Baryonic Vortex Lattice and Other Topological Phases in Low Energy QCD

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Motivation: Phases at Finite Density and Magnetic Field via χPT

Low Energy Regime: Chiral Symmetry Breaking

$$U(1)_V \times SU(N_f)_L \times SU(N_f)_R \to U(1)_V \times SU(N_f)_V$$

EFT: $SU(N_f = 2)$ Chiral Perturbation Theory (χPT)

$$\mathcal{L}_{\text{chiral}} = \frac{f_\pi^2}{2} \text{Tr}\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right), \quad \Sigma = \begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix}$$

- Electromagnetic $U(1)$: $A_\mu = A_\varphi(\rho)\hat{\varphi}$ for $\vec{B} = B\hat{z}$; $\mathcal{L}_{\text{EM}} = \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$
- Density: $A_\mu^{I/B} \equiv \left(\mu_{I/B}, \vec{0}\right)$ for isospin/baryon chemical potentials;

➢ **Wess-Zumino-Wittern term for Anomaly** $\mathcal{L}_{\text{WZW}} = \left(A_\mu^B + qA_\mu\right) j_B^\mu$

Issue: what is the ground state / phase? Key: $-\mu_B N_B$.

μ_I: Pion Condensate → Abrikosov Vortex Lattice

μ_B: Chiral Soliton Lattice vs. (Baryon) Skyrmion Crystal

Inspiration (i): $\pi^{\pm}$ Vortex in μ_I

Charged Pion $\pi^{\pm}$ Condensate $\xrightarrow{B}$ Superconducting Vortex

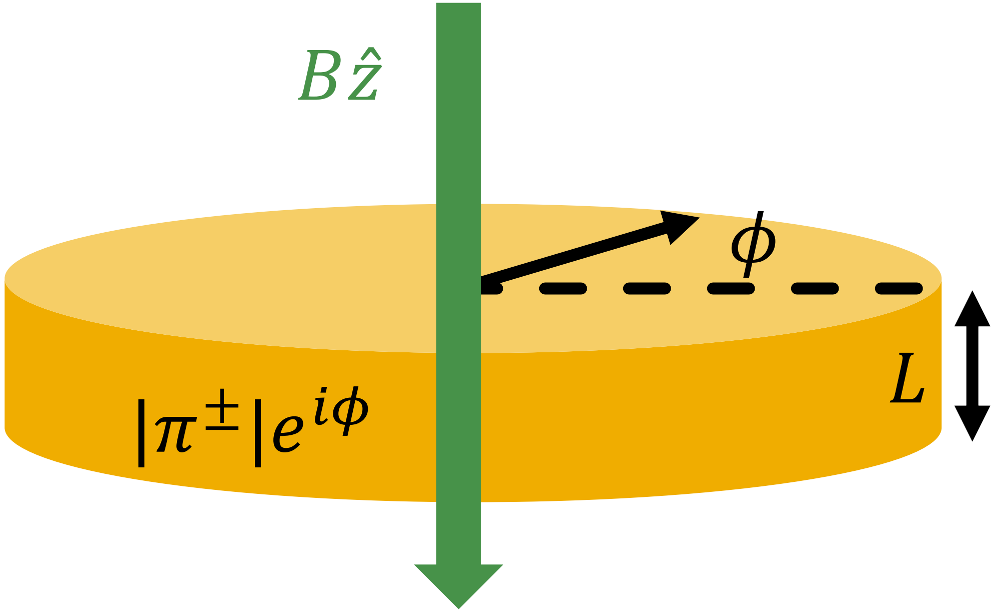
Dynamics: minimizing free energy $\mathcal{E}$.

A portion in $\mathcal{E}$ favors $|\pi^{\pm}|$:

$$V_{\rm eff} = -\frac{f_\pi^2}{2}\mu_I^2\,|\phi_2|^2$$

External magnetic field exceeds critical value:

$$B_{\rm ext} > \frac{\mathcal{E}}{L\Phi_0} \Rightarrow \text{Type II SC}$$



P. Adhikari, E. Leeser and J. Markowski, Mcd. Phys. Lett. A 38 (2023) 2350078
M.S. Grønli and T. Brauner, Eur. Phys. J. C 82 (2022) 354 …

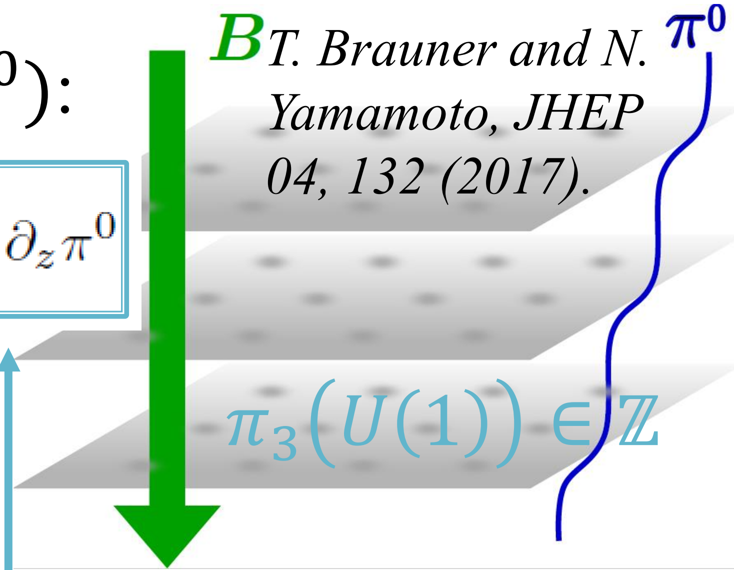
Inspiration (ii): π^0 DW in μ_B

Neutral Pion $\pi^0 \xrightarrow{B}$ Chiral Soliton Lattice / π^0 domain wall

π^0 Hamiltonian w/ $\Sigma \to \exp(i\tau^3\pi^0)$:

$$\mathcal{E} \to \frac{f_\pi^2}{2}\left(\partial_z \pi^0\right)^2 + m_\pi^2 f_\pi^2 \left(1 - \cos\pi^0\right) \boxed{-\frac{\mu_B B}{4\pi^2}\partial_z \pi^0}$$

Solution is a Sine-Gordon soliton



WZW term $\mathcal{L}_{\mathrm{WZW}} = \left(A_\mu^B + qA_\mu\right) j_B^\mu$

topological charge = baryon number

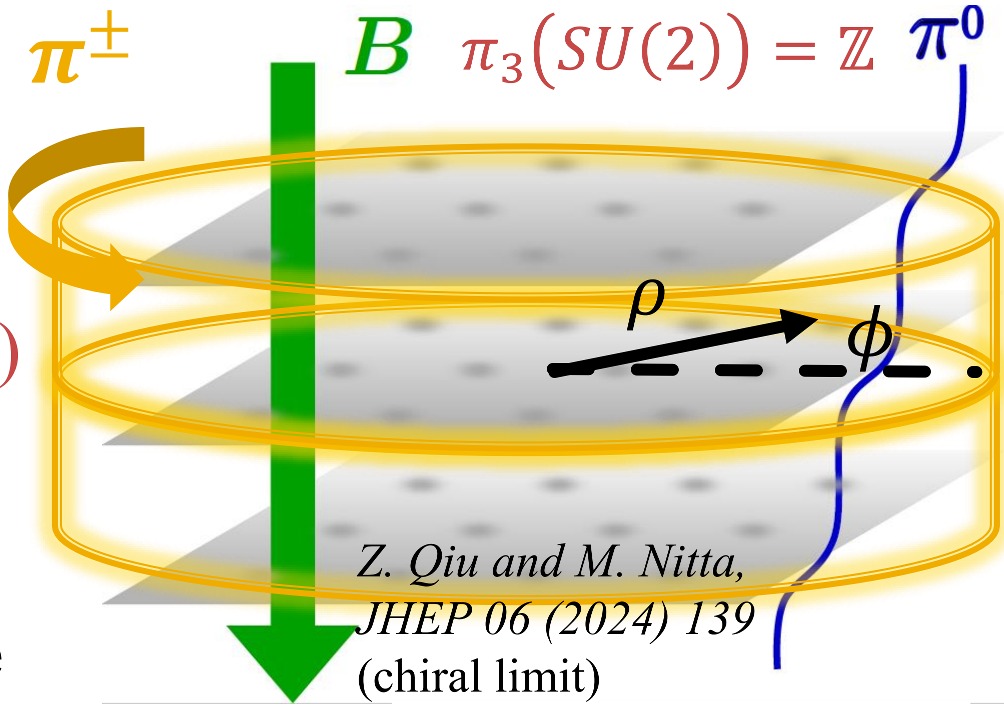
$$j_B^\mu = -\frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\left\{\mathrm{tr}[(\Sigma D_\nu \Sigma^\dagger)(\Sigma D_\alpha \Sigma^\dagger)(\Sigma D_\beta \Sigma^+)] \right.$$
$$\left. \boxed{-\frac{3ie}{2}F_{\nu\alpha}\,\mathrm{tr}[Q(\Sigma D_\beta \Sigma^+ + D_\beta \Sigma^\dagger \Sigma)]}\right\}$$

j_B^0 can also be from $\pi_3(SU(2)) \in \mathbb{Z}$ i.e., Skyrmion

Baryon as Vortex Skyrmion in $\mu_{B/I}$

- $\boldsymbol{\mu_B}$ reduces energy by **WZW term** $-\mu_B \int j_B^0 d^3x$

Vortex-Skyrmion
$= \pi^0$ *chiral soliton*
$+ \pi^{\pm}$ *vortex*
(homotopic to Skyrmion)



Z. Qiu and M. Nitta, JHEP 06 (2024) 139
(chiral limit)

- $\boldsymbol{\mu_I}$ requests the boundary condition of **pion condensate**

$$\Sigma = \begin{pmatrix} \phi_1(\rho, z) & -|\phi_2|(\rho, z)\, e^{-i\varphi} \\ |\phi_2|(\rho, z)\, e^{i\varphi} & \phi_1^*(\rho, z) \end{pmatrix}$$

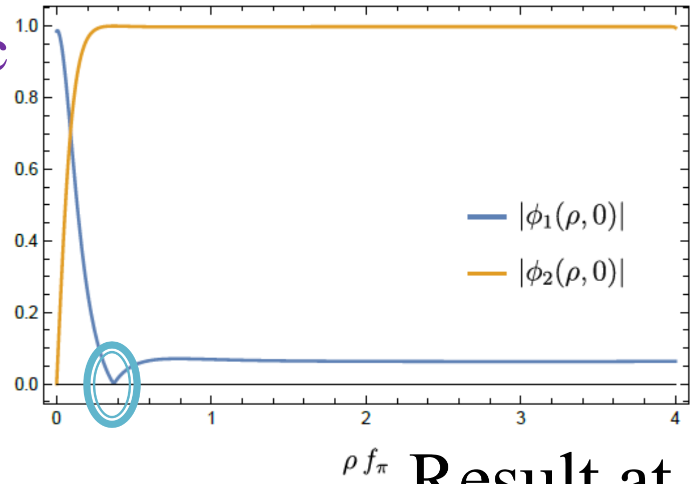
$$|\phi_1|(\rho \to \infty) = m_\pi^2/\mu_I^2$$

$$|\phi_2|(\rho \to \infty) = \sqrt{1 - m_\pi^4/\mu_I^4}$$

Profile (i): Magnitude/Abs

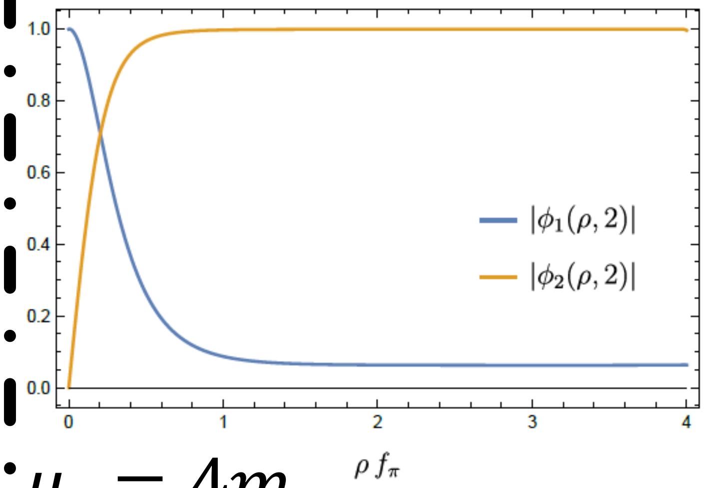
"Baryonic Vortex" solution

π^0 zeros: global (ungauged) vortex



(a)

Result at $\mu_I = 4m_\pi$



(b)

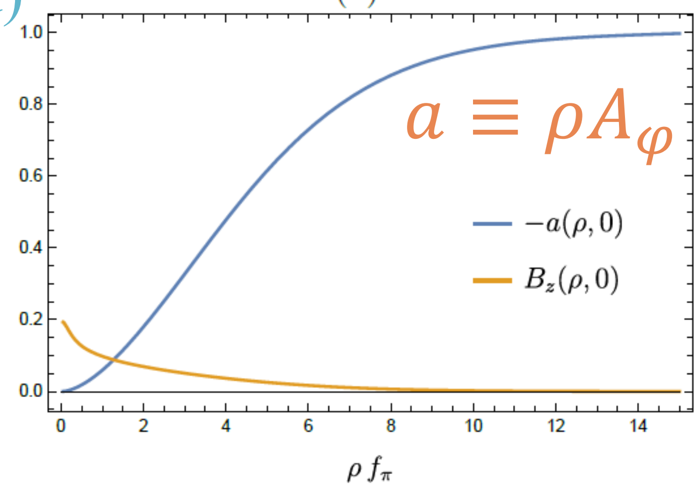
Right Panel @ $z = 2$

Y. Hamada, M. Nitta, and Z. Qiu JHEP 02 (2026) 200

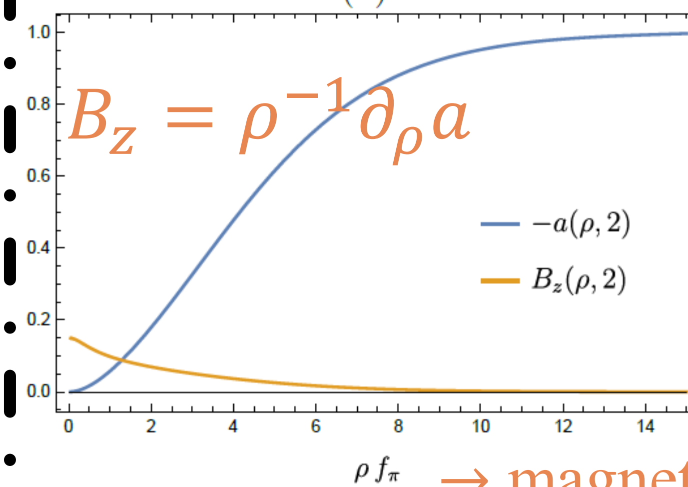
$a \equiv \rho A_\varphi$

$B_z = \rho^{-1}\partial_\rho a$

Left Panel @ $z = 0$



(c)



(d)

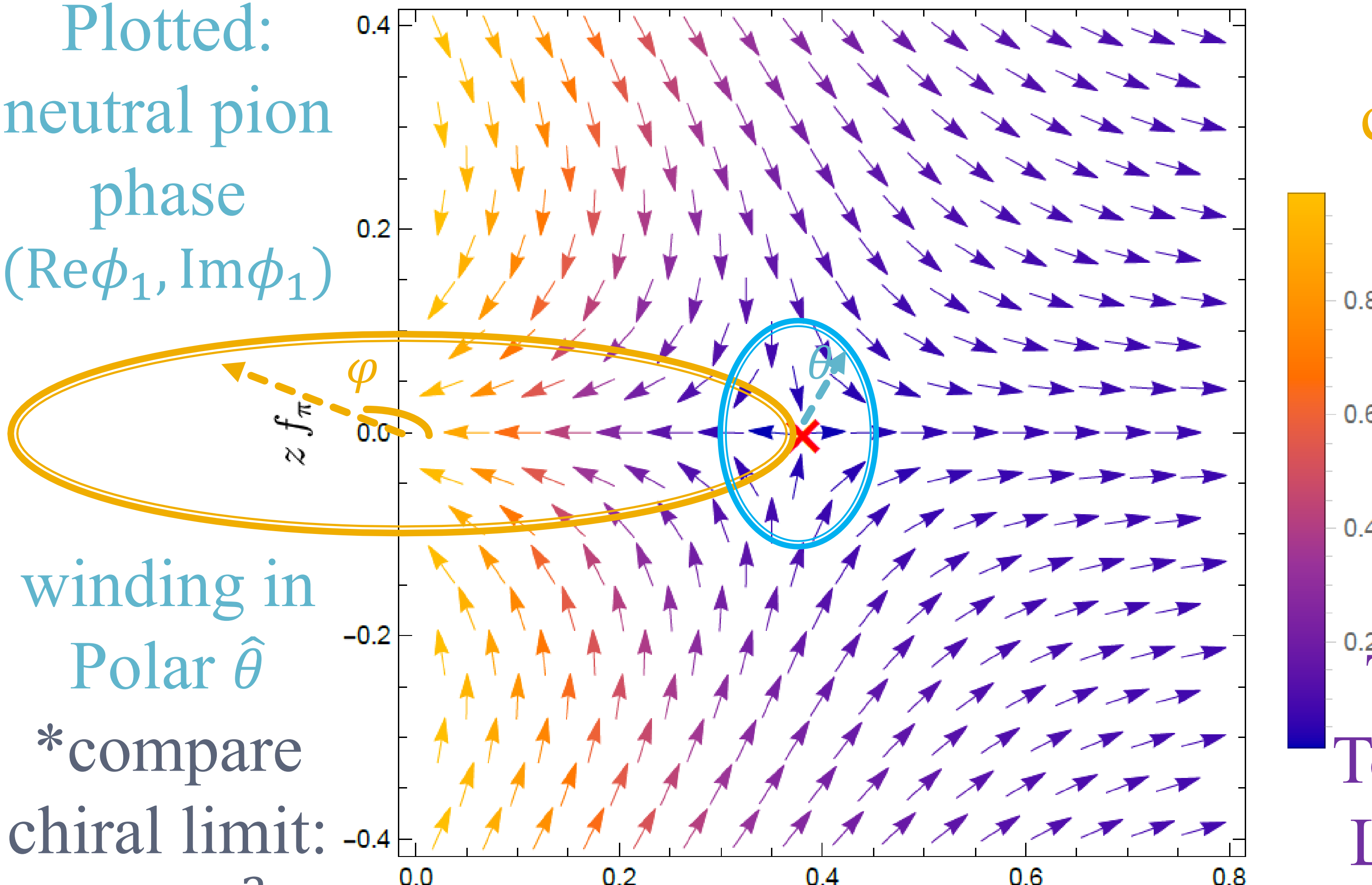
$\pi^\pm$ local vortex: type-II → magnetic flux

Profile (ii): Phase/Arg

Plotted: neutral pion phase $(\mathrm{Re}\phi_1, \mathrm{Im}\phi_1)$

winding in Polar $\hat{\theta}$

*compare chiral limit: $\mathrm{Arg}(\phi_1) = \frac{2\pi}{L} z$



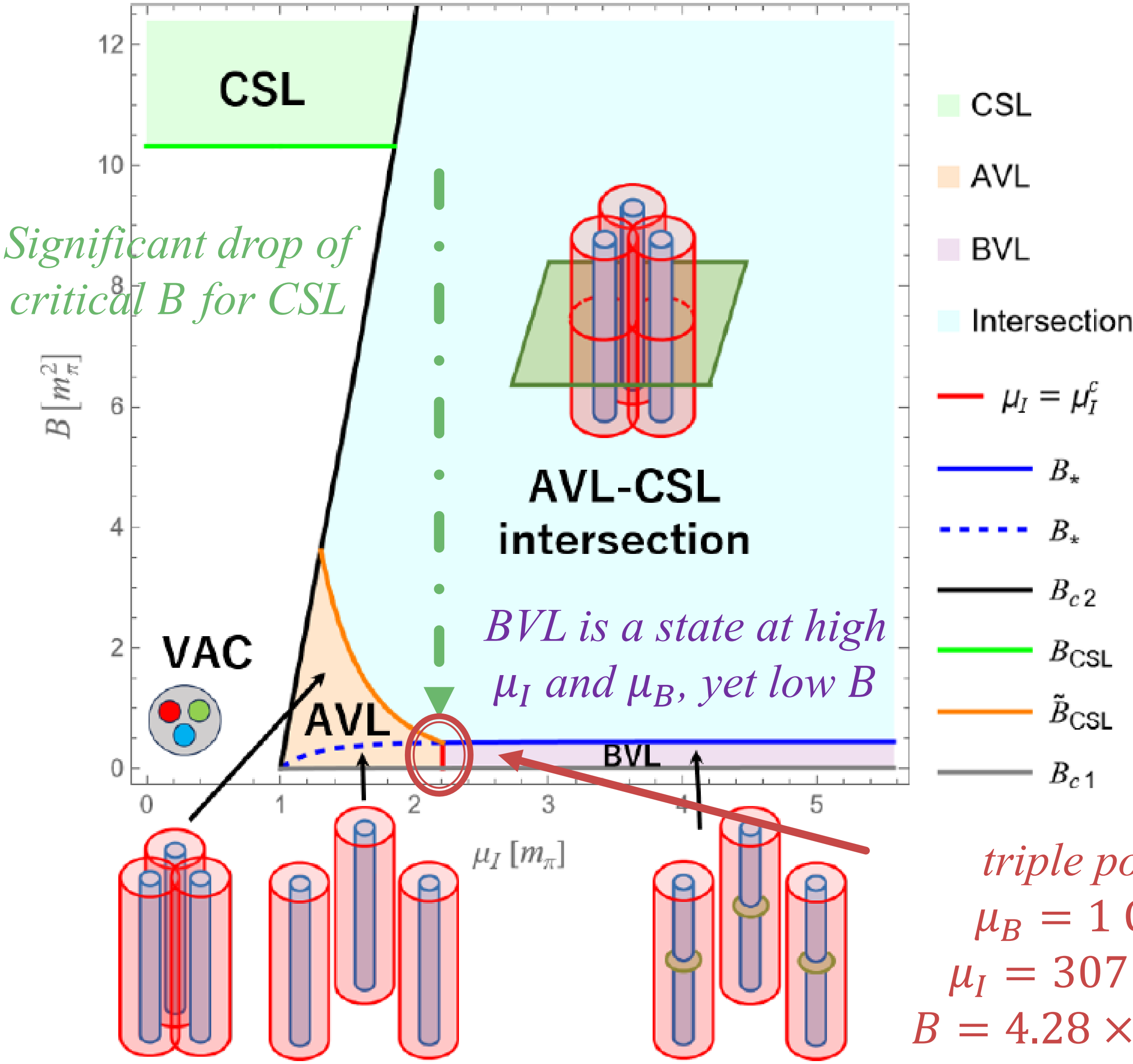
Nitta-Qiu JHEP 06, 139 (2024).

$(0.4 f_\pi^{-1} \sim 0.85\mathrm{fm})$

Unplotted: charged pion vortex $\mathrm{Arg}(\phi_2) \equiv \varphi$ winding in azimuthal $\hat{\varphi}$

Together: Topological Linking → Baryon Number!

Intersection and Phase Diagram



triple point:
$\mu_B = 1$ GeV,
$\mu_I = 307$ MeV,
$B = 4.28 \times 10^{17}$ G

Hamada-Nitta-Qiu
arXiv: 2602.11762

CSL: Domain Wall of π^0, by μ_B and B

AVL: Abrikosov Vortex Lattice of $\pi^{\pm}$, by μ_I and B.

BVL: Baryonic Vortex Lattice (proposed by us), in μ_I, μ_B and B.

Intersection:

- **BVL lattice period < penetration depth → nearly uniform B accomodates CSL**
- **AVL with $\mu_B B$ → induces CSL**
- **CSL with μ_I, B → incurring AVL**

Conclusion & Outlook

Conclusion:
We find a topological soliton homotopic to Skyrmion but composed of linking pion vortices. It carries a baryon number and preserves a magnetic flux, thus dubbed "Baryonic Vortex".
It leads to the novel phase structure involving Baryonic Vortex Lattice and CSL-AVL intersection, at finite isospin/baryon chemical potentials and magnetic field.

Outlook:

- *Phase transition towards traditional Skyrme Crystal?*
- *Application in explaining Neutron Star magnetic field?*

Acknowledgement

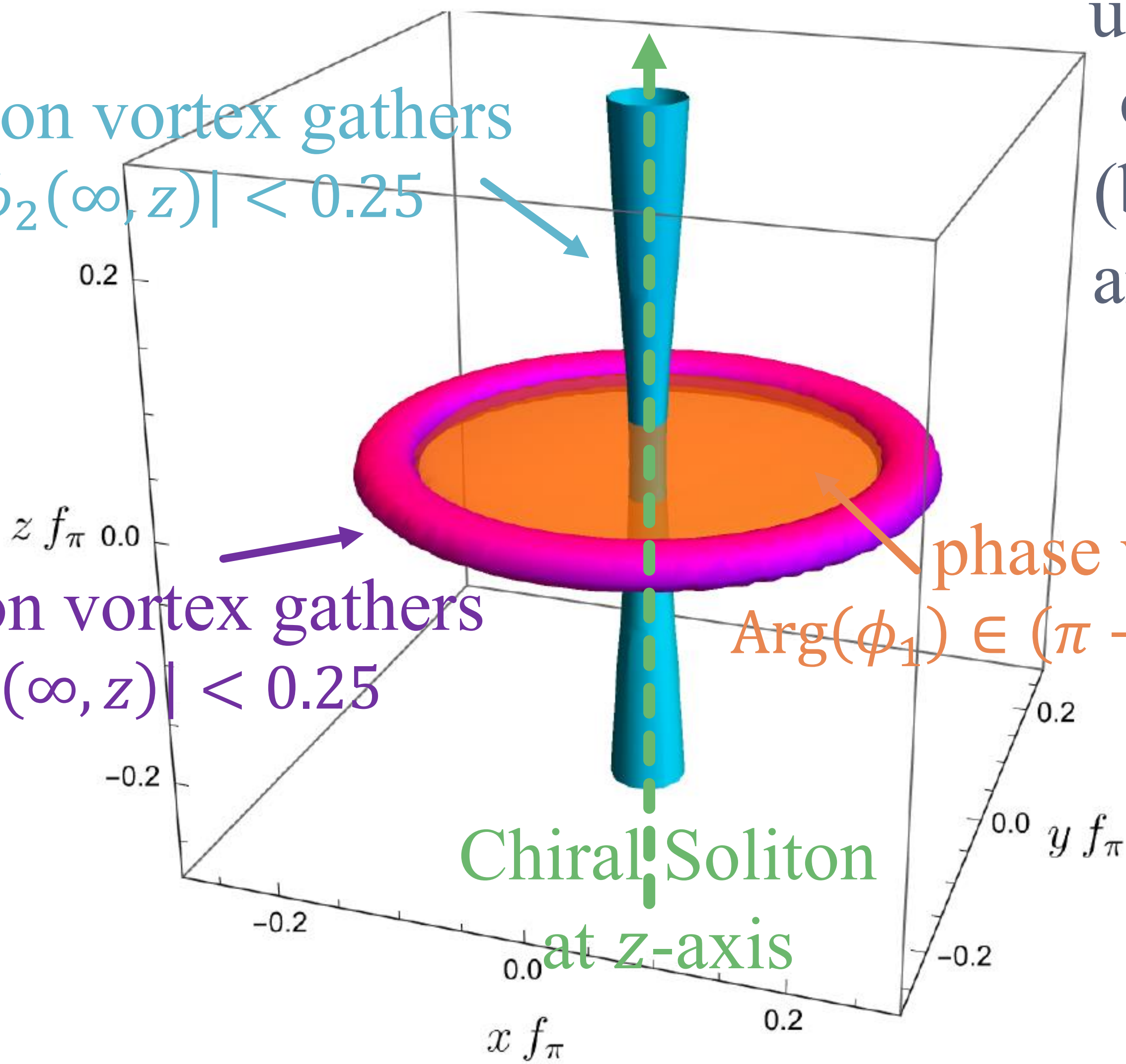
Thank you for your

time and attention

Profile (iii): 3D/Level Surfaces

neutral pion vortex gathers
$|\phi_2|/|\phi_2(\infty, z)| < 0.25$

uniform pion condensate (both neutral and charged) at $r \to \infty$



charged pion vortex gathers
$|\phi_1|/|\phi_1(\infty, z)| < 0.25$

phase varies
$\mathrm{Arg}(\phi_1) \in (\pi - 0.5, \pi + 0.5)$

Chiral Soliton
at z-axis