

iTHEM<sup>®</sup>

RIKEN Center for Interdisciplinary  
Theoretical and Mathematical Sciences



RIKEN's  
Programs for  
Junior Scientists



# Chiral gauge theories

$$S = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \psi^\dagger \bar{\sigma}^\mu D_\mu \psi + \chi^\dagger \bar{\sigma}^\mu D_\mu \chi$$

Chiral: Weyl fermions, only left (right)-handed

- Phenomenology:

- SM and BSM physics (GUTs, preon models, ...)

[Georgi, Glashow '74](#) [Raby, Dimopoulos, Susskind '79](#) [Bars, Yankielowicz '81](#)

[Bolognesi, Konishi, Orso \[2605.10416\]](#) [Cacciapaglia, Sannino, Wagner \[2605.08294\]](#) ...

- Theory:

- understand QFT
- conjectured **rich** and **unexplored** dynamics, a challenge

- ★ Exotic condensates

- ★ Tumbling: several patterns of SB

- ★ Symmetric Mass Generation

[Wang, You \[2204.14271\]](#)  
[Karasik, Önder, Tong \[2208.07842\]](#)  
[Tong \[2104.03997\]](#)

	$SU(N_c)$	$SU(N_c - 4)$	$U(1)$
$\psi$	$\bar{\square}$	$\square$	$-(N_c - 2)$
$\chi$	$\square$	1	$N_c - 4$

**Bars-Yankielowicz (BY)**

	$SU(N_c)$	$SU(N_c + 4)$	$U(1)$
$\psi$	$\bar{\square}$	$\square$	$-(N_c - 2)$
$\chi$	$\square \square$	1	$N_c + 4$

# What do we know?

## Method

Monte Carlo Lattice simulations

Nielsen, Ninomiya '80 '81 Eichten, Preskill '86

Most Attractive Channel

Raby, Dimopoulos, Susskind '79

Dimopoulos, Raby, Susskind '80 Eichten, Feinberg '82

Large N

Eichten, Peccei, Preskill, Zeppenfeld '86

Karasik, Önder, Tong [2208.07842]

Anomaly mediated SUSY breaking

Csáki, Murayama, Telem [2105.03444]

Anomaly matching, discrete symmetries and higher forms

't Hooft '80 Bolognesi, Konishi, Luzio [2101.02601]

Bars, Yankielowicz '81

## Major challenges

Doubler and sign problems

Too naive

Finite N

Non-SUSY limit

Not sufficiently constraining

## Results

*Color-flavor-locked* IR with  $\langle \chi\psi \rangle$

If confinement, no dSB

Conformal theories for few  $N < 13$  and for larger,  $\langle \chi\psi \rangle$  and  $\langle \chi\chi \rangle$

Massless  $\mathcal{B} \sim \langle \chi\psi\psi \rangle$  baryons with no dSB, or *color-flavor-locked* phase

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## Major challenges

Realistic models

Finite N

Non-SUSY limit

Not sufficiently constraining

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**In conclusion:**  
no good understanding of the IR

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Anomaly m  
Csáki, Murayama,

Anomaly matching, discrete  
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Bars, Yankielowicz '81

## Major challenges

Doublet-triplet splitting

Finite N

Not sufficiently constraining

## Results

flavor-locked IR with  $\langle \chi\psi \rangle$

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$<13$  and

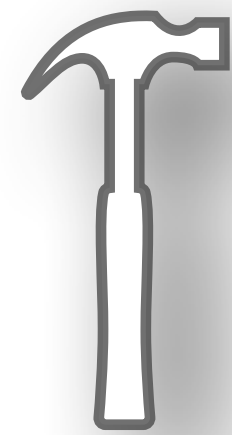
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A new path:

Functional methods



# Functional Renormalisation Group

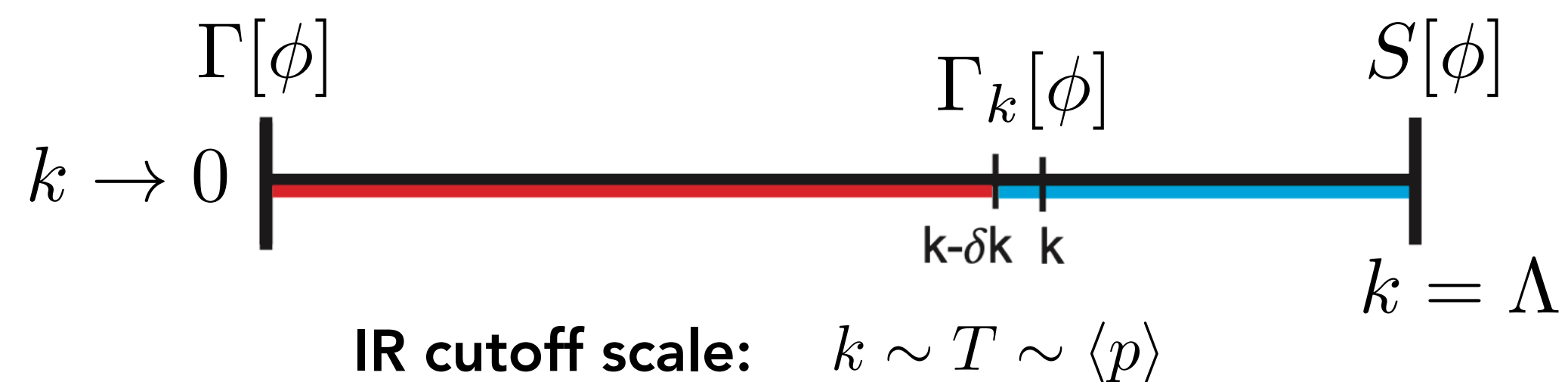
- Progressive integration of momentum shells:

$$\int [\mathcal{D}\phi]_{p>k} = \int \mathcal{D}\phi \exp(-\Delta S_k[\phi]) \quad \Delta S_k[\phi] = \int_p \phi(p) R_k(p) \phi(-p)$$

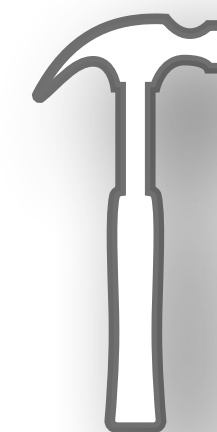
- **Effective average action:**

$$\Gamma_k[\phi] = \int_x J(x)\phi(x) - \mathcal{W}_k[J] - \Delta S_k[\phi] \quad \text{Wetterich '89}$$

- Average action of fields over a  $k^{-d}$  space-time volume
- Kadanoff's block-spinning idea in the continuum limit



# Functional Renormalisation Group



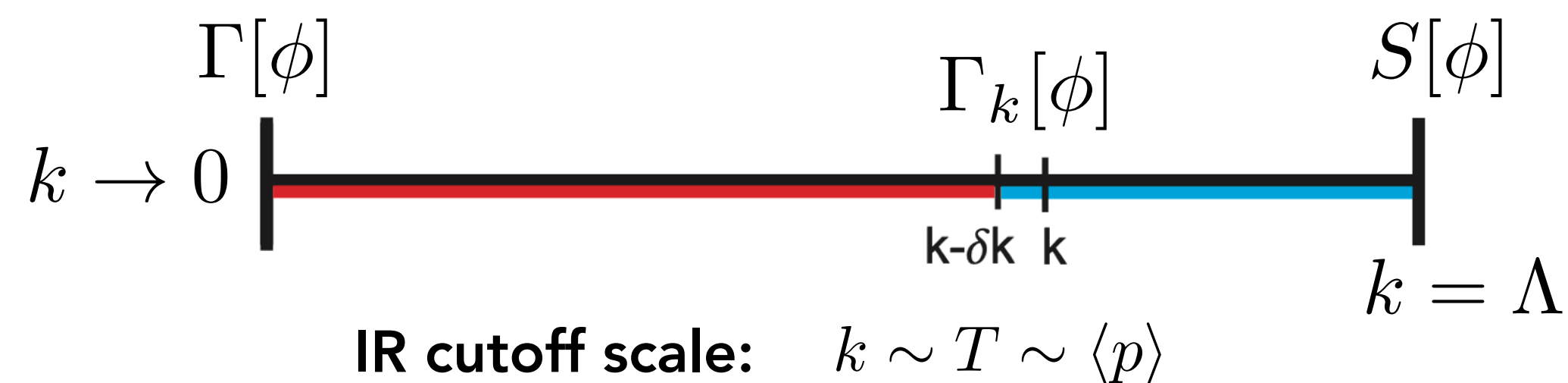
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Flow equation:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[ \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k \right] = \frac{1}{2} \text{Diagram}$$

$$\partial_t \equiv k \partial_k$$

Wetterich '93

- Exact
- Non-perturbative
- One loop
- Analytic regulators
- Versatile
- UV-IR finite
- Systematic truncation schemes
- Chiral regulators

$$R_{\psi/\chi, k}(p^2) = i Z_{\psi/\chi} \bar{\sigma}_\mu p_\mu r_{\psi/\chi}(x)$$

$$r_{\psi/\chi}(x) = (1/\sqrt{x} - 1) \theta(1 - x)$$

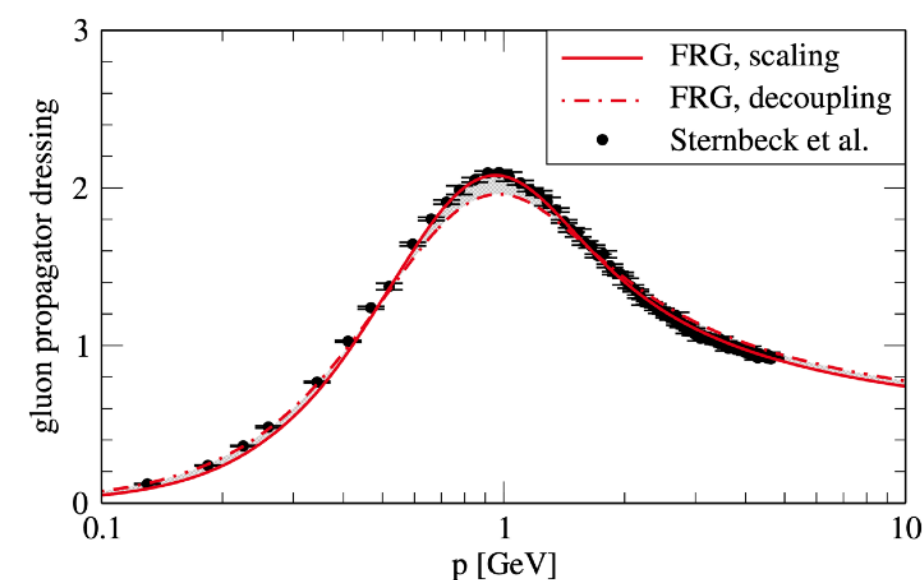
...

# Some results in gauge-fermion theories

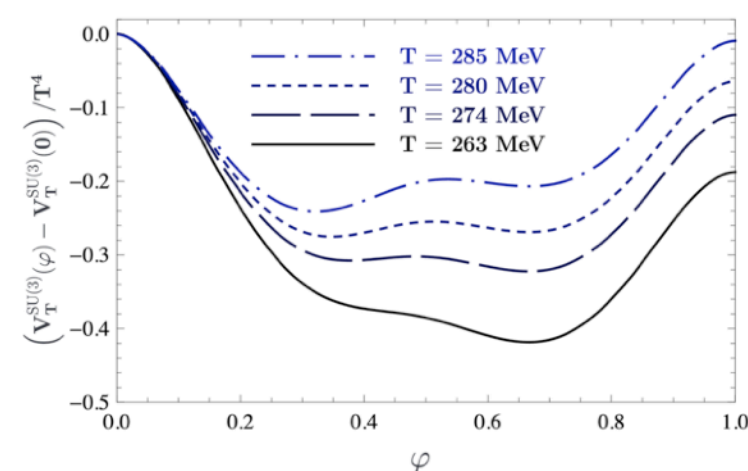
a.k.a. QCD

## Colour confinement

Cyrol,Fister,Mitter,Pawlowski [1605.01856]

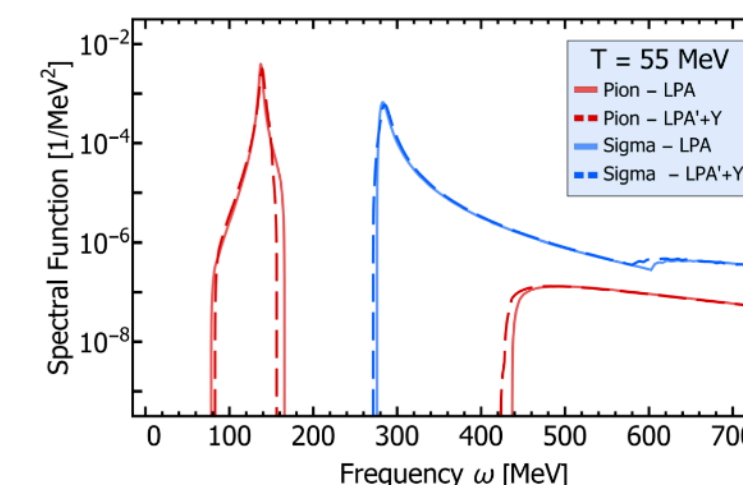
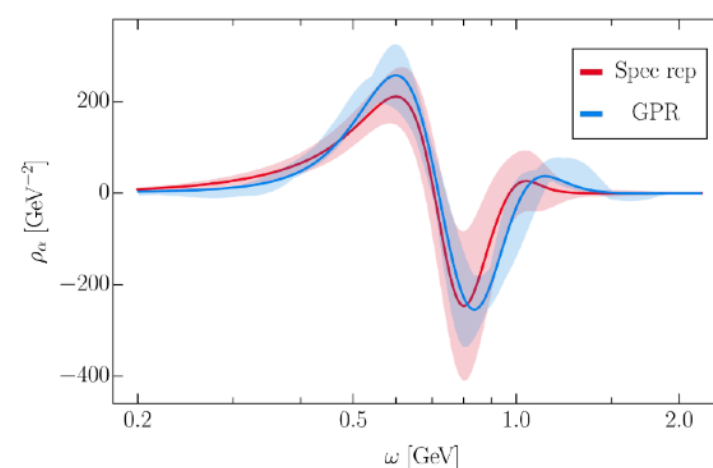


Fister,Pawlowski [1301.4163]



## Real time properties

Horak,Pawlowski,Turnwald,Urban,Wink,  
Zafeiropoulos[1711.07444]



Pawlowski,Wink,Strodthoff[1711.07444]

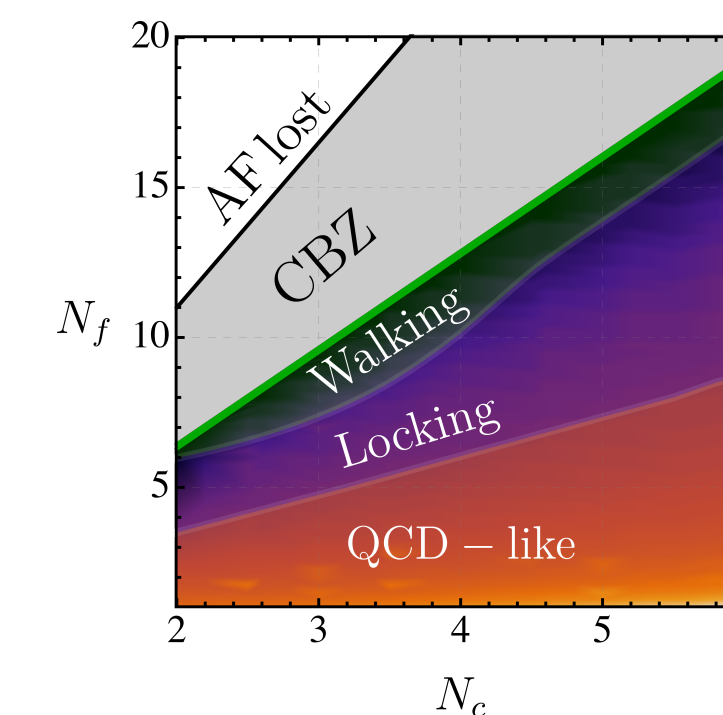
## Many flavour and colours

Gies,Jaeckel[hep-ph/0507171]

Braun,Gies[0912.4168]

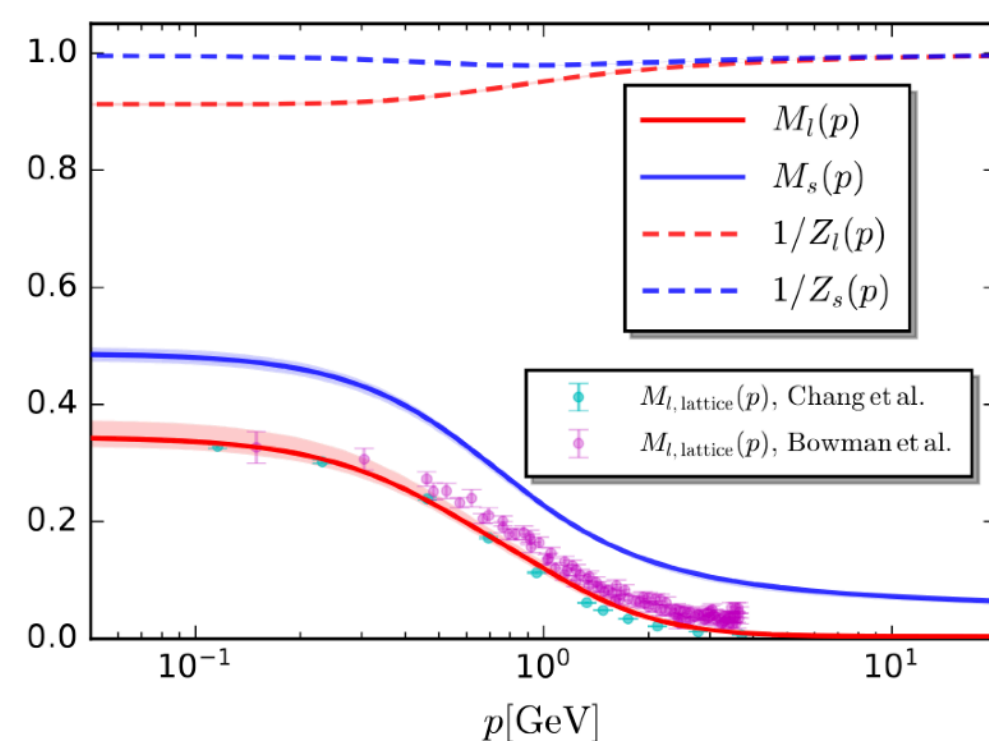
Braun,Fischer,Gies[1012.4279]

Goertz,APG,Pawlowski[2412.12254]

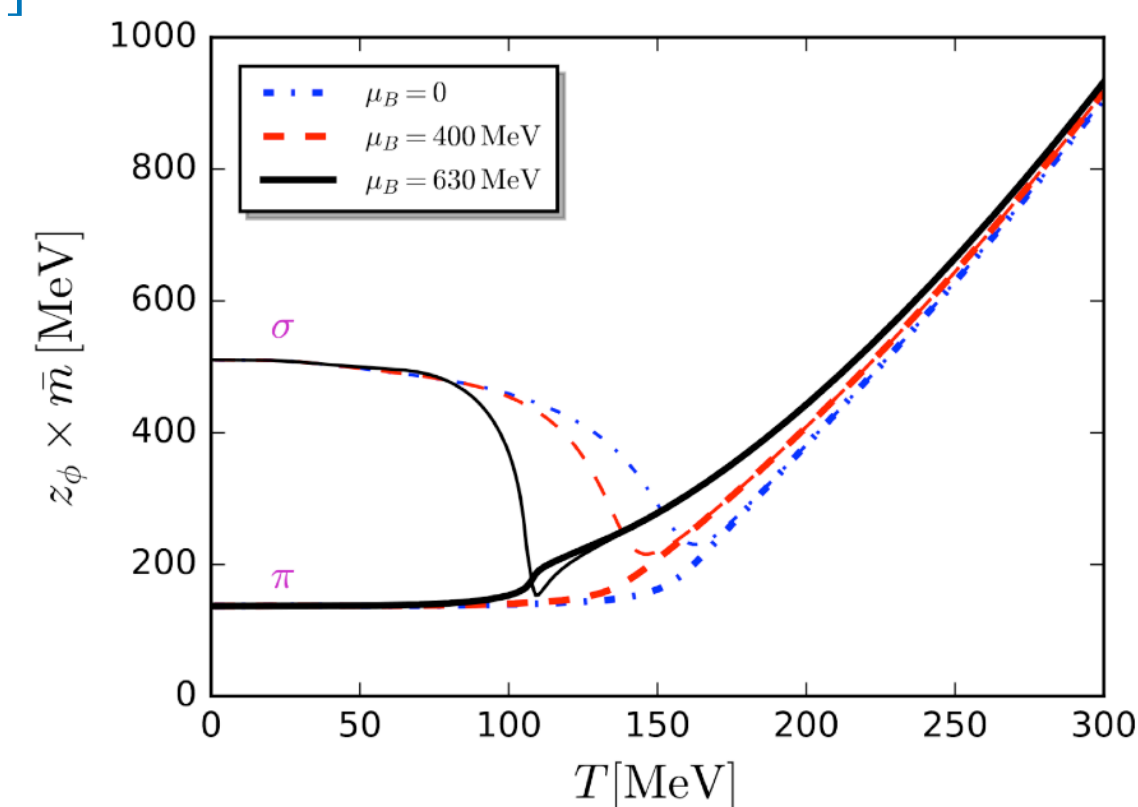


## Dynamical chiral symmetry and bound states

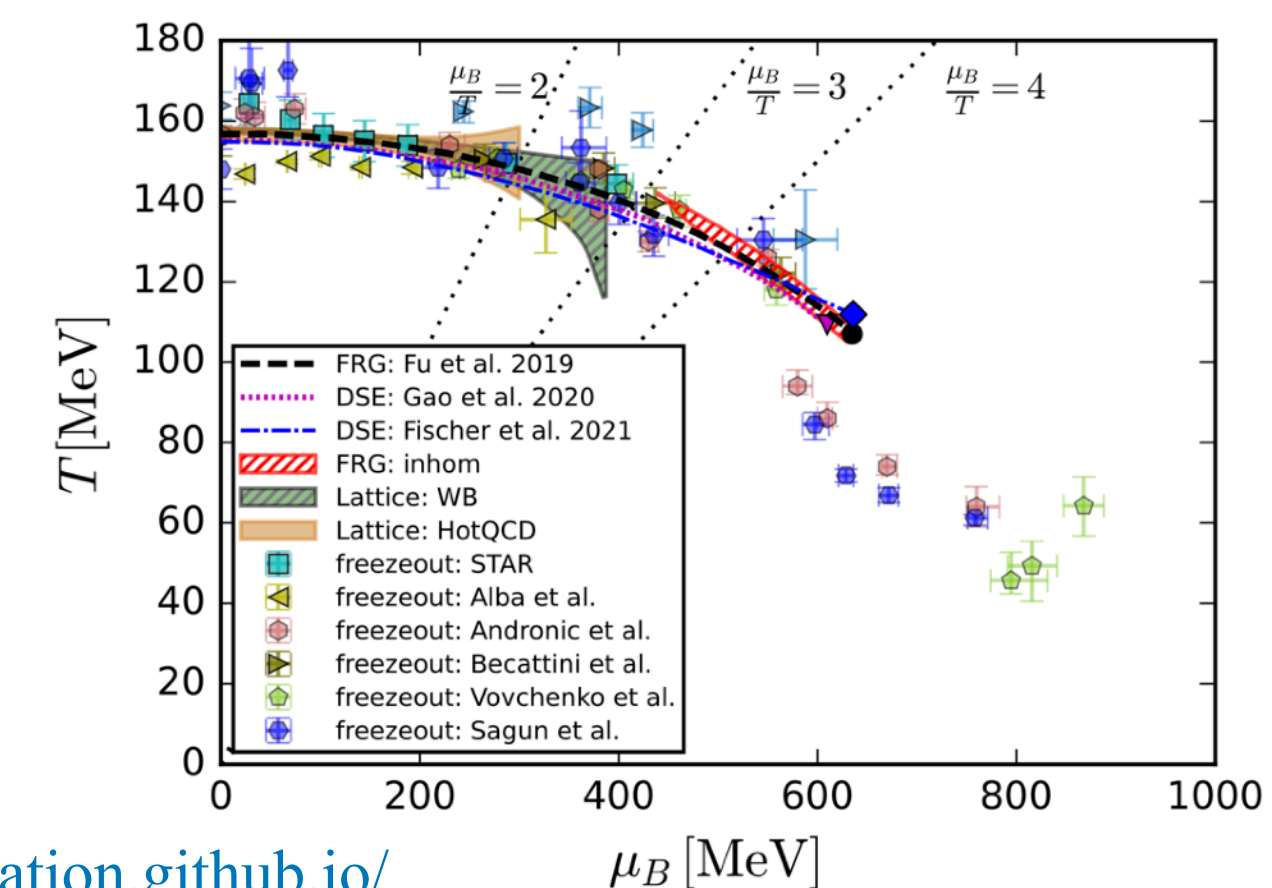
Fu,Huang,Pawlowski,Tan,Zhou[2502.14388]



Fu,Pawlowski,Rennecke [1909.02991]



## Finite temperature and chemical potential



<https://fqcd-collaboration.github.io/>

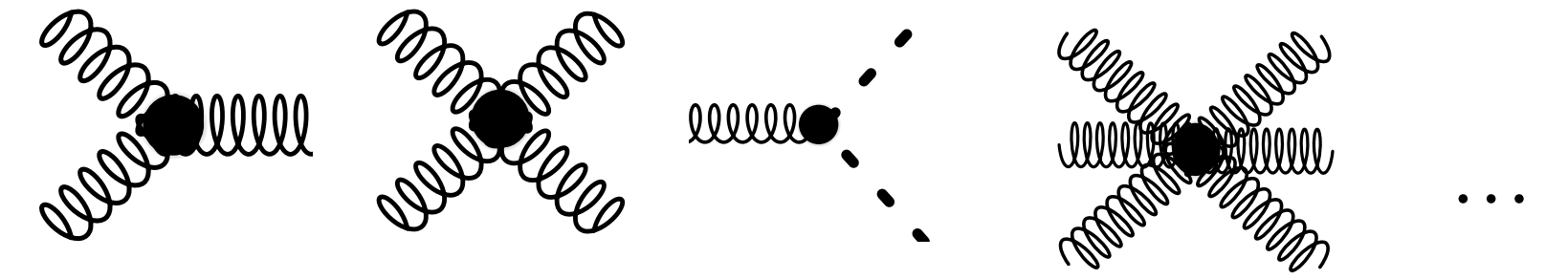
# Effective action for gauge-fermion theories

$$\Gamma[A_\mu, \bar{c}, c, \chi^\dagger, \chi, \psi^\dagger, \psi]$$

# Effective action for gauge-fermion theories

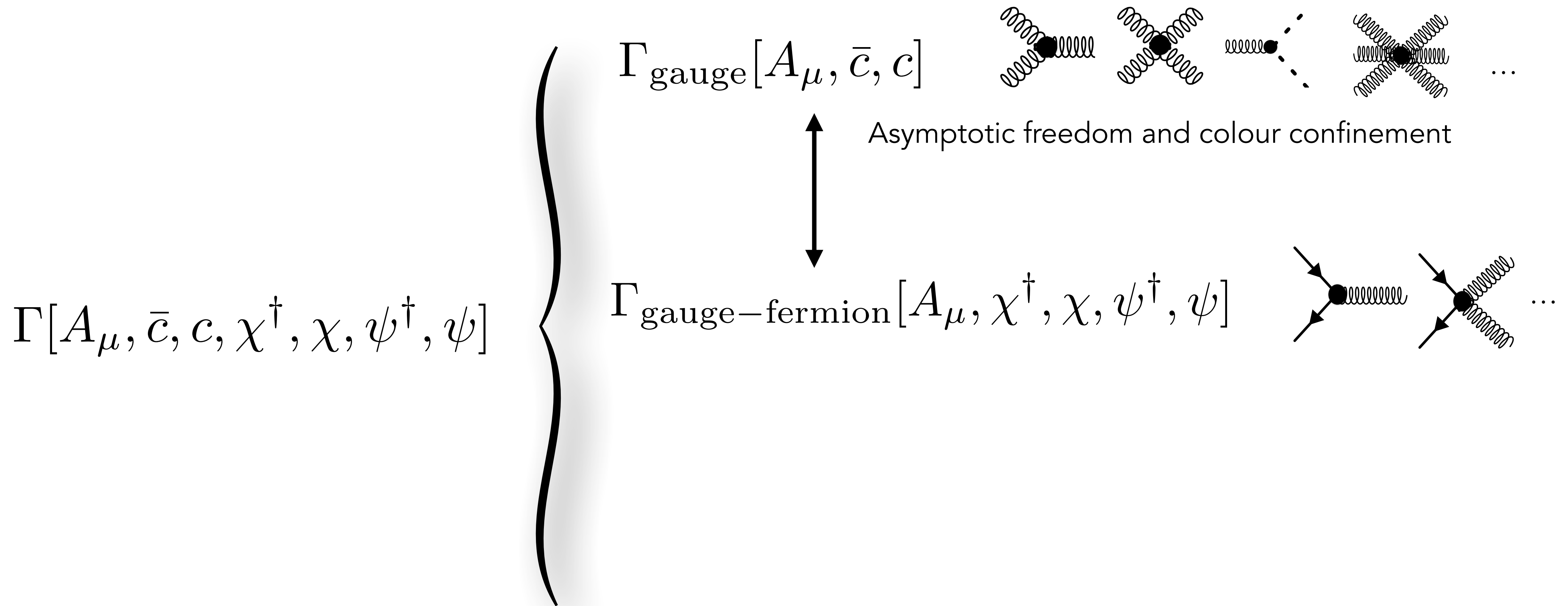
$$\Gamma[A_\mu, \bar{c}, c, \chi^\dagger, \chi, \psi^\dagger, \psi]$$

$$\Gamma_{\text{gauge}}[A_\mu, \bar{c}, c]$$

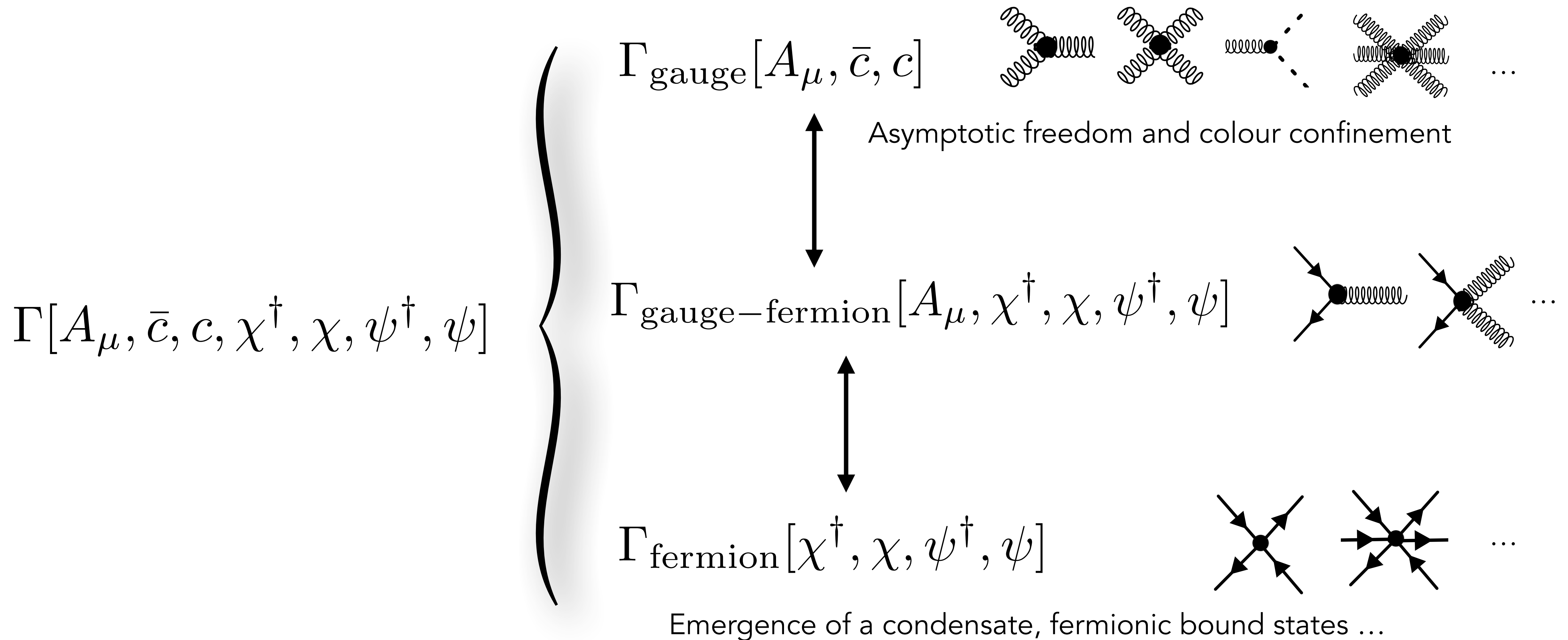


Asymptotic freedom and colour confinement

# Effective action for gauge-fermion theories



# Effective action for gauge-fermion theories



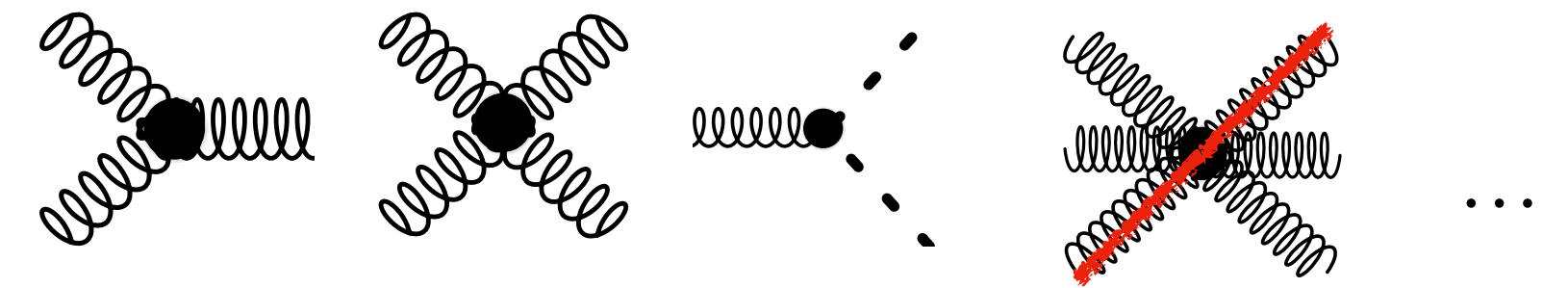
# Effective action for gauge-fermion theories

**Truncation** of the effective action here:

- Keep canonically (marginal) relevant operators and four-fermion interactions
- Minimal setup to describe dSB and confinement

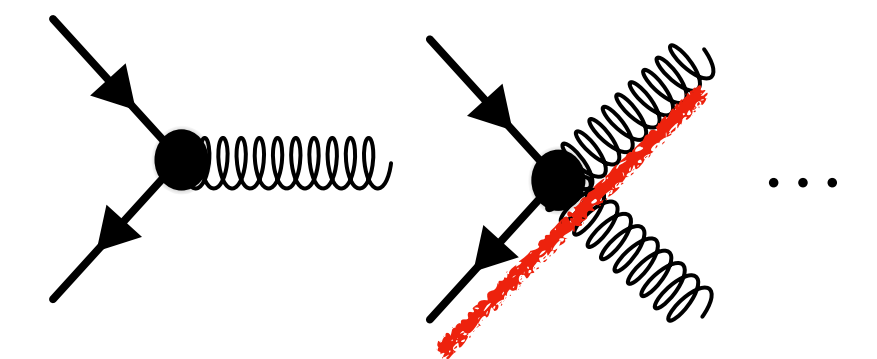
$$\Gamma[A_\mu, \bar{c}, c, \chi^\dagger, \chi, \psi^\dagger, \psi]$$

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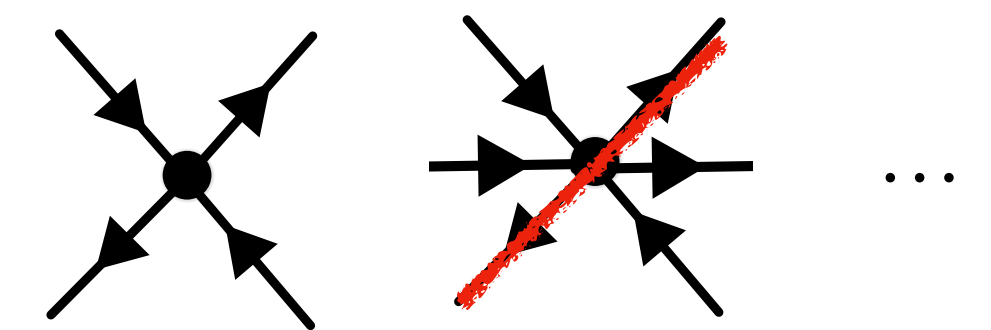


Asymptotic freedom and colour confinement

$$\Gamma_{\text{gauge-fermion}}[A_\mu, \chi^\dagger, \chi, \psi^\dagger, \psi]$$



$$\Gamma_{\text{fermion}}[\chi^\dagger, \chi, \psi^\dagger, \psi]$$



Emergence of a condensate, fermionic bound states ...

# Signatures of colour confinement and the gluon mass gap

Alkofer,von Smekal'96'00 Pawłowski,Litim,Nedelko,von Smekal[hep-th/0312324] Fischer, Maas, Pawłowski[0810.1987] Cyrol,Fister,Mitter,Pawłowski [1605.01856]...

- **Gluon mass gap** generated by **quantum fluctuations**

$$\Gamma_k^{(AA)}(p^2) = Z_{A,k}(p)(p^2 + m_{\text{gap},k}^2) = \hat{Z}_{A,k}(p) p^2$$

- How we know its confinement?

• **Kugo-Ojima criterion satisfied**

[Kugo, Ojima'79](#) [Nakanishi,Ojima'90](#)

- Sufficient conditions: **Massive spectrum of physical states** in the **Hilbert space** and a **global BRST** transformation

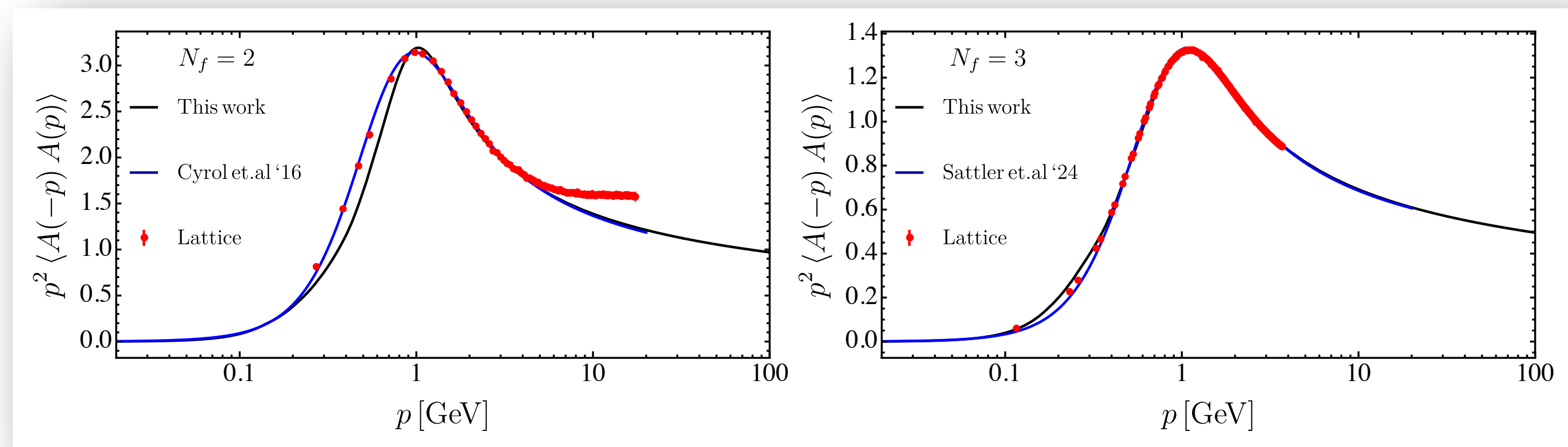
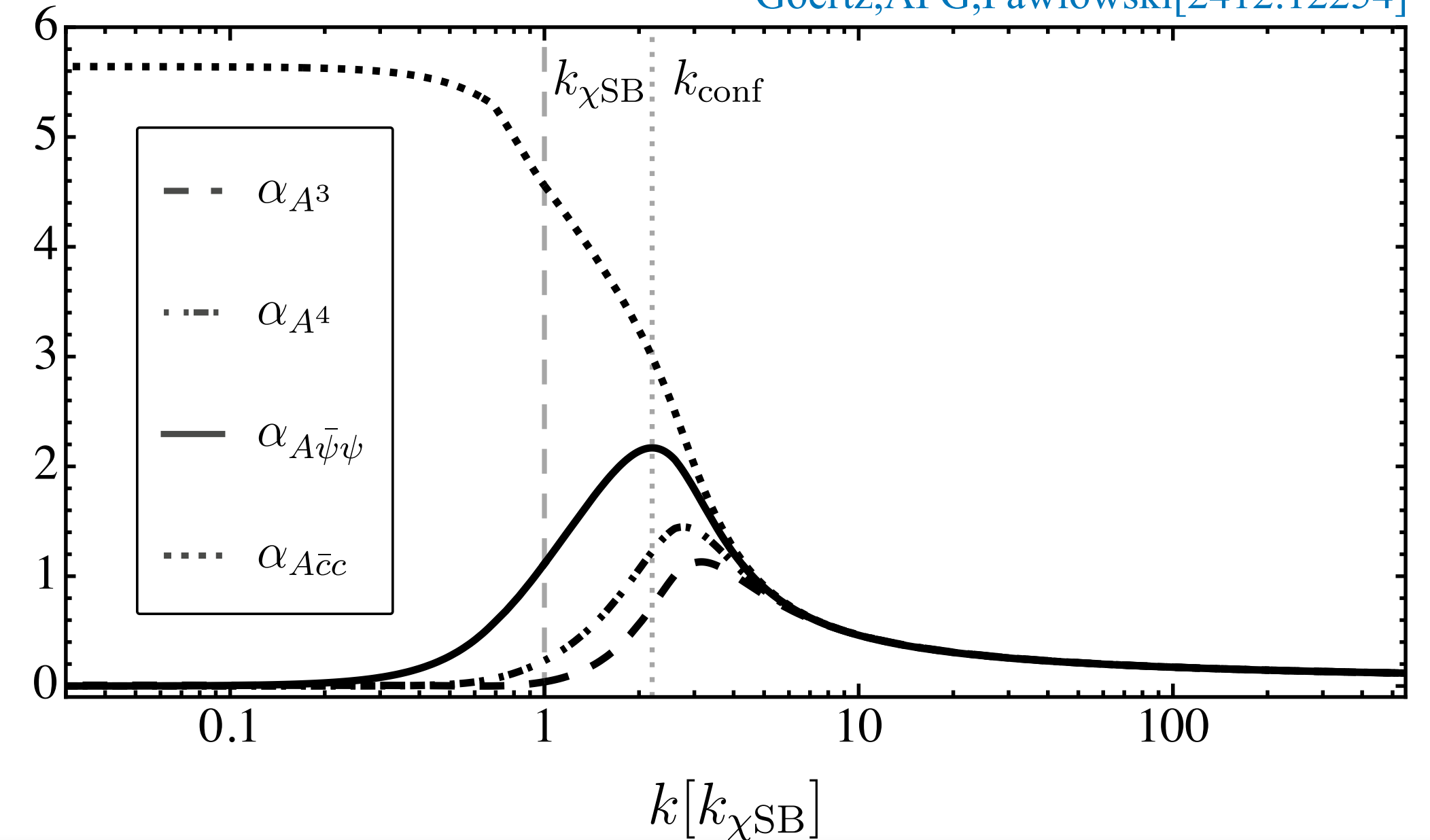
- **Unique** IR scaling of correlation functions

$$\lim_{p \rightarrow 0} Z_A(p^2) \propto (p^2)^{-2\kappa} \quad \lim_{p \rightarrow 0} Z_c(p^2) \propto (p^2)^\kappa$$

[Kugo\[hep-th/9511033\]](#)

$N_c = 3 \quad N_f = 2$

[Goertz,APG,Pawłowski\[2412.12254\]](#)



# Dynamical symmetry breaking in the effective action

a chiral QCD example

$$S = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \bar{\psi} \gamma_\mu D_\mu \psi$$

# Dynamical symmetry breaking in the effective action

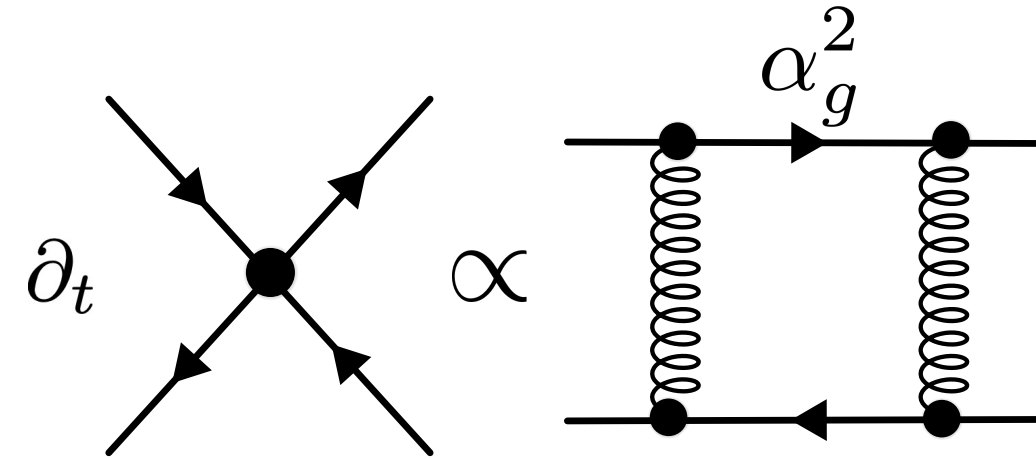
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$$\Gamma = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \lambda_i (\bar{\psi} \mathcal{T}_i \psi)^2 + \kappa_i (\bar{\psi} \mathcal{T}_i \psi)^3 + \dots$$

# Dynamical symmetry breaking in the effective action

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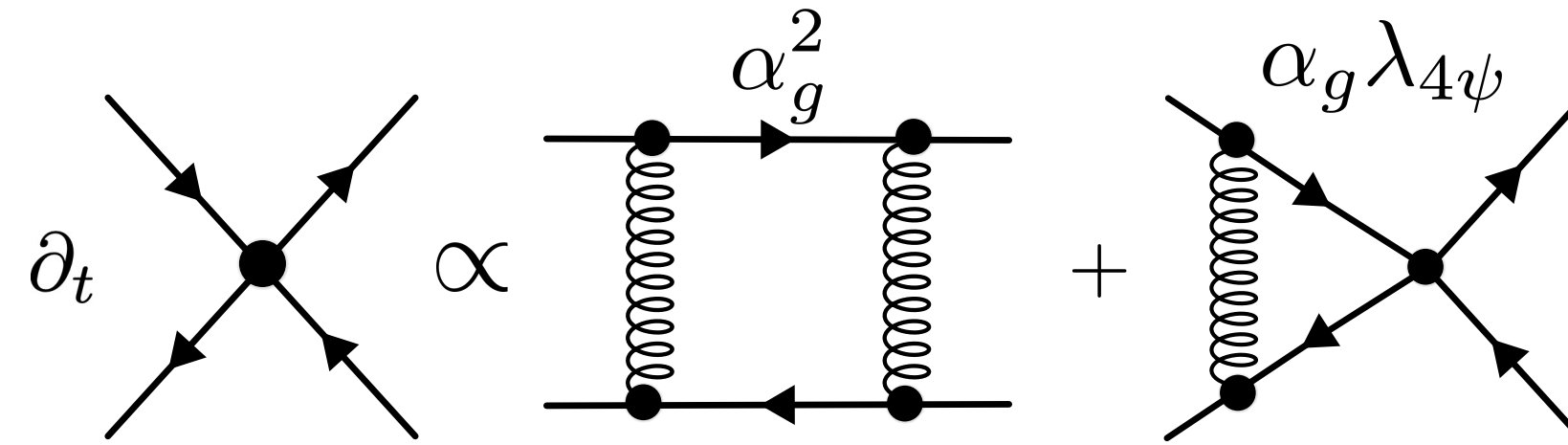
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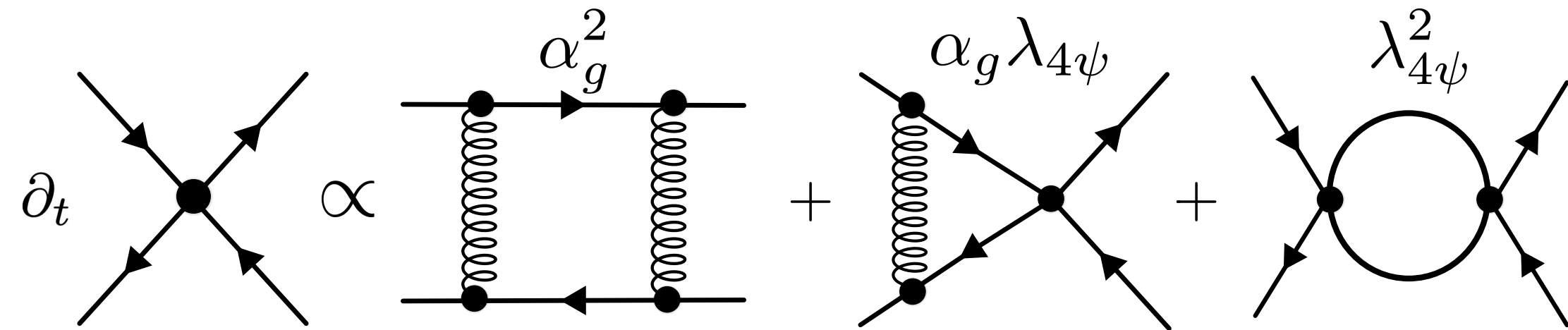
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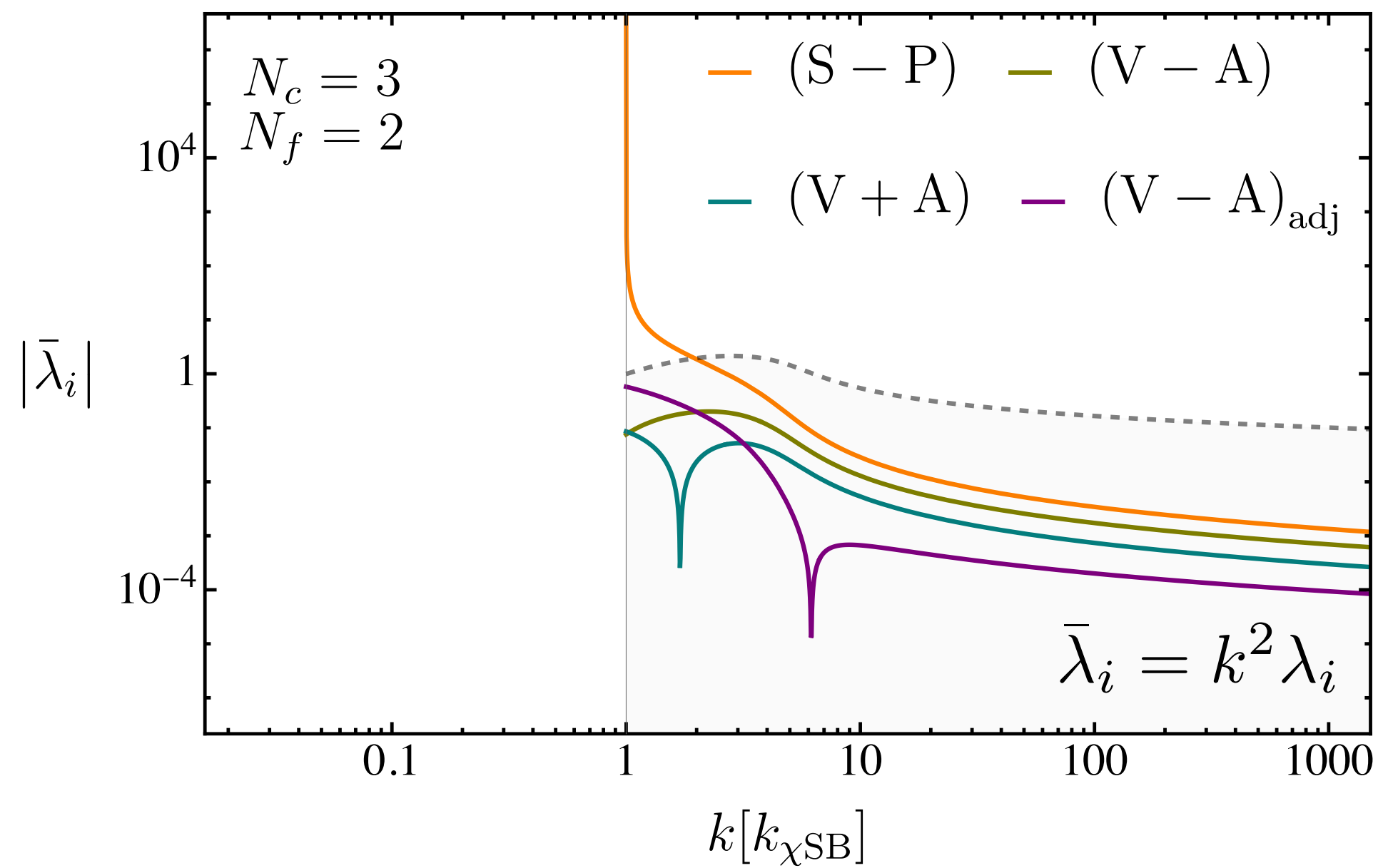
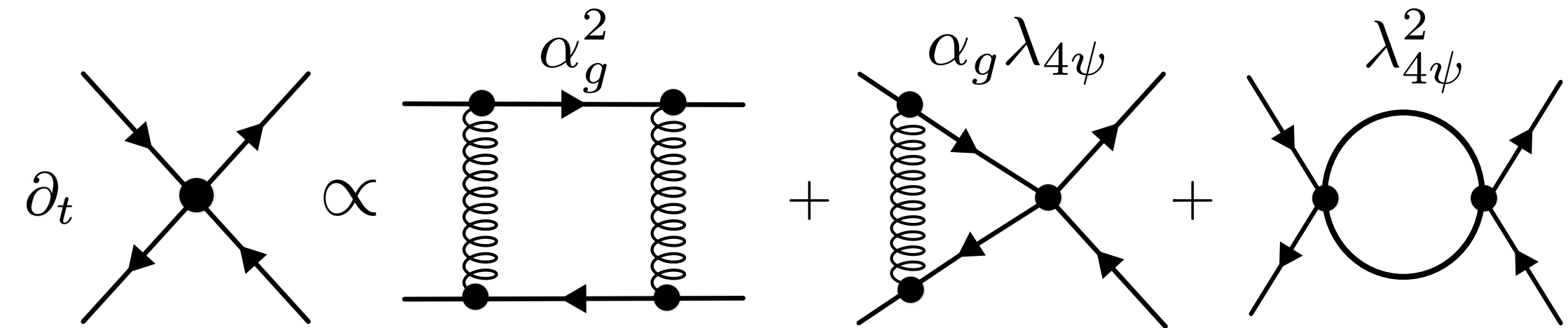
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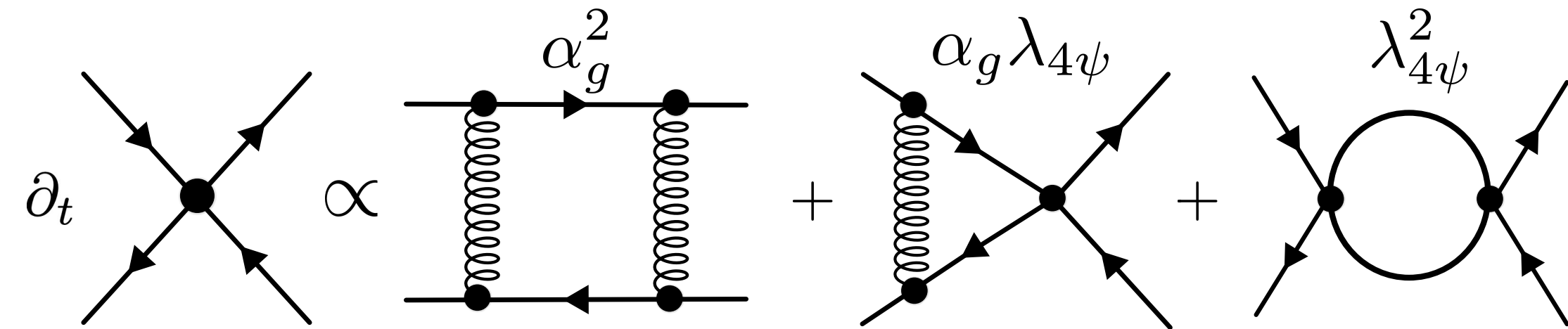
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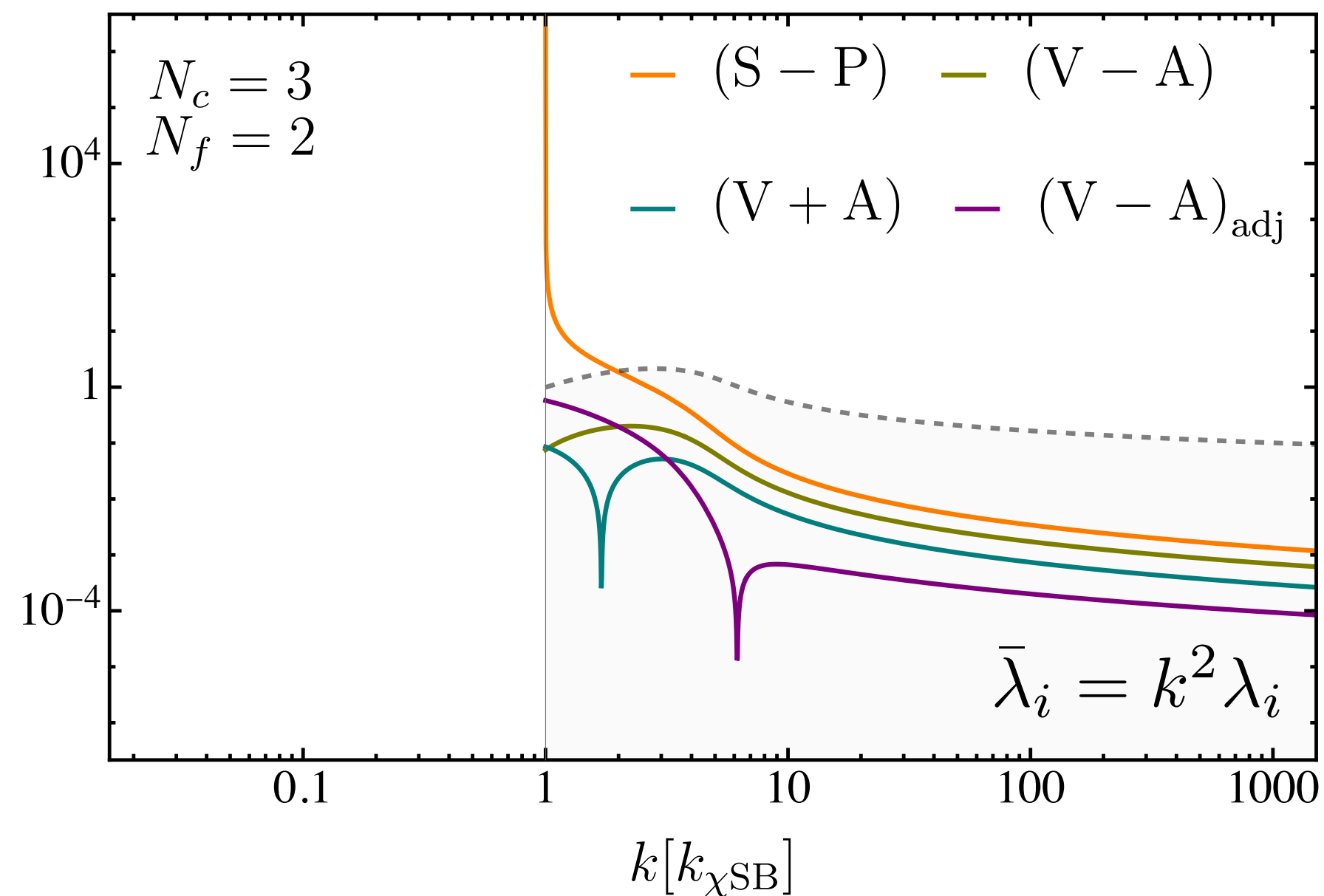
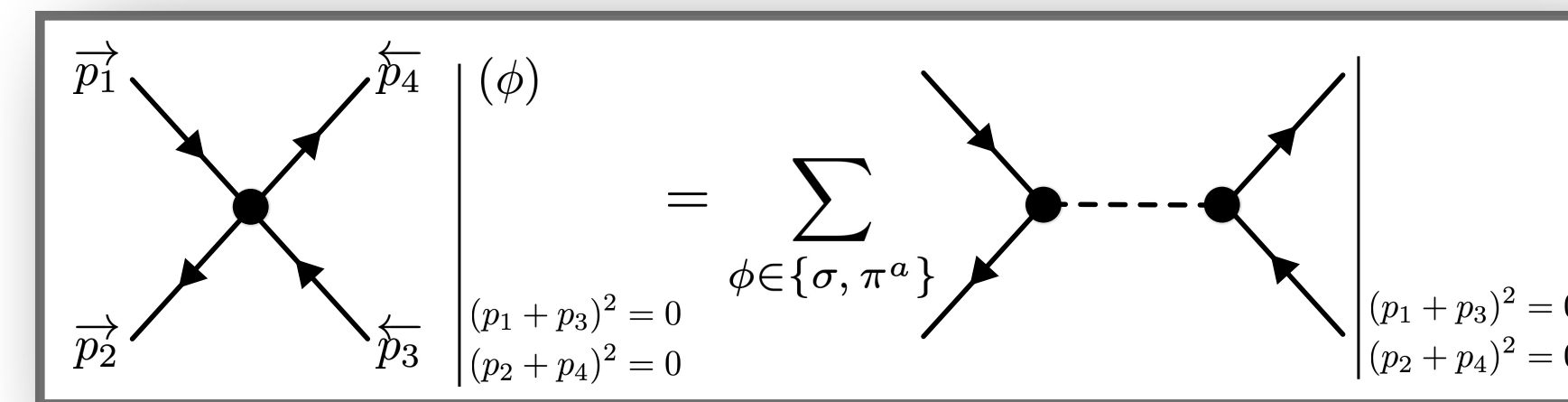
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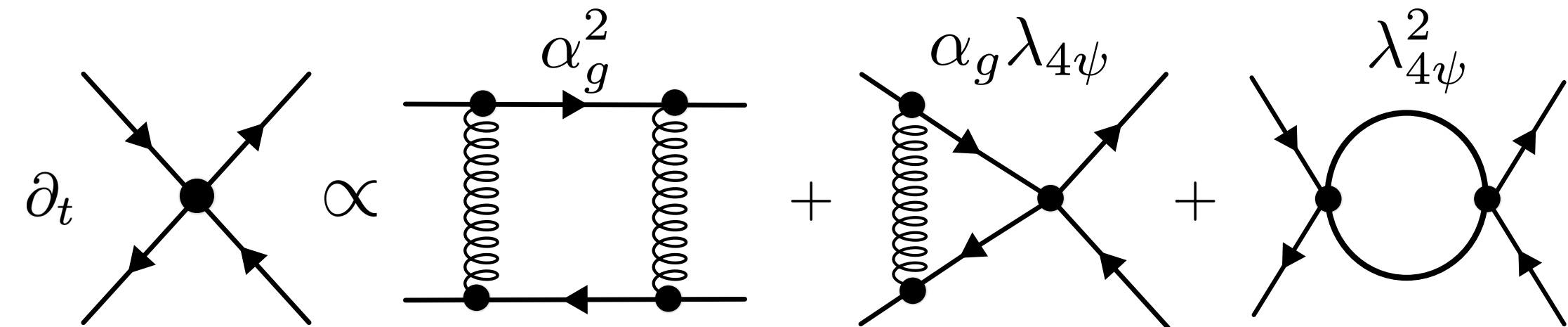
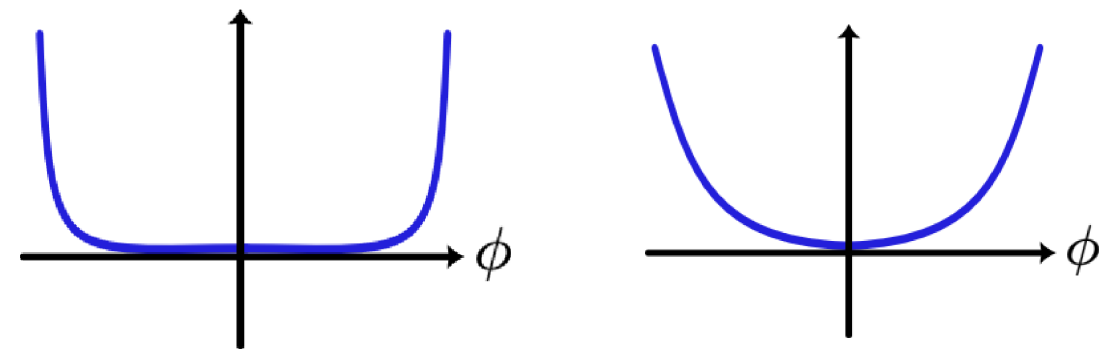
Stratonovich'57 Hubbard'59



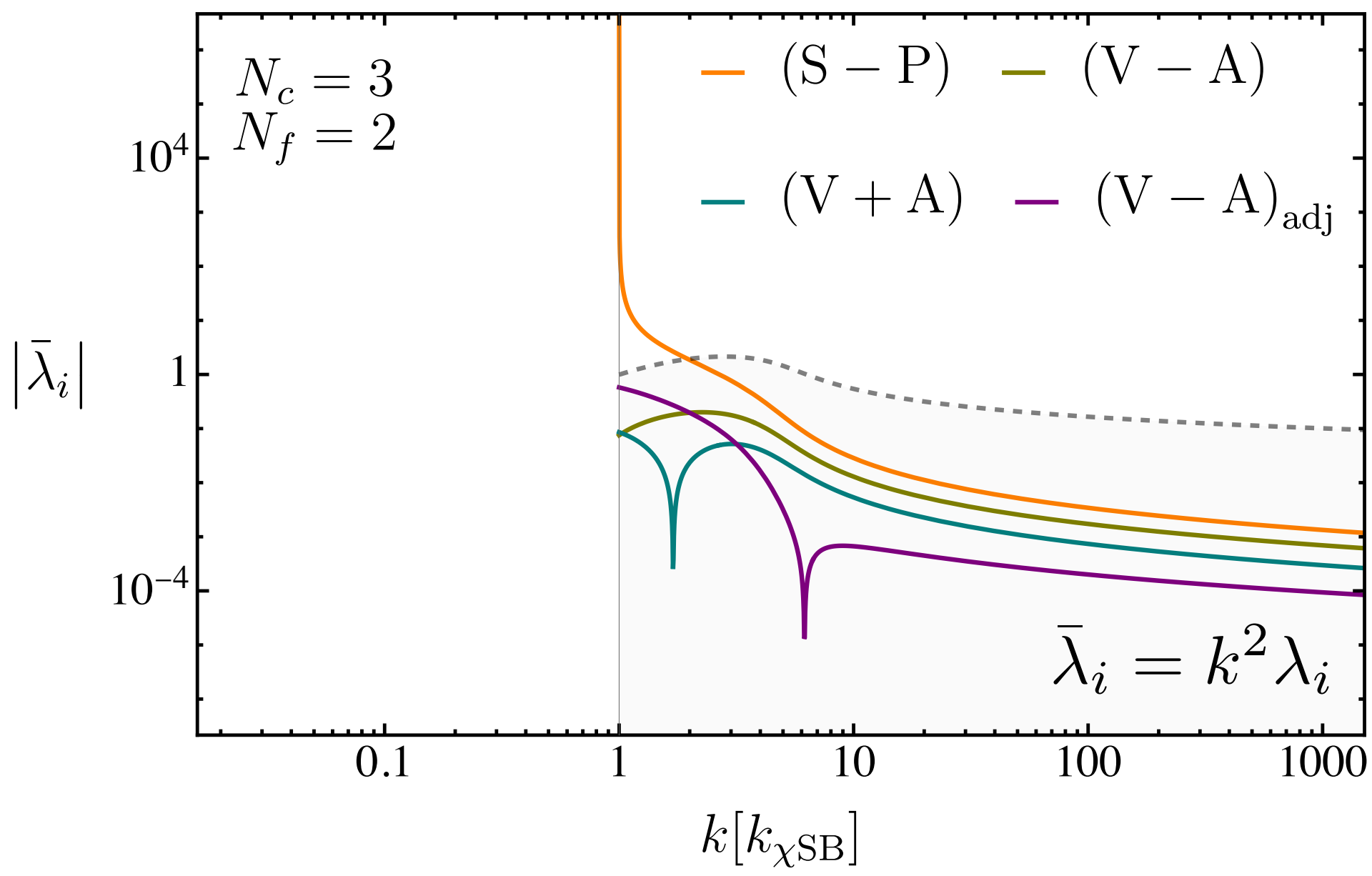
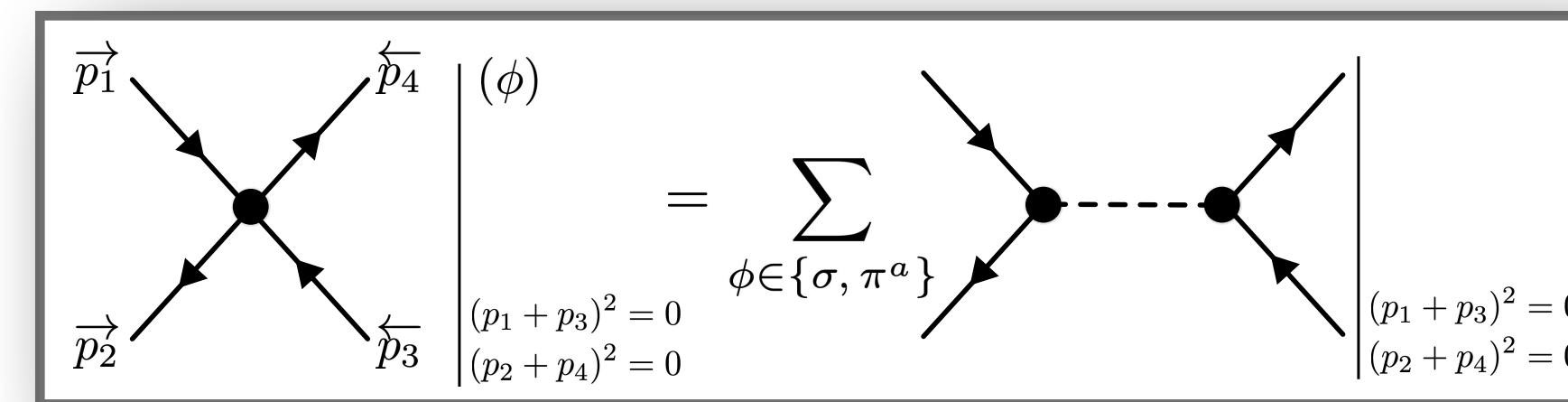
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Stratonovich'57 Hubbard'59



$$\lambda_i \propto 1/m_{\phi_i}^2$$

$$m_{\phi_i}^2 = \partial_{\phi_i^2} V(\phi_i)$$

$$\lambda_i \rightarrow \infty \quad m_{\phi_i}^2 = 0$$

Infinite correlation length:

$$\xi \rightarrow \infty$$

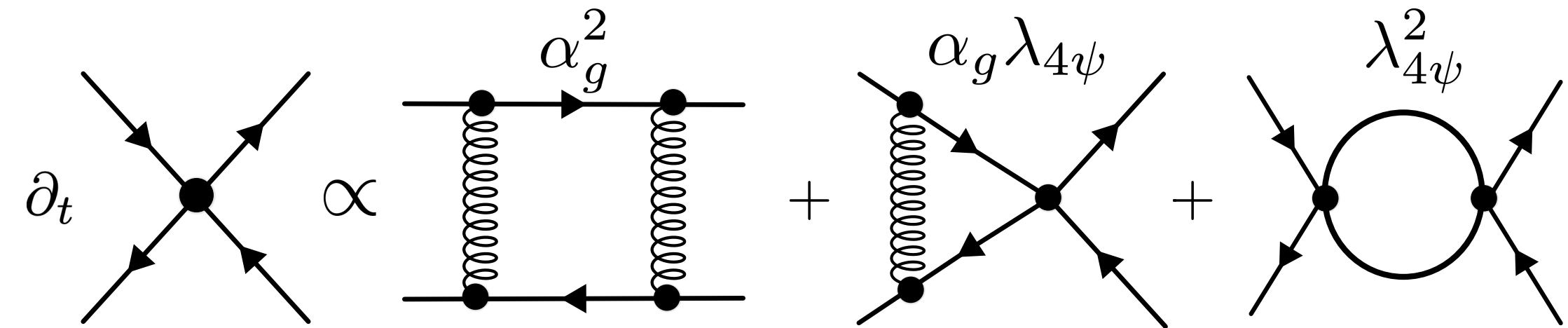
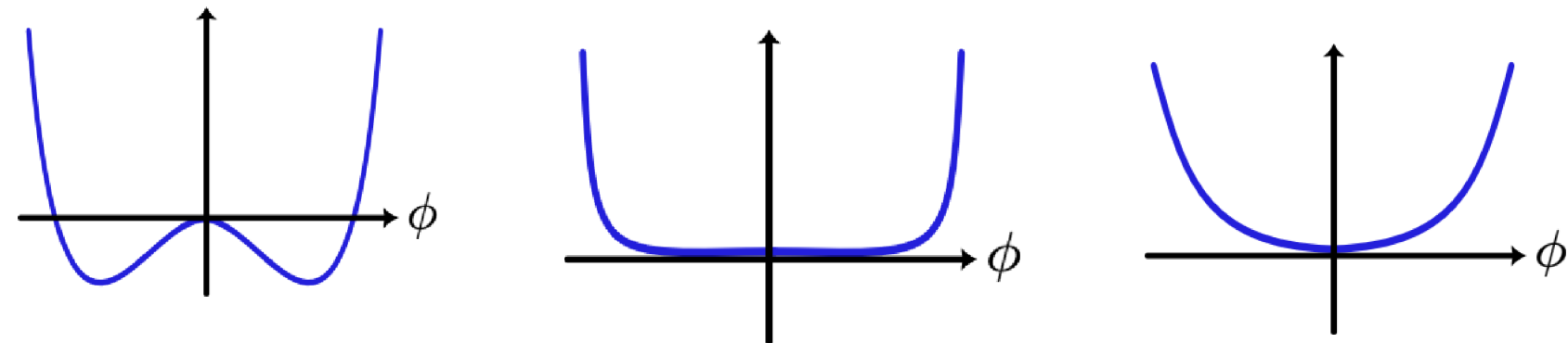
Phase transition

Nambu, Jona-Lasinio'60'61

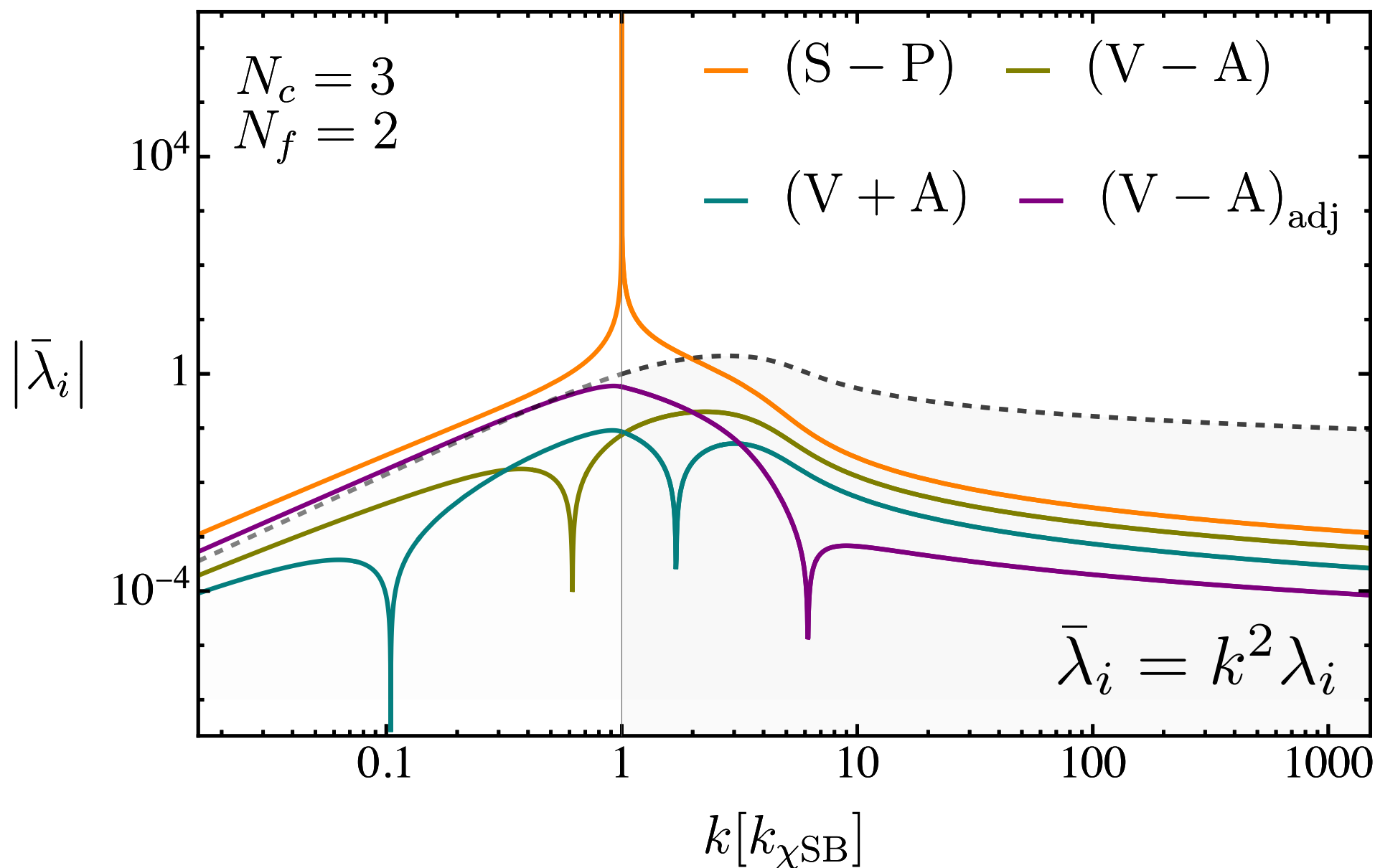
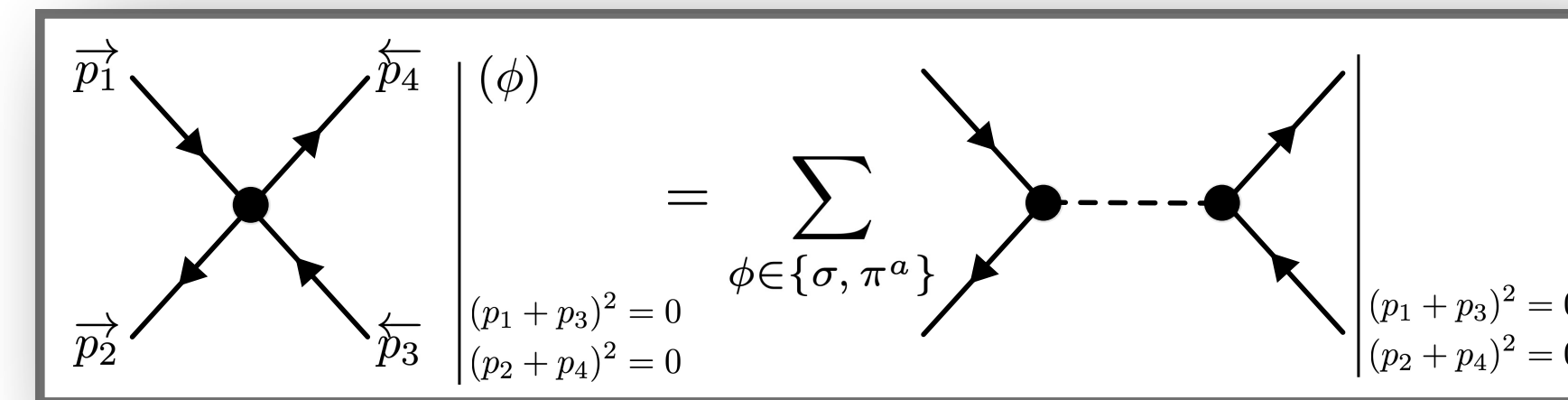
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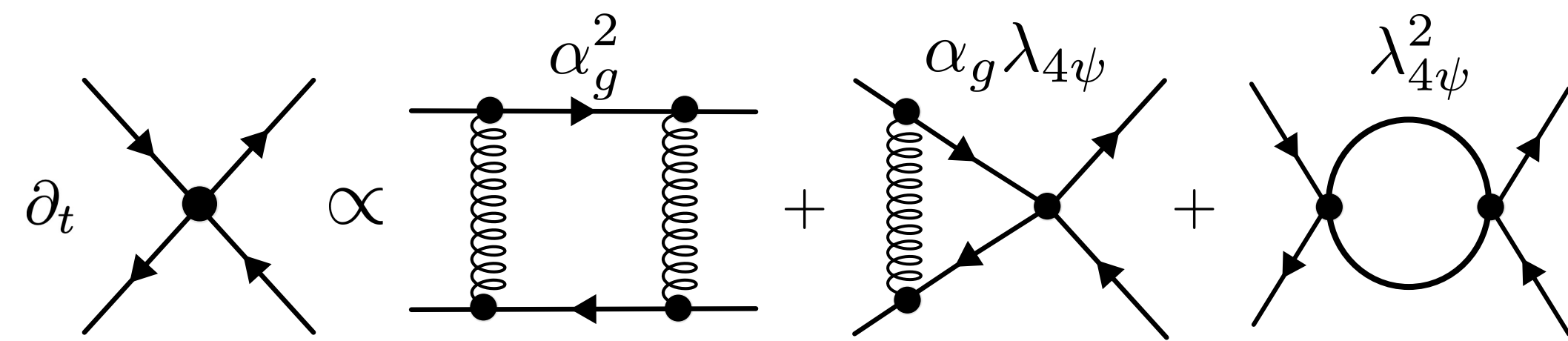
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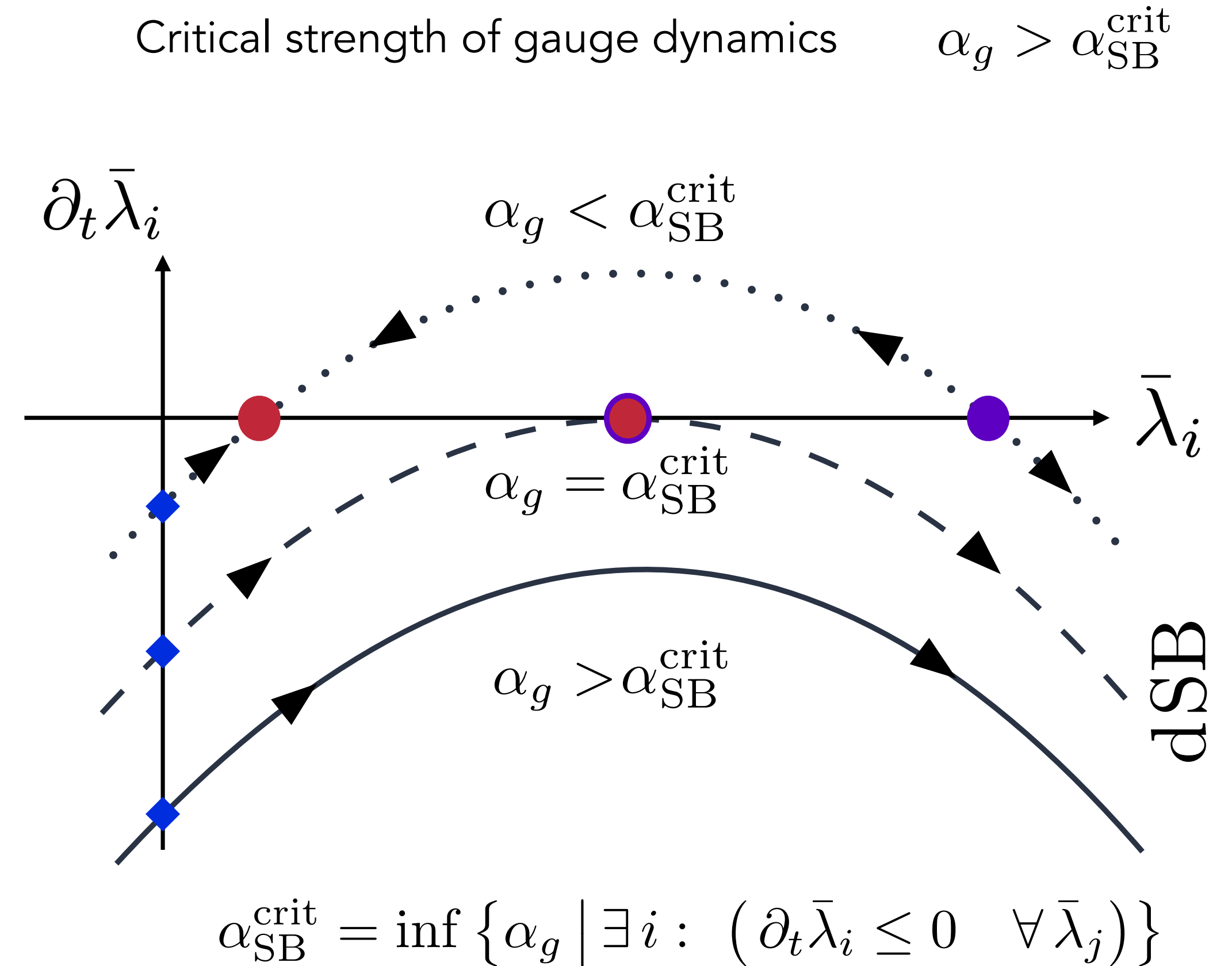
Nambu, Jona-Lasinio'60'61

# Mechanism underlying dSB

Nambu, Jona-Lasinio '60 '61 Kosterlitz '74 Miransky '85 Jungnickel, Wetterich [hep-ph/9505267] Gies, Jaeckel [hep-ph/0507171] B. Kaplan, Lee, T. Son, Stephanov [0905.4752] Braun [1108.4449]



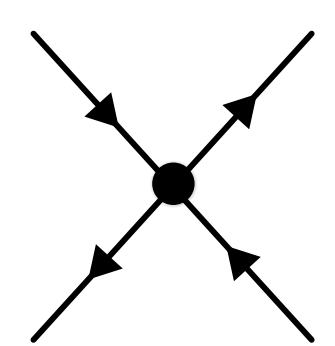
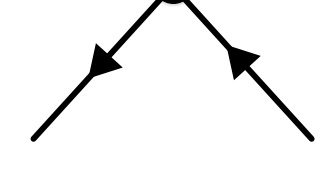
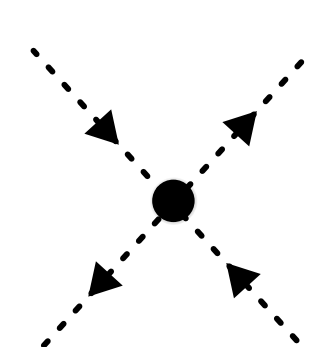
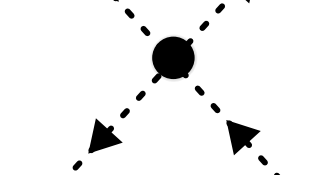
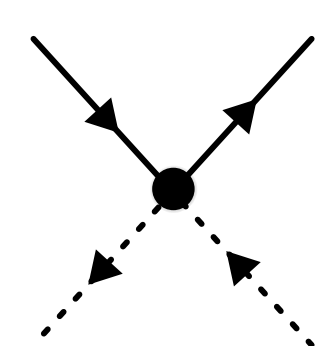
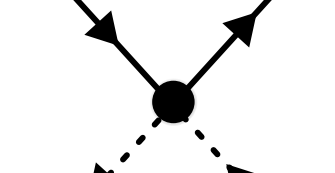
$$\partial_t \bar{\lambda}_i \propto 2 \bar{\lambda}_i + c_{A,i} \cdot \alpha_g^2 + c_{B,ij} \cdot \alpha_g \bar{\lambda}_j + c_{C,ijk} \cdot \bar{\lambda}_j \bar{\lambda}_k + \dots$$

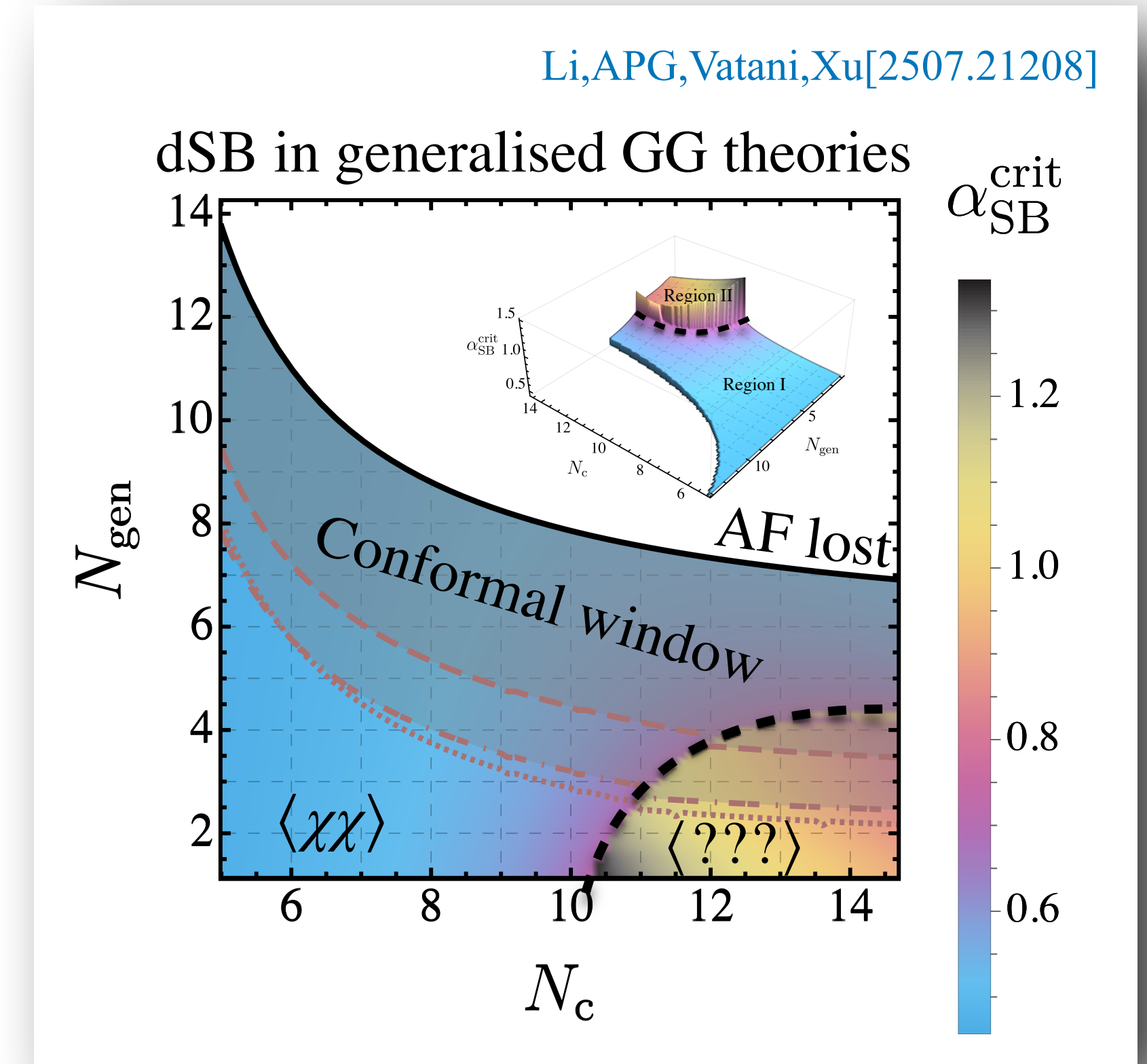


Goertz, APG, Pawłowski [2412.12254]  
Li, APG, Vatani, Xu [2507.21208]

# Fierz-complete four-fermion basis in chiral gauge theories

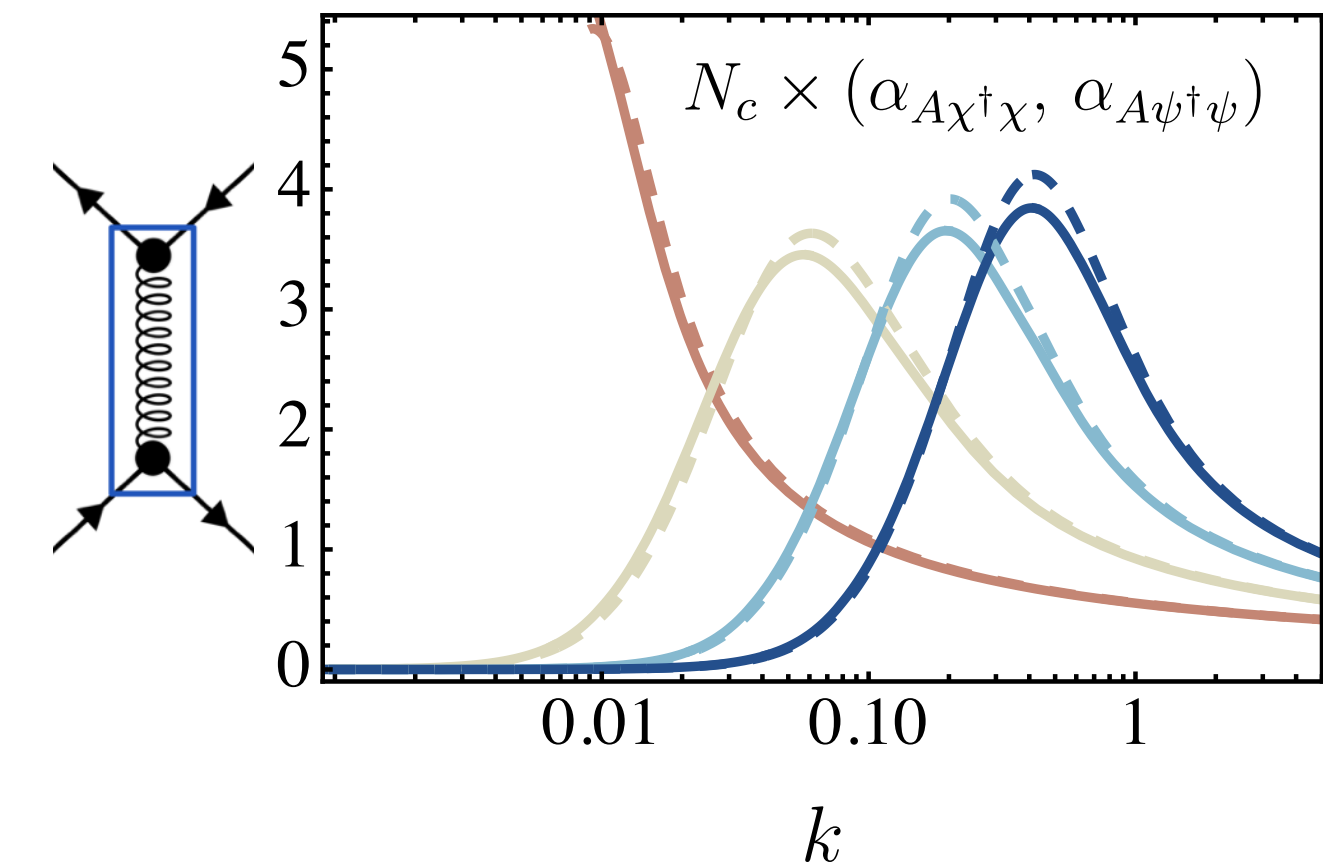
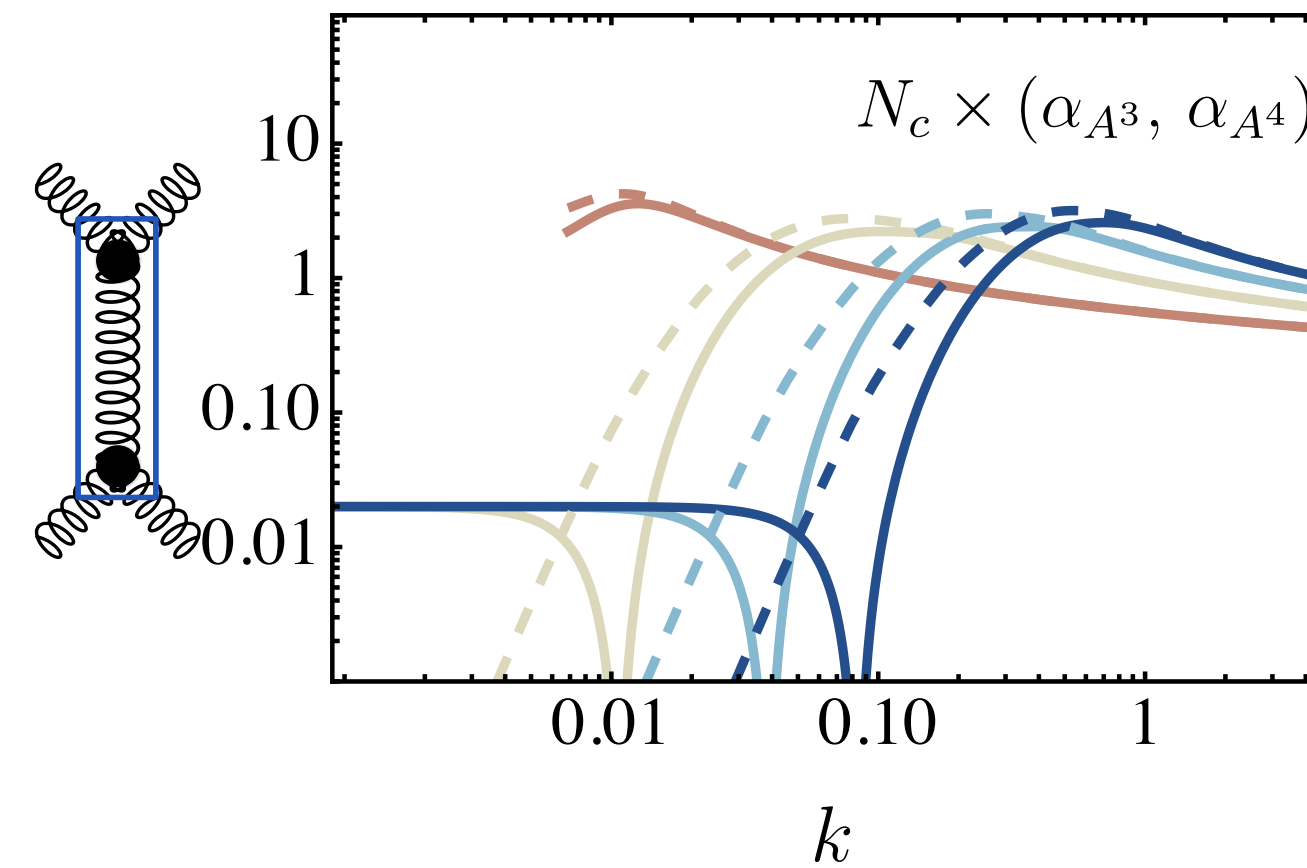
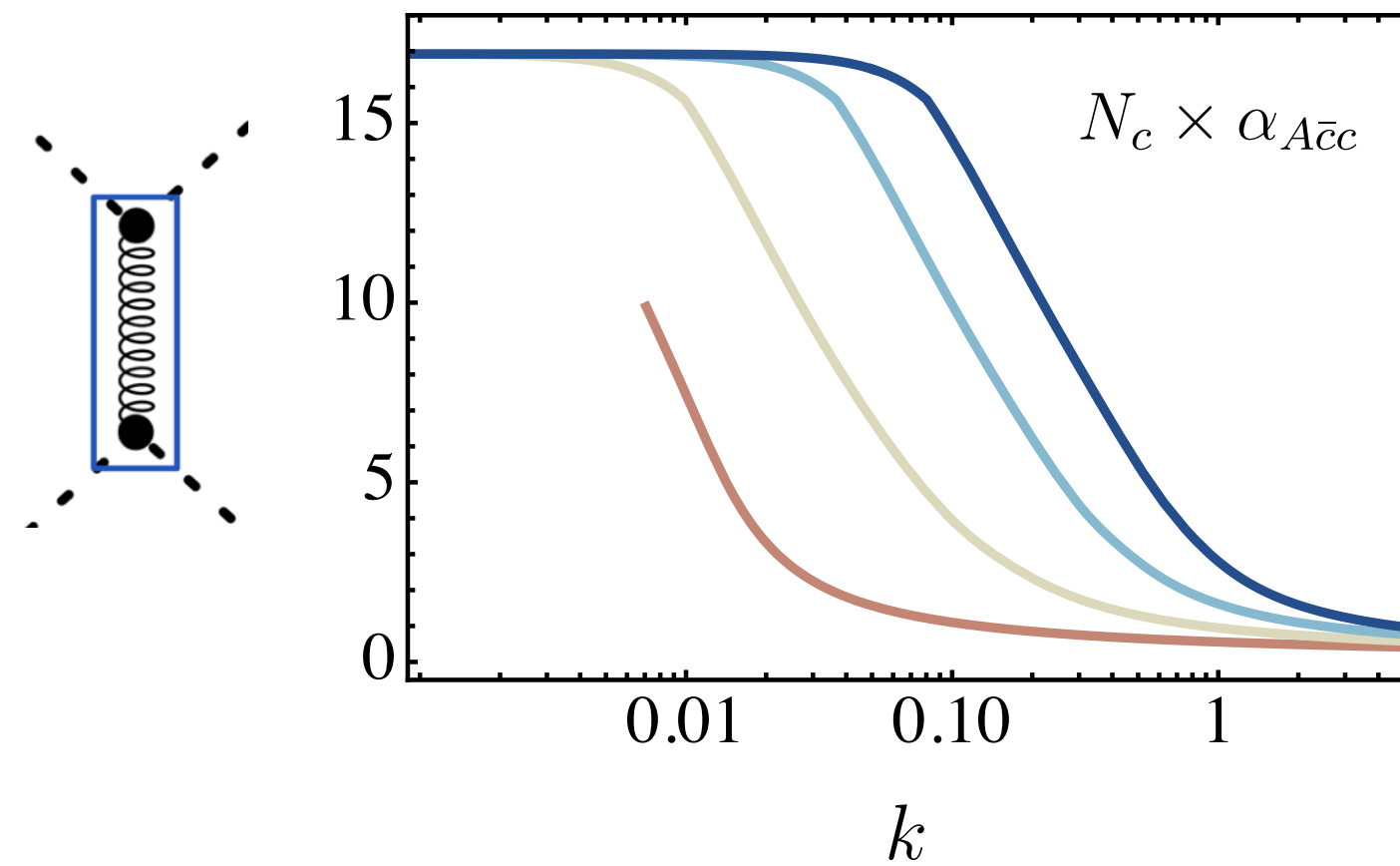
$$\Gamma_{\text{fermion},k}[\chi^\dagger, \chi, \psi^\dagger, \psi] = - \int_x \underbrace{Z_\psi^2 \sum_{i=1}^2 \lambda_i \mathcal{O}_i^\psi}_{\text{blue}} + \underbrace{Z_\chi^2 \sum_{i=4}^5 \lambda_i \mathcal{O}_i^\chi}_{\text{green}} + \underbrace{Z_\psi Z_\chi \sum_{i=6}^7 \lambda_i \mathcal{O}_i^{\chi\psi}}_{\text{red}}$$

	$\left\{ \begin{aligned} \mathcal{O}_1^\psi &= (\psi^\dagger \bar{\sigma}^\mu \psi)(\psi^\dagger \bar{\sigma}^\mu \psi) \\ \mathcal{O}_2^\psi &= (\psi^\dagger f_1 \bar{\sigma}^\mu \psi_{f_2})(\psi^\dagger f_2 \bar{\sigma}^\mu \psi_{f_1}) \end{aligned} \right.$
	
	$\left\{ \begin{aligned} \mathcal{O}_4^\chi &= (\chi^\dagger \bar{\sigma}^\mu \chi)(\chi^\dagger \bar{\sigma}^\mu \chi) \\ \mathcal{O}_5^\chi &= (\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi) \end{aligned} \right.$
	
	$\left\{ \begin{aligned} \mathcal{O}_6^{\chi\psi} &= (\psi^\dagger \bar{\sigma}^\mu \psi)(\chi^\dagger \bar{\sigma}^\mu \chi) \\ \mathcal{O}_7^{\chi\psi} &= (\psi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \psi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi) \end{aligned} \right.$
	

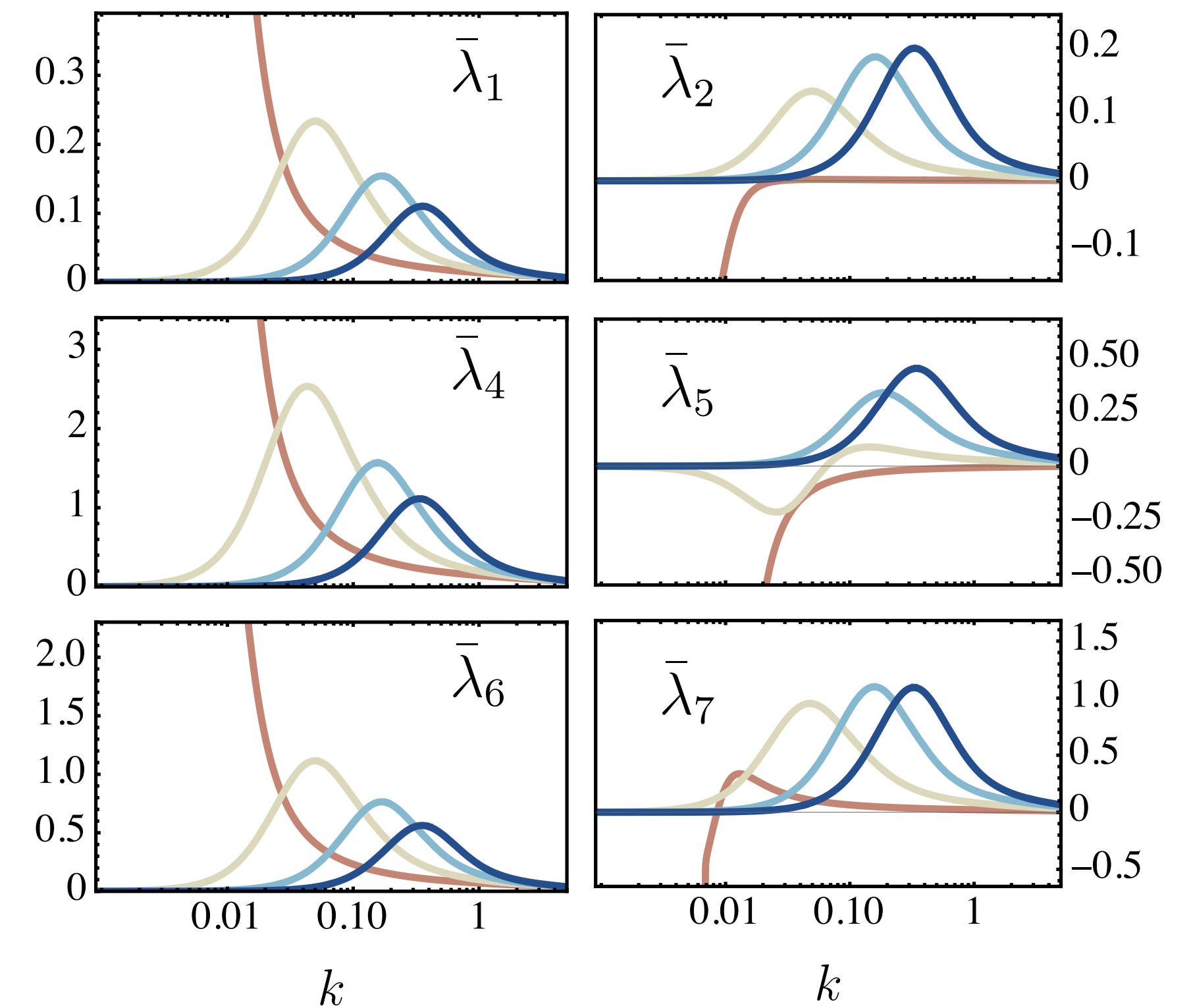


# Colour-confinement and dSB in BY

Li,APG,Vatani[2603.19355]

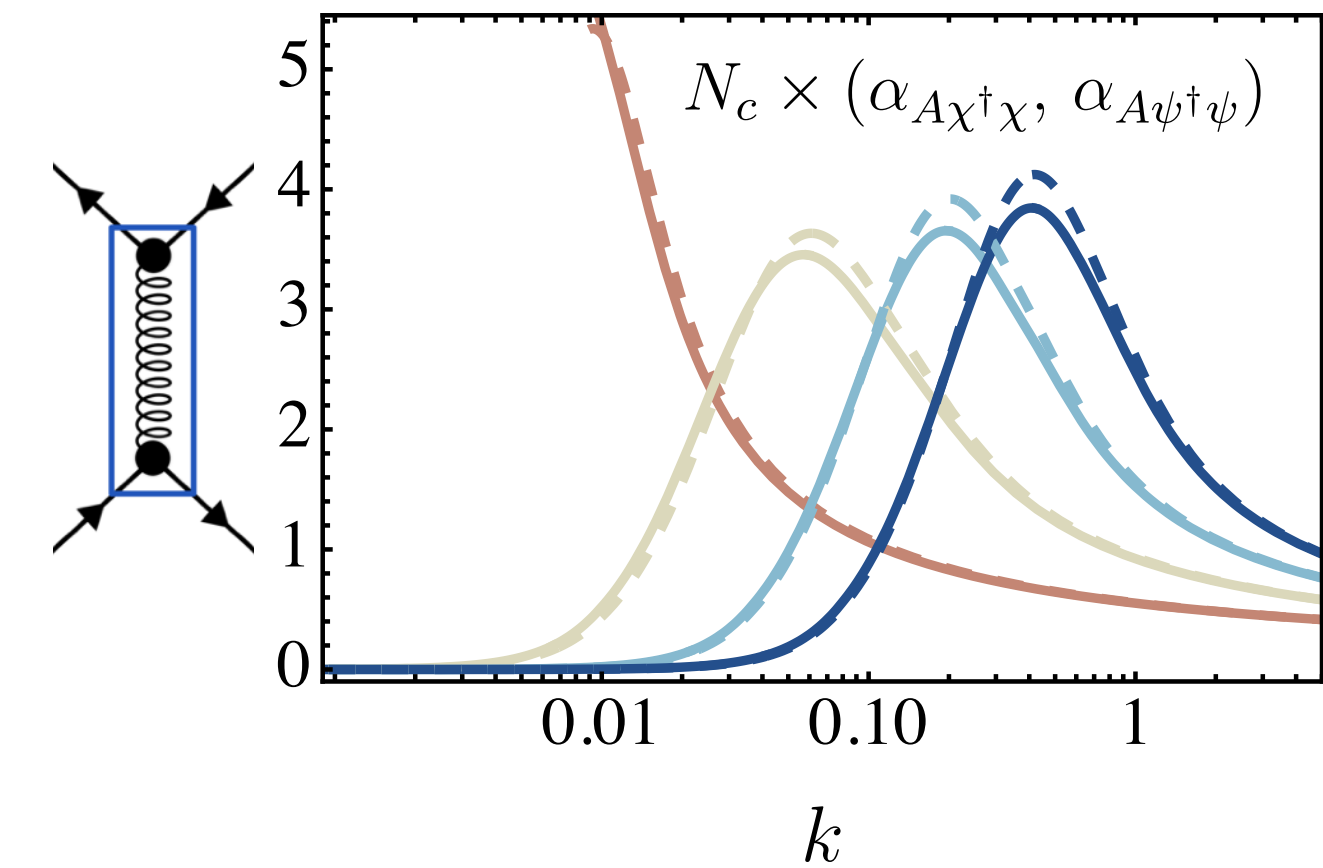
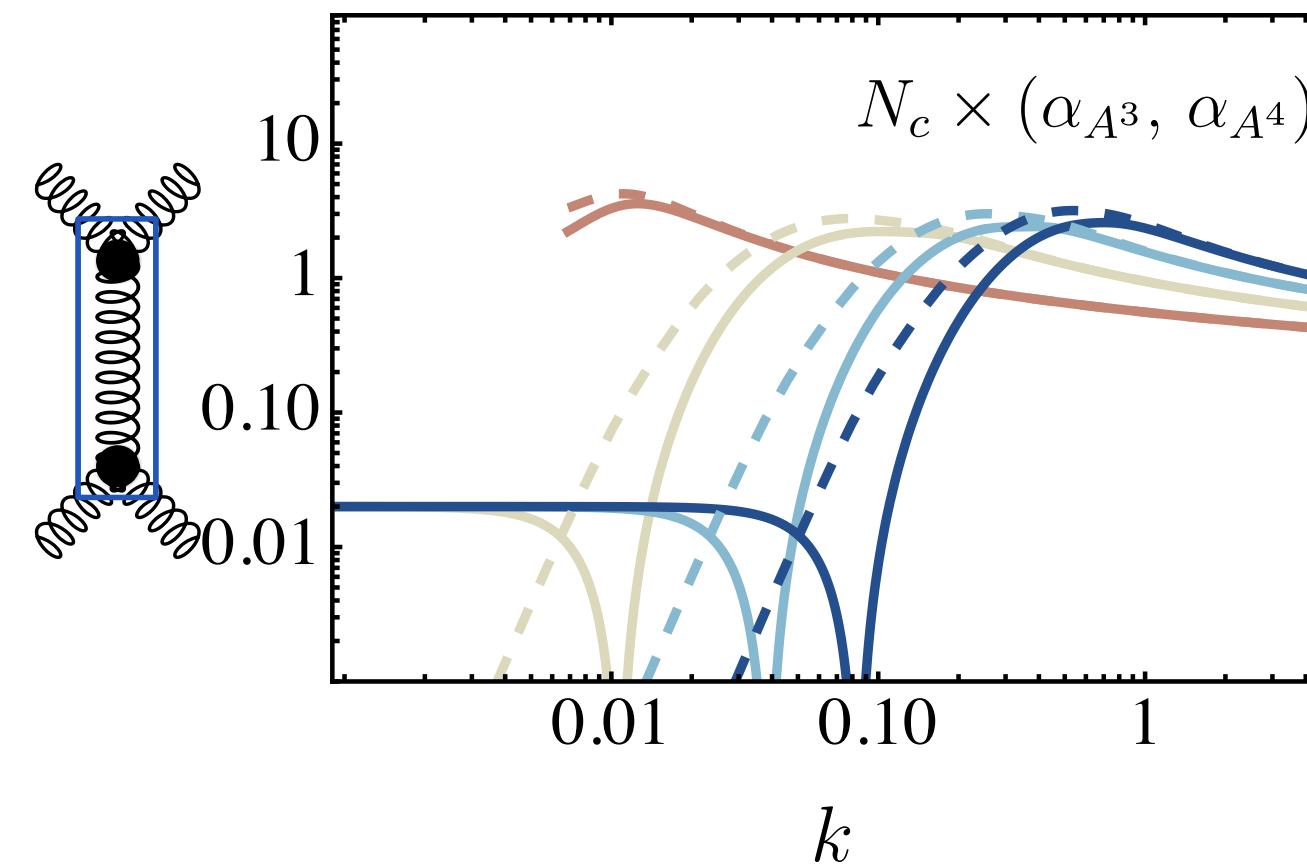
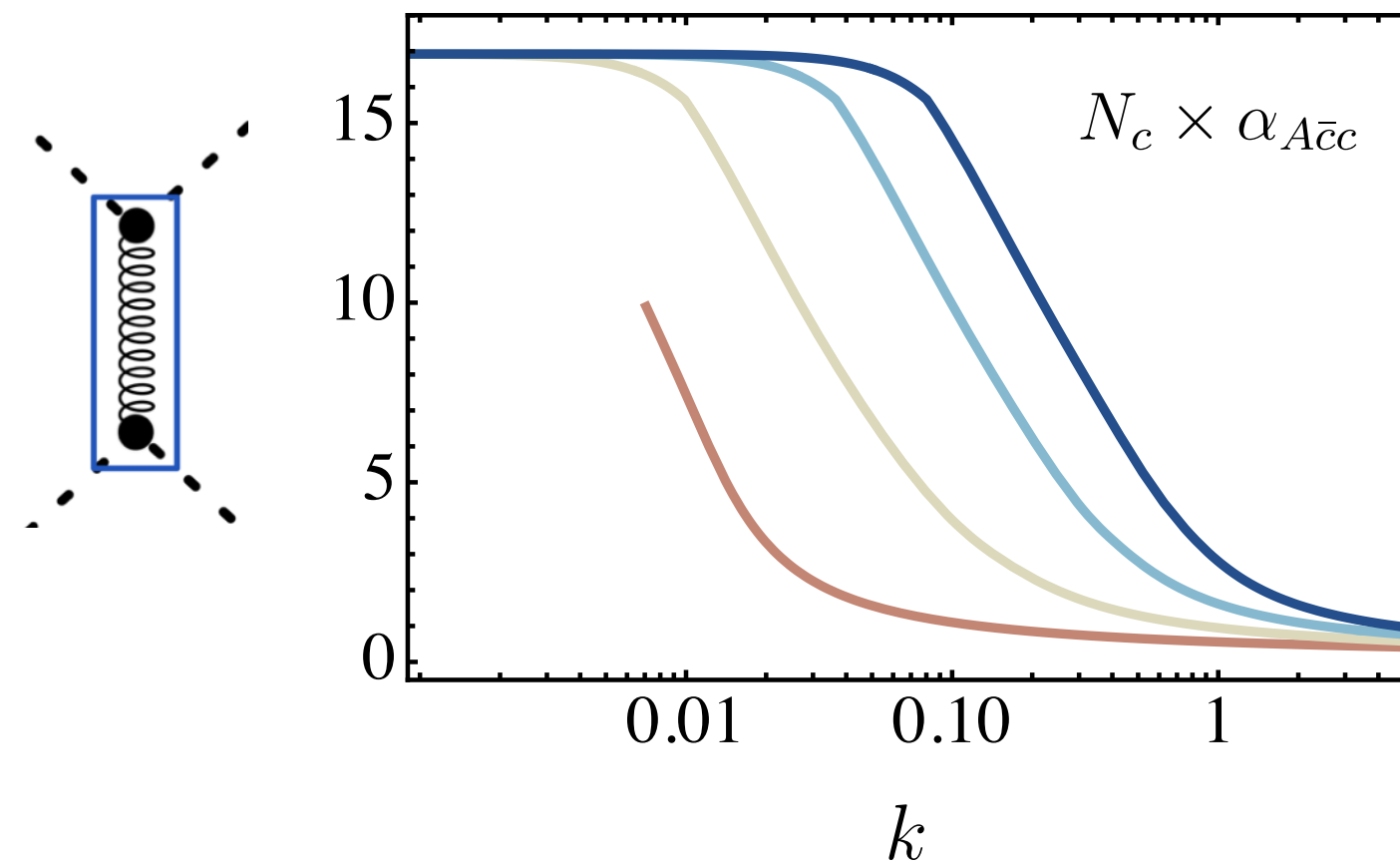


$N_c$  : — 3 — 4 — 5 — 6



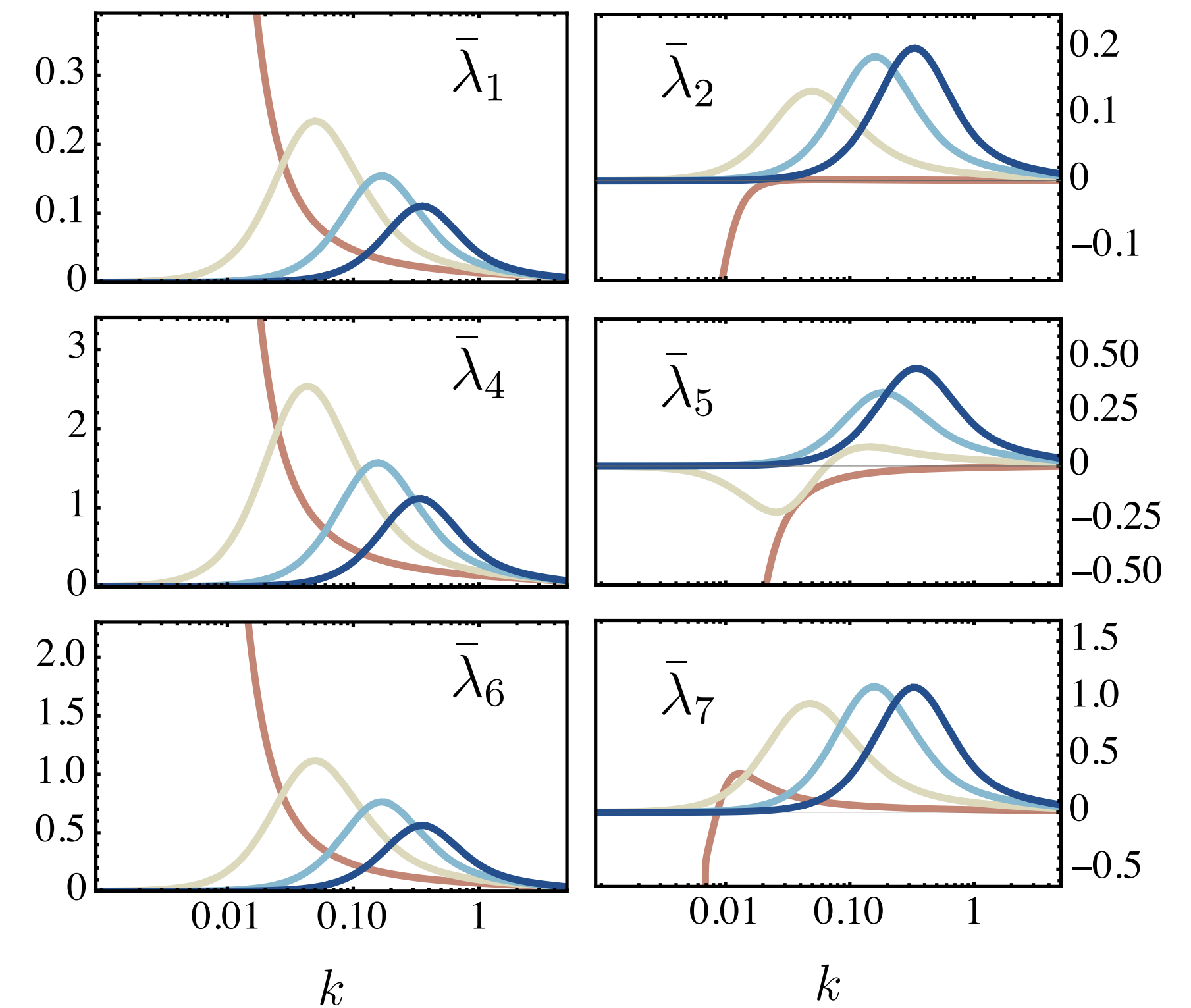
# Colour-confinement and dSB in BY

Li,APG,Vatani[2603.19355]

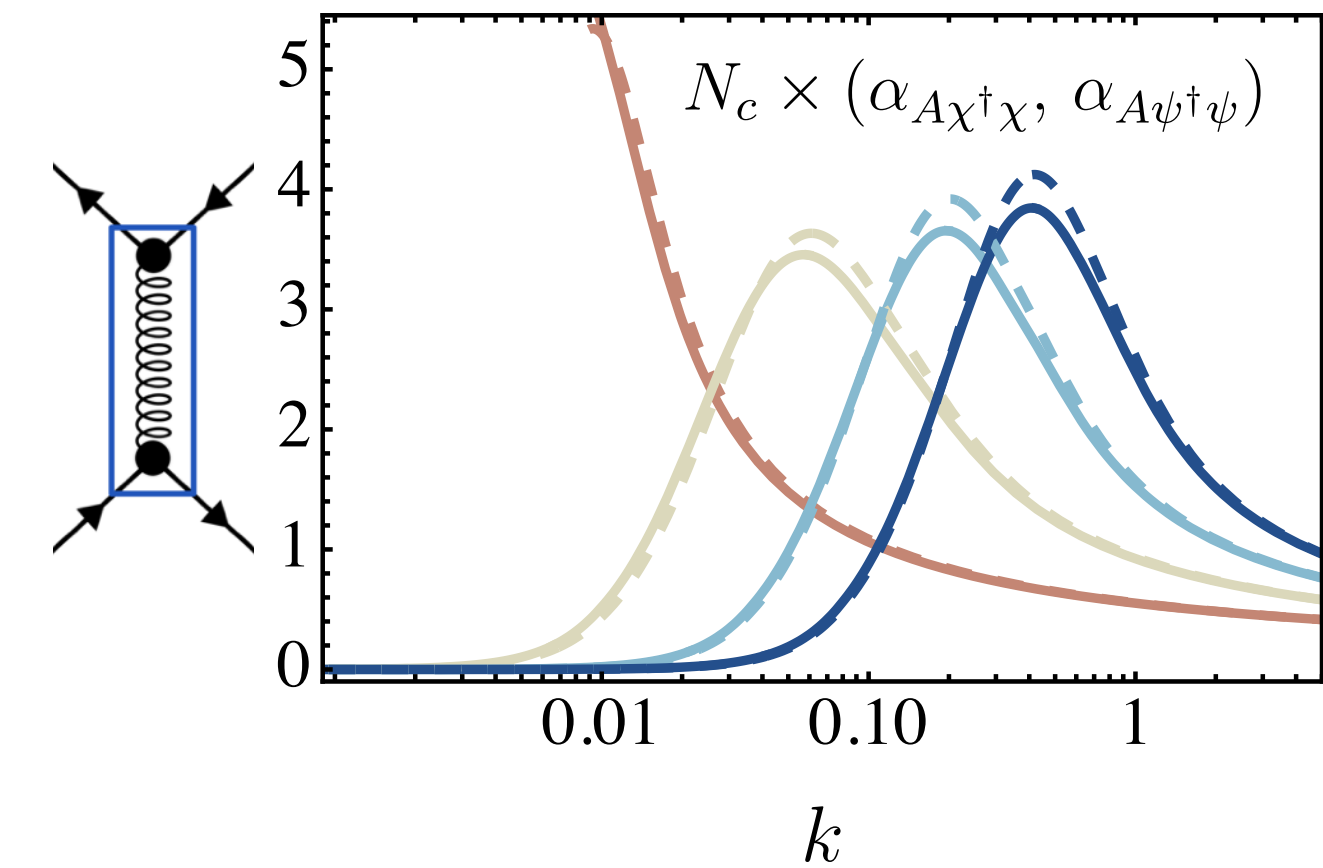
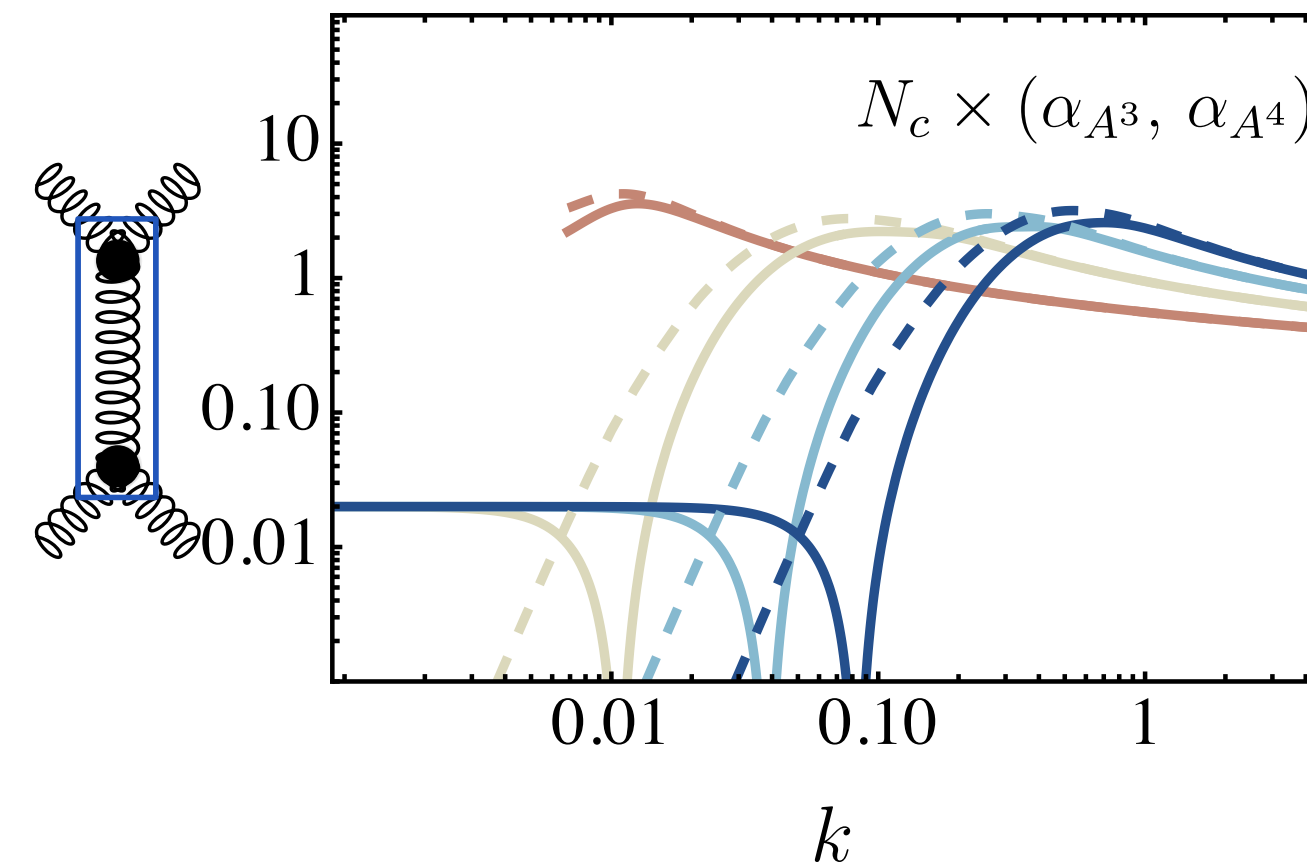
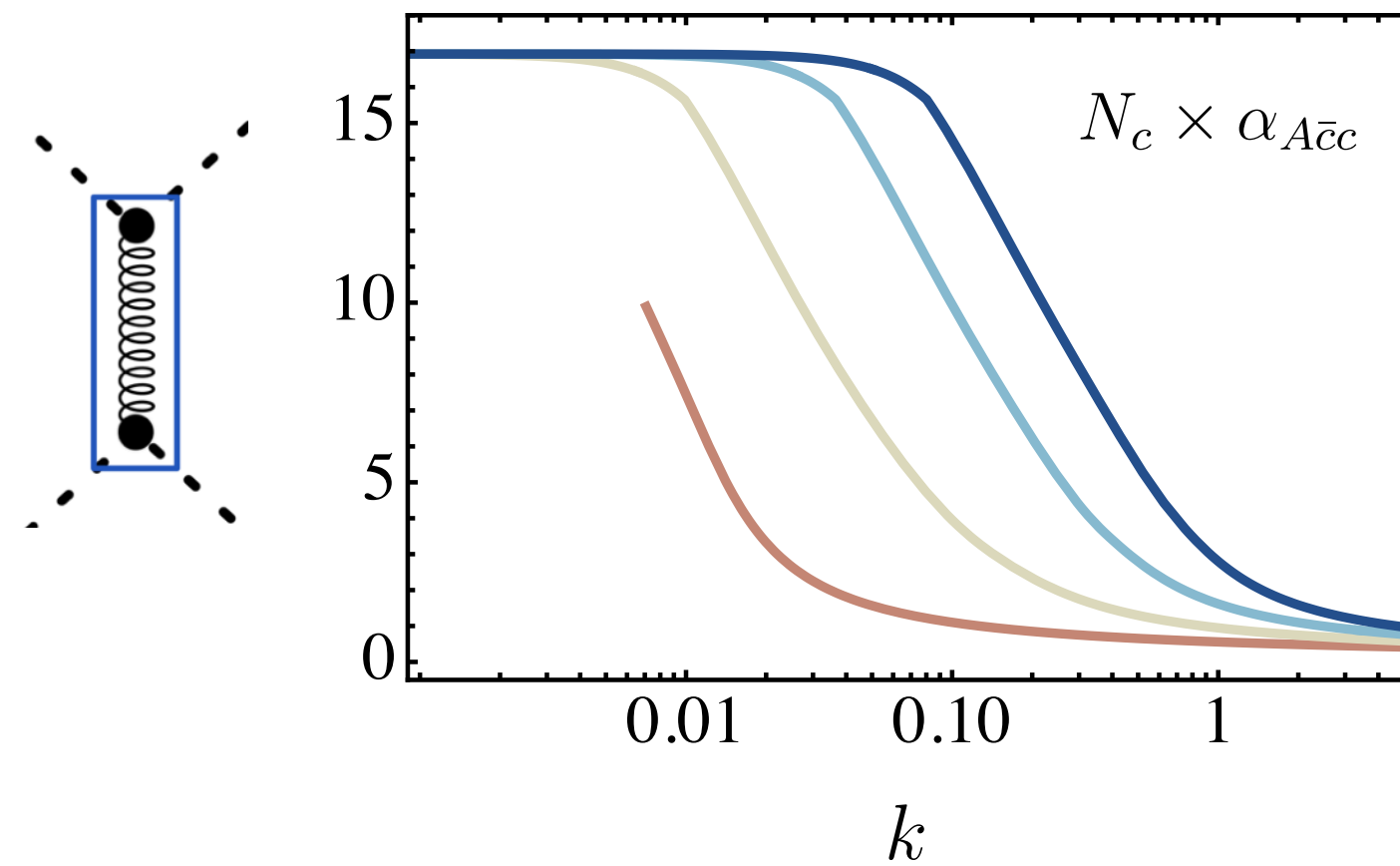


$N_c$  : — 3 — 4 — 5 — 6

Signatures of confinement in the Landau gauge



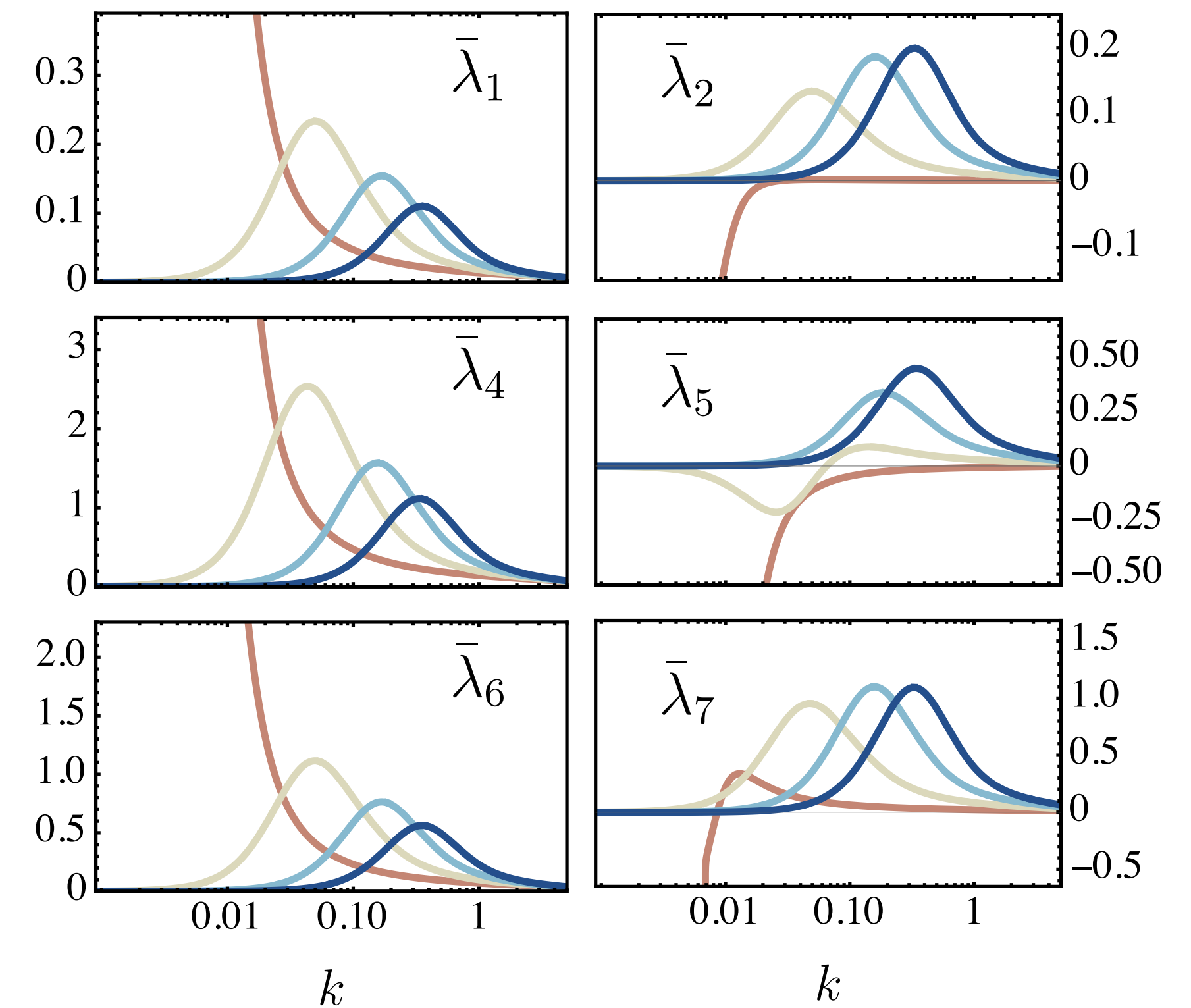
# Colour-confinement and dSB in BY



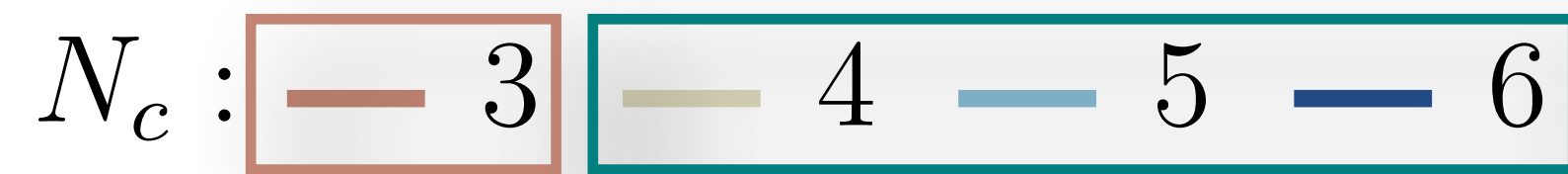
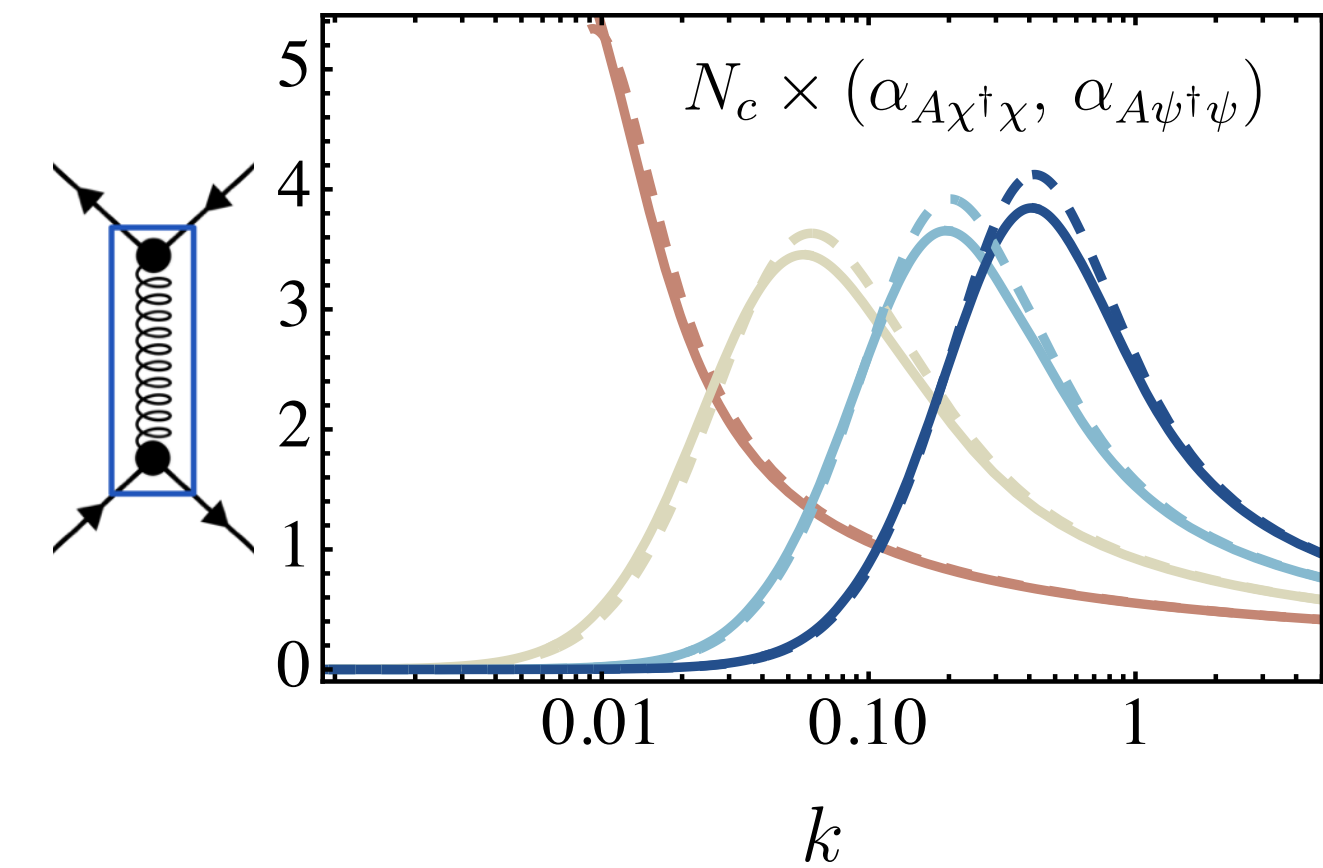
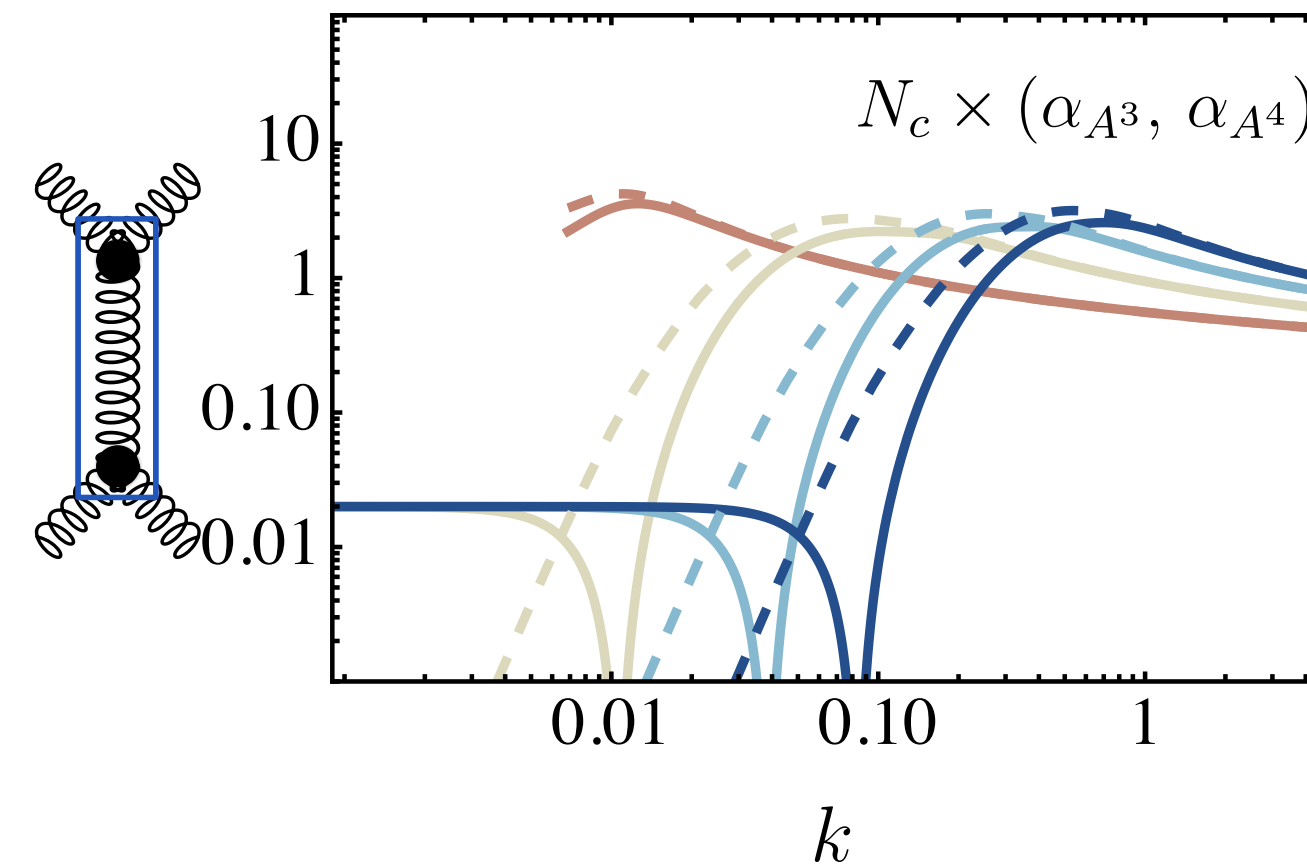
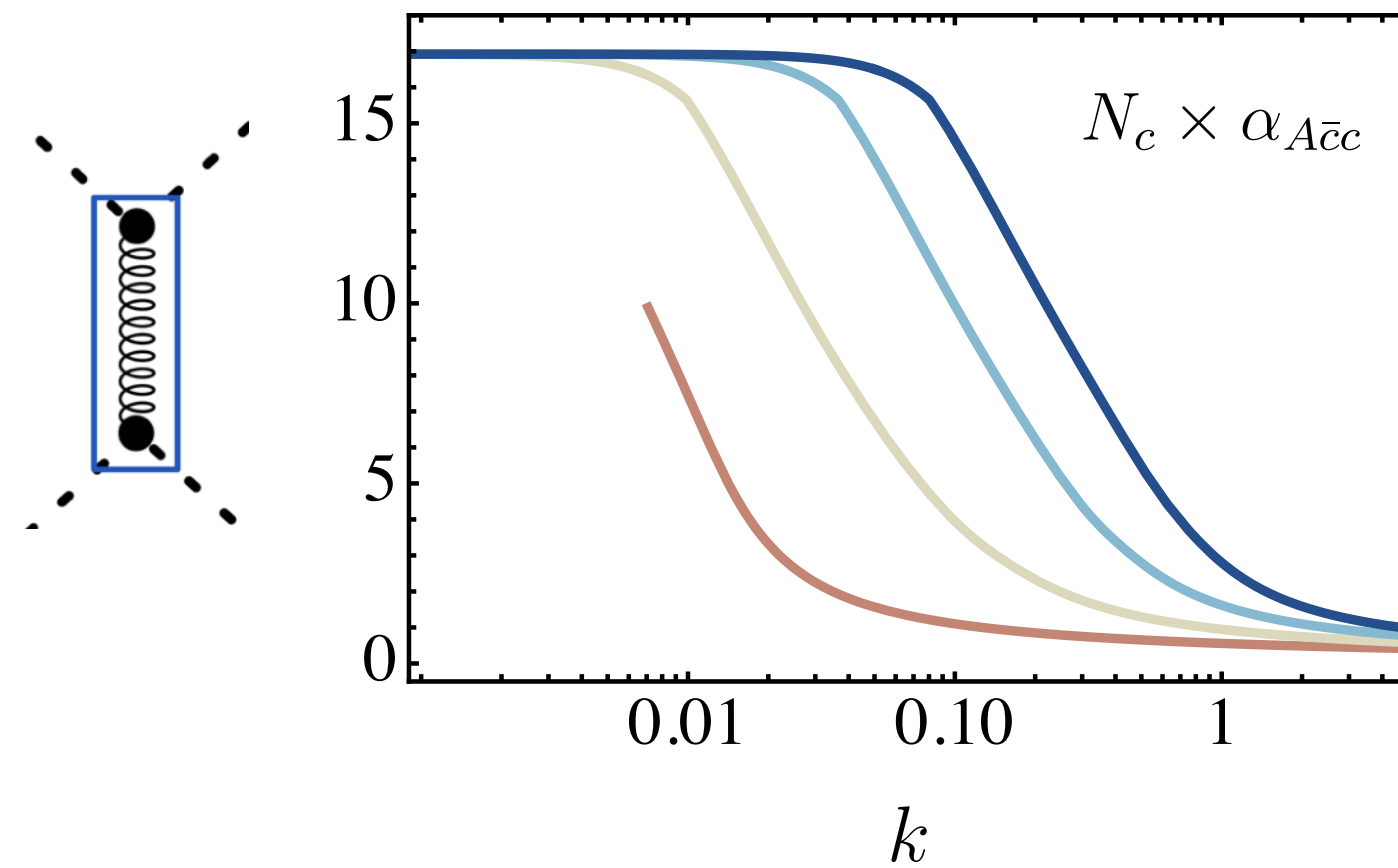
Divergence  $\rightarrow$  dSB

No divergence  $\rightarrow$  no dSB

Signatures of confinement in the Landau gauge



# Colour-confinement and dSB in BY



Divergence  $\rightarrow$  dSB

No divergence  $\rightarrow$  no dSB

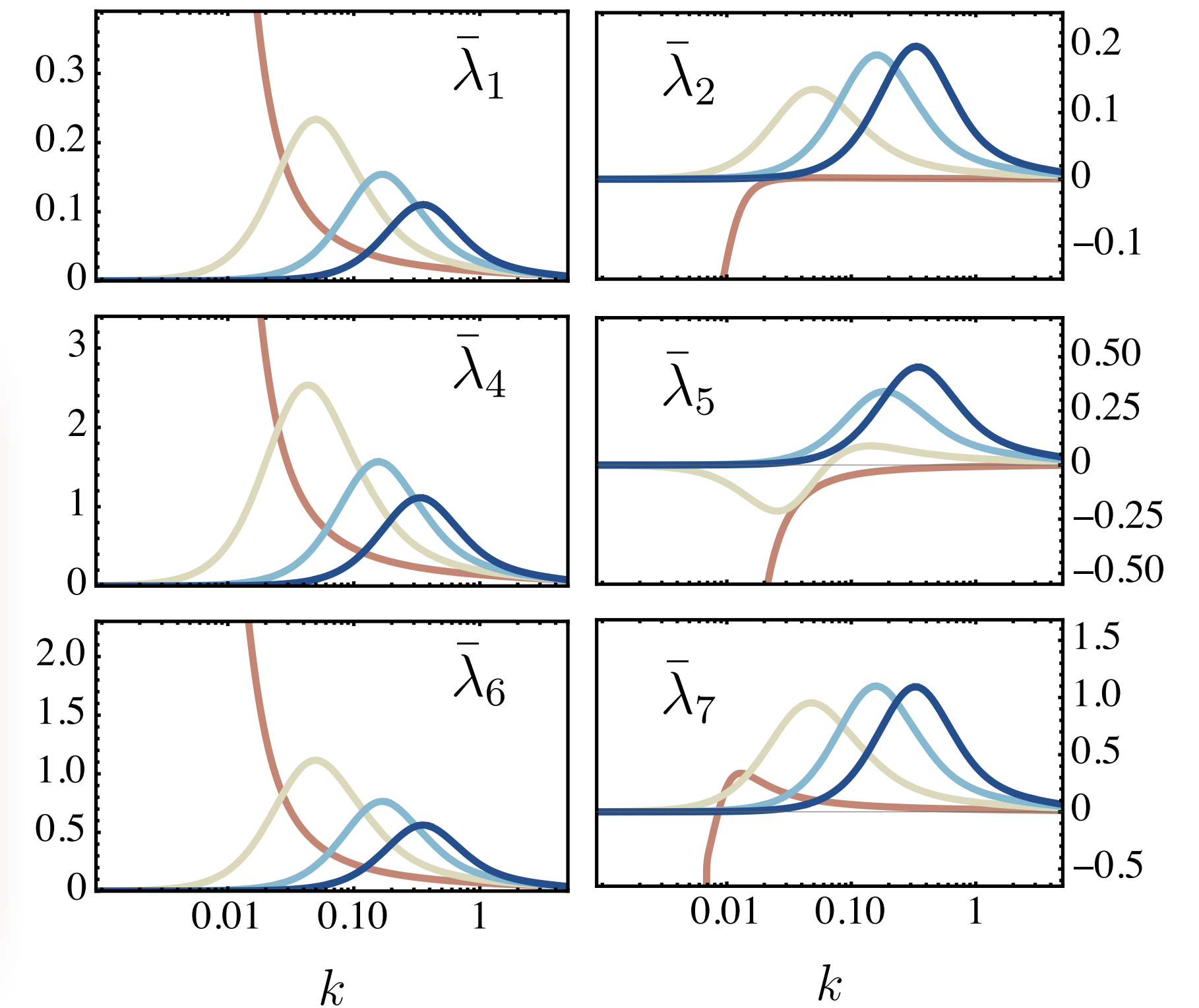
Signatures of confinement in the Landau gauge

Dominant four-fermion operators:

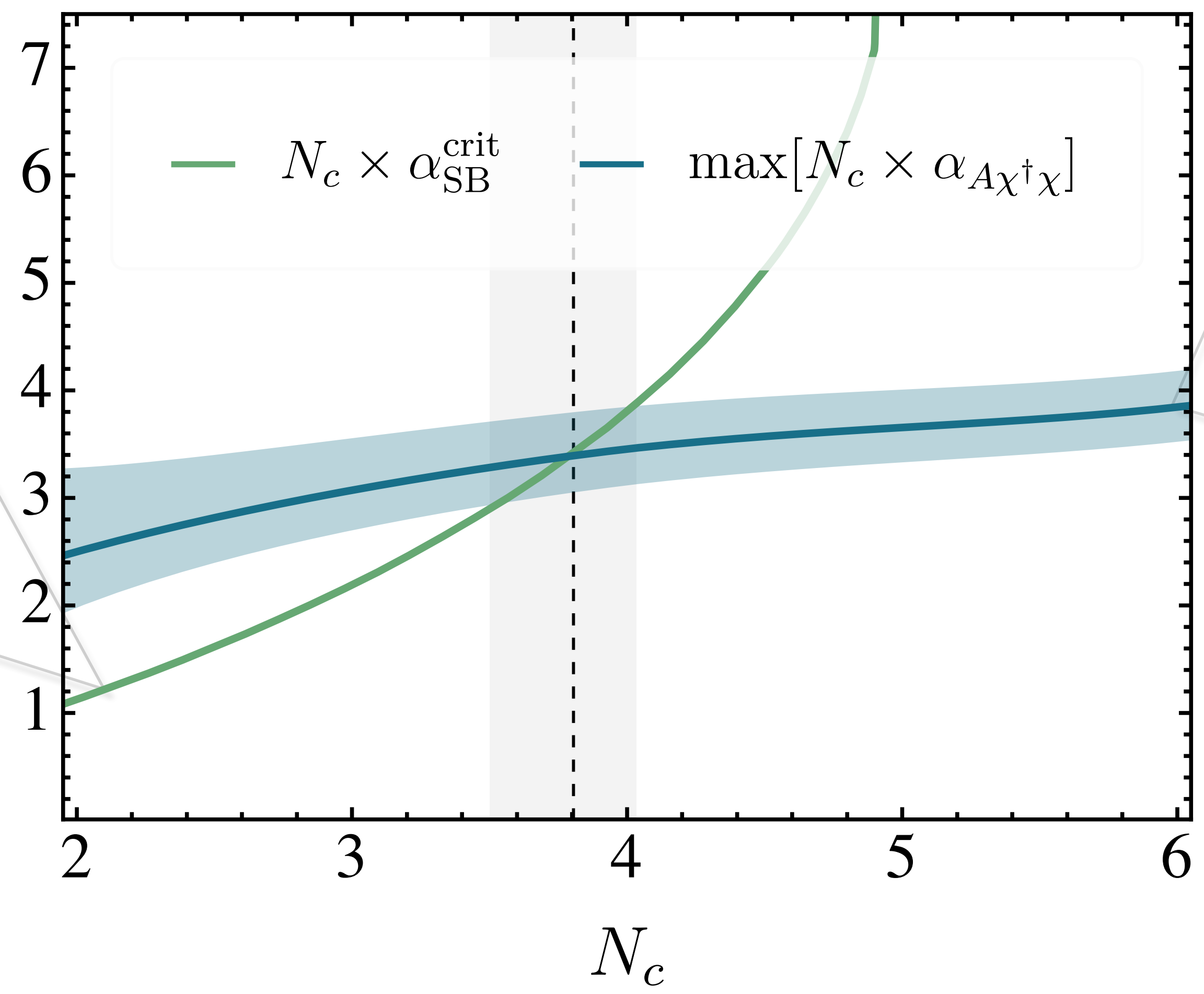
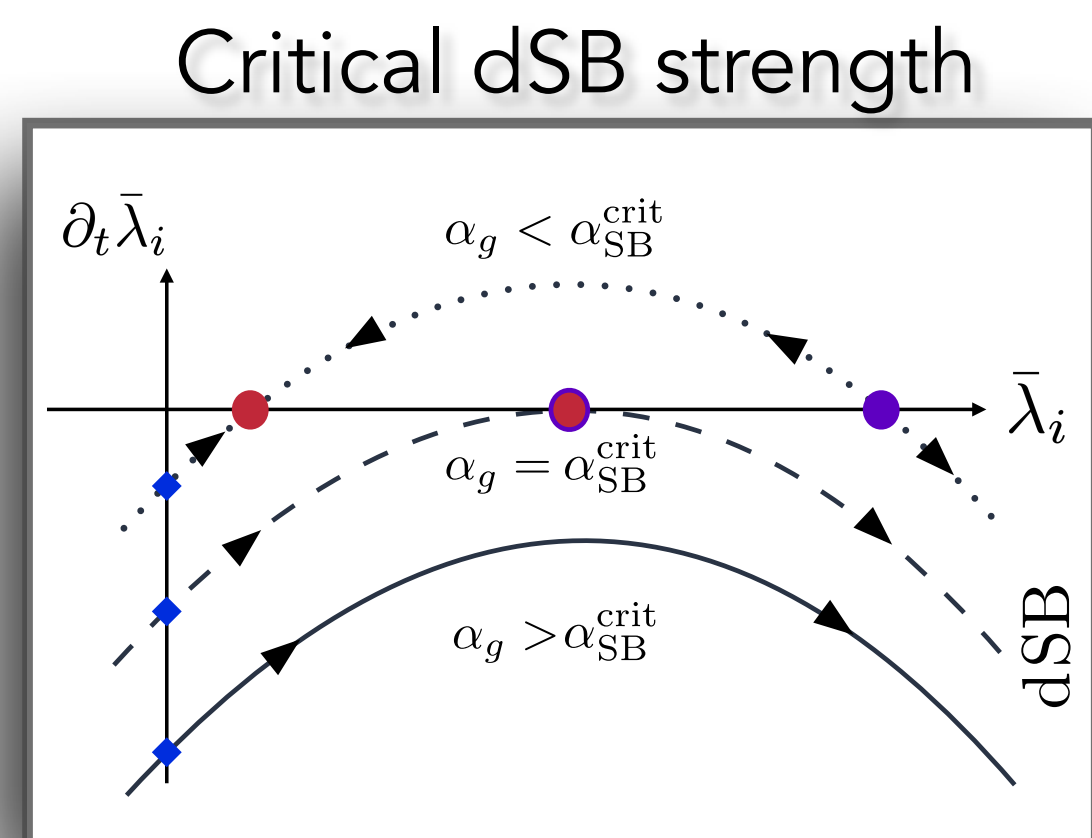
$$\mathcal{O}_4^\chi = (\chi^\dagger \bar{\sigma}^\mu \chi)(\chi^\dagger \bar{\sigma}^\mu \chi)$$

$$\mathcal{O}_5^\chi = (\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)$$

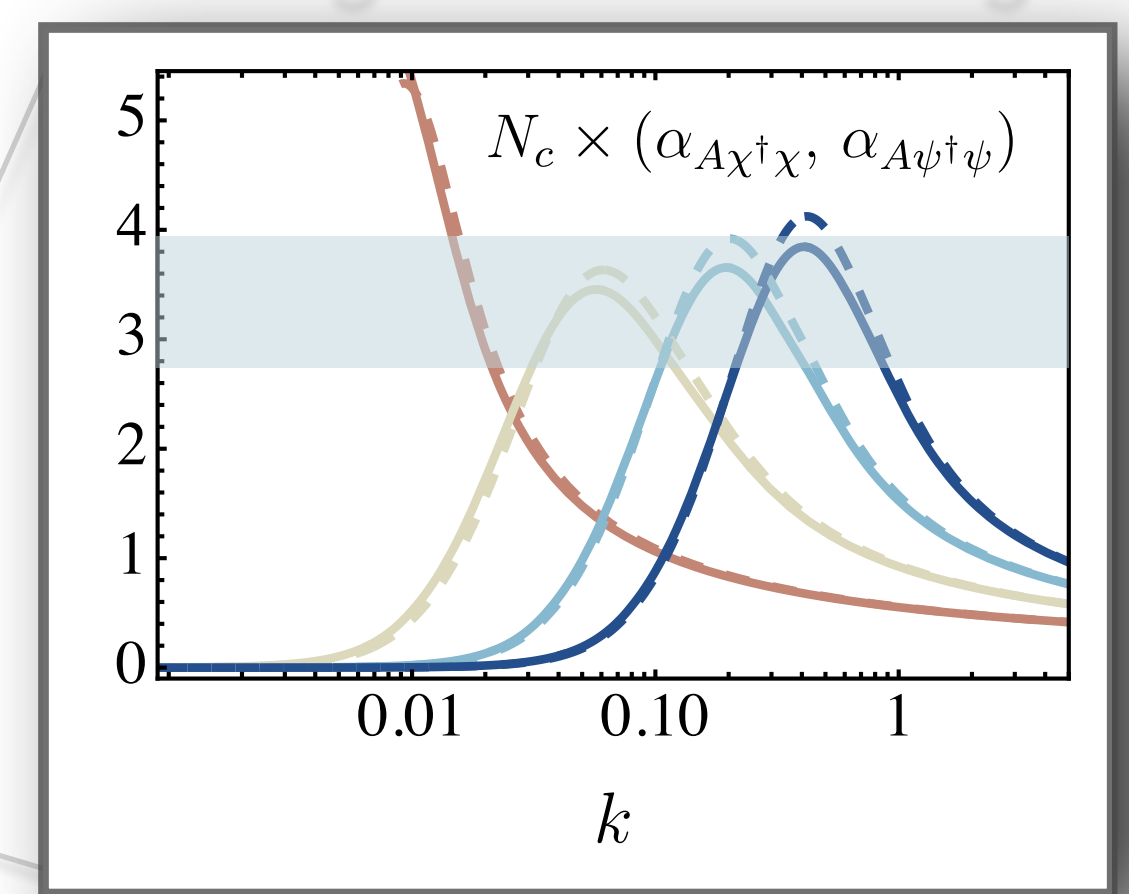
Diquark-like condensate:  $\sim \langle \chi\chi \rangle$



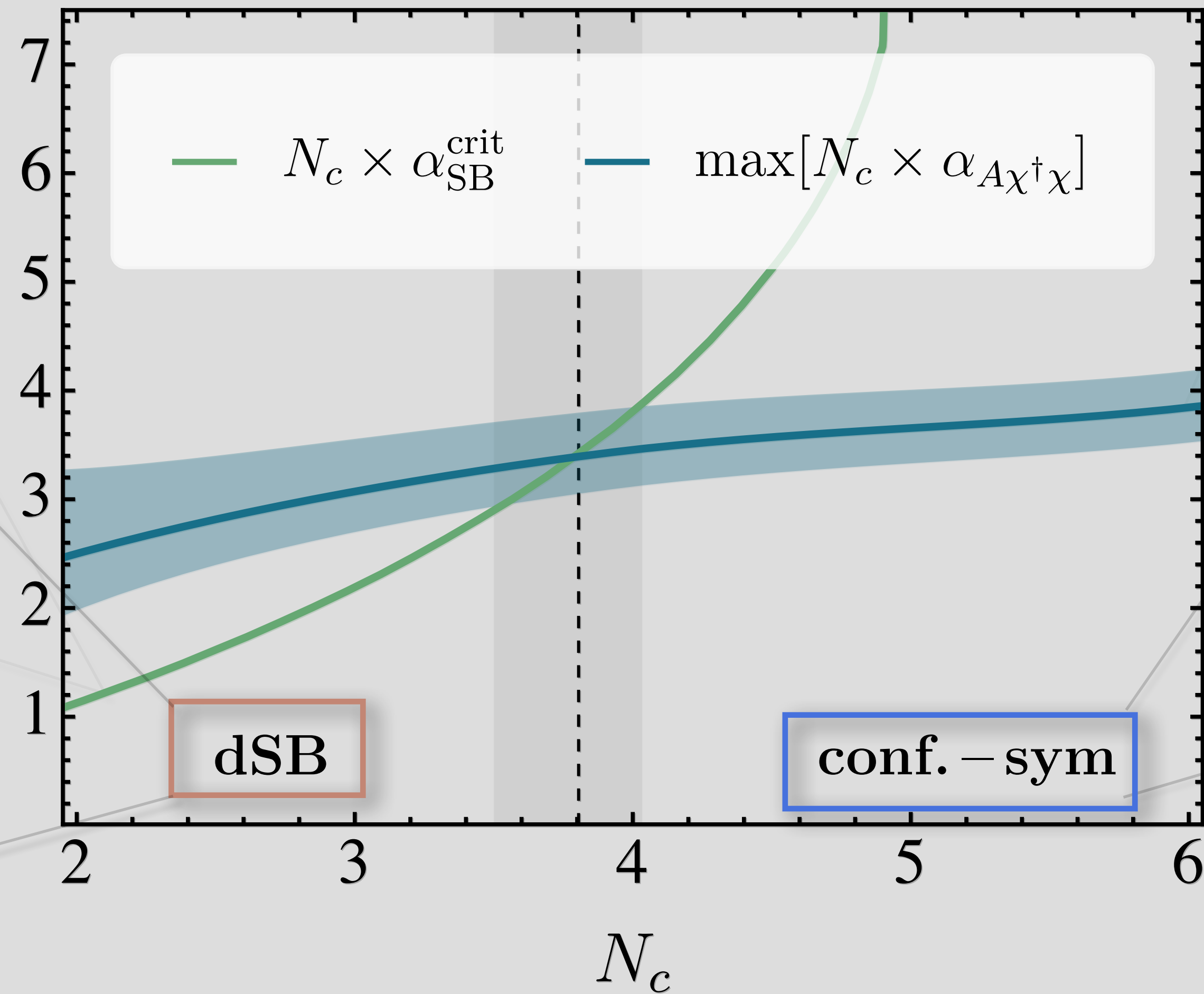
# Phase diagram



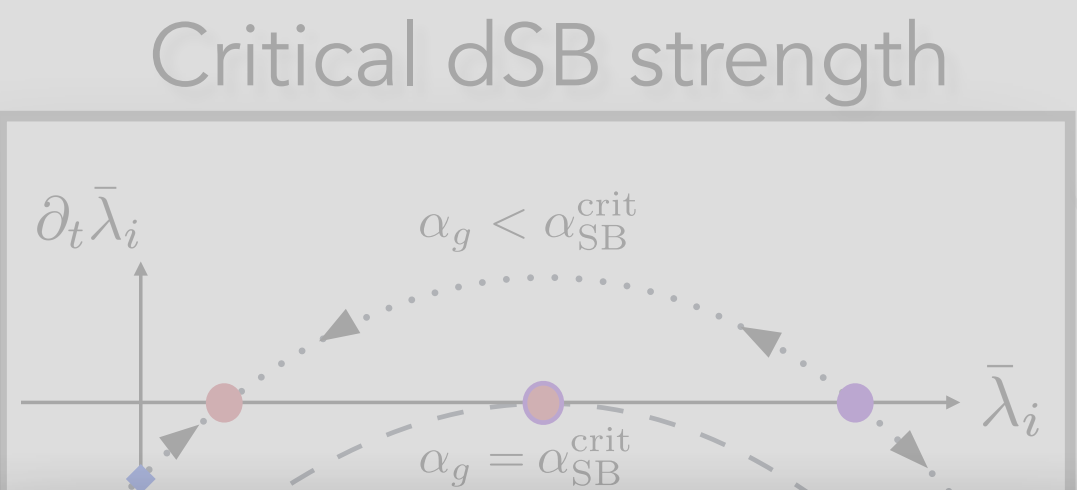
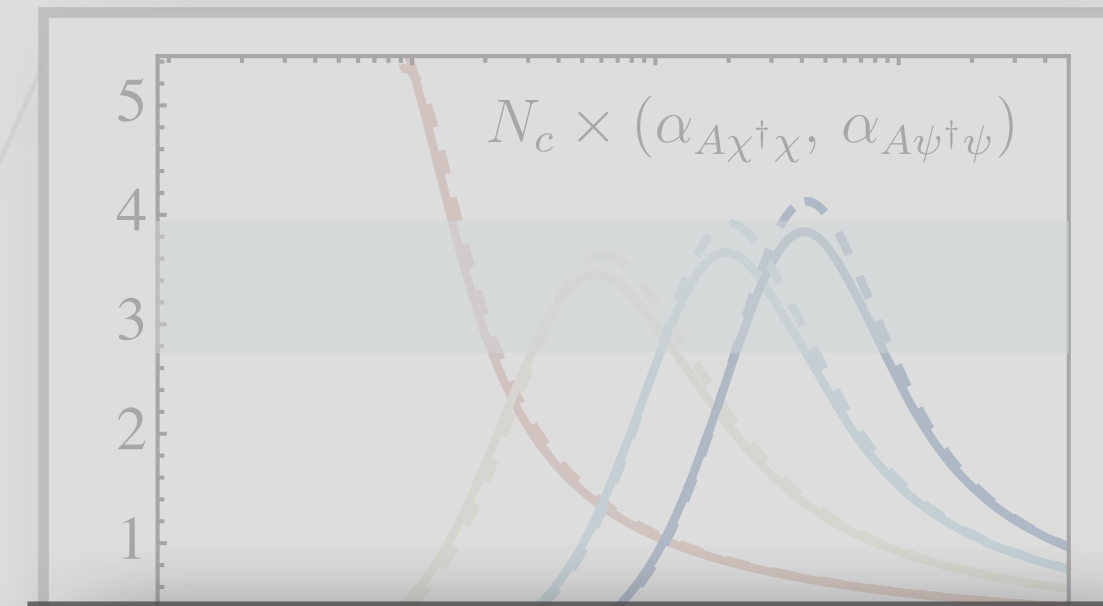
Gauge-fermion strength



# Phase diagram



Gauge-fermion strength



**Highest condensate**  
 Di-quark like  $\sim \langle \chi\chi \rangle$   
 (Partial) higgsing of gluons,  
 symmetry tumbling, ... (tbd)

**Confinement without dSB**  
 Bars, Yankielowicz '81  
 Massless baryons saturate  
 the anomaly  
 Raby, Dimopoulos, Susskind '79  
 Eichten, Peccei, Preskill, Zeppenfeld '86  
 What is this dynamics?  
 SMG?, ...  
 Wang, You [2204.14271]  
 Karasik, Önder, Tong [2208.07842]

# Conclusions

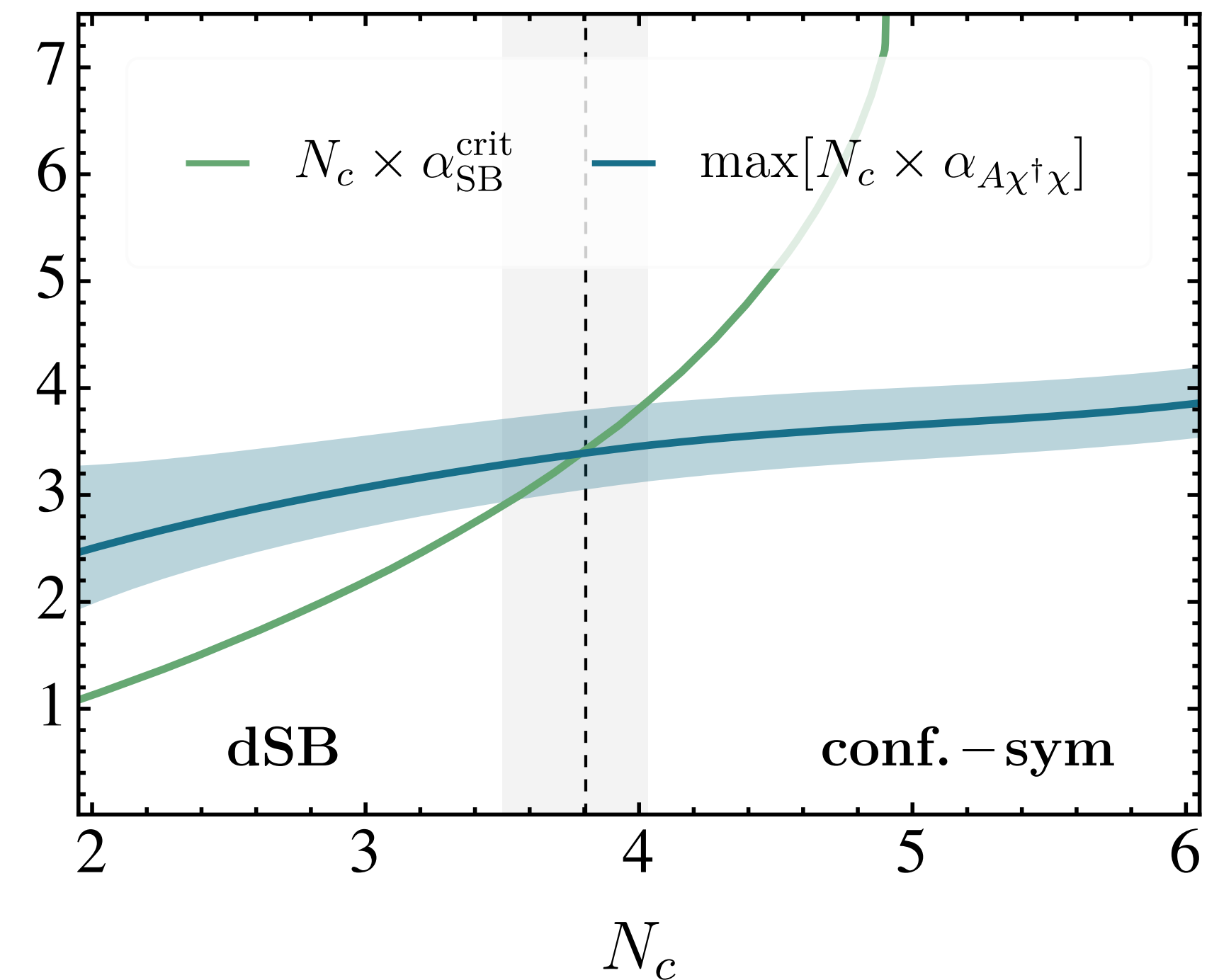
First steps: dynamical emergence of confinement and dSB in the BY class of chiral gauge theories

- fRG approach: approximate, first-principles and nonperturbative
  - Well-defined criteria for confinement (Kugo-Ojima) and dSB (FP merger)
- Phase structure:
  1. dSB,  $\langle \chi\chi \rangle$  condensate, tumbling
  2. Confinement without dSB
    - ★ Novel dynamics and features to be explored deeper
    - ★ Massless hadrons, SMG, ?, ...
- Promising path towards systematically studying chiral gauge theories

Raby,Dimopoulos,Susskind '79  
Eichten,Peccei,Preskill,Zeppenfeld'86

Wang,You[2204.14271]  
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Li,APG,Vatani,Xu[2507.21208] Li,APG,Vatani[2603.19355]



# Conclusions

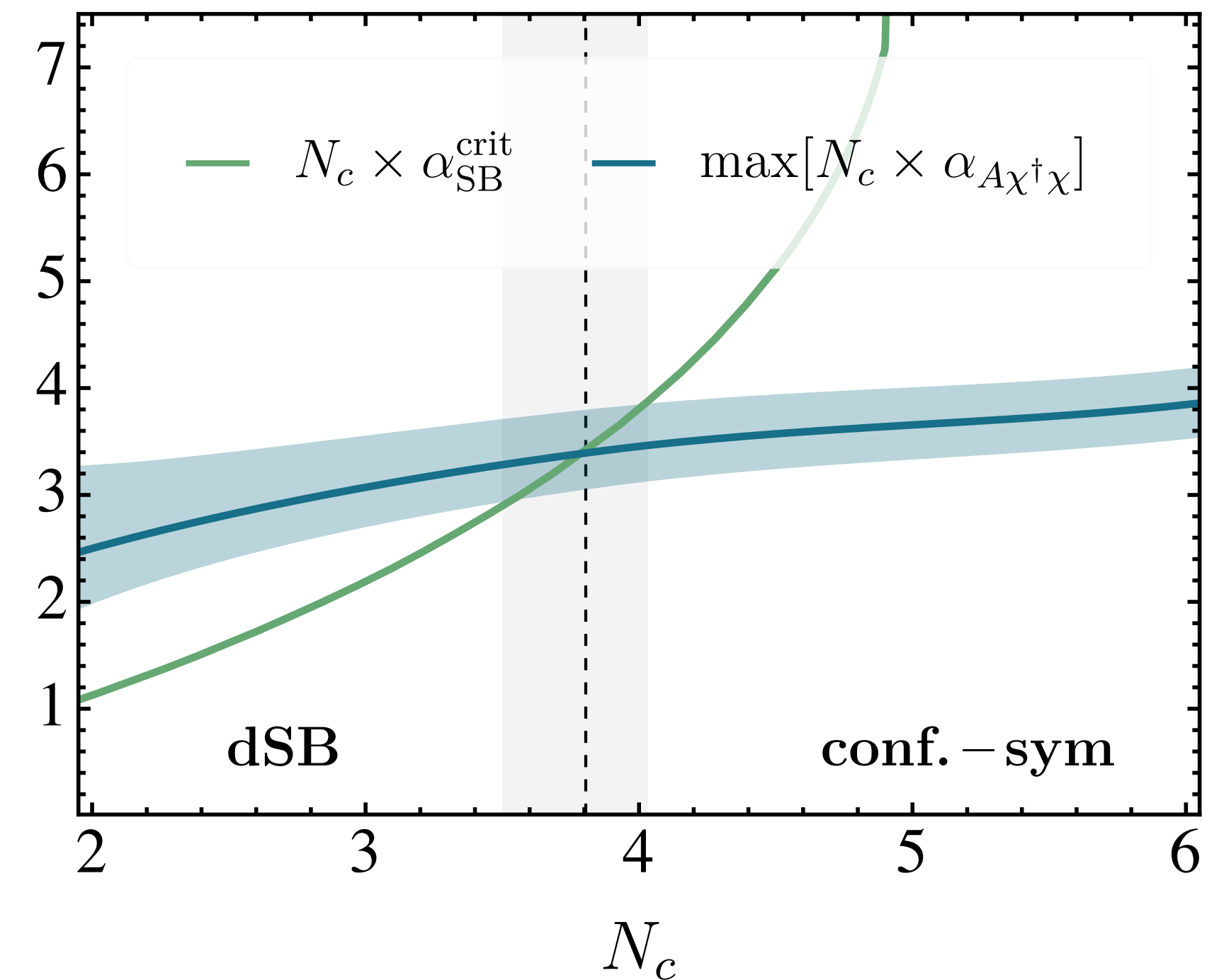
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Thank you for your attention!