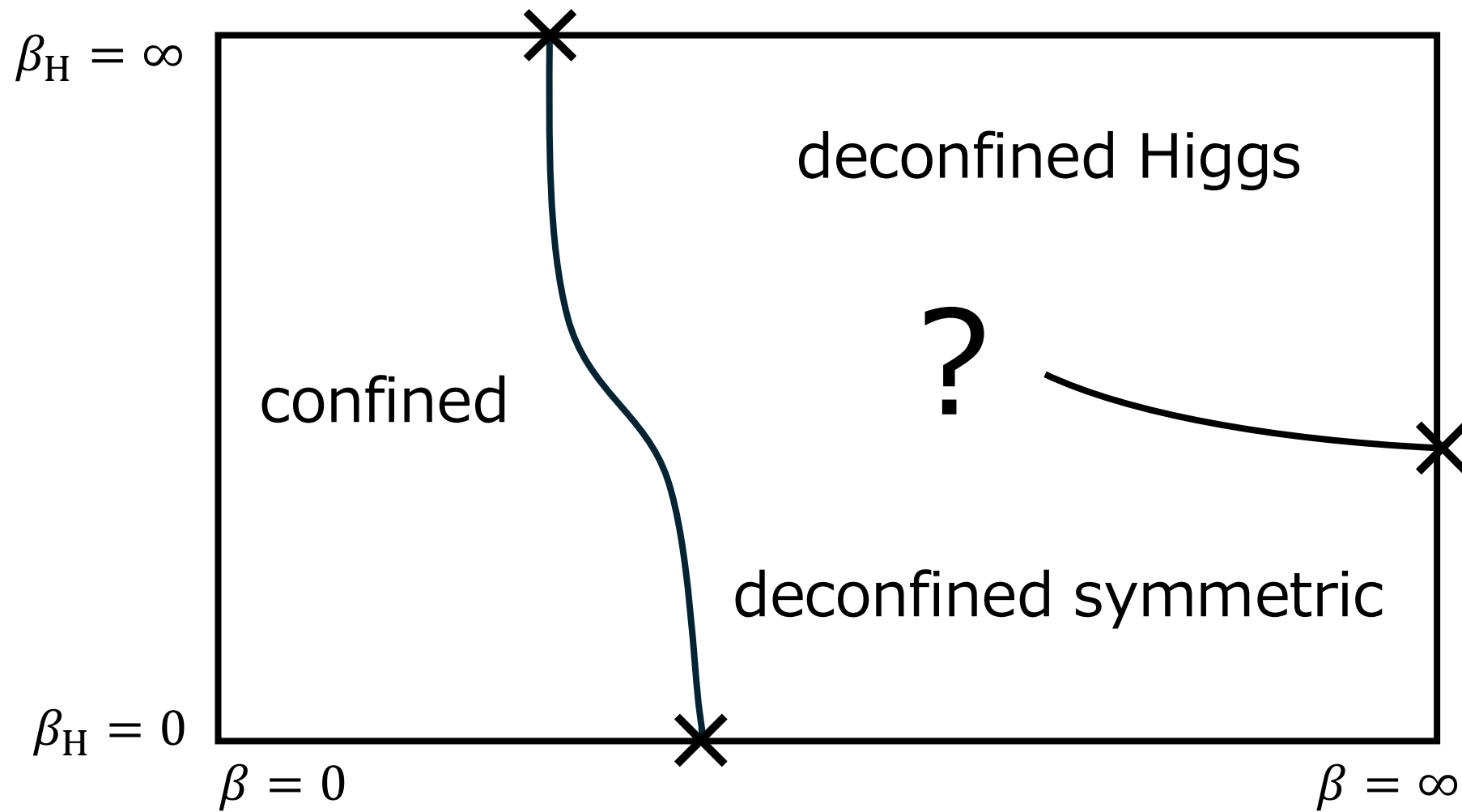
confined

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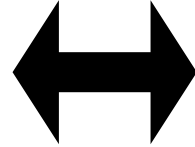
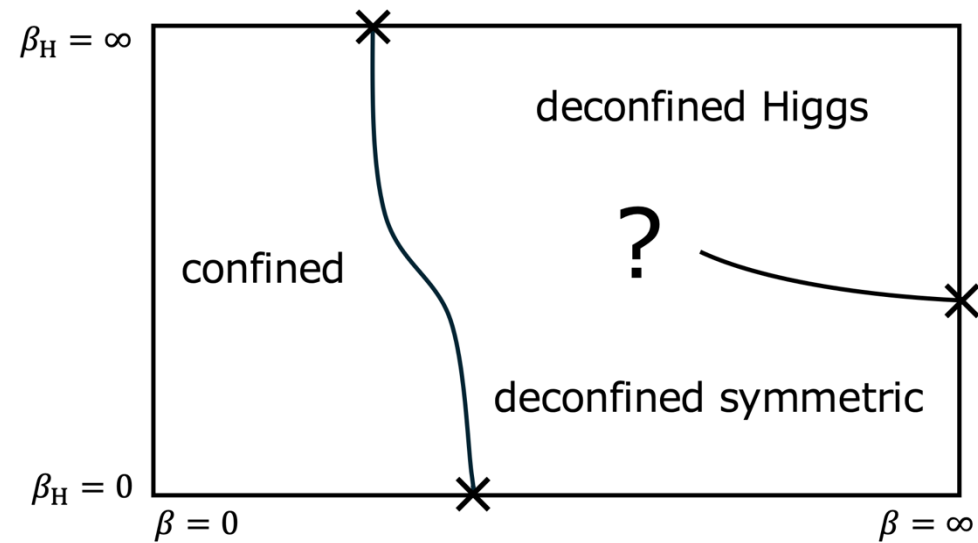
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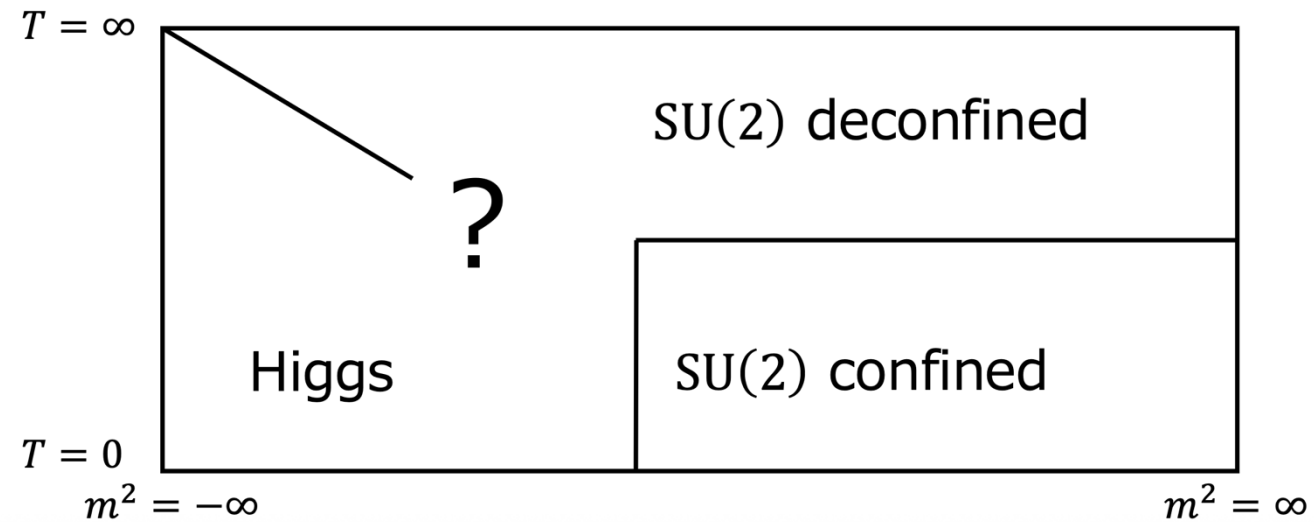


# We qualitatively expect the correspondence

the phase diagram of  
this lattice model



the phase diagram of  
the original adjoint Higgs theory



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# Center-destabilizing deformation

$$S'[U, \phi] = S[U, \phi] - c \sum_{\vec{x}:\text{spatial}} |\text{tr } P(\vec{x})|^2$$

$c$  : deformation parameter

# Center-destabilizing deformation

$$S'[U, \phi] = S[U, \phi] - c \sum_{\vec{x}:\text{spatial}} |\text{tr } P(\vec{x})|^2$$

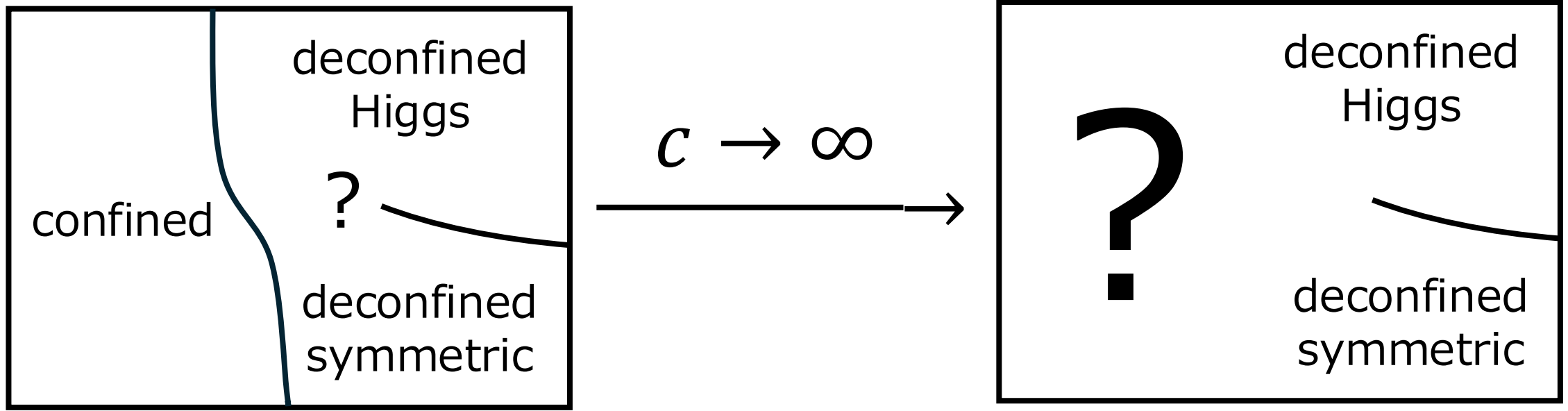
$c$  : deformation parameter

$$c \rightarrow \infty \quad \Rightarrow \quad P \rightarrow \underline{\pm} I$$

(center-destabilizing)

# Assumption

No phase transition  
in deconfined symmetric and deconfined Higgs phase  
under center-destabilizing deformation



After a dimensional reduction (taking  $N_t = 1$ ), we have three kinds of fields.

▶  $U_m(\vec{x}) \in SU(2) \quad m = 1, 2, 3$

▶  $U_4(\vec{x}) \in \mathbb{Z}_2$  Polyakov loop  $\text{tr}(P) = \text{tr}(U_4)$   
(center-destabilizing)

▶  $\phi^i(\vec{x}) \quad \sum_{i=1}^3 (\phi^i)^2 = 1$

# The three-dimensional model

$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}[U]$$

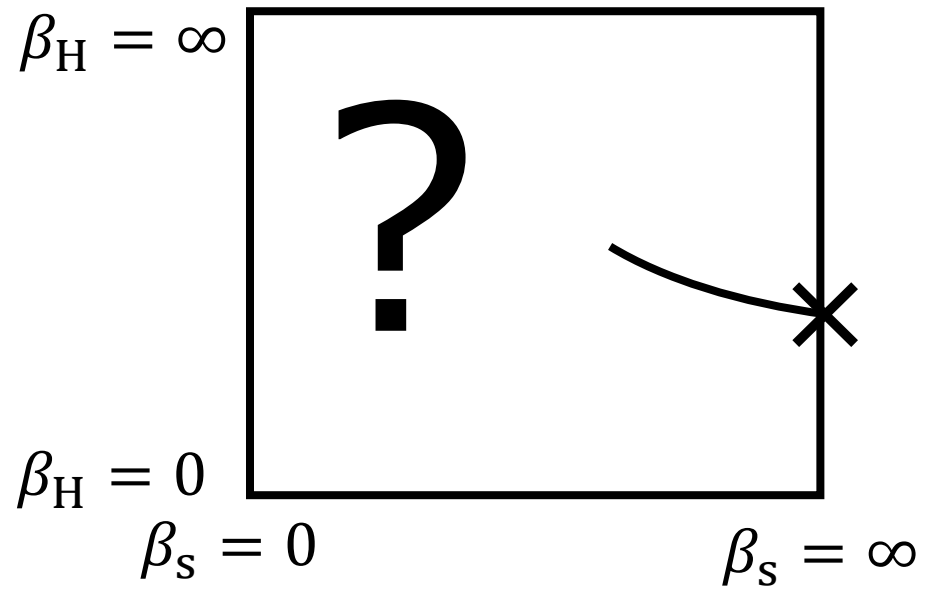
$$- \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x}) U_4(\vec{x} + \hat{e}_m)]$$

$$- \frac{\beta_H}{2} \sum_{\vec{x}, m} \phi^i(\vec{x}) \phi^j(\vec{x} + \hat{e}_m) \text{tr}[U_m(\vec{x}) \sigma^i U_m^\dagger(\vec{x}) \sigma^j]$$

We keep  $\beta_t$  large so that  $\langle \text{tr}(U_4) \rangle \neq 0$ .

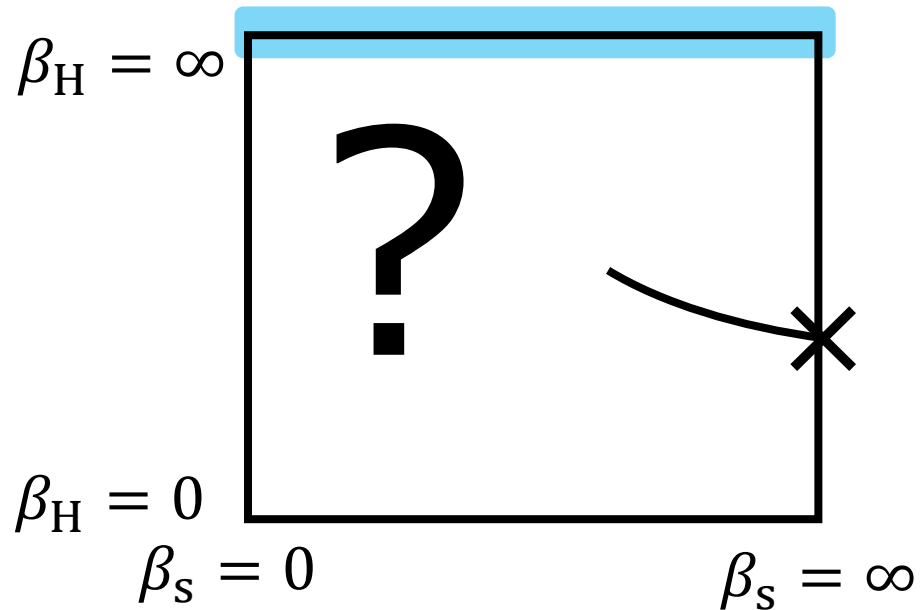
$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

$$- \frac{\beta_H}{2} \sum_{\vec{x}, m} \phi^i(\vec{x})\phi^j(\vec{x} + \hat{e}_m) \text{tr}[U_m(\vec{x})\sigma^i U_m^\dagger(\vec{x})\sigma^j]$$



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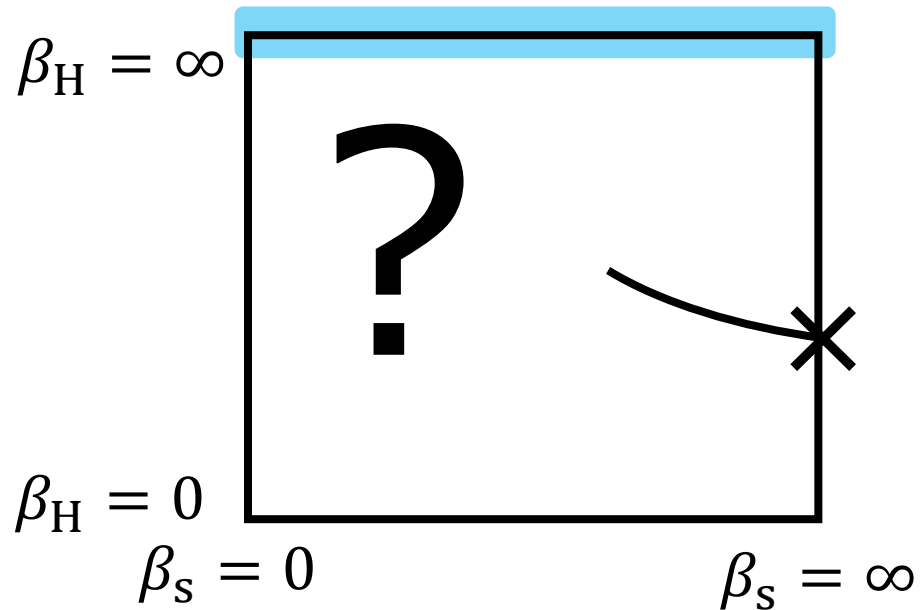
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$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}^{U(1)}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

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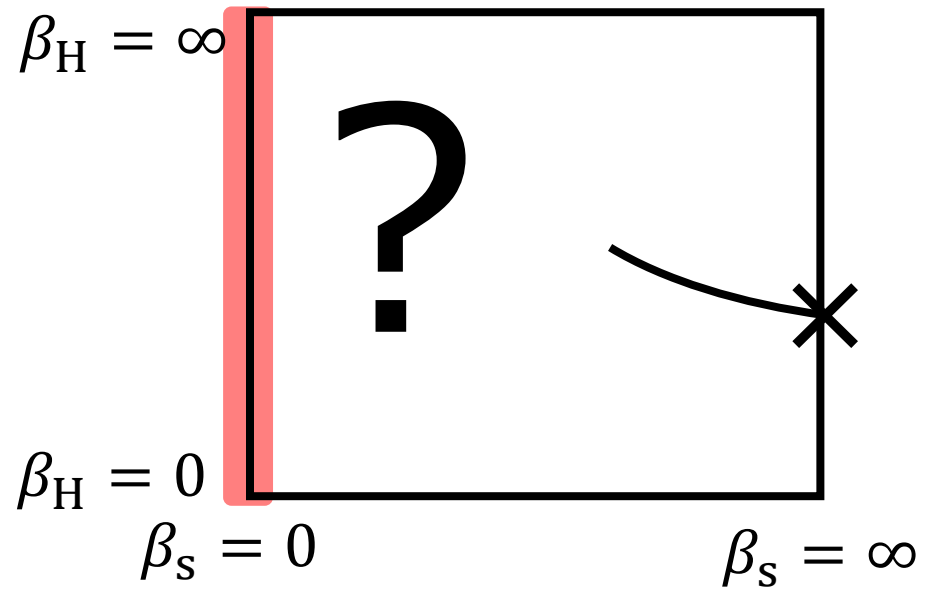


$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}^{U(1)}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

► No phase transition

$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

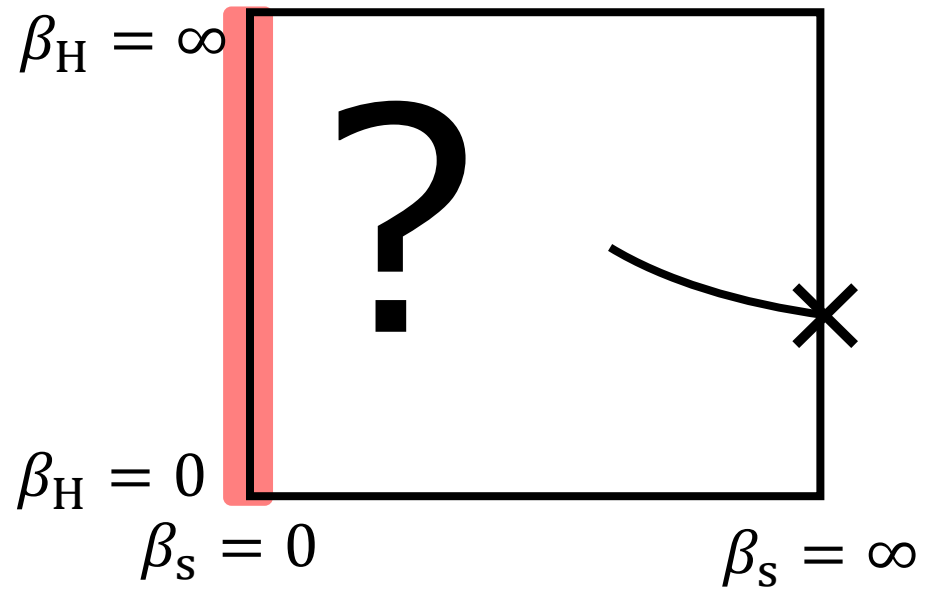
$$- \frac{\beta_H}{2} \sum_{\vec{x}, m} \phi^i(\vec{x})\phi^j(\vec{x} + \hat{e}_m) \text{tr}[U_m(\vec{x})\sigma^i U_m^\dagger(\vec{x})\sigma^j]$$



$$S_{3d}[U, \phi] = -\frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)] - \frac{\beta_H}{2} \sum_{\vec{x}, m} \phi^i(\vec{x})\phi^j(\vec{x} + \hat{e}_m) \text{tr}[U_m(\vec{x})\sigma^i U_m^\dagger(\vec{x})\sigma^j]$$

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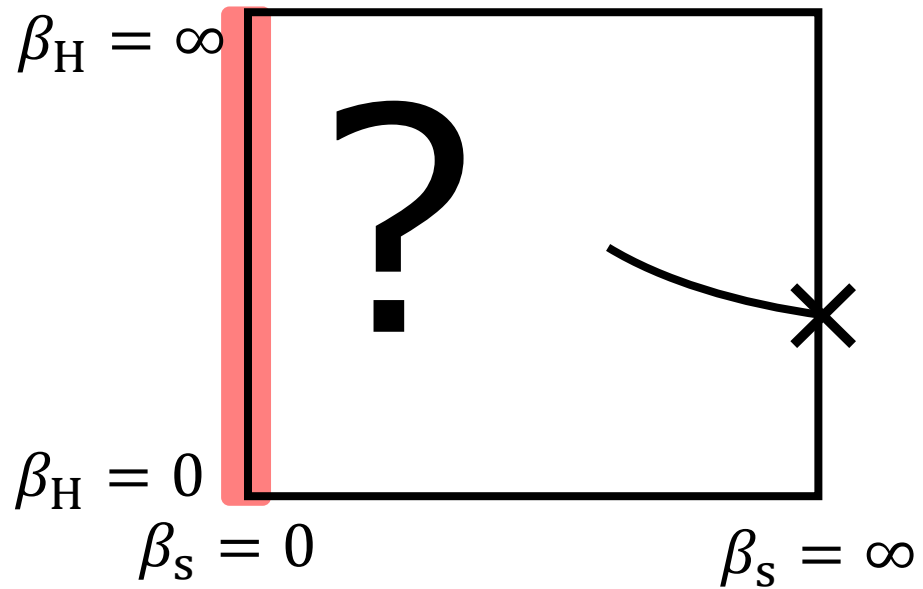


$$S_{3d}[U, \phi] = -\frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)] - \frac{\beta_H}{2} \sum_{\vec{x}, m} \phi^i(\vec{x})\phi^j(\vec{x} + \hat{e}_m) \text{tr}[U_m(\vec{x})\sigma^i U_m^\dagger(\vec{x})\sigma^j]$$

$\xrightarrow{\hspace{10em}}$   $\text{tr}[U_m(\vec{x})\sigma^y U_m^\dagger(\vec{x})\sigma^y]$

$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

$$- \frac{\beta_H}{2} \sum_{\vec{x}, m} \phi^i(\vec{x})\phi^j(\vec{x} + \hat{e}_m) \text{tr}[U_m(\vec{x})\sigma^i U_m^\dagger(\vec{x})\sigma^j]$$



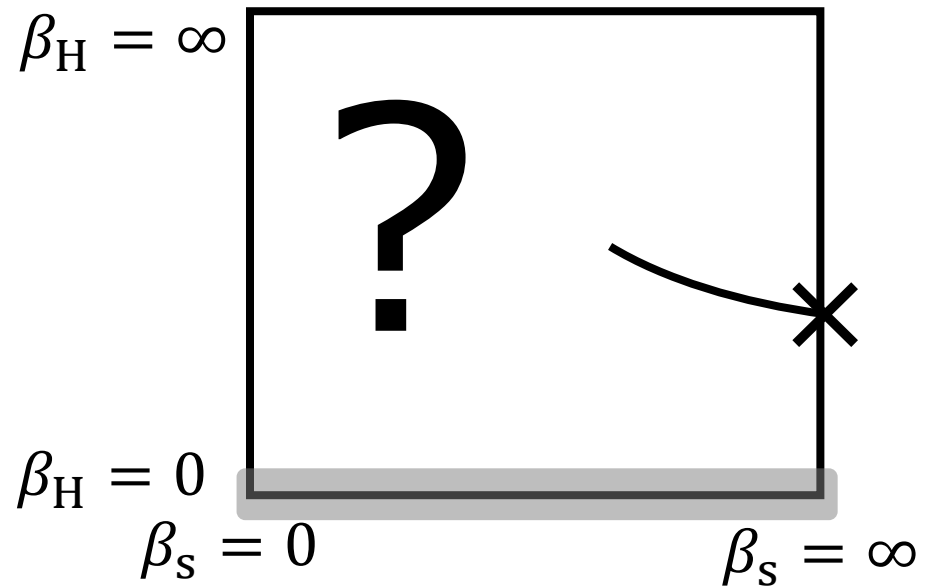
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$\text{tr}[U_m(\vec{x})\sigma^y U_m^\dagger(\vec{x})\sigma^y]$

▶ No phase transition

$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

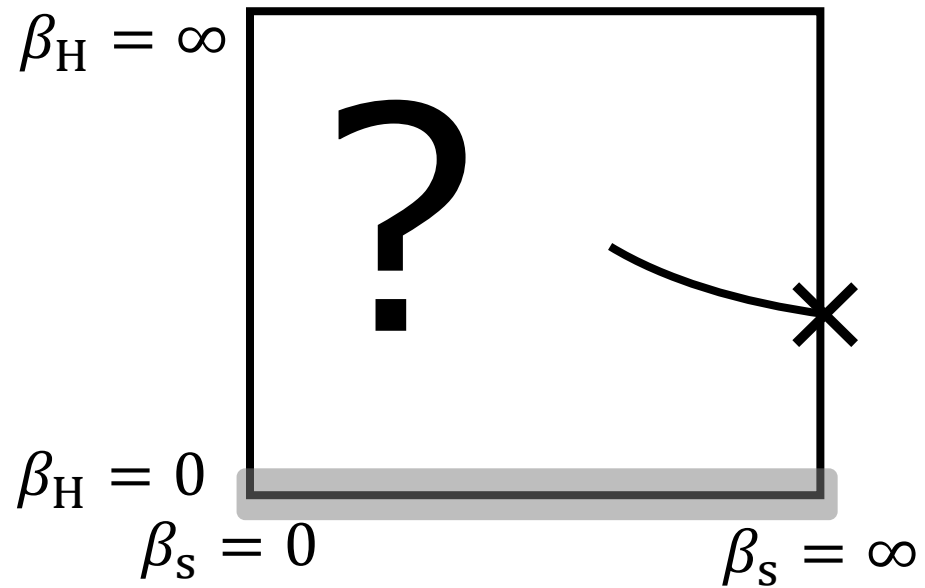
$$- \frac{\beta_H}{2} \sum_{\vec{x}, m} \phi^i(\vec{x})\phi^j(\vec{x} + \hat{e}_m) \text{tr}[U_m(\vec{x})\sigma^i U_m^\dagger(\vec{x})\sigma^j]$$



$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

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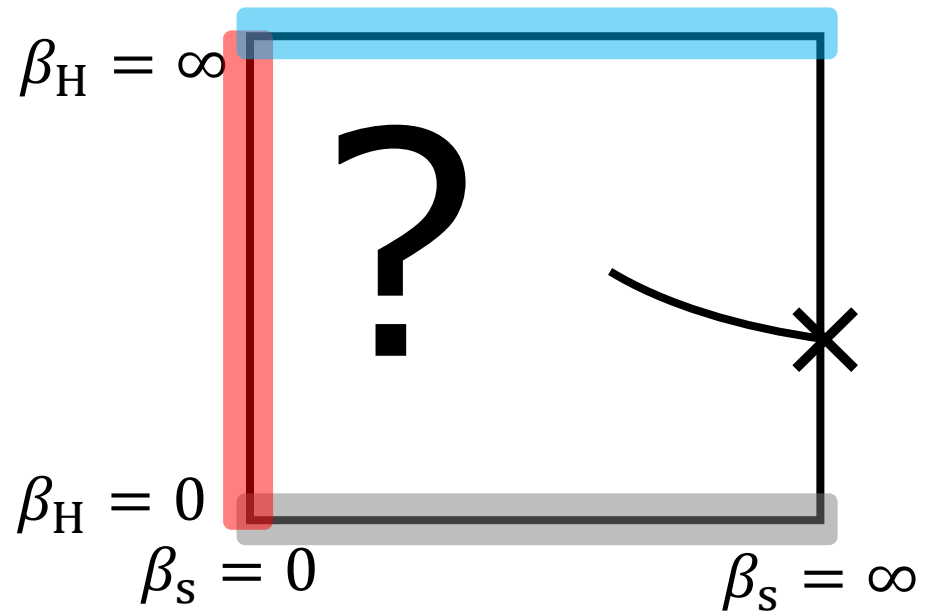


$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

► No phase transition

$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

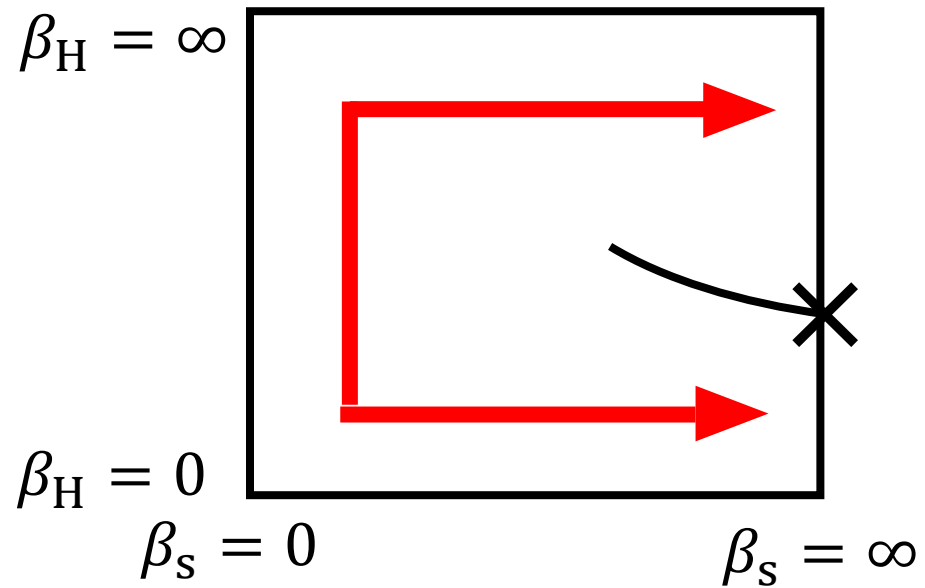
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No phase transition

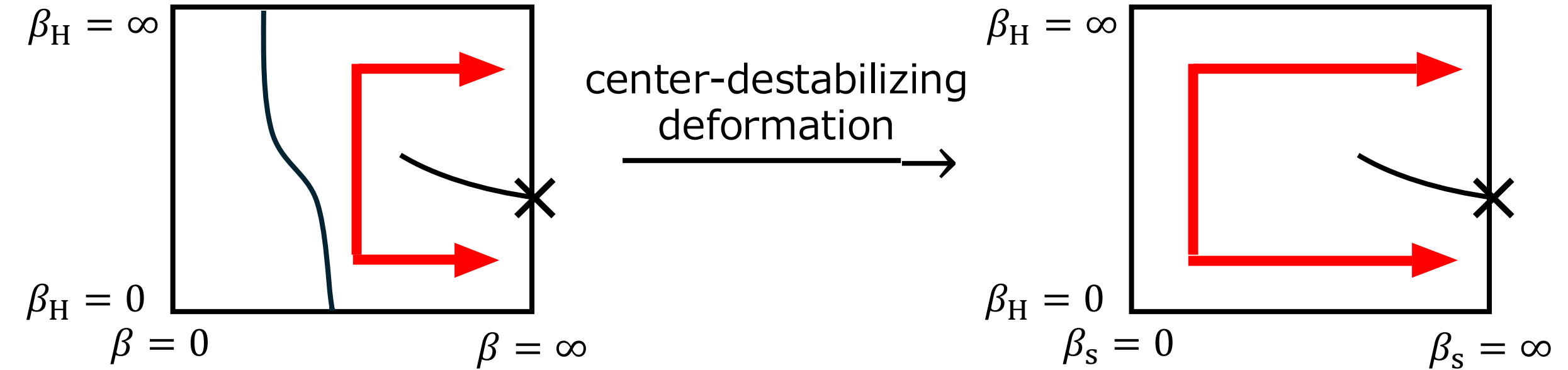
$$S_{3d}[U, \phi] = \frac{\beta_s}{2} S_{3d \text{ Plaquette}}[U] - \frac{\beta_t}{2} \sum_{\vec{x}, m} \text{tr}[U_4(\vec{x})U_4(\vec{x} + \hat{e}_m)]$$

$$- \frac{\beta_H}{2} \sum_{\vec{x}, m} \phi^i(\vec{x})\phi^j(\vec{x} + \hat{e}_m) \text{tr}[U_m(\vec{x})\sigma^i U_m^\dagger(\vec{x})\sigma^j]$$



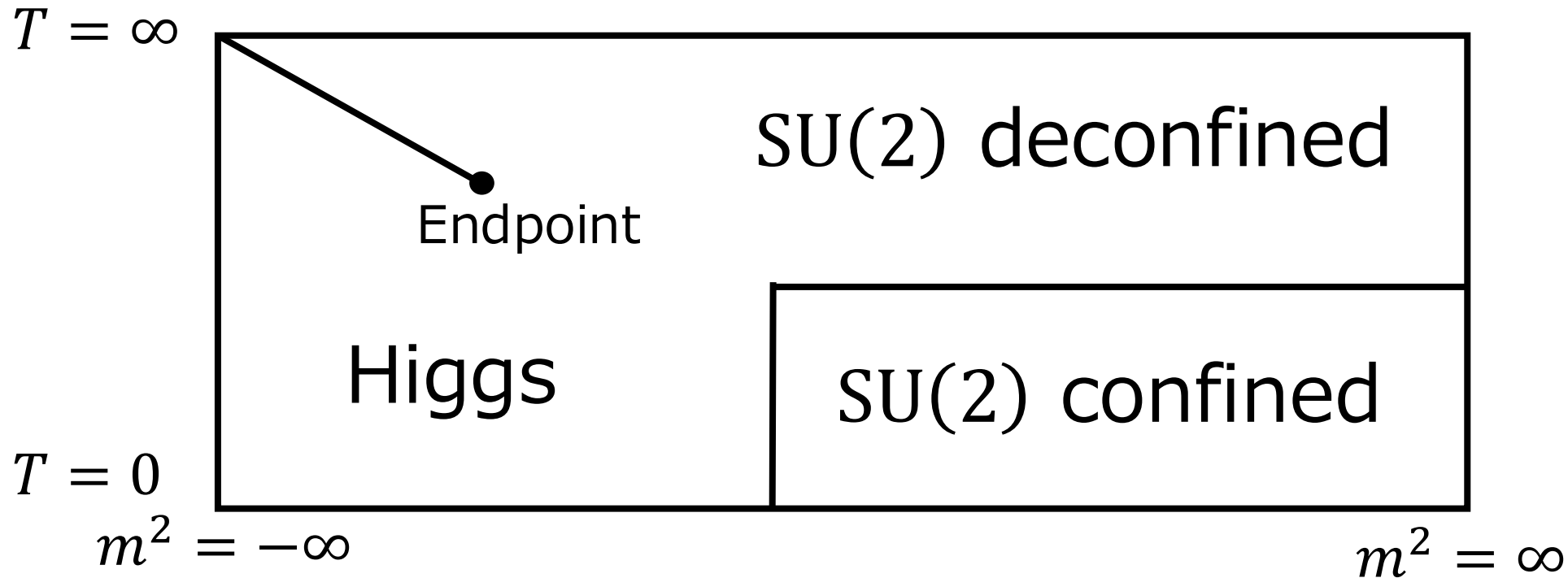
Continuity

If no phase transition occurs under center-destabilization,  
it means the continuity in the original phase diagram.



- Motivation
- Our setup and proposal
- Lattice model and its schematic phase diagram
- Center-destabilizing deformation
- **Summary and Discussion**

In this talk, I discussed the possibility of deconfinement-Higgs continuity.



However, the center-destabilizing deformation ignores the temporal dynamics.

Thus, an analysis **with full dynamics** is ultimately required.

We also performed a preliminary **Monte Carlo analysis**, which is currently consistent with this continuity.

However, several issues remain, such as evaluating the finite-volume scaling.